Hard QCD
from the Tevatron to the LHC

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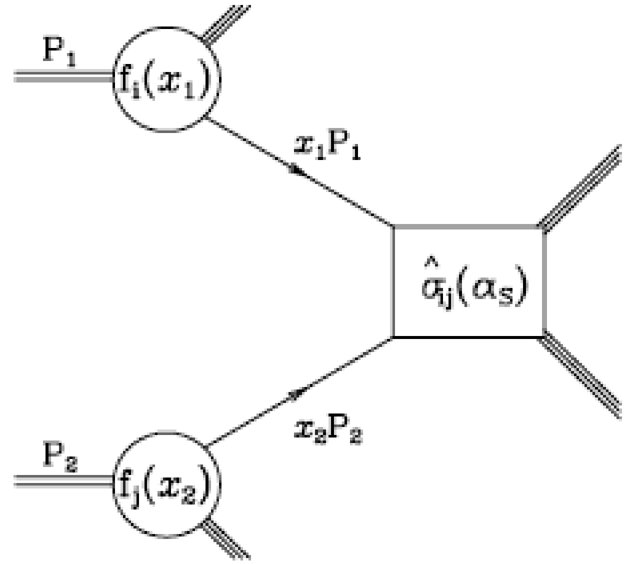
Fermilab

Menu

- Theoretical setup.
- Status of α_s
- Calculation of tree diagrams.
- Necessity of NLO and loop corrections.
- Progress with calculation of one loop diagrams.
- Recent phenomenological results.

QCD improved parton model

Hard QCD cross section is represented as the convolution of a short distance cross-section and non-perturbative parton distribution functions.
Physical cross section is formally independent of $\mu_{F\text{ and }}\mu_R$



Physical cross section

Parton distribution function

Renormalization scale μ_R

$$\sigma(P_1,P_2) = \sum_{i,j} \int dx_1 dx_2\, f_i(x_1,\mu_F) f_j(x_2,\mu_F)\, \hat{\sigma}_{ij}(p_1,p_2,\alpha_S(\mu_R),Q^2,\mu_R,\mu_F).$$

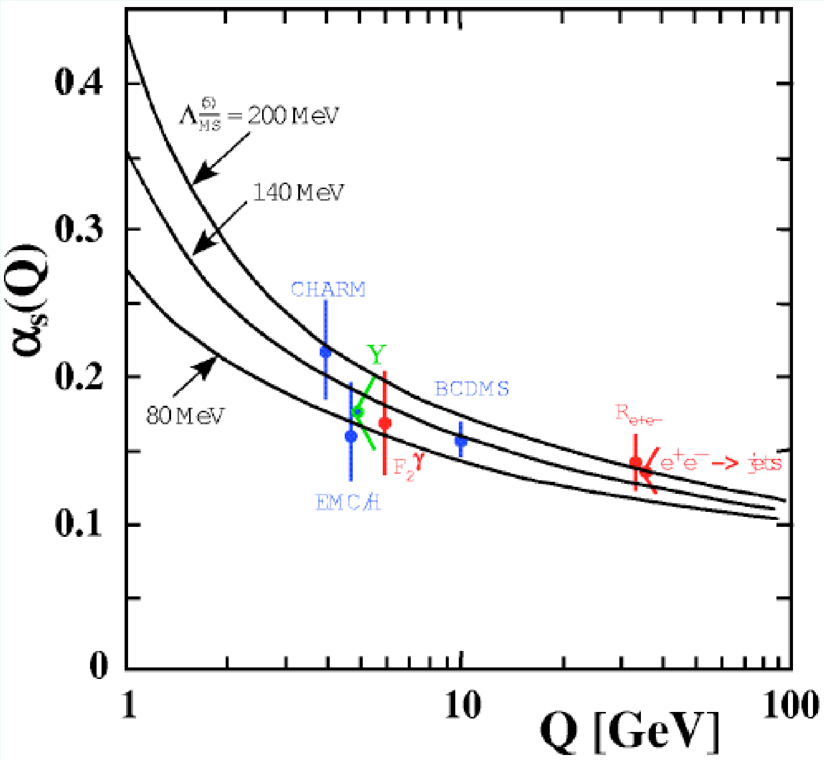
Factorization scale μ_F

Short distance cross section, calculated as a perturbation series in α_S

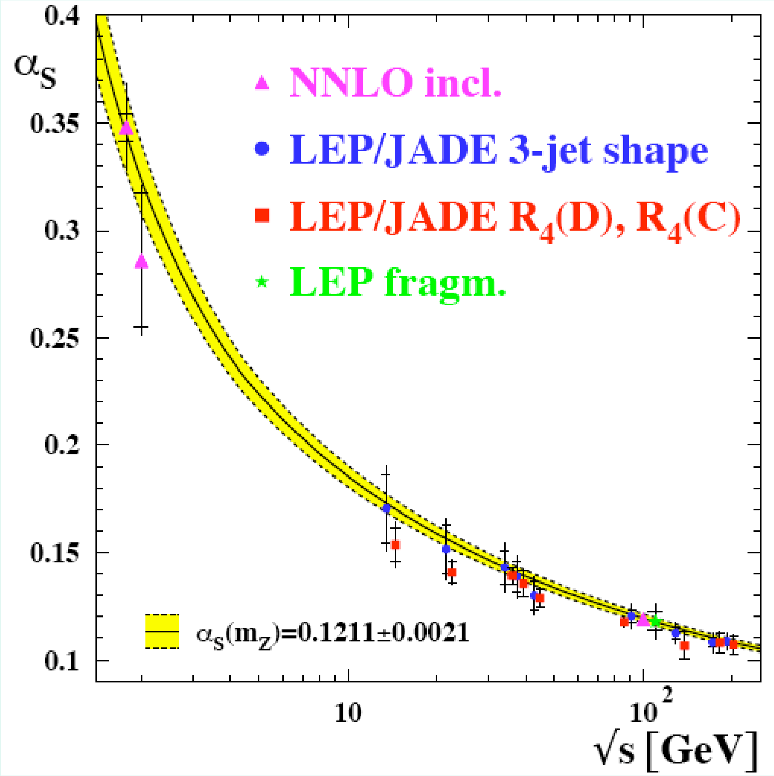
α_S

α_S is small at high energies because of the property of asymptotic freedom.

The role of LEP in determining the size of α_S has been crucial

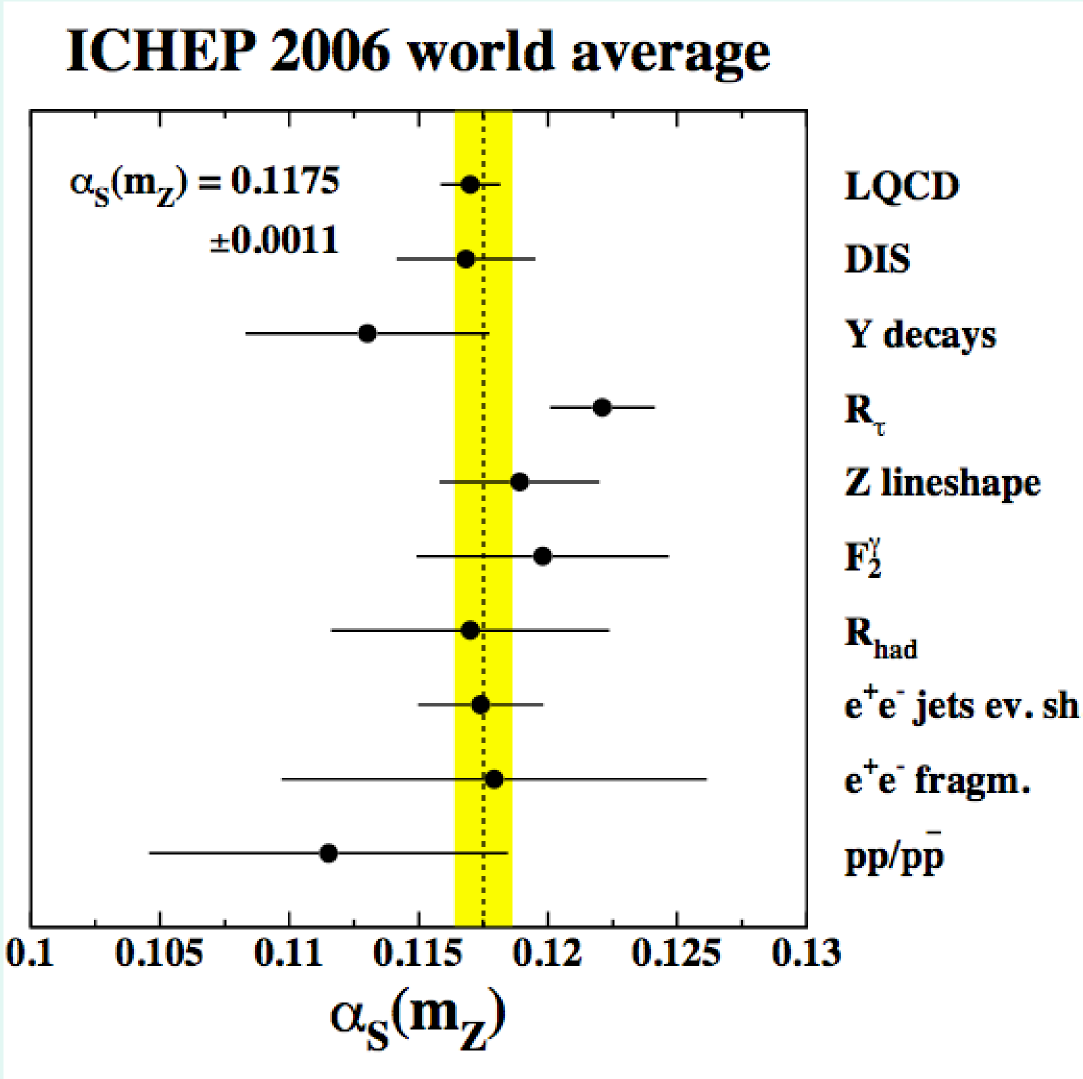


G. Altarelli 1989



S. Kluth EPS, 2007

S Kluth, hep-ex/0609020



2006 World average $\alpha_s(M_z) = 0.1175 \pm 0.0011$

α_S from jet shape measurements

LEP event shapes   91.2-206. NLO+NLLA

α_S=0.1202 ±0.0005(stat) ±0.0008(exp) ±0.0019(soft) ±0.0049(hard)

Theoretical errors dominate the total error, especially in the case of the general event shapes. Therefore an improved theoretical calculation is required.

Four jet rate from LEP

LEP R_4 91.2-206. NLO+NLLA

α_S=0.1175 ±0.0002(stat) ±0.0010(exp) ±0.0014(soft) ±0.0015(hard)

$R_4(y) = \sigma_{4\text{-jet}}(y)/\sigma_{tot}$ is the four jet rate.

S Kluth hep-ex/0609020

New NNLO and N^3LL results for thrust

The thrust is defined as the maximum of directed momentum $$T = \max_{\mathbf{n}} \frac{\sum_i \mathbf{p}_i \cdot \mathbf{n}}{\sum_i |\mathbf{p_i}|}$$

Defining τ=1-T, perturbative expansion of thrust is

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} = \delta(\tau) + \frac{\alpha_S}{2\pi}A(\tau) + \left(\frac{\alpha_S}{2\pi}\right)^2 B(\tau) + \left(\frac{\alpha_S}{2\pi}\right)^3 C(\tau)$$

A=LO, Farhi, 1977

B=NLO, RKE, Ross, Terrano 1981

C=NNLO, Gehrmann-De Ridder, Gehrmann, Glover, Heinrich 2007

Expression for thrust contains large logarithms for small τ

$$R(\tau) = \int_0^\tau d\tau' \frac{1}{\sigma_0}\frac{d\sigma}{d\tau'} \quad \lim_{\tau\to 0} \quad 1 + \frac{2\alpha_S}{3\pi}\left[-2\ln^2\tau - 3\ln\tau\right] + \ldots$$

$\alpha_S^n \ln^{2n}\tau$	:	LL	
$\alpha_S^n \ln^{2n-1}\tau$	:	NLL	Catani et al 1993
$\alpha_S^n \ln^{2n-2}\tau$	:	N^2LL	Becher, Schwartz, 2008
$\alpha_S^n \ln^{2n-3}\tau$	:	N^3LL	Becher, Schwartz, 2008

NNLO+N^{3}LL result for α_S

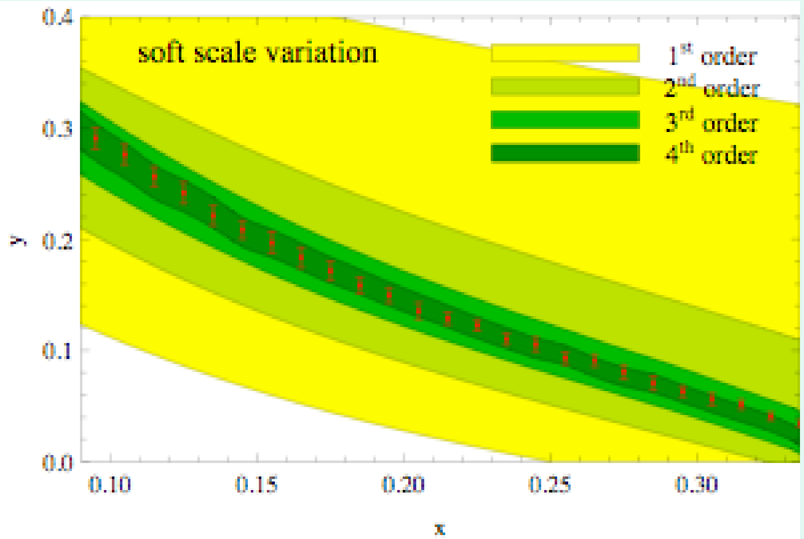
Becher and Schwartz arXiv 0803.0342v1 have a new NNLO+N^{3}LL result using Aleph & Opal data

$$\alpha_s=0.1172+0.0010(\text{stat})+0.0008(\text{sys})+0.0012(\text{had})+0.0012(\text{pert})$$

Perturbative error is for the first time smaller than the other uncertainties at each energy.

Result is in agreement with the world average.

α_S is in known to about 2% from a single measurement.



Parton distribution functions

Measurement of the non-perturbative parton distributions at lower energies allow extrapolations to higher values of μ and lower values of x using the DGLAP equation

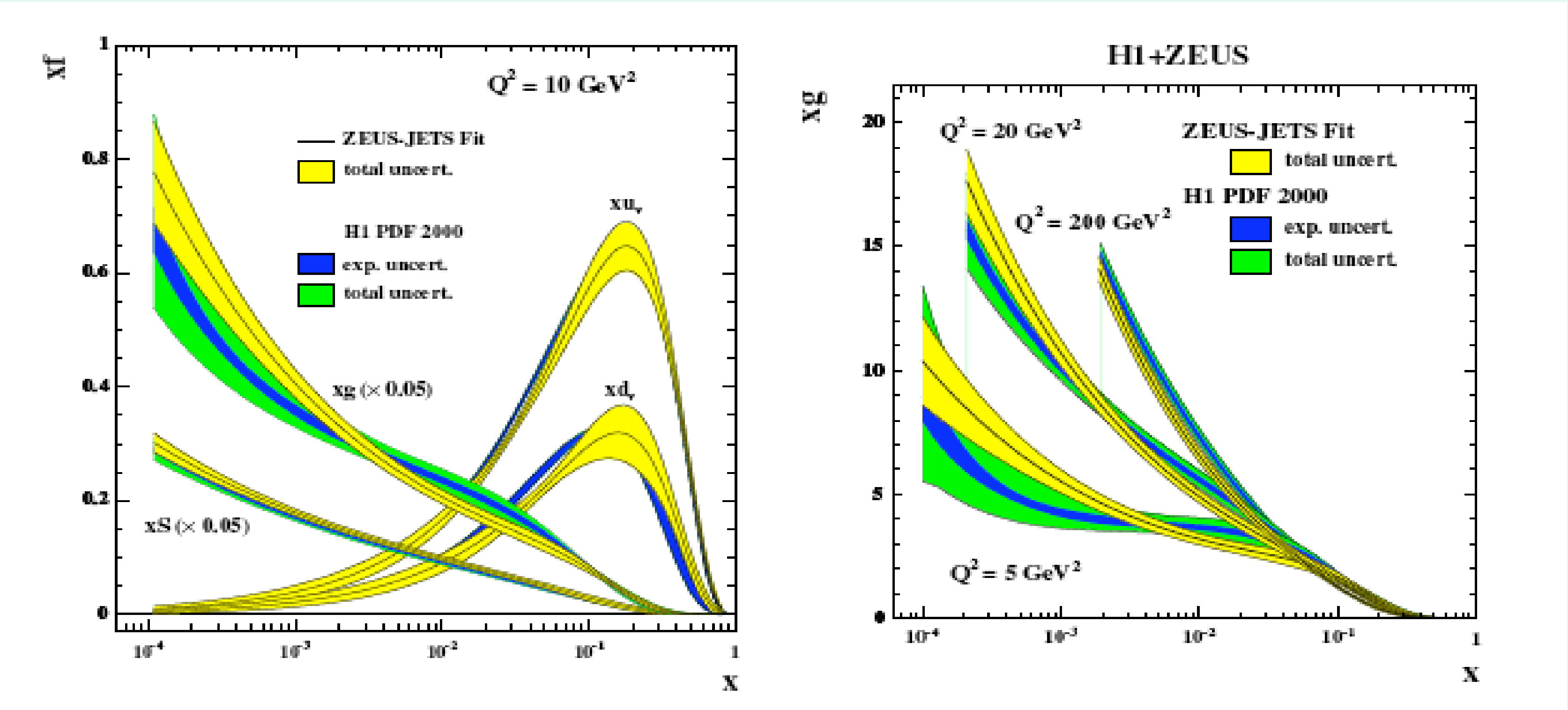
$$\frac{\partial}{\partial \ln \mu^2} f_i(x,t) = \frac{\alpha_S(\mu)}{2\pi} \int_x^1 \frac{d\xi}{\xi} P_{ij}\left(\frac{x}{\xi}, \alpha_S(\mu)\right) f_j(\xi,\mu)$$

The evolution kernel is calculable as a perturbation series in α_s

$$P_{ij}(z,\alpha_S) = \underbrace{P_{ij}^{(0)}(z)}_{\text{LO}} + \frac{\alpha_S}{2\pi} \underbrace{P_{ij}^{(1)}(z)}_{\text{NLO}} + \left(\frac{\alpha_S}{2\pi}\right)^2 \underbrace{P_{ij}^{(2)}(z)}_{\text{NNLO}} \ldots$$

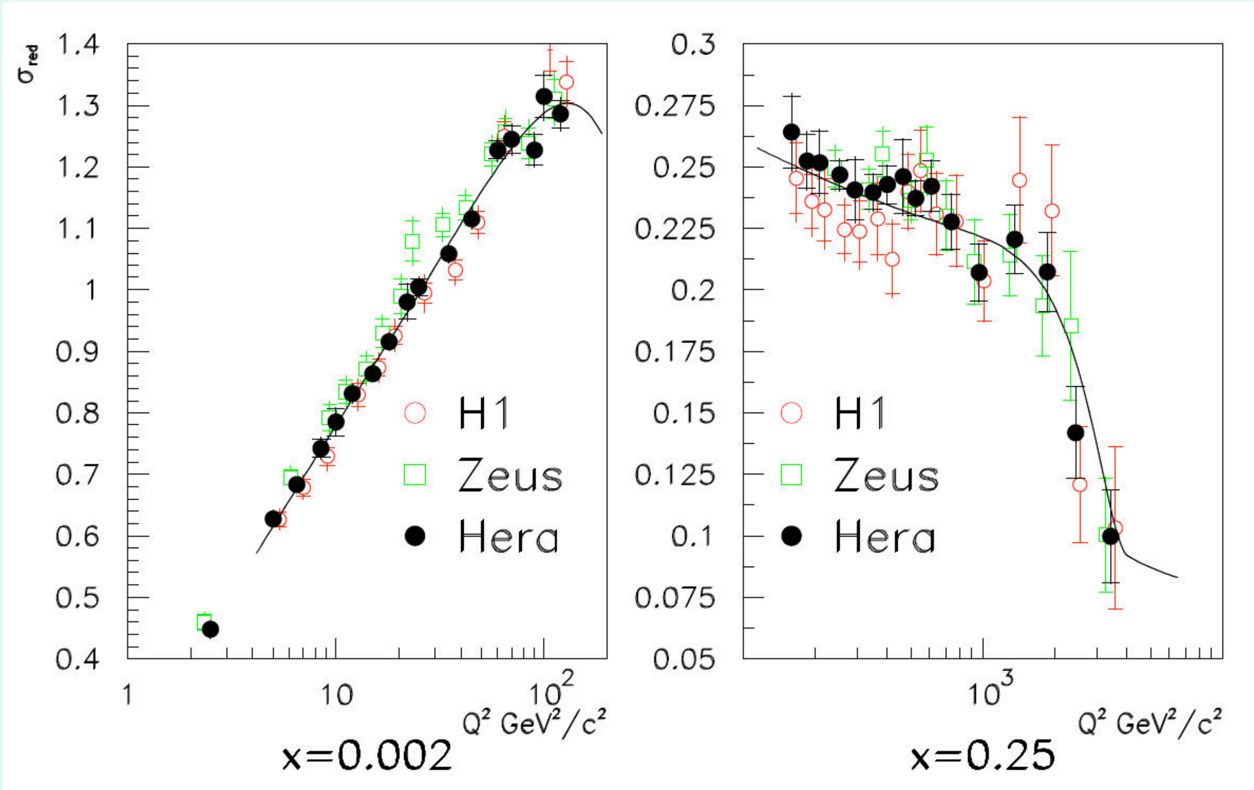
NNLO is known completely. (Moch et al, hep-ph/0403192)

Comparison of H1 and Zeus



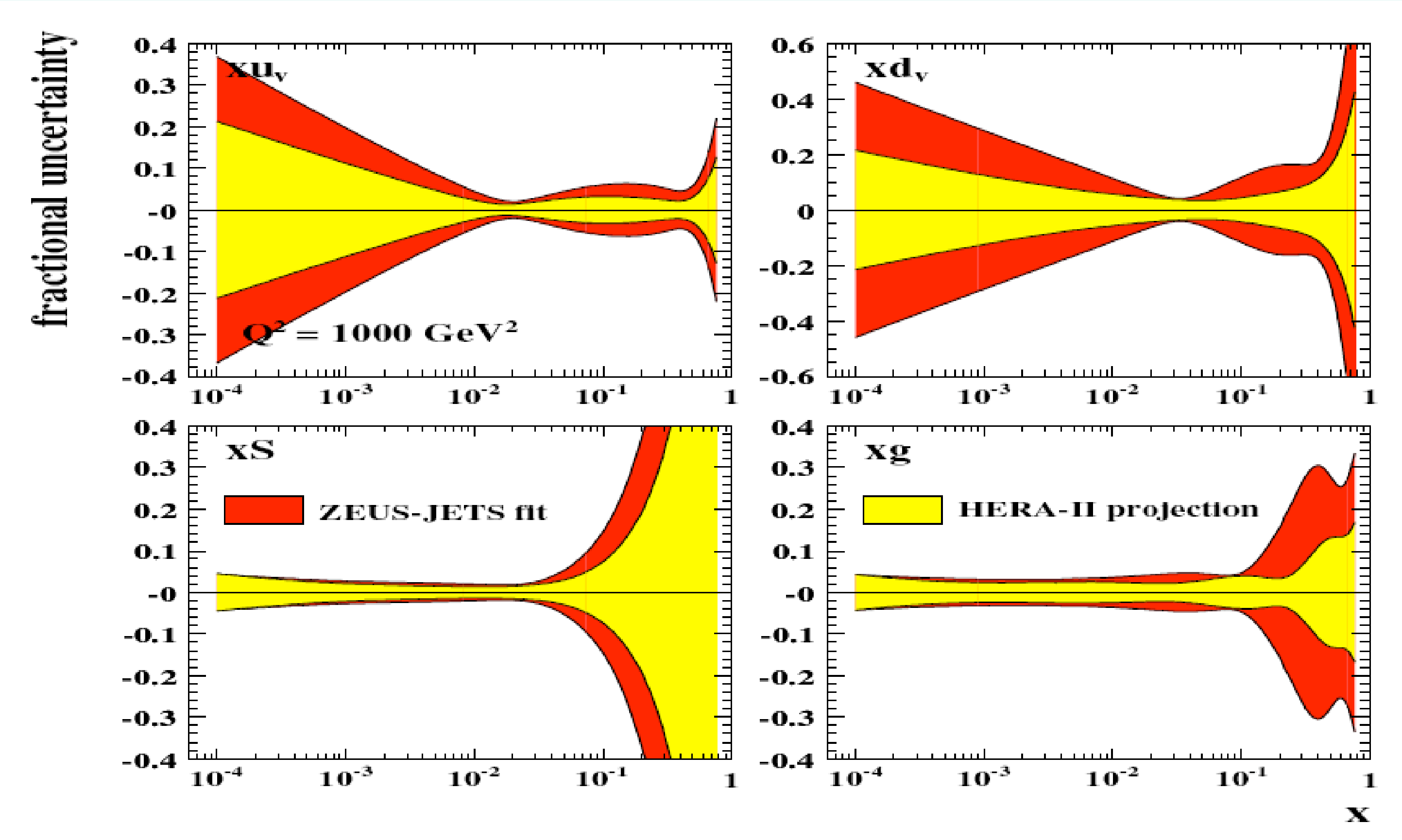
Some of the differences are understood (inclusion of BCDMS at large x (ZEUS) ; inclusion of jet data for mid x gluon (H1))

Combination of ZEUS and H1 data (taken from Rik Yoshida, Fermilab JETP seminar, 11/30/2007)



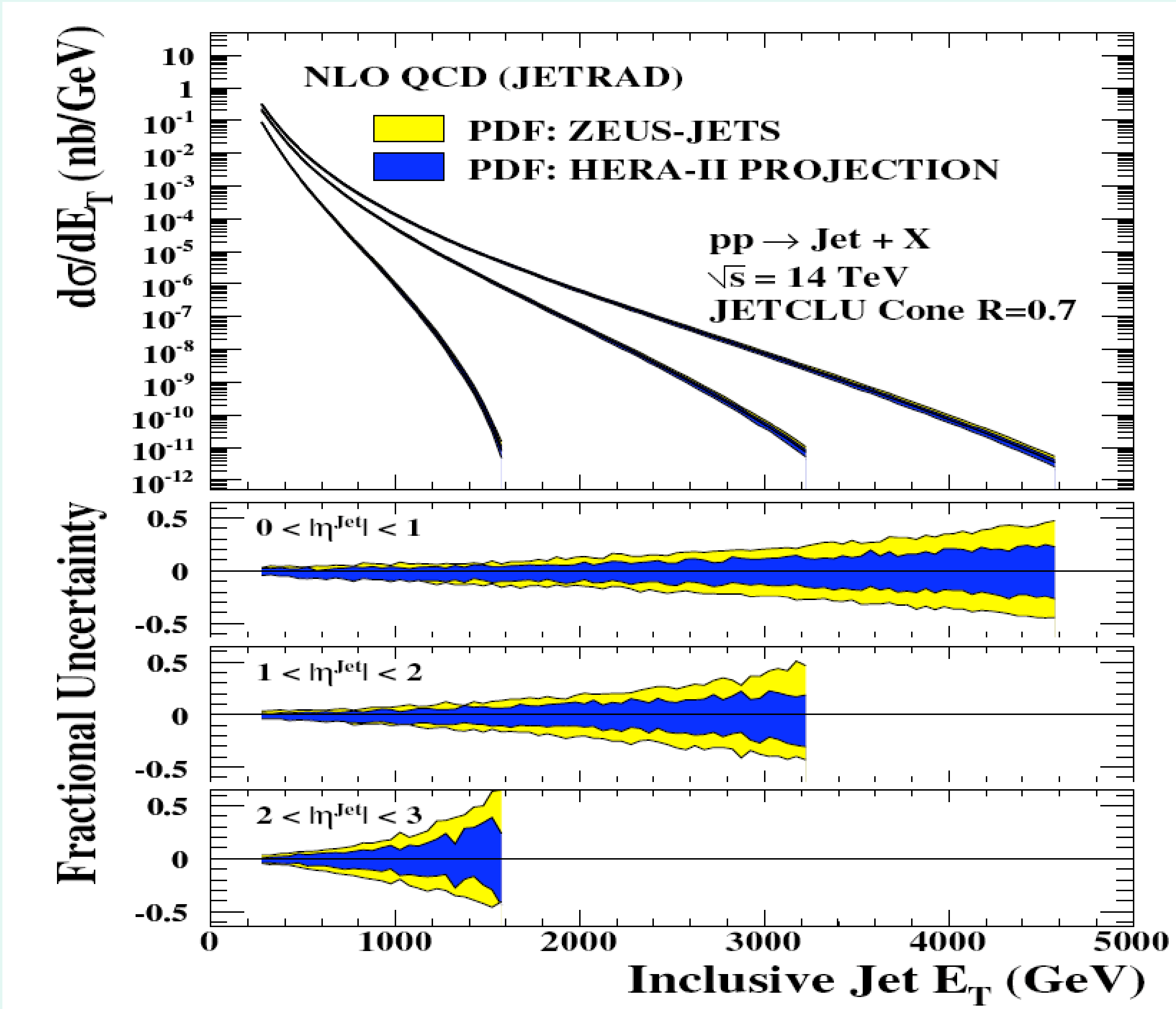
The improvement is (much) better than √2!
How is this possible?
"Systematics correlated across the kinematic plane, but uncorrelated between experiments cancel." (Yoshida)

Projected parton model uncertainties after HERAII



HERA for LHC workshop,hep-ph/0601012

…and consequent improvement on uncertainty of jet cross section

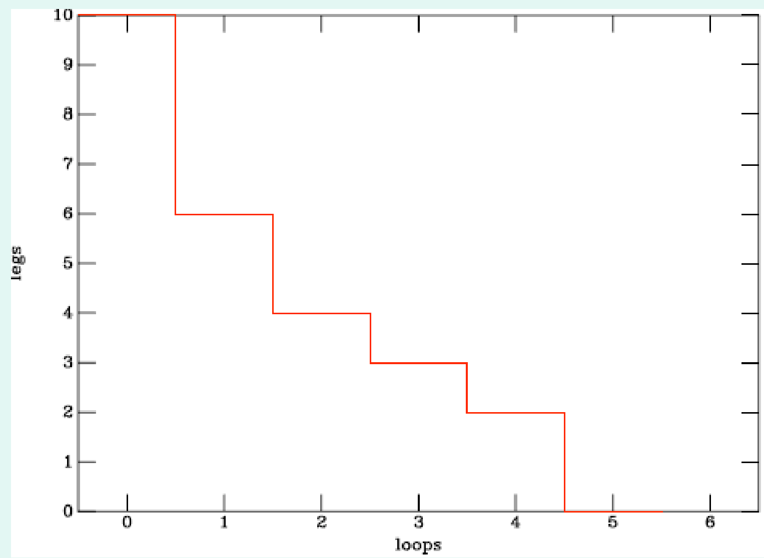


Short-distance cross section

For a hard process the short distance cross section can be calculated in various approximations.

Leading order (LO) tree graphs $\alpha_S{}^n$
Next-to-leading order (NLO) $\alpha_S{}^{n+1}$
Next-to-next_to_leading order (NNLO) $\alpha_S{}^{n+2}$

Current state of the art can calculate large number of loops and small number of legs or a smaller number of legs and a larger number of loops.



Tree graphs

The calculation of any tree graph is essentially solved.

- Madgraph/Helas
- Berends-Giele recursion
- MHV based recursion
- BCF on-shell recursion
- Comparison of methods

Stelzer, Long, Maltoni, …

Madgraph/Madevent

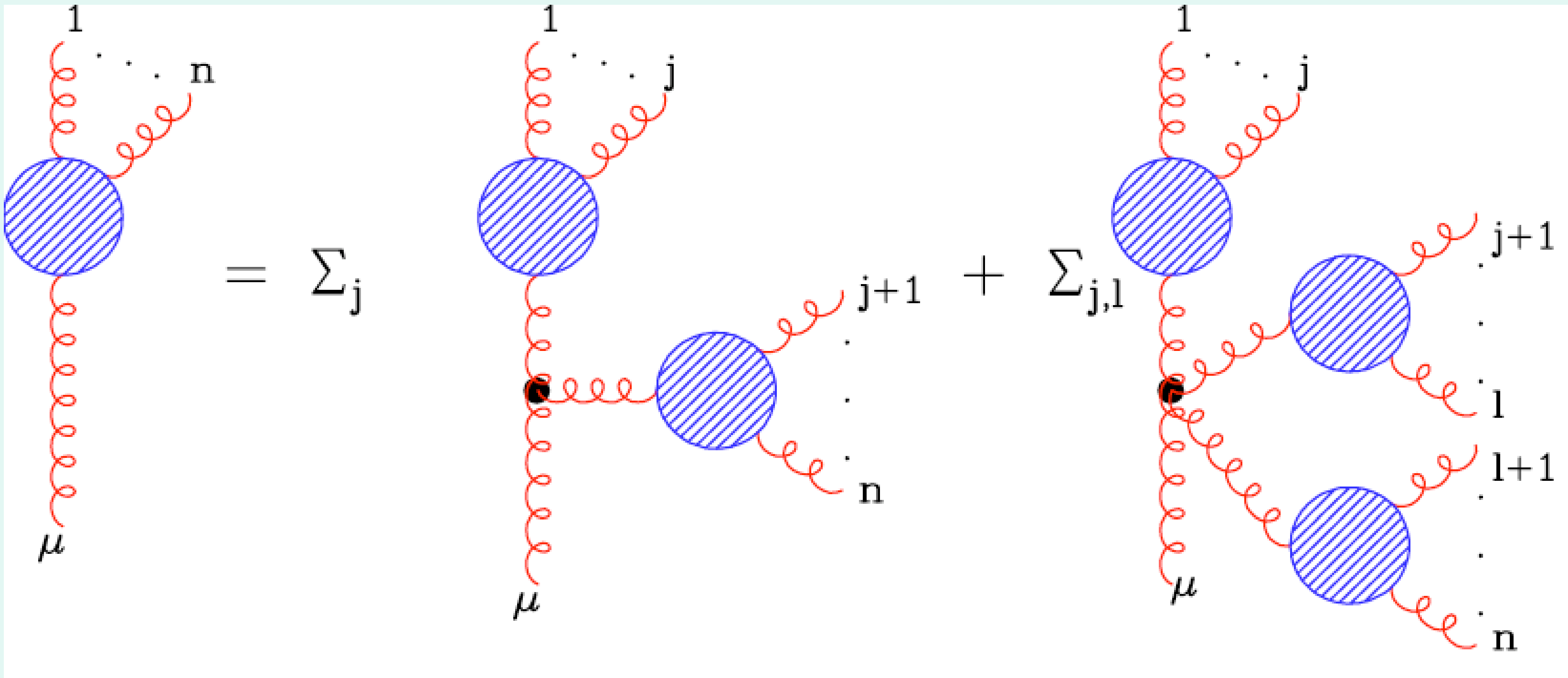
- Generation of Topologies
- Generation of Feynman diagrams corresponding to those topologies and physics model
- Generation of color and symmetry factors
- Generation of Helas code (Murayama, Watanabe, Hagiwara)
- Madevent uses the squared diagrams as a basis for multi-channel integration,

$$f = \sum_{i=1}^{n} f_i \qquad f_i = \frac{|A_i|^2}{\sum_i |A_i|^2} |A_{\rm tot}|^2$$

Peak structure of f_i is the same as that of individual diagram A_i.
Remapping can be done to remove peaking structure.
Code is easily parallelizable.

Berends-Giele recursion

Building blocks are non-gauge invariant color-ordered off-shell currents. Off-shell currents with n legs are related to off-shell currents with fewer legs (shown here for the pure gluon case).



Despite the fact that it is constructing the complete set of Feynman diagrams, BG recursion is a very economical scheme.

On-shell recursion

- Brief description of two on-shell recursion methods which allow the recycling of tree graph amplitudes
- Sewing together MHV amplitudes.
- BCFW recursion.

Spinor techniques

Denote spinors for light-like vectors as follows

$$|k+\rangle \quad = \quad \text{right-handed spinor for massless vector } k$$
$$|k-\rangle \quad = \quad \text{left-handed spinor for massless vector } k$$

Polarization vectors are given by

$$\varepsilon_\mu^+ = \frac{\langle q^-|\gamma_\mu|k^-\rangle}{\sqrt{2}\langle qk\rangle}, \quad \varepsilon_\mu^- = \frac{\langle q^+|\gamma_\mu|k^+\rangle}{\sqrt{2}[kq]}$$

Satisfy all the requirements of a polarization vector

$$\varepsilon_i^2 = 0, \quad k \cdot \varepsilon(k,q) = 0, \quad \varepsilon^+ \cdot \varepsilon^- = -1$$

Equivalent notations

$$\epsilon^{ab}\lambda_{ja}\lambda_{lb} \equiv \langle jk\rangle \equiv \langle k_j^-|k_l^+\rangle = \sqrt{2k_j \cdot k_l}e^{i\phi}$$
$$\epsilon^{\dot{a}\dot{b}}\tilde{\lambda}_{j\dot{a}}\tilde{\lambda}_{l\dot{b}} \equiv [jk] \equiv \langle k_j^+|k_l^-\rangle = -\sqrt{2k_j \cdot k_l}e^{-i\phi}$$

MHV amplitudes

Consider the five gluon amplitude and decompose 5-gluon amplitude into color ordered sub -amplitude

$$A = \mathrm{Tr}\{t^{a1}t^{a2}t^{a3}t^{a4}t^{a5}\}m(1,2,3,4,5) + \text{permutations}$$

Two of the color stripped amplitudes vanish

$$\begin{aligned} m(g_1^+, g_2^+, g_3^+, g_4^+, g_5^+) &= 0 \\ m(g_1^-, g_2^+, g_3^+, g_4^+, g_5^+) &= 0 \end{aligned}$$

The maximum helicity violating color -stripped amplitude

$$m(g_1^-, g_2^-, g_3^+, g_4^+, g_5^+) = \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle}$$

$\langle ij\rangle, [ij]$ useful because QCD amplitudes have square root singularities.

Parke & Taylor and Berends & Giele

MHV amplitudes (continued)

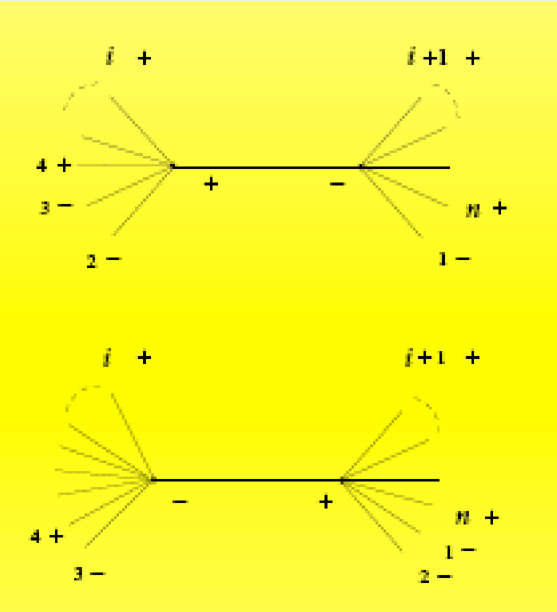
The generalization to the case with two adjacent positive helicity gluons and n-2 negative helicity gluons.

$$m(g_1^-, g_2^-, g_3^+, \ldots g_n^+) = \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle \ldots \langle n1\rangle}$$

Remember that the spinor products $\sim \sqrt{(2p_i \cdot p_j)}$

MHV amplitudes (continued)

Based on intuition from twistors, Cachazo, Svrcek, Witten (2004) proposed an MHV calculus involving sewing together off-shell continuations of MHVamplitudes joined by scalar propagators. The resulting theory yields more compact results than Feynman rules.



BCFW recursion

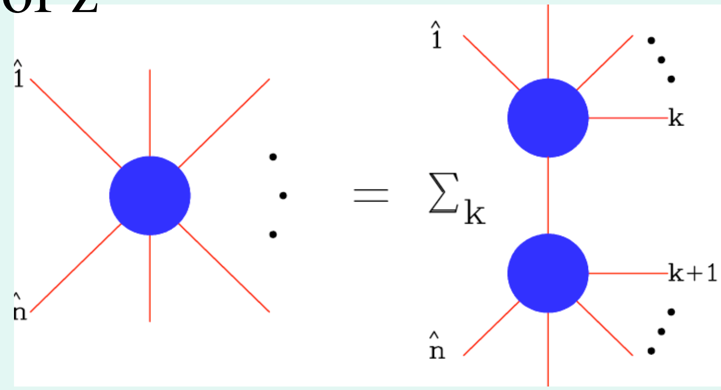
Complex momentum flows in at 1 and exits at n.

$$p_1^\mu \to \hat{p}_1^\mu = p_1^\mu - \frac{z}{2}\langle 1^-|\gamma^\mu|n^-\rangle$$

$$p_n^\mu \to \hat{p}_n^\mu = p_n^\mu + \frac{z}{2}\langle 1^-|\gamma^\mu|n^-\rangle$$

Perform shift of momenta 1 and n conserving masslessness and overall conservation of momentum.

This defines the amplitude as a function of z

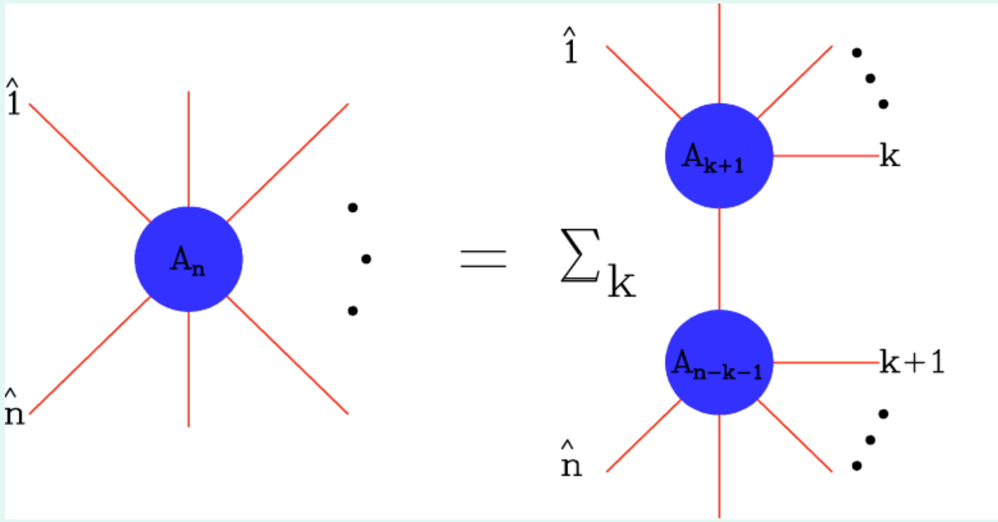


By Cauchy's theorem, if contour at infinity vanishes

$$0 = \frac{1}{2\pi i}\oint \frac{dz}{z} A(z) = A(0) + \sum_k \mathrm{Res}\left[\frac{A(z)}{z}\right]_{z=z_k}$$

Poles (other than the pole at 0) define the k-th term in the recursion relation.

BCFW recursion relations (cont)



A_{k+1} and $A_{n\text{-}k\text{-}1}$ are on-shell tree level amplitudes for processes with fewer legs and momenta shifted by a complex quantity. Thus for the sixgluon amplitude (220 diagrams) we obtain:-

$$A_6(1^+,2^+,3^+,4^-,5^-,6^-) = \frac{i}{\langle 2^-|(6+1)|5^-\rangle}$$

$$\times \quad \left[\frac{(\langle 6^-|(1+2)|3^-\rangle)^3}{\langle 61\rangle\langle 12\rangle[34][45]s_{612}} + \frac{(\langle 4^-|(5+6)|1^-\rangle)^3}{\langle 23\rangle\langle 34\rangle[56][61]s_{561}}\right]$$

Dinsdale, Ternick, Weinzierl, cf. Duhr, Hoche & Maltoni

Comparison of speed for numerical evaluation

Tree level amplitude with n external gluons may be written as

$$\mathcal{A}_n(k_1^{\lambda_1},\ldots,k_n^{\lambda_n}) = g^{n-2} \sum_{\sigma\in S_n/Z_n} 2\,\mathrm{Tr}\left(T^{a_{\sigma(1)}}\ldots T^{a_{\sigma(n)}}\right) A_n\left(k_{\sigma(1)}^{\lambda_{\sigma(1)}},\ldots,k_{\sigma(n)}^{\lambda_{\sigma(n)}}\right)$$

Leading color matrix element squared is given by

$$\mathcal{M}_n = \sum_{\lambda_1,\ldots,\lambda_n} \left| A_n\left(k_1^{\lambda_1},\ldots,k_n^{\lambda_n}\right)\right|^2$$

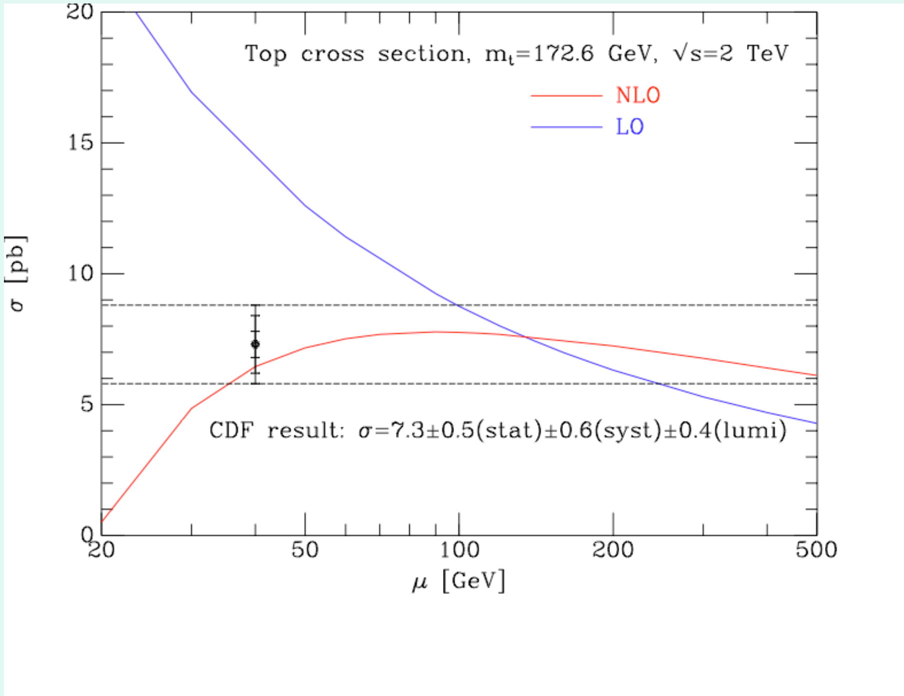
CPU time in seconds to calculate $\mathcal{M}_n$ using the various methods

n	4	5	6	7	8	9	10	11	12
Berends-Giele	0.00005	0.00023	0.0009	0.003	0.011	0.030	0.09	0.27	0.7
Scalar	0.00008	0.00046	0.0018	0.006	0.019	0.057	0.16	0.4	1
MHV	0.00001	0.00040	0.0042	0.033	0.24	1.77	13	81	—
BCF	0.00001	0.00007	0.0003	0.001	0.006	0.037	0.19	0.97	5.5

Conclusion on calculation of tree graphs

- For calculation of six jet rate, BCFW shows a modest advantage in computer speed over BG (for leading color amplitudes).
- Once full color information is included even this advantage is removed. (Duhr, Hoche, Maltoni)
- Berends-Giele off-shell recursion is universal, fast enough and simple to program.

Why NLO?



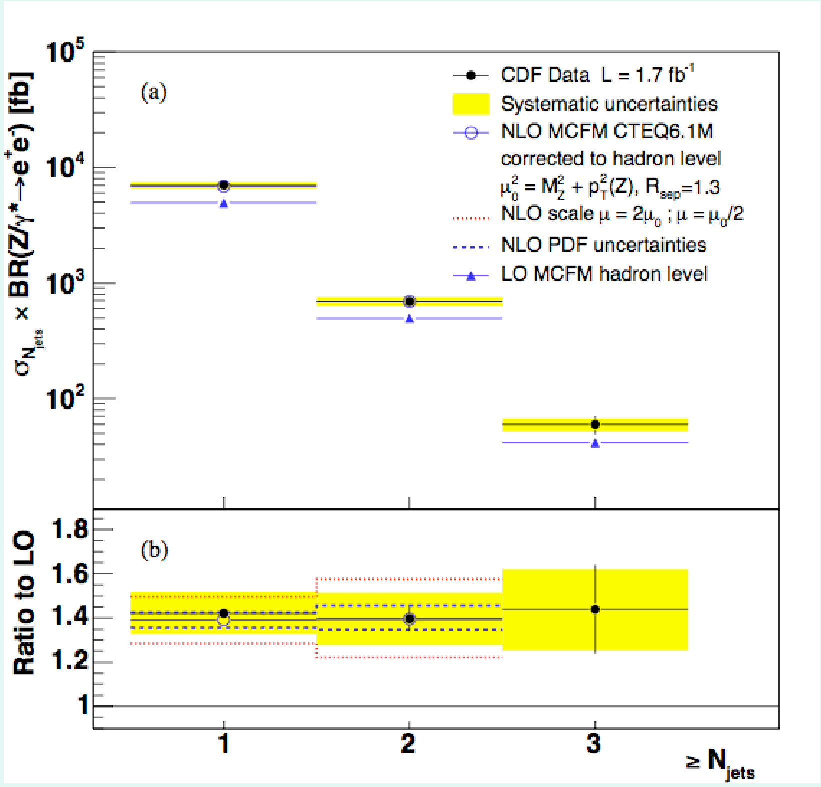
In order to get ~10% accuracy we need to include NLO.

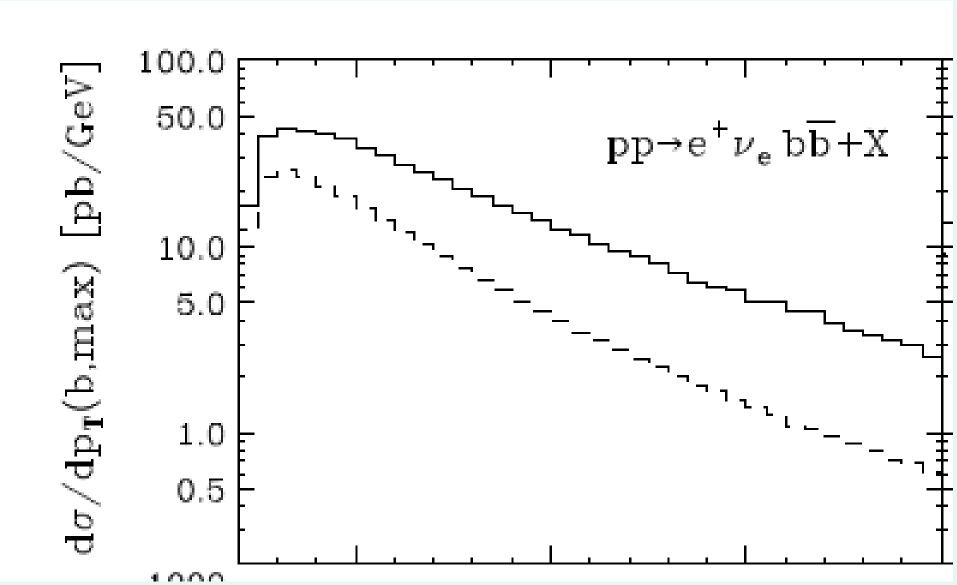
- Less sensitivity to unphysical input scales, (eg. renormalization and factorization scales)
- NLO first approximation in QCD which gives an idea of suitable choice for μ.
- NLO has more physics, parton merging to give structure in jets, initial state radiation, more species of incoming partons enter at NLO.
- A necessary prerequisite for more sophisticated techniques which match NLO with parton showering.

Isn't it just an overall K-factor?

Sometimes…..

but not always





NLO-solid, LO-dashed

Z+jet production at the Tevatron

Wbb production at the LHC

An experimenter's wishlist

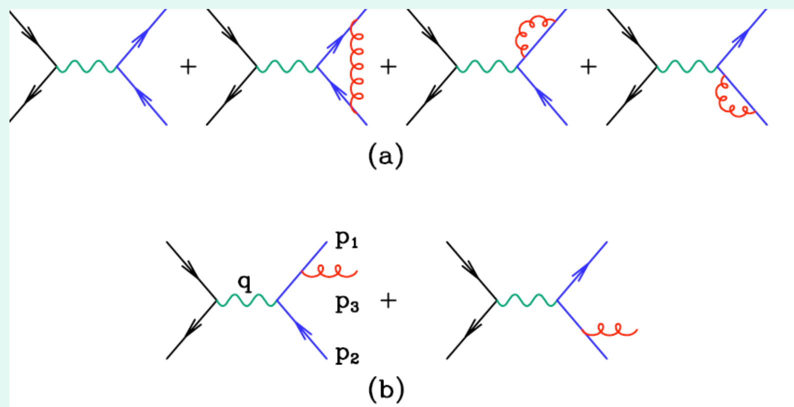
Run II Monte Carlo Workshop

Single Boson	Diboson	Triboson	Heavy Flavour
$W+\leq 5j$	$WW+\leq 5j$	$WWW+\leq 3j$	$t\bar{t}+\leq 3j$
$W+b\bar{b}\leq 3j$	$W+b\bar{b}+\leq 3j$	$WWW+b\bar{b}+\leq 3j$	$t\bar{t}+\gamma+\leq 2j$
$W+c\bar{c}\leq 3j$	$W+c\bar{c}+\leq 3j$	$WWW+\gamma\gamma+\leq 3j$	$t\bar{t}+W+\leq 2j$
$Z+\leq 5j$	$ZZ+\leq 5j$	$Z\gamma\gamma+\leq 3j$	$t\bar{t}+Z+\leq 2j$
$Z+b\bar{b}+\leq 3j$	$Z+b\bar{b}+\leq 3j$	$ZZZ+\leq 3j$	$t\bar{t}+H+\leq 2j$
$Z+c\bar{c}+\leq 3j$	$ZZ+c\bar{c}+\leq 3j$	$WZZ+\leq 3j$	$t\bar{b}\leq 2j$
$\gamma+\leq 5j$	$\gamma\gamma+\leq 5j$	$ZZZ+\leq 3j$	$b\bar{b}+\leq 3j$
$\gamma+b\bar{b}\leq 3j$	$\gamma\gamma+b\bar{b}\leq 3j$		single top
$\gamma+c\bar{c}\leq 3j$	$\gamma\gamma+c\bar{c}\leq 3j$		
	$WZ+\leq 5j$		
	$WZ+b\bar{b}\leq 3j$		
	$WZ+c\bar{c}\leq 3j$		
	$W\gamma+\leq 3j$		
	$Z\gamma+\leq 3j$		

Ingredients for a NLO calculation

Example $e^+e^- \Rightarrow 2$ jets

- Born process LO
- Interference of one-loop with LO
- Real radiation (also contributes to the two jet rate in the region of soft or collinear emission).



What is the bottle-neck?

- Consider for example W+n jets.
  (W+4 jets is a background to top production).
- W+n (LO) and W+(n+1)-parton amplitudes known since 1989 Berends et al.
- Subtraction method understood 1980.
  Ellis, Ross & Terrano, Catani & Seymour
- NLO parton evolution known since 1980.

  Curci, Furmanski & Petronzio
- Bottleneck is the calculation of one loop amplitudes. In fact only the one-loop amplitudes for W+1 jet and W+2 jets are known.
  Bern et al (1997); Campbell, Glover & Miller (1997).

Theoretical digression on the calculation of one loop amplitudes

- The classical paradigm for the calculation of one-loop diagrams was established in 1979.
- Complete calculation of one-loop scalar integrals
- Reduction of tensors one-loop integrals to scalars.

Nuclear Physics B153 (1979) 365–401

SCALAR ONE-LOOP INTEGRALS

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Received 16 January 1979

Nuclear Physics B160 (1979) 151–207

ONE-LOOP CORRECTIONS FOR e^+e^- ANNIHILATION INTO $\mu^+\mu^-$ IN THE WEINBERG MODEL

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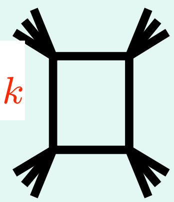
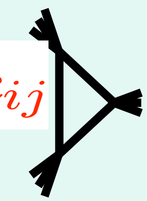
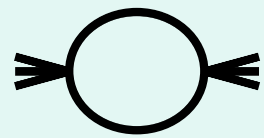
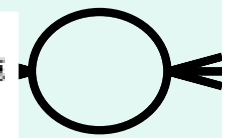
Received 22 March 1979

Neither will be adequate for present-day purposes.

Basis set of scalar integrals

Any one-loop amplitude can be written as a linear sum of boxes, triangles, bubbles and tadpoles

$\mathcal{A}_N(\{p_i\}) = \sum d_{ijk}$  $+ \sum c_{ij}$  $+ \sum b_i$  $+ \sum_i a_i$ 

In addition, in the context of NLO calculations, scalar higher point functions, can always be expressed as sums of box integrals. Passarino, Veltman - Melrose ('65)

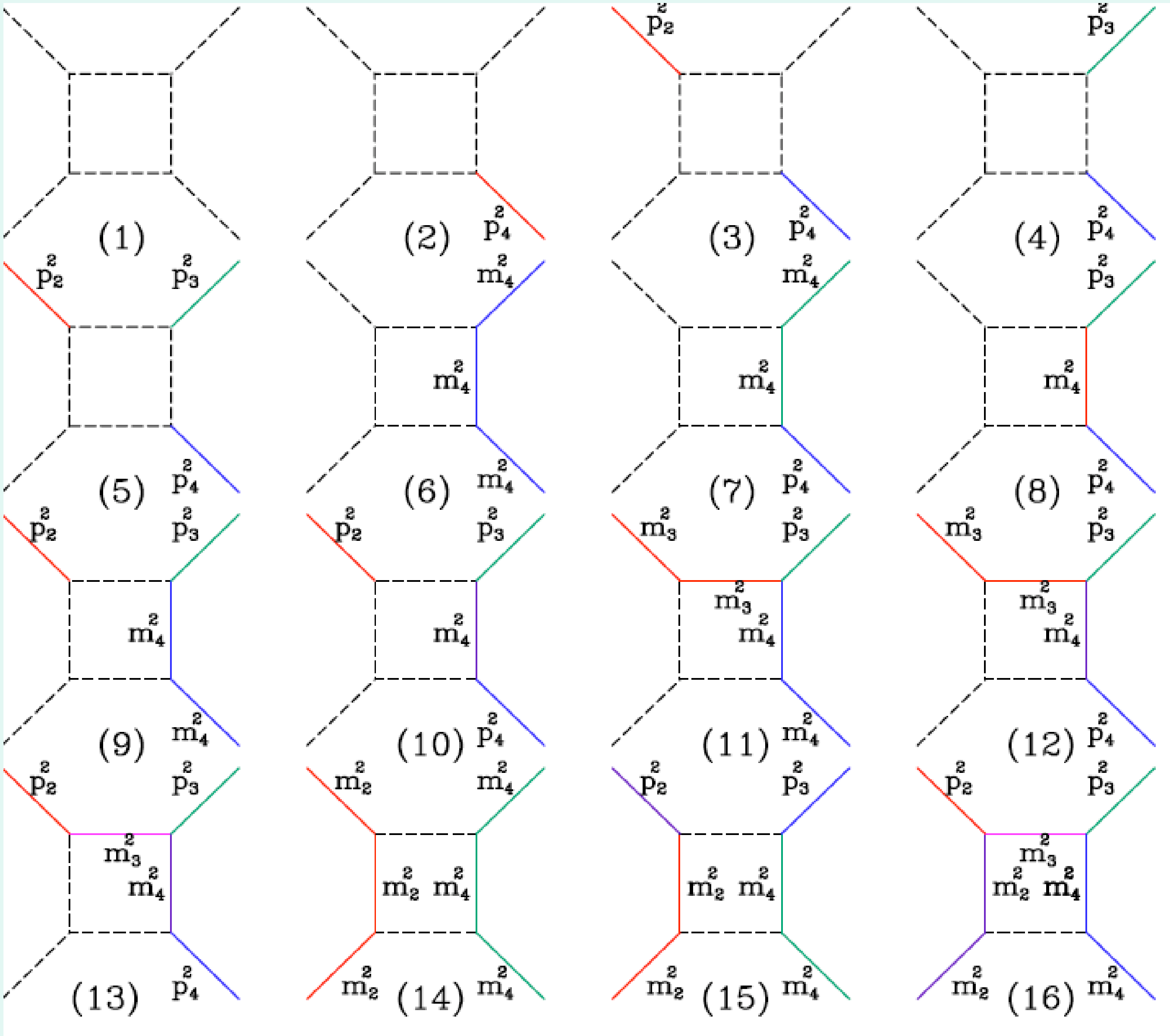
- Scalar hexagon can be written as a sum of six pentagons.
- For the purposes of NLO calculations, the scalar pentagon can be written as a sum of five boxes.
- In addition to the 'tH-V integrals we need integrals containing infrared and collinear divergences.

Scalar one-loop integrals

- ‘t Hooft and Veltman’s integrals contain internal masses; however in QCD many lines are (approximately) massless. The consequent soft and collinear divergences are regulated by dimensional regularization.
- So we need general expressions for boxes, triangles, bubbles and tadpoles, including the cases with one or more vanishing internal masses.

Basis set of sixteen divergent box integrals

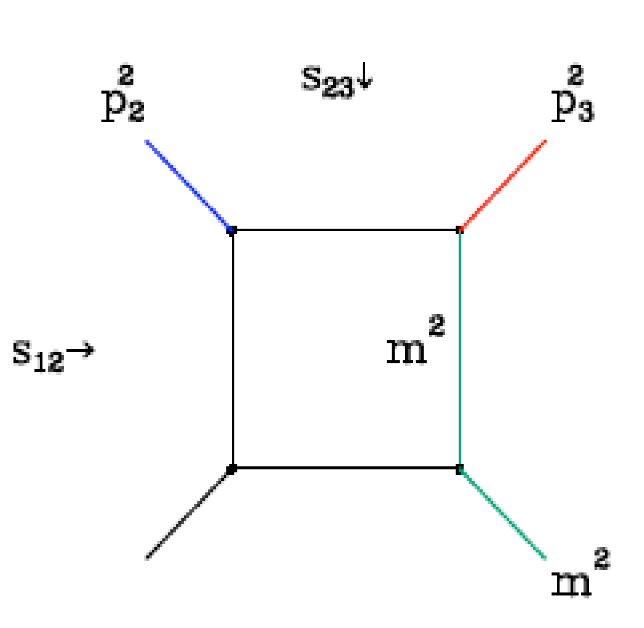
RKE, Zanderighi



QCDLoop

- Analytic results are given for the complete set of divergent box integrals at http://qcdloop.fnal.gov
- Fortran 77 code is provided which calculates an arbitrary scalar box, triangle, bubble or tadpole integral.
- Finite integrals are calculated using the ff library of Van Oldenborgh. (Used also by Looptools)
- For divergent integrals the code returns the coefficients of the Laurent series $1/\varepsilon^2$, $1/\varepsilon$ and finite.
- Problem on one-loop scalar integrals is completely solved numerically and analytically!

Example of box integral from qcdloop.fnal.gov



Basis set of 16 basis integrals allows the calculation of any divergent box diagram.
Result given in the spacelike region.
Analytic continuation as usual by
$s_{ij} \Rightarrow s_{ij} + i\,\varepsilon$

$$I_4^{\{D=4-2\epsilon\}}(0,p_2^2,p_3^2,m^2;s_{12},s_{23};0,0,0,m^2) = \frac{1}{s_{12}(s_{23}-m^2)}\left[\frac{1}{2\epsilon^2} - \frac{1}{\epsilon}\ln\left(\frac{s_{12}}{p_2^2}\frac{(m^2-s_{23})}{\mu m}\right) + \mathrm{Li}_2\left(1+\frac{(m^2-p_3^2)(m^2-s_{23})}{m^2 p_2^2}\right) + 2\,\mathrm{Li}_2\left(1-\frac{s_{12}}{p_2^2}\right) + \frac{\pi^2}{12} + \ln^2\left(\frac{s_{12}}{p_2^2}\frac{(m^2-s_{23})}{\mu m}\right)\right] + \mathcal{O}(\epsilon)\,.$$

Limit $p_3^2 = 0$ can be obtained from this result, (limit $p_2^2 = 0$ cannot)

Determination of coefficients of scalar integrals

Feynman diagrams + Passarino-Veltman reduction cannot be the answer as the number of legs increases. There are too many diagrams with cancellations between them.

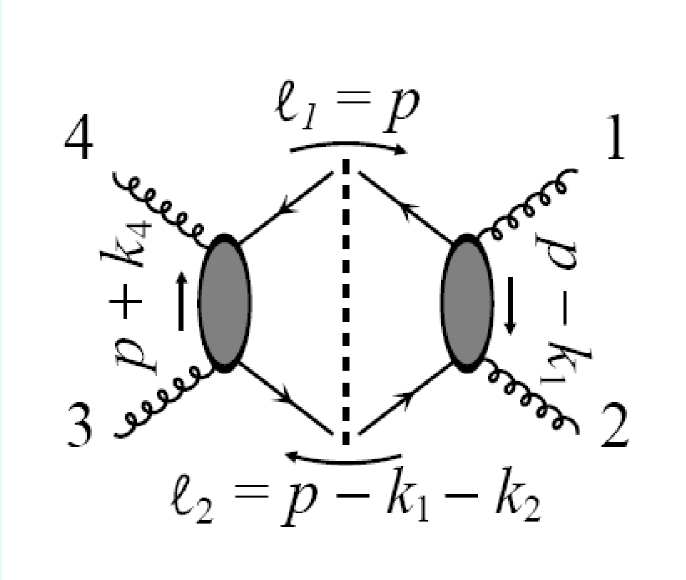
Process	Amplitude	# of diagrams at 1 loop
$t\bar{t}$	$t\bar{t}gg$	30
$t\bar{t}$+1 jet	$t\bar{t}ggg$	341
$t\bar{t}$+2 jets	$t\bar{t}gggg$	4341
$t\bar{t}$+3 jets	$t\bar{t}ggggg$	63800

Semi-numerical methods based on unitarity may be the answer. Note however that the majority of complete n-leg calculations for n>4 are based on Passarino-Veltman.

Basic setup for one-loop diagrams, use of unitarity

$$T^\dagger - T = -2iT^\dagger T$$

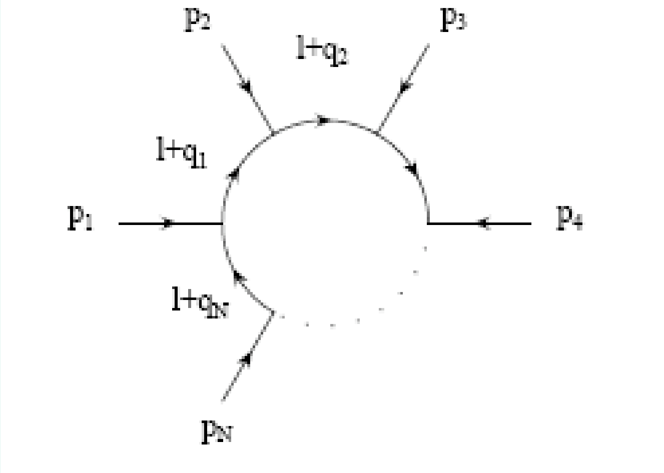
The use of unitarity allows us to recycle tree graph results

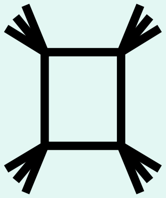
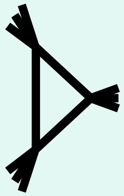
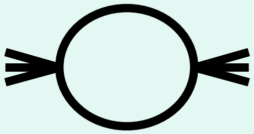


$$-i\,\mathrm{Disc}\,A_4(1,2,3,4)\Big|_{s-\mathrm{cut}} = \int \frac{d^4p}{(2\pi)^4}\, 2\pi\delta^{(+)}(\ell_1^2 - m^2)\, 2\pi\delta^{(+)}(\ell_2^2 - m^2) \times A_4^{\mathrm{tree}}(-\ell_1, 1, 2, \ell_2)\, A_4^{\mathrm{tree}}(-\ell_2, 3, 4, \ell_1)\,,$$

Bern, Dixon, Kosower
Britto, Cacahazo and Feng

Decomposing one-loop N-point amplitudes in terms of master integrals



$\mathcal{A}_N(\{p_i\}) = \sum d_{ijk}$  $+ \sum c_{ij}$  $+ \sum b_i$ 

Any Feynman amplitude can be expressed as a sum of scalar boxes, triangles, bubbles and tadpoles (not shown). In the context of NLO calculations, scalar higher point functions, can always be expressed as sums of box integrals.

Passarino&Veltman('79), Melrose ('65)

Decomposing in terms of ■▲●…

$$
\begin{aligned}
\mathcal{A}_N(p_1, p_2, \ldots, p_N) \; = & \sum_{1 \le i_1 < i_2 < i_3 < i_4 \le N} d_{i_1 i_2 i_3 i_4}(p_1, p_2, \ldots, p_N) I_{i_1 i_2 i_3 i_4} \\
+ & \sum_{1 \le i_1 < i_2 < i_3 \le N} c_{i_1 i_2 i_3}(p_1, p_2, \ldots, p_N) I_{i_1 i_2 i_3} \\
+ & \sum_{1 \le i_1 < i_2 \le N} b_{i_1 i_2}(p_1, p_2, \ldots, p_N) I_{i_1 i_2} \\
+ & \sum_{1 \le i_1 \le N} a_{i_1}(p_1, p_2, \ldots, p_N) I_{i_1} \; ,
\end{aligned}
$$

$$
I_{i_1 \cdots i_M} = \int [d\, l] \, \frac{1}{d_{i_1} \cdots d_{i_M}}
$$

- Without the integral sign, the identification of the coefficients is straightforward.
- Determine the coefficients of a multipole rational function.

Residues of poles and unitarity cuts

Define residue function

$$\text{Res}_{ij\cdots k}\left[F(l)\right] \equiv \left(d_i(l)d_j(l)\cdots d_k(l)F(l)\right)\Big|_{l=l_{ij\cdots k}}$$

We can determine the d-coefficients, then the c-coefficients and so on

$$\begin{aligned}
\overline{d}_{ijkl}(l) &= \text{Res}_{ijkl}\Big(\mathcal{A}_N(l)\Big) \\
\overline{c}_{ijk}(l) &= \text{Res}_{ijk}\left(\mathcal{A}_N(l) - \sum_{l\neq i,j,k}\frac{\overline{d}_{ijkl}(l)}{d_i d_j d_k d_l}\right) \\
\overline{b}_{ij}(l) &= \text{Res}_{ij}\left(\mathcal{A}_N(l) - \sum_{k\neq i,j}\frac{\overline{c}_{ijk}(l)}{d_i d_j d_k} - \frac{1}{2!}\sum_{k,l\neq i,j}\frac{\overline{d}_{ijkl}(l)}{d_i d_j d_k d_l}\right) \\
\overline{a}_i(l) &= \text{Res}_i\left(\mathcal{A}_N(l) - \sum_{j\neq i}\frac{\overline{b}_{ij}(l)}{d_i d_j} - \frac{1}{2!}\sum_{j,k\neq i}\frac{\overline{c}_{ijk}(l)}{d_i d_j d_k} - \frac{1}{3!}\sum_{j,k,l\neq i}\frac{\overline{d}_{ijkl}(l)}{d_i d_j d_k d_l}\right)
\end{aligned}$$

Algebraic reduction, subtraction terms

- Ossola, Papadopoulos and Pittau showed that there is a systematic way of calculating the subtraction terms at the integrand level.
- We can re-express the rational function in an expansion over 4,3,2, and 1 propagator terms.
- The residues of these pole terms contain the l-independent master integral coefficients plus a finite number of spurious terms.

$$\mathcal{A}_N(p_1, p_2, \ldots, p_N|l) = \frac{\mathcal{N}(p_1, p_2, \ldots, p_N; l)}{d_1 d_2 \cdots d_N}$$

$$= \sum_{1 \le i_1 < i_2 < i_3 < i_4 \le N} \frac{\overline{d}_{i_1 i_2 i_3 i_4}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} + \sum_{1 \le i_1 < i_2 < i_3 \le N} \frac{\overline{c}_{i_1 i_2 i_3}(l)}{d_{i_1} d_{i_2} d_{i_3}} + \sum_{1 \le i_1 < i_2 \le N} \frac{\overline{b}_{i_1 i_2}(l)}{d_{i_1} d_{i_2}} + \sum_{1 \le i_1 \le N} \frac{\overline{a}_{i_1}(l)}{d_{i_1}}$$

van Neerven-Vermaseren basis

Example: solve for the box coefficients by setting $d_i = d_j = d_k = d_l = 0$

We find two complex solutions $l_\pm^\mu = V_4^\mu \pm i\sqrt{V_4^2 - m_l^2} \times n_1^\mu$

$$V_4^\mu = -\frac{1}{2}(q_i^2 - m_i^2 + m_l^2)\, v_1^\mu - \frac{1}{2}(q_j^2 - q_i^2 - m_j^2 + m_i^2)\, v_2^\mu - \frac{1}{2}(q_k^2 - q_j^2 - m_k^2 + m_j^2)\, v_3^\mu$$

$$v_1^\mu = \frac{\delta^{\mu k_2 k_3}_{k_1 k_2 k_3}}{\Delta(k_1,k_2,k_3)};\ v_2^\mu = \frac{\delta^{k_1 \mu k_3}_{k_1 k_2 k_3}}{\Delta(k_1,k_2,k_3)};\ v_3^\mu = \frac{\delta^{k_1 k_2 \mu}_{k_1 k_2 k_3}}{\Delta(k_1,k_2,k_3)};\ n_1^\mu = \frac{\varepsilon^{\mu k_1 k_2 k_3}}{\sqrt{\Delta(k_1,k_2,k_3)}}$$

$$v_i \cdot k_j = \delta_{ij}$$

The most general form of the residue is $\overline{d}_{ijkl}(l) = d_{ijkl} + \tilde{d}_{ijkl}\, l \cdot n_1$

$$d_{ijkl} = \frac{\mathrm{Res}_{ijkl}\Big(\mathcal{A}_N(l^+)\Big) + \mathrm{Res}_{ijkl}\Big(\mathcal{A}_N(l^-)\Big)}{2}$$

$$\tilde{d}_{ijkl} = \frac{\mathrm{Res}_{ijkl}\Big(\mathcal{A}_N(l^+)\Big) - \mathrm{Res}_{ijkl}\Big(\mathcal{A}_N(l^-)\Big)}{2i\sqrt{V_4^2 - m_l^2}}$$

Reduction at the integrand level

Spurious terms: residual l-dependence

Finite number of spurious terms:
1 (box), 6 (triangle) 8 (bubble)

2 unknowns

7 unknowns

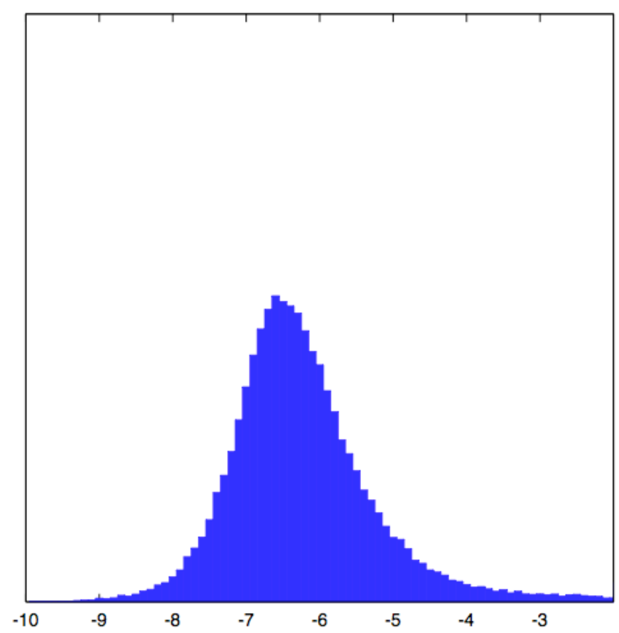
Products of tree amplitudes

Suitable for numerical evaluation
for cut-constructible part

Ellis, Giele, Kunszt

Result for six gluon amplitude

- Results shown here for the cut-constructible part
- The relative error for the finite part of the 6-gluon amplitude compared to the analytic result, for the (+ + - - - -) helicity choice. The horizontal axis is the log of the relative error, the vertical axis is the number of events in arbitrary linear units.
- For most events the error is less than 10^{-6}, although there is a tail extending to higher error.

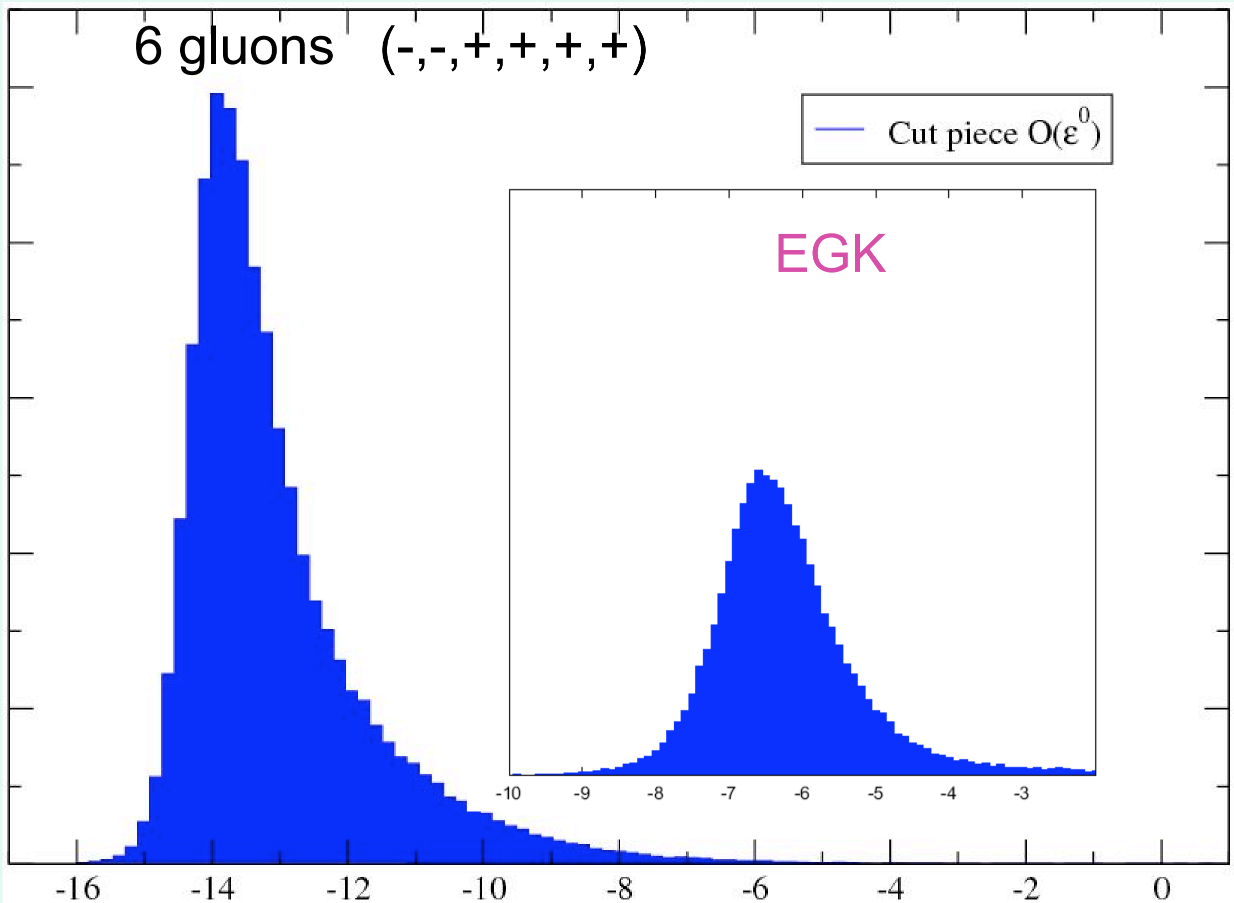


Black-hat=C. Berger, Z. Bern, L Dixon, F. Febres Cordero, D. Forde, H. Ita, D. Kosower, D.Maître

First BlackHat results

0.03 sec/pt

no. of pts



log(rel. error)

Giele, Kunszt,Melnikov, arXiv 0801.2237

Extension to full amplitude

- Keep dimensions of virtual unobserved particles integer and perform calculations in more than one dimension.
- Arrive at non-integer values D=4-2ε by polynomial interpolation.
- Results for six-gluon amplitudes agree with original Feynman diagram calculation of RKE, Giele, Zanderighi.

$\lambda_1, \lambda_2, \ldots, \lambda_6$	$\Delta^{\rm cut}$	$\Delta^{\rm rat}$	Δ
- - + + + +	-19.481065+78.147162 i	28.508591-74.507275 i	9.027526+3.639887 i
- + - + + +	-241.10930+27.176200 i	250.27357-25.695269 i	9.164272+1.480930 i
- + + - + +	5.4801516-12.433657 i	0.19703574+0.25452928 i	5.677187-12.179127 i
- - - + + +	15.478408-2.7380153 i	2.2486654+1.0766607 i	17.727073-1.661354 i
- - + - + +	-339.15056-328.58047 i	348.65907+336.44983 i	9.508509+7.869351 i
- + - + - +	31.947346+507.44665 i	-17.430910-510.42171 i	14.516436-2.975062 i

Recent phenomenological results

- MCFM
- Higgs + 2 jets
- WW+1jet
- Z-bbar and W-bbbar
- ttbar +jet

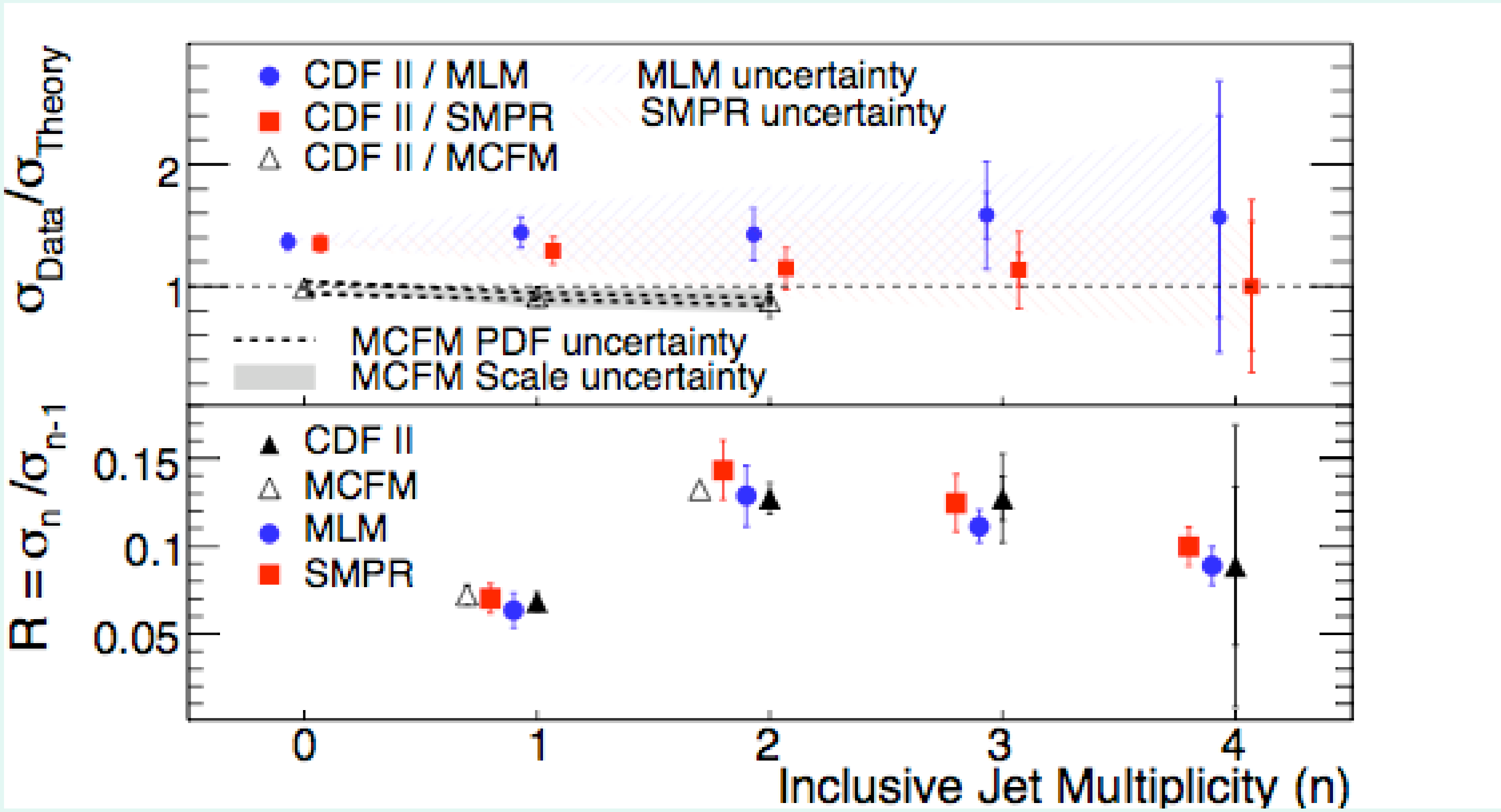
MCFM
A NLO parton level generator

- pp ➔ W/Z
- pp ➔ W+Z, WW, ZZ
- pp ➔ W/Z + 1 jet
- pp ➔ W/Z +2 jets
- pp ➔ t W
- pp ➔ tX (s&t channel)
- pp ➔ tt
- pp ➔ W/Z+H
- pp (gg) ➔ H
- pp➔(gg) ➔ H + 1 jet
- pp➔(gg) ➔ H + 2 jets
- pp(VV) ➔ H +2 jets
- pp ➔ W/Z +b , W+c
- pp ➔ W/Z +bb

Processes calculated at NLO, but no automatic procedure for including new processes.

Code available at http://mcfm.fnal.gov (used for many Tevatron analyses).

W+n jet rates from CDF



Both uncertainty on rates and deviation of Data/Theory from 1 are smaller than other calculations. “Berends” ratio agrees well for all calculations but only available for n=2 from MCFM

John Campbell

Campbell, Ellis, Zanderighi

Higgs+2 jets at NLO

- Calculation performed using an effective Lagrangian, valid in the large m_t limit.

$$\mathcal{L}_{\rm eff} = \frac{1}{4} A(1+\Delta) H G^a_{\mu\nu} G^{a\,\mu\nu}$$

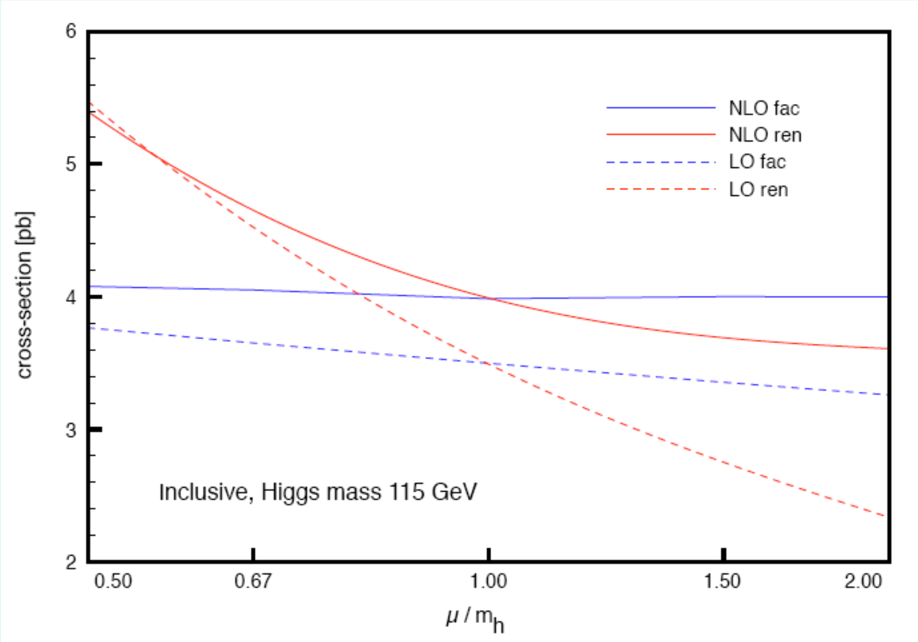
Three basic processes at lowest order.

$$\begin{aligned} &A) \quad 0 \to H q\bar{q} q' \bar{q}' g \,, \\ &B) \quad 0 \to H q\bar{q} g g g \,, \\ &C) \quad 0 \to H g g g g g \,. \end{aligned}$$

Higgs + 2 jet continued

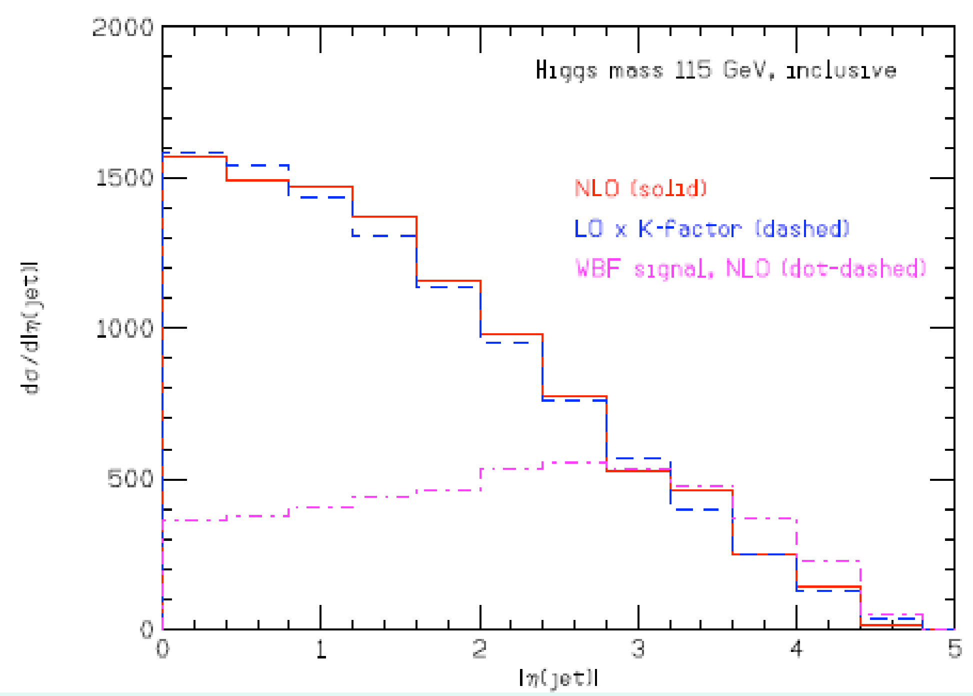
- NLO corrections are quite mild, increasing LO cross section by 15%
- NLO cross section contains a considerable residual scale uncertainty.

Higgs mass	115 GeV	160 GeV
σ_{LO} [pb]	3.50	2.19
σ_{NLO} [pb]	4.03	2.76
σ_{WBF} [pb]	1.77	1.32



Higgs + 2 jets rapidity distribution versus WBF

- Shape of NLO result, similar to LO in rapidity.
- WBF shape is quite different at NLO.



Campbell et al, arXiv:0710.1832 [hep-ph]

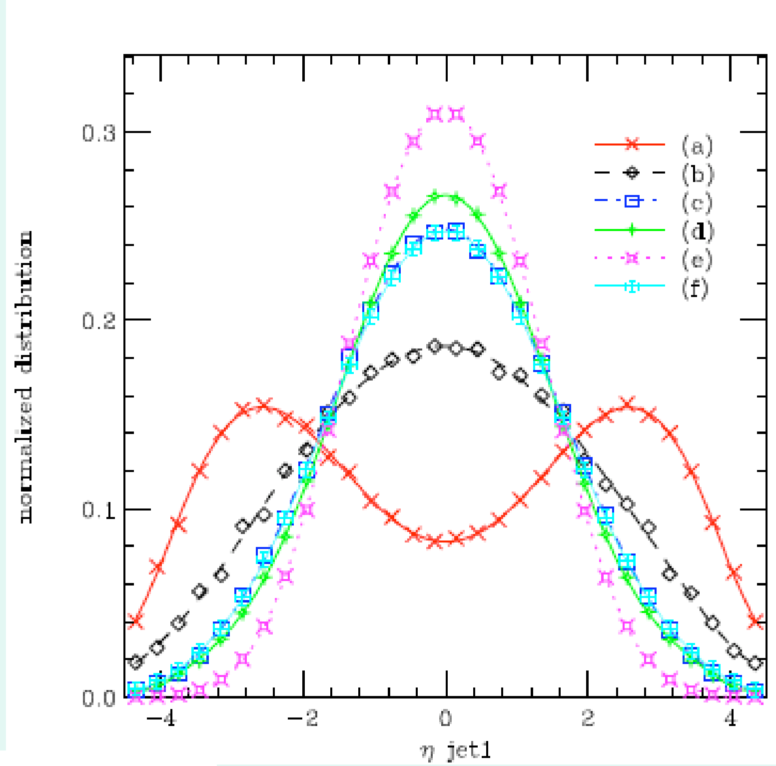
Dittamaier et al, arXiv:0710.1577 [hep-ph]

WW+1jet

WW+1 jet and impact on
Higgs->WW + 1 jet search

Rates with cuts I+II

Process	σ_{LO} (fb)	σ_{NLO} (fb)
(a) $H \to WW$ (WBF)	10.6	10.6
(b) $H \to WW$ (gluon fusion)	8.6	18.0
(c) WW+jet	11.7	20.2
(d) $W + t$	7.8	7.6
(e) $t\bar{t}$	12.7	-
(f) ZZ+jet	0.44	-



Standard Cuts I : $P_{\rm t,j1} > 30$ GeV, $|\eta_{\rm j1}| < 4.5$

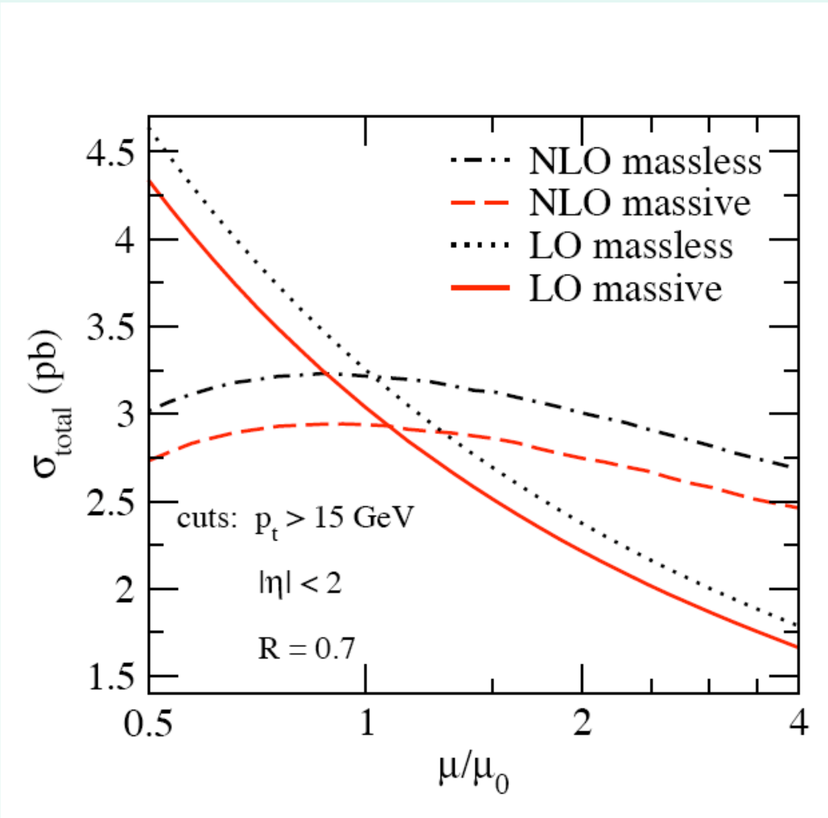
$$P_{\rm t,miss} > 30\ {\rm GeV}\,, \quad P_{{\rm t},\ell_1} > 20\ {\rm GeV}\,, \quad P_{{\rm t},\ell_2} > 10\ {\rm GeV}\,, \quad |\eta_{\ell_1(\ell_2)}| < 2.5\,.$$

Cuts II: $|\eta_{\rm j_1}| > 1.8$ $|\eta_{\rm j_2}| > 2.5$ $\phi_{\ell_1,\ell_2} < 1.2$ $m_{\ell_1,\ell_2} < 75$ GeV

Febres Cordero, Reina, Wackeroth arXiv:0801.2374v1

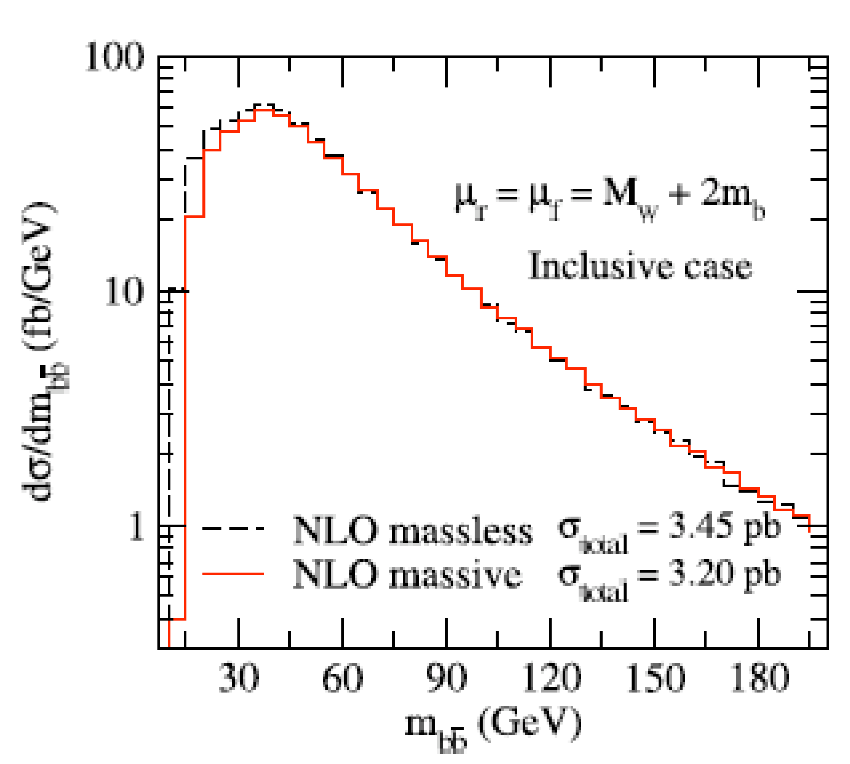
New results on Zbb

New results have recently become available for Zbb



Uncertainty on exclusive NLO cross-section ~10%

They complement the earlier results for Wbb, hep-ph/0606102

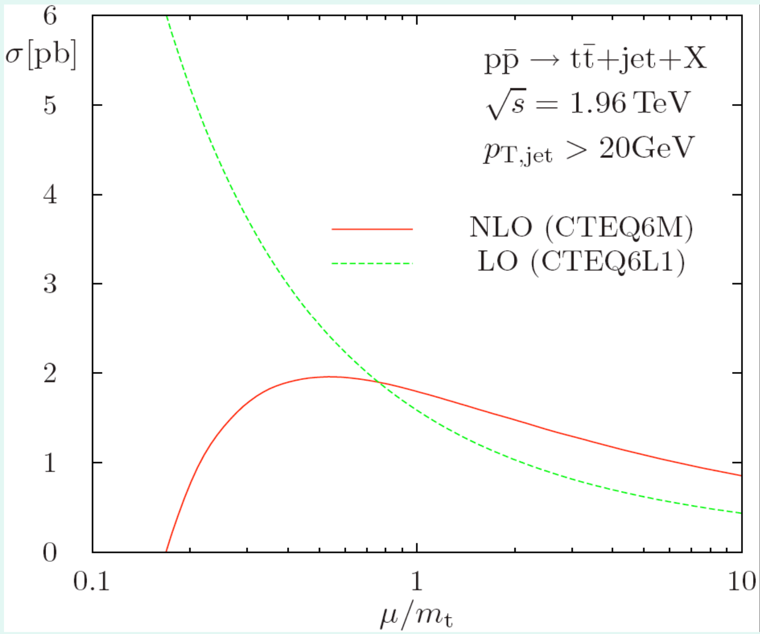


Neglecting m_b overestimates total cross section 8-10%

Dittmaier, Uwer, Weinzierl, hep-ph/0703120

NLO correction to tt + jet

- Calculation which involves pentagon diagrams, performed using traditional methods, (Passarino-Veltman reduction).
- With p_T>20GeV jet cut tt+1jet cross section is a sizeable fraction of total ttbar rate
- Code not yet available.



Compare CDF
σ_{tt}**=7.3±0.5(stat)±0.6(syst)±0.4(lumi)pb**

Summary

- $\alpha_S(M_Z)$ is known to < 2%
- Parton distributions are known well enough to predict most cross sections to 20%, ($0.005<x<0.3$)
- Calculation of tree graphs is a solved problem, for all practical purposes. Berends-Giele recursion is numerically the best method.
- Open theoretical problem is thus the calculation of one-loop amplitudes. There is currently great intellectual fervor regarding the calculation of one-loop corrections.
- Unitarity based methods have achieved important results for one-loop diagrams, but not all semi-numerical methods have been tested in real physical calculations.
- Remaining challenge is to assemble into a program with efficient phase space sampling.
- The hope is to have several semi-automatic methods of calculating one-loop amplitudes (time scale about 1 year).