Recent advances in PI/PID controller tuning

Ramon Vilanova
School of Engineering,
Universitat Autónoma de Barcelona, Barcelona

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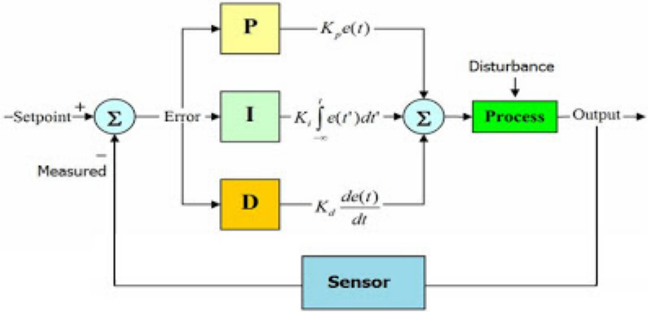
PI/PID controller, ..., very well known !!



The major concern is how to choose appropriate values for the three parameters.

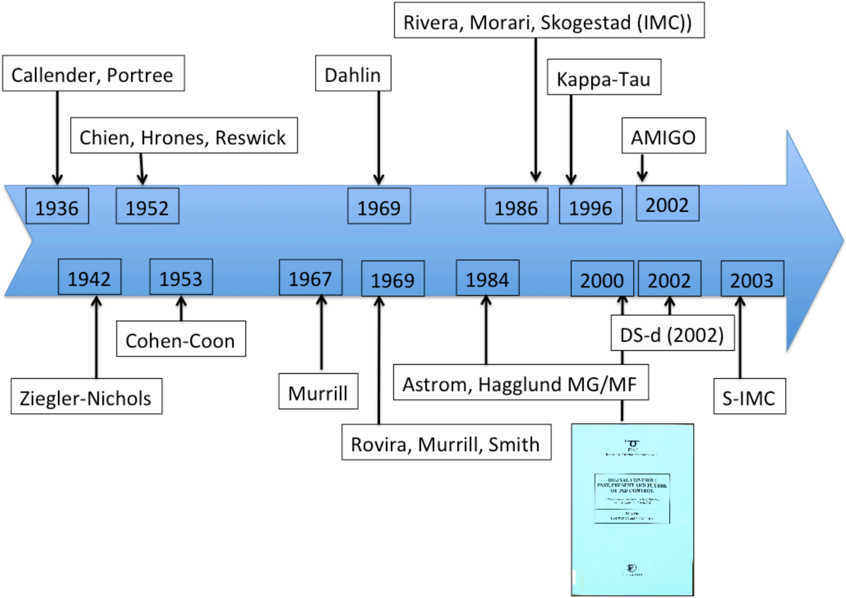
.... it seems an easy problem, but has attracted control engineers for more than 70 years.

In the last O'Dwyer, A. (†) compilation, a total of 1731 tuning rules are collected:

- On the basis of controller structure
  - PI (33%),
  - PID ideal (23%),
  - PID 'real' (14%),
  - 2DoF PI/PID (21%),
  - other... (9%)
- On the basis of process model
  - stable first order (37%),
  - stable second order (17%),
  - integral (19%),
  - unstable (11%),
  - other (6 %).

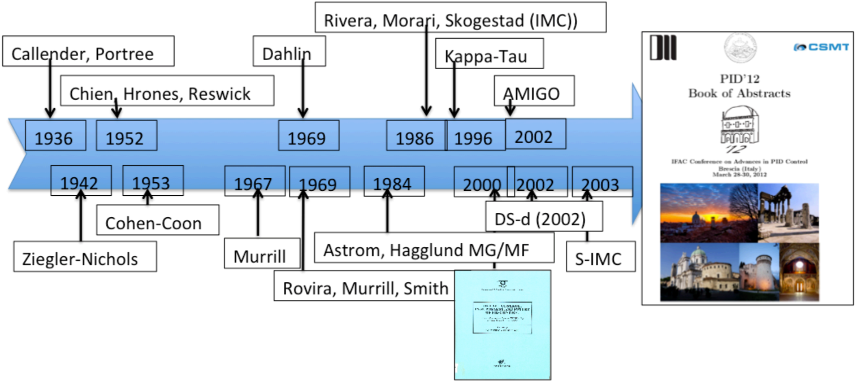
(†) O'Dwyer, A. (2009) - "*Handbook of PI and PID Controller Tuning Rules*", 3rd Edition, Imperial College Press, London, UK.

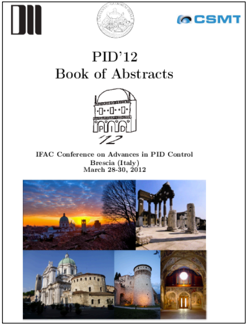
Tuning rules, some important milestones



- Past, Present and Future of PID Control, Terrassa, Spain, 2000
- $\approx$ 100 presentations
- State of the art on tuning / applications
- Some initial works on
  - Robustness
  - Fractional PID control
  - PID 2-DoF
  - Data-driven (iterative control)
  - Optimisation

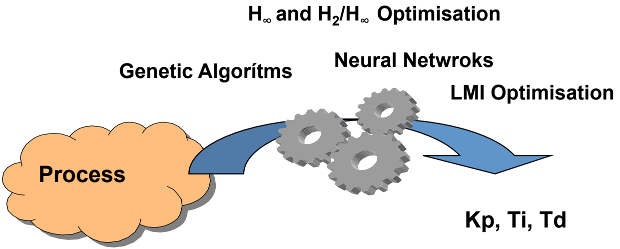
Tuning rules, some important milestones





- IFAC Conference on PID Control, Brescia, Italy, 2012
- $\approx$ 130 presentations
- State of the art on tuning / applications
- Some incipient approaches from PID00 consolidate as full sessions
  - Robustness
  - Tuning Rules
  - Fractonal PID COntrol
  - Data-driven
  - Optimisation
  - Multivariable
  - ...
- Not so much interest on approaches such as fuzzy, adaptive, self-tuning,...

Even there is the possibility to tackle the PI/PID controller design problem from practically any of the existing design approaches



What has been a characteristic feature of PI/PID controllers that has distinguished these controllers from the rest has been the formulation and generation of Tuning Rules.

When referring to PI/PID tuning it is unavoidable to mention the (most?) widely applied tuning choice

Trans. ASME, **64**, 759-768 (Nov. 1942).

Optimum Settings for Automatic Controllers

By J.G. ZIEGLER[1] and N. B. NICHOLS[2] • ROCHESTER, N. Y.

In this paper, the three principle control effects found in present controllers are examined and practical names and units of measurement are proposed for each effect.

varying its output air pressure, repositions a diaphragm-operated valve. The controller may be measuring temperature, pressure, level, or any other variable, but we will completely divorce the

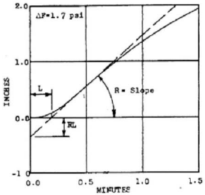


FIG. 8 REACTION CURVE

Reset-Rate Determination From Reaction Curve. Since the period of oscillation at the ultimate sensitivity proves to be 4 times the lag. A substitution of 4 L for P_u in previous equations for optimum reset rate gives an equation expressing this reset rate in terms of lag. For a controller with proportional and auto-matic-reset responses, the optimum settings become

$$Sensitivity = \frac{0.9}{R_1 L} \text{ psi per i}$$

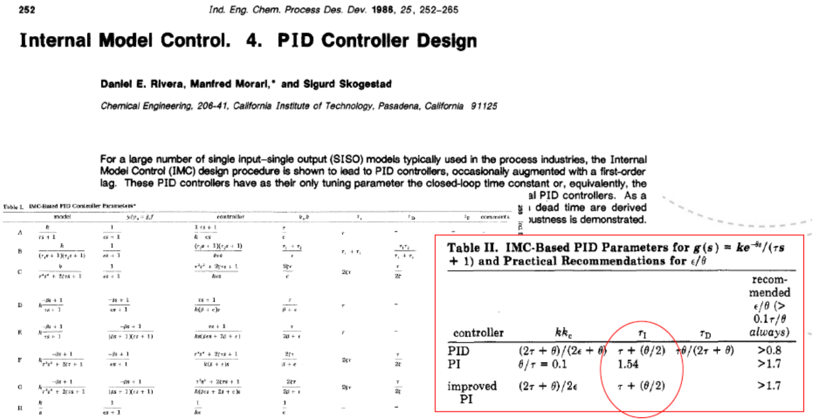
$$\text{Reset Rate} = \frac{0.3}{L} \text{ per min}$$

At these settings the period will be about creased, by both the lowering of sensitivity and the addition of automatic reset.

	K_p	T_i	T_d
P controller	$\frac{1}{LR/U}$	∞	0
PI controller	$\frac{0.9}{LR/U}$	$3.3L$	0
PID controller	$\frac{1.2}{LR/U}$	$2L$	$0.5L = \frac{T_i}{4}$

- Quite aggressive tuning
- Poor stability robustness
- Load disturbance attenuation was the main concern
- No tuning parameter (automatic tuning)

Following ZN, we could probably agree that the major success in PI/PID has been the one provided by the IMC formulation



252 *Ind. Eng. Chem. Process Des. Dev.* **1986**, *25*, 252–265

Internal Model Control. 4. PID Controller Design

Daniel E. Rivera, Manfred Morari,* and Sigurd Skogestad

Chemical Engineering, 206-41, California Institute of Technology, Pasadena, California 91125

For a large number of single input–single output (SISO) models typically used in the process industries, the Internal Model Control (IMC) design procedure is shown to lead to PID controllers, occasionally augmented with a first-order lag. These PID controllers have as their only tuning parameter the closed-loop time constant or, equivalently, the ...al PID controllers. As a ... dead time are derived ...oustness is demonstrated.

Table I. IMC-Based PID Controller Parameters*

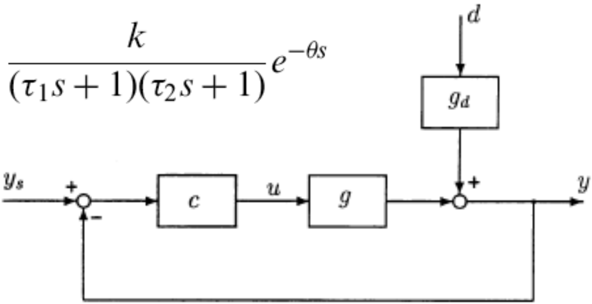
	model	$y/y_s = \tilde{g}_+f$	controller	$k_c k$	τ_I	τ_D	τ_F	comments
A	$\frac{k}{\tau s+1}$	$\frac{1}{\epsilon s+1}$	$\frac{1}{k}\frac{\tau s+1}{\epsilon s}$	$\frac{\tau}{\epsilon}$	τ	–	–	–
B	$\frac{k}{(\tau_1 s+1)(\tau_2 s+1)}$	$\frac{1}{\epsilon s+1}$	$\frac{(\tau_1 s+1)(\tau_2 s+1)}{k\epsilon s}$	$\frac{\tau_1+\tau_2}{\epsilon}$	$\tau_1+\tau_2$	$\frac{\tau_1\tau_2}{\tau_1+\tau_2}$		
C	$\frac{k}{\tau^2 s^2+2\zeta\tau s+1}$	$\frac{1}{\epsilon s+1}$	$\frac{\tau^2 s^2+2\zeta\tau s+1}{k\epsilon s}$	$\frac{2\zeta\tau}{\epsilon}$	$2\zeta\tau$	$\frac{\tau}{2\zeta}$		
D	$k\frac{-\beta s+1}{\tau s+1}$	$\frac{-\beta s+1}{\epsilon s+1}$	$\frac{\tau s+1}{k(\beta+\epsilon)s}$	$\frac{\tau}{\beta+\epsilon}$	τ	–		
E	$k\frac{-\beta s+1}{\tau s+1}$	$\frac{-\beta s+1}{(\beta s+1)(\epsilon s+1)}$	$\frac{\tau s+1}{ks(\beta\epsilon s+2\beta+\epsilon)}$	$\frac{\tau}{2\beta+\epsilon}$	τ	–		
F	$k\frac{-\beta s+1}{\tau^2 s^2+2\zeta\tau s+1}$	$\frac{-\beta s+1}{\epsilon s+1}$	$\frac{\tau^2 s^2+2\zeta\tau s+1}{k(\beta+\epsilon)s}$	$\frac{2\zeta\tau}{\beta+\epsilon}$	$2\zeta\tau$	$\frac{\tau}{2\zeta}$		
G	$k\frac{-\beta s+1}{\tau^2 s^2+2\zeta\tau s+1}$	$\frac{-\beta s+1}{(\beta s+1)(\epsilon s+1)}$	$\frac{\tau^2 s^2+2\zeta\tau s+1}{k(\beta\epsilon s+2\beta+\epsilon)s}$	$\frac{2\zeta\tau}{2\beta+\epsilon}$	$2\zeta\tau$	$\frac{\tau}{2\zeta}$		
H	$\frac{k}{s}$	$\frac{1}{\epsilon s+1}$	$\frac{1}{k\epsilon}$	$\frac{1}{\epsilon}$	–	–		

Table II. IMC-Based PID Parameters for $g(s) = ke^{-\theta s}/(\tau s + 1)$ and Practical Recommendations for ϵ/θ

controller	kk_c	τ_I	τ_D	recommended ϵ/θ (> $0.1\tau/\theta$ always)
PID	$(2\tau + \theta)/(2\epsilon + \theta)$	$\tau + (\theta/2)$	$\tau\theta/(2\tau + \theta)$	>0.8
PI	$\theta/\tau = 0.1$	1.54		>1.7
improved PI	$(2\tau + \theta)/2\epsilon$	$\tau + (\theta/2)$		>1.7

- Too many different tuning rules
- Poor disturbance rejection (set-point is the main concern)

$$g(s) = \frac{k}{(\tau_1 s + 1)(\tau_2 s + 1)} e^{-\theta s}$$



$$\frac{y}{y_s} = \frac{g(s)c(s)}{g(s)c(s) + 1}$$

$$c(s) = \frac{1}{g(s)} \frac{1}{\frac{1}{(y/y_s)_{\text{desired}}} - 1}$$

$$\left(\frac{y}{y_s}\right)_{\text{desired}} = \frac{1}{\tau_c s + 1} e^{-\theta s}$$

The controller finally gets the expression

$$c(s) = \frac{(\tau_1 s + 1)(\tau_2 s + 1)}{k} \frac{1}{(\tau_c + \theta)s}$$

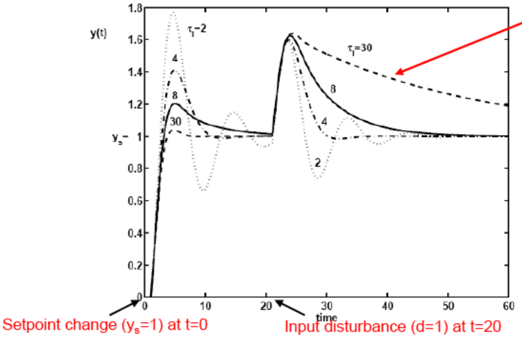
That corresponds to a series PID with parameters

$$Kc = \frac{1}{k} \frac{\tau_I}{(\tau_c + \theta)} \quad \tau_I = \tau_1 \quad \tau_D = \tau_2$$

or a PI if the initial process model is of first order ($\tau_2 = 0$)

$$Kc = \frac{1}{k} \frac{\tau_I}{(\tau_c + \theta)} \quad \tau_I = \tau_1$$

Effect of changing the integral time for PI control on slow process.

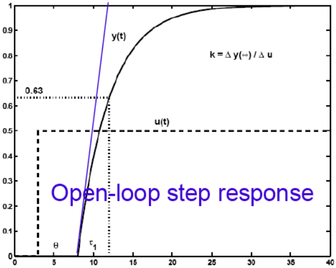


$\tau_I = \tau_1 = 30$

$$G(s) = k \frac{e^{-\theta s}}{\tau_1 s + 1}$$

Should find a compromise value. -> The S-IMC tuning

It is on 2003 that S. Skogestad proposes the S-IMC method. Specially good for PI control of FOPDT processes.



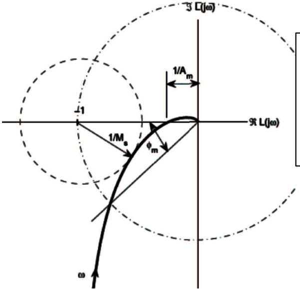
S-IMC Tuning rule

$$K_c = \frac{1}{k} \cdot \frac{\tau_1}{(\tau_c + \theta)}$$
$$\tau_I = \min(\tau_1, 4(\tau_c + \theta))$$

$$G(s) = k \frac{e^{-\theta s}}{\tau_1 s + 1}$$

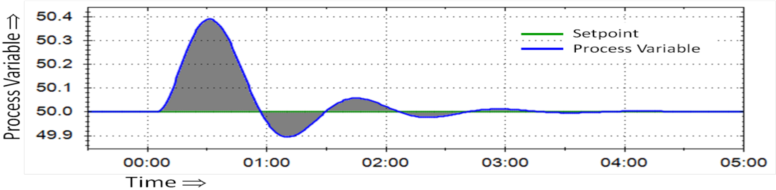
Default choice $\tau_c = \theta$. However

- S-IMC $\approx$ IMC for processes with small time τ_I ($\tau_I = \tau_1$)
- S-IMC $\approx$ Ziegler-Nichols for large τ_1 if $\tau_c = 0$ (aggressive tuning)



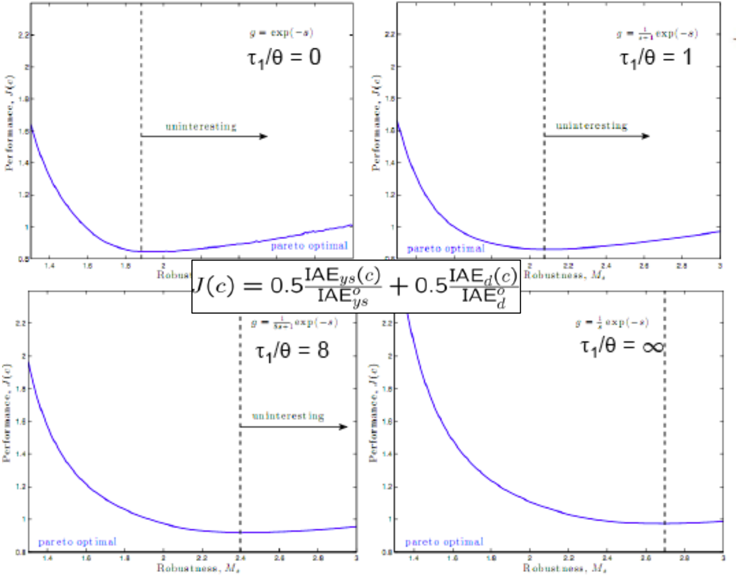
$$M_S \doteq \max_\omega |S(j\omega)| = \max_\omega \left| \frac{1}{1 + C(j\omega)P(j\omega)} \right|,$$
$$= \frac{1}{\min_\omega |1 + L(j\omega)|},$$

$$MG \geq \frac{M_S}{M_S - 1} \quad MF \geq 2\sin^{-1}\left(\frac{1}{M_S}\right)$$

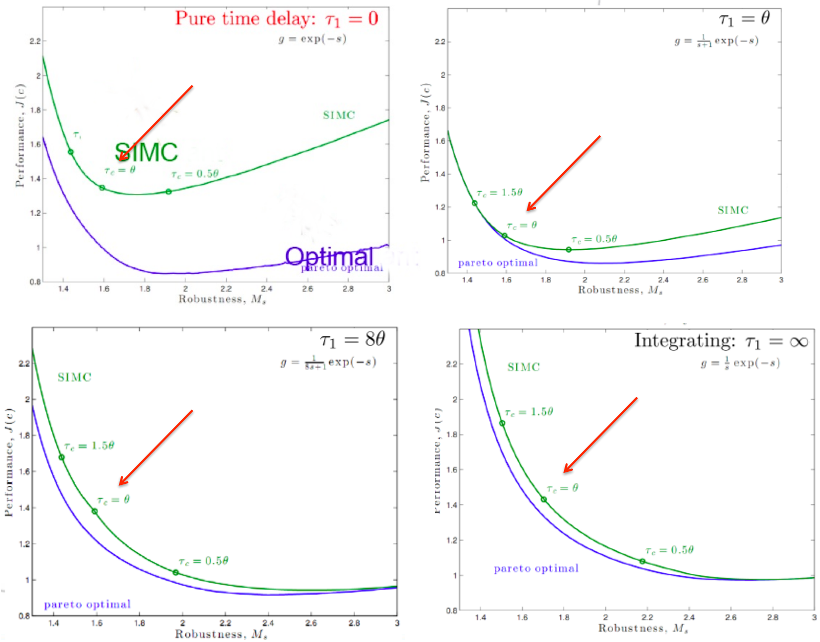


IAE = Integrated absolute error $= \int |y - y_s| dt$, for step change in y_s or d

Pareto-Optimal *tradeoff* for Robustness and Performance



$$J(c) = 0.5\frac{\mathrm{IAE}_{ys}(c)}{\mathrm{IAE}^o_{ys}} + 0.5\frac{\mathrm{IAE}_{d}(c)}{\mathrm{IAE}^o_{d}}$$


Pure time delay: $\tau_1 = 0$
$g = \exp(-s)$
SIMC
SIMC
$\tau_c = \theta$
$\tau_c = 0.5\theta$
Optimal
pareto optimal
Performance, $J(c)$
Robustness, M_s
$\tau_1 = \theta$
$g = \frac{1}{s+1}\exp(-s)$
$\tau_c = 1.5\theta$
$\tau_c = \theta$
$\tau_c = 0.5\theta$
SIMC
pareto optimal
$\tau_1 = 8\theta$
$g = \frac{1}{8s+1}\exp(-s)$
SIMC
$\tau_c = 1.5\theta$
$\tau_c = \theta$
$\tau_c = 0.5\theta$
pareto optimal
Integrating: $\tau_1 = \infty$
$g = \frac{1}{s}\exp(-s)$
SIMC
$\tau_c = 1.5\theta$
$\tau_c = \theta$
$\tau_c = 0.5\theta$
pareto optimal

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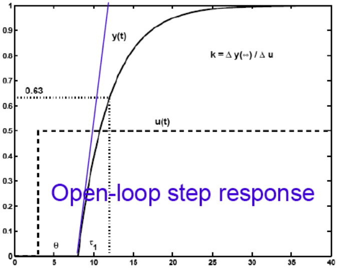
Specially good for PI control of FOPDT processes.



S-IMC Tuning rule

$$K_c = \frac{1}{k} \cdot \frac{\tau_1}{(\tau_c + \theta)}$$

$$\tau_I = \min(\tau_1, 4(\tau_c + \theta))$$

$$G(s) = k \frac{e^{-\theta s}}{\tau_1 s + 1}$$

Simple and "good enough"

- Varying τ_c gives (almost) Pareto-optimal tradeoff between performance (IAE) and robustness (Ms)
- $\tau_c = \theta$ is a good "default" choice
- Not possible to do much better with any other PI controller!!

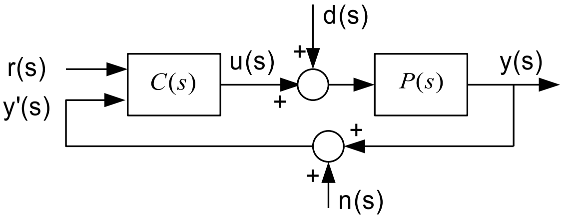
With S-IMC a change in the way tuning is though was introduced. There has been a revival of PID but with an increasing requirements setup:

- Servo/Regulation Performance
- Moderate usage of control action
- Robustness
- Simple and clear formulations
- Include derivative term
- Measurement noise attenuation
- *tradeoff* (smoothness/robustness/reactivity)

Global setup

There arises the need to rethink how to conjugate all these considerations, possible interactions among them and to express this in a simple way.

From a standard control loop block diagram



What we should do is to look after the performance regarding the three external signals (r, d y n) as well as guaranteeing a minimum level of robustness to process variations.

It is a great advantage if this compromise can be decided by the user by a tuning parameter. (†)

(†) K.J. Astrom, T.Hagglund, The future of PID control, Control Engineering Practice, 9 (2001) 1163-1175

It is easy to find design formulations giving primary importance to one of such aspects, therefore driving the design (tuning) approach:

- Robustness:
  - Specific indexes: MG, MF, Parametric stability, ...
  - Aggregated indexes: M_S, M_T, $\|\cdot\|_\infty$, ...

$$MG \geq \frac{M_S}{M_S - 1} \quad MF \geq 2\sin^{-1}\left(\frac{1}{M_S}\right)$$

- Performance:
  - Optimizing some index of performance (ISE, IAE, ITAE,...)

$$I = \int_0^\infty t^n |e(t)|^m dt$$

  - Parametrisation of the closed-loop (IMC, λ-tuning, etc)

$$T(s) \longrightarrow \frac{1}{\lambda s + 1}$$

- Do they really reflect the desired closed-loop?
- Does it make sense to focus the design in just one of them?

As an example, we take a tuning proposal that considers exclusively to minimize performance on the IAE sense :

- First order process models: $P(s) = \frac{K_p e^{-\theta s}}{\tau_1 s+1}$

Proposed tuning relations for FOPTD process

Tuning parameter	Set point change	Load change
K_C	$\frac{0.4967}{K_P}\left(\frac{\theta}{\theta+\tau_1}\right)^{-1.2299}$	$\frac{0.5249}{K_P}\left(\frac{\theta}{\theta+\tau_1}\right)^{-1.2787}$
τ_I	$0.67390\left(\frac{\theta}{\theta+\tau_1}\right)^{-1.317}$	$\theta\left(-1.9167\left(\frac{\theta}{\theta+\tau_1}\right)+2.1356\right)$
τ_D	$\tau_1\left(1.138\left(\frac{\theta}{\theta+\tau_1}\right)^2+0.1992\left(\frac{\theta}{\theta+\tau_1}\right)\right)$	$\tau_1\left(1.1321\left(\frac{\theta}{\theta+\tau_1}\right)^2+0.1788\left(\frac{\theta}{\theta+\tau_1}\right)\right)$

- Second order process models: $P(s) = \frac{K_p e^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)}$

Proposed tuning relations for SOPTD process

Tuning parameter	Set point change	Load change
K_C	$\frac{0.5723}{K_P}\left(\frac{\theta}{\theta+\tau_1+\tau_2}\right)^{-1.0409}$	$\frac{0.6202}{K_P}\left(\frac{\theta}{\theta+\tau_1+\tau_2}\right)^{-0.9931}$
$\frac{\tau_I\tau_1}{\theta}$	$0.2476(\theta+\tau_1+\tau_2)\left(\frac{\theta}{\theta+\tau_1+\tau_2}\right)^{-1.6501}$	$(\theta+\tau_1)\left(13.81\left(\frac{\theta}{\theta+\tau_1+\tau_2}\right)^2-14.906\left(\frac{\theta}{\theta+\tau_1+\tau_2}\right)+4.566\right)$
$\frac{\tau_D\tau_1}{(\tau_2+\theta)\theta}$	$0.0943\left(\frac{\theta}{\theta+\tau_1+\tau_2}\right)^{-1.4636}$	$0.0921\left(\frac{\theta}{\theta+\tau_1+\tau_2}\right)^{-1.4849}$

(†) C. R. Madhuranthakam, A. Elkamel, and H. Budman. "*Optimal tuning of PID controllers for FOPDT, SOPDT and SOPDT with lead processes.* Chemical Engineering and Processing, 47:251-264, 2008

For a comparative basis, the *uSORT* tuning provides simultaneously tuning for PI/PID on the basis of FOPTD/SOPTD process models, also ensuring certain levels of robustness and minimizing IAE criteria.

Process: $P(s) = \frac{K\mathrm{e}^{-Ls}}{(Ts+1)(aTs+1)} \;\; a \in (0,1) \;\; \tau_L = L/T$

Controller: $u(s) = K_p\left(\beta + \frac{1}{T_i s}\right) r(s) - K_p\left(1 + \frac{1}{T_i s} + \frac{T_d s}{0.1 T_d s + 1}\right) y(s)$

Performance: $J_e \doteq \int_0^\infty |e(t)|\,\mathrm{d}t = \int_0^\infty |y(t) - r(t)|\,\mathrm{d}t$

Performance degradation: $F_p \doteq \frac{J_e^o}{J_e}, \;\; F_p \leq 1$

$$J_{F_p} \doteq J(\theta_c, F_p^t) = \left|\frac{J_e^o}{J_e(\theta_c)} - F_p^t\right| \quad t.q. \;\; M_S = M_S^t \in \{2, 1.8, 1.6, 1.4\}$$

What is provided is a unique set of equations for the different combinations of process model and controller.

Set-Point

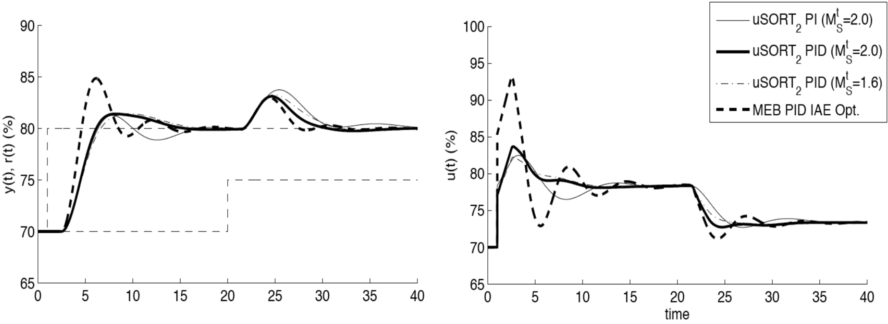
$$\kappa_p \doteq K_p K = a_0 + a_1 \tau_L^{a_2},$$
$$\tau_i \doteq \frac{T_i}{T} = \frac{b_0 + b_1 \tau_L + b_2 \tau_L^2}{b_3 + \tau_L},$$
$$\tau_d \doteq \frac{T_d}{T} = c_0 + c_1 \tau_L^{c_2}.$$

Regulation

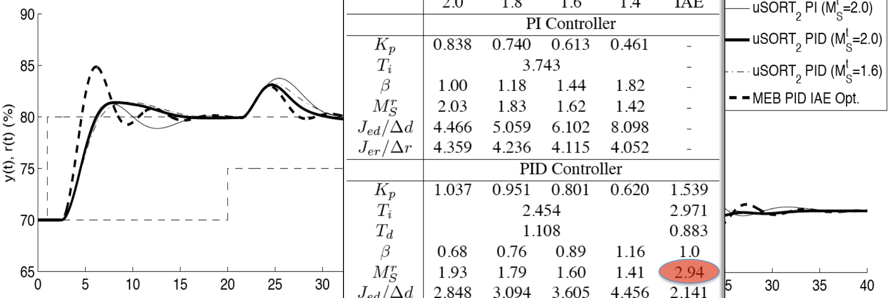
$$\kappa_p \doteq K_p K = a_0 + a_1 \tau_L^{a_2},$$
$$\tau_i \doteq \frac{T_i}{T} = b_0 + b_1 \tau_L^{b_2},$$
$$\tau_d \doteq \frac{T_d}{T} = c_0 + c_1 \tau_L^{c_2}.$$

- Tuning relations for PI/PID are the same
- Constants a_i, b_i, c_i are determined by the process model parameters.
- Robustness just imposes changes on the proportional gain(a_i therefore determined by the desired robustness level).
- For the regulator case, they are also available equations for β.

If we compare performance in terms of time responses of *uSORT* tuning and the optimal *MEB*



If we compare performance in terms of time responses of *uSORT* tuning and the optimal *MEB*

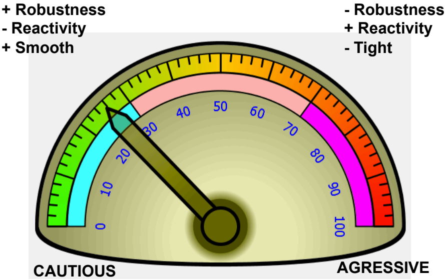


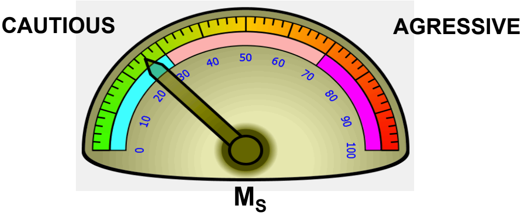
	$uSORT_2$ M_S^d 2.0	1.8	1.6	1.4	MEB IAE
	PI Controller				
K_p	0.838	0.740	0.613	0.461	-
T_i	3.743				-
β	1.00	1.18	1.44	1.82	-
M_S^r	2.03	1.83	1.62	1.42	-
$J_{ed}/\Delta d$	4.466	5.059	6.102	8.098	-
$J_{er}/\Delta r$	4.359	4.236	4.115	4.052	-
	PID Controller				
K_p	1.037	0.951	0.801	0.620	1.539
T_i	2.454				2.971
T_d	1.108				0.883
β	0.68	0.76	0.89	1.16	1.0
M_S^r	1.93	1.79	1.60	1.41	2.94
$J_{ed}/\Delta d$	2.848	3.094	3.605	4.456	2.141
$J_{er}/\Delta r$	4.325	4.396	4.534	4.702	4.289

It is easy to find design formulations giving primary importance to one of such aspects, therefore driving the design (tuning) approach:

- Robustness:
  - Specific indexes: MG, MF, Parametric stability, ...
  - Aggregated indexes: M_S, M_T, $\|\cdot\|_\infty$, ...
- Performance:
  - Optimising some index of performance (ISE, IAE, ITAE,...)
  - Parametrisation of the closed-loop (IMC, λ-tuning, etc)

- They are not independent aspects.
- Tuning strategy should combine them on the most appropriate way
- Attention should be focused on methods that do integrate them on a more qualitative way.





$$p_i(\tau, M_s^d) = a_i(M_s^d)\tau^{b_i(M_s^d)} + c_i(M_s^d)$$

$$\begin{aligned} K_p K &= a_1 \tau^{b_1} + c_1 \\ \frac{T_i}{T} &= a_2 \tau^{b_2} + c_2 \\ \frac{T_d}{T} &= a_3 \tau^{b_3} + c_3 \end{aligned}$$

Just a short consideration regarding the PID implementation in use:

- Do we have freedom to choose the particular implementation for the PID or are we constrained by the vendor/deployed algorithm?
- If we use a PI there is no problem, all PI are identical. But this is not the same for a PID
- A tuning rule may be developed by assuming a particular formulation for a PID that may be different from the one implemented.

PID configuration

What are the implications of using a particular PID implementation?

PID 2DoF Standard

$$u(s) = K_p \left(\beta + \frac{1}{T_i s} \right) r(s) + K_p \left(1 + \frac{1}{T_i s} + \frac{T_d s}{\alpha T_d s + 1} \right) y(s)$$

PID 2DoF Paralel

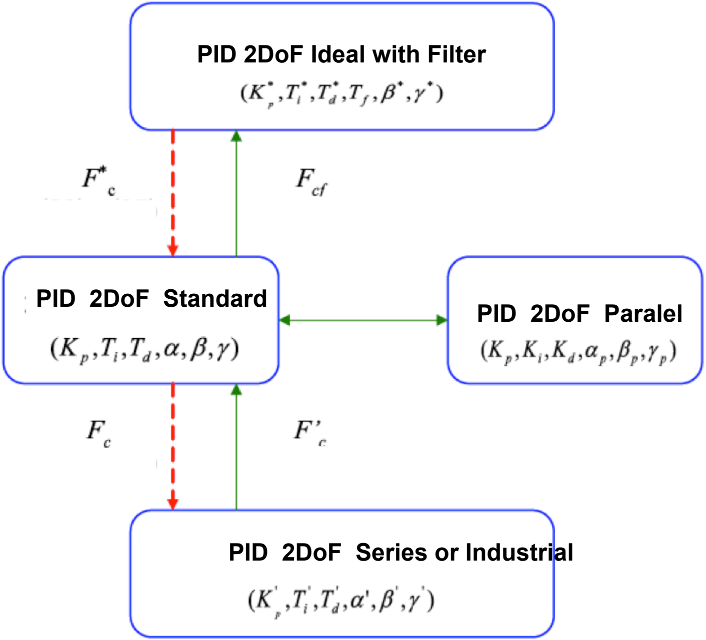
$$u(s) = \left(\beta_p K_p + \frac{K_i}{s} \right) r(s) - \left(K_p + \frac{K_i}{s} + \frac{K_d s}{\alpha_p K_d s + 1} \right) y(s)$$

PID 2DoF Series (or Industrial PID)

$$u(s) = K_p' \left(\beta' + \frac{1}{T_i' s} \right) r(s) - K_p' \left(1 + \frac{1}{T_i' s} \right) \left(\frac{T_d' s + 1}{\alpha' T_d' s + 1} \right) y(s)$$

PID 2Dof Ideal with filter

$$u(s) = K_p^* \left(\beta^* + \frac{1}{T_i^* s} \right) r(s) - K_p^* \left(1 + \frac{1}{T_i^* s} + T_d^* s \right) \left(\frac{1}{T_f s + 1} \right) y(s)$$


PID 2DoF Ideal with Filter
$(K^*_p, T^*_i, T^*_d, T_f, \beta^*, \gamma^*)$
F^*_c
F_{cf}
PID 2DoF Standard
$(K_p, T_i, T_d, \alpha, \beta, \gamma)$
PID 2DoF Paralel
$(K_p, K_i, K_d, \alpha_p, \beta_p, \gamma_p)$
F_c
F'_c
PID 2DoF Series or Industrial
$(K'_p, T'_i, T'_d, \alpha', \beta', \gamma')$

- This is just a snapshot of a new family of tuning rules that address the problem in a more qualitative and global way.
- New approaches attempt to extend the presented ideas by considering the use of more elaborated models (therefore opening the door to better performance)
- One of the latest approaches integrates signal filtering (r and y) as an integral part of te PID tuning. Therefore helping the PID and getting superior performance (measurement noise !!).
- All tuning methods provide an initial tuning that usually has to be manually adjusted: fine tuning: the fragility issue enters into play

Think of Advanced Control also as Advanced PID Control

Help the PID with appropriate filtering and suitable, analytical, model based, tuning.

Recent advances in PI/PID controller tuning

Ramon Vilanova
School of Engineering,
Universitat Autónoma de Barcelona, Barcelona

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