Finite Temperature Quark-Gluon Vertex with a Magnetic Field in the Hard Themal Loop Approximation

M. Loewe

Pontificia Universidad Católica de Chile (PUC) and Centro Científico y Tecnológico de Valparaíso (CCTVAL)

Valparaíso, March 2015

This talk is based on the following article:

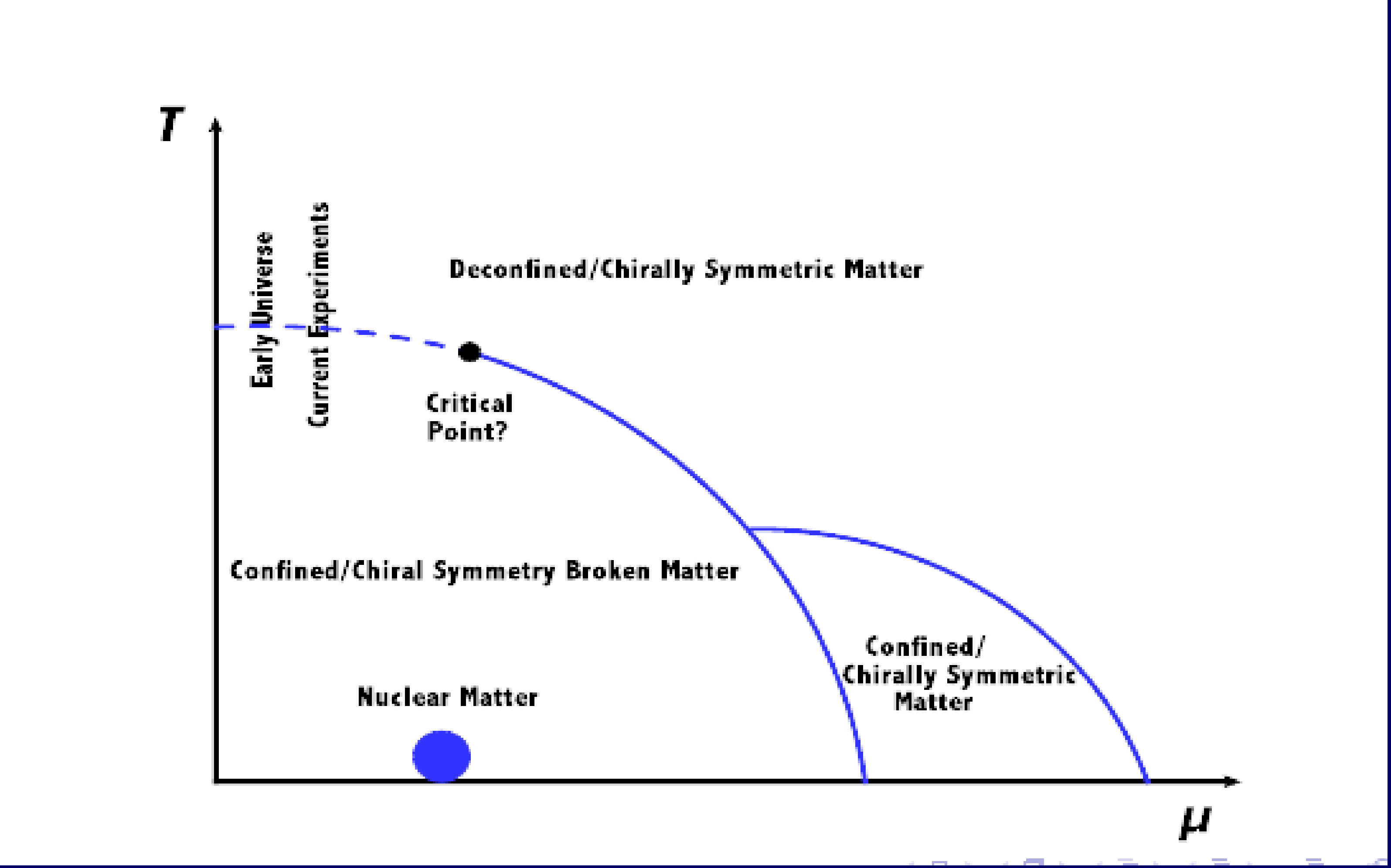
Finite temperature Quark-Gluon Vertex with a magnetic field in the hard thermal loop approximation.

A. Ayala, J. J. Cobos-Martínez, M. Loewe, M. Elena Tejeda-Yeomans, and R. Zamora

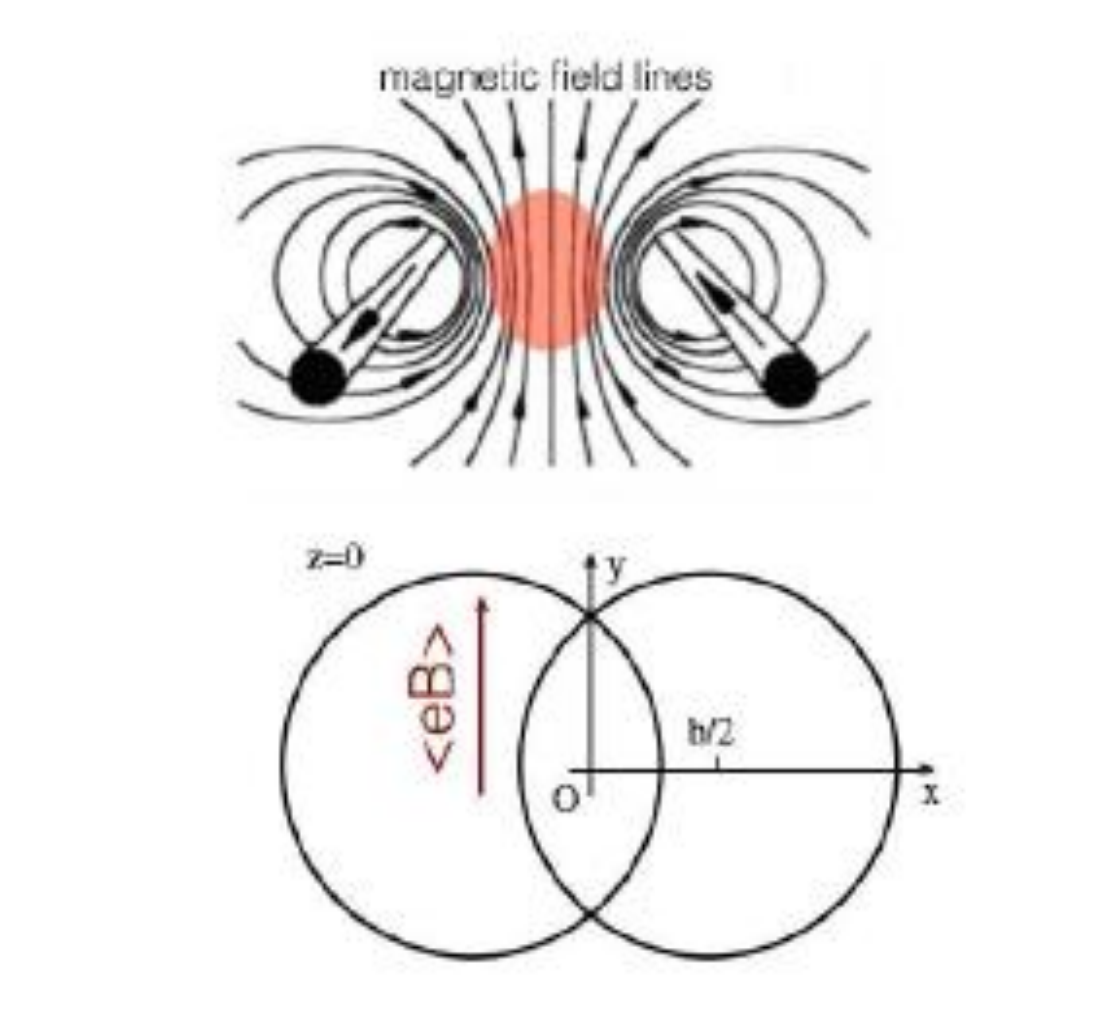
Physical Review D 91, 016007 (2015)

I acknowledge support form : FONDECYT 1130056 and 1120770 (CHILE)

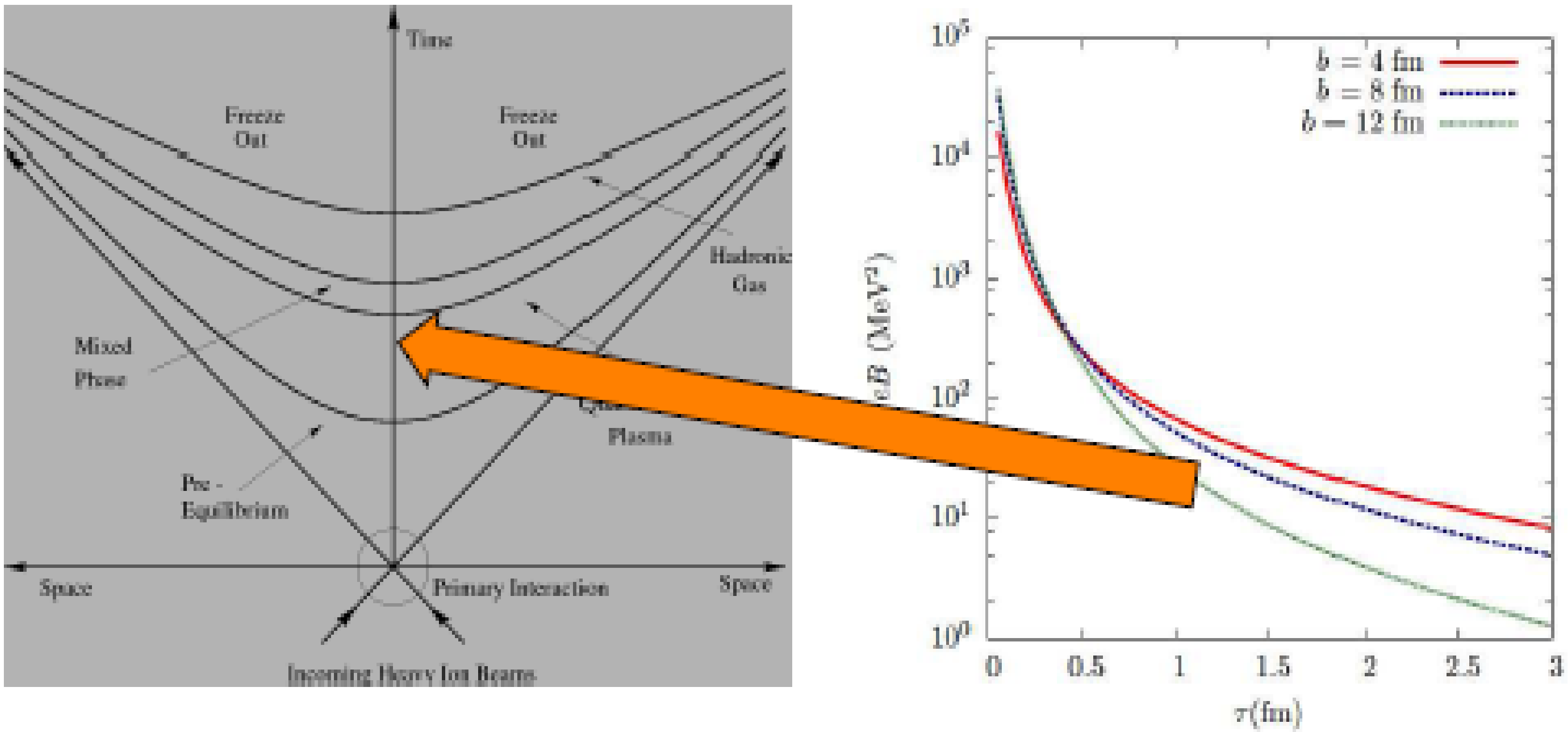
Phase Diagram: Which will be the influence of an external Magnetic Field B?



Huge Magnetic fields are produced in perhipheral heavy-ion collisions



The intensity of the magnetic field decreases very rapidly as function of time



D. E. Kharzeev, L. D. McLerran, H. J. Warringa, Nucl. Phys. **A** 803 (2008) 227-253

- By the time the quarks and gluon thermalize the temperature becomes the largest of the energy scales. This means that the weak magnetic field approximation seems to be appropriate

($eB << T^2$).

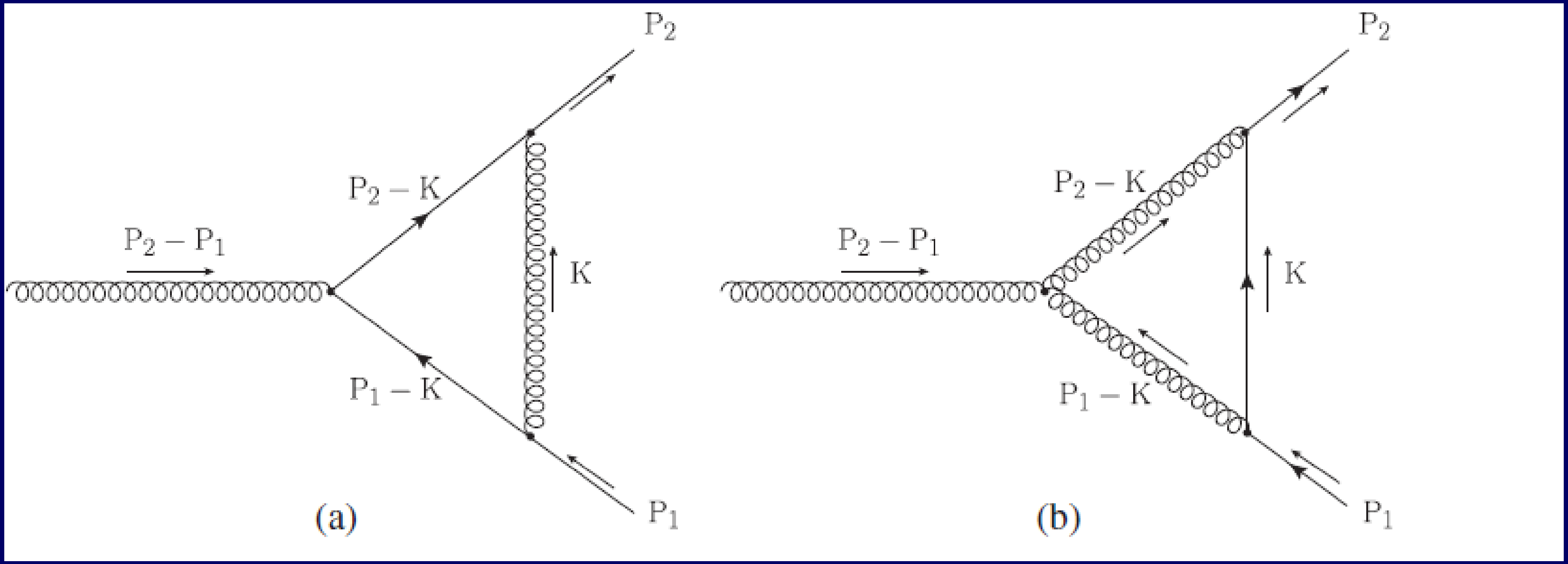
Recently, using the linear σ-model, we have shown the occurence of magnetic anti-catalysis (in agreement with results from the lattice). T_c (for chiral restoration) diminishes as function of B.

A. Ayala, M. Loewe, and R. Zamora, Phys. Rev. D **91**, 016002 (2015).

- The idea is to consider the behavior of the QCD coupling constant (not an effective model) as function of the external B field as well as on temperature.

E. Ferrer, V.de la Incera and X. J. Wen (arXiiv: 1407.3503) have considered this problem for the the strong magnetic field scenario (no temperature), finding an anisotropic behavior where the coupling decreases with B .

- We will use the Schwinger propagator, in the weak magnetic field approximation, for the (charged) quarks in the triangle diagrams



- Some technical details: The fermion propagator in the presence of an external homogeneous magnetic field B is given by

$$S(x,x') = \Phi(x,x') \int \frac{d^4 p}{(2\pi)^4} e^{-ip\cdot(x-x')} S(k)$$

where

$$\Phi(x,x') = \exp\left\{ iq \int_{x'}^{x} d\xi^{\mu} \left[A_{\mu} + \frac{1}{2} F_{\mu\nu} (\xi - x')^{\nu} \right] \right\}$$

is a phase.

It can be shown that for one or two fermion propagators, stemming from a common vertex, this phase factor can be gauged away. So, we will not take it into account.

S(k) is the famous propagator for a charged fermion (q is the absolute value of the charge (in units of e) in the presence of a constant magnetic field pointing in the z direction.

(Schwinger, 1951)

$$\vec{B} = B\hat{z}.$$

$$S(k) = -i\int_0^\infty \frac{ds}{\cos(qBs)} e^{is\left(k_\parallel^2 - k_\perp^2 \frac{\tan(qBs)}{qBs} - m^2\right)} \times \left\{ [\cos(qBs) + \gamma_1\gamma_2 \sin(qBs)](m + k_\parallel) - \frac{k_\perp}{\cos(qBs)} \right\}$$

We use the notation

$$(a \cdot b)_\parallel = a_0 b_0 - a_3 b_3,$$
$$(a \cdot b)_\perp = a_1 b_1 + a_2 b_2.$$

The expansion of S(k) for small values of qB, up to the order O(qB), takes the form

$$S(K) = \frac{m - \not{K}}{K^2 + m^2} - i\gamma_1\gamma_2 \frac{m - \not{K}_{\parallel}}{(K^2 + m^2)^2}(qB)$$

T.-K. Chyi, C.-W. Hwang, W. F. Kao, G. L. Lin, K.-W. Ng, and J.-J. Tseng, Phys. Rev. D **62**, 105014 (2000).

So, we have to insert this propagator in order to get the thermo-magnetic correction to the QCD coupling

For the first diagram, (the QED-like vertex correction) we have

$$\delta\Gamma_\mu^{(\mathrm{a})} = -ig^2(C_F - C_A/2)(qB)T\sum_n \int \frac{d^3k}{(2\pi)^3} \times \gamma_\nu[\gamma_1\gamma_2 \not{K}_\parallel \gamma_\mu \not{K}\tilde{\Delta}(P_2 - K) + \not{K}\gamma_\mu\gamma_1\gamma_2\not{K}_\parallel\tilde{\Delta}(P_1 - K)]\gamma_\nu \times \Delta(K)\tilde{\Delta}(P_2 - K)\tilde{\Delta}(P_1 - K),$$

$$C_F = \frac{N^2 - 1}{2N},$$

corresponding to the fundamental and adjoint representations of the $SU(N)$ Casimir operators

$$C_A = N,$$

Form the second diagram (the genuine non-abelian vertex co rrection) we have

$$\delta\Gamma_\mu^{(b)} = -2ig^2\frac{C_A}{2}(qB)T\sum_n\int\frac{d^3k}{(2\pi)^3} \times[-\not{K}\gamma_1\gamma_2\not{K}_{\|}\gamma_\mu + 2\gamma_\nu\gamma_1\gamma_2\not{K}_{\|}\gamma_\nu K_\mu - \gamma_\mu\gamma_1\gamma_2\not{K}_{\|}\not{K}] \times\tilde{\Delta}(K)^2\Delta(P_1-K)\Delta(P_2-K)$$

In both diagrams the following notation has been used

$$\tilde{\Delta}(K) \equiv \frac{1}{\tilde{\omega}_n^2 + k^2 + m^2},$$

$$\Delta(K) \equiv \frac{1}{\omega_n^2 + k^2},$$

- Where the Matsubara frequencies are given by

$$\tilde{\omega}_n = (2n+1)\pi T \text{ and } \omega_n = 2n\pi T$$

We are in the Euclidean space

$$K_\mu = (k_4, \vec{k}) = (-\omega, \vec{k})$$

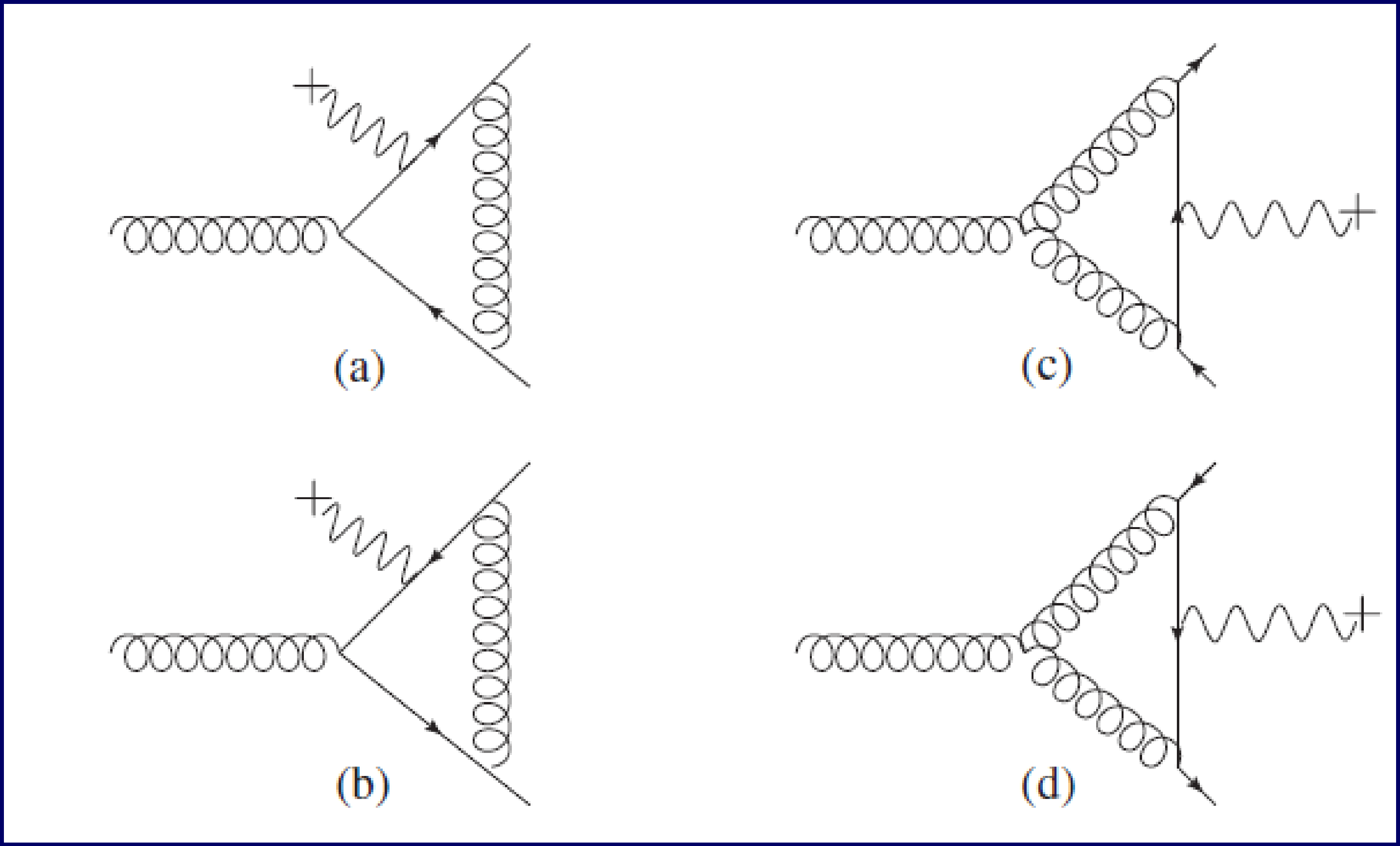
According to the philosophy of the HTL approximation, we neglect the mass terms in the numerator. Remember that

$$\gamma_4 = i\gamma_0$$

$$\{\gamma_\mu, \gamma_\nu\} = -2\delta_{\mu\nu}$$

$$\gamma_5 = \gamma_4\gamma_1\gamma_2\gamma_3$$

The wawy lines with a cross denote the insertion of the magnetic propagators



- Further on, by introducing

$$u_\mu = (1,0,0,0)$$
$$b_\mu = (0,0,0,1)$$

It can easily shown that

$$\gamma_1\gamma_2 \not{K}_\parallel = \gamma_5[(K\cdot b)\not{u} - (K\cdot u)\not{b}]$$

In the spirit of the HTL approx., P_1 and P_2 are small and can be taken of the same order. Thus, to extract the leading temperature behavior, we may approximate

$$[\gamma_1\gamma_2\not{K}_\parallel\gamma_\mu\not{K}\tilde{\Delta}(P_2-K) + \not{K}\gamma_\mu\gamma_1\gamma_2\not{K}_\parallel\tilde{\Delta}(P_1-K)]$$
$$\simeq [\gamma_1\gamma_2\not{K}_\parallel\gamma_\mu\not{K} + \not{K}\gamma_\mu\gamma_1\gamma_2\not{K}_\parallel]\tilde{\Delta}(P_1-K)$$

- We can play the same game with the second diagram and we can combine both in a very compact expression

$$
\begin{aligned}
\delta\Gamma_\mu &= \delta\Gamma_\mu^{(\mathrm{a})} + \delta\Gamma_\mu^{(\mathrm{b})} \\
&= 2i\gamma_5 g^2 C_F (qB) G_\mu(P_1, P_2).
\end{aligned}
$$

where

$$
\begin{aligned}
G_\mu(P_1, P_2) = 2T\sum_n \int \frac{d^3k}{(2\pi)^3} \{ & (K\cdot b)\not{K} u_\mu - (K\cdot u)\not{K} b_\mu \\
& + [(K\cdot b)\not{u} - (K\cdot u)\not{b}]K_\mu \} \\
& \times \tilde{\Delta}^2(K)\Delta(P_1 - K)\Delta(P_2 - K).
\end{aligned}
$$

- I will not present here the techniques we employed for carrying on the sums over the Matsubara frequencies. Looking for the leading terms

$$
\begin{aligned}
\delta\Gamma_\mu(P_1, P_2) = {} & 4i\gamma_5 g^2 C_F M^2(T, m, qB) \\
& \times \int \frac{d\Omega}{4\pi} \frac{1}{(P_1 \cdot \hat{K})(P_2 \cdot \hat{K})} \\
& \times \{(\hat{K} \cdot b)\hat{\not{K}} u_\mu - (\hat{K} \cdot u)\hat{\not{K}} b_\mu \\
& + [(\hat{K} \cdot b)\not{u} - (\hat{K} \cdot u)\not{b}]\hat{K}_\mu\},
\end{aligned}
$$

where $\hat{K} = (-i, \hat{k})$

$P_1 = (-\omega_1, \vec{p}_1)$, and $P_2 = (-\omega_2, \vec{p}_2)$

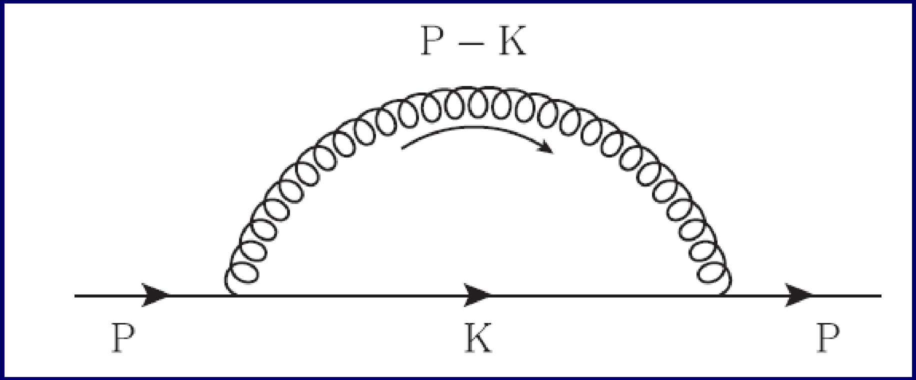
- And where the function $M^2(T, m, qB)$ is given by

$$M^2(T,m,qB) = \frac{qB}{16\pi^2}\left[\ln(2) - \frac{\pi}{2}\frac{T}{m}\right]$$

It is important and interesting to notice that our vertex correction satisfies a QED-like Ward-identity. In fact

$$(P_1 - P_2)\cdot \delta\Gamma(P_1, P_2) = \Sigma(P_1) - \Sigma(P_2)$$

Where Σ(p) corresponds to the self-energy correction to the quark propagator



$$\Sigma(P) = 2i\gamma_5 g^2 C_F M^2(T, m, qB) \int \frac{d\Omega}{4\pi} \frac{[(\hat{K}\cdot b)\not{u} - (\hat{K}\cdot u)\not{b}]}{(P\cdot\hat{K})}$$

This remarkable result shows that even in the presence of a magnetic field, provided the temperature is the highest energy scale, the thermomagnetic correction to the vertex is gauge invariant.

For computing the thermo-magnetic dependence of the Coupling we have to deal with

$$J_{\alpha i}(P_1, P_2) \equiv \int \frac{d\Omega}{4\pi} \frac{\hat{K}_\alpha \hat{K}_i}{(P_1 \cdot \hat{K})(P_2 \cdot \hat{K})}$$

For the sake of simplicity we choose the configuration where the momenta (vector components) make an angle $\theta_{12} = \pi$.
It turns out that only the 00 and 33 components (after returning To Minkowski) survive. These are the paralell components.

$$\delta\vec{\Gamma}_{||}(p_0) = \left(\frac{2}{3p_0^2}\right) 4g^2 C_F M^2(T, m, qB)\vec{\gamma}_{||}\Sigma_3$$

- where

$$\vec{\gamma}_{\|} = (\gamma_0, 0, 0, -\gamma_3)$$

and we have rearranged the γ matrices introducing the spin operator Σ_3

$$\Sigma_3 = i\gamma_1\gamma_2 = \frac{i}{2}[\gamma_1, \gamma_2]$$

It is natural to consider P_0 = T and to take m as the thermal fermionic mass.

$$p_0 = T,$$
$$m^2 = m_f^2 = \frac{1}{8} g^2 T^2 C_F$$

- As it is well known, the purely thermal corrections to the quark-gluon vertex is given by (Le Bellac’s book)

$$\delta\Gamma_\mu^{\text{therm}}(P_1, P_2) = -m_f^2 \int \frac{d\Omega}{4\pi} \frac{\hat{K}_\mu \hat{\not{K}}}{(P_1 \cdot \hat{K})(P_2 \cdot \hat{K})}$$

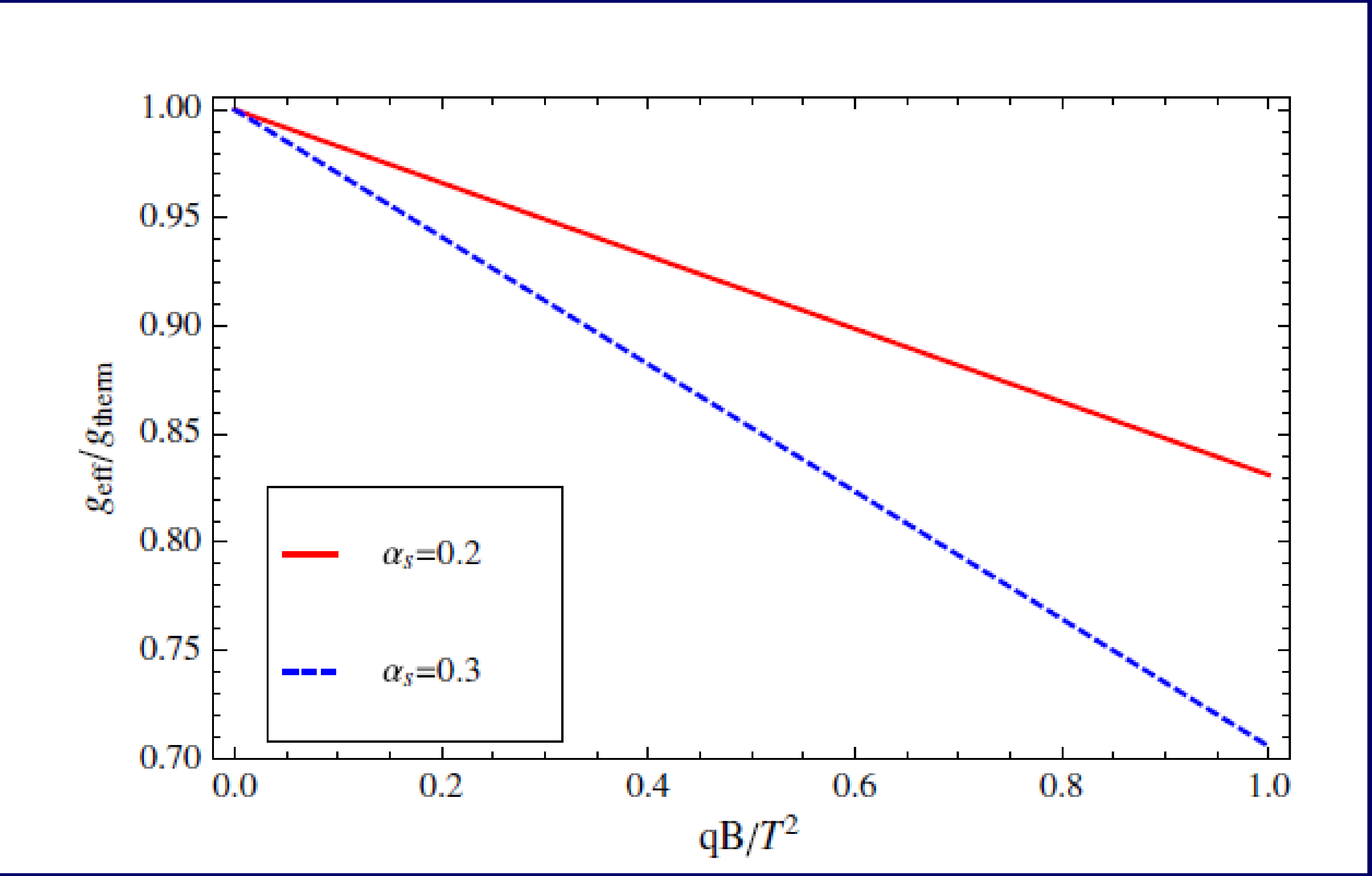
From this analysis we extracted the effective thermo-magnetic coupling (from the effective longitudinal vertex)

$$g_{\text{eff}} = g\left[1 - \frac{m_f^2}{T^2} + \left(\frac{8}{3T^2}\right) g^2 C_F M^2(T, m_f, qB)\right]$$

Where we have used a spin proyection 1 for Σ_3 and we calculated in the static limit.

- In the next figure we show the behavior of $g_{eff}(T,B)$ normalize with the thermal coupling

$$g_{\text{therm}} \equiv g\left(1 - \frac{m_f^2}{T^2}\right)$$



Note that the effective thermo-magnetic coupling

g_{eff} decreases as function of the magnetic field. The decrease becomes more relevant for higher values of α_s.

For the considered values it becomes 15%-25% smaller than the purely thermal correction for

$qB \sim T^2 \sim 1$.

This decreasing behavior of the thermo-magnetic

effective coupling was extremely important for getting magnetic anti-catalysis in effectives models

CONCLUSIONS

- We have computed the thermo-magnetic correction (TMC) to the quark-gluon vertex for weak magnetic field in the HTL approximation
- This vertex satisfies a QED-like Ward identity with the quark self-energy.
- The TMC is proportional to the spin componente in the direction of the magnetic field and affects only the longitudinal components.
- The effective coupling decreases if the strength of the magnetic field increases.