The secret life of rational terms

Yu-tin Huang
w
WeiMing Chen, David McGady (1307.4065, 1402.7062)

IAS

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Prelude

$$\mathcal{A}_n = C_n \, (\text{branch cuts}) + R_n$$

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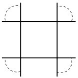
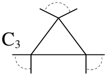
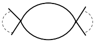
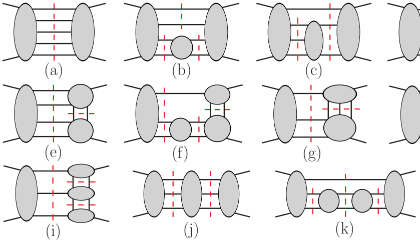
$= C_4$  C_3  C_2  $+ \underbrace{R_n}_{\color{red}?}$

Prelude

$\mathcal{A}_n \quad =$



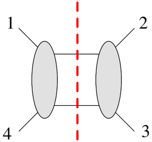
$+ \underbrace{R_n}_{?}$

How do we see chiral theories are sick?

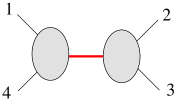
Can it be framed without Feynman diagrams, gauge redundancy, path integral, symmetry violating/preserving regulator ... ?

Gauge anomalies

Branch cuts



Factorization

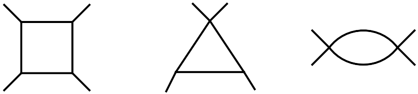


Consider 1-loop 4-pt

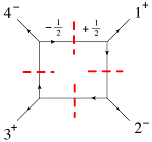
$$\mathcal{A}_4 = Tr(1234)A(1234) + Tr(1342)A(1342) + Tr(1423)A(1423) + \text{flip}$$

The color-ordered amplitude can be conveniently written as:

$$A(1234) = C_4 I_4 + C_{3s} I_{3s} + C_{3t} I_{3t} + C_{2s} I_{2s} + C_{2t} I_{2t} + R$$



$$-\frac{t^4 s^2}{u^4}$$



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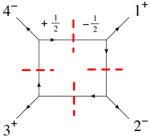

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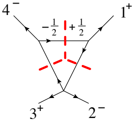

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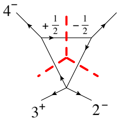
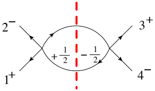
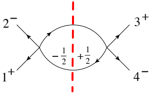


$$\frac{t(su - 6st - 2ut)}{6u^3}$$



$$\frac{t(4s^2 + 2t^2 - 7su)}{6u^3}$$



Four-dimensional prelude: (Parity-even)

Parity-even:

$$A^{\text{even}}(1^+2^-3^+4^-) = A^{\text{tree}}\Bigg\{ -\frac{st(s^2+t^2)}{2u^4}\left(\log^2 x + \pi^2\right)$$
$$+\left[\left(\frac{s-t}{3u} - \frac{st(s-t)}{u^3}\right)\right]\log x - \frac{(-s)^{-\epsilon}+(-t)^{-\epsilon}}{3\epsilon} + R^{\text{even}}\Bigg\}$$

$$x = \frac{s}{t}, \quad u = (p_1+p_3)^2$$

Locality requires these spurious poles to be absent.

$$A^{\text{even}}\Big|_{u\to 0} = A^{\text{tree}}\left\{ -\frac{s^2}{u^2} - \frac{s}{u} + \mathcal{O}(u^0)\right\}.$$

$$A^{\text{tree}} R^{\text{even}} = -A^{\text{tree}}\frac{st}{u^2}$$

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$$A^{\text{odd}}\Big|_{u\to 0} = A^{\text{tree}}\left\{-\frac{s}{u}+\mathcal{O}(u^0).\right\}$$

$$A^{\text{tree}}R^{\text{odd}} = A^{\text{tree}}\frac{s-t}{2u}$$

The cancelation of spurious singularities introduces a new factorization channel !

$$Res_s[A^{\text{tree}}\frac{s-t}{2u}] = \frac{1}{2}A_3A_3, \quad Res_t[A^{\text{tree}}\frac{s-t}{2u}] = -\frac{1}{2}A_3A_3$$

with inconsistent factorization

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$$\mathcal{R}^{\text{odd}} = \frac{\langle 24\rangle^2[13]^2}{2stu}[(s-t)Tr(1234)+(u-s)Tr(1342)+(t-u)Tr(1423) \\ +(s-u)Tr(1243)+(u-t)Tr(1324)+(t-s)Tr(1432)]\,.$$

Let us consider the residue for the $s\to 0$

$$\frac{\langle 24\rangle^2[13]^2}{2su}[-Tr(1234)-Tr(1342)+Tr(1243)+Tr(1432)]$$

$$Tr(1432)-Tr(1234)+(1\leftrightarrow 2)=d^{1a4}f^{23}{}_a+d^{13a}f^{24}{}_a+d^{1a2}f^{34}{}_a+(1\leftrightarrow 2)=0\,.$$

$$d^{abc}f^{de}{}_a=0\,.$$

The non-abelian box-anomaly

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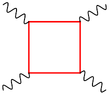
Anomaly cancellation in $D > 4$

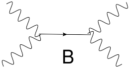


$$R^{anom} = -\frac{1}{18}\left[\left(\frac{(\epsilon_1\cdot k_2)}{s}+\frac{(\epsilon_1\cdot k_3)}{u}+\frac{(\epsilon_1\cdot k_4)}{t}\right)F_2\wedge F_3\wedge F_4+(\text{cyclic})\right]$$

The GS mechanism

$$\mathcal{L}=\epsilon^{abcdef}B_{ab}tr(F_{cd}F_{ef})+H_{abc}H^{abc},\quad H_{abc}=d_{[a}B_{bc]}+tr(A_{[a}d_bA_{c]})$$

$$\delta_v B_{ab}=0,\qquad \delta_v B_{ab}=[\Lambda, B_{ab}]$$

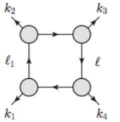
$\delta\Big($  $\Big) = F\wedge F\wedge F$

The computation:

$$(\lambda_\ell)^A{}_a = \begin{pmatrix} -\kappa\lambda_j & \frac{(1-y)}{c}\lambda_j + \lambda_i \\ c\tilde{\lambda}_j + y\tilde{\lambda}_i\,, & \frac{\tilde{\kappa}}{c}\tilde{\lambda}_i \end{pmatrix}, \quad \kappa = \frac{m}{\langle ij\rangle}, \quad \tilde{\kappa} = \frac{\tilde{m}}{[ij]},$$

$$(\tilde{\lambda}_\ell)_{A\dot{a}} = \begin{pmatrix} \kappa'\lambda_j & \frac{(1-y)}{c}\lambda_j + \lambda_i \\ -c\tilde{\lambda}_j - y\tilde{\lambda}_i & \frac{\tilde{\kappa}'}{c}\tilde{\lambda}_i \end{pmatrix}, \quad \kappa' = \frac{\tilde{m}}{\langle ij\rangle}, \quad \tilde{\kappa}' = \frac{m}{[ij]},$$

$(k^{(6)})^2 = (k^{(4)})^2 + m\tilde{m}$ with $m = k_4 + ik_5, \quad \tilde{m} = m^*$



$$(\ell+k_3)^2\,(\ell_1+k_1)^2 A((-\ell_1)_a\, 2_{2\dot{1}}\, 3_{2\dot{2}}\, \ell_b)\; A((-\ell)^b\, 4_{1\dot{1}}\, 1_{1\dot{2}}\, \ell_1^a)$$

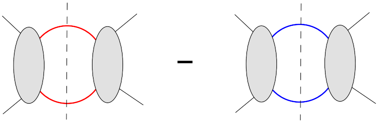
$$\to C_4 = \left(\frac{2m\tilde{m}}{t} - 1\right)\langle 23\rangle^2[24]^2\left(\frac{m\tilde{m}}{u} + \frac{st}{2u^2}\right)$$

$$m\tilde{m} = -16\,\frac{Gram(\ell, k_1, k_2, k_3, k_4')}{(stu)^2}$$

with $k_4' = \epsilon^{\mu\nu\rho\sigma}k_{1\,\mu}k_{2\,\nu}k_{3\,\rho}$

Six-dimensions (Parity-odd)

$$A^{\text{odd}}(1,2,3,4) = C_4 I_4 + C_{3s} I_{3s} + C_{3t} I_{3t} + C_{2s} I_{2s} + C_{2t} I_{2t} + R^{\text{odd}}$$



$$C_4 = \frac{(s-t)}{6u^2} F^4 , \quad C_{3s} = -\frac{(s-t)}{6tu^2} F^4 , \quad C_{2s} = \frac{F^4}{stu} ,$$

$$F^4 \equiv \langle 4_d | p_2 p_3 | 4_{\dot d}] (\langle 1_a | 2_{\dot b}] \langle 2_b | 3_{\dot c}] \langle 3_c | 1_{\dot a}] + \langle 2_b | 1_{\dot a}] \langle 1_a | 3_{\dot c}] \langle 3_c | 2_{\dot b}]) + (\sigma_i)\text{cyclic} ,$$

$$F^4 = \Big((\epsilon_4 \cdot p_1) s - (\epsilon_4 \cdot p_3) t \Big) F_1 \wedge F_2 \wedge F_3 + (\sigma_i)\text{cyclic}$$

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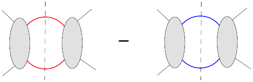

$$C_4 = \frac{(s-t)}{6u^2}F^4, \quad C_{3s} = -\frac{(s-t)}{6tu^2}F^4, \quad C_{2s} = \frac{F^4}{stu},$$

$$\begin{aligned} I_3[K^2] &= \frac{1}{2\epsilon} + \frac{1}{2}\left(3 - \gamma_E - \log[K^2]\right), \\ I_2[K^2] &= -\frac{K^2}{6\epsilon} + \frac{K^2}{18}\left(-8 + 3\gamma_E + 3\log[K^2]\right), \\ I_4(1,2,3,4) &= -\frac{\log^2 x + \pi^2}{2u}. \end{aligned}$$

$$A^{\text{odd}}(1,2,3,4) = F^4\left(\frac{(t-s)(\pi^2+\log^2 x)}{12u^3} + \frac{\log x}{3u^2} + \frac{s-t}{18stu} + R^{\text{odd}}\right)$$

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$$\mathcal{A}_4 \xrightarrow{u=0} -\frac{F^4}{18ut}str(T^4)+\mathcal{O}(u^0), \quad \mathcal{A}_4 \xrightarrow{s=0} -\frac{F^4}{18su}str(T^4)+\mathcal{O}(s^0)$$
$$\mathcal{A}_4 \xrightarrow{t=0} -\frac{F^4}{18ts}str(T^4)+\mathcal{O}(t^0)\,, \tag{1}$$

There are no factorization channels if $str(T^4)=0$, no $R^{\rm odd}$

$$str(T^4)\neq 0 \rightarrow \begin{cases} str(T^4)=tr(tt)tr(tt) \\ Res_u[-\frac{F^4}{18ut}]=A_3A_3 \end{cases}$$

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Six-dimensions

What is A_3 in $Res_u[-\frac{F^4}{18ut}] = A_3 A_3$?

$$\langle 1_a | 2_{\dot a}] \equiv u_{1a}\tilde u_{2\dot a}$$

$$u_{ia}w_{ib} - u_{ib}w_{ia} = \epsilon_{ab}, \qquad \tilde u_{i\dot a}\tilde w_{i\dot b} - \tilde u_{i\dot b}\tilde w_{i\dot a} = \epsilon_{\dot a\dot b}$$

The definitions are invariant under the following rescaling and shift symmetries:

$$\begin{aligned}(u_i, w_i) &\to (\alpha u_i, \alpha^{-1} w_i), & (\tilde u_i, \tilde w_i) &\to (\alpha^{-1}\tilde u_i, \alpha\tilde w_i),\\ w_i &\to w_i + b_i u_i, & \tilde w_i &\to \tilde w_i + \tilde b_i \tilde u_i\,.\end{aligned}$$

A_3 involving one (anti-) chiral two-form and two vector fields are unique:

$$A^B_{(ad),b\dot b,c\dot c} = \left(\Delta_{(a|bc}u_{1|d)}\right)\tilde u_{2\dot b}\tilde u_{3\dot c}\,, \quad A^{\bar B}_{(\dot a\dot d),b\dot b,c\dot c} = u_{2b}u_{3c}\left(\tilde\Delta_{(\dot a|\dot b\dot c}\tilde u_{1|\dot d)}\right),$$

where

$$\Delta_{abc} \quad = \quad (u_{1a}u_{2b}w_{3c} + u_{2b}u_{3c}w_{1a} + u_{3c}u_{1a}w_{2b})\,,$$

Six-dimensions

What is A_3 in $Res_t[-\frac{F^4}{18ut}] = A_3 A_3$?

$$A^{B}_{(ad),b\dot{b},c\dot{c}} = \left(\Delta_{(a|bc}u_{1|d)}\right)\tilde{u}_{2\dot{b}}\tilde{u}_{3\dot{c}}\,,\quad A^{\bar{B}}_{(\dot{a}\dot{d}),b\dot{b},c\dot{c}} = u_{2b}u_{3c}\left(\tilde{\Delta}_{(\dot{a}|\dot{b}\dot{c}}\tilde{u}_{1|\dot{d})}\right),$$

$$\begin{aligned} &A^{B}_{3L(ab)}\;A^{B}_{3R}{}^{(ab)} - A^{\bar{B}}_{3L(\dot{a}\dot{b})}\;A^{\bar{B}}_{3R}{}^{(\dot{a}\dot{b})} \\ &= -\frac{F^4}{s} \end{aligned}$$

The result from the unitarity cut contains a two-form factorization channel !

Six-dimensions

$$str(T^4) \neq 0 \to str(T^4) = tr(tt)tr(tt)$$

We are not done yet

$$\mathcal{A}_4 \xrightarrow{u=0} -\frac{F^4}{18ut}\left(tr(t_1t_2)(t_3t_4) + tr(t_1t_3)(t_2t_4) + tr(t_1t_4)(t_3t_2)\right) + \mathcal{O}(u^0)\,.$$

Clearly only $tr(t_1t_3)(t_2t_4)$ makes sense for the u channel pole

$$\mathcal{R} = F^4 tr\left(tr(t_1t_2)(t_3t_4)\frac{u-t}{18stu} + tr(t_1t_3)(t_2t_4)\frac{t-s}{18stu} + tr(t_1t_4)(t_3t_2)\frac{s-u}{18stu}\right)\,.$$

With the above rational term, we now find:

$$\mathcal{A}_4 \xrightarrow{u=0} -\frac{F^4}{6ut}tr(t_1t_3)(t_2t_4) + \mathcal{O}(u^0), \quad \mathcal{A}_4 \xrightarrow{s=0} -\frac{F^4}{6su}tr(t_1t_2)(t_3t_4) + \mathcal{O}(s^0)$$

$$\mathcal{A}_4 \xrightarrow{t=0} -\frac{F^4}{6ts}tr(t_1t_4)(t_3t_2) + \mathcal{O}(t^0)\,.$$

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$$str(T^4) \neq 0 \to str(T^4) = tr(tt)tr(tt)$$

We are not done yet

$$\mathcal{A}_4 \xrightarrow{u=0} -\frac{F^4}{18ut}\left(tr(t_1t_2)(t_3t_4) + tr(t_1t_3)(t_2t_4) + tr(t_1t_4)(t_3t_2)\right) + \mathcal{O}(u^0)\,.$$

Clearly only $tr(t_1t_3)(t_2t_4)$ makes sense for the u channel pole

$$\mathcal{R} = F^4 tr\left(tr(t_1t_2)(t_3t_4)\frac{u-t}{18stu} + tr(t_1t_3)(t_2t_4)\frac{t-s}{18stu} + tr(t_1t_4)(t_3t_2)\frac{s-u}{18stu}\right)\,.$$

With the above rational term, we now find:

$$\mathcal{A}_4 \xrightarrow{u=0} -\frac{F^4}{6ut}tr(t_1t_3)(t_2t_4) + \mathcal{O}(u^0),\quad \mathcal{A}_4 \xrightarrow{s=0} -\frac{F^4}{6su}tr(t_1t_2)(t_3t_4) + \mathcal{O}(s^0)$$

$$\mathcal{A}_4 \xrightarrow{t=0} -\frac{F^4}{6ts}tr(t_1t_4)(t_3t_2) + \mathcal{O}(t^0)\,.$$

Six-dimensions

- Is there a pure two-form exchange tree-amplitude?
- What is $\mathcal{R}$ in traditional approach?

Six-dimensions

Can we construct a parity-odd rational 4 gluon amplitude whose factorization channels include the two-form exchange?

$$A^{B}_{3L(ab)}\,A^{B}_{3R}{}^{(ab)} - A^{\bar{B}}_{3L(\dot{a}\dot{b})}\,A^{\bar{B}}_{3R}{}^{(\dot{a}\dot{b})}$$
$$= -\frac{F^4}{s}$$

t-channel exchange also involves an s (u) channel pole:

$$\text{cyclic sym} \;\rightarrow\; A^{B}_{4} = \frac{F^4(s-t)}{stu}$$

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But then we would have inconsistent residue

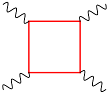
$$Res_u\left[A^B_4\right] = \frac{-2F^{(4)}}{s},\; Res_s\left[A^B_4\right] = \frac{-F^{(4)}}{u},\; Res_t\left[A^B_4\right] = \frac{F^{(4)}}{u}\,.$$

Parity-odd two-form exchange is only consistent if combined with a chiral fermion loop

Six-dimensions

$$\mathcal{R} = F^4 tr\left(tr(t_1 t_2)(t_3 t_4)\frac{u-t}{18stu} + tr(t_1 t_3)(t_2 t_4)\frac{t-s}{18stu} + tr(t_1 t_4)(t_3 t_2)\frac{s-u}{18stu}\right) .$$

Where did this factor come from?



$$R^{anom} = -\frac{1}{18}\left[\left(\frac{(\epsilon_1\cdot k_2)}{s} + \frac{(\epsilon_1\cdot k_3)}{u} + \frac{(\epsilon_1\cdot k_4)}{t}\right) F_2\wedge F_3\wedge F_4 + (\text{cyclic})\right]$$

The GS mechanism

$$\mathcal{L} = \epsilon^{abcdef} B_{ab} tr(F_{cd}F_{ef}) + H_{abc}H^{abc}, \quad H_{abc} = d_{[a}B_{bc]} + tr(A_{[a}d_b A_{c]})$$

Adding the GS term for $tr(t_1 t_3)tr(t_2 t_4)$

$$\begin{aligned} R^{anom} + GS &= \frac{-1}{18}\left[\left(\frac{(\epsilon_1\cdot k_2)}{s} - 2\frac{(\epsilon_1\cdot k_3)}{u} + \frac{(\epsilon_1\cdot k_4)}{t}\right) F_2\wedge F_3\wedge F_4 + (\text{cyclic})\right] \\ &= F^4\frac{t-s}{18stu} \end{aligned}$$

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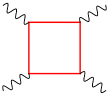

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D-dimensions

$$D=6 \quad R^{anom} = \frac{-1}{3*4!Gr_{(1,2,3)}}\Big\{[tu(\epsilon_1\cdot k_2)+st(\epsilon_1\cdot k_3)+su(\epsilon_1\cdot k_4)]$$
$$F_2\wedge F_3\wedge F_4+(\text{cyclic})\Big\}$$

$$D=8 \quad R^{anom} = \frac{-1}{4*5!Gr_{(1,2,3,4)}}\Big\{[G_{(1234)}+G_{(1345)}+G_{(1235)}+G_{(1245)}]|_{k_1\to e_1}$$
$$\times F_2\wedge\cdots\wedge F_5+(\text{cyclic})\Big\}$$

$$D=10 \quad R^{anom} = \frac{-1}{5*6!Gr_{(1,2,3,4,5)}}\Big\{[G_{(12345)}+G_{(13456)}+G_{(12356)}$$
$$+G_{(12456)}+G_{(12346)}]|_{k_1\to e_1}\times F_2\wedge\cdots\wedge F_6+(\text{cyclic})\Big\}$$

$$GS\sim\left(\frac{\epsilon_n\cdot k_1}{s_{n1}}+\frac{\epsilon_n\cdot k_{n-1}}{s_{nn-1}}\right)F_1\wedge F_2\wedge\cdots F_{n-1}+(\text{cyclic})$$

$$C_n=(R^{\text{anom}}+GS)(\prod_{i=1}^{n}s_{i,i+1})/Gram(1,2,\cdots,n-1)$$

Summary

There is no gauge anomaly per se..

- Rational terms holds the key to locality.
- For chiral theories, rational terms hold locality for ransom
- Anomaly cancellation + GS mechanism $\rightarrow$ off-shell way of obtaining *R*
- Rational terms occur only for $\mathrm{D} = \mathrm{even}, \mathrm{n} = \mathrm{D}/2 + 1.$
- Similar construction has been applied to gravitational, mixed anomaly.

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D dimensional unitarity cuts

Are there rational terms that are local and unitary by itself ? $\rightarrow$ It has an integrand but vanishing cuts

$$\int d\ell^D d\mu^{-2\epsilon} \frac{(\mu^2)^n}{\cdots(\ell^2+\mu^2)((\ell+K)^2+\mu^2+)\cdots}$$

These terms are detectable by embedding the theory in higher-dimensions

Half-maximal sugra

D=6 N=(1,1) sugra, $(\eta_b, \tilde{\eta}_{\dot{b}}) = (-\frac{1}{2}, -\frac{1}{2})$ Sp(2)$\times$Sp(2)

$$A_{a\dot{a}}(\ \eta_b, \tilde{\eta}_{\dot{b}}\ ) = v_{a\dot{a}} + \cdots + \eta^b \tilde{\eta}^{\dot{b}} g_{(ba);(\dot{b}\dot{a})} + \cdots$$

$$M_4 = \frac{\delta^4(Q)\delta^4(\tilde{Q})\langle 1_{a_1} 2_{a_2} 3_{a_3} 4_{a_4}\rangle [1_{\dot{a}_1} 2_{\dot{a}_2} 3_{\dot{a}_3} 4_{\dot{a}_4}]}{stu}$$

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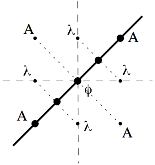
project this to D=4 $\rightarrow$ N=4 SUGRA + 2 N=4 SYM



$$\begin{aligned}
A^- = A_{1\dot{1}} &= g^- + \cdots + \eta^{+\frac{1}{2}} \tilde{\eta}^{+\frac{1}{2}} h^{-2} + \eta^{-\frac{1}{2}} \tilde{\eta}^{-\frac{1}{2}} t + \cdots \\
A^+ = A_{2\dot{2}} &= g^+ + \cdots + \eta^{-\frac{1}{2}} \tilde{\eta}^{-\frac{1}{2}} h^{+2} + \eta^{-\frac{1}{2}} \tilde{\eta}^{-\frac{1}{2}} \bar{t} + \cdots \\
A^{v_i} = A_{2\dot{1}},\ A_{1\dot{2}} &= \phi + \cdots + \eta^{+\frac{1}{2}} \tilde{\eta}^{+\frac{1}{2}} g^- + \eta^{-\frac{1}{2}} \tilde{\eta}^{-\frac{1}{2}} g^+ + \cdots
\end{aligned}$$

$(t, \bar{t})$ are charged under the U(1), SU(1,1)/U(1)

Half-maximal sugra

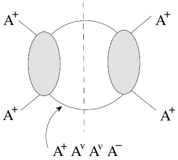
The U(1) duality symmetry:

U(1) rotates $(t, \bar{t})$ as well as (g^-, g^+) has an interpretation of electric/magnetic duality rotation:

$$\langle A^- A^- A^- A^- \rangle = \langle A^- A^- A^- A^+ \rangle = \langle A^+ A^+ A^+ A^- \rangle = \langle A^+ A^+ A^+ A^+ \rangle = 0$$

Explicit computation yields

$\langle A^+ A^+ A^+ A^+ \rangle \quad =$



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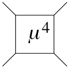

$$\frac{\delta^4(Q)\delta^4(\tilde{Q})[12]^2[34]^2}{s^2}$$

$$\rightarrow \quad (2 + n_v)\langle h_1^{+2} h_2^{+2} \bar{t}_3 \bar{t}_4 \rangle = [12]^4$$

Agrees with color-kinematic duality Carrasco, Kallosh, Roiban and Tseytlin 1303.6219

Rational term breaks U(1) global symmetry $\rightarrow$ soft-scalar limit $\neq 0$

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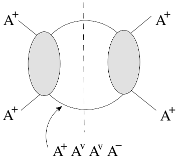

$$= \boxed{\mu^4}\ \frac{\delta^4(Q)\delta^4(\tilde{Q})[12]^2[34]^2}{s^2}$$

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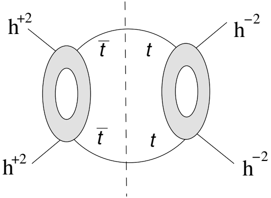
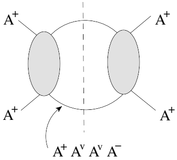
This anomalous term is detectable through unitarity cuts Carrasco, Kallosh, Roiban and Tseytlin 1303.6219



Contributes to UV-divergences at four-loops Bern, Dennen, Davies, A Smirnov and V Smirnov 1309.2498

$\langle A^+ A^+ A^+ A^+ \rangle \quad =$



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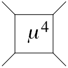

$$\frac{\delta^4(Q)\delta^4(\tilde{Q})[12]^2[34]^2}{s^2}$$

$\mathcal{N}$ supergravity has potential global anomalies for $\mathcal{N} \leq 4$

$$\mathcal{A}_n = \underbrace{C_n\,(\text{branch cuts})}_{\text{UV, IR - divergences}} + \underbrace{R_n}_{\text{Local anomalies}} + \underbrace{R'_n}_{\text{Global anomalies}}$$