# Chiral transitions with magnetic fields of low magnitude

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-We are interested on The effect of a magnetic background on Phase transitions in QFT.

-The effective model to be used is the Abelian Higgs model with fermions.

-The gauge field is Classical, i.e. Is an external parameter.

-The cases Considered are: $qB < |m_b|^2 < T^2$ , and $|m_b|^2 < qB < T^2$.

1-loop effective potential

$$\mathcal{L} = (D_\mu \phi)^\dagger D^\mu \phi + i \bar{\psi} \gamma^\mu D_\mu \psi + \mu^2 \phi^\dagger \phi - \frac{\lambda}{4} (\phi^\dagger \phi)^2 - \frac{g}{\sqrt{2}} (\phi \bar{\psi} \psi + c.c.),$$

Where

$$D_\mu = \partial_\mu + i q A_\mu , \quad A^\mu = \frac{B}{2} (0, -y, x, 0).$$

$$\phi(x) = \frac{1}{\sqrt{2}} [\sigma(x) + i \chi(x)]$$

$$\sigma \rightarrow \sigma + v$$

After the shift, the lagrangian can be written as

$$\mathcal{L} = -\frac{1}{2}\left[\sigma(\partial_\mu + i q A_\mu)^2 \sigma\right] - \frac{1}{2}\left(\frac{3\lambda v^2}{4} - \mu^2\right)\sigma^2 - \frac{\lambda}{16}\sigma^4$$

$$-\frac{1}{2}\left[\chi(\partial_\mu + i q A_\mu)^2 \chi\right] - \frac{1}{2}\left(\frac{\lambda v^2}{4} - \mu^2\right)\chi^2 - \frac{\lambda}{16}\sigma^4$$

$$+\frac{\mu^2}{2} v^2 - \frac{\lambda}{16} v^4 + i\bar{\psi}(\partial_\mu + i q A_\mu)\psi - g v \bar{\psi}\psi + \mathcal{L}_I$$

Thus, the tree Term is given by

$$V^{(\text{Tree})} = -\frac{1}{2}\mu^2 v^2 + \frac{\lambda}{16} v^4.$$

With the minima

$$v_0 = 2\mu/\sqrt{\lambda}$$

Since

$$V \propto \mathrm{tr}\ln D^{-1} \propto \frac{dV}{dm^2} = \mathrm{tr}\, D$$

$$\Rightarrow V_b^1 = \frac{T}{2}\sum_n \int dm_b^2 \int \frac{d^3k}{(2\pi)^3} \int_0^\infty \frac{ds}{\cosh(qBs)}\, e^{-s\left(\omega_n^2 + k_3^2 + k_\perp^2 \frac{\tanh(qBs)}{qBs} + m_b^2\right)}$$

$$\omega_n = 2\pi n$$

$$V_b^1 = \frac{1}{2}\frac{2qB}{4\pi}\sum_l \int \frac{dk_3}{2\pi}\left[\omega_l + 2T\ln\left(1-e^{-\omega_l/T}\right)\right] \equiv V_b^{1,vac} + V_b^{1,matt}$$

$$\omega_l = \sqrt{k_3^2 + m_b^2 + 2\left(l+\tfrac{1}{2}\right)qB}$$

The same for the fermions

$$V_f^1 = -\frac{2qB}{4\pi}\sum_{l,r}\int \frac{dk_3}{2\pi}\left[\omega_{l,r} + 2T\ln\left(1+e^{-\omega_{l,r}/T}\right)\right] \equiv V_f^{1,vac} + V_f^{1,matt}.$$

$$\omega_{l,r} = \sqrt{k_3^2 + m_f^2 + 2\left[l + (1+r)/2\right]qB}$$

$qB < |m_b|^2, m_f^2$ case:

The sum over the Landau levels can be expressed as

$$S_b = \sum_l (2qB)\, g_l$$

$$g_l = \int \frac{dk_3}{2\pi} \left[ \omega_l + 2T \ln\left(1 - e^{-\omega_l/T}\right)\right]$$

Using the Euler-Maclaurin formula

$$S_b \simeq \int dy\, g(y) - \frac{1}{2} \frac{B_2 h^2}{2!} \left[ g'(y=\infty) - g'(y=0)\right]$$

With $h = 2qB$, we have

$$V_b^1 = \frac{m_b^4}{64\pi^2} \left[ \ln\left(\frac{m_b^2}{2\mu^2}\right) - \frac{1}{2} + \ln\left(\frac{(4\pi T)^2}{m_b^2}\right) - 2\gamma_E + \frac{3}{2}\right] - \frac{\pi^2 T^4}{90} + \frac{m_b^2 T^2}{24} - \frac{m_b^3 T}{12\pi}$$
$$- \frac{(qB)^2}{192\pi^2} \left[ \ln\left(\frac{m_b^2}{2\mu^2}\right) + 1 + \ln\left(\frac{(4\pi T)^2}{m_b^2}\right) - 2\gamma_E - \frac{2\pi T}{m_b} + \zeta(3)\left(\frac{m_b}{2\pi T}\right)^2 - \frac{3}{4}\zeta(5)\left(\frac{m_b}{2\pi T}\right)^4\right].$$

Notice that there terms which can given an imaginary contribution for $m_b^2 < 0$

$$V_b^1 = \frac{m_b^4}{64\pi^2}\left[\ln\left(\frac{m_b^2}{2\mu^2}\right) - \frac{1}{2} + \ln\left(\frac{(4\pi T)^2}{m_b^2}\right) - 2\gamma_E + \frac{3}{2}\right] - \frac{\pi^2 T^4}{90} + \frac{m_b^2 T^2}{24} - \underline{\frac{m_b^3 T}{12\pi}}$$

$$- \frac{(2B)^2}{192\pi^2}\left[\ln\left(\frac{m_b^2}{2\mu^2}\right) + 1 + \ln\left(\frac{(4\pi T)^2}{m_b^2}\right) - 2\gamma_E - \underline{\frac{2\pi T}{m_b}} + \zeta(3)\left(\frac{m_b}{2\pi T}\right)^2 - \frac{3}{4}\zeta(5)\left(\frac{m_b}{2\pi T}\right)^4\right].$$

The sum over Landau levels For the fermions

$$S_f = \sum_{l,r} (2qB)\, g_{l,r}$$

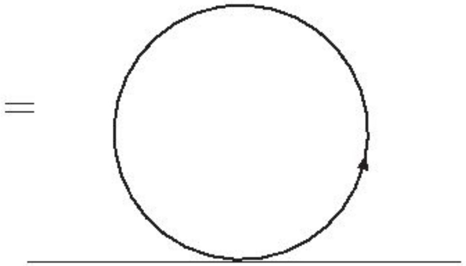
$$g_{l,r} = \int \frac{dk_3}{2\pi} \left[ \omega_{l,r} + 2T \ln\left(1 + e^{-\omega_{l,r}/T}\right) \right]$$

$$V_f' = -\frac{m_f^4}{16\pi^2} \left[ \ln\left(\frac{m_f^2}{2\mu^2}\right) - \frac{1}{2} + \ln\left(\frac{(\pi T)^2}{m_f^2}\right) - 2\gamma_E + \frac{3}{2} \right]$$

$$- \frac{7\pi^2 T^4}{180} + \frac{m_f^2 T^2}{12} - \frac{(qB)^2}{24\pi^2} \left[ \ln\left(\frac{m_f^2}{2\mu^2}\right) + 1 + \ln\left(\frac{(\pi T)^2}{m_f^2}\right) - 2\gamma_E \right].$$

In order to consider the leading corrections inte infrared regime, we need to include Rings diagrams for $\omega_n = 0$

$$V_b^{ring} = \frac{T}{2}\int \frac{d^3k}{(2\pi)^3} \ln\left[1 + \Pi \Delta_B(\omega_n = 0, k)\right],$$

$\Pi =$ 

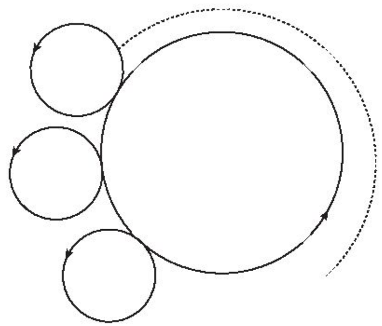
$$\Pi = \frac{\lambda T^2}{12} + N_f\, g^2 \frac{T^2}{6},$$

$$V_b^{ring} = \frac{T}{2}\int \frac{d^3k}{(2\pi)^3} \left[\ln(\Delta_B^{-1} + \Pi) - \ln(\Delta_B^{-1})\right]$$

$$\equiv V_{bI}^{ring} - V_{bII}^{ring}$$

$$V_{bII}^{ring} = \frac{T}{2}\int \frac{d^3k}{(2\pi)^3} \ln \Delta_B^{-1} \approx \left(\frac{T}{8\pi}\right)\left[\frac{8}{3}(m_b^2 + \Lambda^2)^{3/2} - \frac{2}{3} m_b^3 + \frac{(qB)^2}{12 m_b}\right].$$

$$V_b^{ring} = \left(\frac{T}{12\pi}\right)\left[m_b^3 - (m_b^2 + \Pi)^{3/2} - \frac{(qB)^2}{8 m_b} + \frac{(qB)^2}{8(m_b^2 + \Pi)^{1/2}}\right]$$



With the Ring contribution, we end with an effective potencial with no complex terms

$$V^{eff} = -\frac{\mu^2}{2} v^2 + \frac{\lambda}{16} v^4 + \sum_{i=\sigma,\chi} \Bigg[ \frac{m^4}{64\pi^2} \left[ \ln\left( \frac{(4\pi T)^2}{2\mu^2} - 2\gamma_E + 1 \right] \right.$$

$$- \frac{\pi^2 T^4}{90} + m_i^2 T^2 - \frac{(m_i^2 + \Pi)^{3/2}}{12\pi} T - \frac{(\lambda B)^2}{192\pi^2} \left[ \ln\left( \frac{(4\pi T)^2}{2\mu^2} \right) \right.$$

$$\left. - 2\gamma_E + 1 - \frac{2\pi T}{(m_i^2 + \Pi)^{1/2}} + \zeta(3)\left( \frac{m_b}{2\pi T} \right)^2 - \frac{3}{4}\zeta(5)\left( \frac{m_b}{2\pi T} \right)^4 \right] \Bigg\}$$

$$- N_f \left\{ \frac{m_f^4}{16\pi^2} \left[ \ln\left( \frac{(\pi T)^2}{2\mu^2} \right) - 2\gamma_E + 1 \right] + \frac{7\pi^2 T^4}{180} \right.$$

$$\left. - \frac{m_f^2 T^2}{12} + \frac{(qB)^2}{24\pi^2} \left[ \ln\left( \frac{(\pi T)^2}{2\mu^2} \right) - 2\gamma_E + 1 \right] \right\}.$$

It is obtained that

$$m_s \longrightarrow (m_s^2 + \Pi)^{1/2}$$

Since

$$\Pi = \frac{\lambda T^2}{12} + N_f \, g^2 \frac{T^2}{6},$$

If $v = 0$, $m_s^2 = 0$,

$$m_s^2 + \Pi \longrightarrow -\mu^2 + \Pi$$

$$\Rightarrow \quad T > \frac{\mu}{\sqrt{\frac{\lambda}{12} + \frac{N_f g^2}{6}}}$$

-Intermediate-field regime $|m_b|^2 < qB < T^2$.

-Bosons

$$V_b^1 = \frac{T}{2} \sum_n \int \frac{d^3k}{(2\pi)^3} \ln[\Delta_B(\omega_n, k)^{-1}]$$

The zero frequency is treated Separately, as was done before

$$\sum_n \Delta_\beta(\omega_n, k) = \sum_{n\neq 0} \Delta_B(\omega_{n\neq 0}, k) + \Delta_B(0, k)$$

For the case $qB, |m_b|^2 < T$, we expand

$$\Delta_B(\omega_n \neq 0, k) \simeq \frac{1}{\omega_n^2 + k^2 + m_b^2}\left[1 - \frac{(qB)^2}{(\omega_n^2 + k^2 + m_b^2)^2} + \frac{2(qB)^2 k_\perp^2}{(\omega_n^2 + k^2 + m_b^2)^3}\right]$$

$$\Delta_B(0, k) = \int_0^\infty \frac{ds}{\cos(qBs)} e^{-is[k_\parallel^2 + k_\perp^2 \frac{\tan qBs}{qBs} + m_b^2 - i\epsilon]}$$

$$V_b^1 = V_{bI}^1 + V_{bII}^1$$

-Intermediate-field regime

-Bosons

$$V_b^1 = \frac{T}{2} \sum_n \int \frac{d^3k}{(2\pi)^3} \ln\left[\Delta_B(\omega_n,k)^{-1}\right]$$

The zero frequency is treated Separately

$$\sum_n \Delta_\beta(\omega_n,k) = \sum_{n\neq 0} \Delta_B(\omega_{n\neq 0},k) + \Delta_B(0,k)$$

For the case

$$\Delta_B(\omega_{n\neq 0},k) \simeq \frac{1}{\omega_n^2+k^2+m_b^2}\left[1 - \frac{(qB)^2}{(\omega_n^2+k^2+m_b^2)^2} + \frac{2(qB)^2 k_\perp^2}{(\omega_n^2+k^2+m_b^2)^3}\right]$$

$$\Delta_B(0,k) = \int_0^\infty \frac{ds}{\cos(qBs)} e^{-is\left[k_z^2 + k_\perp^2 \frac{\tan qBs}{qBs} + m_b^2 - i\epsilon\right]}$$

$$V_b^1 = V_{bI}^1 + V_{bII}^1$$

$$V^{1}_{b\,\mathrm{I}} = \frac{T}{2} \sum_{n\neq 0} \int dm_i^2 \int \frac{d^3k}{(2\pi)^3} \Delta_B(\omega_{n\neq 0}, k)$$

$$V^{1}_{b\,\mathrm{II}} = \frac{T}{2} \int dm_b^2 \int \frac{d^3k}{(2\pi)^3} \Delta_B(\omega_{n=0}, k)$$

Nonzero modes

$$V^1_b = \frac{m_b^2 T^2}{24} + \frac{m_b^4}{64\pi^2}\left[\ln\left(\frac{(4\pi T)^2}{2M^2}\right) - 2\gamma_E + 1\right]$$

$$- \frac{(\varphi B)^2}{192\pi^2}\left[\zeta(3)\left(\frac{m_b}{2\pi T}\right)^2 - \frac{3}{4}\zeta(5)\left(\frac{m_b}{2\pi T}\right)^4\right].$$

Zero mode plus ring

$$V_b^{ring} = \frac{T}{2} \int dm_b^2 \int \frac{d^3k}{(2\pi)^3} \frac{d}{dm_b^2} \Big\{ \ln \left[ \Delta_B(\omega_n = 0, k)^{-1} + \Pi \right]$$

$$- \ln \left[ \Delta_B(\omega_n = 0, k)^{-1} \right] \Big\} .$$

$$V_{b4}^1 + V_b^{ring} = \frac{T(2eB)^{3/2}}{8\pi^2} \zeta\left(-\frac{1}{2}, \frac{1}{2} + \frac{m_b^2 + \Pi}{2eB}\right).$$

For the fermions, we work in the weak field limit ( $qB \ll T^2$ ).

$$V_f^1 = -2T \sum_n \int dm_f \int \frac{d^3k}{(2\pi)^3} \left\{ \frac{1}{\tilde{\omega}_n^2 + k^2 + m_f^2} + \frac{2(qB)^2 k_\perp^2}{(\tilde{\omega}_n^2 + k^2 + m_f^2)^4} \right\}$$

$$= -\frac{m_f^4}{16\pi^2}\left[\ln\left(\frac{(\pi T)^2}{2\tilde{m}^2}\right) - 2\gamma_E\right] - \frac{7\pi^2 T^4}{180} + \frac{m_f^2 T^2}{12}$$

$$- \frac{(qB)^2}{24\pi^2}\left[\ln\left(\frac{(\pi T)^2}{2\tilde{m}^2}\right) - 2\gamma_E\right].$$

$$V^{eff} = -\frac{\mu^2}{2}v^2 + \frac{\lambda}{16}v^4 + \sum_{i=\sigma,\pi}\left\{\frac{m_i^4}{64\pi^2}\left[\ln\left(\frac{(4\pi T)^2}{2m^2}\right) - 2\gamma_E + 1\right] - \frac{\pi^2 T^4}{90}\right.$$

$$+ \frac{m_i T^2}{24} + \frac{T(2qB)^{3/2}}{8\pi}\zeta\left(-\frac{1}{2}, \frac{1}{2} + \frac{m_i^2 + \Pi}{2qB}\right) - \frac{(qB)^2}{192\pi^2}\left[\ln\left(\frac{(4\pi T)^2}{2m^2}\right)\right.$$

$$\left.\left. - 2\gamma_E + 1 + \zeta(3)\left(\frac{m_i^2}{2\pi T}\right)^2 - \frac{3}{4}\zeta(5)\left(\frac{m_i}{2\pi T}\right)^4\right]\right\} - N_f\left\{\frac{m_f^4}{16\pi^2}\left[\ln\left(\frac{(\pi T)^2}{2m^2}\right) - \gamma_E + 1\right]\right.$$

$$\left. + \frac{7\pi^2 T^4}{180} - \frac{m_f^2 T^2}{12} + \frac{(qB)^2}{24\pi^2}\left[\ln\left(\frac{(\pi T)^2}{2m^2}\right) - 2\gamma_E + 1\right]\right\}$$

Notice that this expression coincides with the low field limit, using the replacement

$$-\frac{(m_i^2+\pi)^{3/2}\,T}{12\pi} - \frac{qB^2}{96\pi^2}\frac{\pi T}{(m_i^2+\Pi)^{1/2}} \to \frac{T(2qB)^{3/2}}{8\pi}\,\zeta\left(-\frac{1}{2},\, \frac{1}{2}+\frac{m_i^2+\Pi}{2qB}\right).$$

The rality condition is

$$-\mu^2+\Pi > qB$$

Graphics

Effective potential as function of $v$ in Units of $\mu$ ($x = v/\mu$), in the low field limit ($b = qB/\mu^2$)

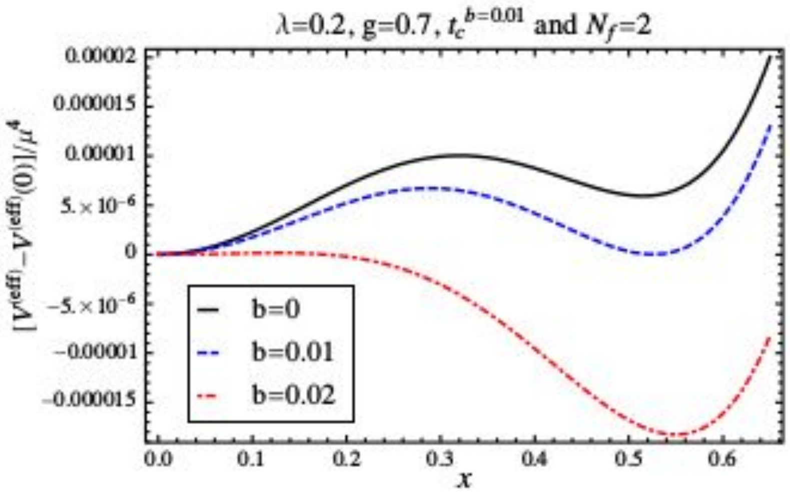



FIG. 1 (color online). Effective potential as a function of $x = v/\mu$ for $b = qB/\mu^2 = 0,\ 0.01,\ 0.02$ for $t_c^{b=0.01} = T_c^{b=0.01}/\mu$ and fixed values of $\lambda = 0.2$, $g = 0.7$, and $N_f = 2$. For the chosen set of parameters, the phase transition is first order.

For the chosen parameters, we have a first order transition

As we increase the field strenght, the transition becomes a first order one

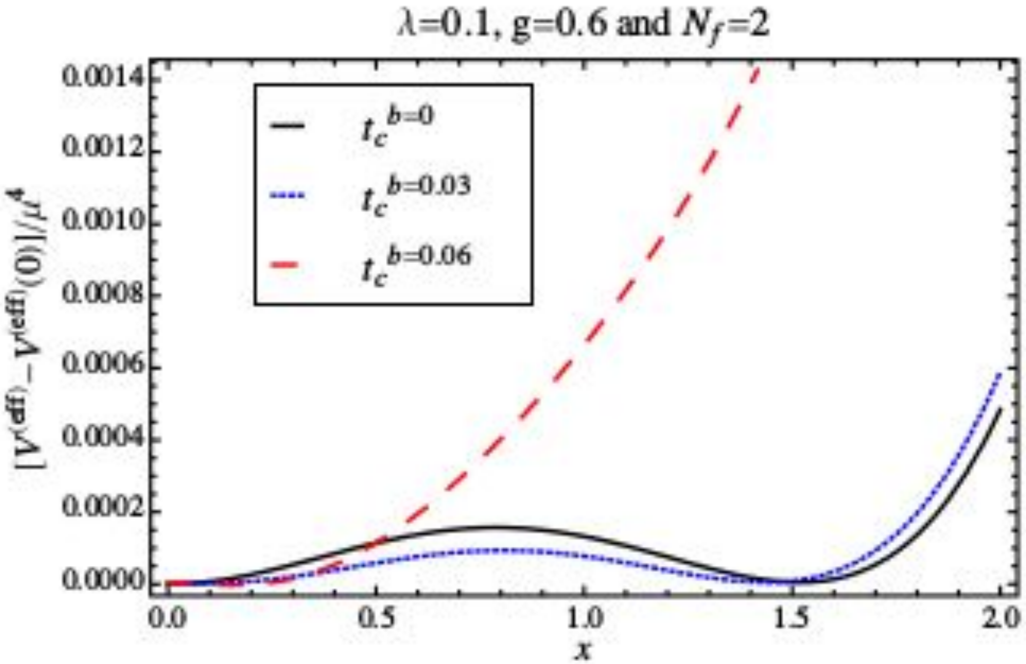



FIG. 2 (color online). Effective potential as a function of $x = v/\mu$ for $b = qB/\mu^2 = 0, 0.03, 0.06$ for fixed values of $\lambda = 0.1$, $g = 0.6$, and $N_f = 2$. Starting from first order, the phase transition becomes second order as we increase the field strength.

Separate contributions of tree level, boson and fermion contribution

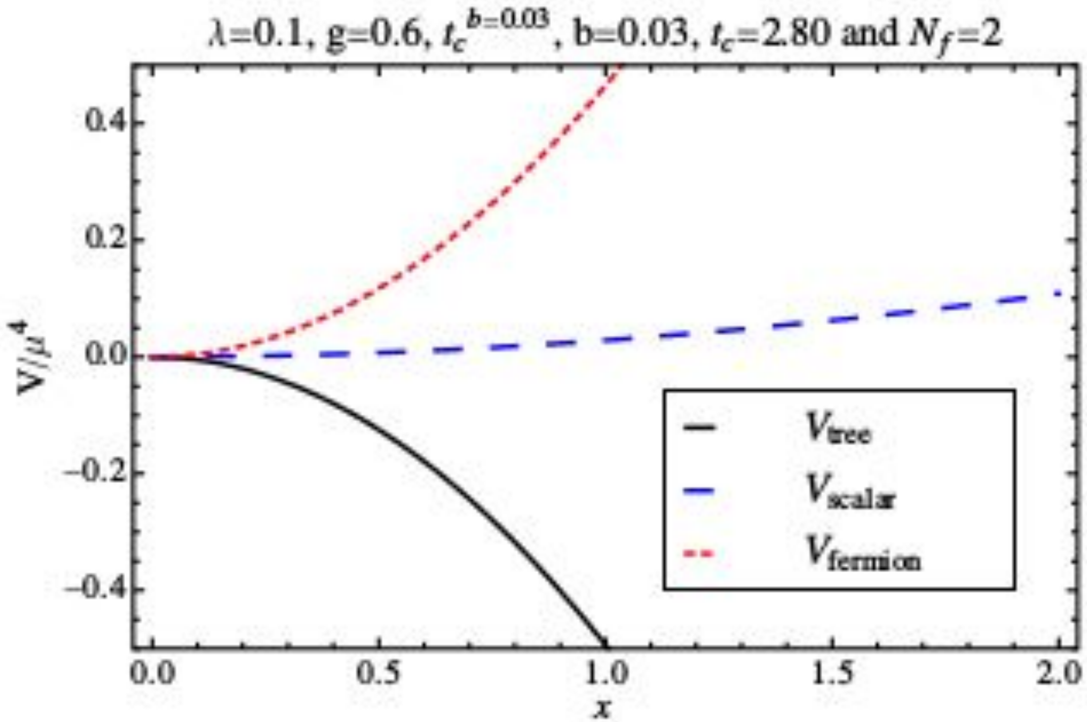



FIG. 3 (color online). Separate contributions from the tree-level, the boson one-loop (ring-corrected), and the fermion one-loop contributions to the effective potential, for $\lambda = 0.1$, $g = 0.6$, $b = 0.03$, and $N_f = 2$ evaluated at the critical temperature $t_c = 2.8$. For these values, the fermion contribution overcomes the boson's, and the combined effect is to produce a small hump that signals a first-order phase transition.

Effect of the magnetic field strenght on the nature of phase transition

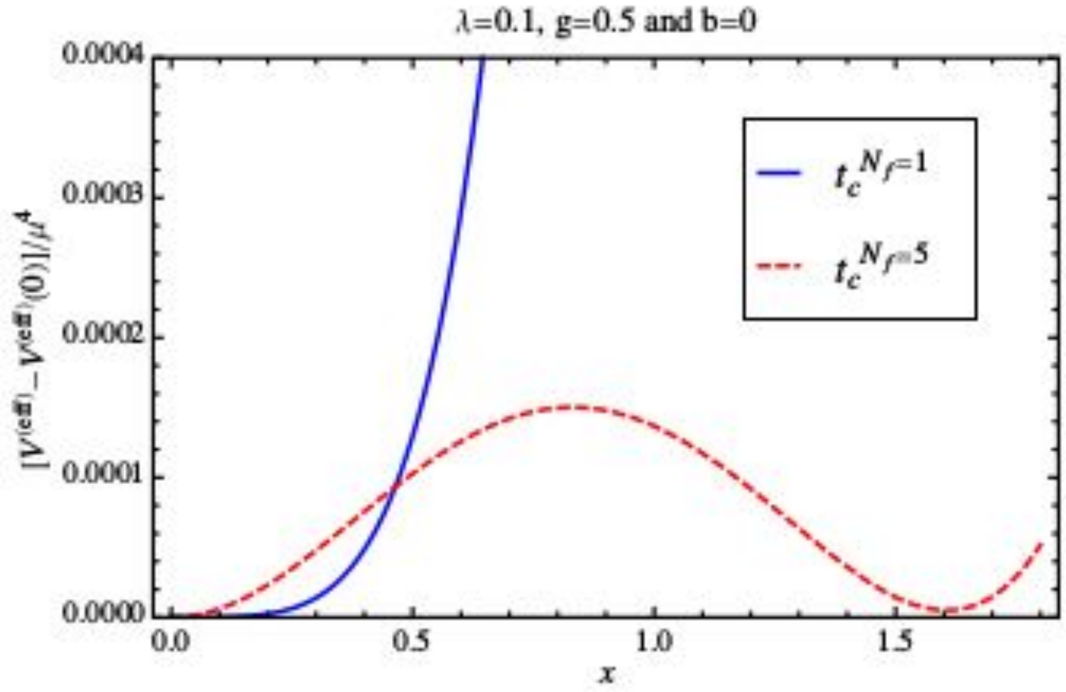
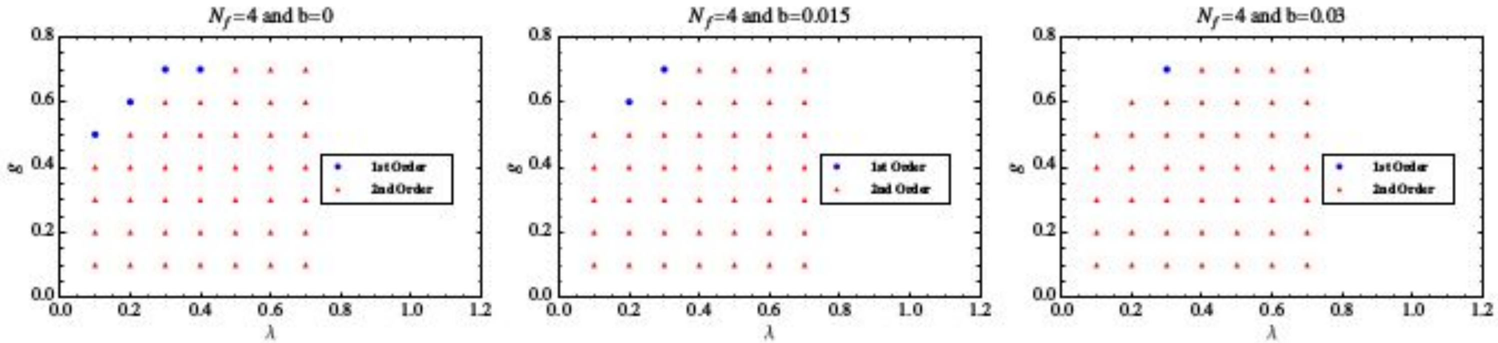



FIG. 4 (color online). Effect of the number of fermions on the order of the phase transition for $\lambda = 0.1$, $g = 0.5$, and $b = 0$. If the phase transition is second order for $N_f = 1$, it becomes first order as we increase the number of fermions to $N_f = 5$.

# Phase diagram as a function of $\lambda$ and $g$




(a) (b) (c)

FIG. 5 (color online). Phase diagram as a function of $\lambda$ and $g$ for different values of $b$: (a) $b = 0$, (b) $b = 0.015$, and (c) $b = 0.03$, with $N_f = 4$. As the field strength grows, the corner of the parameter space that allows a first-order phase transition disappears.

Critical temperature as function of b (= $qB/\mu^2$)

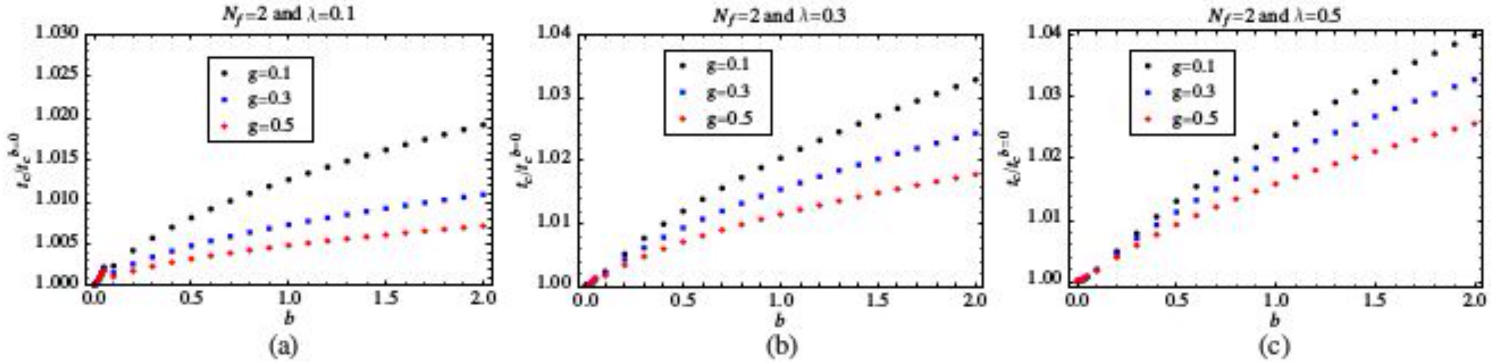



FIG. 6 (color online). Critical temperature $t_c$ as a function of the field strength $b$ in the weak- ($b \leq 0.05$) and intermediate-field ($0.05 < b \leq 2$) regimes for $N_f = 2$, three values of $g = 0.1,\ 0.3,\ 0.5$, and (a) $\lambda = 0.1$, (b) $\lambda = 0.3$, and (c) $\lambda = 0.5$. The critical temperature is always a monotonically increasing function of the field strength. The curves join more smoothly as the couplings grow.

# Summary and conclusions

- the system is well behaved when the plasma screening is taked into account

-the system has first or second order transition, dependind on the value of the parameters.

-the first order transition is caused by the fermion contribution and happen when the coupling $g$ is larger than $\lambda$.

-magnetic fields make first order transition to become second order.

-Critical temperature is a monotonic increasing function of the field strenght.