THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

Cluster of Excellence PRISMA
Precision Physics, Fundamental Interactions and Structure of Matter

SUBLEADING EFFECTS OF NUCLEON STRUCTURE IN HYDROGEN

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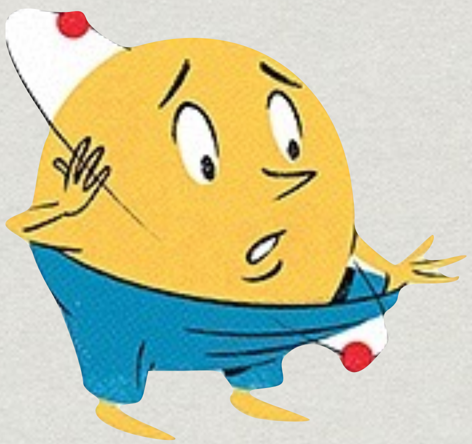
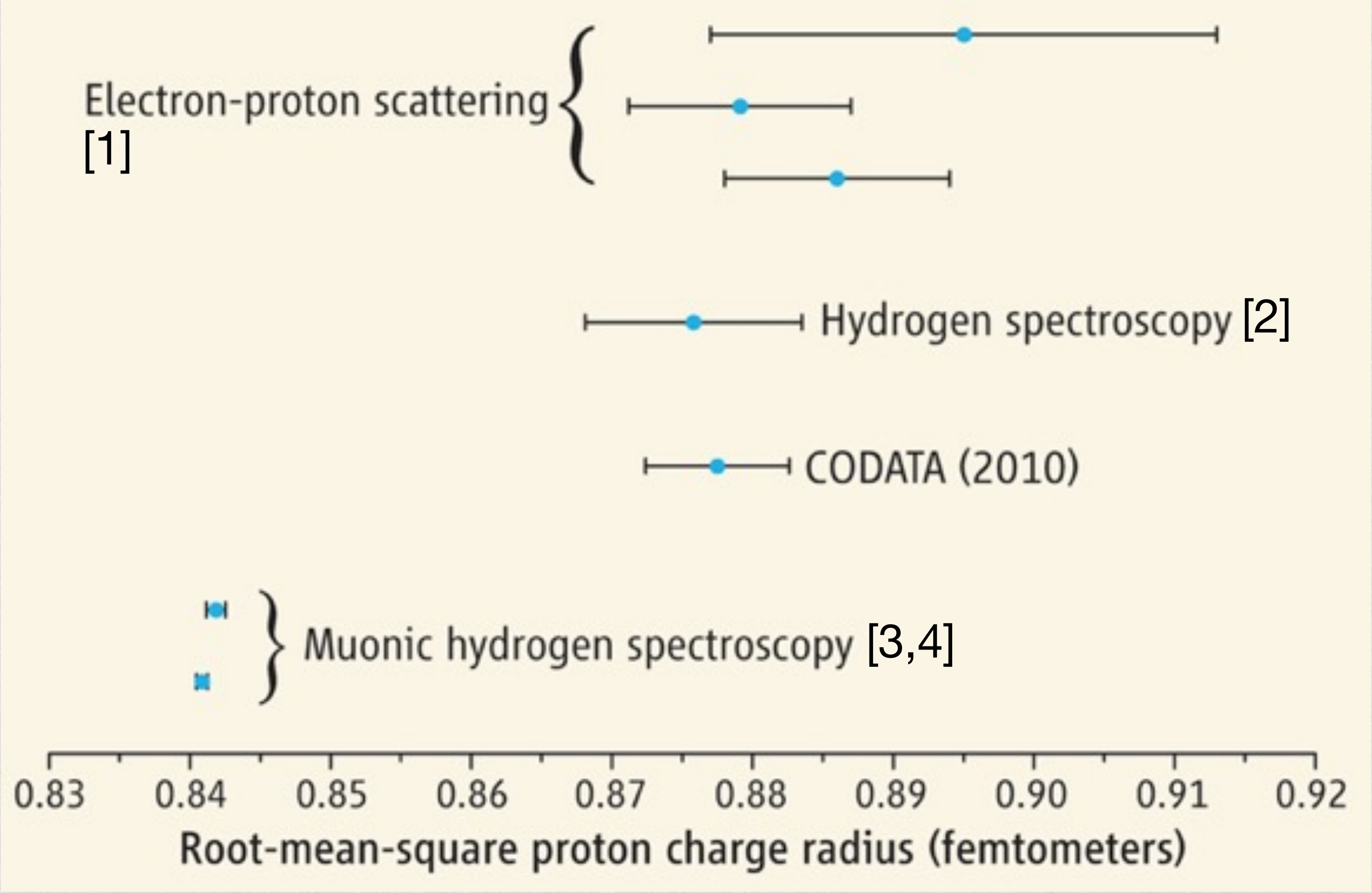
Proton Size Puzzle





[1] J. C. Bernauer *et al.*, Phys. Rev. Lett. **105**, 242001 (2010).

[2] P. J. Mohr, B. N. Taylor and D. B. Newell, Rev. Mod. Phys. **84**, 1527 (2012).

[3] R. Pohl, A. Antognini *et al.*, Nature **466**, 213 (2010).

[4] A. Antognini *et al.*, Science **339**, 417 (2013).

eight standard-deviation discrepancy (7.9 σ) !!!

$$[R_E^{\mu\mathrm{H}} = 0.84087(39)\,\mathrm{fm}] \longleftrightarrow [R_E^{\mathrm{CODATA\ 2010}} = 0.8775(51)\,\mathrm{fm}]$$

How Does the Proton Structure Affect the Hydrogen Lamb Shift?

- finite-size contribution:

becomes appreciable in μH

$$\Delta E_{nS}(\mathrm{LO}) = \frac{2(Z\alpha)^4 m_r^3}{3n^3} R_E^2 \qquad \Delta E_{nS}(\mathrm{NLO}) = -\frac{(Z\alpha)^5 m_r^4}{3n^3} R_{E(2)}^3$$

$$R_E^2 = -6 \lim_{Q^2 \to 0} \frac{\mathrm{d}}{\mathrm{d}Q^2} G_E(Q^2)$$

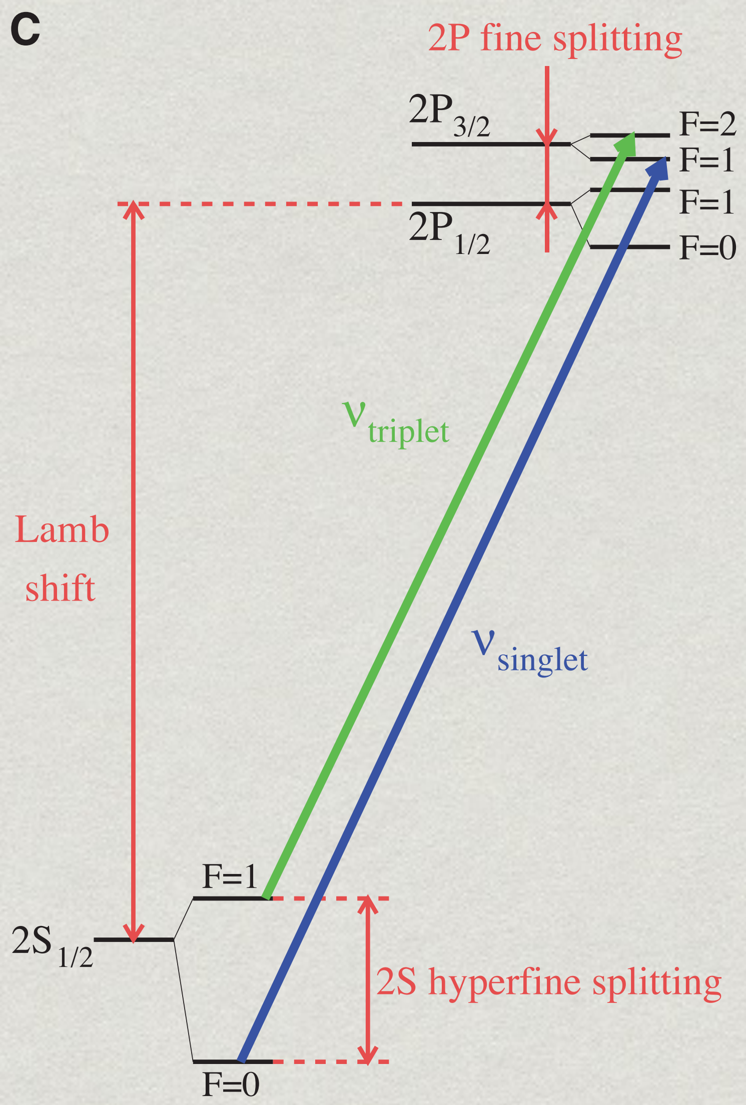
$$R_{E(2)}^3 = \frac{48}{\pi} \int_0^\infty \frac{\mathrm{d}Q}{Q^4} \left\{ G_E^2(Q^2) - 1 + \frac{1}{3} \langle r^2 \rangle_E \, Q^2 \right\}$$

- a tiny, non-smoothness of the electric form factor $G_E(Q^2)$ at scales comparable to the inverse Bohr radius can break down this expansion

→ missing "soft" physics to explain the puzzle ?!

μH Spectroscopy

$$\Delta E_L^{\text{th}} = 206.0336(15)\ \text{meV} - 5.2275(10)\ R_E^2[\text{fm}^2] + \Delta E_{\text{TPE}}$$



radiative, relativistic and recoil effects

finite size effects: proton structure

two-photon exchange (TPE) effects, including the proton polarizability

$$\Delta E_{\text{TPE}} = 0.0332(20)\ \text{meV}$$

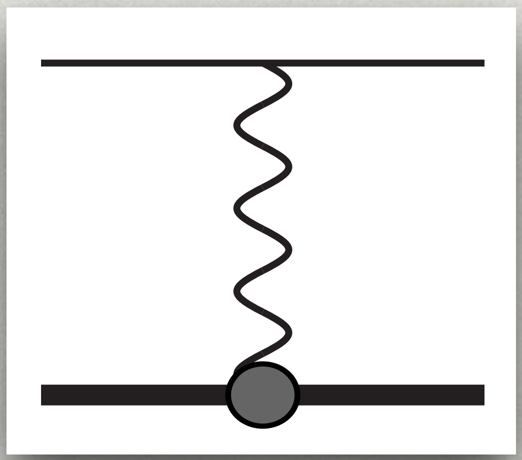
$$\Delta E_L^{\text{exp}} = 202.3706(23)\ \text{meV}$$

A. Antognini *et al.*, Science **339**, 417 (2013).

Finite-Size Effects

- one-photon exchange correction to the Coulomb potential:

$$\Delta V_{\mathrm{size}} = \left(2E_k\, 2E_{k'}\, 2E_p\, 2E_{p'}\right)^{-1/2} \bar{u}(\boldsymbol{k}')(-e\gamma^\mu)u(\boldsymbol{k})\,\Delta_{\mu\nu}(q)\,\bar{N}(\boldsymbol{p}')e\Gamma^\nu N(\boldsymbol{p})$$



electromagnetic vertex:

$$\Gamma^\mu = Z\gamma^\mu F_1(Q^2) - \frac{1}{2M}\gamma^{\mu\nu}q_\nu F_2(Q^2)$$

where $Q^2 = -q^2$

photon propagator:

$$-\frac{1}{q^2}\left[g_{\mu\nu} - \frac{1}{\boldsymbol{q}^2+t}\left(q^\mu q^\nu - \chi^\mu q^\nu - \chi^\nu q^\mu\right)\right]$$

in Coulomb gauge $\chi = (0, \boldsymbol{q})$

Form Factors

- Pauli & Dirac form factors: F_1, F_2

once- subtracted dispersion relation:

$$\begin{pmatrix} F_1(Q^2) \\ F_2(Q^2) \end{pmatrix} = \begin{pmatrix} 1 \\ \varkappa \end{pmatrix} + \frac{q^2}{\pi} \int_{t_0}^{\infty} \frac{\mathrm{d}t}{t(t-q^2)} \,\mathrm{Im} \begin{pmatrix} F_1(t) \\ F_2(t) \end{pmatrix}$$

- Sachs form factors: G_E, G_M

$$\mathrm{Im}\; G_E(t) = \mathrm{Im}\; F_1(t) + \frac{t}{(2M)^2} \mathrm{Im}\; F_2(t)$$

$$\mathrm{Im}\; G_M(t) = \mathrm{Im}\; F_1(t) + \mathrm{Im}\; F_2(t)$$

Moments of Charge Distribution

- charge distribution: $\rho_E(r) = \int \frac{\mathrm{d}\vec{q}}{(2\pi)^3} G_E(\vec{q}^{\,2}) e^{-i\vec{q}\vec{r}}$

$\rightarrow$

$$\begin{aligned} \langle r^N \rangle_E &\equiv \int \mathrm{d}\vec{r}\, r^N \rho_E(r) \\ &= \frac{(N+1)!}{\pi} \int_{t_0}^{\infty} \mathrm{d}t\, \frac{\mathrm{Im}\, G_E(t)}{t^{1+N/2}} \end{aligned}$$

$$\langle r^2 \rangle_E = -6 \lim_{Q^2 \to 0} \frac{\mathrm{d}}{\mathrm{d}Q^2} G_E(Q^2)$$

$$\langle r^3 \rangle_E = \frac{48}{\pi} \int_0^{\infty} \frac{\mathrm{d}Q}{Q^4} \left\{ G_E(Q^2) - 1 + \frac{1}{6} \langle r^2 \rangle_E\, Q^2 \right\}$$

- Zemach moments: $\langle r^N \rangle_{E(2)} \equiv \int \mathrm{d}\vec{r}\, r^N \rho_{E(2)}(r)$ with $\rho_{E(2)}(r) = \int \frac{\mathrm{d}\vec{q}}{(2\pi)^3} G_E^2(\vec{q}^{\,2}) e^{-i\vec{q}\vec{r}}$

- Zemach radius: $\langle r \rangle_Z \equiv \int \mathrm{d}\vec{r}\, r \rho_Z(r)$ with $\rho_Z(r) = \int \frac{\mathrm{d}\vec{q}}{(2\pi)^3} G_E(\vec{q}^{\,2}) G_M(\vec{q}^{\,2}) e^{-i\vec{q}\vec{r}}$

FH, VP: in preparation

S-Wave Potentials

$$V_C(\boldsymbol{r}) = -\frac{Z\alpha}{r}$$

$$V_Y(\boldsymbol{r},t) = \frac{1}{\pi}\int_{t_0}^{\infty}\frac{\mathrm{d}t}{t}\,\frac{Z\alpha}{r}e^{-r\sqrt{t}}\,\mathrm{Im}\;G_E(t)$$

$$V_{\mathrm{rel.C.}}(\boldsymbol{r}) = \frac{Z\alpha}{8m_r^2}\,4\pi\delta(\boldsymbol{r})$$

$$V_{\mathrm{rel.E_{kin.}}}(\boldsymbol{r}) = -\frac{\boldsymbol{p}^4}{8m_r^3}$$

$$V_1(\boldsymbol{r}) = Z\alpha\left[\frac{\pi\delta(\boldsymbol{r})}{mM} - \frac{1}{mM}\frac{\boldsymbol{p}^2}{r}\right]$$

$$V_2(\boldsymbol{r},t) = \frac{Z\alpha}{\pi}\int_{t_0}^{\infty}\frac{\mathrm{d}t}{t}\left\{-\frac{1}{8}\left(\frac{1}{m_r^2}+\frac{2}{mM}\right)4\pi\,\delta(\boldsymbol{r}) + \frac{1}{8m_r^2}\frac{t\,e^{-r\sqrt{t}}}{r}\right.$$
$$\left. + \frac{1}{4mM}e^{-r\sqrt{t}}\left[\left(4+2r\sqrt{t}\right)\frac{\boldsymbol{p}^2}{r} + \frac{t^{3/2}}{2}\right]\right\}\mathrm{Im}\;G_E(t)$$

HFS

$$V_3(\boldsymbol{r}) = Z\alpha\left\{\frac{1+\kappa}{3mM}\left[f(f+1)-\frac{3}{2}\right]4\pi\delta(\boldsymbol{r})\right\}$$

$$V_4(\boldsymbol{r},t) = -\frac{Z\alpha}{\pi}\int_{t_0}^{\infty}\frac{\mathrm{d}t}{t}\left\{\frac{1}{3mM}\left[f(f+1)-\frac{3}{2}\right]\left(4\pi\,\delta(\boldsymbol{r}) - \frac{te^{-r\sqrt{t}}}{r}\right)\right\}\mathrm{Im}\;G_M(t)$$

electronic vacuum polarization:

$$\mathrm{Im}\;\Pi\left(4m_e^2 t\right) = -\frac{\alpha}{3}\left(1+\frac{1}{2\,t}\right)\sqrt{1-\frac{1}{t}}$$

P-Wave Potentials

$$V_C(\boldsymbol{r}) = -\frac{Z\alpha}{r}$$

$$V_Y(\boldsymbol{r},t) = \frac{1}{\pi}\int_{t_0}^{\infty}\frac{\mathrm{d}t}{t}\frac{Z\alpha}{r}e^{-r\sqrt{t}}\mathrm{Im}\ G_E(t)$$

$$V_{\mathrm{rel.C.}}(\boldsymbol{r}) = \frac{Z\alpha}{2m_r^2}\frac{\boldsymbol{L}\cdot\boldsymbol{s}}{r^3}$$

$$V_{\mathrm{rel.E_{kin.}}}(\boldsymbol{r}) = -\frac{\boldsymbol{p}^4}{8m_r^3}$$

$$V_1(\boldsymbol{r}) = -Z\alpha\left[\frac{1}{2M^2}\frac{\boldsymbol{L}\cdot\boldsymbol{s}}{r^3} + \frac{1}{mM}\left(\frac{\boldsymbol{p}^2}{r} - \frac{1}{r^3}\right)\right]$$

$$V_2(\boldsymbol{r},t) = \frac{Z\alpha}{\pi}\int_{t_0}^{\infty}\frac{\mathrm{d}t}{t}\left\{\frac{1}{8m_r^2}\frac{t\,e^{-r\sqrt{t}}}{r} - \left(\frac{1}{m_r^2} - \frac{1}{M^2}\right)e^{-r\sqrt{t}}(1+r\sqrt{t})\,\frac{\boldsymbol{L}\cdot\boldsymbol{s}}{2r^3}\right.$$
$$\left. + \frac{1}{4mM}e^{-r\sqrt{t}}\left[-(1+r\sqrt{t})\,\frac{4}{r^3} + \frac{t^{3/2}}{2} + (4+2r\sqrt{t})\,\frac{\boldsymbol{p}^2}{r}\right]\right\}\mathrm{Im}\ G_E(t)$$

HFS

$$V_3(\boldsymbol{r},t) = Z\alpha\left\{\frac{1}{r^3}\left[\frac{1/2+\kappa}{M^2}\boldsymbol{L}\cdot\boldsymbol{S} + \frac{1+\kappa}{mM}\left(\boldsymbol{L}\cdot\boldsymbol{S} + \frac{4}{5}\boldsymbol{s}\cdot\boldsymbol{S} - \frac{3}{5}\{\boldsymbol{L}\cdot\boldsymbol{s},\boldsymbol{L}\cdot\boldsymbol{S}\}\right)\right]\right\}$$

$$V_4(\boldsymbol{r},t) = -\frac{Z\alpha}{\pi}\int_{t_0}^{\infty}\frac{\mathrm{d}t}{t}\left\{\frac{e^{-r\sqrt{t}}}{r^3}(1+r\sqrt{t})\left[\frac{\boldsymbol{L}\cdot\boldsymbol{S}}{M^2} + \frac{1}{mM}\left(\boldsymbol{L}\cdot\boldsymbol{S} + \frac{4}{5}\boldsymbol{s}\cdot\boldsymbol{S} - \frac{3}{5}\{\boldsymbol{L}\cdot\boldsymbol{s},\boldsymbol{L}\cdot\boldsymbol{S}\}\right)\right]\right.$$
$$\left. - \frac{2}{5mM}\frac{te^{-r\sqrt{t}}}{r}\left(\boldsymbol{s}\cdot\boldsymbol{S} + \frac{1}{2}\{\boldsymbol{L}\cdot\boldsymbol{s},\boldsymbol{L}\cdot\boldsymbol{S}\}\right)\right\}\mathrm{Im}\ G_M(t)$$

$$V_5(\boldsymbol{r},t) = \frac{Z\alpha}{\pi}\int_{t_0}^{\infty}\frac{\mathrm{d}t}{t}\frac{\boldsymbol{L}\cdot\boldsymbol{S}}{2M^2}\frac{e^{-r\sqrt{t}}}{r^3}(1+r\sqrt{t})\,\mathrm{Im}\ G_E(t)$$

Structure of Hydrogen Spectra

- $\langle V_3 \rangle$: $\Delta E_{nS,\mathrm{HFS}} = \dfrac{E_F}{n^3}$

Fermi - Energy:
$$E_F = \frac{8}{3}\frac{Z\alpha}{a^3}\frac{1+\kappa}{mM}$$

- $\langle V_4 \rangle + \langle V_Y \rangle \langle V_3 + V_4 \rangle$:

$$\Delta E_{nS,\mathrm{HFS}} \approx \frac{8E_F}{an^3}\frac{1}{\pi}\int_0^\infty \frac{\mathrm{d}Q}{Q^2}\left[\frac{G_E(Q^2)G_M(Q^2)}{1+\kappa} - 1\right]$$
$$= -\frac{2E_F}{an^3}\langle r \rangle_Z$$

- $\langle V_{\mathrm{rel.\ C.}} \rangle + \langle V_1 \rangle$:

$$\Delta E_{2P_{3/2}-2P_{1/2}} = \frac{Z\alpha}{32a^3}\left[\frac{1}{m_r^2} - \frac{1}{M^2}\right] = 8.329 \text{ meV}$$

- $\langle V_2 \rangle_{eVP}$:

$$\Delta E_{\text{ eVP, }2P} = \frac{Z\alpha}{a^3}\frac{1}{\pi}\int_1^\infty \frac{\mathrm{d}t}{t}\left\{\frac{1}{32m_r^2}\frac{t/\lambda^2}{\left[1+\sqrt{t}/\lambda\right]^4} + \frac{1}{16mM}\frac{1+2\sqrt{t}/\lambda}{\left[1+\sqrt{t}/\lambda\right]^4}\right.$$
$$\left. + \frac{\boldsymbol{L}\cdot\boldsymbol{s}}{48}\left(\frac{1}{M^2} - \frac{1}{m_r^2}\right)\frac{1+3\sqrt{t}/\lambda}{\left[1+\sqrt{t}/\lambda\right]^3}\right\} \mathrm{Im}\,\Pi(4m_e^2 t) \qquad \lambda = 1/2am_e$$
$$= \begin{cases} -2.811\ \mu\text{eV} & j = 1/2 \\ 0.229\ \mu\text{eV} & j = 3/2, \end{cases}$$

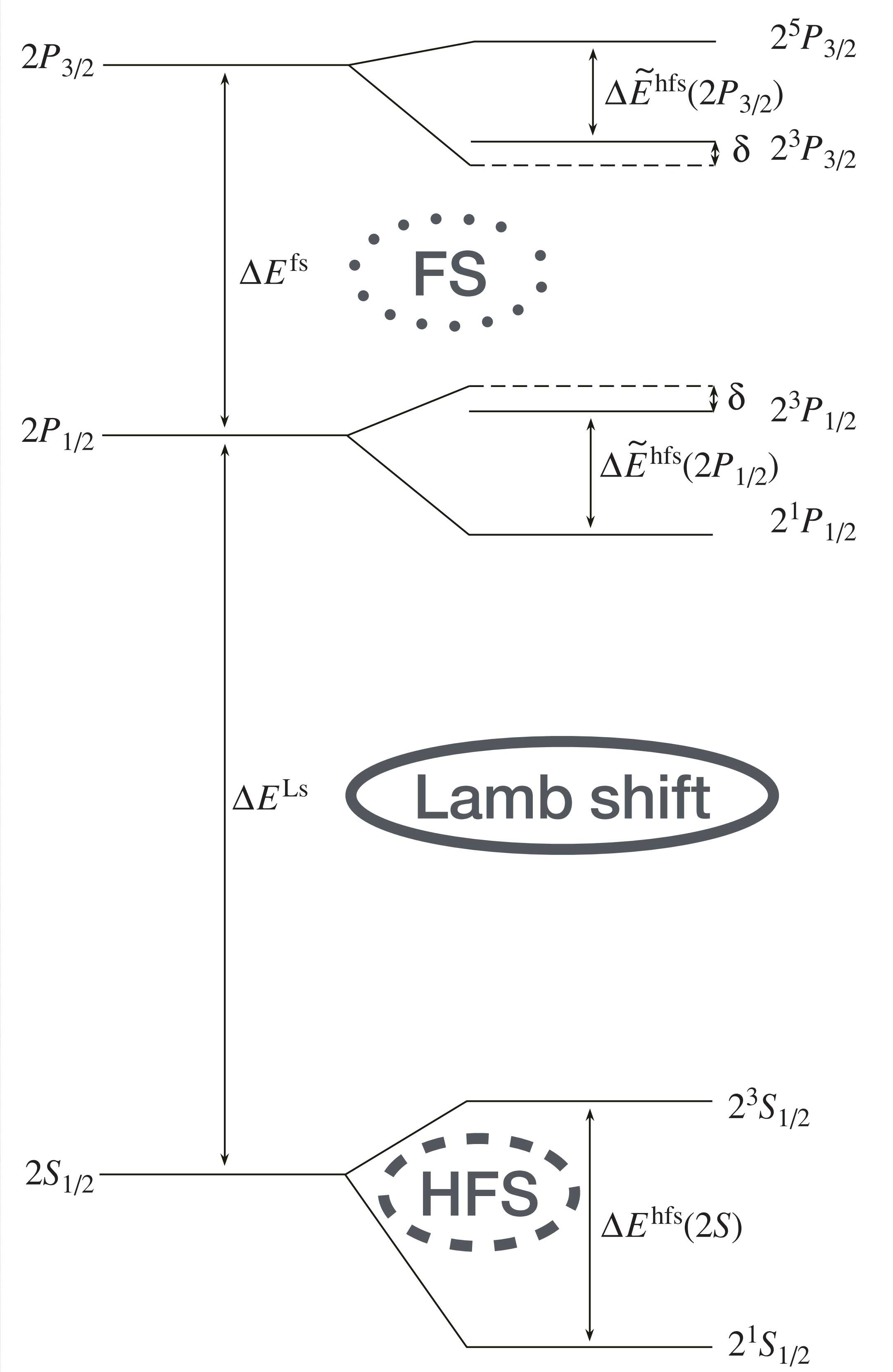


Fig. 4. Structure of the S- and P-wave energy levels in muonic hydrogen for $n = 2$.

Martynenko, Physics of Atomic Nuclei **71**, 1, 125-135 (2008)

Lamb Shift (1)

Yukawa potential:

$$V_Y(r) = \frac{Z\alpha}{r}\frac{1}{\pi}\int_{t_0}^{\infty}\frac{\mathrm{d}t}{t}\,e^{-r\sqrt{t}}\,\mathrm{Im}\,G_E(t)$$

electric form factor (FF) correction to the Coulomb potential $-\frac{Z\alpha}{r}$

- contribution of $V_Y(r)$ at 1st order perturbation theory (PT):

$$\begin{aligned}\Delta E^{\mathrm{FF}(1)}_{2P-2S} &= \langle 2P_{1/2}\,|V_Y|\,2P_{1/2}\rangle - \langle 2S_{1/2}\,|V_Y|\,2S_{1/2}\rangle \\ &= -\frac{(Z\alpha)^4 m_r^3}{2\pi}\int_{t_0}^{\infty}\mathrm{d}t\,\frac{\mathrm{Im}\,G_E(t)}{(\sqrt{t}+Z\alpha m_r)^4} \\ &= -\frac{(Z\alpha)^4 m_r^3}{12}\sum_{k=0}^{\infty}\frac{(-Z\alpha m_r)^k}{k!}\langle r^{k+2}\rangle_E \\ &= -\frac{(Z\alpha)^4 m_r^3}{12}\left[\langle r^2\rangle_E - Z\alpha m_r\langle r^3\rangle_E\right] + O(\alpha^6)\end{aligned}$$

convergence radius of the expansion is limited by t_0

Lamb Shift (2)

1st order PT:

$$\Delta E^{\mathrm{FF}(1)}_{2P-2S} = -\frac{(Z\alpha)^4 m_r^3}{12}\left[\langle r^2\rangle_E - Z\alpha m_r \langle r^3\rangle_E\right]$$

- 2nd order PT Lamb shift, to $O(\alpha^5)$:

$$\begin{aligned}\Delta E^{\mathrm{FF}(2)}_{2P-2S} &= -\delta E^{FF(2)}_{2S}\\ &= -\frac{1}{2\pi}\int_0^\infty \mathrm{d}Q\,\frac{|\langle V_{\mathrm{Y}}\rangle|^2}{E_2 - E_k}\\ &\cong (Z\alpha)^5 m_r^4\,\frac{2}{\pi}\int_0^\infty \mathrm{d}k\left\{\frac{1}{\pi}\int_{t_0}^\infty \frac{\mathrm{d}t}{t}\,\frac{1}{t+Q^2}\,\mathrm{Im}\;\; G_E(t)\right\}^2\\ &= (Z\alpha)^5 m_r^4\,\frac{2}{\pi}\int_0^\infty \frac{\mathrm{d}Q}{Q^4}\,\left\{G_E(Q^2)-1\right\}^2\\ &= -\frac{(Z\alpha)^5 m_r^4}{12}\left[\langle r^3\rangle_E - \frac{1}{2}\langle r^3\rangle_{E(2)}\right]\end{aligned}$$

J. L. Friar, Annals Phys. **122**, 151 (1979).

$$\Delta E_{nS}(\mathrm{LO}) = \frac{2(Z\alpha)^4 m_r^3}{3n^3} R_E^2 \qquad \Delta E_{nS}(\mathrm{NLO}) = -\frac{(Z\alpha)^5 m_r^4}{3n^3} R^3_{E(2)}$$

arXiv:1502.03721

Lamb shift: To expand or not ?!

$$\Delta E^{\mathrm{FF}(1)}_{2P-2S} = -\frac{(Z\alpha)^4 m_r^3}{2\pi}\int_{t_0}^{\infty}\mathrm{d}t\,\frac{\mathrm{Im}\;G_E(t)}{(\sqrt{t}+Z\alpha m_r)^4}$$

$$= -\frac{(Z\alpha)^4 m_r^3}{12}\left[\langle r^2\rangle_E - Z\alpha m_r \langle r^3\rangle_E\right] + O(\alpha^6)$$

$$\frac{1}{\pi}\int_{t_0}^{\infty}\mathrm{d}t\,W(t)\,\mathrm{Im}\;G_E(t) = \int_0^{\infty}\mathrm{d}Q\,w(Q)\,G_E(Q^2)$$

Stieltjes integral transform:

$$W(t) = \int_0^{\infty}\mathrm{d}Q\,\frac{w(Q)}{t+Q^2}$$

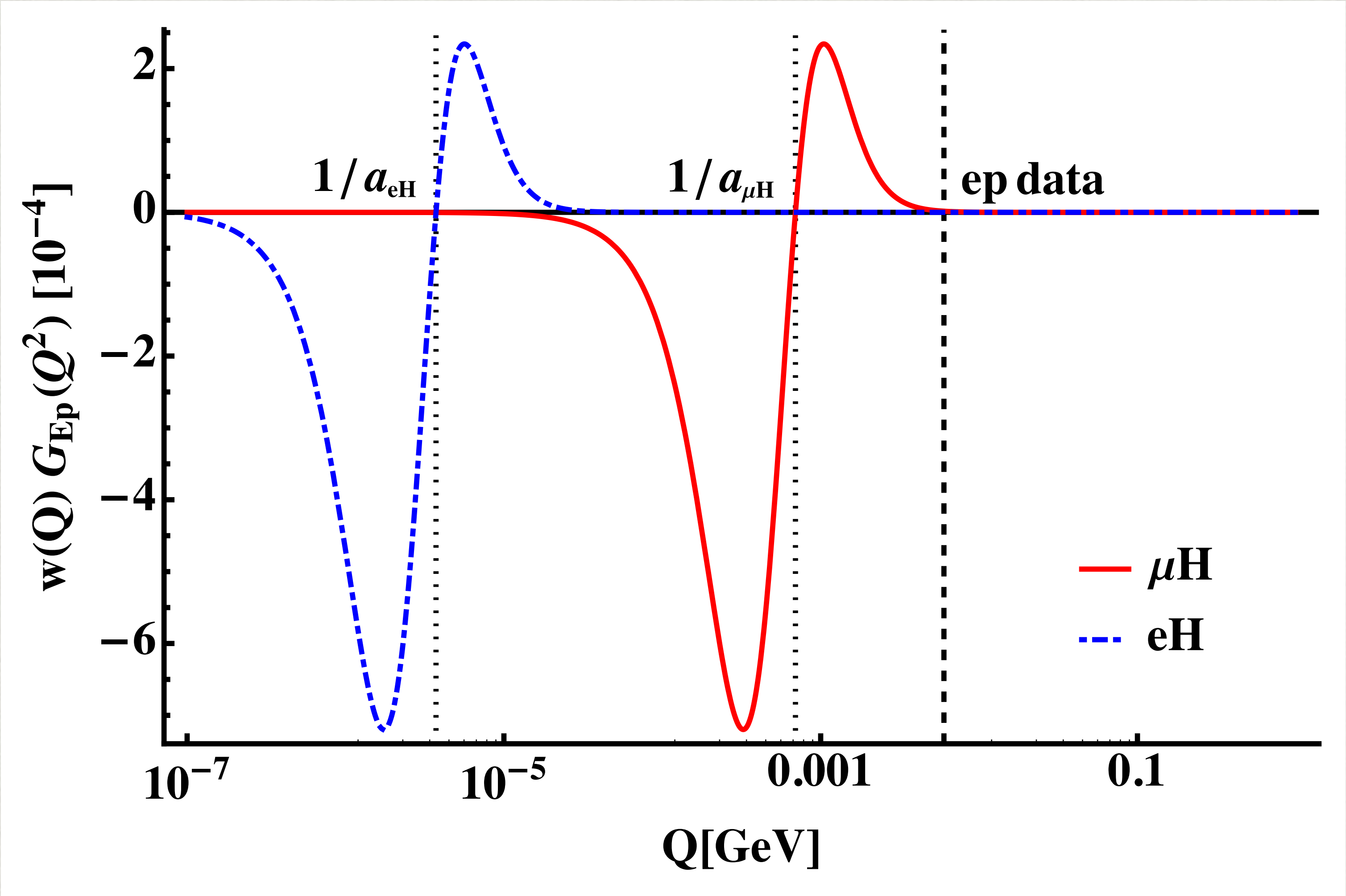
inverse Stieltjes transform:

$$w(Q) = \frac{Q}{i\pi}\lim_{\varepsilon\to 0}\left\{W(-Q^2-i\varepsilon) - W(-Q^2+i\varepsilon)\right\}$$

$$E^{\mathrm{FF}(1)}_{2P-2S} = \int_0^{\infty}\mathrm{d}Q\,w(Q)\,G_E(Q^2)$$ with $$w(Q) = -\frac{4}{\pi}(Z\alpha)^5 m_r^4\,Q^2\,\frac{(Z\alpha m_r)^2 - Q^2}{[(Z\alpha m_r)^2+Q^2]^4}$$

alternatively: $E^{\mathrm{FF}(1)}_{2P-2S} = -\frac{1}{3}\pi(Z\alpha)^4 m_r^3\int_0^{\infty}\mathrm{d}r\,r^4 e^{-r/a}\rho_E(r)$ with $\rho_E(r) = \frac{1}{(2\pi)^2\,r}\int_{t_0}^{\infty}\mathrm{d}t\,\mathrm{Im}\;G_E(t)\,e^{-r\sqrt{t}}$

$a = 1/(Z\alpha m_r)$ Bohr radius



arXiv:1502.03721

$$w(Q) = -\frac{4}{\pi}(Z\alpha)^5 m_r^4\, Q^2\, \frac{(Z\alpha m_r)^2 - Q^2}{[(Z\alpha m_r)^2 + Q^2]^4}$$

Dipole FF:

$$G_{Ep} = (1 + Q^2/0.71\mathrm{GeV}^2)^{-2}$$

- large cancellations around the Bohr radius scale

→ small variation in the FF around Bohr radius scale may lead to significant effects !!!

⇒
- decompose FF into the “smooth” and “non-smooth” part
- missing effect in the FF between 0 and 50 MeV ???

A Toy Model "Resolving the Puzzle"

- We assume the electric FF to have a smooth part $\overline{G}_E$ and a non-smooth part $\widetilde{G}_E$: $G_E = \overline{G}_E\,(1 + \widetilde{G}_E)$

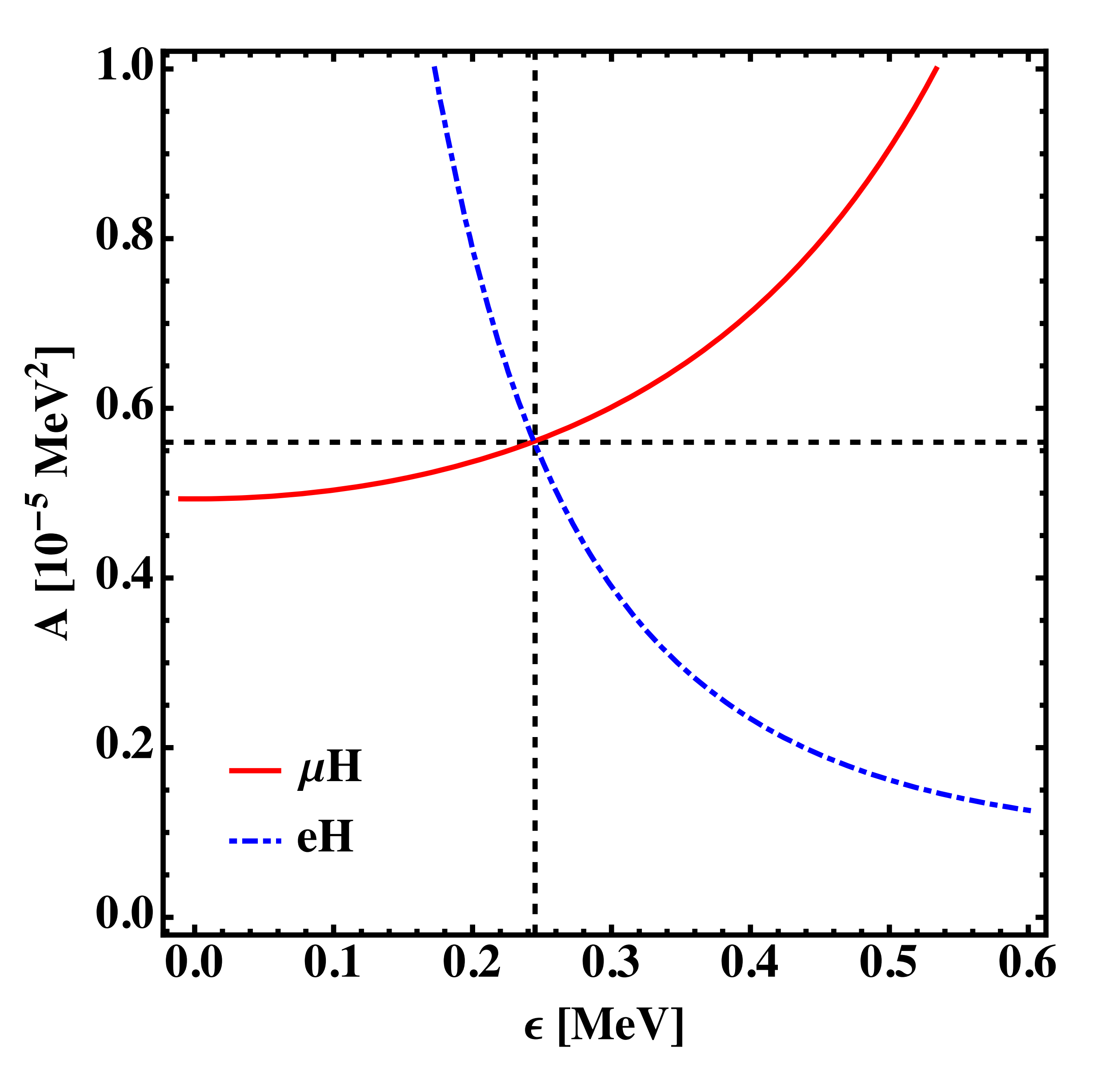
  - chain-fraction fit of Arrington and Sick:
    J. Arrington and I. Sick, Phys. Rev. C76: 035201 (2007).

$$\overline{G}_E(Q^2) = \cfrac{1}{1 + \cfrac{3.478\,Q^2}{1 - \cfrac{0.140\,Q^2}{1 - \cfrac{1.311\,Q^2}{1 + \cfrac{1.128\,Q^2}{1 - 0.233\,Q^2}}}}}$$

  - fluctuation: $\widetilde{G}_E(Q^2) = \frac{A}{\pi}\left\{\frac{\epsilon^2}{\left(Q^2 - Q_0^2\right)^2 + \epsilon^4} - \frac{\epsilon^2}{Q_0^4 + \epsilon^4}\right\}$

    Breit-Wigner type of peak around Q_0^2 with width given by $2\,\epsilon^2$

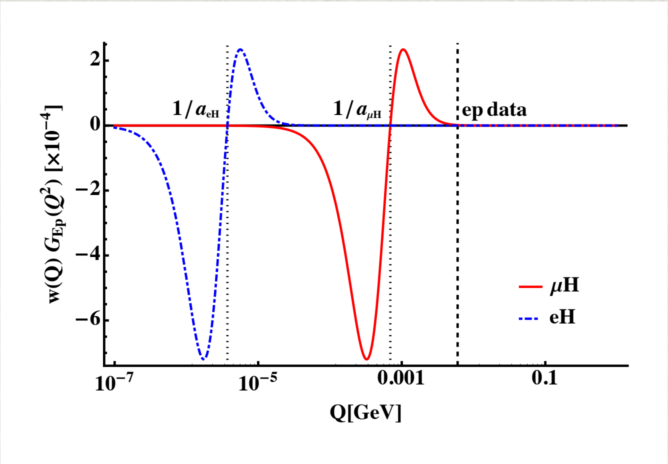
arXiv:1502.03721



$$E^{\mathrm{FF}(exp.)}_{2P-2S}(eH) = -0.62250(724)\,\mathrm{neV}$$
$$E^{\mathrm{FF}(exp.)}_{2P-2S}(\mu H) = -3650(2)\,\mu\mathrm{eV}$$

$$\widetilde{\langle r^2\rangle}_E \equiv -6\frac{\mathrm{d}}{\mathrm{d}Q^2}\widetilde{G}_E(Q^2)\Big|_{Q^2=0} = -\frac{12AQ_0^2\epsilon^2}{\pi(Q_0^4+\epsilon^4)^2}$$

$$\widetilde{\langle r^3\rangle}_E \equiv \frac{48}{\pi}\int_0^\infty \frac{\mathrm{d}Q}{Q^4}\left\{\widetilde{G}_E(Q^2)+\frac{1}{6}\widetilde{\langle r^2\rangle}_E Q^2\right\}$$
$$= -\frac{12iA}{\pi}\left\{\frac{1}{(-Q_0^2-i\epsilon^2)^{5/2}}-\frac{1}{(-Q_0^2+i\epsilon^2)^{5/2}}\right\}$$
$$= \frac{24A}{\pi Q_0^5}+O(\epsilon^2/Q_0^2)$$



$$Q_0 = \frac{1}{2a_{\mu H}}\sqrt{5+\sqrt{17}} \simeq 1.04657\ \mathrm{MeV}$$

	Eq.	$\overline{G}_E$	$\widetilde{G}_E$	G_E
$\langle r^2\rangle_E[\mathrm{fm}^2]$	(6a)	$(0.9014)^2$	$-(0.1945)^2$	$(0.8802)^2$
$\langle r^3\rangle_E[\mathrm{fm}^3]$	(13)	$(1.052)^3$	$(6.369)^3$	$(6.379)^3$
Lamb-shift, expanded	(12)			
$E^{\mathrm{FF}(1)}_{2P-2S}(eH)[\mathrm{neV}]$		-0.65690	0.03452	-0.62238
$E^{\mathrm{FF}(1)}_{2P-2S}(\mu H)[\mu\mathrm{eV}]$		-4202	4913	711
Lamb-shift, exact	(21a)			
$E^{\mathrm{FF}(1)}_{2P-2S}(eH)[\mathrm{neV}]$		-0.65691	0.03451	-0.62239
$E^{\mathrm{FF}(1)}_{2P-2S}(\mu H)[\mu\mathrm{eV}]$		-4202	551	-3651

TABLE I: Lamb shift and moments corresponding to our model FF, with $\epsilon = 0.245$ MeV, $A = 5.6\times10^{-6}$ MeV^2, and Q_0 set by Eq. (24). $|\widetilde{G}_E| < 3\times10^{-5}$

Outlook & Conclusion

- We reproduce the standard finite-size corrections to hydrogen spectra, applying a dispersive formalism.

- We show that the finite-size effects of the nuclear charge distribution on the Lamb shift is not always expandable.

  - convergence radius of the Taylor expansion of $G_E(Q^2)$ has to be much larger than the inverse Bohr radius of the given hydrogen-like system

- We show how tiny, milli-percent changes in the proton electric form factor at a MeV scale would be able to explain the puzzle.

  - one needs to know all the "soft" (below several MeV) contributions to proton electric form factor to pcm accuracy