An overview of Model Predictive Control (MPC)

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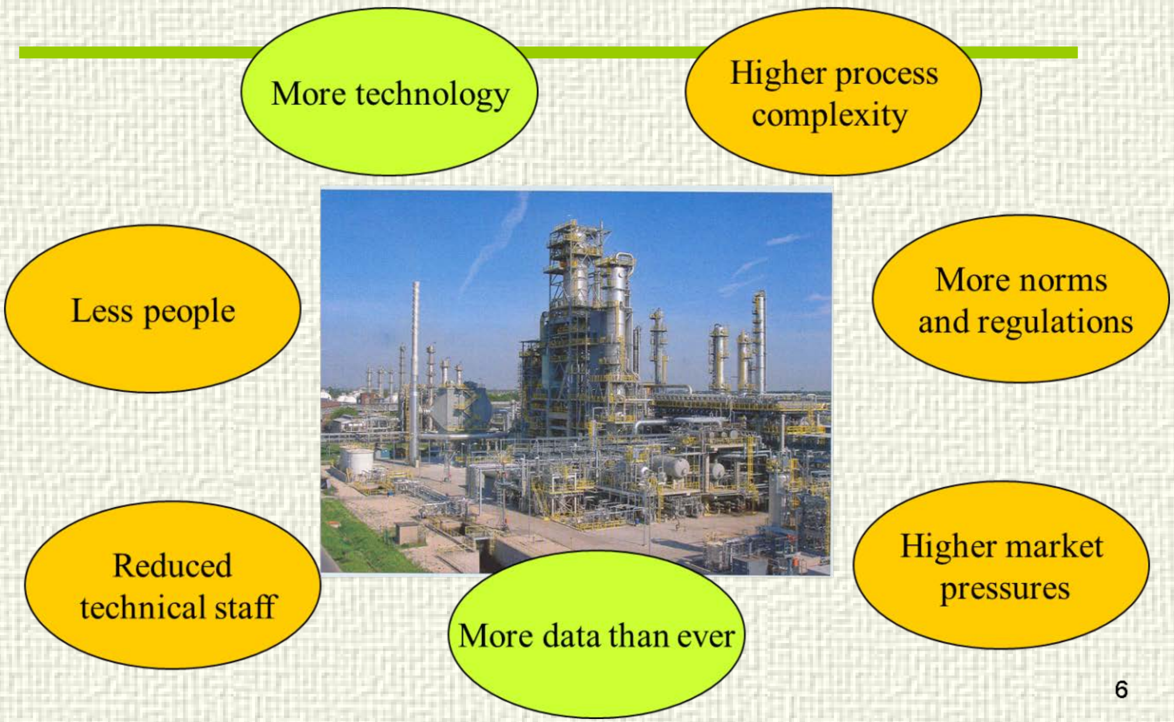
Outline

- ✓ Introduction to MPC
  - – Motivation
  - – Basic controllers
  - – Constraints
  - – Multivariable MPC
  - – Implementation methodology, MHE, Identification
  - – Software and industrial tools
  - – Economic optimization
- ✓ Non linear MPC
- ✓ Industrial examples

Why MPC?

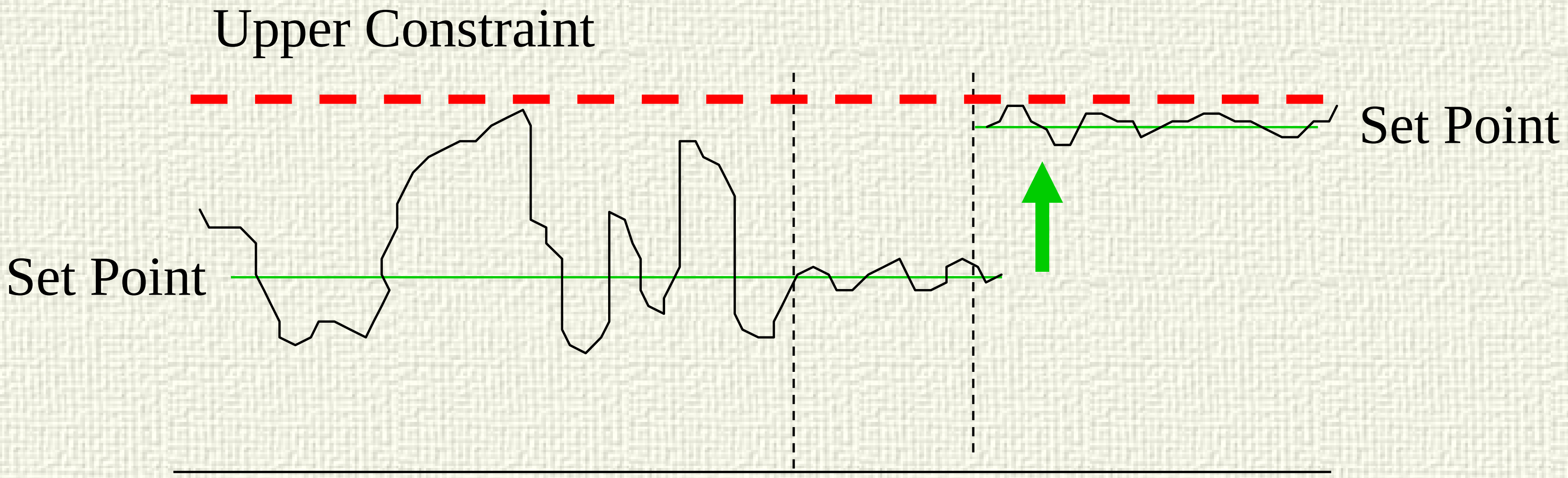


Increasing demands on product quality, costs optimisation, productivity, flexibility to adapt to a wide range of operating conditions, security, care of the environment,..

Larger and more complex plants

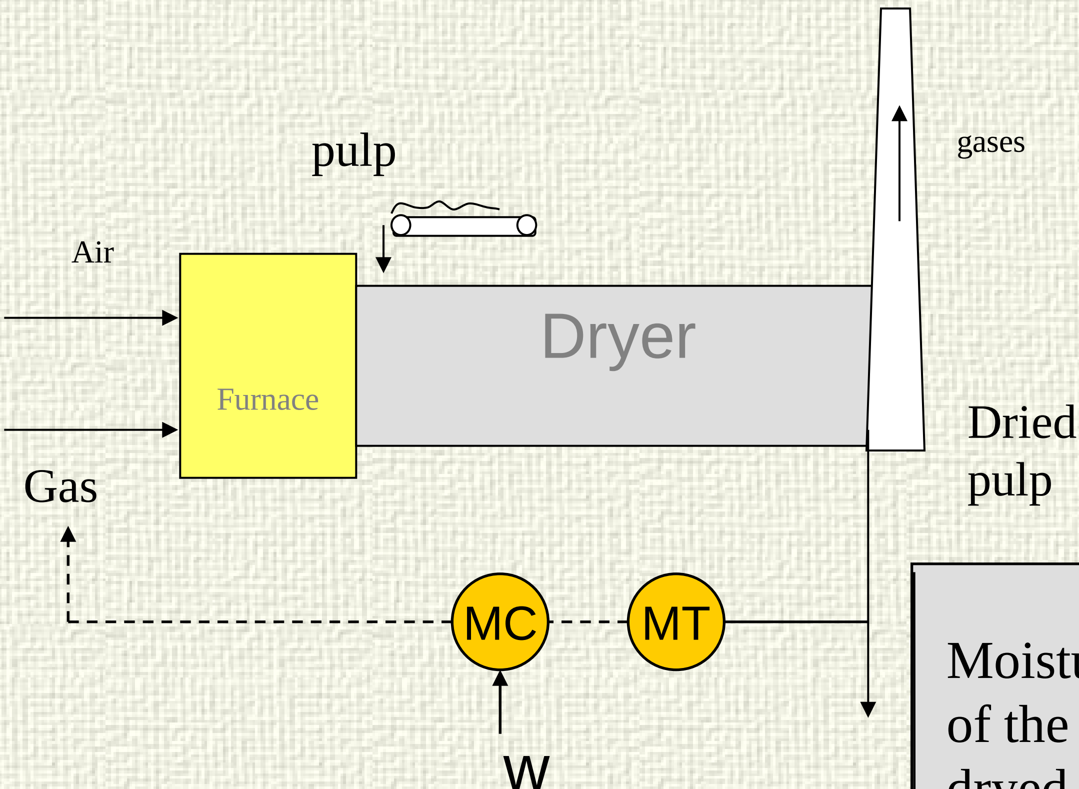
- ✓ Increasing pressure on the control systems to fulfil new and stronger specifications.
- ✓ Increasing needs to rationalise upper level decisions linked to plant economy, and to integrate and improve all different operating levels.

Economic optimization and Control

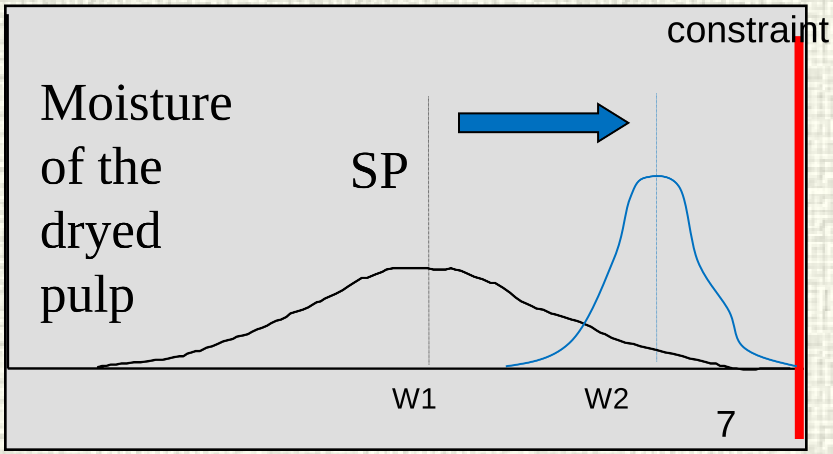


- Better control means smaller changes around the prescribed set points ⇨ better product quality
- Reduced variance allows to move set points respecting the constraints, giving room for optimization

Economic optimization and Control

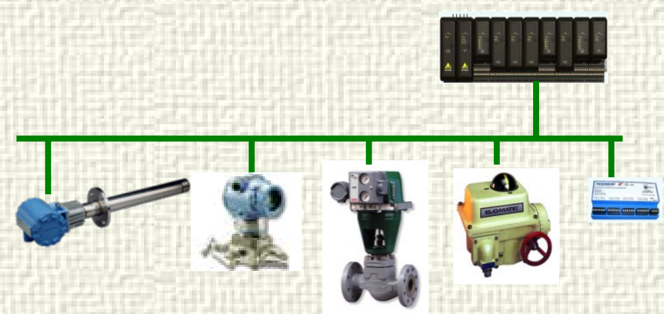


Improving control allows to move the moisture set point to a more profitable value, respecting the maximum moisture constraint

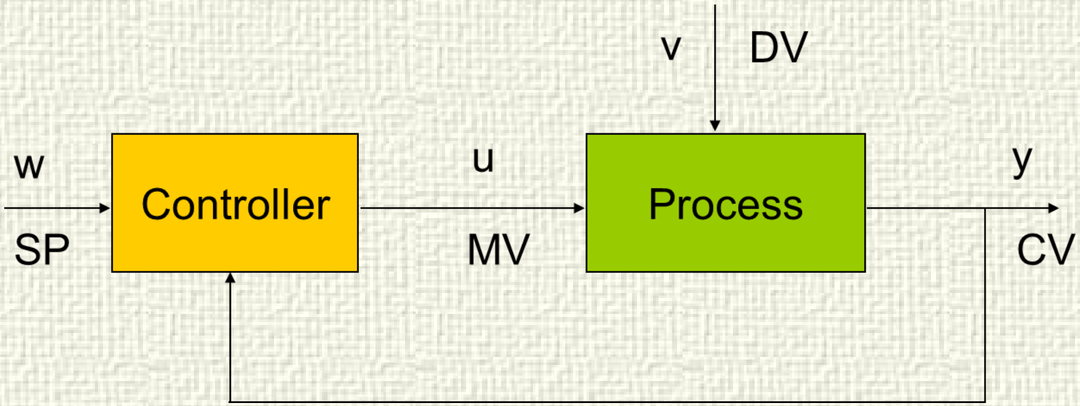


What is the situation of process control?

- Mature technology
- Basic architectures and functions have not changed very much
- How is it able to deal with the problems at plant-wide scale?



Conventional control



$$e(t) = w(t) - y(t)$$

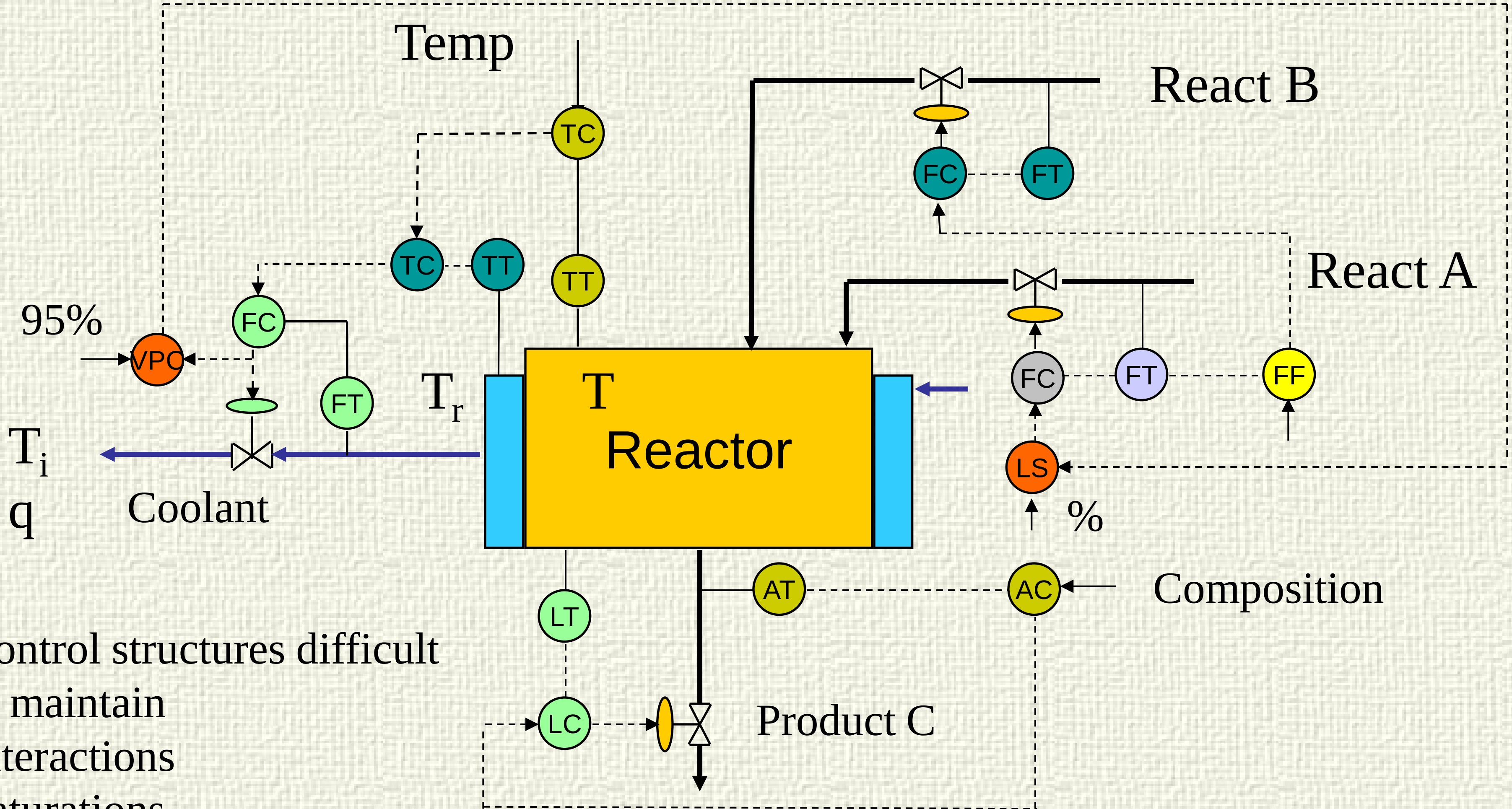
$$u(t) = K_p\left(e(t) + \frac{1}{T_i}\int e(\tau)d\tau + T_d\frac{de}{dt}\right)$$

Most of the loops implement PID controllers

Signal based

Not very efficient with:
process delays
Non-minimum phase or unstable processes
Large disturbances
Loop interaction…

Conventional control



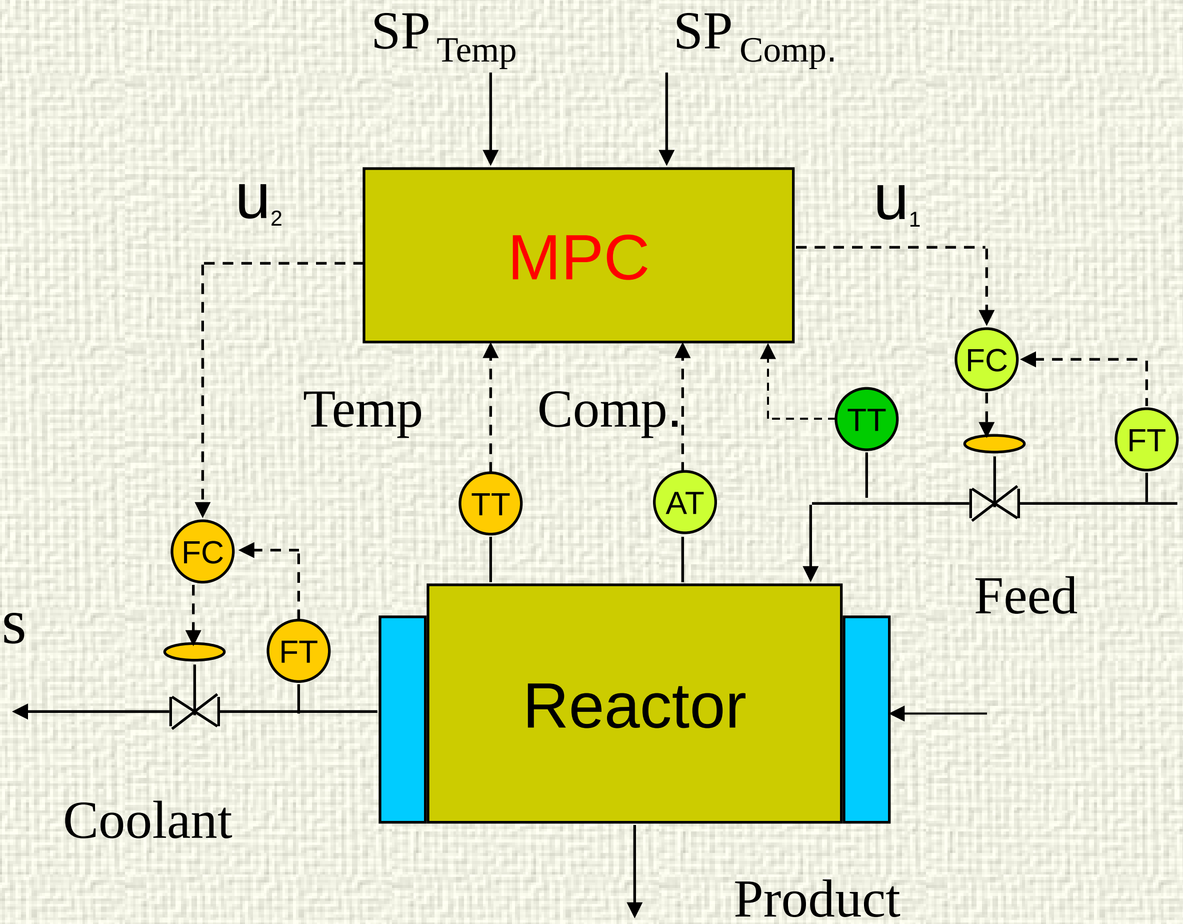
Control structures difficult to maintain
Interactions
Saturations

What is required….

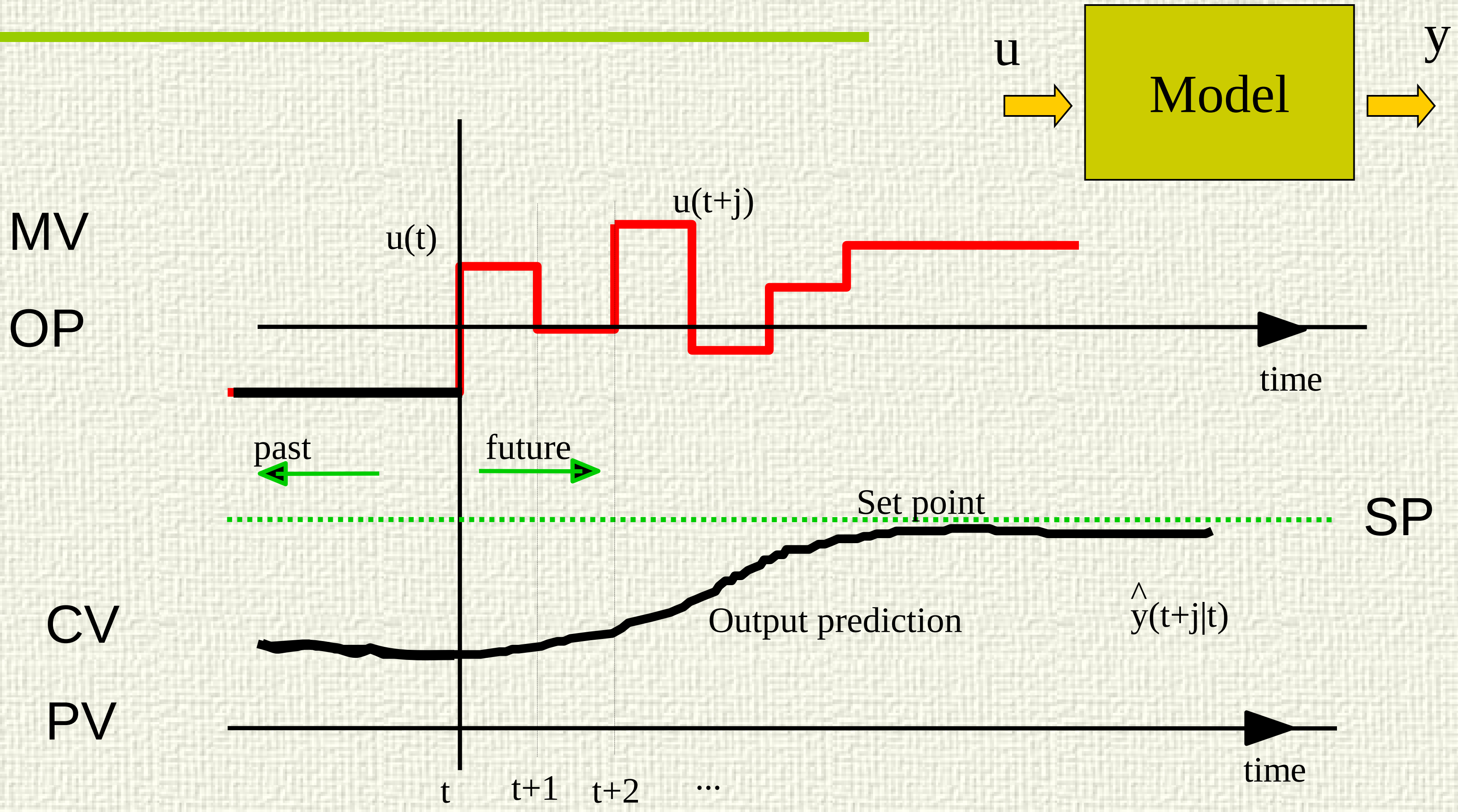
- Model based
  - Predicts the future process behaviour, taking into account complex dynamics and measurable disturbances
- Multivariable
  - Able to deal with multiple manipulated and controlled variables simultaneously
- Constraints
  - On manipulated and controlled variables
- Economic optimization
  - The process can be driven with an economic target

Multivariable Predictive Control

MPC considers all manipulated and controlled variables simultaneously, as well as the measured disturbances and constraints.
It handles all interactions, disturbances and constraints using a process model in an optimal way, improving control performance.

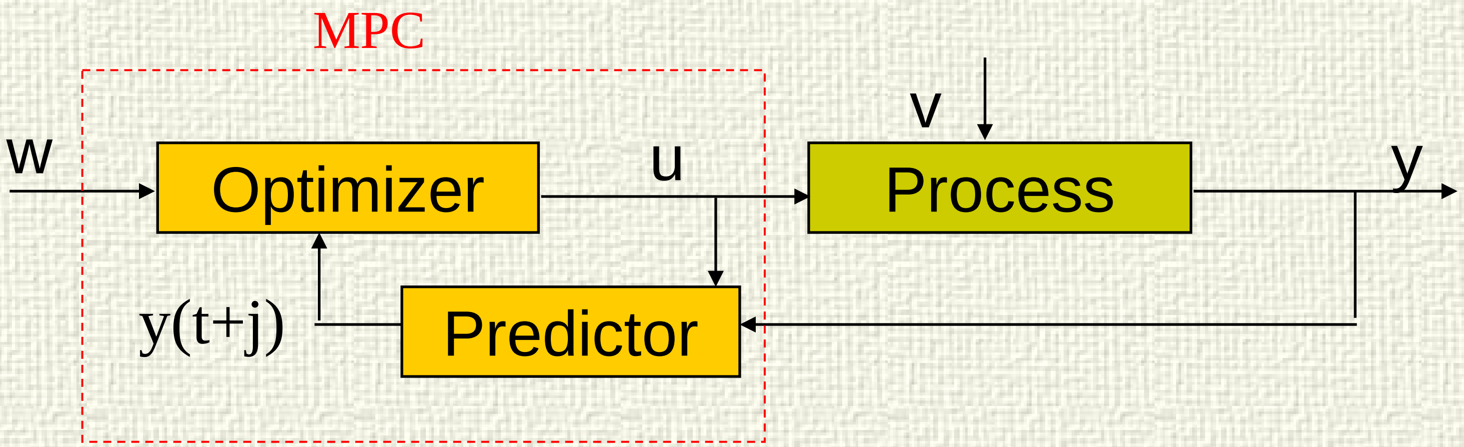


Model Predictive Control

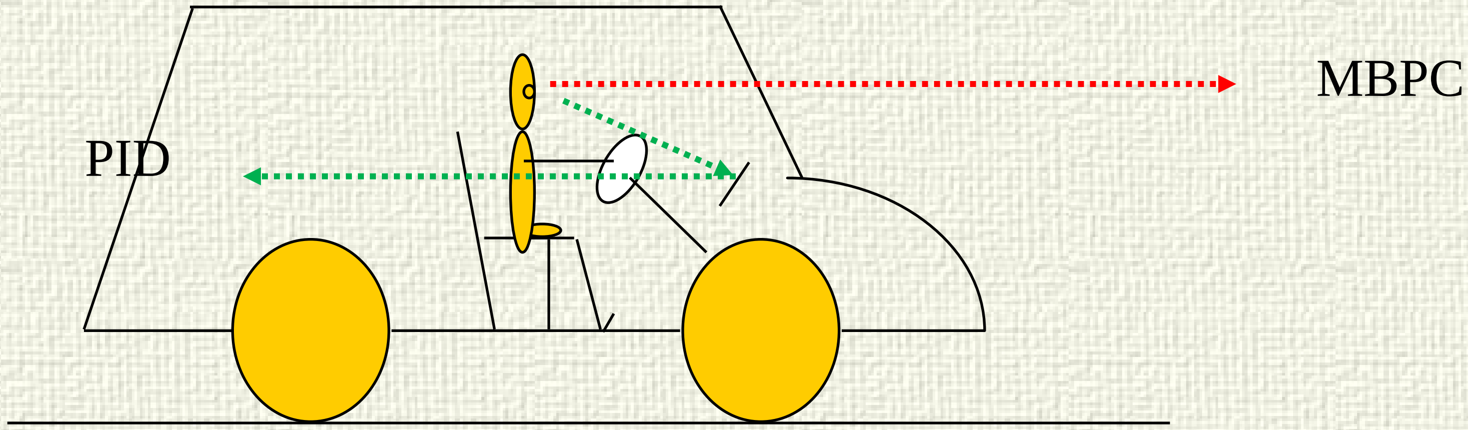


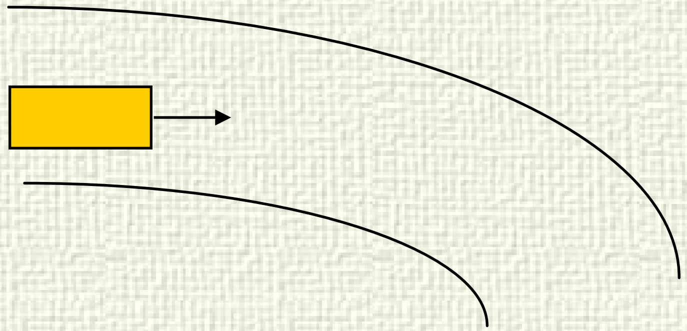
Model Predictive Control (MPC) MBPC

- Control strategy based in the on-line use of a model to predict the future behaviour of the process output over a certain temporal horizon, as a function of the future control actions
- The best control action is selected using an optimization procedure
- Many methods sharing some common principles (DMC, GPC, EPSAC, PFC, PPC, RMPC,..)

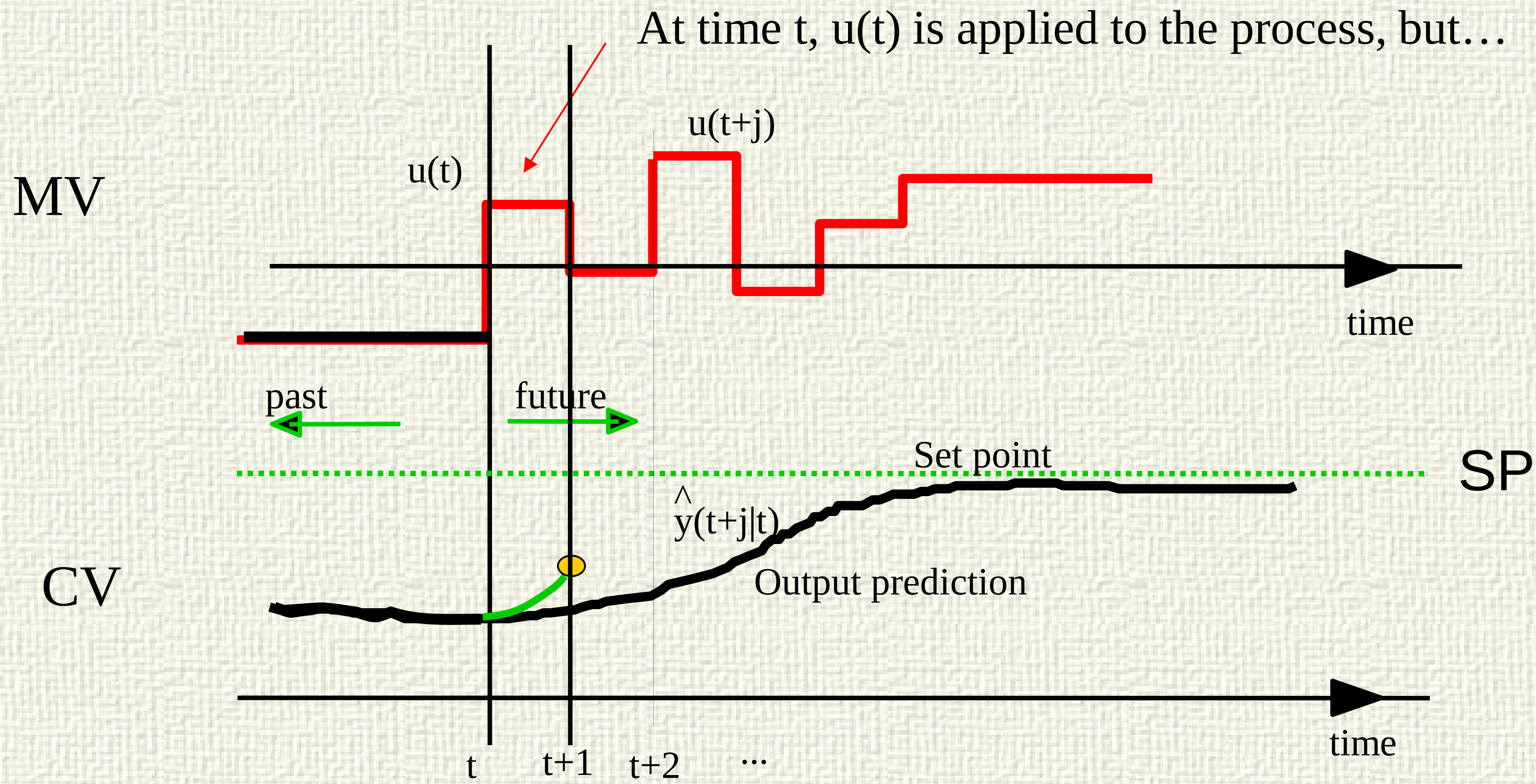


MPC





MPC, control implementation

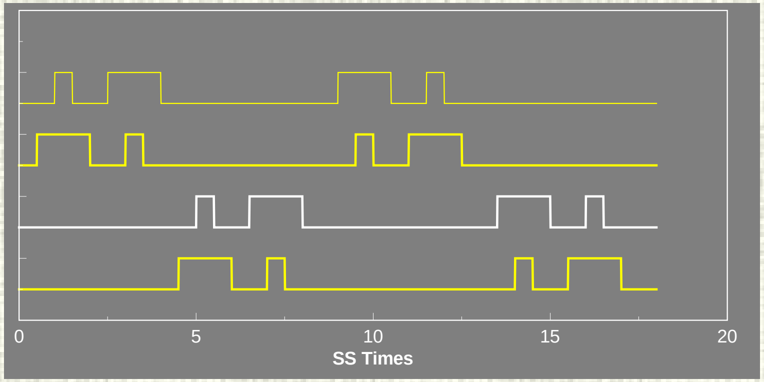


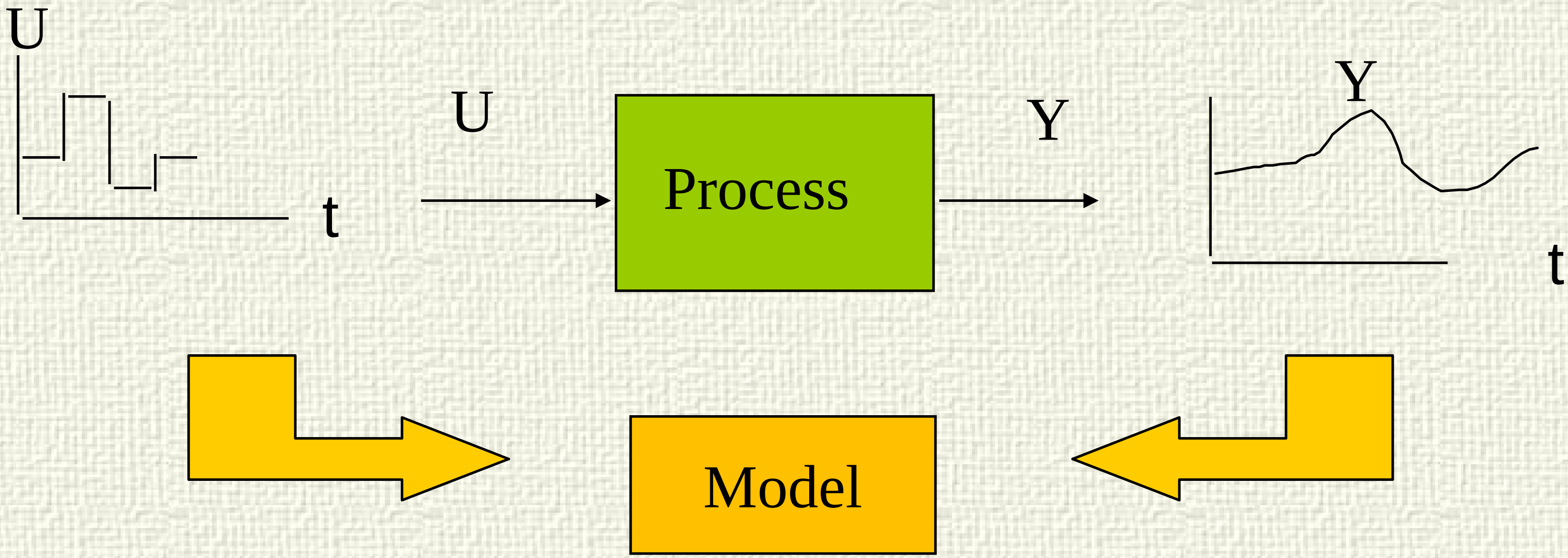
Moving horizon policy

…at time t +1, the whole process is repeated with the information available

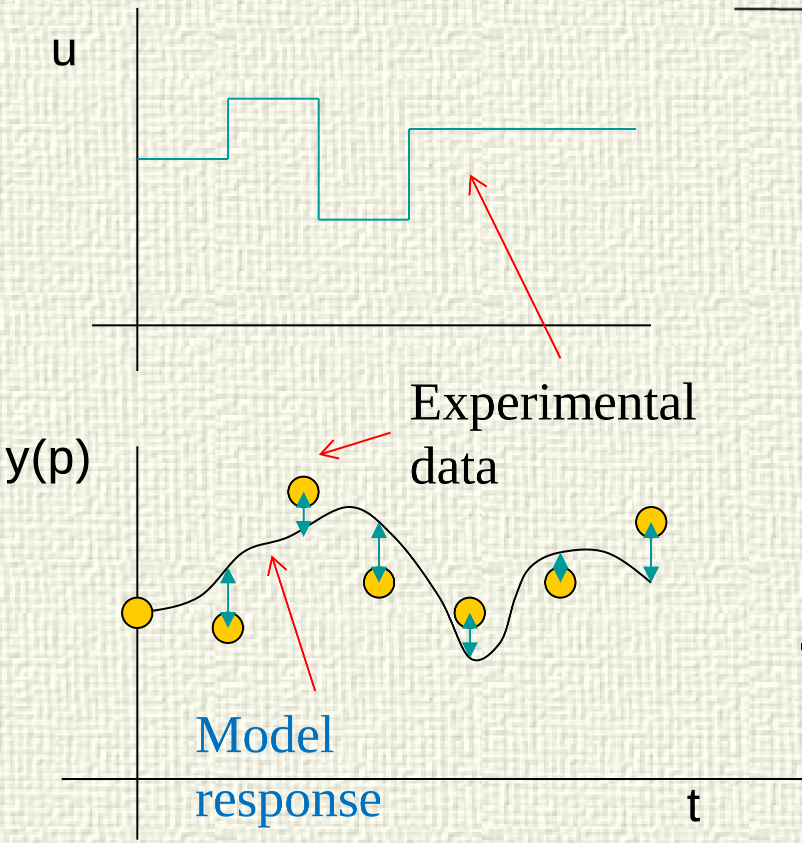
How to obtain models?

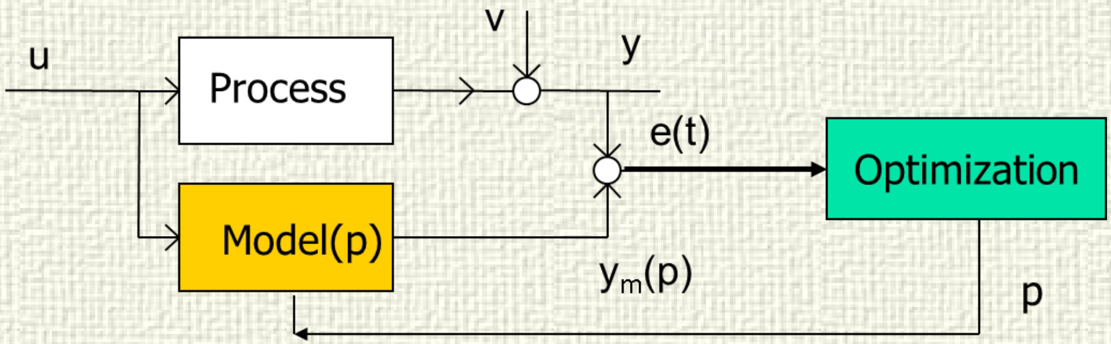
The model can be formulated using physical laws, but normally is computed from input output data obtained with experiments





Identification





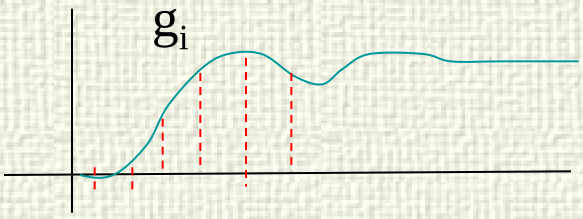
Model parameters p can be selected in order to minimize an error function:

$$J = \frac{1}{N}\sum_{t=1}^{N} e(t)^2 = \frac{1}{N}\sum_{t=1}^{N}[y(t) - y_m(u,p,t)]^2$$

Different types of models according to the process and target

DMC Dynamic Matrix Control

$$\Delta z(t) = z(t) - z(t-1)$$



Model: step response (linear) + disturbance

$$y(t) = \sum_{i=1}^{\infty} g_i \Delta u(t-i) + n(t)$$

Predictions:

$$y(t+j) = \sum_{i=1}^{j} g_i \Delta u(t+j-i) + \sum_{i=j+1}^{\infty} g_i \Delta u(t+j-i) + n(t+j)$$

Disturbance model:

$$n(t+j) = n(t) = y_p(t) - \sum_{i=1}^{\infty} g_i \Delta u(t-i)$$

Predictions:

$$\hat{y}(t+j) = \sum_{i=1}^{j} g_i \Delta u(t+j-i) + \sum_{i=j+1}^{\infty} g_i \Delta u(t+j-i) +$$

$$+ y_p(t) - \sum_{i=1}^{\infty} g_i \Delta u(t-i)$$

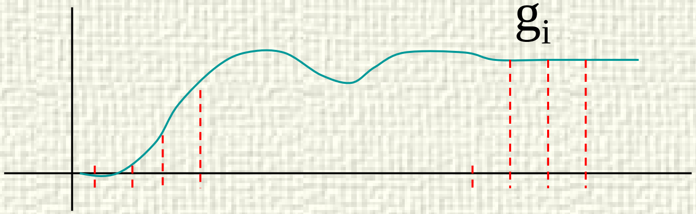
DMC, Dynamic Matrix Control

$$\hat{y}(t+j) = \sum_{i=1}^{j} g_i \Delta u(t+j-i) + \sum_{i=j+1}^{\infty} g_i \Delta u(t+j-i) + y_p(t) - \sum_{i=1}^{\infty} g_i \Delta u(t-i)$$

$$p_j = y_p(t) + \sum_{i=j+1}^{\infty} g_i \Delta u(t+j-i) - \sum_{i=1}^{\infty} g_i \Delta u(t-i) =$$

$$= y_p(t) + g_{j+1}\Delta u(t-1) + g_{j+2}\Delta u(t-2) + \ldots - g_1 \Delta u(t-1) - g_2 \Delta u(t-2) - \ldots.$$

$$p_j = y_p(t) + \sum_{i=1}^{\infty} (g_{j+i} - g_i)\Delta u(t-i)$$



In asymptotically stable systems:

$$g_{j+i} - g_i \cong 0 \quad d_{j+i} - d_i \cong 0 \quad i > N, \qquad j = N1, \ldots, N2$$

$$p_j = y_p(t) + \sum_{i=1}^{N} (g_{j+i} - g_i)\Delta u(t-i)$$

Free response of the system at time t

DMC, Dynamic Matrix Control

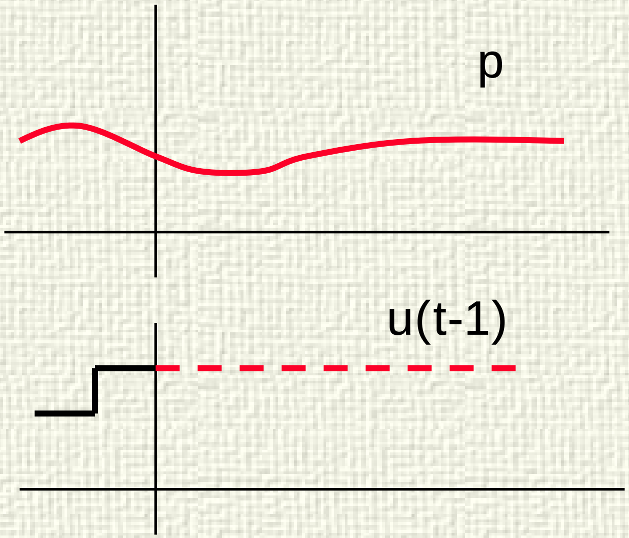
Predictions: $\hat{y}(t+j) = \sum_{i=1}^{j} g_i \Delta u(t+j-i) + p_j$ Forced + free response

$$G_j(q^{-1}) = g_1 q^{-1} + \ldots\ldots + g_j q^{-j}$$

$$\hat{y}(t+j) = G_j(q^{-1})\Delta u(t+j) + p_j$$

$$p_j = y_p(t) + \sum_{i=1}^{N} (g_{j+i} - g_i)\Delta u(t-i)$$

p free response



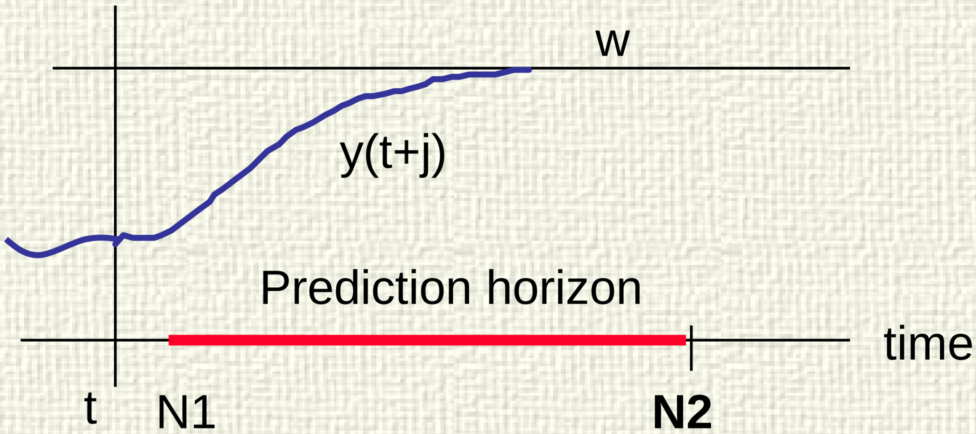
Notice that, in steady state, $\Delta u = 0$ and $p_j = y_p$, so, the predictions correspond to the real process output, eliminating a possible offset

DMC, optimal choice of u

Min Cost function:

$$J = \sum_{j=N1}^{N2} \left[\hat{y}(t+j) - w(t+j)\right]^2 + \sum_{j=0}^{Nu-1} \left[\beta \Delta u(t+j)\right]^2$$

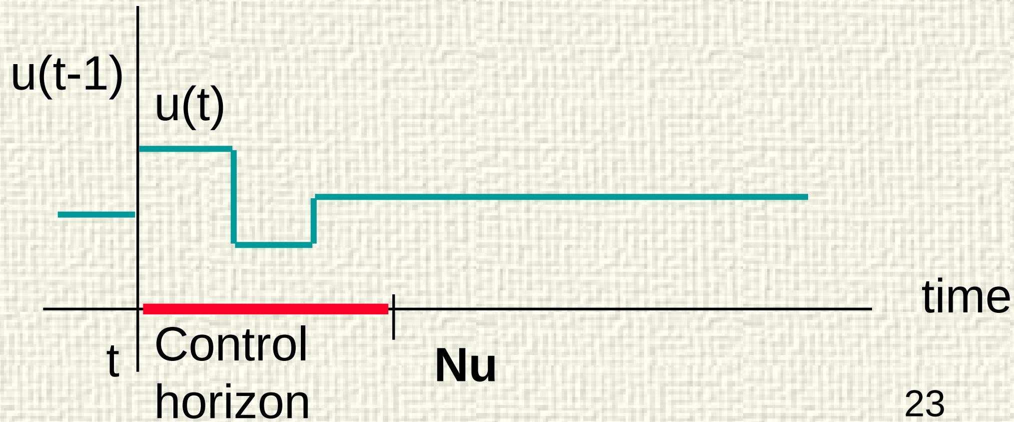
$$\hat{y}(t+j) = G_j(q^{-1})\Delta u(t+j) + p_j$$



Control structuring:

$\Delta u(t+j) = 0 \quad j \geq Nu$

Nu control horizon



DMC offset

DMC predictions: $$\hat{y}(t+j) = G_j(q^{-1})\Delta u(t+j) + p_j$$

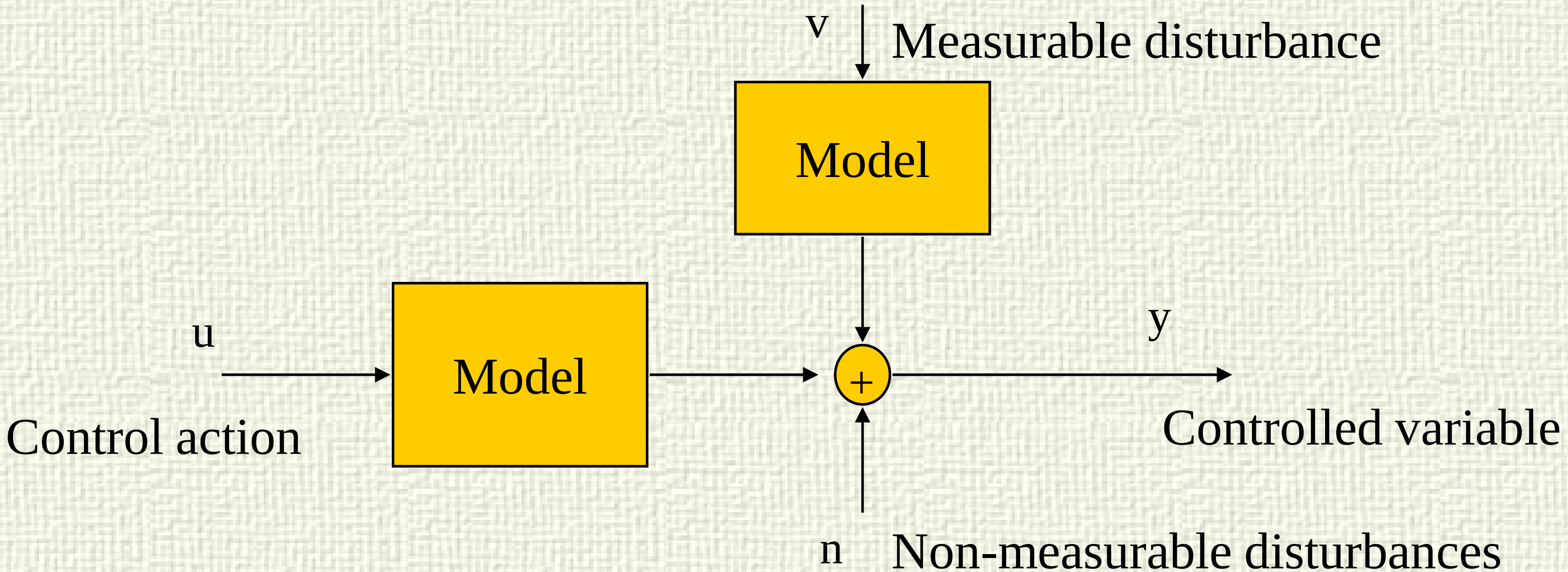
In steady state: $$\hat{y} = p_{ss}$$

And as $$p_j = y_p(t) + \sum_{i=1}^{N}(g_{j+i} - g_i)\Delta u(t-i) \Rightarrow p_{ss} = y_p$$

$$\Rightarrow \hat{y} = y_p$$

So, the predictions in steady state are unbiased and, if the optimization drives the predictions to w, then the process output y_p will be equal to the set point w, providing **offset free control**.

Measurable disturbances: incorporates feedforward compensation

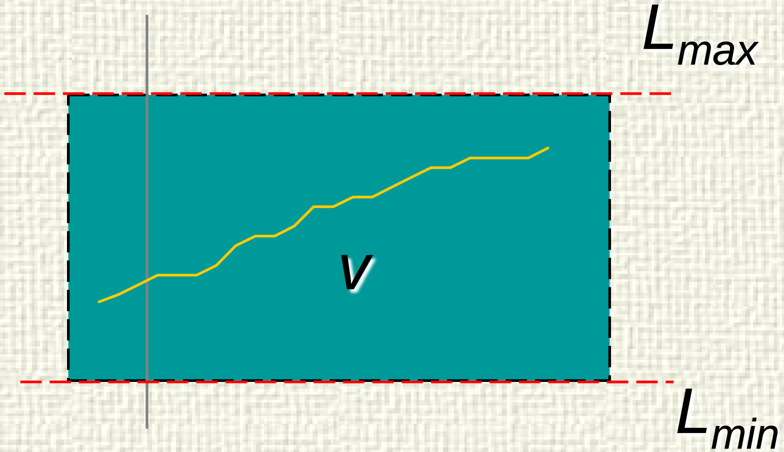


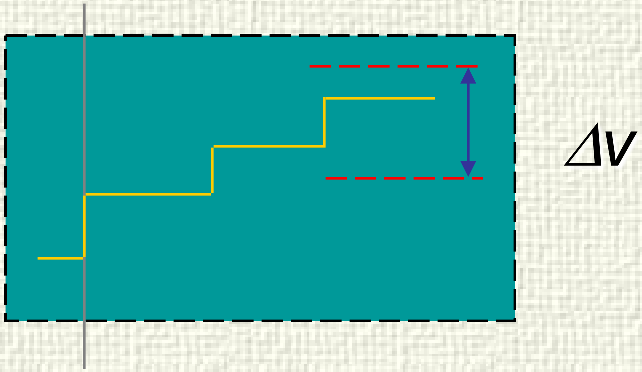
DMC model:

$$y(t) = \sum_{i=1}^{\infty} g_i \Delta u(t-i) + \sum_{i=1}^{\infty} d_i \Delta v(t-i) + n(t)$$

The effect of measurable disturbances is incorporated in the free response p_j

Constraints





Range of control signals: $U_m \leq u(t+j) \leq U_M$

Rate of change of u: $D_m \leq \Delta u(t+j) \leq D_M$

Range of controlled variables: $L_m \leq \hat{y}(t+j) \leq L_M$

Should be added to the computation of the optimal decisions

Constrained DMC

$$\min_{\Delta u} J = \sum_{j=N1}^{N2}\left[\hat{y}(t+j) - w(t+j)\right]^2 + \sum_{j=0}^{Nu-1}\left[\beta\Delta u(t+j)\right]^2$$

A QP problem has to be solved on-line every sampling period

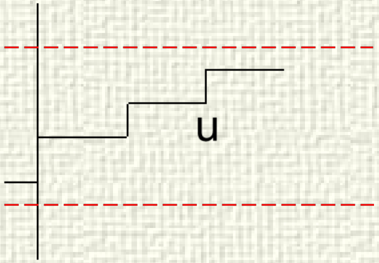
$$D_m \le \Delta u(t+j) \le D_M$$

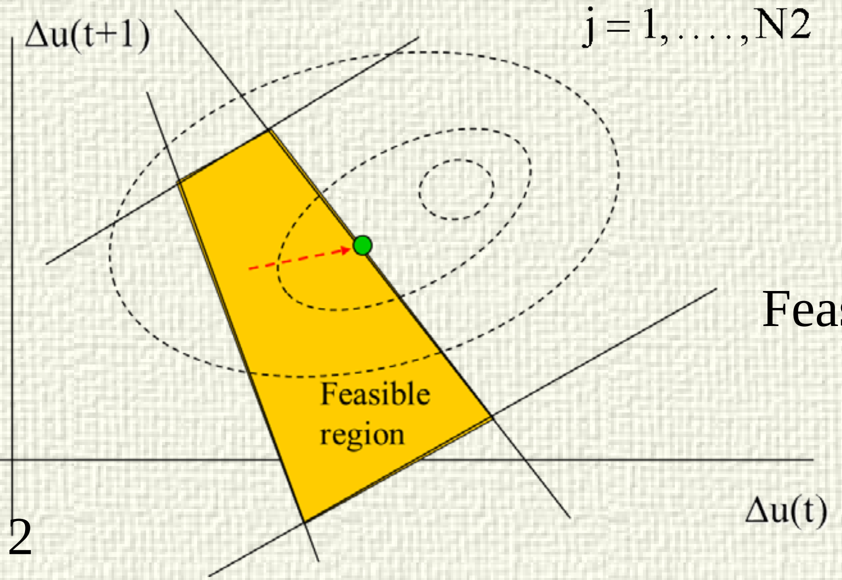
$$U_m \le u(t-1) + \sum_{i=0}^{j}\Delta u(t+j) \le U_M$$

$$j = 0, \ldots, Nu$$

$$L_m \le \hat{y}(t+j) = \sum_{k=1}^{j} g_k \Delta u(t-k+j) + p(t+j) \le L_M$$

$$j = 1, \ldots, N2$$





$N_u = 2$

Feasibility

Efficient numerical software

$$\min_{x} \quad \mathbf{c}\cdot\mathbf{x} + \frac{1}{2}\mathbf{x}\cdot\mathbf{Q}\cdot\mathbf{x}$$

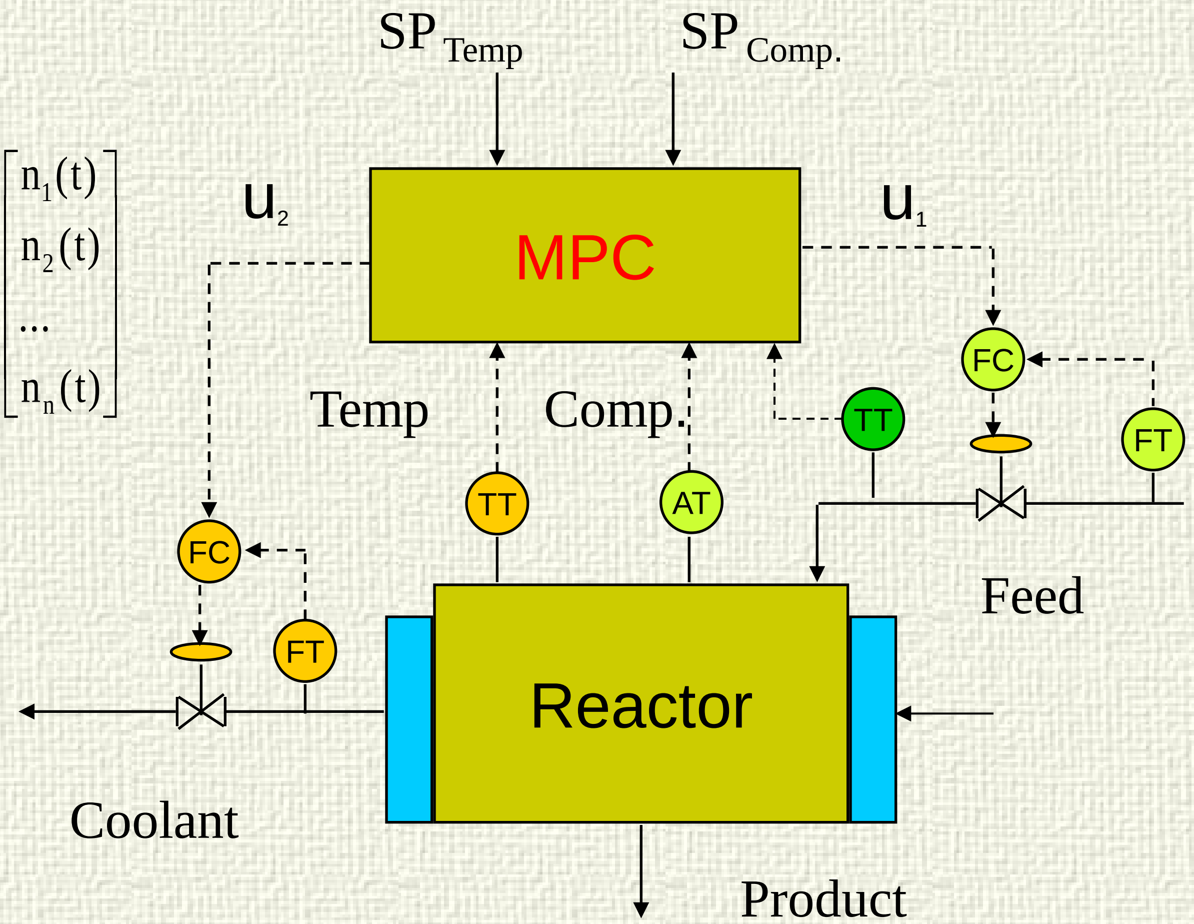
$$\mathbf{A}\mathbf{x} \le \mathbf{b}$$

$$\mathbf{x} \ge \mathbf{0}$$

Multivariable formulation

$$\begin{bmatrix} y_1(t) \\ y_2(t) \\ \dots \\ y_n(t) \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} & \dots M_{1m} \\ M_{21} & M_{22} & \dots M_{2m} \\ \dots & & \\ M_{n1} & M_{n2} & \dots M_{nm} \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \\ \dots \\ u_m(t) \end{bmatrix} + \begin{bmatrix} n_1(t) \\ n_2(t) \\ \dots \\ n_n(t) \end{bmatrix}$$

A model relating the controlled variables with the manipulated ones and the measurable disturbances is needed



Multivariable MPC

Predictions:

$$\hat{y}_1(t+j) = \sum_{i=1}^{j} g_{i11}\Delta u_1(t-i+j) + \sum_{i=1}^{j} g_{i12}\Delta u_2(t-i+j) + p_1(t+j)$$

$$\hat{y}_2(t+j) = \sum_{i=1}^{j} g_{i21}\Delta u_1(t-i+j) + \sum_{i=1}^{j} g_{i22}\Delta u_2(t-i+j) + p_2(t+j)$$

$$\hat{y}_3(t+j) = \sum_{i=1}^{j} g_{i31}\Delta u_1(t-i+j) + \sum_{i=1}^{j} g_{i32}\Delta u_2(t-i+j) + p_3(t+j)$$

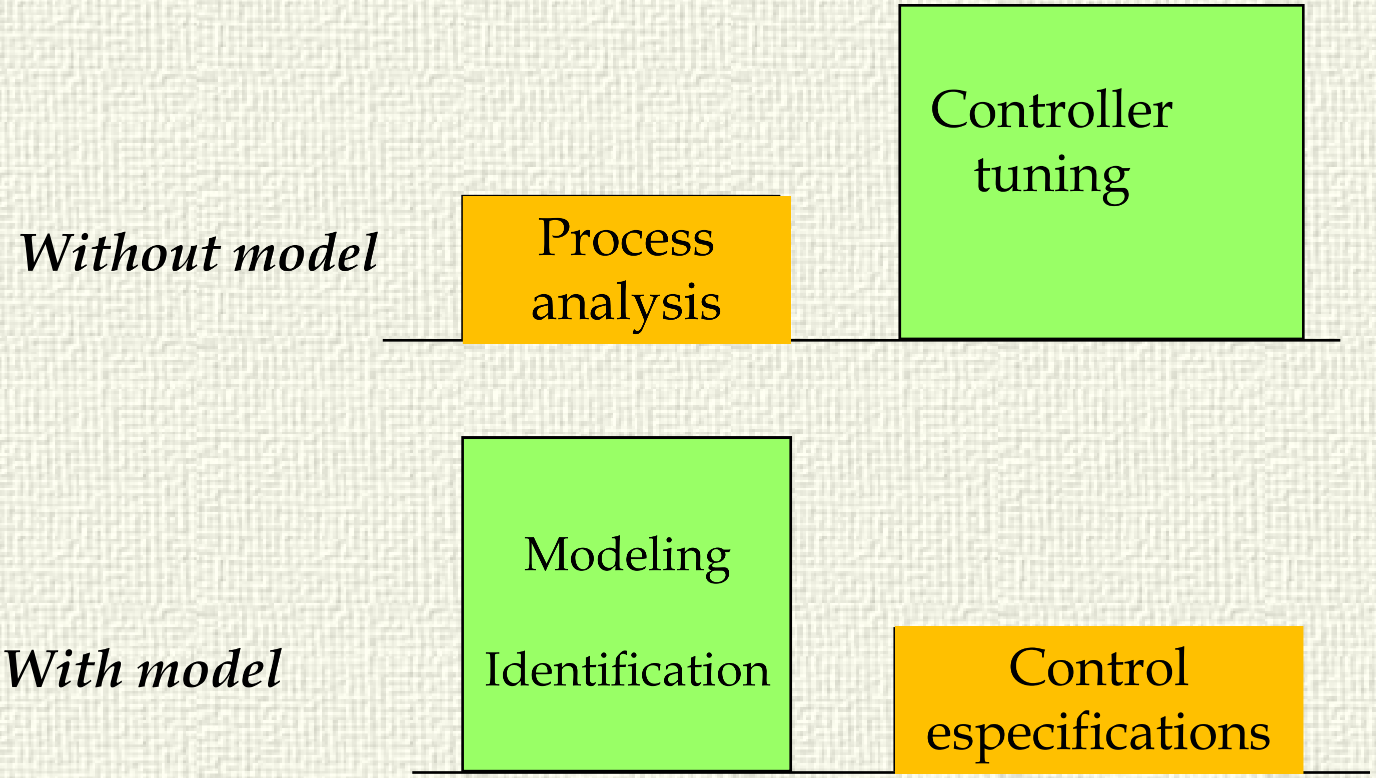
Min Cost function:

$$J = \sum_{j=N1}^{N2} \left[\gamma_1(\hat{y}_1(t+j) - r_1(t+j))\right]^2 + \left[\gamma_2(\hat{y}_2(t+j) - r_2(t+j))\right]^2 + \sum_{j=0}^{NU-1} \left[\beta_1 \Delta u_1(t+j)\right]^2 + \left[\beta_2 \Delta u_2(t+j)\right]^2$$

γ Equal concern errors β move suppression factors

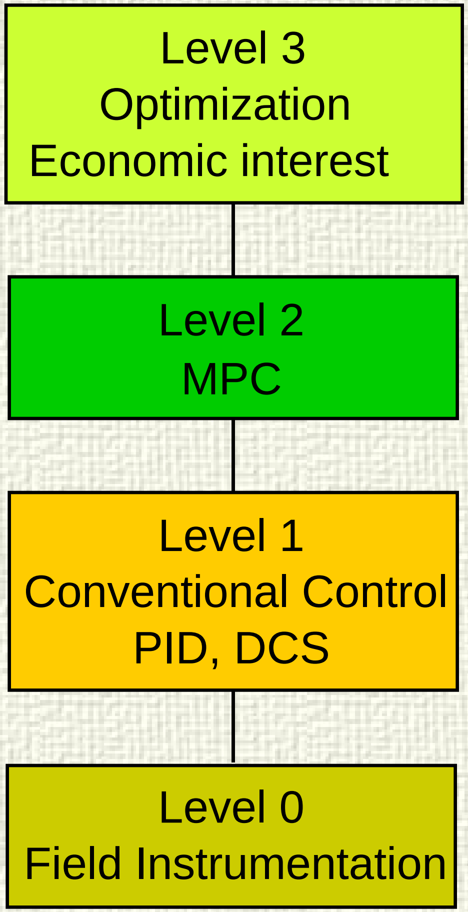
With constraints: QP problem solved every sampling time

Time and efforts

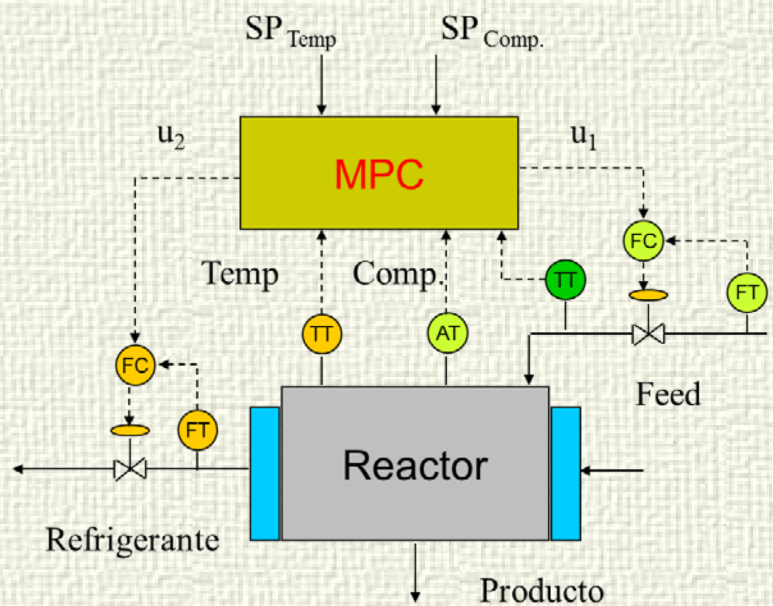


Able to deal with complex dynamical systems, with interaction, constraints, different number of MV and CV, disturbances, …. and basic ideas easy to understand

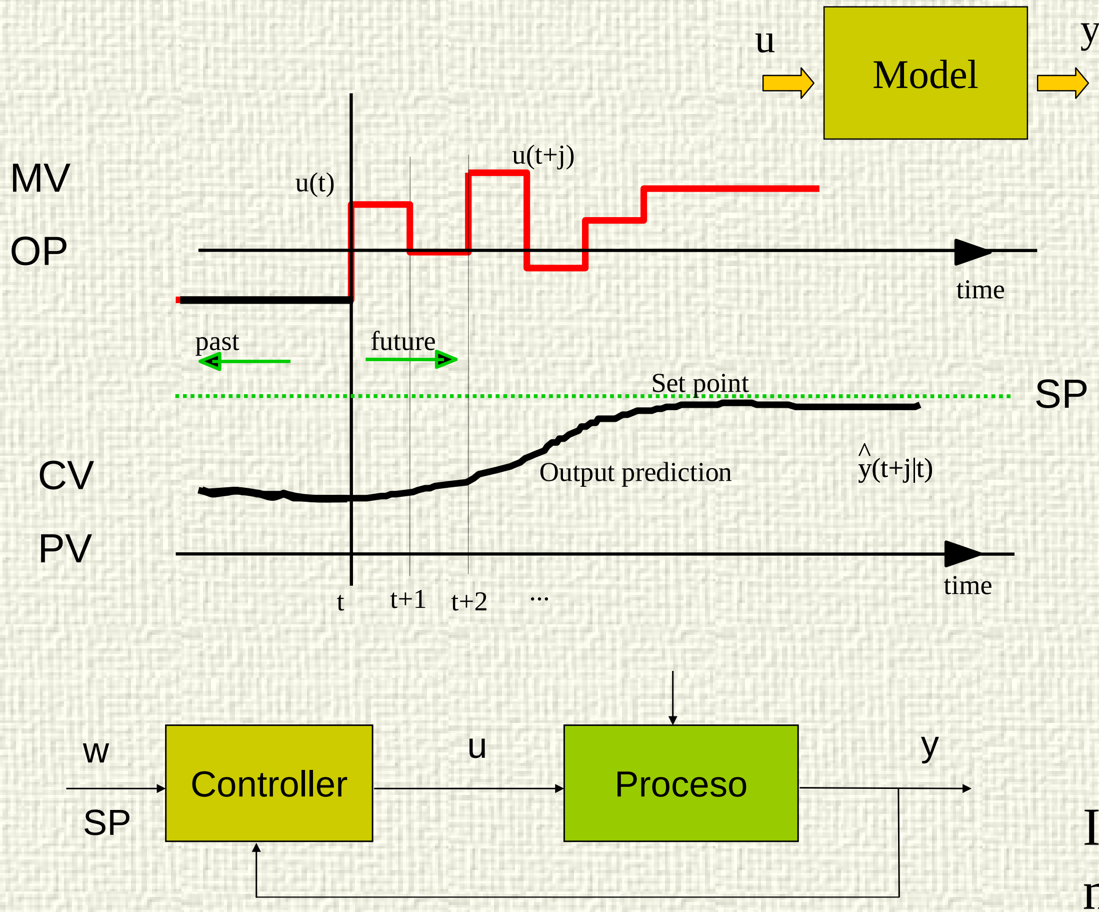
Where is MPC placed in the control hierarchy?



In order to implement properly
the actions taken in one level,
the lower levels must work correctly.



MPC – Optimal control



$u(t)=kx(t)$

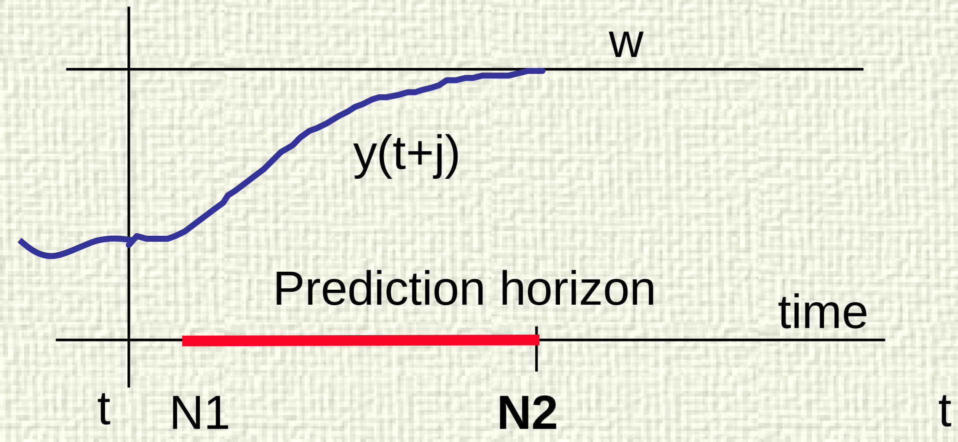
MPC is **NOT** optimal control

An open loop optimization problem is solved every sampling time, starting in the current state

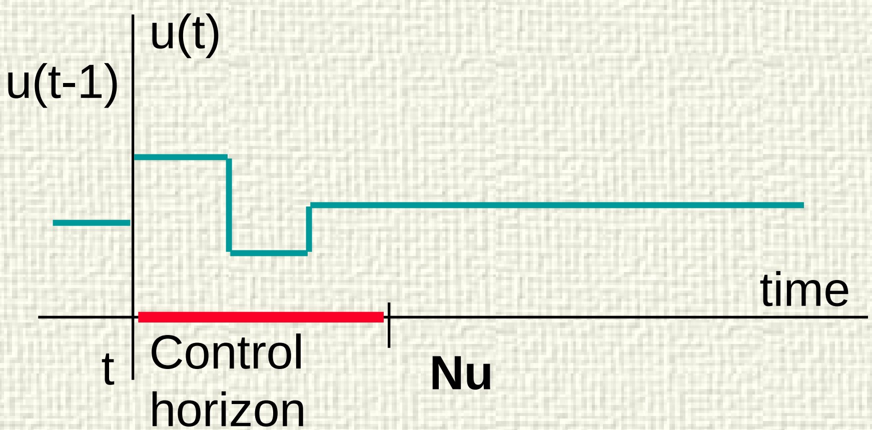
u(t), u(t+1), u(t+2) are considered independent variables in the optimization problem

In optimal control u(t+j) are not independent and the target is the control law (k)

Is MPC always stable en closed loop?



With finite horizon N2, there is no guarantee that, even with a perfect model, the application of the “optimally computed” control signals u(t) of MPC leads to a stable closed loop system



Infinite horizon (or equivalent formulations) si required to provide nominal stability guarantee

Can MPC be applied to fast processes?

State space model

$$x(t+1) = A\mathbf{x}(t) + Bu(t)$$
$$y(t) = C\mathbf{x}(t)$$

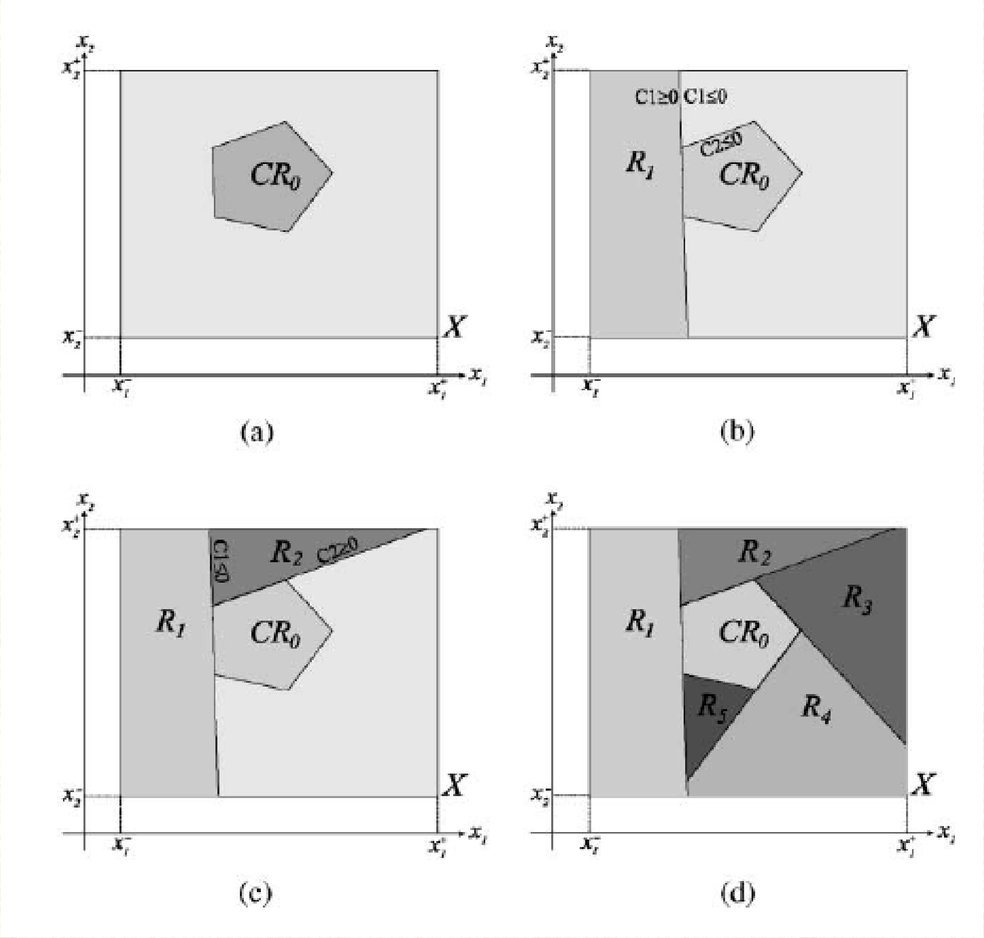
The solution of the MPC problem depends on the current process state

$$\hat{x}(t+j) = A^k x(t) + \sum_{k=0}^{j-1} A^k Bu(t+j-1-k)$$

Multi-parametric quadratic programming

$$V(x(t)) = \frac{1}{2} x'(t) Y x(t) + \min_U \left\{ \frac{1}{2} U' H U + x'(t) F U, \right.$$
$$\left. \text{s.t.} \quad GU \leq W + Ex(t) \right\},$$

Multiparametric MPC

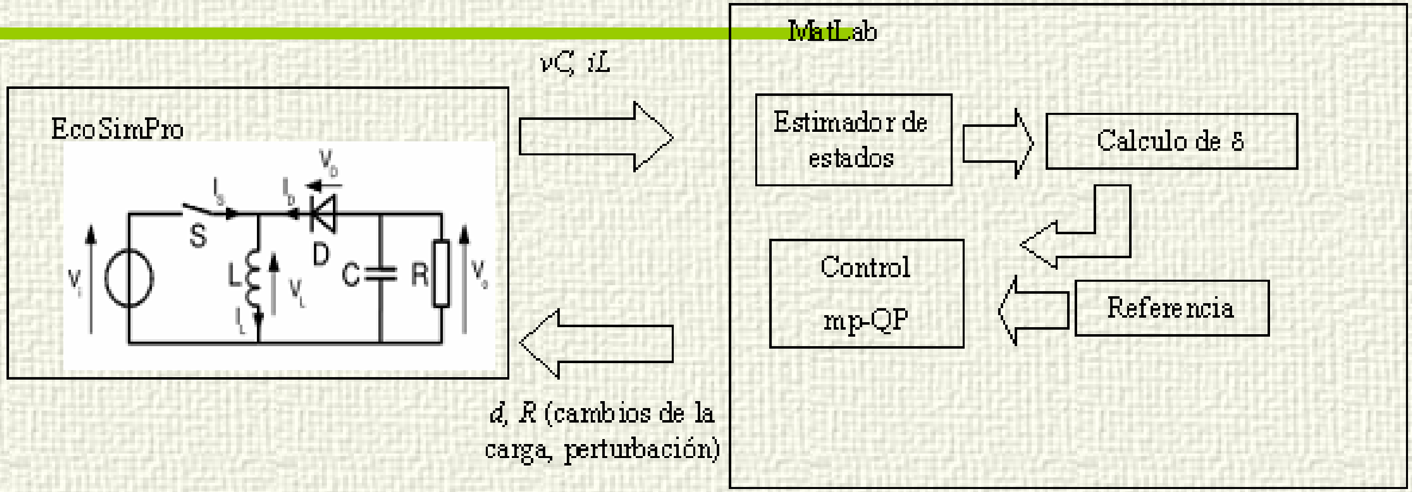


It is possible to obtain a closed loop expression for the control action as a function of the current state

Regions at which an explicit control formula applies, that can be computed off-line

Problems with the number of regions

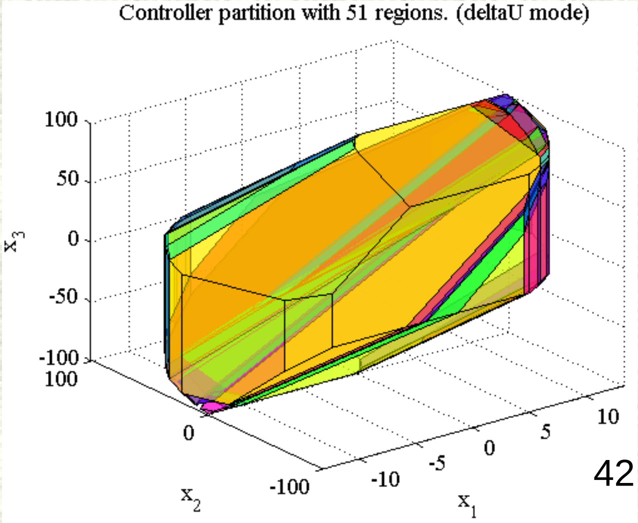
Example: BB converter



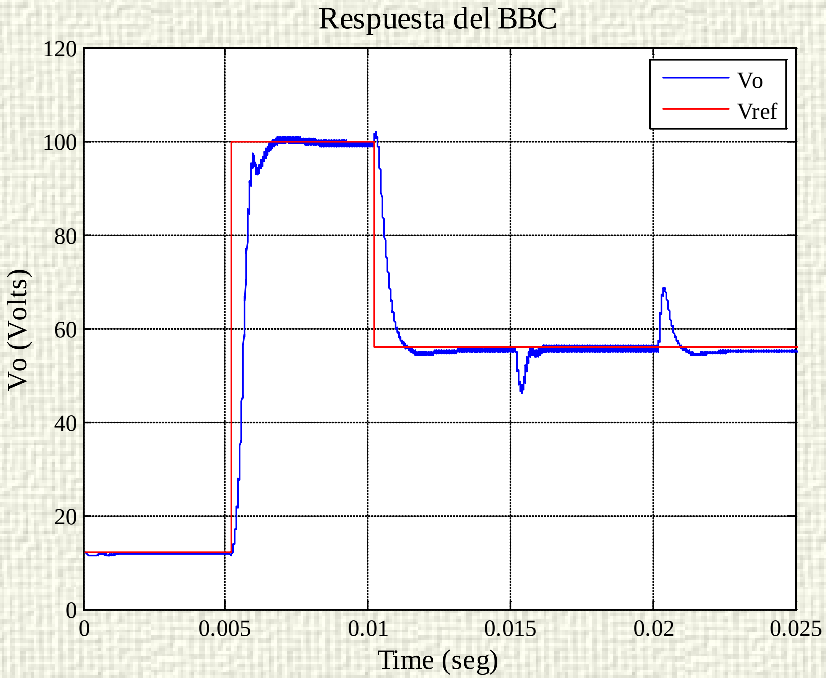
Changes in the input with disturbances δ = *Vo*(*t*)-*Vo Est* (*t/t*)

$$\begin{pmatrix} il(t+1) \\ vc(t+1) \\ d(t+1) \\ \delta(t+1) \end{pmatrix} = \begin{bmatrix} A & B & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \begin{pmatrix} il(t) \\ vc(t) \\ d(t) \\ \delta(t) \end{pmatrix} + \begin{bmatrix} B \\ I \\ 0 \end{bmatrix} \Delta d$$

$$Vo = [0 \quad -1 \quad 0 \quad 1] \begin{pmatrix} il(t) \\ vc(t) \\ d(t) \\ \delta(t) \end{pmatrix}$$

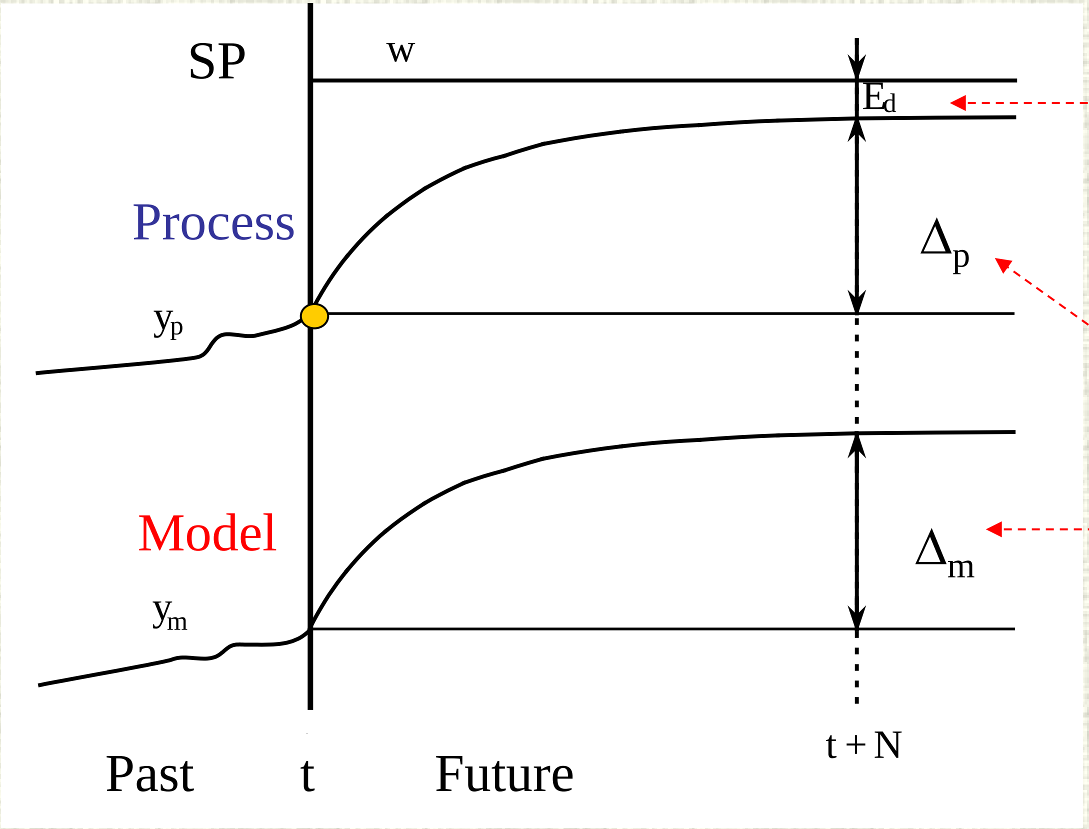


BB converter



Tiempo (mseg)	*V* Referencia	Carga *R*
0	-12	80
5	-100	80
10	-56	80
15	-56	40
20	-56	120

Can MPC replace PID?



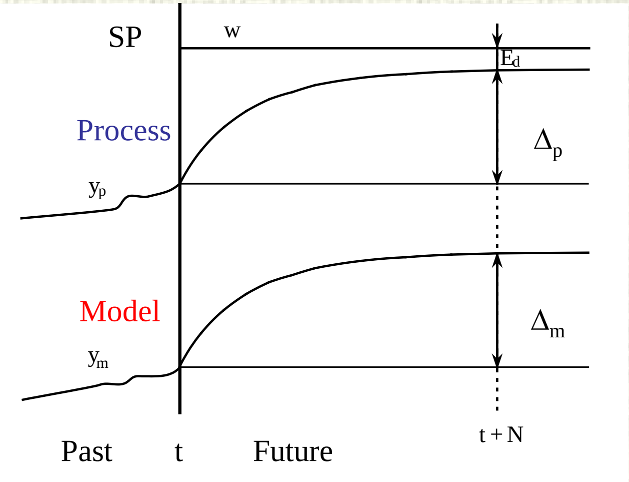
Aim: Decrease the error in the future until a certain percentage of the current error w – y(t)

This implies to change the process output by Δ_p

Use the model to compute the control u(t) that provides a change in the model output Δ_m (u) = Δ_p

Key design equation: $\Delta_m (u) = \Delta_p$

Design equation



First order model

$$\Delta_p = (1-\lambda^N)(w - y_p(t))$$

$$\Delta y_m = \left(e^{-\frac{N T_s}{\tau}} - 1\right) y_p(t) - k\,u(t)\left(e^{-\frac{N T_s}{\tau}} - 1\right)$$

design equation: $\Delta_m(u) = \Delta_p$

Explicit solution for the control signal

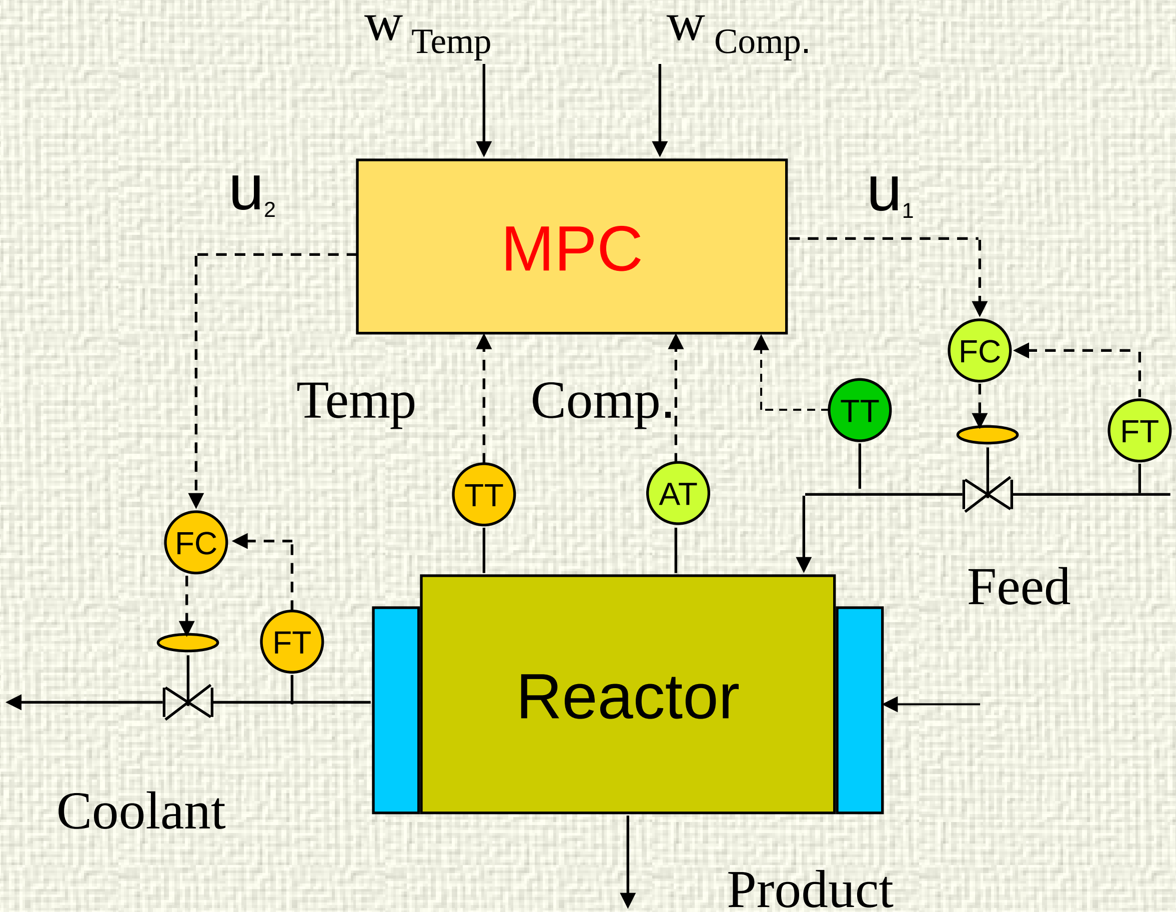
PLC implementable

$$u(t) = \frac{\left(e^{-\frac{N T_s}{\tau}} - 1\right) y_p(t) - (1-\lambda^N)[w(t) - y_p(t)]}{K\left(e^{-\frac{N T_s}{\tau}} - 1\right)}$$

Economic optimization

Which is the most profitable operation point?

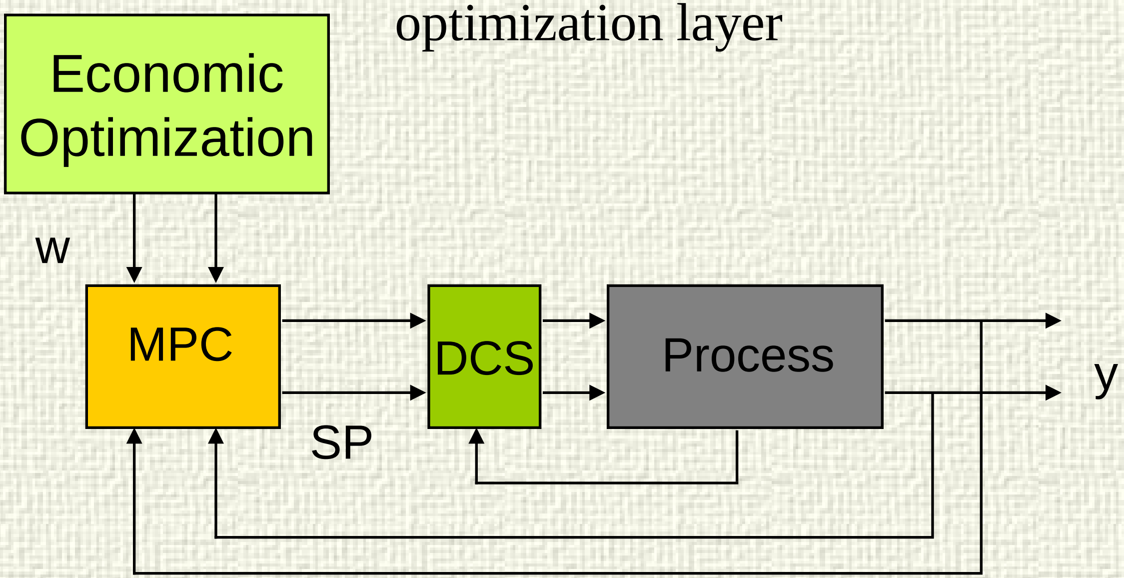
How to adjust the set points w_{Temp}, w_{Comp} to this point?



Economic optimization

The economic optimization takes advantage of the MPC model and constraints, using them quite often to compute the local optimal targets

Most of commercial MPC incorporate an economic optimization layer



Economic optimization

Economic cost function:

(Product*concentration*price-

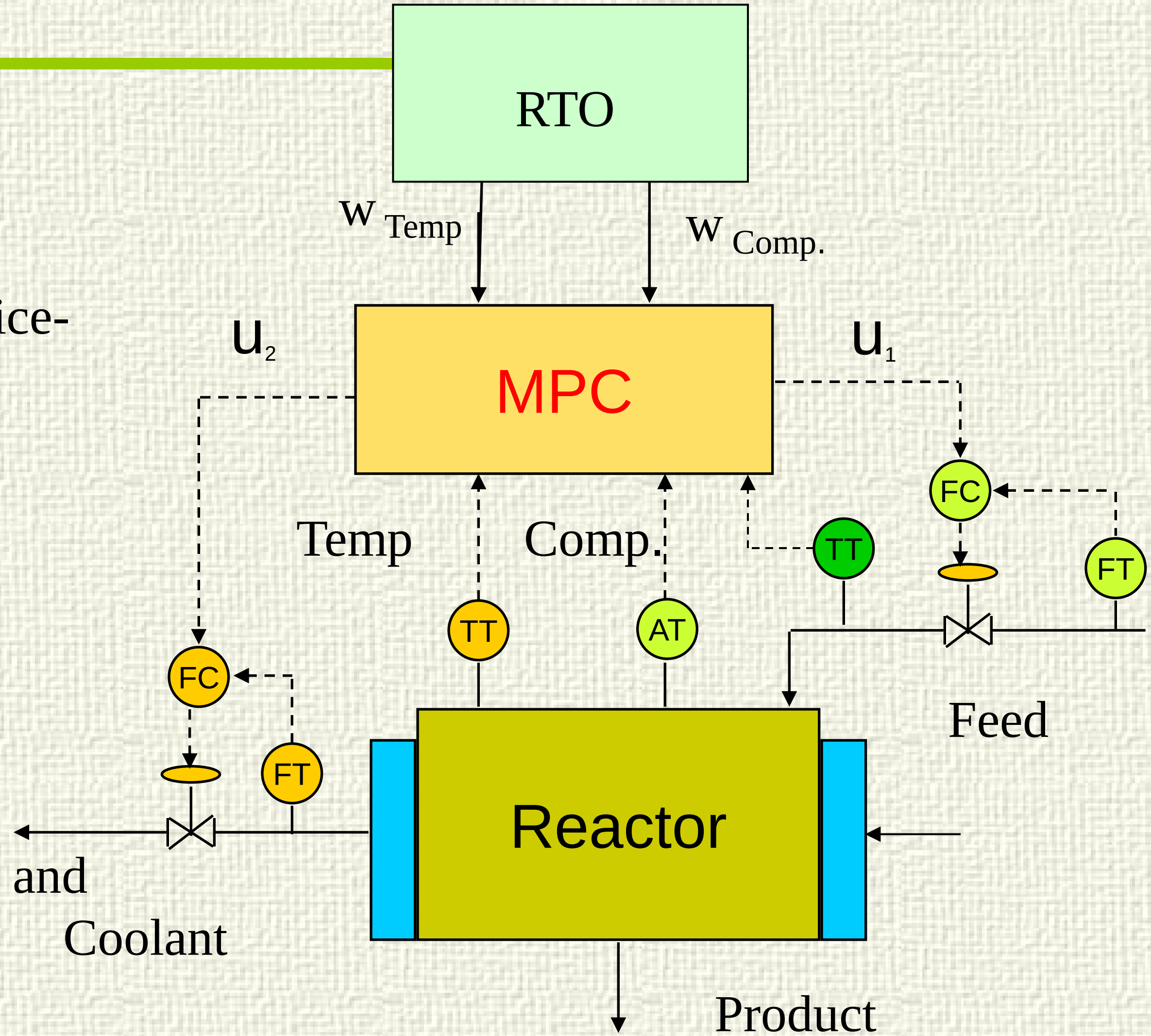
– row material*price –

-flow refrigerant*price)*

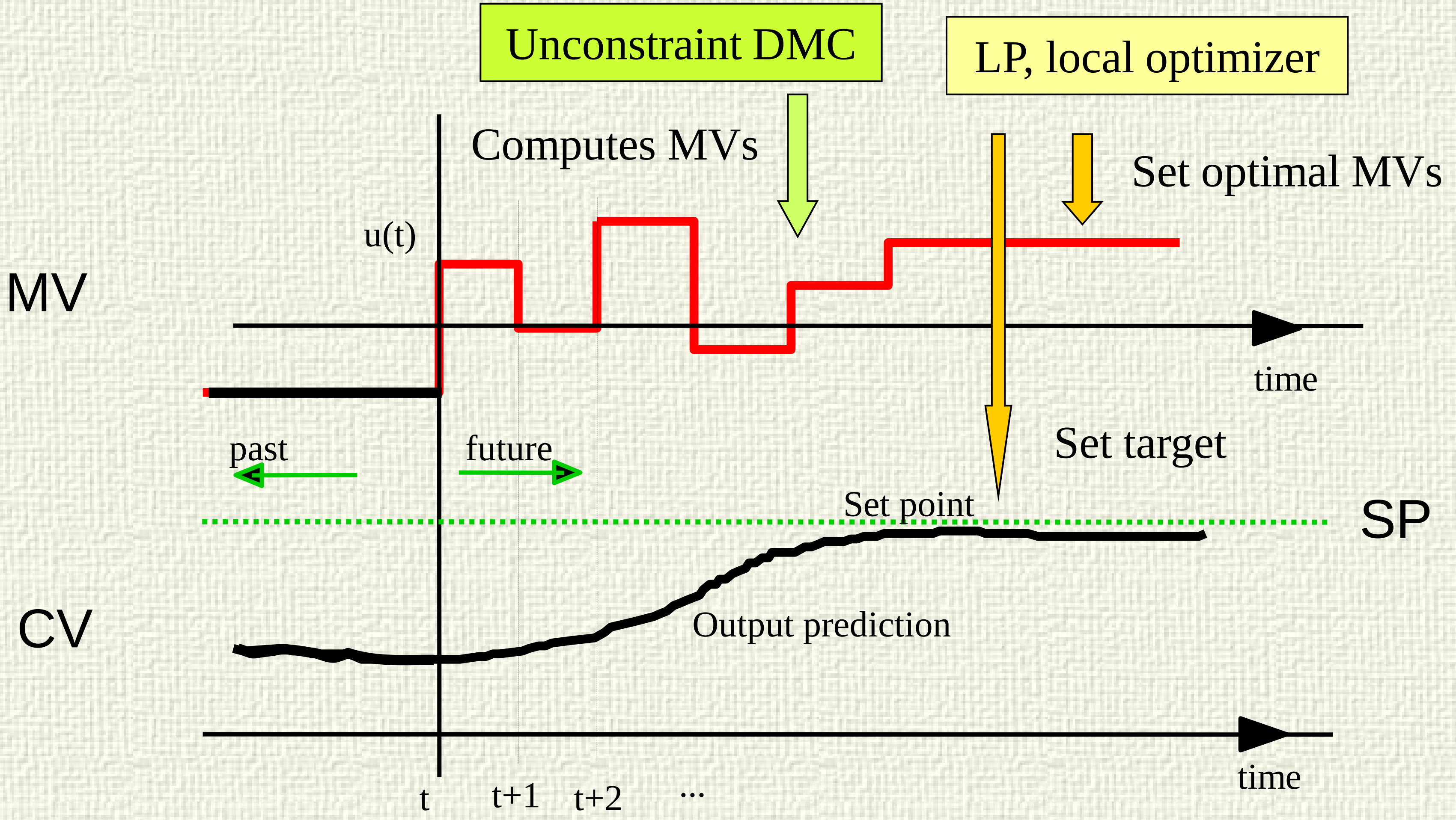
period of time

Max Benefit

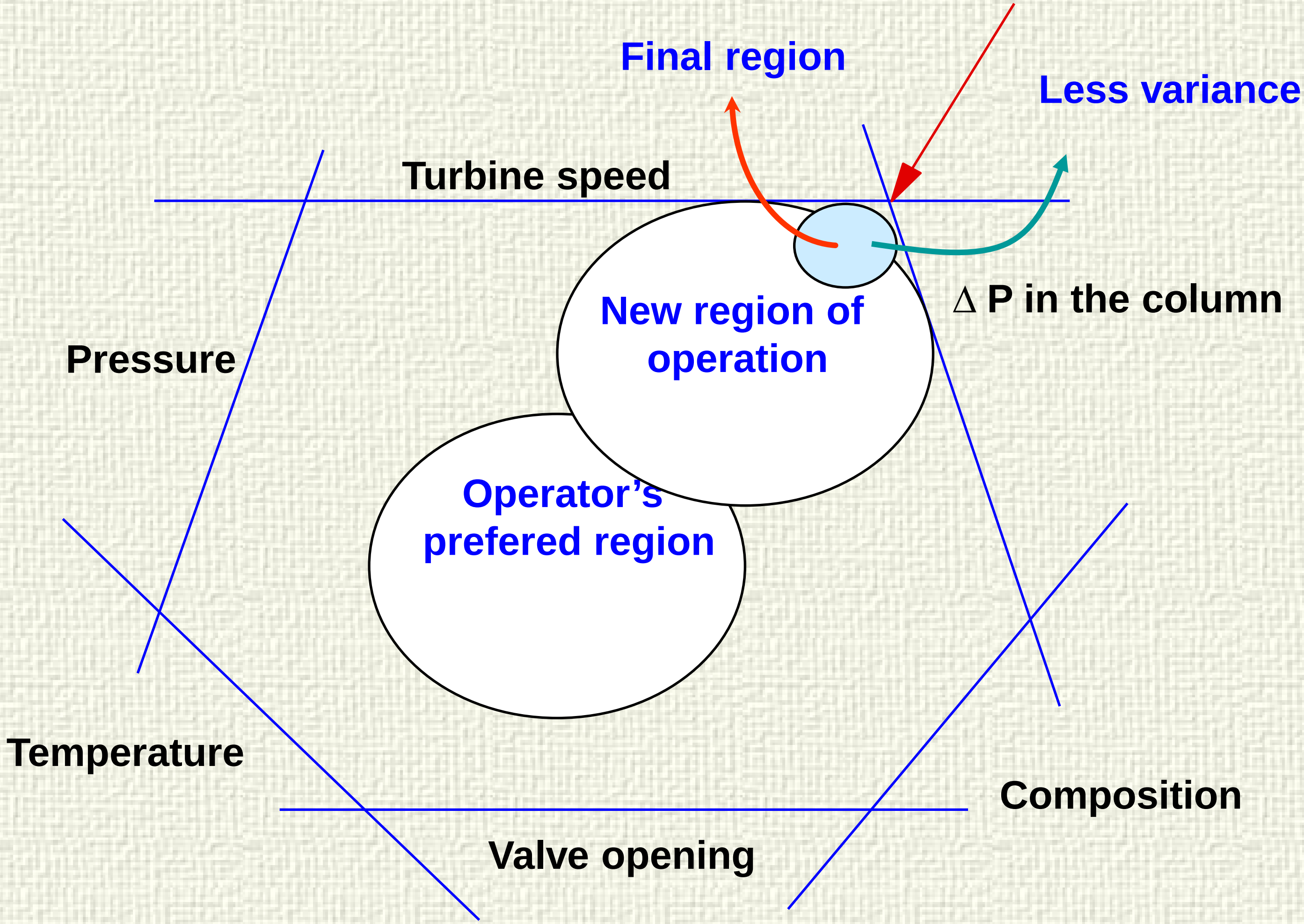
under operating constraints and
dynamical model



DMC Plus (Aspen)



Operating with constraints



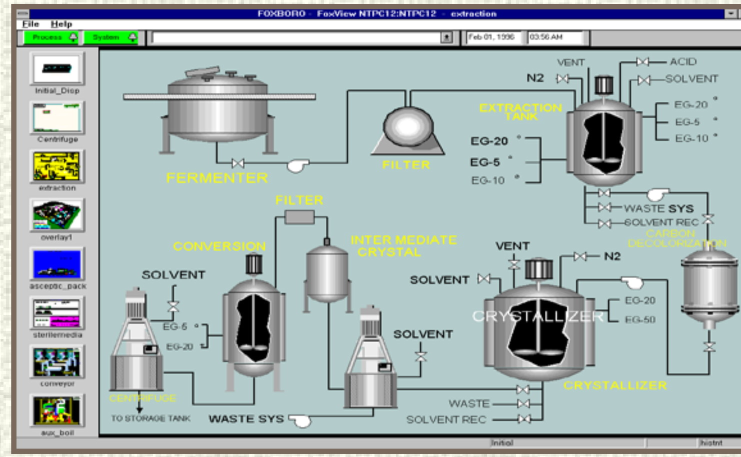
Key to success…

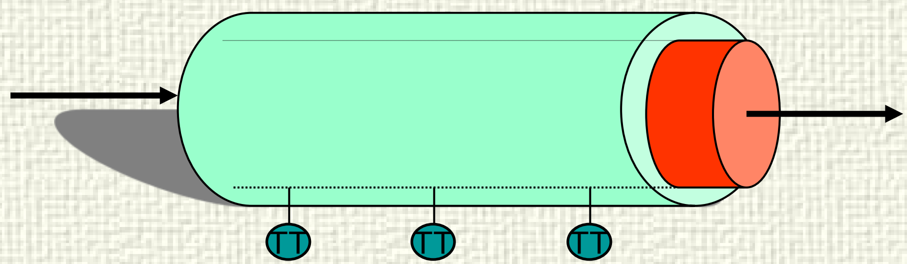
- Easy to understand by the users
- Can be applied to multivariable processes with different number of manipulated and controlled variables
- Compensates measurable disturbances and delays
- Takes into account constraints on MV and CV
- Can be applied to processes with difficult dynamics: delays, inverse response, unstable systems, slow processes,…
- Opens the door to economic optimization of the process operation

HITO

Industrial MPC problems

- ✓ Found often in practice
- ✓ Hybrid:
  - On/off
  - Logic
  - Continuous - batch
- ✓ Batch units
- ✓ Distributed parameter
- ✓ Population balance
- ✓ Start-up / Shut down
- ✓ Main problems:
  - Large scale
  - Variability
  - Model reduction





Non-linear MPC (NMPC)

$$\min_{u(t),x_0,t_f} \quad J(\mathbf{u}) = \int_{t_0}^{t_f} L(\mathbf{x},\mathbf{u})dt$$

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x},\mathbf{u},\mathbf{z}), \quad \mathbf{x}(t_0) = \mathbf{x}_0$$

$$\mathbf{h}(\mathbf{x},\mathbf{u},\mathbf{z}) = \mathbf{0}$$

$$\mathbf{g}(\mathbf{x},\mathbf{u},\mathbf{z}) \leq \mathbf{0}$$

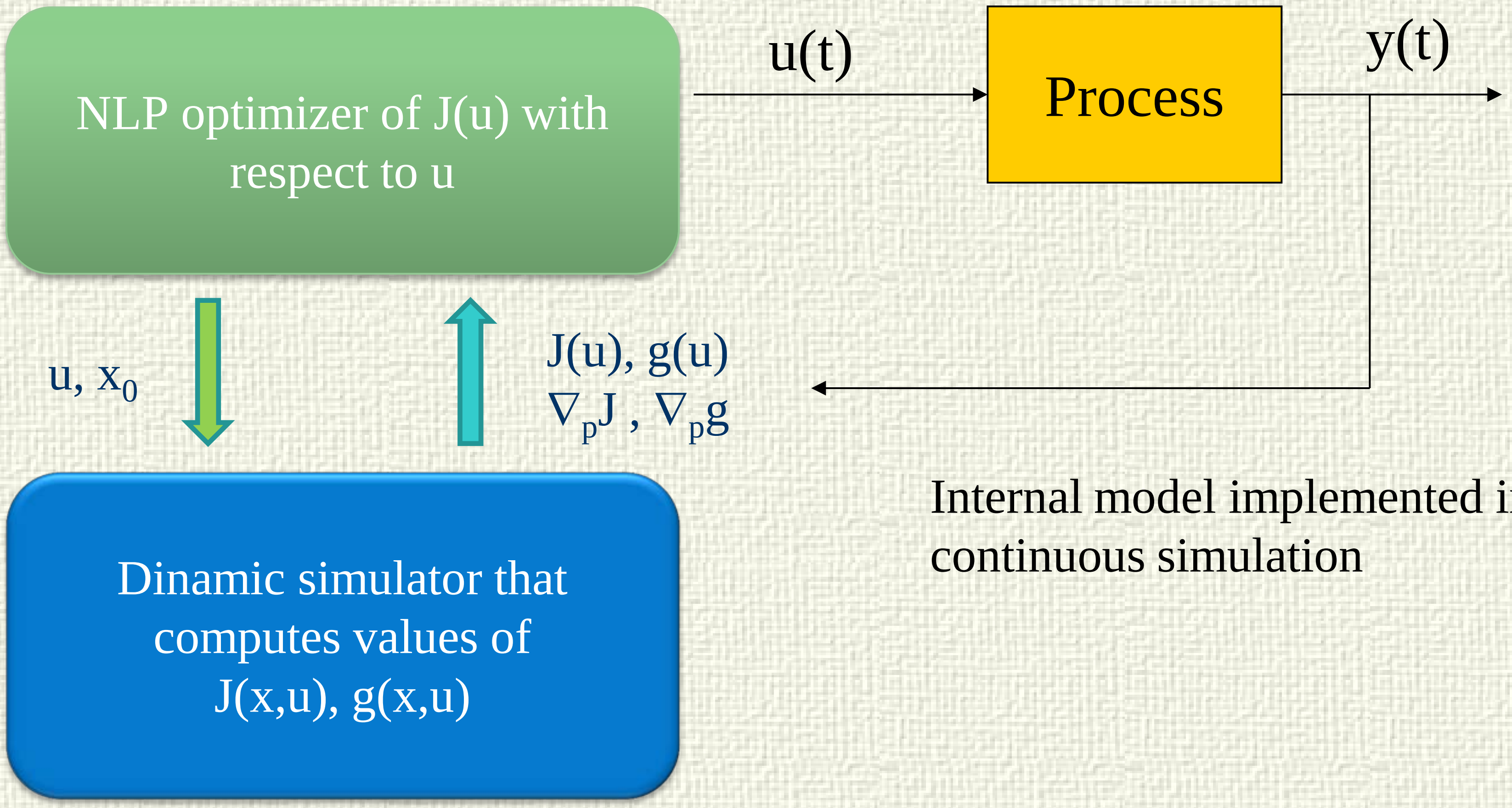
Dynamic optimization problem solved every sampling time

How to solve the dynamic optimization problem?

How to estimate the state?

Different types of models (first principles, Volterra series, NN, Wiener, Hammerstein ….) and cost functions

NMPC



Internal model implemented in a continuous simulation

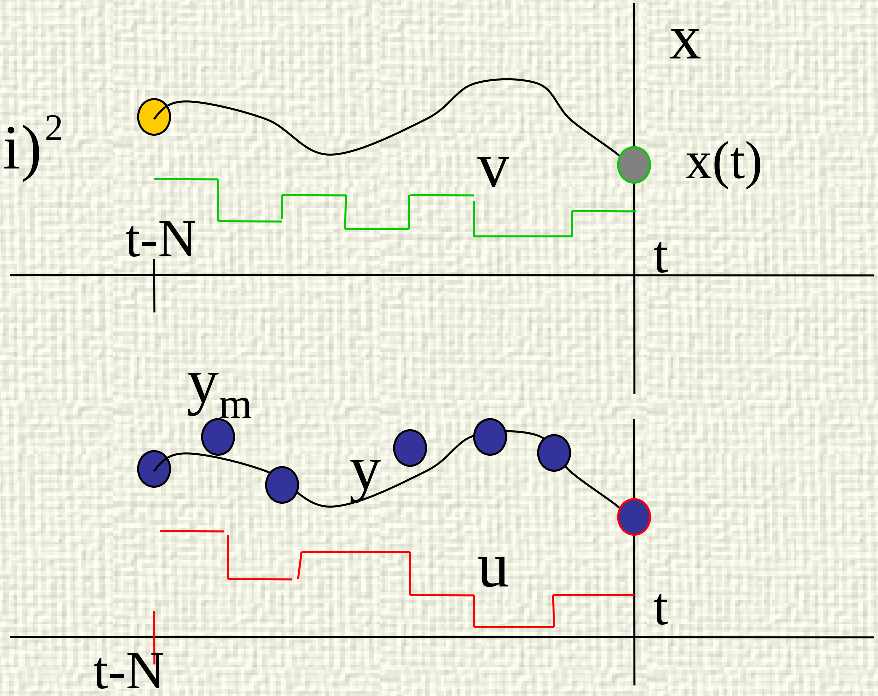
Moving Horizon Estimation (MHE)

$$\min_{x_{t-N}, v_i} \sum_{i=0}^{N} [y(t-i) - y_m(t-i)]^2 + \gamma v(t-i)^2$$

$$\dot{x} = f(x(t), u(t), v(t))$$

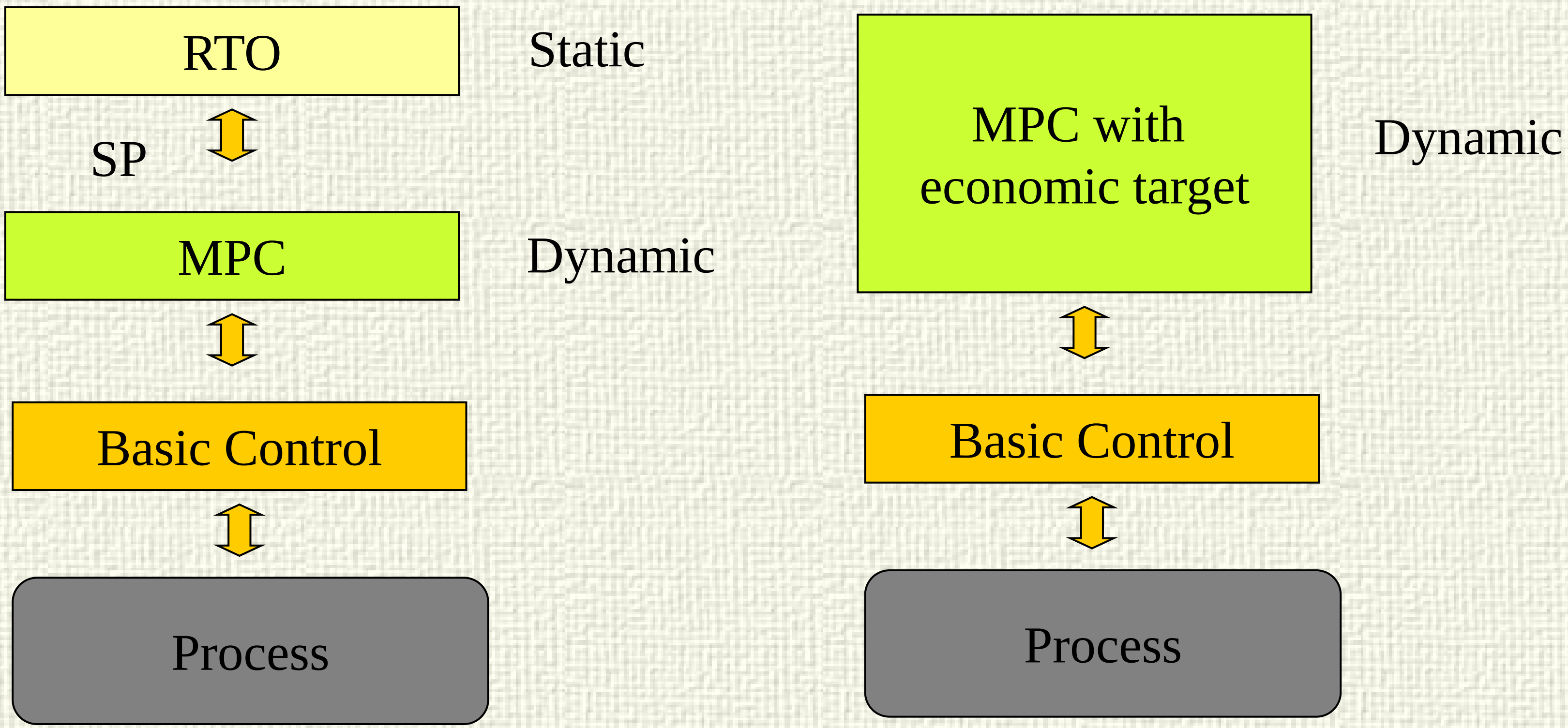
$$y(t) = g(x(t), u(t))$$

$$l_v \le v(t-i) \le L_V$$



Which initial state at time t-N and minimal disturbances vt-i) would drive the process in the closest way to the actual output trayectory if the control actions were the ones actually applied ?

RTO embedded



Example: Optimal transient

$$V\frac{dc_A}{dt} = q(c_{Ai} - c_A) - V\beta e^{-E/RT} c_A$$

$$V\rho c_p \frac{dT}{dt} = q\rho c_p (T_i - T) - Vkc_A H - UA(T - T_r)$$

$$V_r\rho_r c_{pr} \frac{dT_r}{dt} = F_r\rho_r c_{pr}(T_{ri} - T_r) + UA(T - T_r)$$

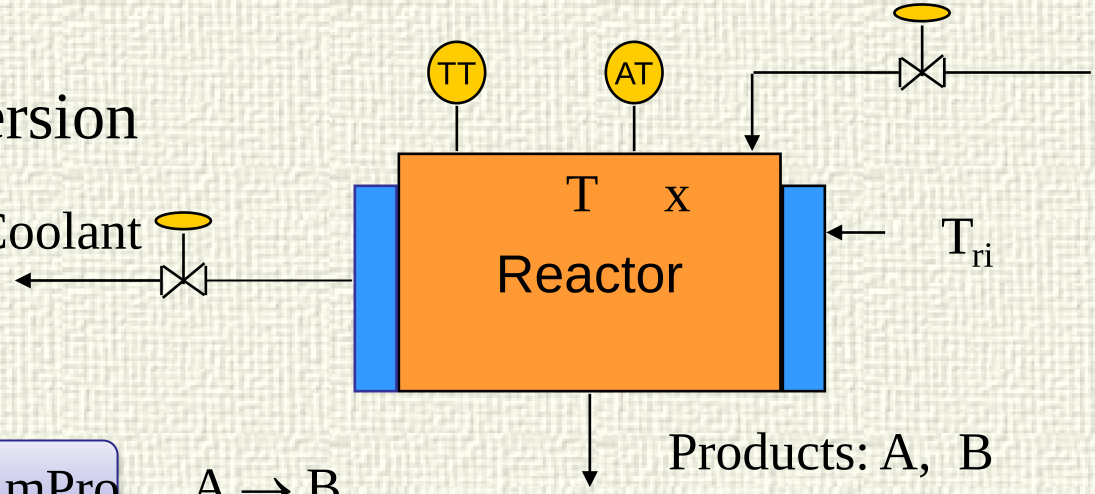
$$c_B = c_{Ai} - c_A$$

$$x = c_B / c_{Ai} \qquad x \text{ conversion}$$

Max $J = q\, c_B$

Row material: Product A

T_i, q, c_Ai



$$T_{min} \le T \le T_{max}$$

$$x_{min} \le x \le 1$$

$$q_{min} \le q \le q_{max}$$

$$F_{r\,min} \le F_r \le F_{r\,max}$$

EcosimPro

$A \rightarrow B$

Products: A, B

Process-Model gap

$$\min_{u} \quad J(u) = \int_0^T L(x,u)dt$$

$$F(\dot{x}, x, u) = 0$$

$$g(x,u) \le 0$$

MPC optimize the model responses

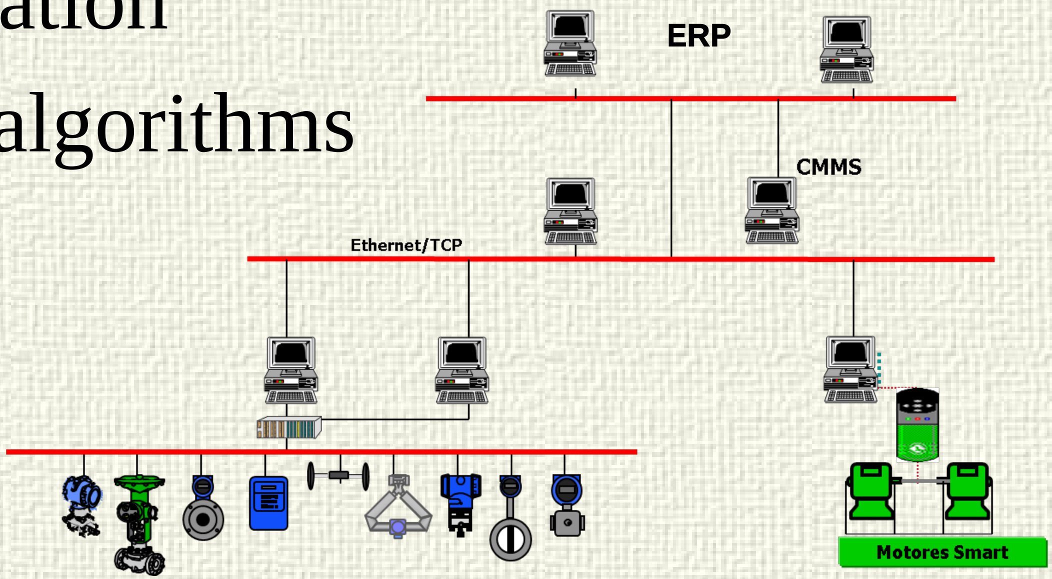
What happens if the model is not correct?

Solutions:

- ✓ **Update the model**
- ✓ **Modify the optimization problem acording to the process data**
- ✓ **Robust / Stochastic MPC**

MPC is possible because of recent technological changes

- ✓ Better hardware
- ✓ Communication networks and information
- ✓ Better Instrumentation
- ✓ Better numerical algorithms
- ✓ New theories
- ✓ Open systems
- ✓ …



NMPC

- ✓ Depends critically on the models and reliable optimization
- ✓ Mature theory on performance and stability
- ✓ Robustness: Key factor for industrial application. Most of the approaches are worst case, but this is conservative and not takes advantage of the plant measurements. Computation time.
- ✓ Many of the practical industrial problems do not fit into the academic continuous formulation.
- ✓ There are not many tools that facilitate the development, testing and implementation of NMPC
- ✓ Applications in the process industry are often unique and the development costs cannot be spread among many applications, unless software tools facilitate re-use

Thank you for your atention

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