NUCLEON POLARIZABILITIES STATUS AND RELEVANCE

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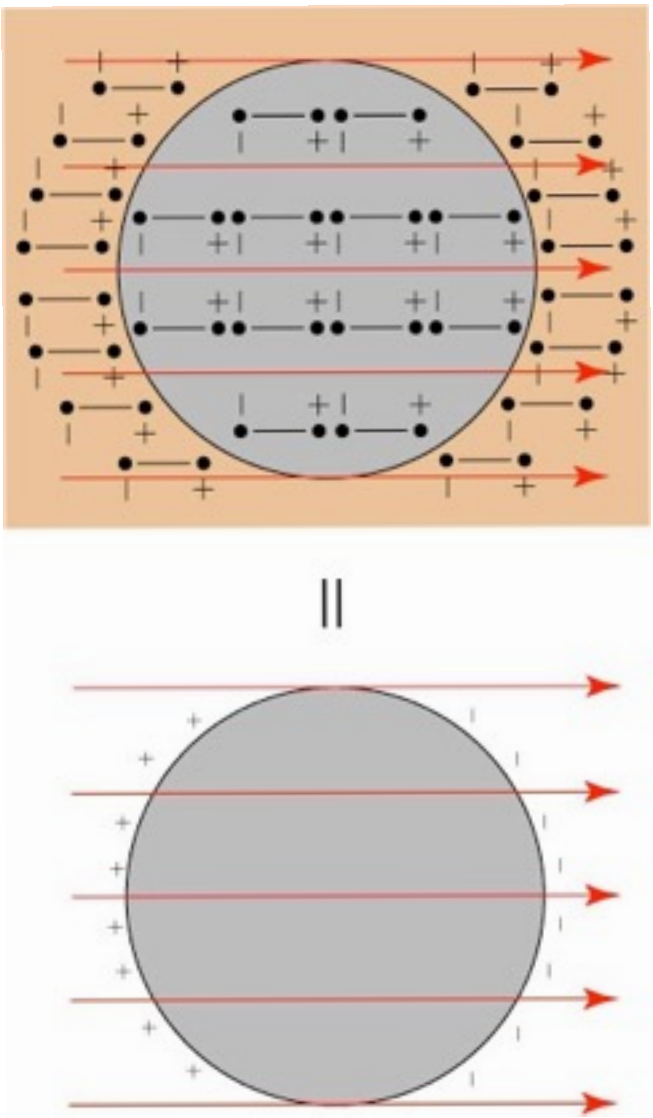
THE LOW-ENERGY FRONTIER
OF THE STANDARD MODEL

The Concept



- induced electric dipole polarization:

$$\vec{P} = \alpha_{E1}\,\vec{E} \qquad \text{(linear dielectric)}$$

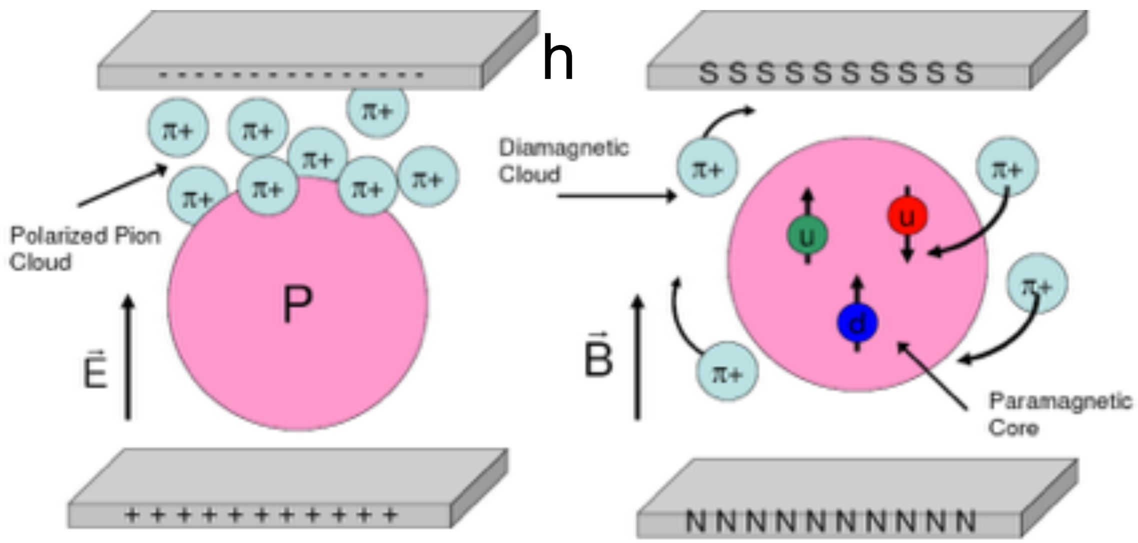
electric polarizability

- for polarization induced by magnetic field:

$$\vec{P} = \beta_{M1}\,\vec{B}$$

magnetic polarizability

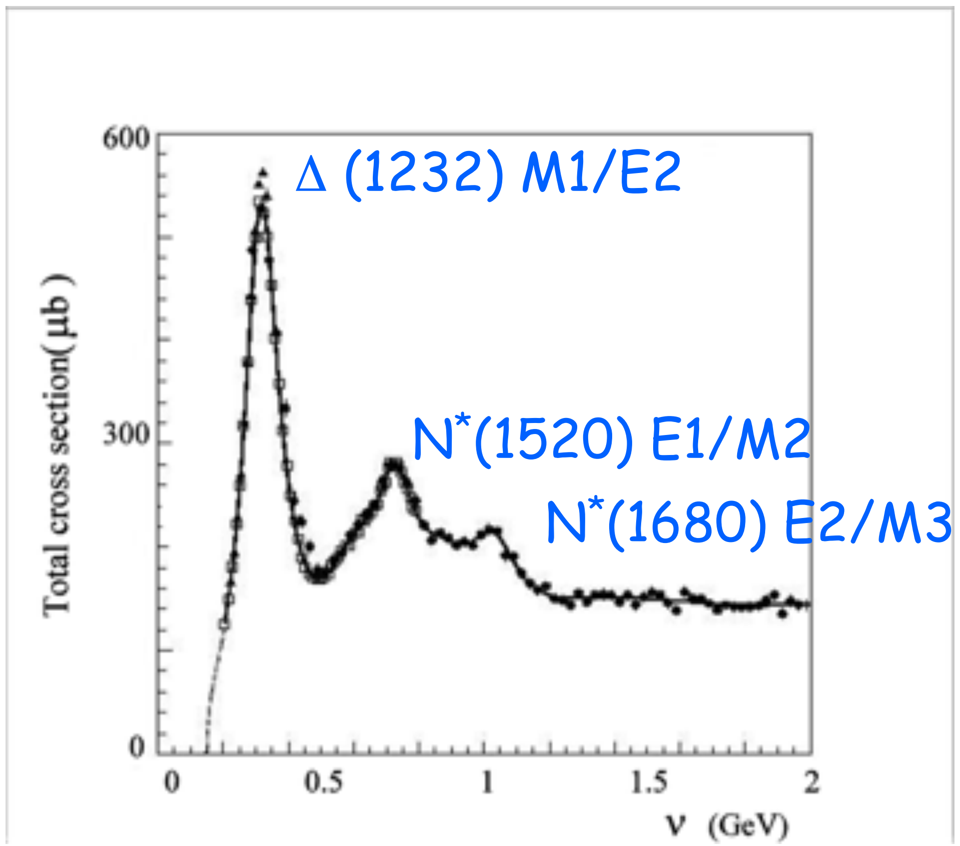
Proton Polarizabilities from Baldin sum rule



$$\alpha_{E1} + \beta_{M1} = \frac{1}{4\pi^2} \int_{\nu_{thr}}^{\infty} d\nu' \frac{\sigma_{tot}(\nu')}{\nu'^2} \simeq 14 \times 10^{-4} \mathrm{fm}^3$$

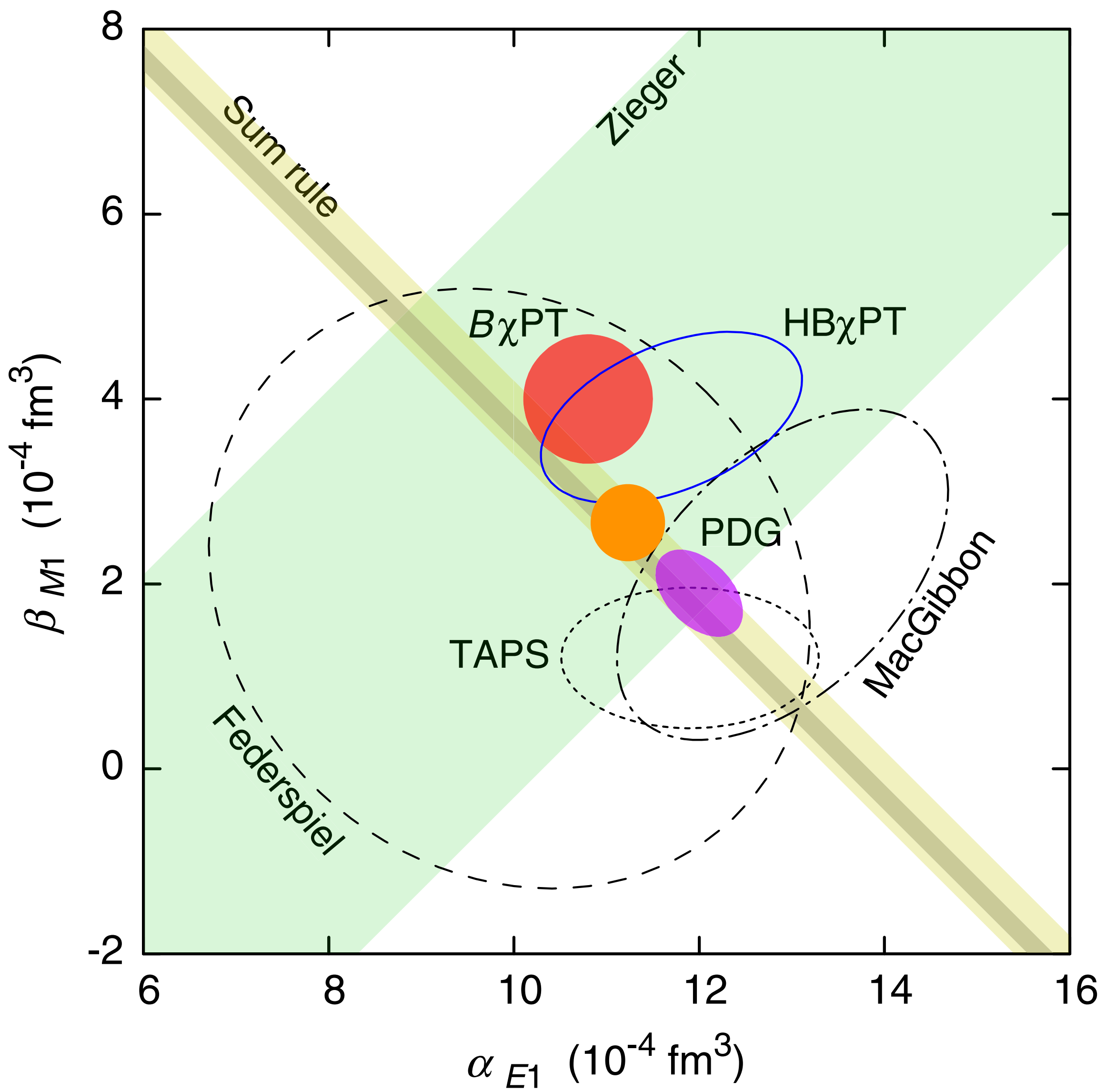
[Baldin sum rule (1960)]

diamagnetic: $\beta_{M1} < 0$
paramagnetic: $\beta_{M1} > 0$



Status

Proton polarizabilities: Status

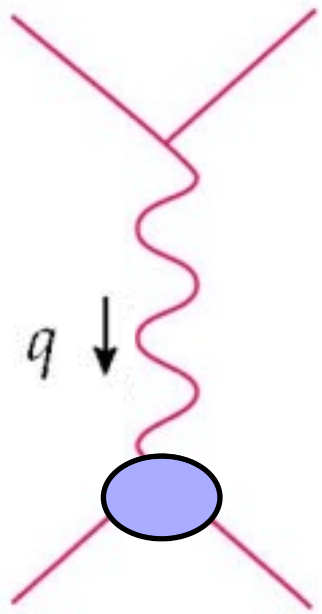


BChPT - Lensky & V. Pascalutsa, EPJC(2010)
HBChPT - Griesshammer, McGovern, Phillips, EPJA (2013)

PDG adjusted values
from 2012 edition (**purple**) to
2014 edition (**orange**)

Relevance

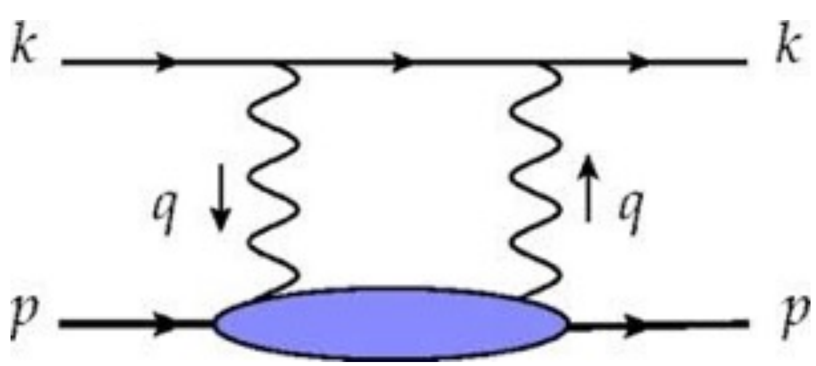
Proton structure in hydrogen spectrum



$$\delta V^{(1\gamma)} = -\frac{4\pi\alpha}{\vec{q}^{\,2}}[G_E(-\vec{q}^{\,2}) - 1] = \frac{2}{3}\pi\alpha\, {\color{red} r_E^2} + O(\vec{q}^{\,2})$$

$$\Delta E_{nl}^{(\mathrm{FS})} = \big\langle nlm|\, \delta V^{(1\gamma)}\, |nlm\big\rangle = \delta_{l0}\frac{2}{3}\pi\alpha\, {\color{red} r_E^2}\, {\color{blue}\frac{\alpha^3 m_r^3}{\pi n^3}} + O(\alpha^5)$$

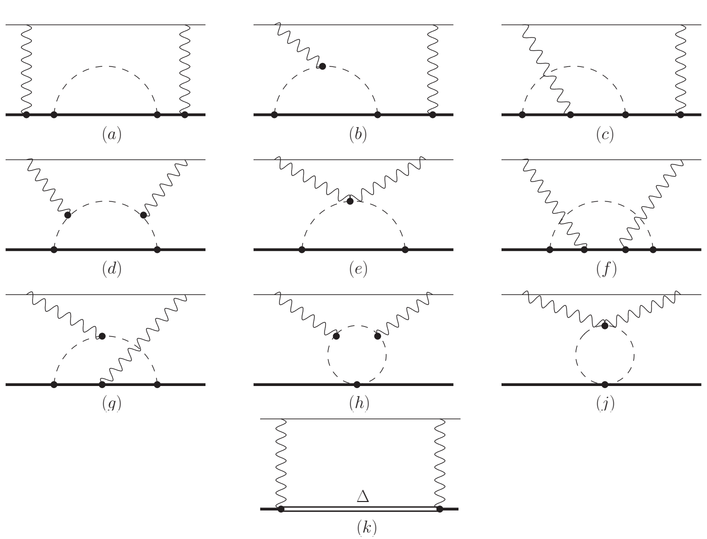
wave function at origin



included in 3rd Zemach moment

$$\delta V^{(2\gamma)} = \delta V^{(2\gamma)}_{\mathrm{elastic}} + \delta V^{(2\gamma)}_{\mathrm{polariz.}}$$

ChPT prediction:
finite (free-LEC free)
result



Polarizability contribution in ChPT

Eur. Phys. J. C (2014) 74:2852
DOI 10.1140/epjc/s10052-014-2852-0

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Chiral perturbation theory of muonic-hydrogen Lamb shift: polarizability contribution

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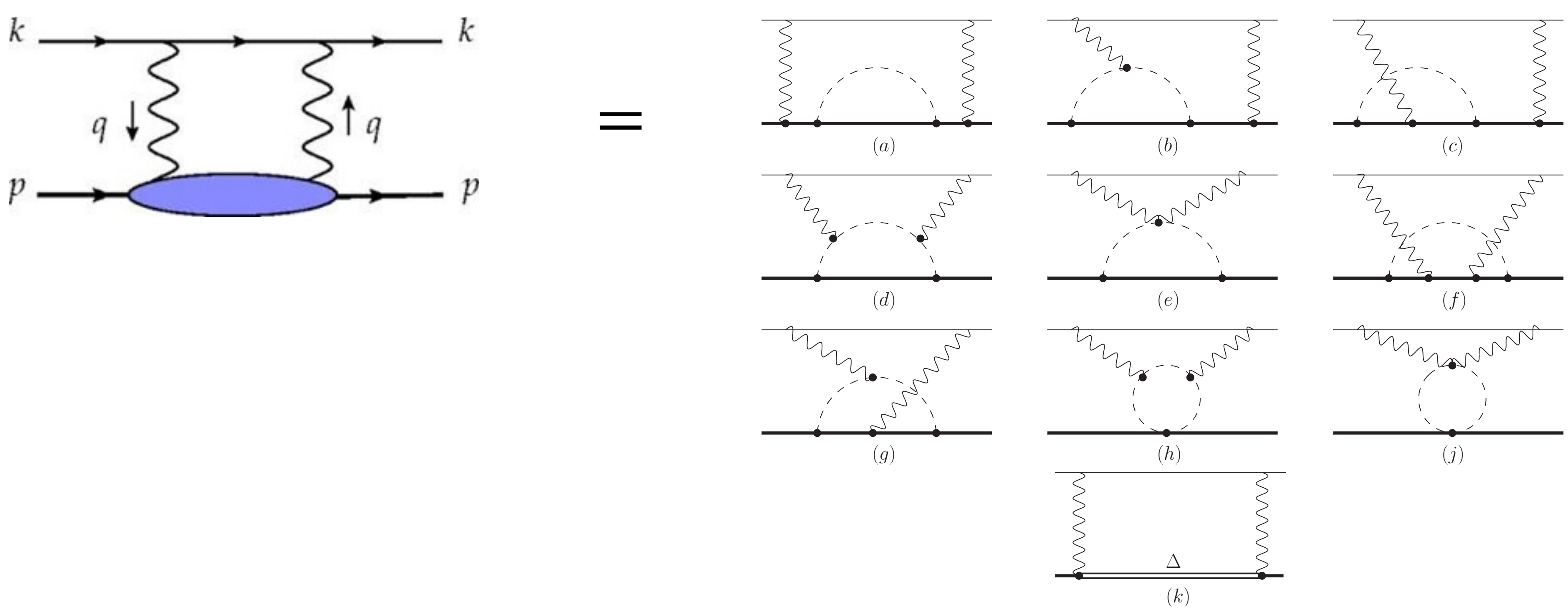


with corrections to elastic proton FFs subtracted, i.e. "polarizability" alone

Proton polarizability effect in mu-H

Heavy-Baryon (HB)ChPT

[Alarcon, Lensky & VP, EPJC (2014)]

(μeV)	Pachucki [9]	Martynenko [10]	Nevado and Pineda [11]	Carlson and Vanderhaeghen [12]	Birse and McGovern [13]	Gorchtein et al. [14]	LO-BχPT [this work]
$\Delta E_{2S}^{(\mathrm{subt})}$	1.8	2.3	–	5.3 (1.9)	4.2 (1.0)	−2.3 (4.6)[a]	−3.0
$\Delta E_{2S}^{(\mathrm{inel})}$	−13.9	−13.8	–	−12.7 (5)	−12.7 (5)[b]	−13.0 (6)	−5.2
$\Delta E_{2S}^{(\mathrm{pol})}$	−12 (2)	−11.5	−18.5	−7.4 (2.4)	−8.5 (1.1)	−15.3 (5.6)	$-8.2(^{+1.2}_{-2.5})$

[a] Adjusted value; the original value of Ref. [14], +3.3, is based on a different decomposition into the 'elastic' and 'polarizability' contributions
[b] Taken from Ref. [12]

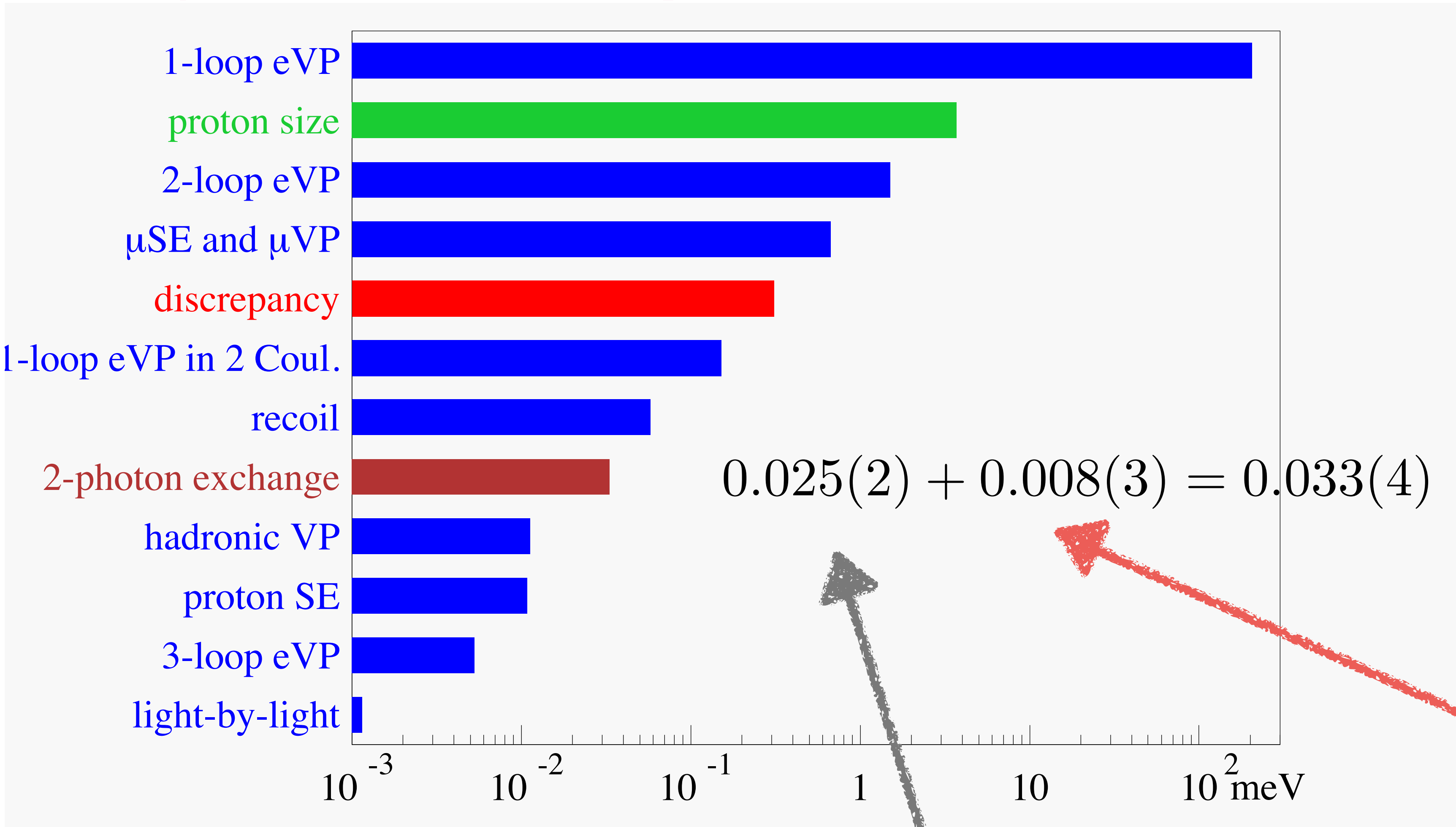
[9] K. Pachucki, Phys. Rev. A **60**, 3593 (1999).
[10] A. P. Martynenko, Phys. Atom. Nucl. **69**, 1309 (2006).
[11] D. Nevado and A. Pineda, Phys. Rev. C **77**, 035202 (2008).
[12] C. E. Carlson and M. Vanderhaeghen, Phys. Rev. A **84**, 020102 (2011).
[13] M. C. Birse and J. A. McGovern, Eur. Phys. J. A **48**, 120 (2012).
[14] M. Gorchtein, F. J. Llanes-Estrada and A. P. Szczepaniak, Phys. Rev. A 87, **052501** (2013).

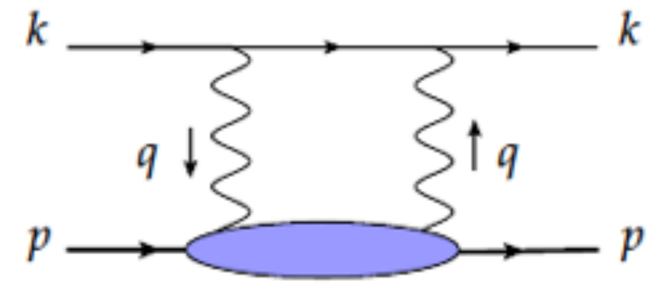
$$\Delta E_{2S}^{(\mathrm{pol})}(\text{LO-HB}\chi\text{PT}) \approx \frac{\alpha_{\mathrm{em}}^5 m_r^3 g_A^2}{4(4\pi f_\pi)^2}\,\frac{m_\mu}{m_\pi}(1-10G+6\log 2) = -16.1\ \mu\mathrm{eV},$$

$G \simeq 0.9160$ is the Catalan constant.

Proton structure in muonic hydrogen Lamb shift

Antognini et al, Ann Phys (2013):





$$0.025(2) + 0.008(3) = 0.033(4)$$

calculable in ChPT @ LO

HB ChPT:

Nevado & Pineda, PRC (2008)

BChPT:

Alarcon, Lensky & Pascalutsa, EPJC (2014)

elastic (3rd Zemach moment):

Carlson & Vanderhaeghen, PRA (2011)
Birse & McGovern, EPJA (2012)

newer analysis:
Karshenboim,1405.6039

Re-evaluation of Baldin sum rule and Kramers-Kronig relation for the proton

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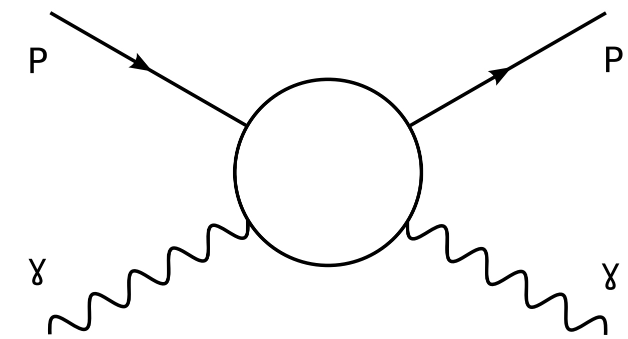
Compton scattering



Forward scattering:

$$T(\nu,\,\theta=0) \quad = \quad \vec{\varepsilon}\,'^{*}\cdot\vec{\varepsilon}\,f(\nu)\;+\;i\vec{\sigma}\cdot(\vec{\varepsilon}\,'^{*}\times\vec{\varepsilon})\,g(\nu)$$

causality	➡	$P\int_{-\infty}^{\infty}\frac{d\nu'}{\nu'-\nu}\,\frac{f(\nu')-f(0)}{\nu'^2} = i\pi\frac{f(\nu)-f(0)}{\nu^2}$
low energy theorem	➡	$f(0) = -\frac{\alpha}{M_p}$
unitarity	➡	$Im f(\nu) = \frac{\nu}{4\pi}\sigma_{tot}(\nu)$
crossing symmetry	➡	$f(-\nu) = f(\nu)$

Kramers—Kronig relation

$$\boxed{Re f(\nu) = -\frac{\alpha}{M_p} + \frac{\nu^2}{2\pi^2} P \int_0^\infty \frac{d\nu'}{\nu'^2 - \nu^2}\, \sigma_{tot}(\nu')}$$

First order in α $\Rightarrow$ $\sigma_{tot} \to \sigma_{p+\gamma\to hadrons}$

Low energy expansion $\Rightarrow$ sum rules for polarizabilities

$$Re f(\nu) = -\frac{\alpha}{M_p} + \nu^2(\alpha_{E1} + \beta_{M1}) + \nu^4\left(\alpha_{E\nu} + \beta_{M\nu} + \frac{1}{12}(\alpha_{E2} + \beta_{M2})\right) + O(\nu^6)$$

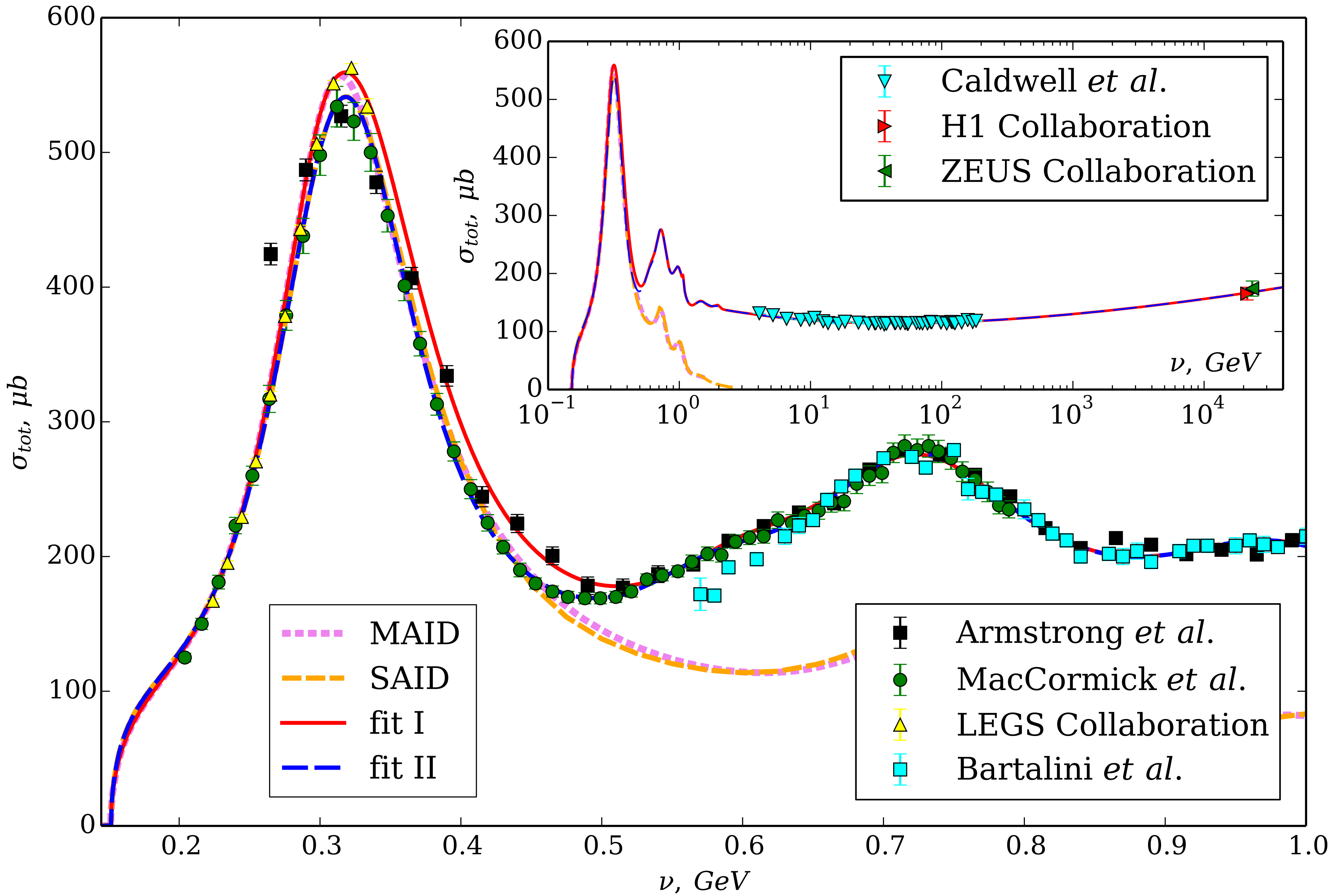
Baldin sum rule:

$$\boxed{\alpha_{E1} + \beta_{M1} = \frac{1}{2\pi^2} \int_{\nu_0}^\infty \frac{d\nu}{\nu^2} \sigma_{abs}(\nu)}$$

4th order:

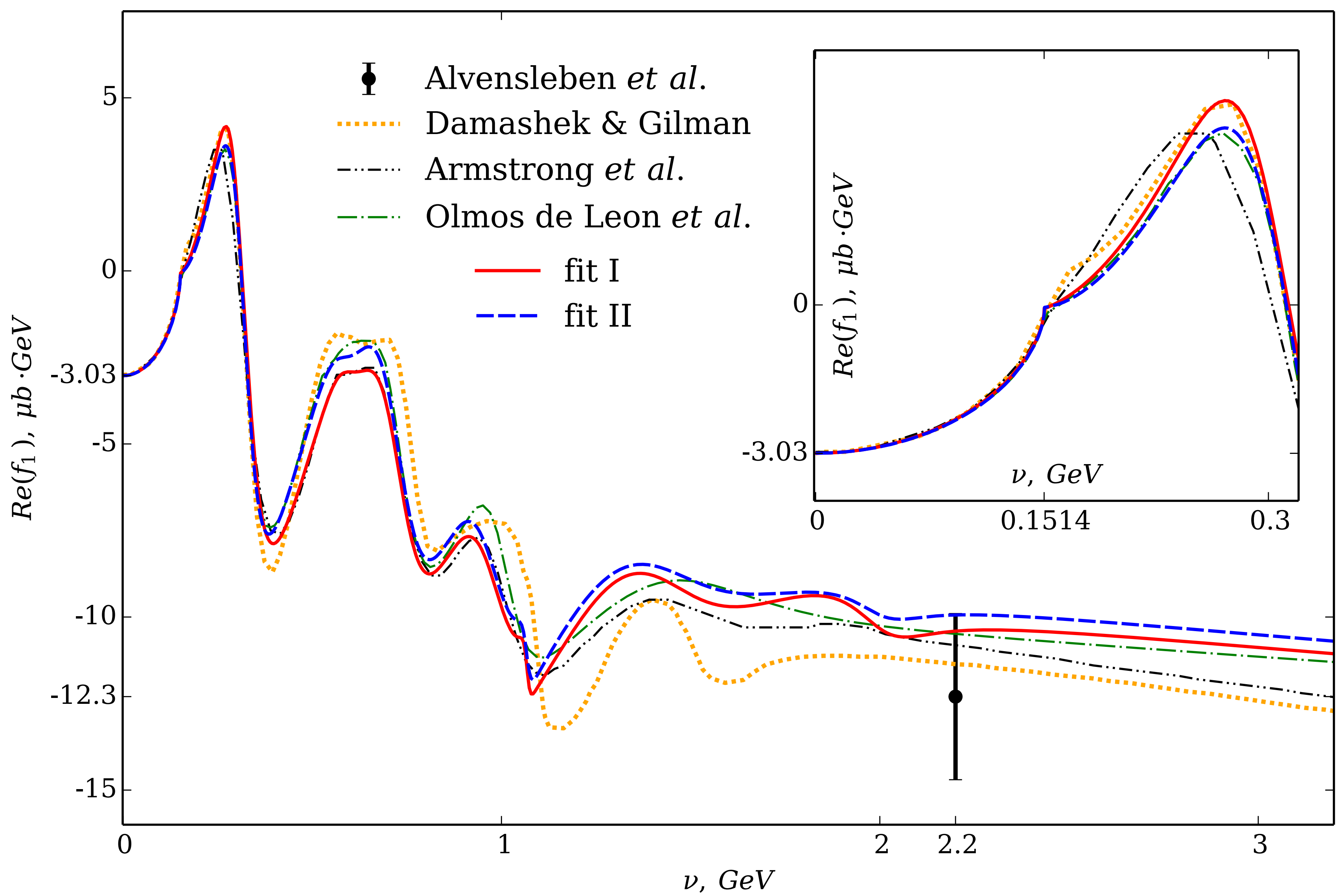
$$\boxed{\alpha_{E\nu} + \beta_{M\nu} + \frac{1}{12}(\alpha_{E2} + \beta_{M2}) = \frac{1}{2\pi^2} \int_{\nu_0}^\infty d\nu \frac{\sigma(\nu)}{\nu^4}}$$

Fitting photoabsorption cross section data

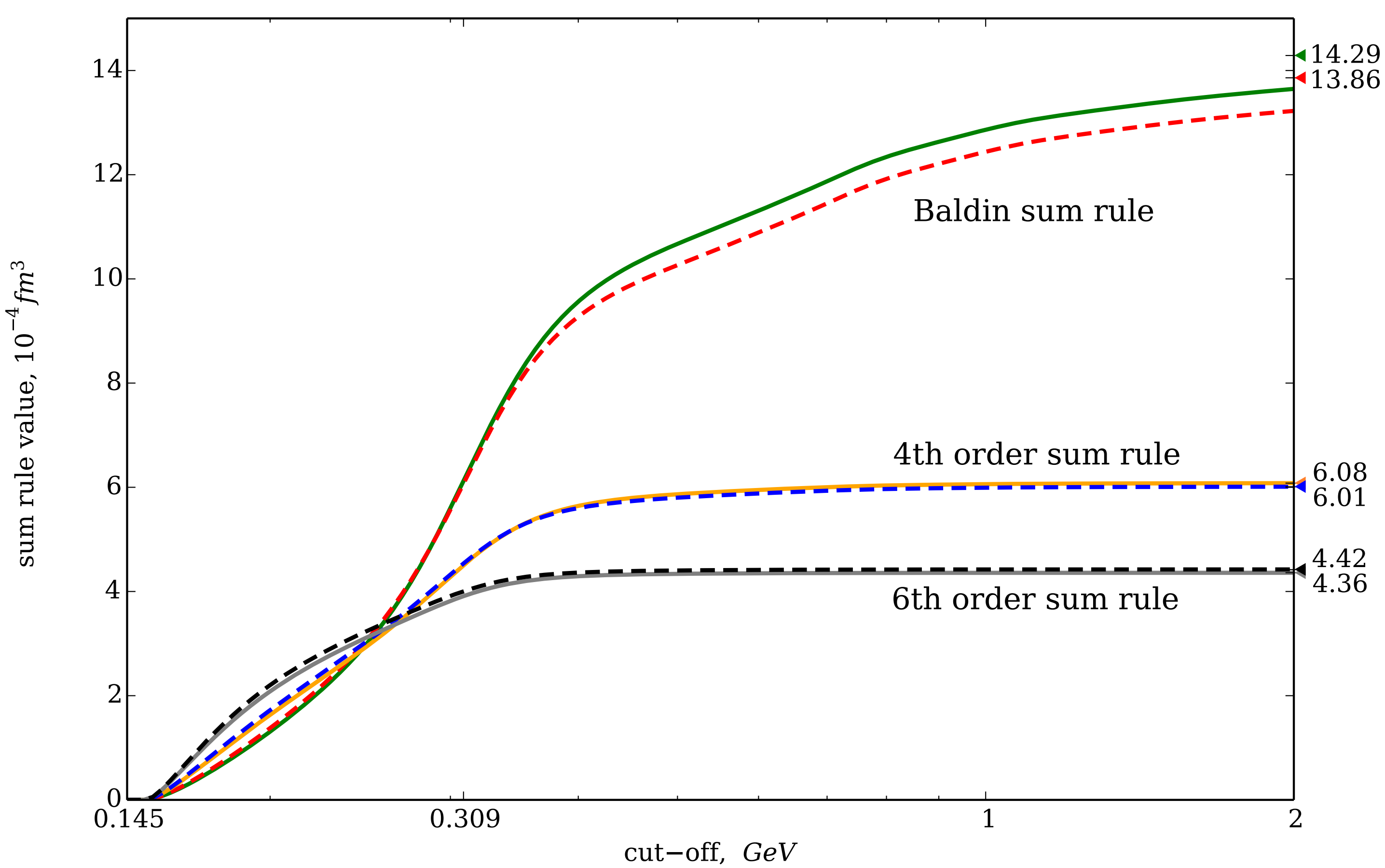


Kramers—Kronig relation: Evaluation

$$Re f(\nu) = -\frac{\alpha}{M_p} + \frac{\nu^2}{2\pi^2} P\int_0^\infty \frac{d\nu'}{\nu'^2-\nu^2}\,\sigma_{tot}(\nu')$$



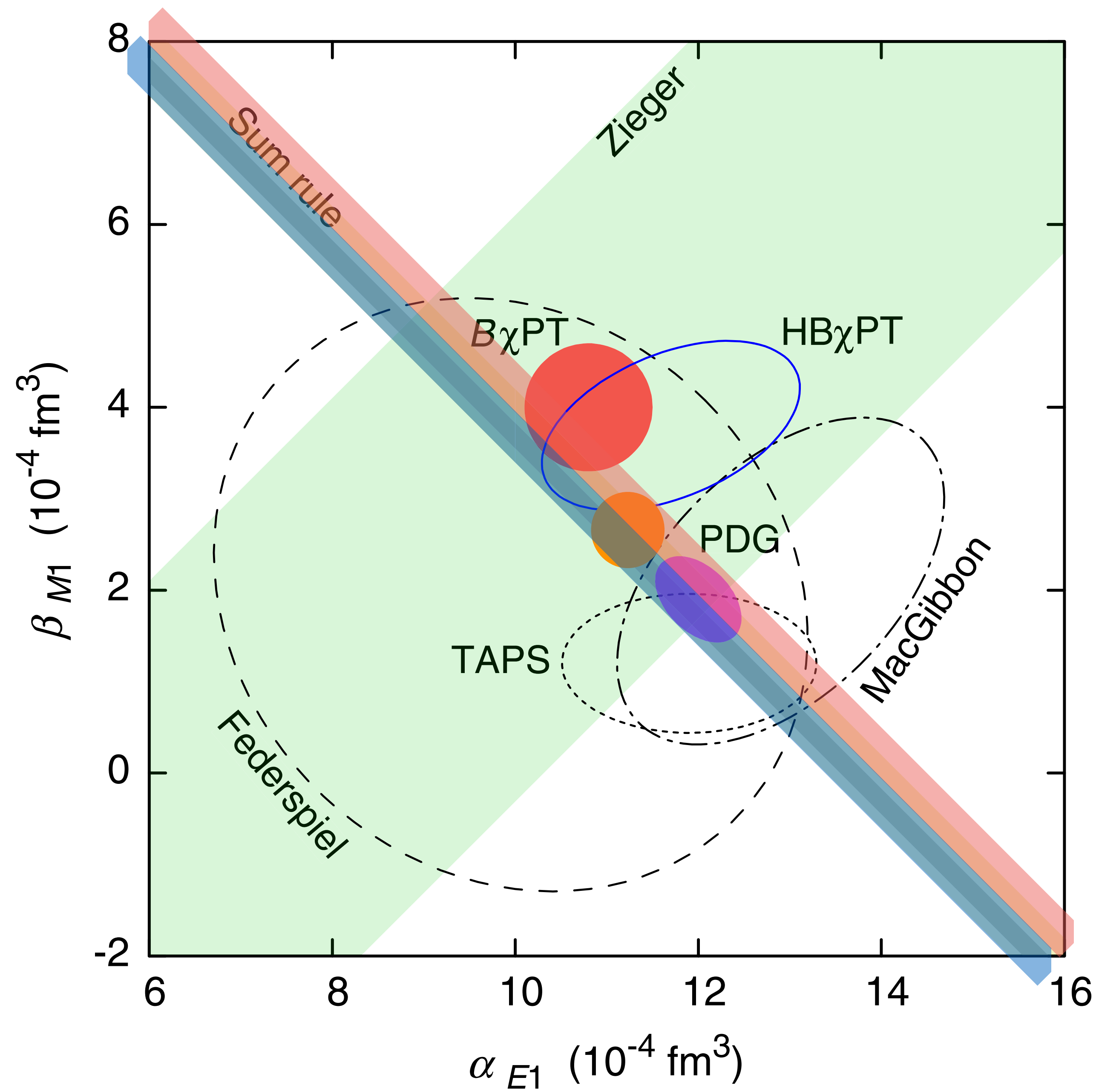
	Baldin SR $[10^{-4}\ \mathrm{fm}^3]$	4th-order SR $[10^{-4}\ \mathrm{fm}^5]$	6th-order SR $[10^{-4}\ \mathrm{fm}^7]$	$f(\nu = 2.2\ \mathrm{GeV})$ $[\mu\mathrm{b} \cdot \mathrm{GeV}]$
Fit I	14.29 ± 0.30	6.08 ± 0.12	4.36 ± 0.09	-10.4
Fit II	13.86 ± 0.21	6.01 ± 0.10	4.42 ± 0.08	-9.9
Armstrong *et al.*	–	–	–	-10.8
Damashek & Gilman	14.2 ± 0.3	–	–	-11.4
Babusci *et al.*	13.69 ± 0.14	–	–	–
Olmos de Leon *et al.*	13.8 ± 0.4	–	–	-10.5
MAID (π channel)	11.63	–	–	–
Alvensleben *et al.*	–	–	–	-12.3 ± 2.4



$$\alpha_{E1} + \beta_{M1} = \frac{1}{2\pi^2} \int_{\nu_0}^{\infty} \frac{d\nu}{\nu^2} \sigma_{abs}(\nu)$$

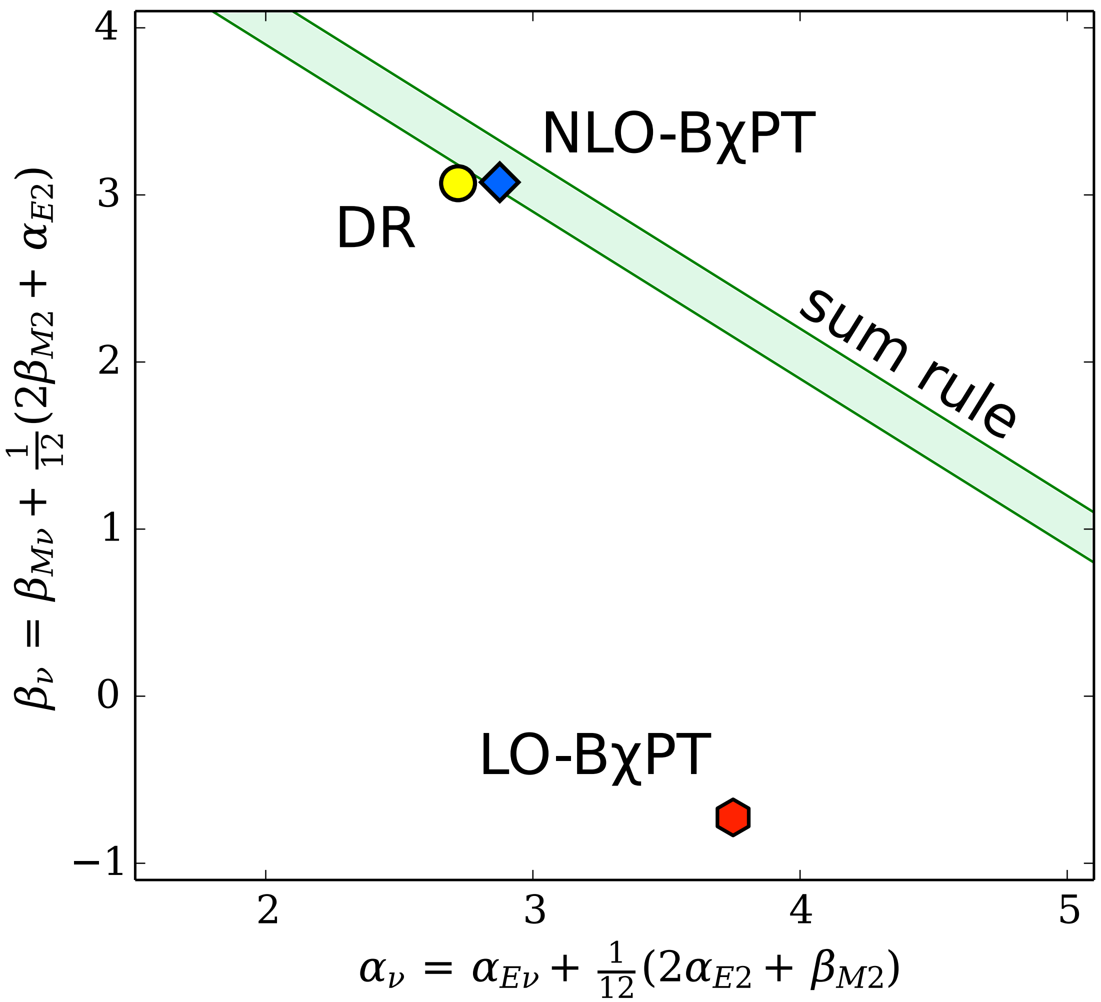
$$\alpha_{E\nu} + \beta_{M\nu} + \frac{1}{12}(\alpha_{E2} + \beta_{M2}) = \frac{1}{2\pi^2} \int_{\nu_0}^{\infty} d\nu \frac{\sigma(\nu)}{\nu^4}$$

Baldin sum rule evaluated with Fit I and II



4th order sum rule

	DR	LO-BχPT	NLO-BχPT
$\alpha_{E\nu}$	-3.75	0.8	-1.3
$\beta_{M\nu}$	9.1	1.8	7.1
α_{E2}	27.6	13.5	17.3
β_{M2}	-22.4	-8.4	-15.5



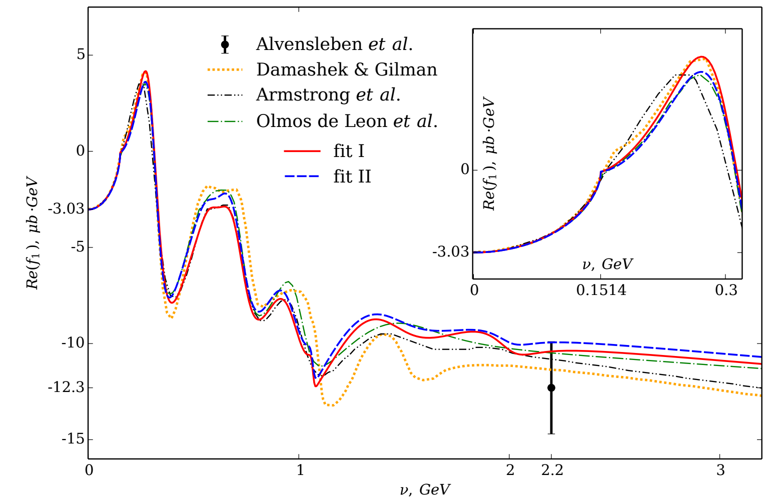
DR D. Babusci, G. Giordano, A. I. L'vov, G. Matone and A. M. Nathan, Phys. Rev. C **58** (1998)

BχPT V. Lensky and V. Pascalutsa, Eur. Phys. J. C **65**, 195 (2010).

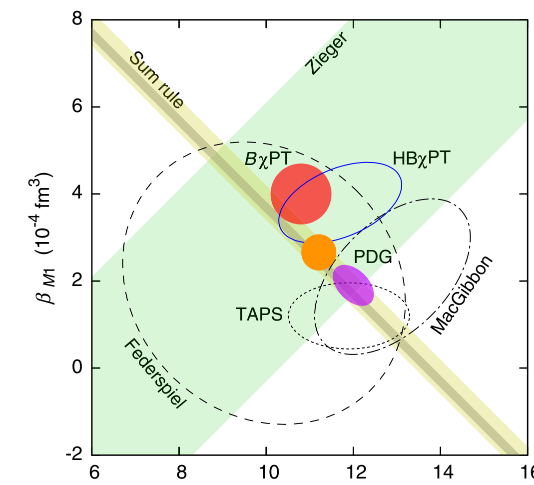
Lensky, McGovern, Pascalutsa (2015).

Summary

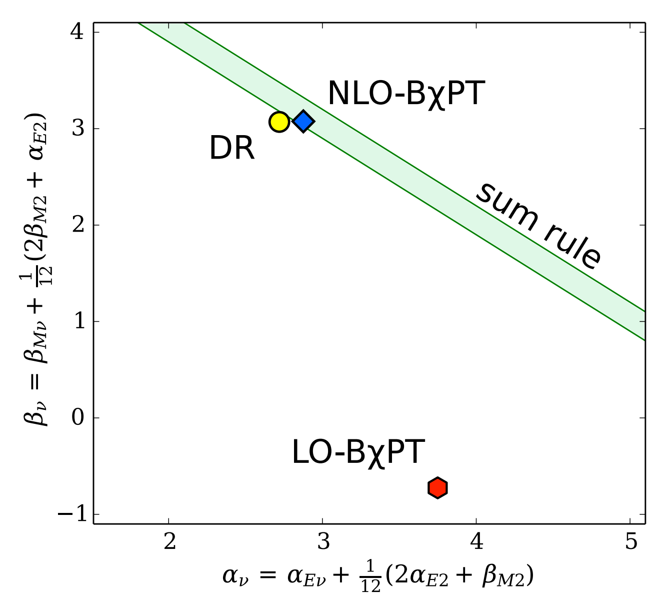
With updated data:



- a forward Compton scattering amplitude was constructed



- Baldin sum rule for proton reevaluated



- the 4th order sum rule obtained

VP: 1307.2340 (2013)