Valparaiso QNP2015

Nucleon PDF inside Compressed Nuclear Matter

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“Is it possible to maintain my volume constant when the pressure increases?”

- an nucleon when entering the compressed medium.

J. Phys. G: Nucl. Part. Phys. 42 (2015) 045109.

Nuclear Entalpies, 1311.3591; Pressure Corrections to the Equation of State in the Nuclear Mean Field, 1205.0431, Acta Phys. Pol. B Proc. Suppl. Vol. 5 No 2 (2012) 375

Introduction

- The aim is to check two approximations of

The nuclear Relativistic Mean Field Model

1. constant nucleon mass
2. no nucleon volumes i compressed NM

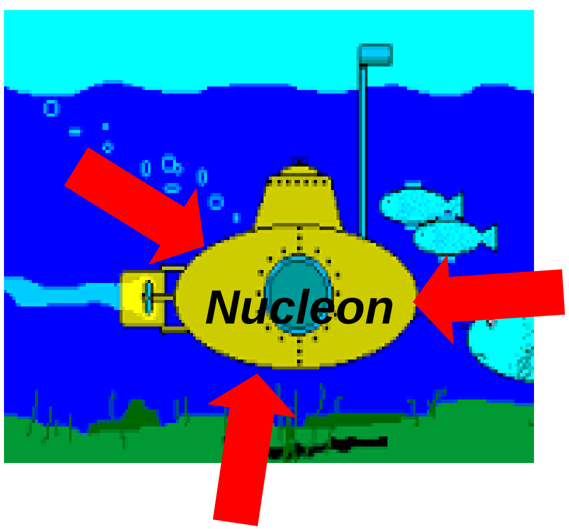
Possible applications in HI colisions and

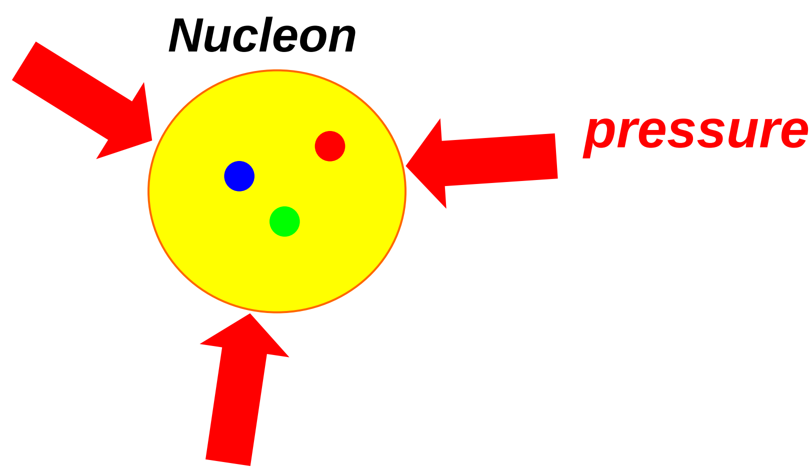
inside neutron stars.

Finite volume effect in compressed medium

Nucleon inside saturated NM

Compressed inside Neutron Star or in H I collision

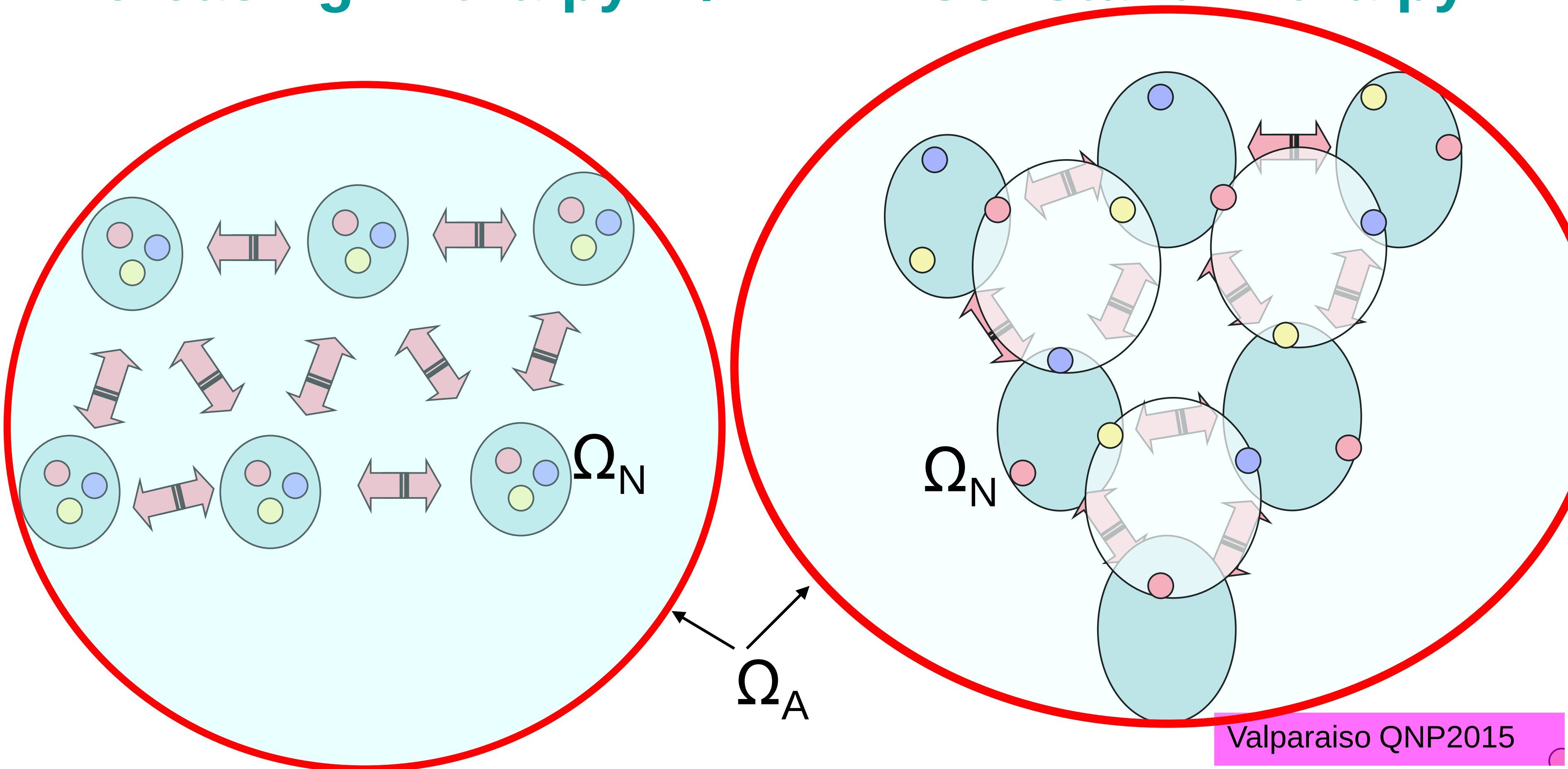




Two Scenarios

for NN repulsion with qq attraction

- **Constant Mass**
  = Increasing Enthalpy 1/R
- **Constant Volume**
  = Constant Enthalpy



Two Scenarios

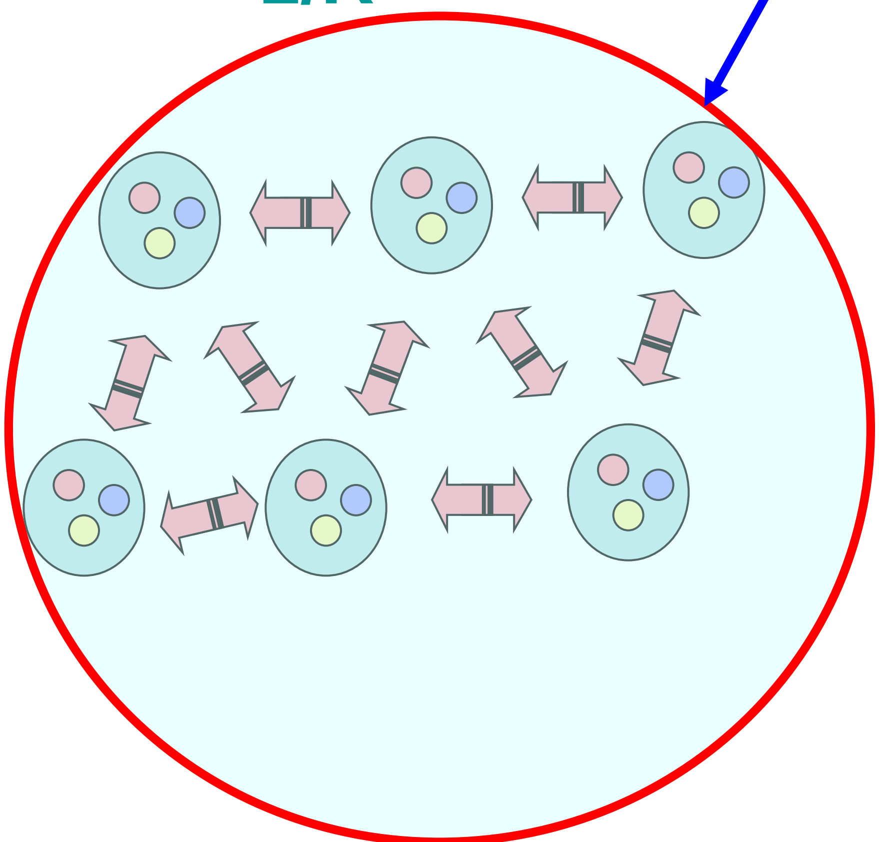
affecting nuclear compressibility K_A^{-1}

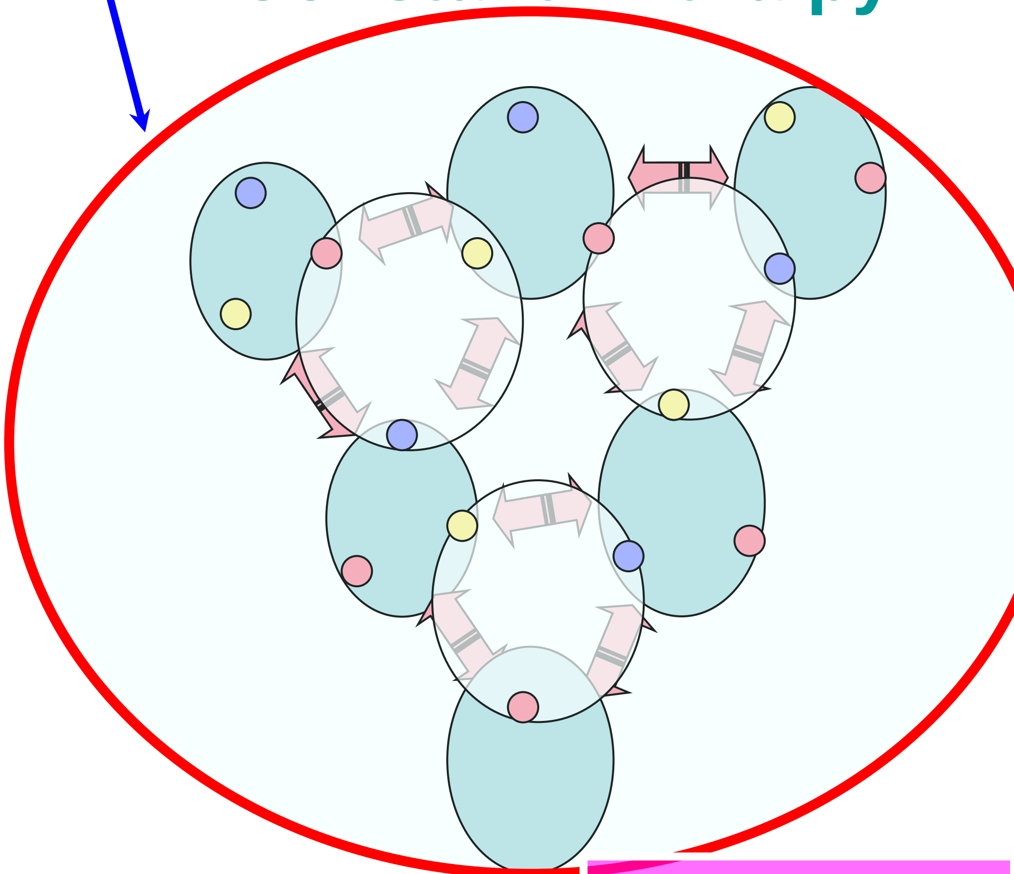
- **Constant Mass**

= Increasing Enthalpy 1/R

- **Constant Volume**

= Constant Enthalpy





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Definitions

- **Enthalpy** is a measure of the total energy of a thermodynamic system. It includes the system's internal energy and thermodynamic potential (a state function), as well as its volume Ω and pressure p_H (the energy required to "make room for it" by displacing its environment, which is an extensive quantity).

$$H_A = E_A + p_H \Omega_A$$ Nuclear Enthalpy (1)

$$H_N = M_{pr} + p_H \Omega_N$$ Nucleon Enthalpy (2)

Specific Enthalpies

$$h_A(\rho) = H_A/E_A = 1 + p_H/(\rho\, \varepsilon(\rho))$$

$$h_N(\rho) = H_N/M_{pr} = 1 + p_H/(\rho_{cp}\, M_{pr}(\rho))$$

(3)

Enthalpy vs Hugenholz - van Hove relation with chemical potential

$$\mu \doteq (\partial M_A/\partial A)_{\Omega_A} \equiv (\partial H_A/\partial A)_{p_H} = \varepsilon_A + \frac{p_H}{\varrho} = H_A/A$$

$$E_F \doteq P^0_N(P_F) = (\partial M_A/\partial A)_{\Omega_A} = \varepsilon_A + p_H/\varrho = \mu$$

Also valid for constant nucleon volumes !!

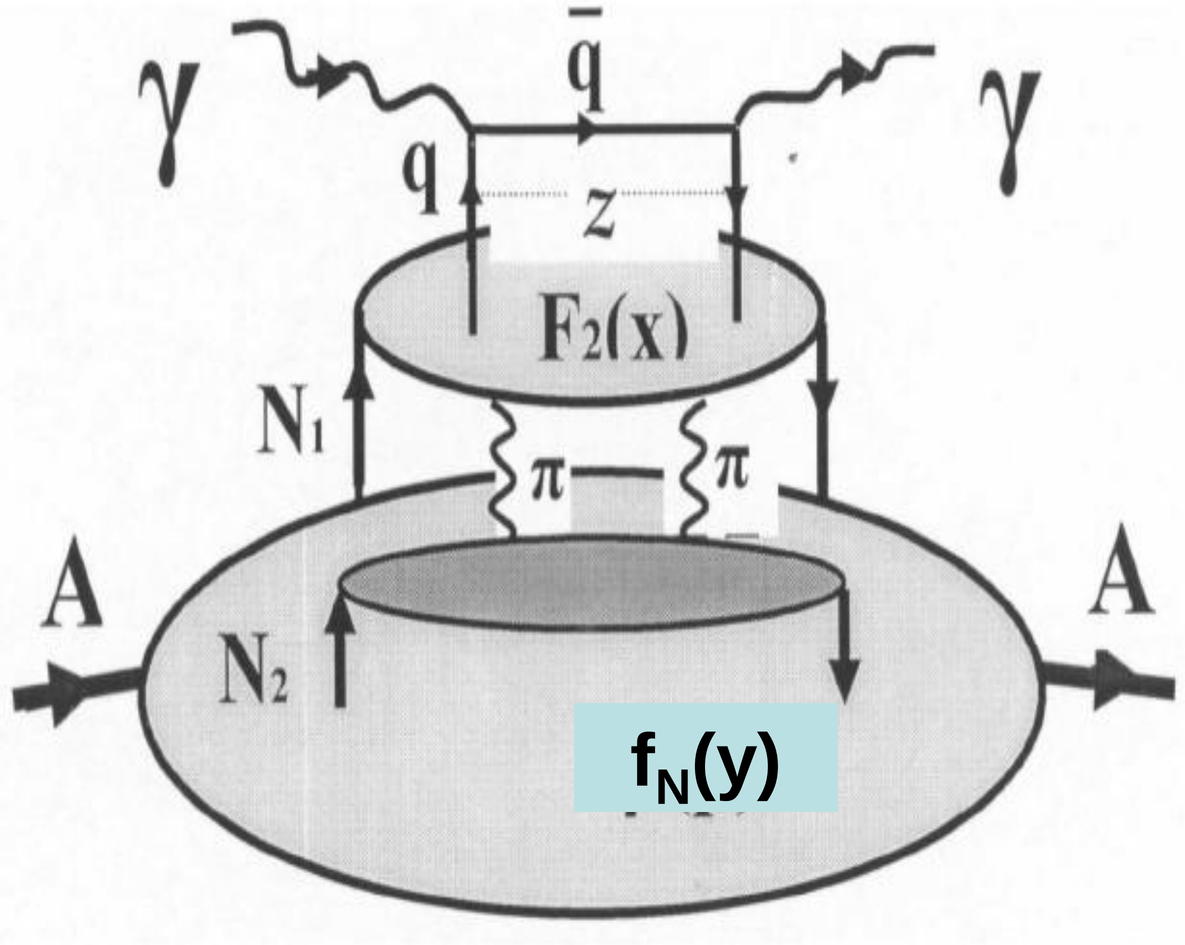
In the NM in equilibrium $p_H = 0$ therefore $H_A = E_A$. Dividing H_A by A we obtain the following relation between single particle enthalpy h_A and $\varepsilon_A = E_A/A$,

$$h_A = \varepsilon_A + p_H/\varrho. \qquad (1a)$$

Please note that the same equation fulfills a Fermi energy $E_F \equiv P^0_N(p_F) = \varepsilon_A + p_H/\varrho$ of nucleon with a Fermi momentum p_F; well-known as the HvH [9] relation, also proven in the self-consistent RMF approach [10]. It turns out that definitions of the Fermi energy or a single particle enthalpy have the same energy balance.

We will argue that the enthalpy rather than the rest energy should be used in the momentum distribution sum rules (MSR) in NM and in a nucleon.

Nuclear convolution model



Light cone variables in the rest frame

$x = k^+/p_N^+$

$y = p_N^+/P_A$

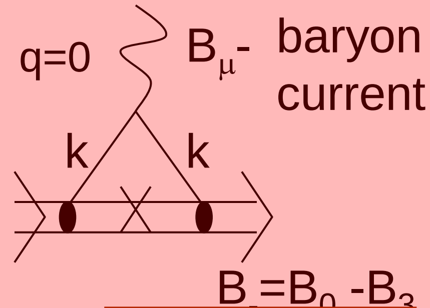
RMF and Momentum Sum Rule

The relativistic nuclear dynamics of nucleons in the nucleus is described by the Light Cone (LC), momentum distribution function $f_N(y)$ (Jaffe) where $y = AP_N^+/P_A^+$, a fraction of longitudinal momentum of A nucleons in the nucleus is Lorentz invariant. Let us now focus our attention on the sum rule for longitudinal momenta $P_N^+ = P_N^0 + P_N^Z$. Do they sum in the rest frame to the nuclear energy E_A, or rather to nuclear enthalpy H_A? To answer this question we can examine the distribution

$$f_N(y) = \int \frac{d^4 P_N}{(2\pi)^4} \delta\left(y - \frac{AP_N^+}{P_A^+}\right) Tr\left[\gamma^+ S(P_N, P_A)\right]. \qquad (4)$$

Frankfurt, Strikman Phys. Reports 160 (1988)

Finally with a good normalization of S_N we have:



Flux Factor

$$f_N(y)=\frac{4}{\varrho}\int_0^{P_F}\frac{S_N(P_N)d^3\boldsymbol{P}_N}{(2\pi)^3}\left(1+\frac{P_N^3}{E_N^*}\right)\delta(y-AP_N^+/P_A^0)=$$

$$=(3/4)[P_A^0/(AP_F)]^3[(AP_F/P_A^0)^2-(y-AE_F/P_A^0)^2]. \quad (5)$$

$P^0_A=E_A=A\varepsilon_A$

and Momentum Sum Rule

$$\frac{1}{A}\int dy\, y f_N(y)=\frac{E_F}{P_A^0}\equiv\frac{\frac{\partial}{\partial A}(E_A)_{\Omega_A}}{P_A^0}=\frac{\varepsilon_A+p_H/\varrho}{P_A^0}. \quad (6)$$

$$\int dy\, y f_N(y)=\frac{E_F}{h_A}=1.\ .$$

Fermi Energy

Enthalpy/A

Bag Model in Compress Medium

$p_H = 0$

$$E^0_{Bag}(R) = \frac{3\omega_0 - Z_0}{R} + \frac{4\pi}{3} B(\varrho_0) R^3 \sim 1/R,$$

$$p_H = p_B = \frac{3\omega_0 - Z_0}{4\pi R^4} - B(\varrho) \quad \rightarrow \quad (B(\varrho) + p_H) R^4 = const$$

$$R = \left[\frac{3\omega_0 - Z_0}{4\pi(B(\varrho) + p_H)}\right]^{1/4},$$

$$M_{pr} = E_{Bag} = 4\pi R^3 \left[\frac{4}{3}(B + p_H) - \frac{p_H}{3}\right] = E^0_{Bag} \frac{R_0}{R} - p_H \Omega_N .$$

$$H_N = E^0_{Bag} \frac{R_0}{R} \sim 1/R. \qquad (7)$$

Nucleon compressibilty

$$K_N^{-1}|_{p_H=0} = 9N_q\frac{\partial p_H}{\partial \varrho_N} = -9\Omega_N^2\frac{\partial p_H}{\partial \Omega_N}$$

and two scenarios

Constant Nucleon Mass

$$K_N^{-1}|_{M_N, R\to R_0} = -3\Omega_N^2\frac{\partial[M_N(R_0/R-1)/\Omega_N]}{\partial\Omega_N} = M_N \simeq 940\ \mathtt{MeV}$$

Constant Nuclear Radius

$$K_N^{-1}|_{\Omega_N} \to \infty.$$

Semi-experimental Value

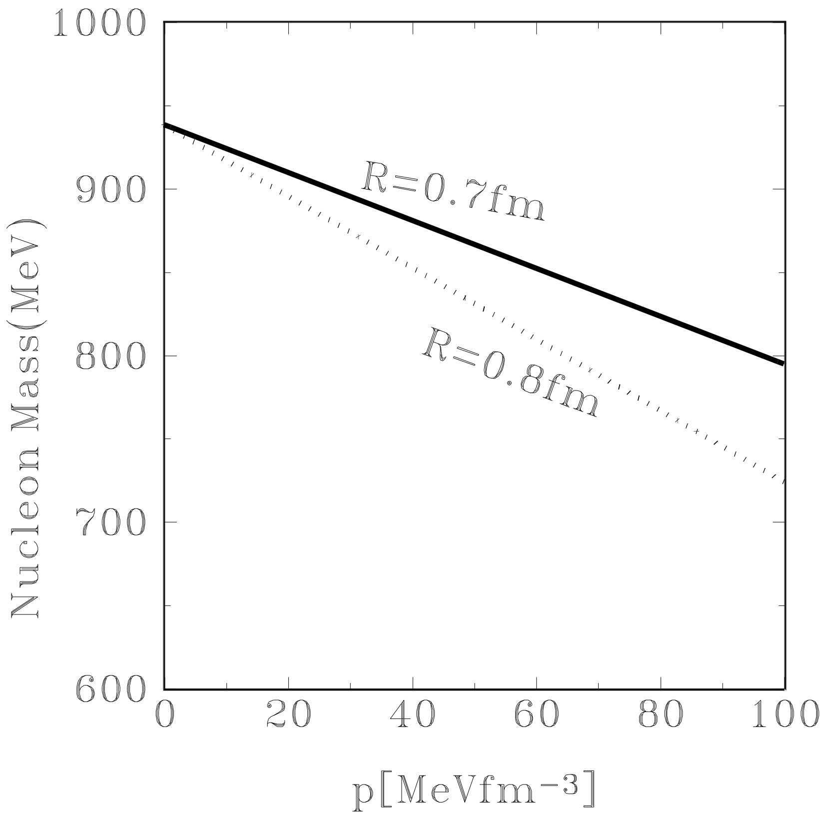
sum rules $K_N^{-1} \Rightarrow M\, E_x^2 \langle r_N^2 \rangle$ (Morsch, Julich, PRL 1995)

$$K_N^{-1} = (1.4 \pm 0.3)\ \mathrm{GeV}$$

From 7Gev/c (α,p) scattering in P_{11} region in SATURN

Nucleon Mass for different nucleon radii in compressed NM

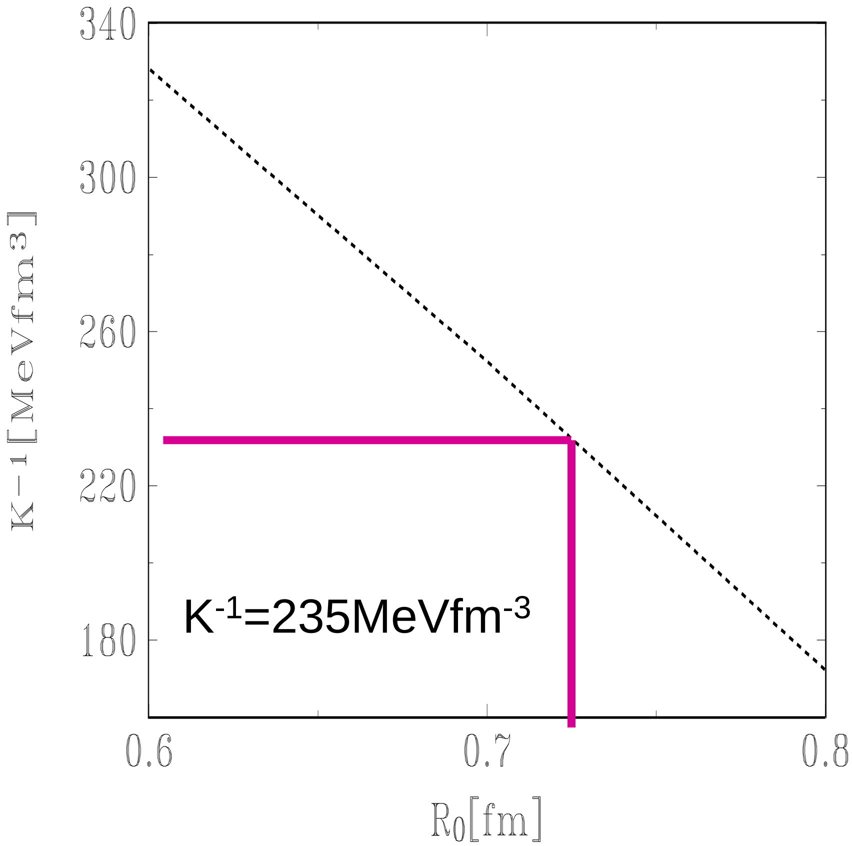
$$M_{pr}(\varrho) = M_N - p_H(\varrho)\Omega_N,$$
$$p_H(\varrho) = \varrho^2 \varepsilon'_A(\varrho)/(1-\varrho\Omega_N).$$



Nuclear compressibility for different constant nucleon radii in compressed NM

$$H_A^T/A = \varepsilon_A - (\partial M_A/\partial\Omega_{A-})_A/\varrho = \varepsilon_A + p_H/\varrho = E_F$$

Our version of Hugenholz-Van Hove relation for finite nucleons in NM

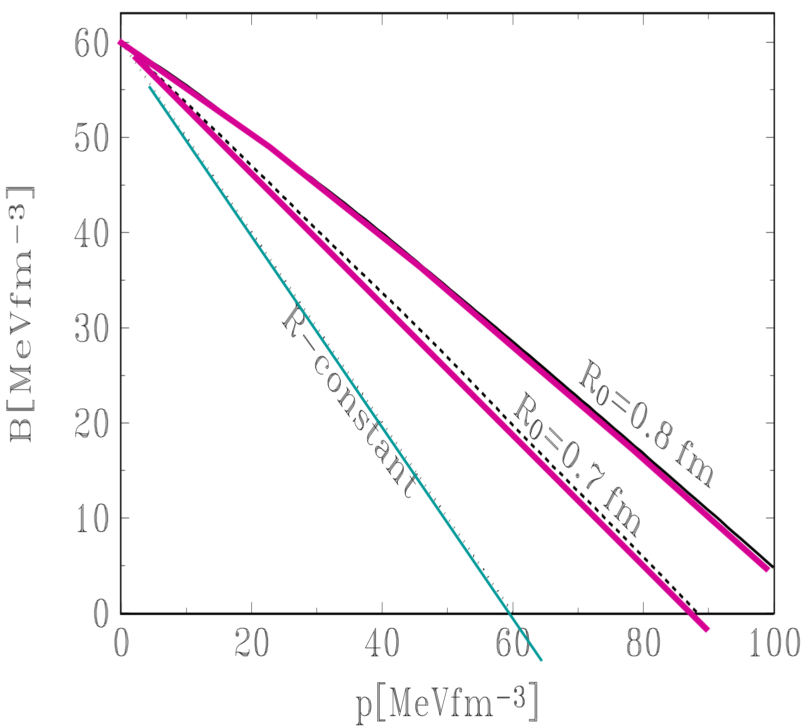


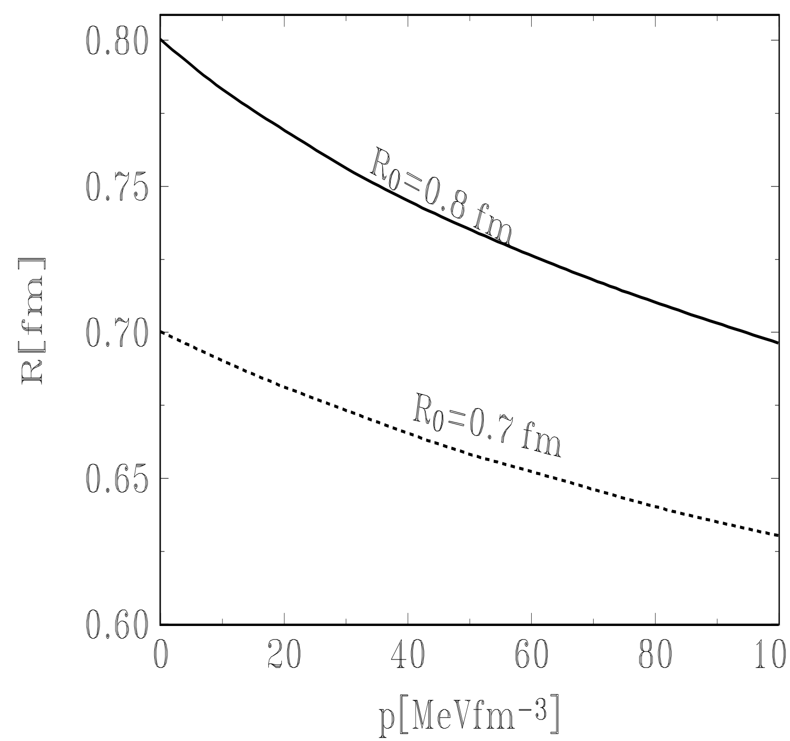
Bag constant in function of nuclear pressure

Nucleon radius in compressed NM
for a constant nucleon mass

$$B = B(\varrho_0)(R_0/R)^4 - p.$$

$$M_N R_0/R = M_N + 4/3\pi R^3 p$$





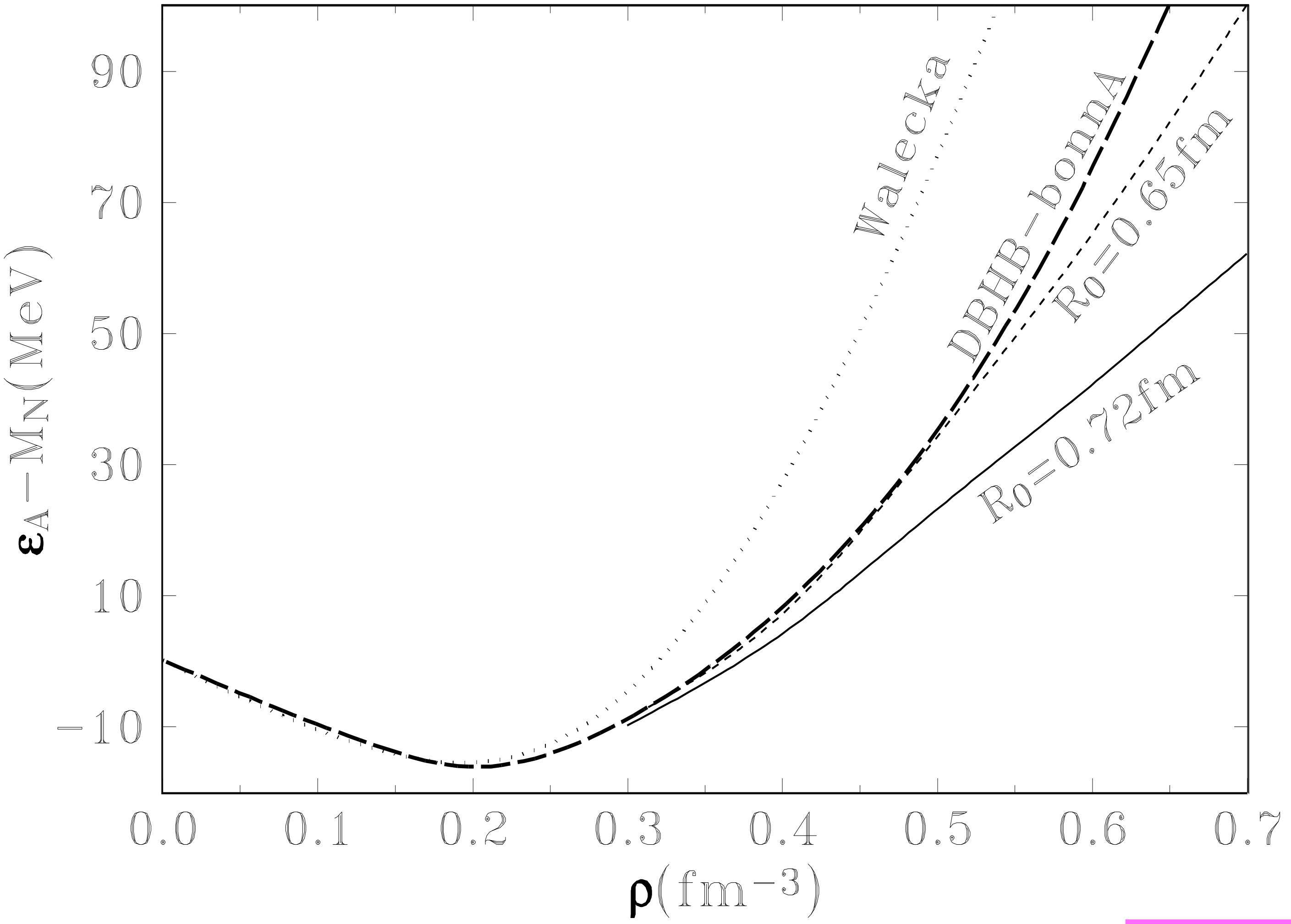
RMF Equation of State for const Enthalpy scenario B

$$\varepsilon_A = C_1^2 \varrho + \frac{C_2^2}{\varrho}(M_{pr} - M_{pr}^*)^2 + \frac{\gamma}{\varrho}\int_0^{P_F} \frac{d^3p}{(2\pi)^3}\sqrt{\boldsymbol{P}_N^2 + M_{pr}^{*2}}$$

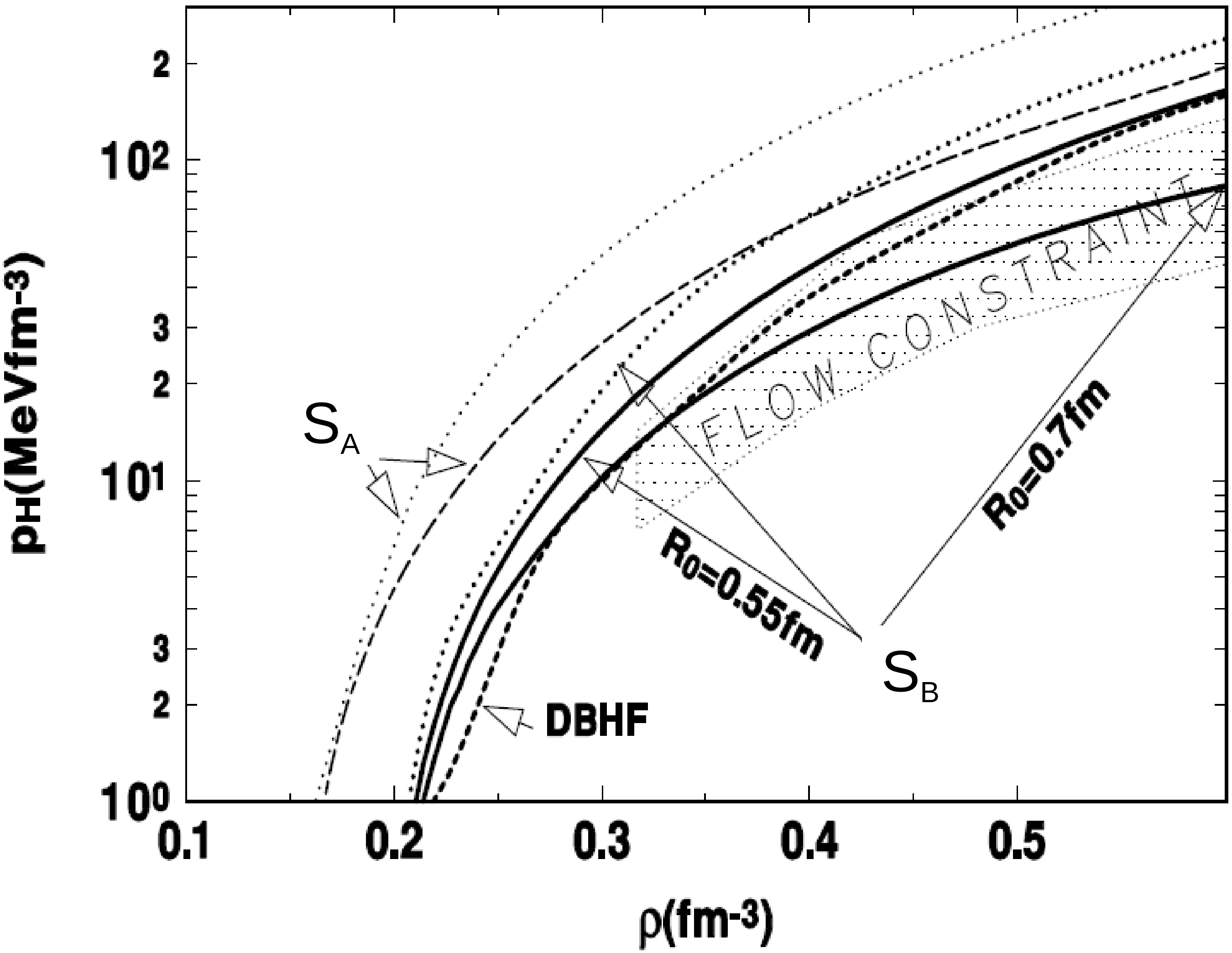
$$M_{pr}^* = M_{pr} - \frac{\gamma}{2C_2^2}\int_0^{P_F} \frac{d^3p}{(2\pi)^3}\frac{M_{pr}^*}{\sqrt{\boldsymbol{P}_N^2 + M_{pr}^{*2}}}, \qquad (8)$$

$$M_{pr}(\varrho) = M_N - p_H(\varrho)\Omega_N$$
$$p_H(\varrho) = \varrho^2 \varepsilon'_A(\varrho)/(1 - \varrho\Omega_N). \qquad (9)$$

Equation of state - different models



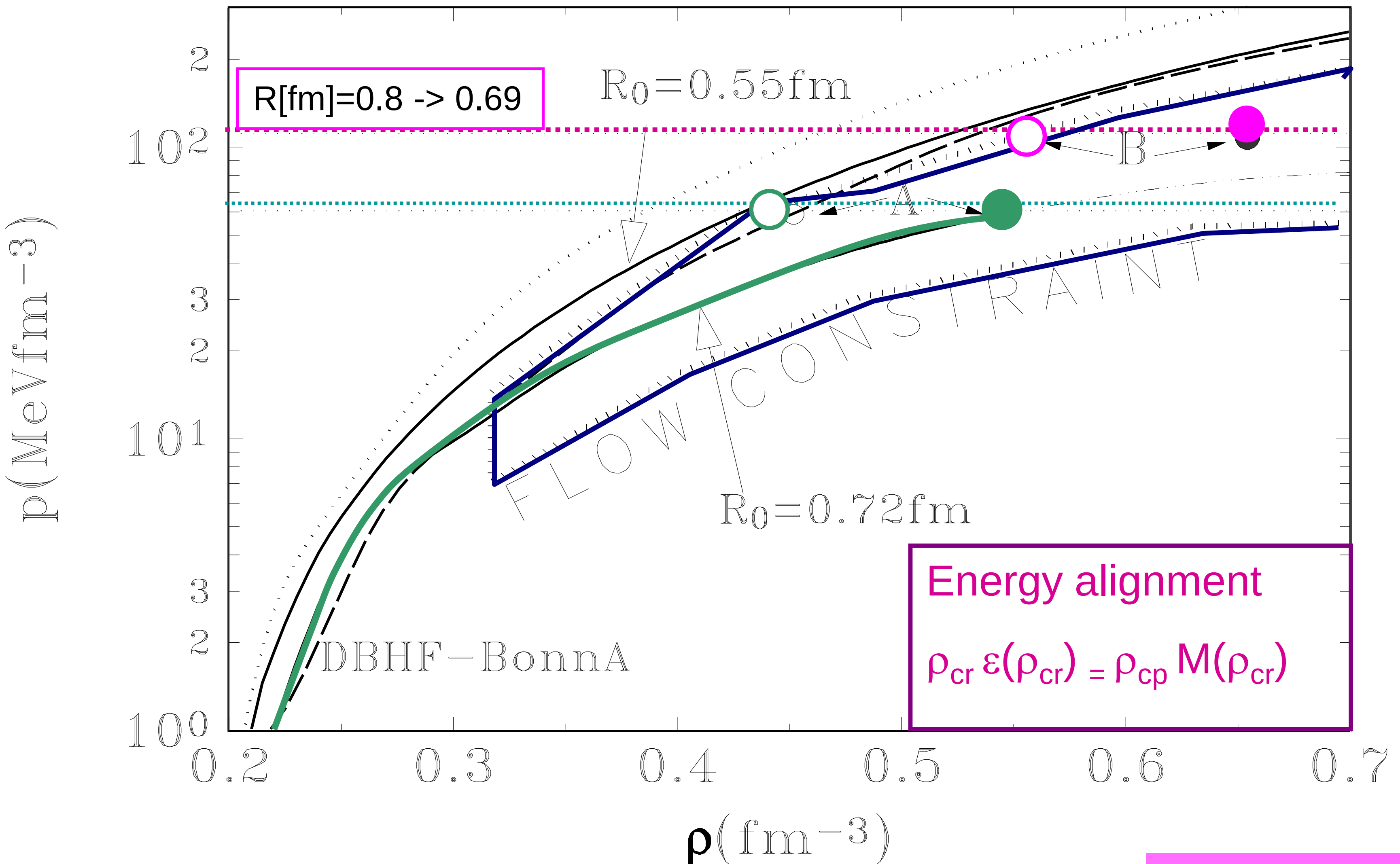
Results



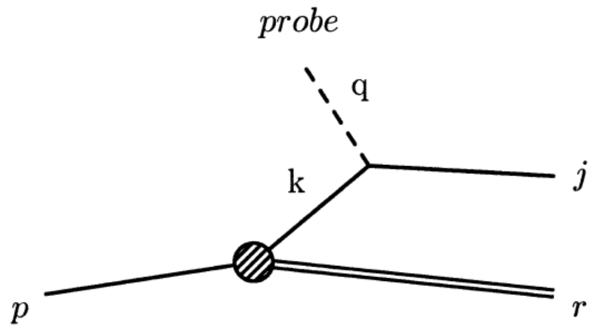
2
10^2
3
2
10^1
3
2
10^0
0.1
0.2
0.3
0.4
0.5
p_H(MeVfm-3)
ρ(fm-3)
S_A
S_B
DBHF
FLOW CONSTRAINT
R0=0.55fm
R0=0.7fm

Two possible scenario of phase transition

A - constant nucleon radius, B - constant nucleon mass



A model for parton distribution



$$f_i(k)dk = N(\sigma_i, m_i) e^{-\frac{(k_0-m_i)^2+k_1^2+k_2^2+k_3^2}{2\sigma_i^2}} dk.$$

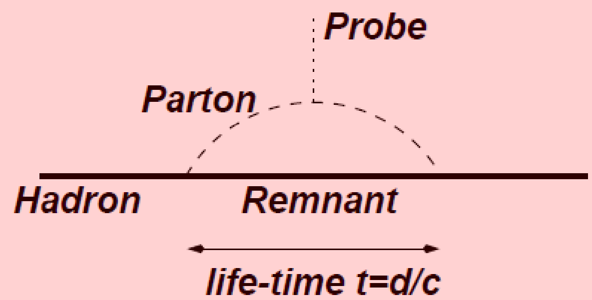
Primodial quark transverse momentum distribution

$\sigma = 1/(2R) \quad k^+ = xp^+$

Kinematical conditions for Monte Carlo technique

$$m_i^2 \leq j^2 < W^2 \quad \text{and} \quad r^2 > \sum_i m_i^2$$

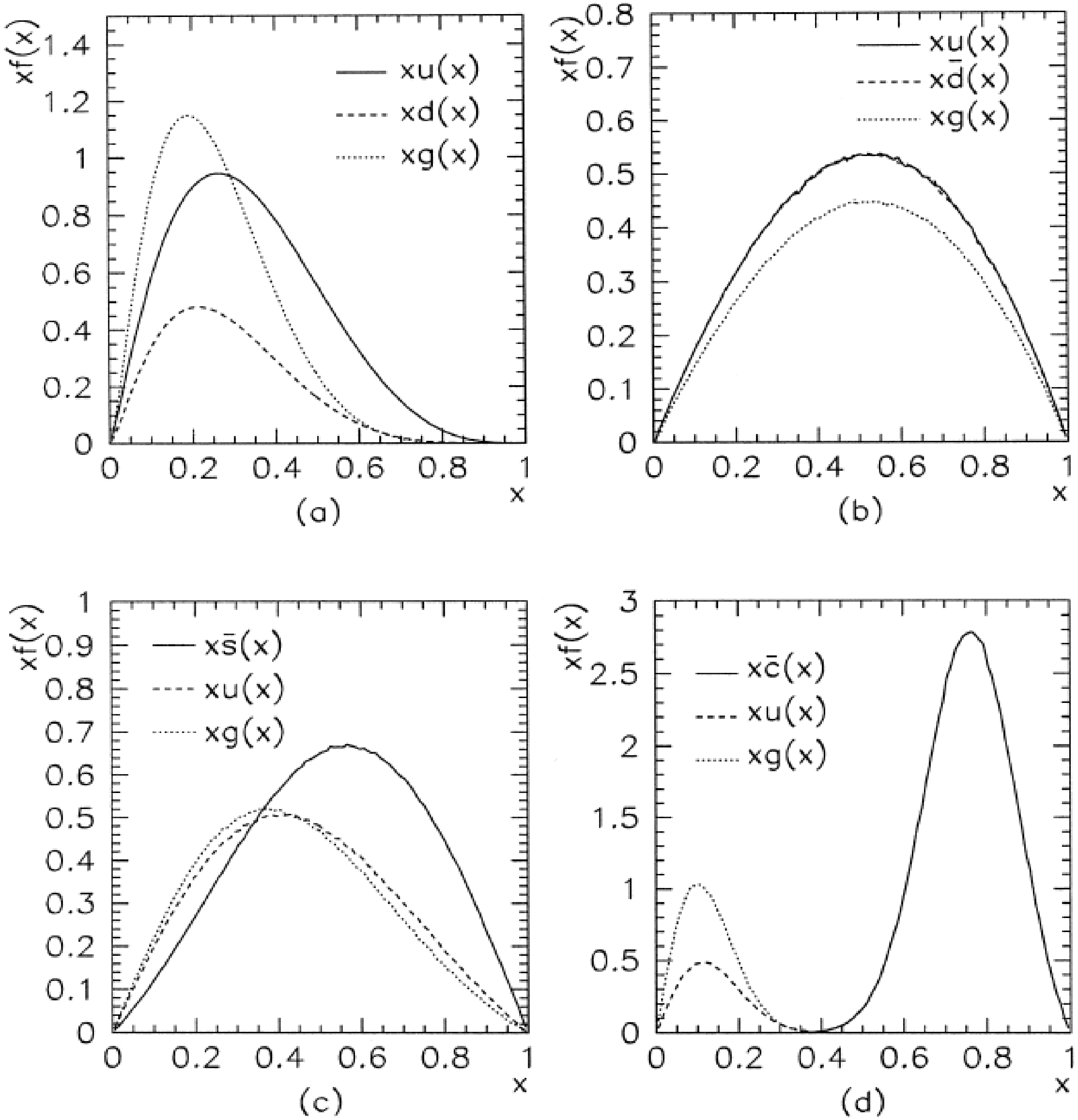
Line cone variables in the nucleon rest frame



Fluctuation of a hadron into a parton

COMPRESSED Nuclear Case

$p^+_{rest} = H_N(R)$



. 2. The valence quark and gluon distributions obtained from the model applied to (a) the proton, (b) the pion, (c) the strange meson K^+, the charm meson $\overline{D}^0$. The Gaussian widths used are 135 MeV for gluons and 150 MeV for q and $\overline{q}$, except $\sigma_u = 180$ MeV in (a)

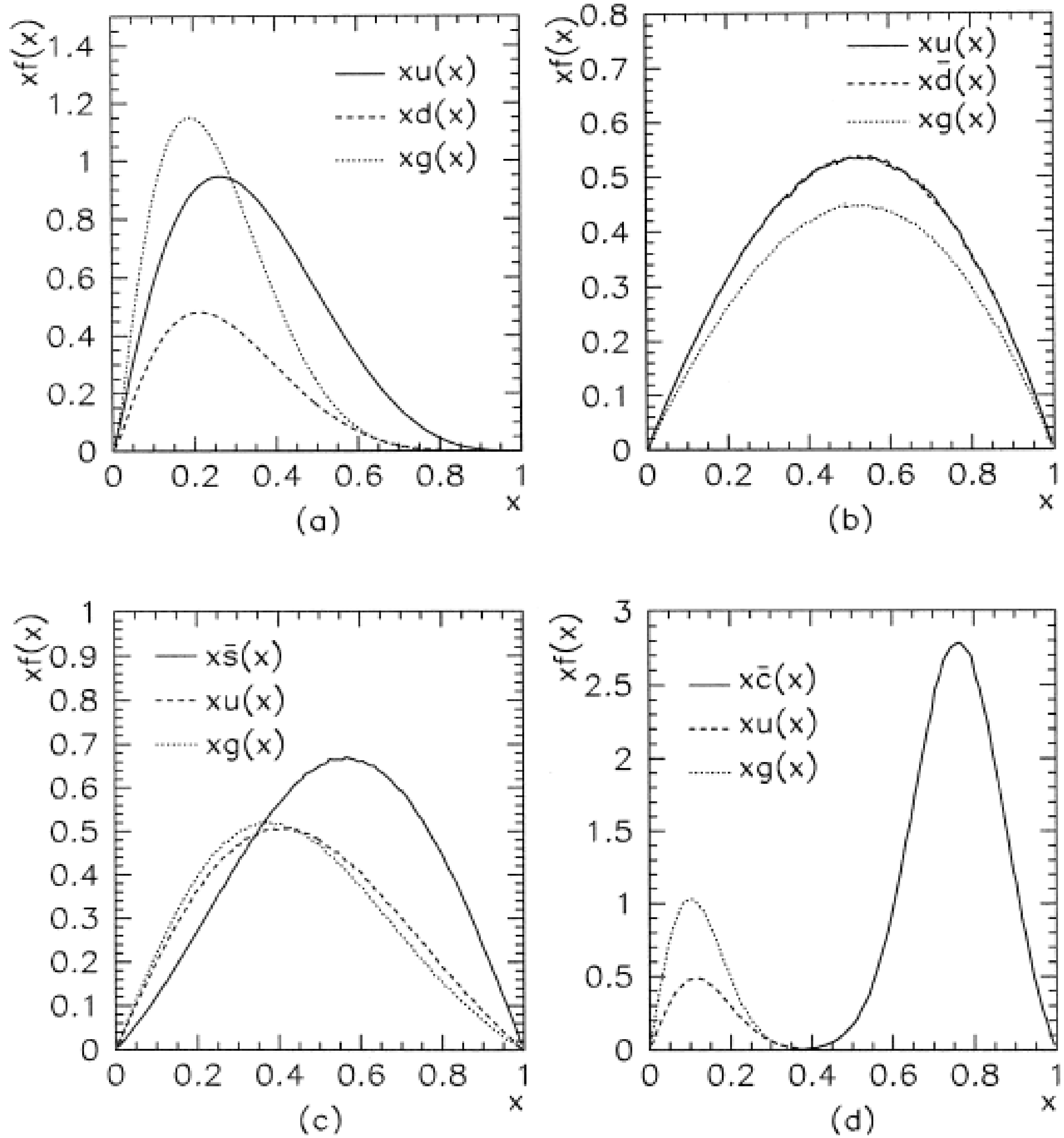


Fig. 2. The valence quark and gluon distributions obtained from the model applied to (a) the proton, (b) the pion, (c) the strange meson K^+, (d) the charm meson $\overline{D}^0$. The Gaussian widths used are 135 MeV for gluons and 150 MeV for q and $\overline{q}$, except $\sigma_u = 180$ MeV in (a)

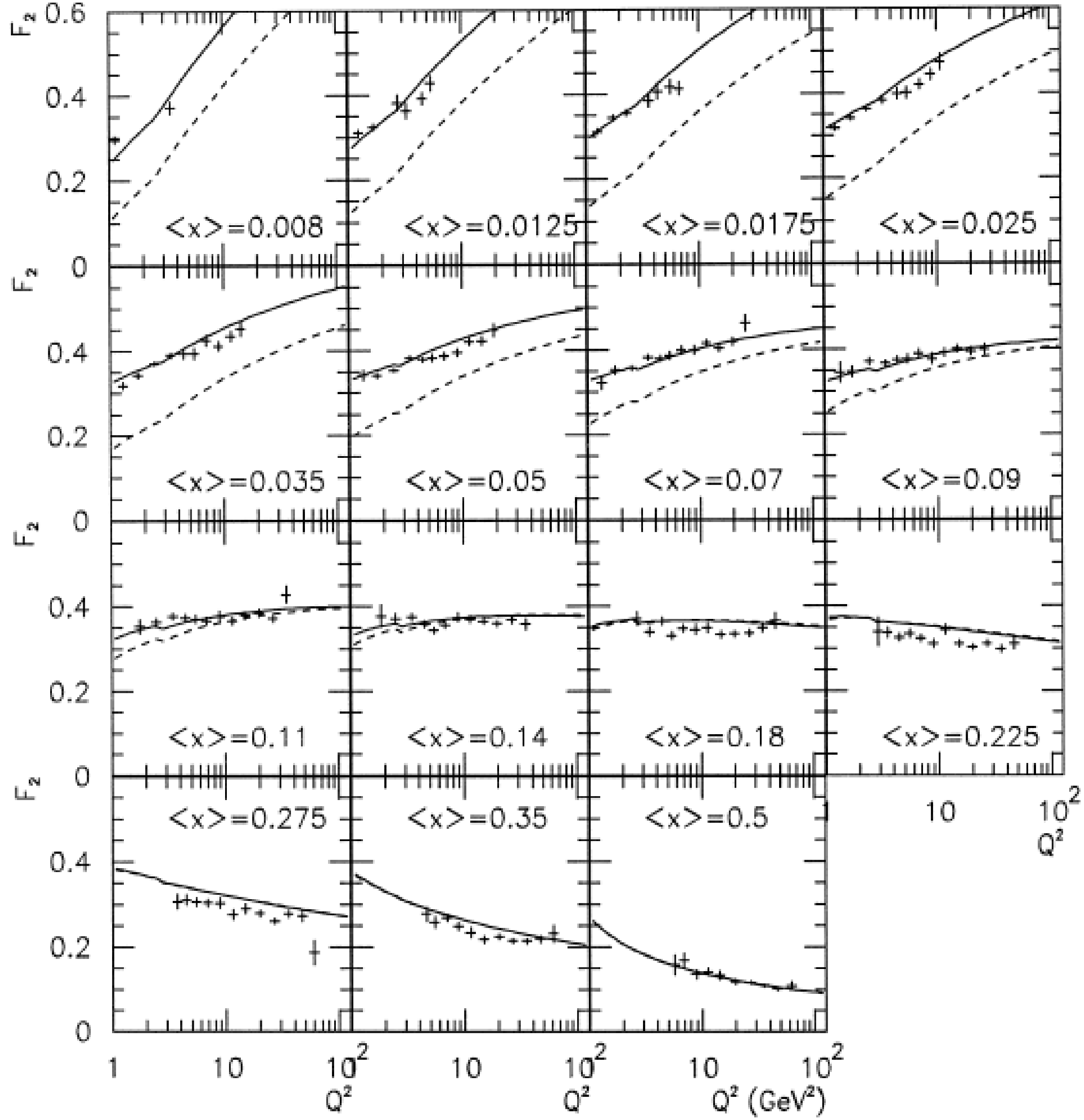
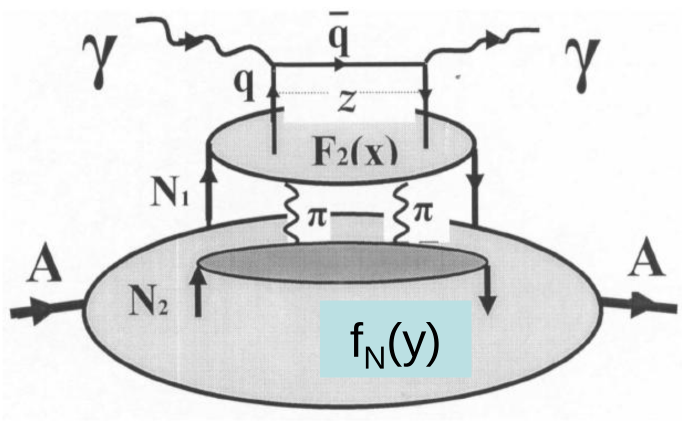
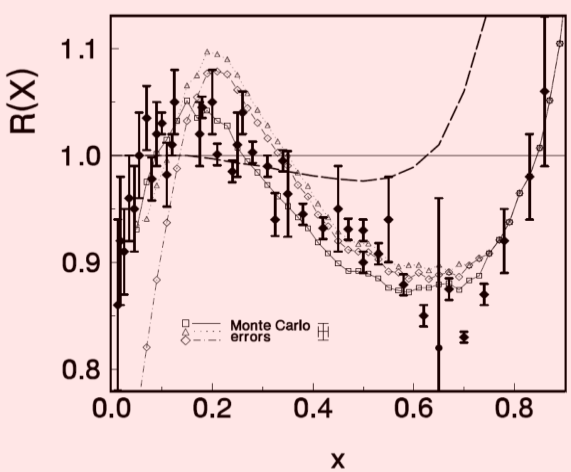


Fig. 3. The DIS structure function F_2 versus Q^2 in bins of x. Fixed target NMC data [7] compared to the model starting from only valence quarks and gluons (dashed) and including also a sea quark component (full). (The small break in the curves at $Q^2 \sim m_c^2$ is due to the charm threshold.)

Nuclear Models - equilibrium

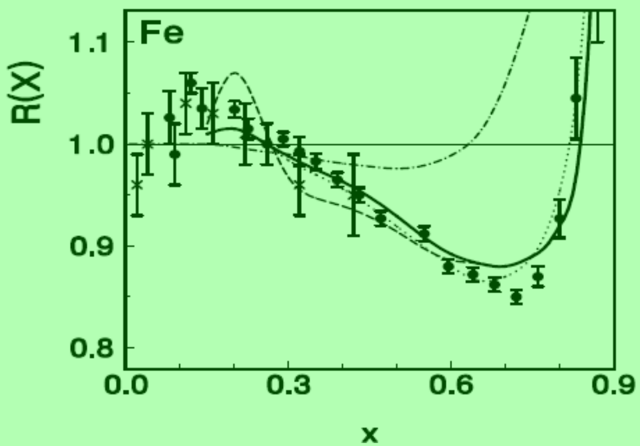


Shifting pion mass

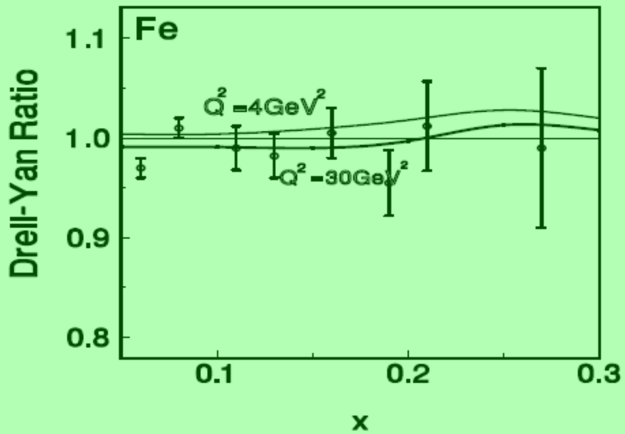


JR G.Wilk PLB 473 (2000)

$$M_x = M_N + \frac{(1 - f(x))}{2} < V_N >$$



Only 1% of nuclear pions



Phys. Rev. C71 (2005)

Toy Model (Edin and Ingelman)

(Neglecting transverse quark momenta)

$$f_i(x) = N'(\tilde{\sigma}_i) \exp\left(-\frac{x^2}{4\tilde{\sigma}_i^2}\right) \operatorname{erf}\left(\frac{1-x}{2\tilde{\sigma}_i}\right)$$

where

$$\tilde{\sigma} = \frac{1}{d_h m_h}$$

In our case $d_h m_h$ => $R^* H_N(R)$ is const.

But the $x = k^+/H_N(R(\rho))$ depends on nucleon density

Finite Nucleon Volumes - Conclusions

A. Constant nucleon mass requires increasing enthalpy

STIFFER EOS

Shift in Bjorken X

B. Constant nucleon volume gives the constant enthalpy with decreasing nucleon mass, lower compressibility

SOFTER EOS

A&B. In both cases the same width of parton distribution because $R*H_N(R)$ const.

The toy model for phase transition

It is easy to show that the equality of these specific enthalpies at a certain density ϱ_{cr}

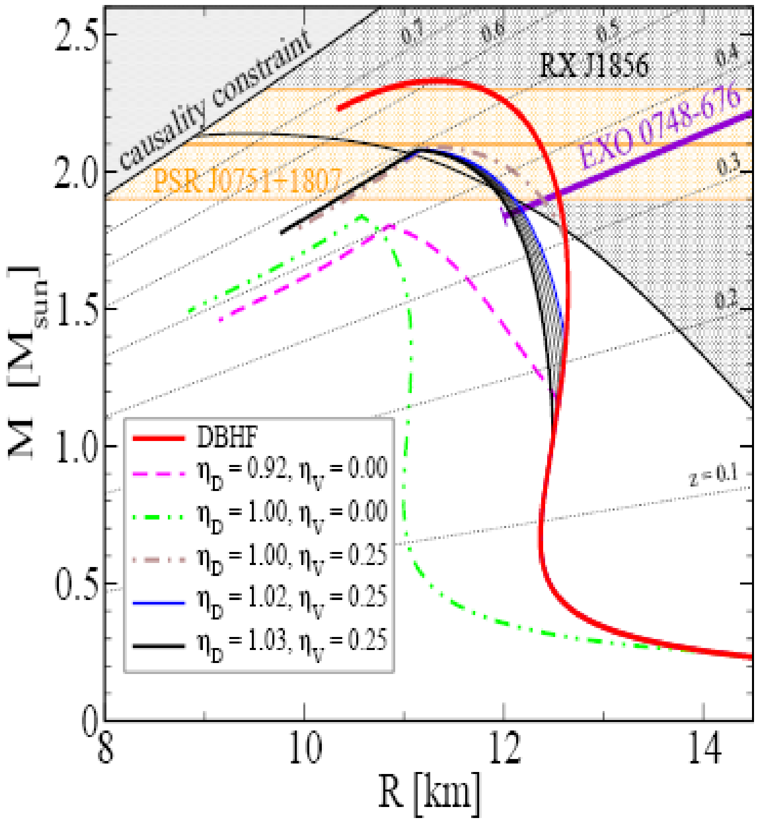
$$h_A^T(\varrho_{cr}) = h_N(\varrho_{cr})$$

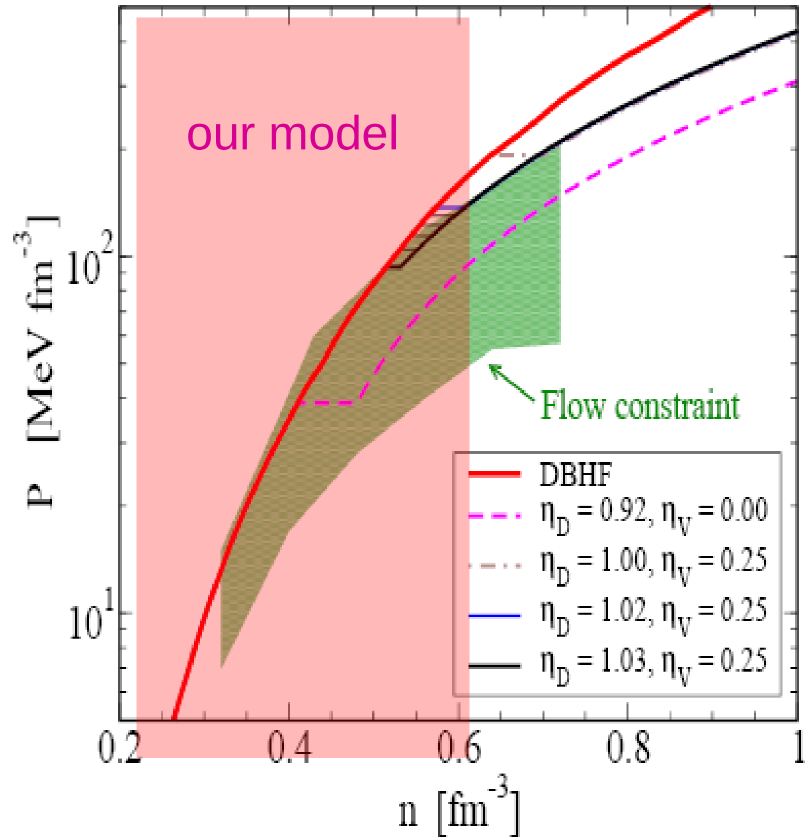
is equivalent to a following condition for the critical density ϱ_{cr}

$$\varrho_{cr}\,\varepsilon_A(\varrho_{cr}) = \varrho_{cp}(\varrho_{cr})\,M_{pr}(\varrho_{cr}).$$

where the alignment of energy densities, outside and inside nucleon, takes place. Another word, energy density $(\varrho\varepsilon_A)$, which includes a space Ω_{A-} between nucleons, reaches the energy density of a quark plasma $(\varrho_{cp}M_{pr})$ inside nucleon therefore an ultimate de-confinement transition to the Quark-Gluon-Plasma (QGP) will take place when condition (5) or (4) is satisfied. This self-consistent condition for the will discussed in two selected regimes: a constant nucleon radius (subsection "A") and a constant nucleon mass (subsection "B").

Mass-Radius constraint and Flow constraint (II)





- Large Mass ($\sim 2\, M_\odot$) and radius ($R \geq 12$ km) $\Rightarrow$ stiff quark matter EoS;
  Note: DU problem of DBHF removed by deconfinement! and: CFL core Hybrids unstable!
- Flow in Heavy-Ion Collisions $\Rightarrow$ not too stiff EoS !
  Note: Quark matter removes violation by DBHF at high densities

Klähn, D.B., Sandin, Fuchs, Faessler, Grigorian, Roepke, Truemper, [arxiv:nucl-th/0609067] (2006) *PRC 74*