Stability of the pion in the chiral quark model

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- Motivation
- Energy momentum tensor form factors of the pion
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Motivation

- Internal structure of hadrons
- Energy-momentum tensor form factors
  [Pagels, H. (1966) Physical Review, 144(4), 1250.]
- GPDs: Polynomiality $\int dx\, x H_\pi^{I=0}(x,\,\xi,t) \;=\; A_{2,0}(t) + 4\xi^2 A_{2,2}(t)$
- Reliability of a model: **pressure**, mass, …
- Chiral quark model: dynamical & explicit χSB
- Low energy theorem: Gravitational LECs

Energy-momentum tensor

$$\mathcal{L}(\phi(x), \partial_\mu \phi(x))$$

$$x_\mu \rightarrow x'_\mu + a_\mu$$

$$\delta\mathcal{L} = a^\nu \partial^\mu \Theta_{\mu\nu} = 0$$

$$\Theta_{\mu\nu} = \begin{bmatrix} \Theta_{00} & \Theta_{01} & \Theta_{02} & \Theta_{03} \\ \Theta_{10} & \Theta_{11} & \Theta_{12} & \Theta_{13} \\ \Theta_{20} & \Theta_{21} & \Theta_{22} & \Theta_{23} \\ \Theta_{30} & \Theta_{31} & \Theta_{32} & \Theta_{33} \end{bmatrix}$$

Flux of μ momentum across
a surface of constant x_ν

- 00 : Energy density where t = constant
- 0i : Energy flux across i surface
- i0 : Momentum density where t = constant
- ij : Momentum flux of i across j surface

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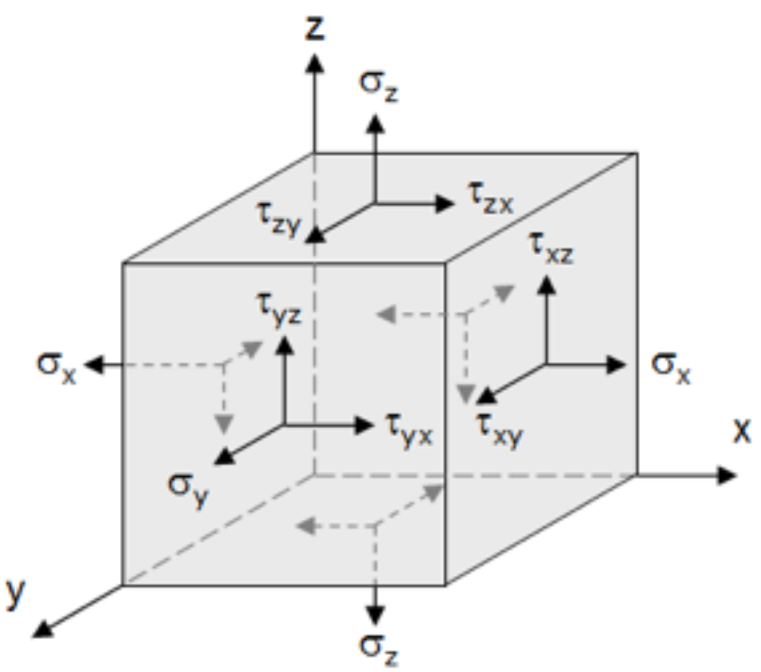

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Energy-momentum tensor: Pion matrix elements

$$\left\langle \pi^a(p')|\Theta_{\mu\nu}(0)|\pi^b(p)\right\rangle$$

$$= \frac{\delta^{ab}}{2}\left[(tg_{\mu\nu} - q_\mu q_\nu)\Theta_1(t) + 2P_\mu P_\nu \Theta_2(t)\right]$$

$$q_\mu = p'_\mu - p_\mu \;\&\; P_\mu = (p'_\mu + p_\mu)/2$$

[H. Pagels, (1966), Phys. Rev, 144(4), 1250.]

Pion mass $\langle \pi^a(p)|\Theta_{00}(0)|\pi^b(p)\rangle = -2m_\pi^2\delta^{ab}$

Stability $\sum_i \langle \pi^a(p)|\Theta_{ii}(0)|\pi^b(p)\rangle = 0$

Chiral Quark Model

$$S_{\rm eff} \;=\; -N_c {\rm Tr} \log \left[i\partial\!\!\!/ + iM\Sigma_5 + im\mathbf{1} \right]$$

[A. Manohar and H. Georgi, Nucl. Phys. B 234, 189 (1984)]

- Effective chiral action in Euclidean space
- Leading order @ Large N_c
- Dynamical χSB: $SU(2)_L \times SU(2)_R \to SU(2)_V$ $\quad \Sigma_5 \;=\; \exp\left(\frac{i\gamma_5 \tau\cdot\pi}{f_\pi}\right)$
- Explicit χSB with isospin symmetry $\quad m = (m_u + m_d)/2$
- Regularization: Pauli-Villars, Proper-time …

Pion EMT Matrix Elements within the Chiral Quark Model

- Quark energy-momentum tensor operator

$$\Theta_{\mu\nu}(x) = \frac{1}{2}\psi^{\dagger}(x)\gamma_{\{\mu}i\overleftrightarrow{\partial}_{\nu\}}\psi(x)$$

- Normalization conditions for the form factors

$$\Theta_2(0) \ = \ 1$$ ➡ Pion mass

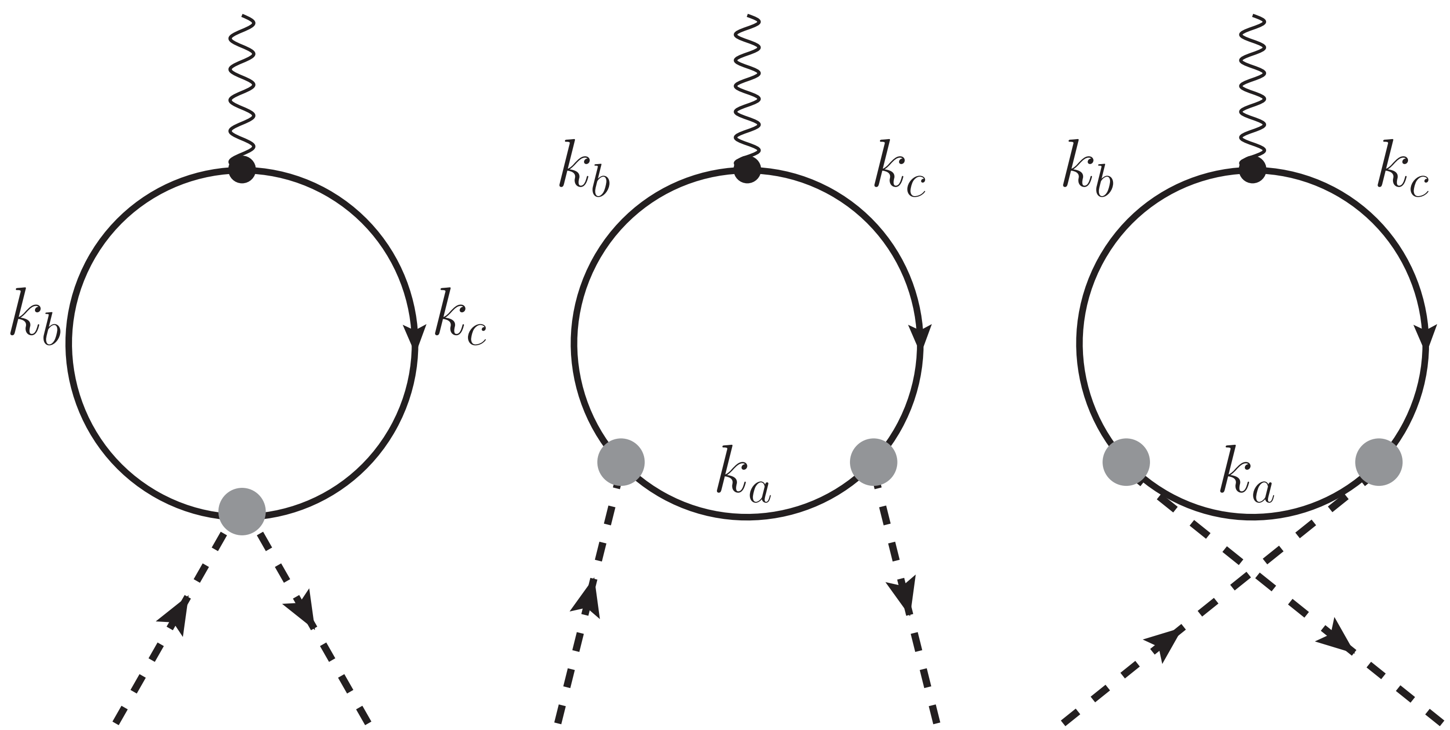
$$\Theta_1(0) - \Theta_2(0) \ = \ \mathcal{O}(m_\pi^2)$$ ➡ Explicit χSB

Calculation of the Matrix Element

$$\langle \pi^a(p')|\Theta_{\mu\nu}(0)|\pi^b(p)\rangle = \delta^{ab}\frac{2N_c}{f_\pi^2}\int d\tilde{k}\sum_i \mathcal{F}_i(k,p,q)_{\mu\nu} + (\mu \leftrightarrow \nu)$$

$$\mathcal{F}_{a\mu\nu} = -\frac{M\overline{M}k_{d\mu}k_{d\nu}}{D_b D_c} \qquad (\overline{M} = m + M)$$

$$\mathcal{F}_{b\mu\nu} = \frac{2M^2 k_{d\nu}}{D_a D_b D_c}\left[-k_{a\mu}\left(k_{bc}+\overline{M}^2\right) + k_{b\mu}\left(k_{ac}+\overline{M}^2\right) + k_{c\mu}\left(k_{ab}+\overline{M}^2\right)\right]$$



$$k_{a\mu} = k_\mu - p_\mu/2 - q_\mu/2$$
$$k_{b\mu} = k_\mu + p_\mu/2 - q_\mu/2$$
$$k_{c\mu} = k_\mu + p_\mu/2 + q_\mu/2$$
$$k_d = k_b + k_c$$
$$k_{ij} = k_i \cdot k_j$$

Pressure and Stability

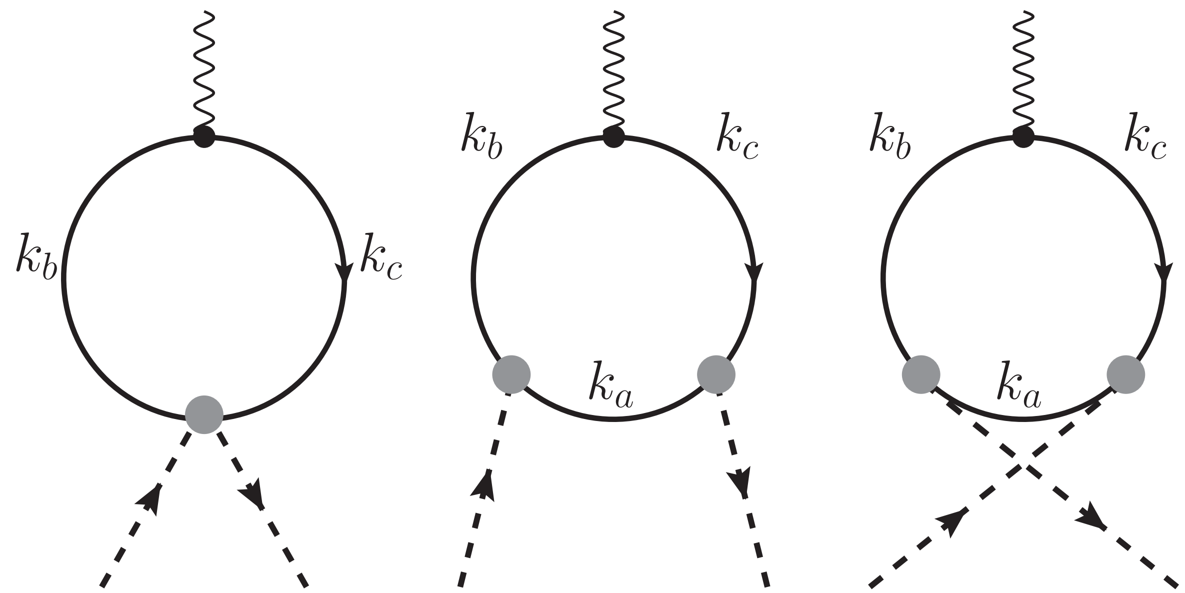
$$\sum_i \langle \pi^a(p)|\Theta_{ii}(0)|\pi^b(p)\rangle \;=\; 0$$

Stability of the pion ⇔ Zero pressure

$$\sum_i \langle \pi^a(p)|\Theta_{ii}(0)|\pi^b(p)\rangle \;=\; \frac{3}{2} t\,\Theta_1(t)\Big|_{t=0}$$

Zero pressure ⇔ Finite form factor

Pressure and Stability



$$\mathcal{P} = \sum_i \langle \pi^a(p) | \Theta_{ii}(0) | \pi^a(p) \rangle$$

$$= \frac{12 N_c m M}{f_\pi^2} \int d\tilde{l} \, \frac{-l^2}{[l^2 + \overline{M}^2]^2} + \frac{12 N_c M^2}{f_\pi^2} \int d\tilde{l} \int_0^1 dx \, \frac{-p^2 l^2}{[l^2 + x(1-x)p^2 + \overline{M}^2]^3}$$

→ Trivially vanishes in the chiral limit

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$$\langle\bar{\psi}\psi\rangle = i\mathrm{Tr}S(x,\,x) = -\int d\tilde{l}\,\frac{8N_c\overline{M}}{[l^2+\overline{M}^2]}$$

$$f_\pi^2 \;=\; \int_0^1 dx\int d\tilde{l}\,\frac{4N_c M\overline{M}}{[l^2+\overline{M}^2+x(1-x)p^2]^2}$$

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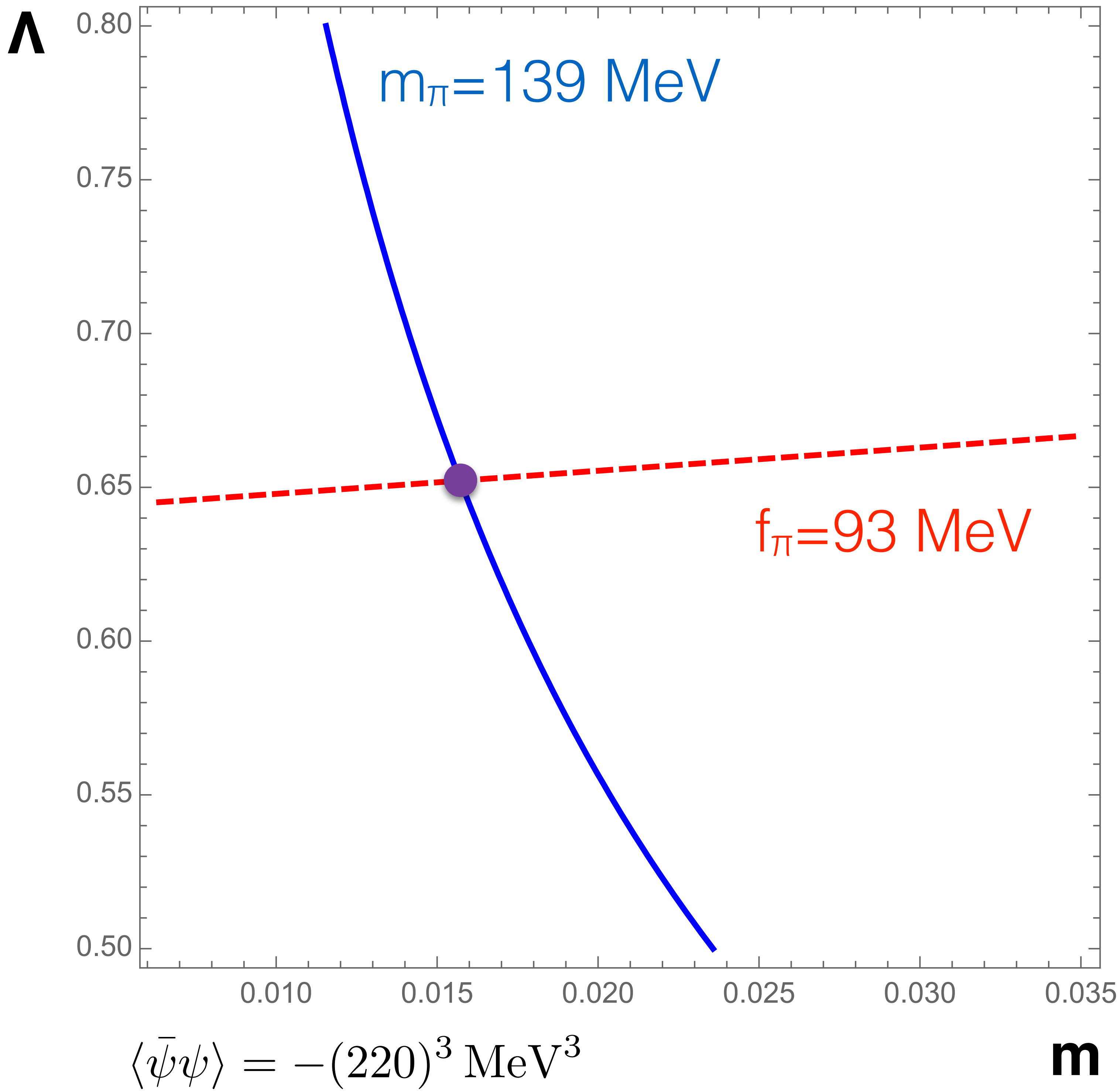
$$= \frac{3M}{f_\pi^2 \overline{M}} \left(m \langle \bar{\psi}\psi \rangle + m_\pi^2 f_\pi^2 \right)$$

- Gell-Mann - Oakes - Renner relation ensures the stability of the pion within the framework of chiral quark model!

$$m_\pi^2 f_\pi^2 = -m \langle \bar{\psi}\psi \rangle + \mathcal{O}(m^2)$$

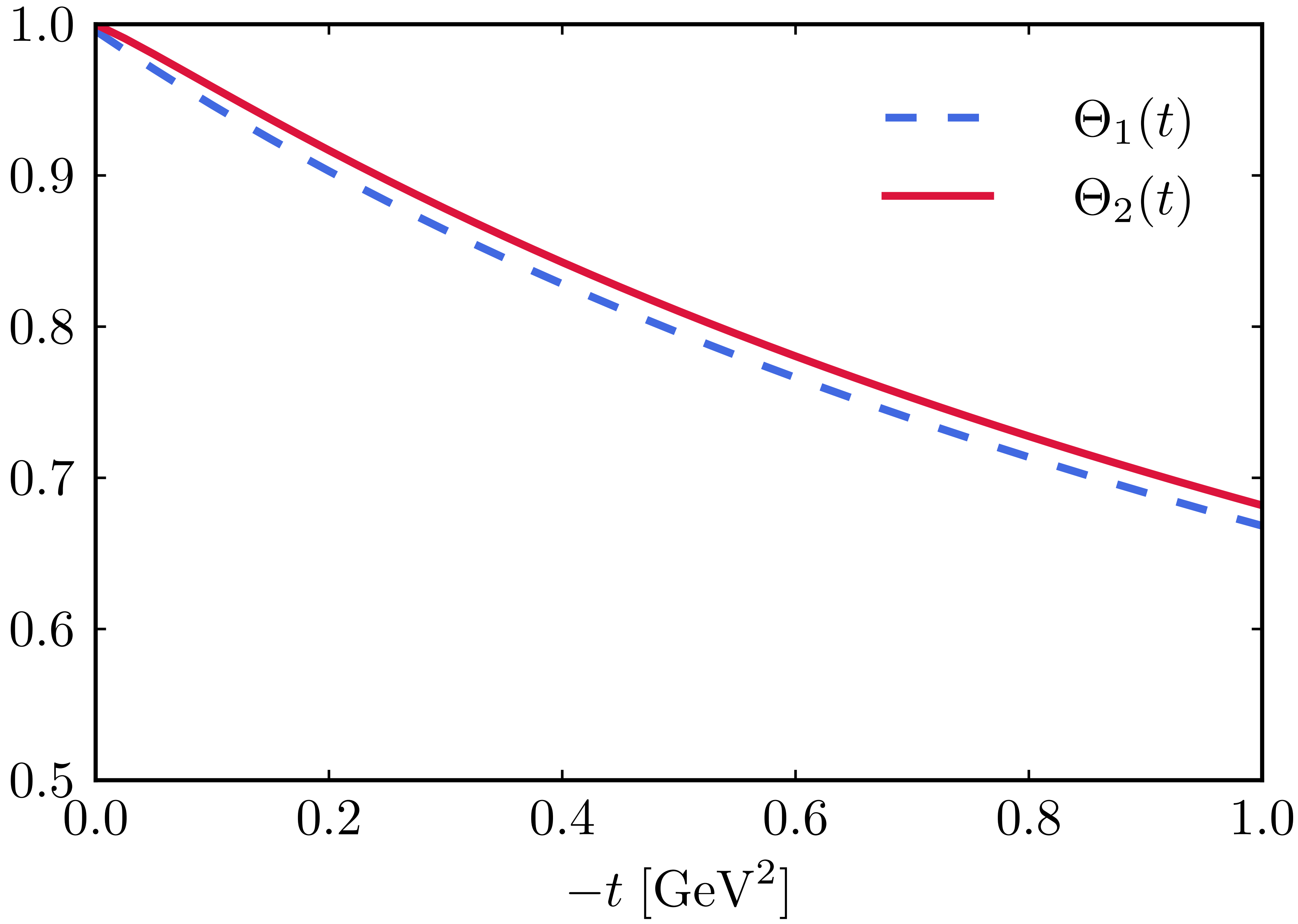
Regularization

M = 350 MeV



$\langle \bar{\psi}\psi \rangle = -(220)^3\,\mathrm{MeV}^3$

Form Factors



Low Energy Constants

$$\Theta_1(q^2) = 1 + \frac{2q^2}{f_\pi^2}(4L_{11} + L_{12}) - \frac{16m_\pi^2}{f_\pi^2}(L_{11} - L_{13}) + \ldots$$

$$\Theta_2(q^2) = 1 - \frac{2q^2}{f_\pi^2}L_{12} + \ldots$$

[J.F. Donoghue and H. Leutwyler, Z.Phys.C(1991) 52, 343]

- Derivative expansion in a curved space leads

$$L_{11} = \frac{N_c}{192\pi^2} = 1.6 \times 10^{-3}$$

$$L_{12} = -2L_{11} = -3.2 \times 10^{-3}$$

$$L_{13} = -\frac{N_c}{96\pi^2}\frac{M}{B_0}\Gamma\left(0, \frac{M^2}{\Lambda^2}\right) = 0.84 \times 10^{-3}$$

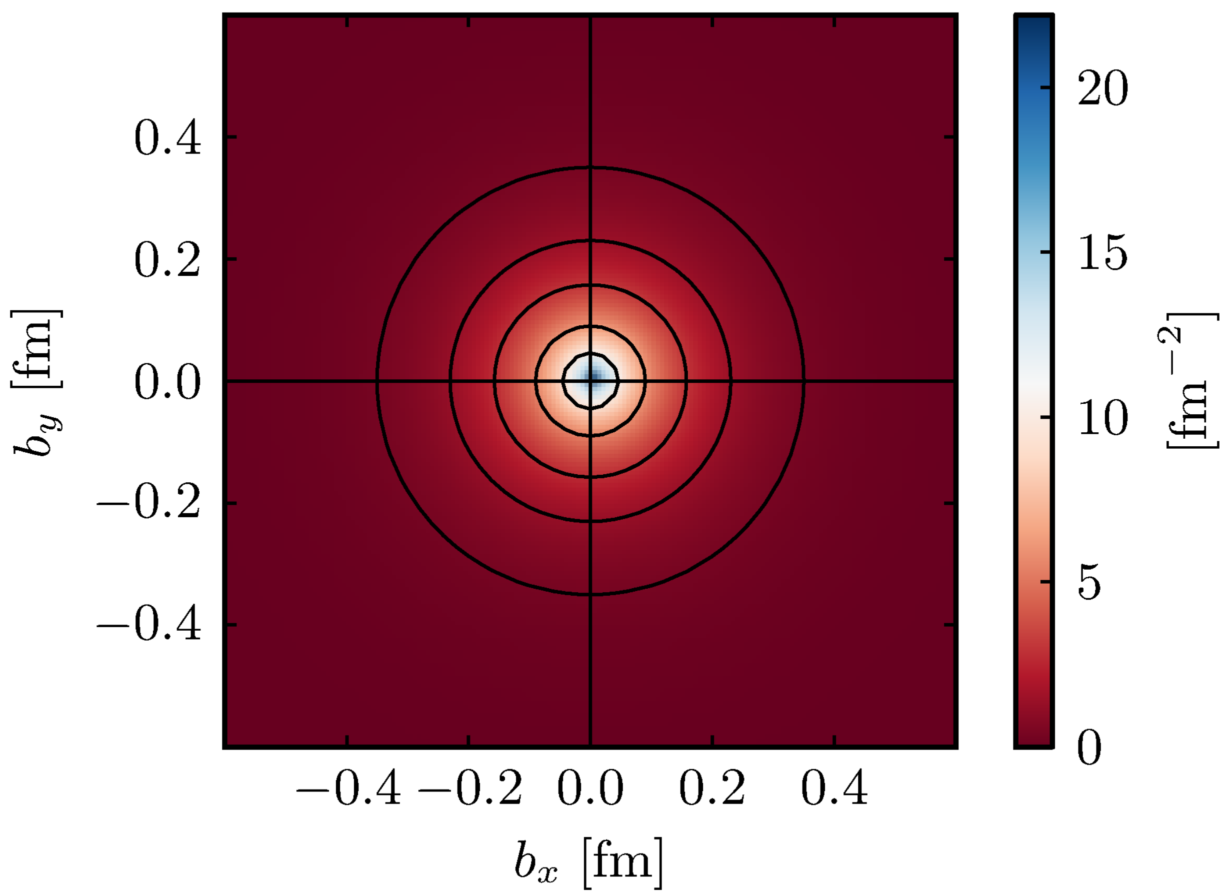
Low Energy Constants

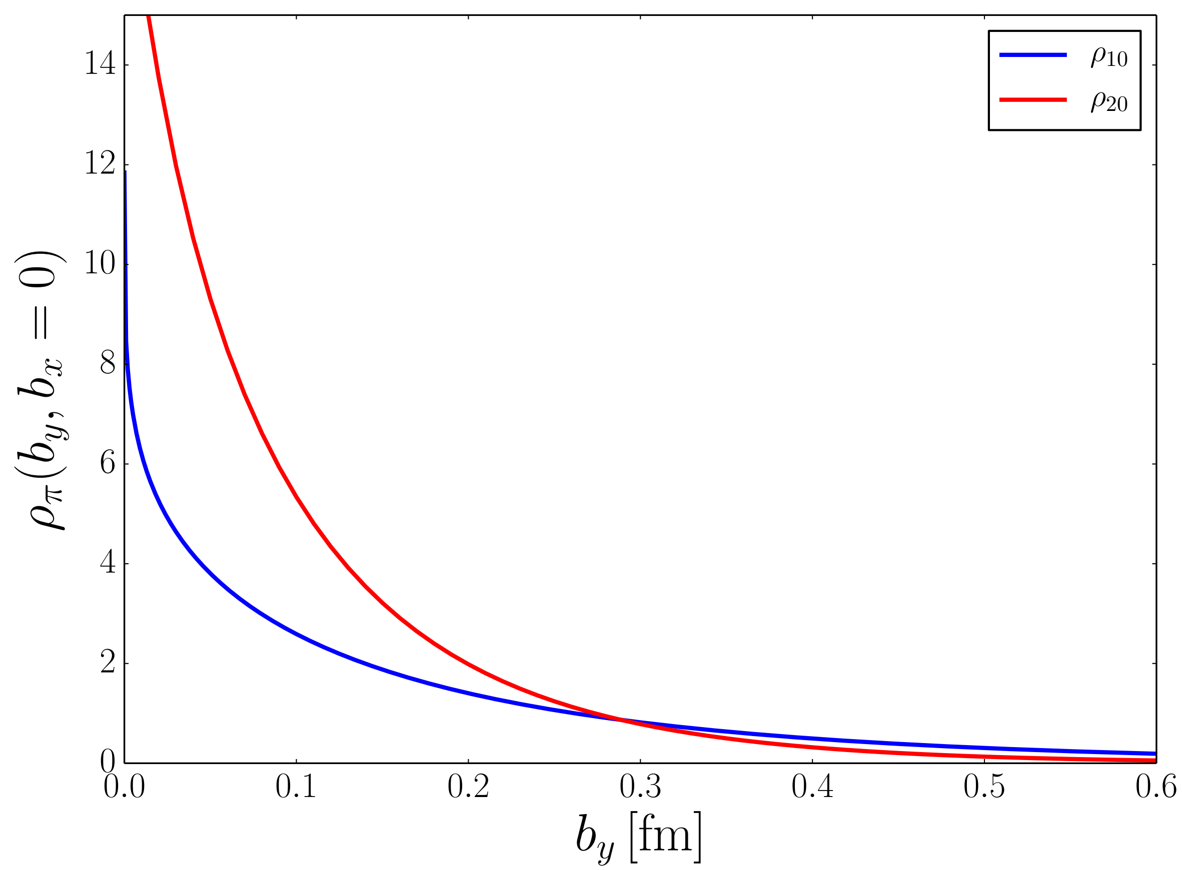
	L_{11}	L_{12}	L_{13}
Present Work	**$1.6*10^{-3}$**	**$-3.2*10^{-3}$**	**$0.84*10^{-3}$**
SQM*	$1.6*10^{-3}$	$-3.2*10^{-3}$	$0.3*10^{-3}$
ChPTh**	$1.4*10^{-3}$	$-2.7*10^{-3}$	$0.9*10^{-3}$

[*Megias *et al*. Phys.Rev. D70 (2004) 034031]
[**J.F. Donoghue and H. Leutwyler, Z.Phys.C(1991) 52, 343]

Transverse Charge Density

$$\rho_{20}(b) = \int_0^\infty \frac{QdQ}{2\pi} J_0(bQ)\Theta_2(t)$$





$$\langle b^2 \rangle_{20} = \int_0^\infty d^2b\, b^2 \rho_{20}(b)$$ →

n=2: $(0.264\ \text{fm})^2$

n=1: $(0.535\ \text{fm})^2$

Summary & Outlook

- EMT Matrix elements within chiral quark model
- Pressure of the pion vanishes due to the GMOR relation: pion stability is deeply rooted in the pattern of χSB
- Model independent approach?
- Form factors, LECs, Transverse charge density

Thank you very much!