

II. Symmetries

A. Space-time (flat but relativistic)

* Minkowski (ct, \vec{x})

metric : $\eta_{\mu\nu} (+ - - -)$

$$ds^2 = c^2 dt^2 - d\vec{x} \cdot d\vec{x}$$

* Symmetry operations

$$x_\mu \rightarrow x'_\mu = x_\mu + \delta x_\mu$$

$$\delta x_\mu = \Lambda_\mu^\nu + \Lambda_{\mu\nu} x^\nu$$

* generators $P_\mu, \Lambda_{\mu\nu}$ satisfy Poincaré Alg

$$[P_\mu, P_\nu] = 0 ,$$

$$[\Lambda_{\mu\rho}, P_\nu] = i(\delta_{\mu\rho} P_\nu - P_\rho \delta_{\mu\nu}) ,$$

$$[\Lambda_{\mu\rho}, \Lambda_{\nu\lambda}] = i(\delta_{\mu\rho} \eta_{\nu\lambda} + \delta_{\mu\lambda} \eta_{\nu\rho} - \delta_{\mu\lambda} \eta_{\nu\rho} - \delta_{\mu\nu} \eta_{\rho\lambda}) .$$

* Wave functions transform

$$\phi(x) \rightarrow \phi(x') = U(a, \lambda) \phi(x)$$

$$U(a, \lambda) = \exp \{ i a \cdot P + i \lambda \cdot J \} \in \text{Poincaré group}$$

* Casimir Invariants

$$P^2 = m^2$$

$$W^2 = w_\mu w^\mu$$

$$w_\mu = \frac{1}{\lambda} g_{\mu\nu\rho} P^\nu \sqrt{-g}$$

\rightarrow In rest frame

$$P_\mu = (m, D); w_\mu = (0, m \vec{s})$$

↑
spin!

status : mass, spin

K.G. - m, spin D

Dirac - m, spin $\frac{1}{2}$

photon - D, spin L

Proca - m, spin L

Rarita-Schwinger - m, spin $\frac{3}{2}$

graviton - D, spin 2

* Dynamics defined by Action

$$S = \int d^4x \mathcal{L}$$

$$\delta S = 0, \text{ inv.}$$

$$\delta \mathcal{L} = \partial_\mu \delta x_\mu \quad (\text{total derivative})$$

\Rightarrow conserved currents, conservation laws

Ex. under translation, energy-momentum
tensor $\Theta_{\mu\nu}$

$$\Theta_{\mu\nu} = g_{\mu\nu} I - \frac{\delta \mathcal{L}}{\delta (\partial_\mu q_\nu)} \partial_\nu q_\mu$$

$$\text{conserved energy} = \int d^3x \Theta_{00}$$

$$\text{conserved momentum} = \int d^3x \Theta_{0i}$$

\Rightarrow Noether principle

Symmetries \Rightarrow invariance \Rightarrow conservation law.

B. Global Internal Symmetries

* Examples

- phase change $U(1)$; baryon number, lepton number
- isospin $SU(2)$: $\begin{pmatrix} p \\ n \end{pmatrix}$; $\Pi = \begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix}$
- Eight-fold Way $SU(3)$: octet & decuplet

* To illustrate

$$L = \bar{\psi}(i\cancel{D} - m)\psi$$

$$\psi \rightarrow \psi' = e^{-i\frac{q}{2}\Lambda} \psi$$

$$\cancel{D}\psi = 0 !!$$

$$j_\mu = \bar{\psi} \gamma_\mu \psi$$

$$Q = \int d^3x \bar{\psi} \gamma^\mu \psi$$

C. Local Internal Symmetries

* If we make $\Lambda = \Lambda(x)$ (local!)

$$j_\mu^A \rightarrow j_\mu^A' = e^{-ig\Lambda(x)} [j_\mu - e g \partial_\mu \Lambda] j_\mu^A$$

will lose inv.

→ to regain sym., must introduce
 A_μ via principle of minimal
 substitution

$$P_{\mu\nu} \rightarrow P_{\mu\nu} - g A_\mu ,$$

$$\underset{F}{A} \rightarrow \underset{F}{A}' = \underset{F}{A} + \underset{F}{g} A$$

$$= U^\dagger \underset{F}{A} U + i \frac{g}{q} U^\dagger \underset{F}{g} U$$

gauge transf! Paulian in Maxwell-
 Faraday

$$\mathcal{L} = \bar{\psi} [i\gamma^\mu - m] \psi - \frac{1}{4} F^2 \quad \left. \right\} \text{QED!}$$

$$A^\mu = \delta_\mu^\nu (j_\nu - g A_\nu)$$

$$F^2 = F_{\mu\nu} F^{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu)^2$$

Means: Forces !!

* For isospin

$$\psi = \begin{pmatrix} \psi_p \\ \psi_n \end{pmatrix}, \text{ each } \psi_p, \psi_n \text{ Dirac spinor}$$

Global sym.

$$\psi \rightarrow \psi' = e^{ig \Lambda^a T^a} \psi \quad ; [T^a, T^b] = i \epsilon^{abc} T^c$$

$$\mathcal{L} = \bar{\psi} (i \not{D} - m) \psi$$

mass matrix \uparrow
 $i \epsilon^{abc} T^c$
 SU(2) algebra

If we make $\Lambda^a = \Lambda^a(x)$, must add

$$A_\mu^a = T^a A_\mu^a,$$

but

$$\rightarrow A_\mu^a \rightarrow A'_\mu^a = \omega A_\mu^a \omega^{-1} + \frac{i}{g} \partial^\nu \omega_\mu^\nu \omega$$

$F_{\mu\nu}^2$ invariant but $F_{\mu\nu}$ cov.

$$F_{\mu\nu} \rightarrow \omega F_{\mu\nu} \omega^{-1}$$

$$F_{\mu\nu}^a = \gamma_\mu^a A_\nu^a - \gamma_\nu^a A_\mu^a + g \epsilon^{abc} A_\mu^b A_\nu^c$$

1954 Yang-Mills
Saw

non-linear, gauge-
field self-interacting

D. Discrete Symmetries

* Charge conjugation C, Parity reversal P and time-reversal T

- charge conjugation

$$U_C^{-1} Q U_C = -Q$$

$$U_C |4\rangle = \gamma_C |4_C\rangle ; \gamma_C = \pm 1$$

$$U_C^2 = U_C^\dagger U_C = 1 ; U_C \text{ unitary?}$$

- Parity $\vec{x} \rightarrow \vec{x}' = -\vec{x}$

$$U_P^{-1} \vec{x} U_P = -\vec{x} ; U_P^{-1} x_0 U_P = x_0$$

$$U_P |4\rangle = \gamma_P |4_P\rangle ; U_P^\dagger U_P = 1$$

$$\gamma_P^2 = 1 ; \gamma_P = \pm 1$$

- Time-reversal $t \rightarrow -t$

$$J^+ x_0 J = -x_0 ; J^+ \vec{x} J = \vec{x}$$

$$J = U_T K \quad [x, p] = i\hbar$$

unitary \downarrow \uparrow complex conj.

* Multiplicatively conserved!

* CPT theorem: Lorentz inv. theory \Rightarrow CPT is conserved!

III Quick Tour to the SM

- * Elec. particle physics - relativity, quantum mechanical

$$\frac{v}{c} \sim 1 \quad \text{relativistic}$$

$$J/t_1 \sim 1 \quad \text{& M.}$$

$$\Rightarrow \text{time for crit.} \sim T_{\text{annihilation}} = \hbar/m_c^2$$

very fast!

$$\sim 10^{-20} \quad \text{for electron}$$

$$\sim 10^{-23} \quad \text{for proton}$$

- * U(1) part of SM worked out in 1950

- T, S, F + D

QED

- * Weak interaction & strong interaction posed more challenge.

A. Weak Interaction (rel. long lifetime compared to strong)

- * Beta decay

$$n \rightarrow p e^- (\bar{\nu}_e)$$

↑ not seen right away

- some willing to consider energy & angular momentum non-conservation!

- Pauli and neutrino concept
- Fermi's Universal interaction

$$H = \sum_i (\bar{\psi}_u \theta_i \psi_p) (\bar{\psi}_e \theta_i \psi_\nu)$$

$$\theta_i = \{I, \gamma_5, \gamma_\mu, \gamma_\mu \gamma_5, \gamma_{\mu\nu}\}$$

- Parity Violation in Weak Interaction - θ -S puzzle

Lee & Yang suggested parity violation and how to look!

- Marshak + Sudarshan, Feynman + Gell-Mann

V-A !

$$H = G_F [\bar{\psi}_u \gamma_\mu (1-\gamma_5) \psi_p] [\bar{\psi}_e \gamma_\mu (1-\gamma_5) \psi_\nu]$$

* Other weak processes

- purely leptonic

$$\mu^+ \rightarrow e^+ \bar{\nu}_e \nu_\mu$$

- non-leptonic

$$\Delta \rightarrow p \pi^-$$

$$K \begin{cases} \pi^- \\ 3\pi^- \end{cases} \quad \left\{ \text{#-S puzzle} \right.$$

- other semi-leptonic

$$K^+ \rightarrow \mu^+ \nu_\mu$$

$$\pi^+ \rightarrow \mu^+ \nu_\mu$$

- strangeness changing

K^+ decay suppressed rel. to π^+ decay
led Cabibbo to propose the mixing
angle named after him. Easier
to see w/ quark hypothesis

* Taken together, weak interaction
via current-current

$$H \sim G_F J_\mu^+ J_\mu^-$$

only charged
currents!

$$J_\mu^L = \bar{\psi}_e \gamma_\mu (1 - \gamma_5) \psi_e + \bar{\psi}_\mu \gamma_\mu (1 - \gamma_5) \psi_\mu$$

$$J_\mu^L = \sin \theta_w J_\mu^+ (ds=1) + \cos \theta_w J_\mu^+ (ds=0)$$

Furthermore

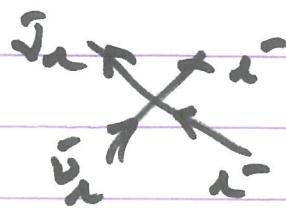
$$J_\mu^L \sim \bar{\psi} \gamma_\mu (1 - 1.15 \gamma_5) \psi$$

strong interaction
breaks w/
weak currents!

* Problems w/ V-A

(1) Unitarity

$$\bar{\nu}_e e^- \rightarrow \bar{\nu}_e e^-$$



$$\sigma = \frac{G_F^2 s}{\pi}; \text{ but } S\text{-wave } \sigma = \frac{16\pi}{s}$$

$$s = E_{CM}^2; \text{ Failure } \frac{G_F^2 s}{\pi} \gtrsim \frac{16\pi}{s}$$

$$\Rightarrow E_{CM} > \sqrt{G_F} \sim \text{few hundred GeV.}$$

(2) Not renormalizable

$$G_F \sim \left(\frac{1}{\text{mass}}\right)^2$$

$$(24)^2 \text{ dim 6}$$

* Alleviate the 1st via ~~$\chi + \bar{\chi}$~~

$$\chi = \begin{array}{c} \bar{\nu}_e \\ \bar{\nu}_e \end{array} \xrightarrow{W^+} \begin{array}{c} \nu_e \\ e^- \end{array}$$

No longer S-wave

But W^+ massive (short-ranged!)

massive force not renormalizable
because no gauge symmetry!

\Rightarrow Weak interaction via massive I \bar{V} B
still problematical!

B. Strong Interaction

- * Yukawa's suggestion of meson exchange to bind protons + neutrons inside the nucleus \Rightarrow pion-nucleon physics

$$L_y \sim g^2 (\bar{\pi} \pi) \psi \cdot \psi = \binom{?}{n}$$

$\bar{\pi} = (\pi^+, \pi^0, \pi^-)$

$$\pi^\pm = \frac{\pi^+ \pm i\pi^0}{\sqrt{2}}$$

- * Nuclear Shell model, central potential

$$V = \frac{-V_0}{1 + \exp\left(\frac{(r-R)}{g}\right)}$$

(Woods-Saxon)

- I do not think this was ever derived from pion-exchange

$$V \sim \frac{-mr}{r}$$

- * Decay of $\Lambda \rightarrow N\bar{\pi}$,

$$\frac{g^2}{4\pi} \sim 15 !!$$

contrast QED

$$\frac{e^2}{4\pi} = \frac{1}{137}$$

perturbation expansion does not make sense

(12)

* Dynamics cannot be solved,
shift to classifying observed
particles.

- Search for rank 2 group.
Successful $SU(3)_c$ of

Gellman + Neuman

(slides off octets!?)

5^1 and decuplet! Success!

- Quark hypothesis and the fundamental representation (Gellman + Zweig)
 - most believed as bookkeeping device (see slide?)
(Gellman)
 - few took it very seriously -
(Zweig)

* Two experiments that "established" reality of quarks

(1) Deep inelastic ep scattering

(2) J/ψ observation \Rightarrow charm!

\Rightarrow quarks as real fundamental particles! Dynamics!

* Flavor is a global symmetry,
No dynamics!

→ must explain quark-parton
observations - free particles
deep inside hadrons

→ No free quarks outside
hadrons

asymptotic freedom and infrared slavery

* Evidences for color!

(1) anti-symmetry of baryon wave func.

Δ^{++} = charge 2, spin $\frac{3}{2}$ fermion
= $u^+ u^+ u^+$ in S config. w/ 3 Δ

syn. w/ flavor, spin, L

∴ for anti-symmetry, new quantum
number $C = \text{color! anti-sym.}$

$$(2) R = \frac{\tau(e^+ e^- \rightarrow \text{hadrons})}{\tau(\pi^+ \pi^- \rightarrow \pi^+ \pi^-)}$$

$$= \sum_i (\theta_i / \epsilon)^2$$

to account for exp'l data

(see slide), 3 colors for each flavor

$$(3) \pi^0 \rightarrow 2\gamma$$



quark (u or d)

$$\Gamma(\pi^0 \rightarrow 2\gamma) = 77 \text{ ev.} \quad \text{exptl.}$$

w/o anomaly, limit of massless quarks, $\Gamma = 0$

$$T_n j_{5p}^3 = F_\pi M_\pi^2 \phi_\pi + \frac{\epsilon}{\pi} \vec{E} \cdot \vec{B}$$

Theory agrees w/ expt. if 3 colors!

\therefore 3 colors for each flavor

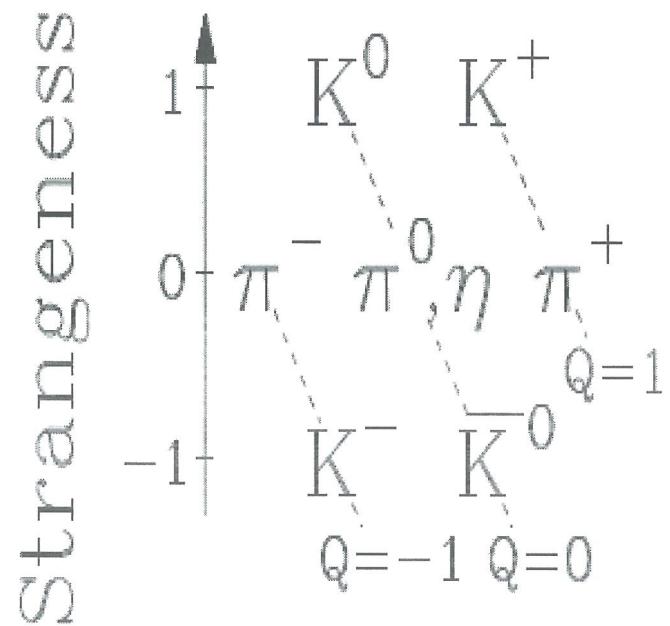
* Asymptotic freedom in 1973,
free quarks well inside proton)

\Rightarrow gauge color! local symmetry
universally

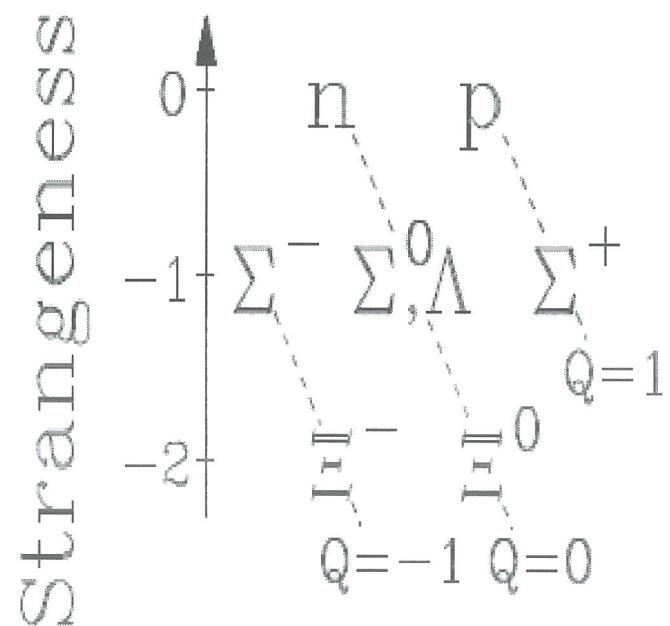
But we accepted proof of
confinement!!

From Rosner: Macmillan Encyclopedia of Physics

Meson octet

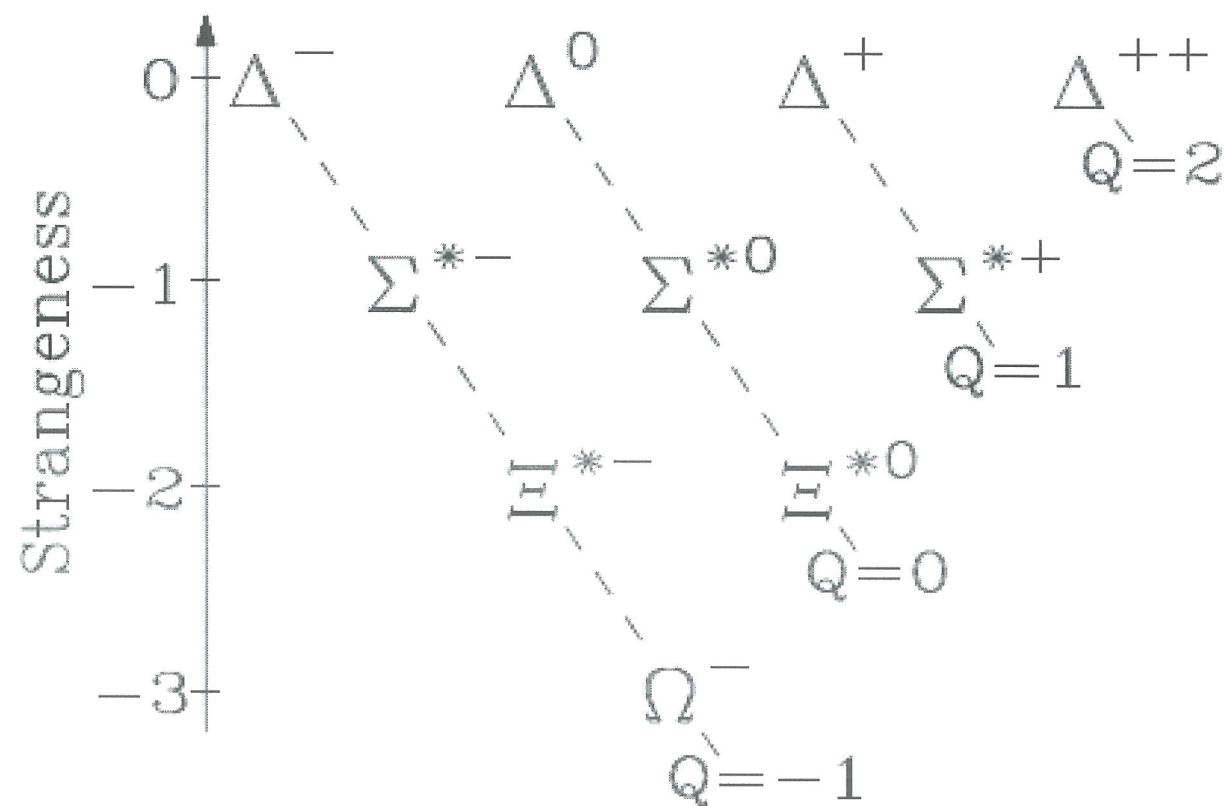


Baryon octet

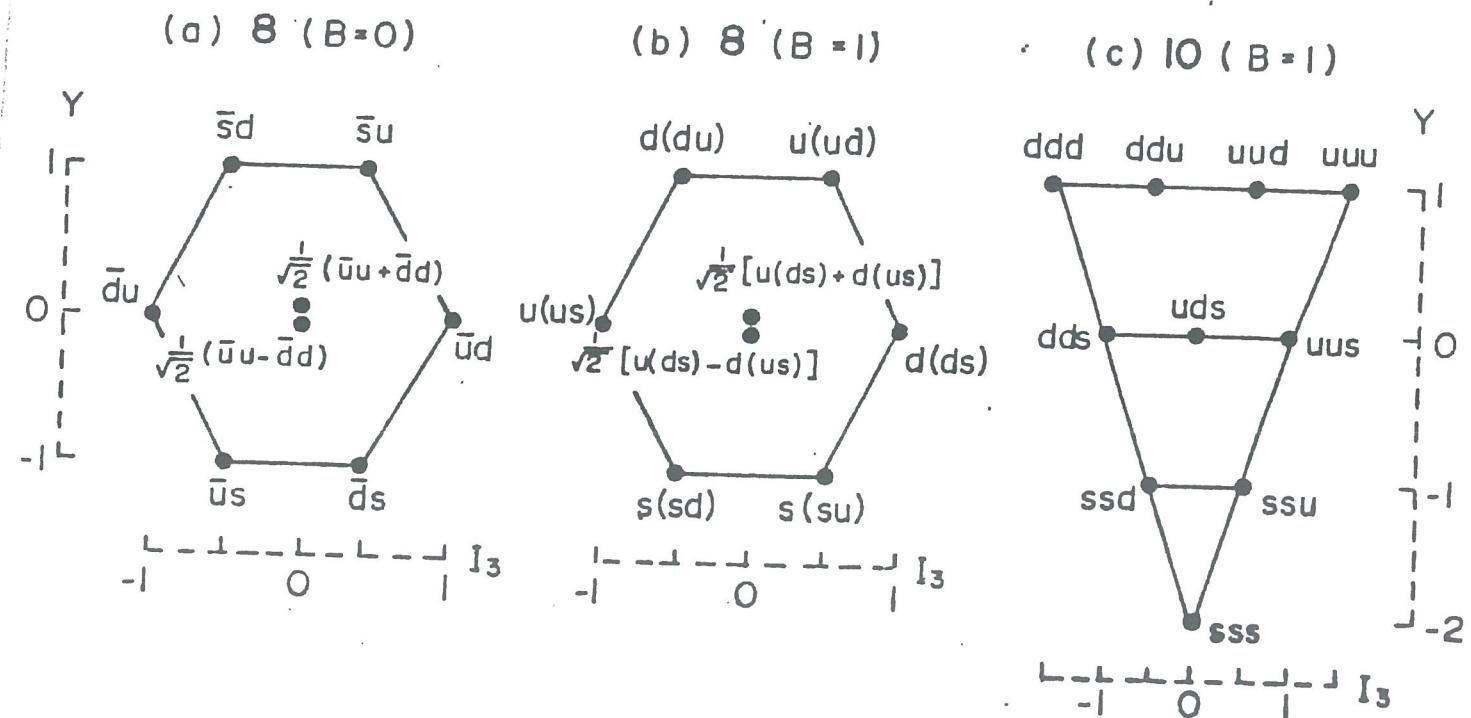


From Rosner: Macmillan Encyclopedia of Physics 2002

Baryon decuplet



28.

Fig. 3.4 Quark contents of 8 and 10

From: Kerson Huang : Quarks

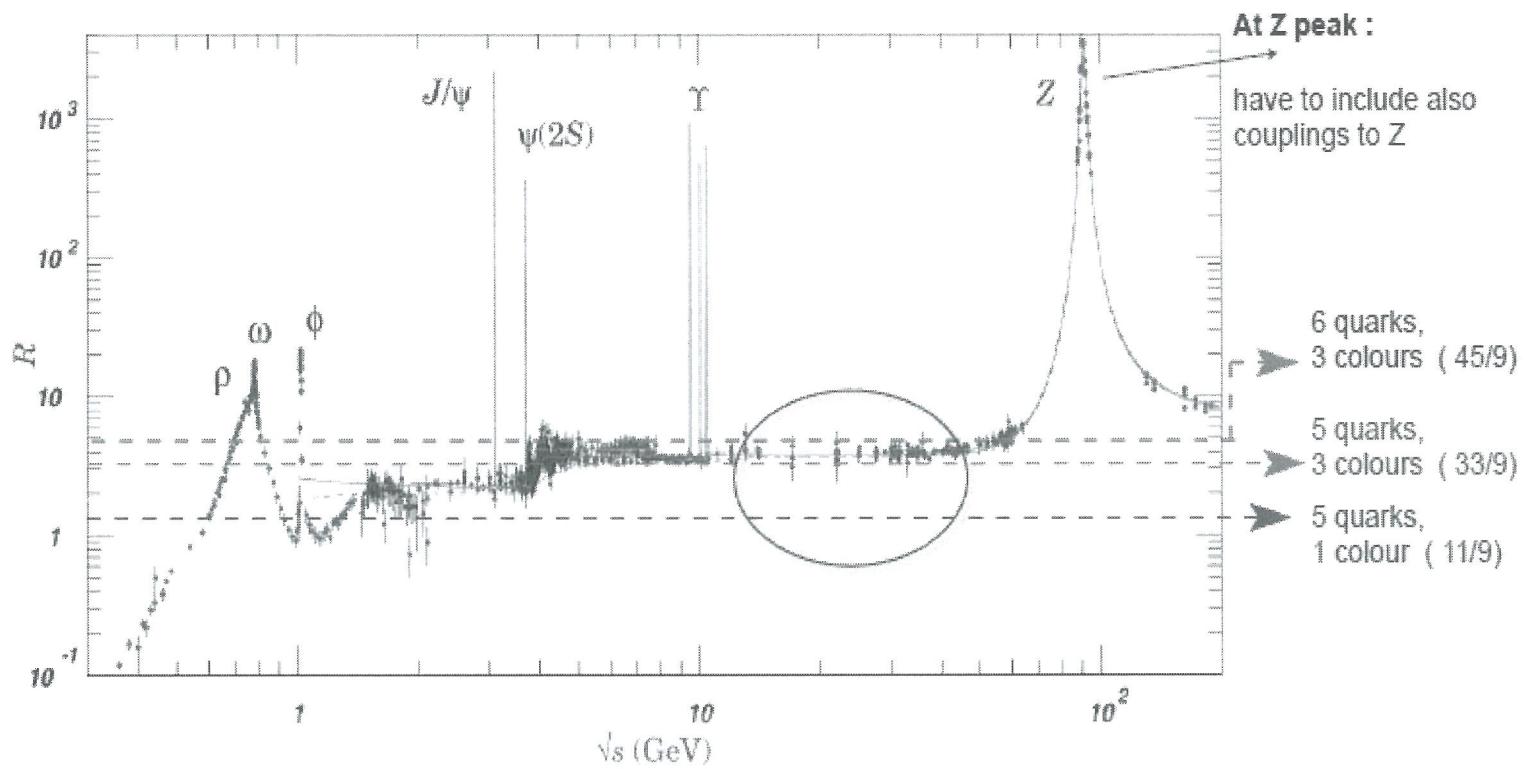


Figure 8.3: Ratio $R = \sigma^{e^+e^- \rightarrow \text{hadrons}} / \sigma^{e^+e^- \rightarrow \mu^+\mu^-}$ as a function of the center of mass energy. As expected by Eq. (8.3), there is roughly no energy dependence besides various resonances. The data confirm that there are three quark colors.