Magnetic sum-rules in the Axial-Axial channel

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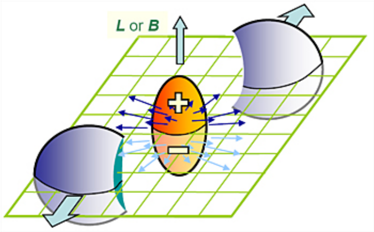
Introduction

Magnetars are the most magnetized stellar objects.
In surface $B \sim 10^3$–10^6 Gauss (10^{-9}–10^{-6} GeV^2)
Even higher inside the star.

Huge magnetic field generated in relativistic heavy ion collisions
$B \sim 10^{11}$–10^{12} Gauss (10^{-2}–1 GeV^2).
Fast screening $\rightarrow$ difficult to measure



Outline

- Introduction to finite energy sum rules (FESR)
- Axial-axial channel at $B = 0$ in the chiral limit
- Axial-axial channel at $B \neq 0$ in the chiral limit
- results
- Conclusions and outlook

Introduction to finite energy sum rules (FESR)

Two current correlator

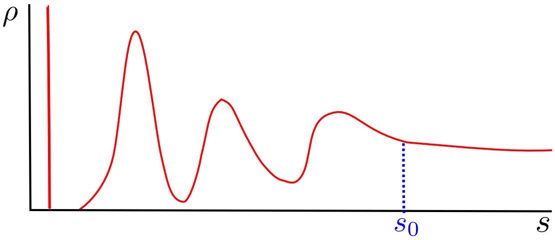
One example

$$\Pi_{\mu\nu}(x-y) = i\langle 0|T J_\mu(x) J_\nu^\dagger(y)|0\rangle$$

The fourier transformation can be written as

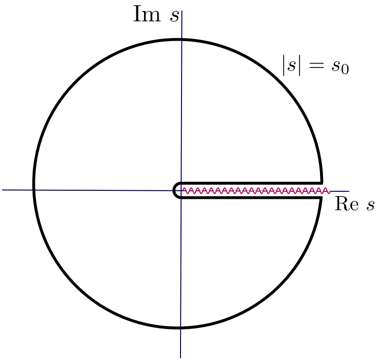
$$\Pi_{\mu\nu}(q) = q_\mu q_\nu \Pi_L(q^2) + (g_{\mu\nu}q^2 - q_\mu q_\nu)\Pi_T(q^2)$$

The hadronic resonances information is contained in the imaginary part of the correlator: $\rho(s) = \frac{1}{\pi}\mathrm{Im}\Pi(s+i\epsilon)$



$s_0 \sim$ 1–4 GeV^2 is the hadronic continuum threshold.

Finite energy sum rules (FESR)



Quark-hadron duality
$\Pi^{\text{Had}} \leftrightarrow \Pi^{\text{QCD}}$

Integration using some analytic function $p(s)$ as a weight function:

Cauchy's theorem

$$\frac{1}{\pi}\int_0^{s_0} ds\, p(s)\, \text{Im}\Pi(s+i\epsilon) \quad = \quad \frac{-1}{2\pi i}\oint_{|s|=s_0} ds\, p(s)\, \Pi(s)$$

$$\text{Hadrons} \quad \leftrightarrow \quad \text{QCD}$$

Hadronic continuum threshold s_0

If s_0 is high enough, QCD can be treated as perturbative + nonperturbative (pQCD + OPE)

When $s_0 = 0$, there are no hadronic resonances:
$\Rightarrow s_0$ acts as a order parameter for deconfinement

At finite temperature

$$\frac{s_0(T)}{s_0(0)} \approx \frac{\langle \bar{q}q \rangle(T)}{\langle \bar{q}q \rangle(0)}$$

Operator product expansion (OPE)

$$\Pi^{\mathrm{QCD}}(s) = \sum_{n=0}^{\infty} C_{2n}(s,\mu) \frac{\langle O_{2n}(\mu) \rangle}{s^n}$$

C_{2n} are the Wilson parameters (dimensionless)
$\langle O_{2n} \rangle$ are vacuum condensates (dimension $2n$)

$$\begin{aligned}
\langle O_0 \rangle &= 1 \\
\langle O_2 \rangle &= 0 \\
\langle O_4 \rangle &\sim \alpha_s \langle G^a_{\mu\nu} G^{a\mu\nu} \rangle,\ m_q \langle \bar{q} q \rangle \\
\langle O_6 \rangle &= \cdots
\end{aligned}$$

FESR strategy

In the chiral limit and neglecting radiative corrections

$$\Pi^{\mathrm{QCD}}(s) \approx C_0 \ln(-s/\mu^2) + \sum_{n=1}^{\infty} C_{2n} \frac{\langle O_{2n} \rangle}{s^n}$$

if we consider the function $p(s) = s^{N-1}$ with $N \geq 1$

$$\frac{1}{\pi} \int_0^{s_0} ds\, s^{N-1}\, \mathrm{Im}\Pi(s + i\epsilon) = -C_0 \frac{s_0^N}{N} - C_{2N} \langle O_{2N} \rangle$$

Advantages:
The OPE condensates can be isolated from the series

Disadvantages:
loose of precision in the absence of radiative corrections and mass effects

Axial-Axial channel at $B = 0$

Hadronic sector

$$\Pi_{\mu\nu}(x-y) = i\langle 0|T\mathcal{A}_\mu(x)\mathcal{A}_\nu^\dagger(y)|0\rangle$$

The Fourier transformation can be written as

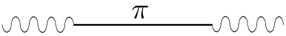
$$\Pi_{\mu\nu}(q) = q_\mu q_\nu \Pi_0(q^2) + (g_{\mu\nu}q^2 - q_\mu q_\nu)\Pi_1(q^2)$$

Π_0 is associated with spin-0 particles (pions)
Π_1 with spin-1 resonances (a_0)

Considering only pions

$$\mathcal{A}_\mu(x) = -f_\pi \partial_\mu \pi^+(x)$$

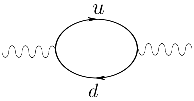
The correlator is proportional to the pion propagator



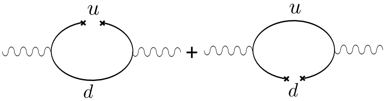
$$\Pi_{\mu\nu}^{\text{Had}}(q) = i f_\pi^2 \frac{i q_\mu q_\nu}{q^2 - m_\pi^2} = q_\mu q_\nu \Pi_0^{\text{Had}}(q^2)$$

QCD sector

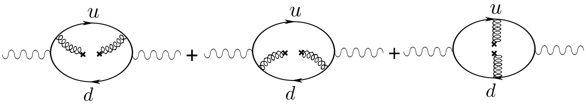
$$\mathcal{A}_\mu(x) = \bar{d}(x)\gamma_\mu\gamma_5 u(x)$$



$$\sim \ln(-s/\mu^2)$$



$$\sim (m_u\langle\bar{u}u\rangle + m_d\langle\bar{d}d\rangle)/s^2$$



$$\sim \alpha_s\langle GG\rangle/s^2$$

Axial-Axial channel at $B \neq 0$ in the chiral limit

Changes in the presence of B

Considering $\boldsymbol{B} = B\hat{\boldsymbol{z}}$. The constants now depend on the magnetic field

$$s_0,\ f_\pi \ \rightarrow \ s_0(B),\ f_\pi(B)$$

The propagators change (Schwinger proper time representation)

$$\begin{aligned} D_\pi(x-y) &\rightarrow e^{ie\Phi(x,y)} D_\pi^B(x-y) \\ S_q(x-y) &\rightarrow e^{ie_q\Phi(x,y)} S_q^B(x-y) \end{aligned}$$

where $\Phi(x,y) = \int_x^y A(\xi)\cdot d\xi$ is a non-local gauge dependent phase

The tensorial structure of the correlator changes in the presence of the magnetic field

$$\begin{aligned} q^2 &\rightarrow q_\parallel^2,\ q_\perp^2 \\ q_\mu &\rightarrow q_{\parallel\mu},\ q_{\perp\mu} \\ g_{\mu\nu} &\rightarrow g_{\parallel\mu\nu},\ g_{\perp\mu\nu},\ \epsilon_{\perp\mu\nu} \end{aligned}$$

Hadronic sector

The axial current includes the magnetic field as a minimal coupling

$$\mathcal{A}_\mu(x) = -f_\pi \mathcal{D}_\mu(x)\pi^+(x) = -f_\pi[\partial_\mu - ieA_\mu(x)]\pi^+(x)$$

$$\begin{aligned}
\tilde{\Pi}^{\text{Had}}_{\mu\nu}(x,y) &= i f_\pi^2 \mathcal{D}_\mu(x)\mathcal{D}^\dagger_\nu(y)[e^{ie\Phi(x,y)} D^B_\pi(x-y)] \\
&= e^{ie\Phi(x,y)} \left[i f_\pi^2 \frac{\partial}{\partial x^\mu}\frac{\partial}{\partial y^\nu} D^B_\pi(x-y) \right] \\
&= e^{ie\Phi(x,y)} \Pi^{\text{Had}}_{\mu\nu}(x-y)
\end{aligned}$$

$$\Pi^{\text{Had}}_{\mu\nu}(q) = q_\mu q_\nu \left[i f_\pi^2 D^B_\pi(q_\parallel^2, q_\perp^2) \right]$$

Perturbative part in QCD sector

$$\tilde{\Pi}^{\rm QCD}_{\mu\nu}(x,y) = -i{\rm tr}\left[e^{ie_u\Phi(x,y)}S^B_u(x-y)\,\gamma_\mu\gamma_5\, e^{ie_d\Phi(y,x)}S^B_d(y-x)\,\gamma_\nu\gamma_5\right]$$

$$= e^{i(e_u-e_d)\Phi(x,y)}\Pi^{\rm QCD}_{\mu\nu}(x-y)$$

with $e_u = \frac{2}{3}e$ and $e_d = -\frac{1}{3}e$

$$\Pi^{\rm QCD}_{\mu\nu}(q) = -i\int \frac{d^4k}{(2\pi)^4}{\rm tr}[S^B_u(k)\gamma_\mu\gamma_5 S^B_d(q+k)\gamma_\nu\gamma_5]$$

Quark-Hadron duality

The correlator in the hadron sector represents spin-0 particles

$$\Pi^{\mathrm{Had}}_{\mu\nu}(q) = q_\mu q_\nu \Pi^{\mathrm{Had}}_0(q^2_\parallel, q^2_\perp)$$

The correlator in the QCD sector possesses a complicated tensorial structure. We project the spin-0 component

$$\frac{q^\mu q^\nu}{q^4}\Pi^{\mathrm{QCD}}_{\mu\nu}(q) = \Pi^{\mathrm{QCD}}_0(q^2_\parallel, q^2_\perp)$$

As the external momentum is not Lorentz invariant, we choose frame $q_\perp = 0$ and $q^2_\parallel \equiv s$

$$\Pi^{\mathrm{Had}}_0(s) \leftrightarrow \Pi^{\mathrm{QCD}}_0(s)$$

Proper time integration – Hadronic sector

The correlator must be integrated in s *after* the proper-time integration

For s in the real axis we split the integral

$$\begin{aligned}
&\frac{1}{\pi}\int_0^{s_0} ds\, p(s)\, \mathrm{Im}\Pi_0^{\mathrm{Had}}(s+i\epsilon) \\
&\quad = \frac{1}{\pi}\int_0^{eB} ds\, p(s)\, \mathrm{Im}\Pi_0^{\mathrm{Had}}(s+i\epsilon) \quad \rightarrow \quad \text{Landau series} \\
&\qquad + \frac{1}{\pi}\int_{eB}^{s_0} ds\, p(s)\, \mathrm{Im}\Pi_0^{\mathrm{Had}}(s+i\epsilon) \quad \rightarrow \quad eB/s \text{ expansion}
\end{aligned}$$

Only the lowest Landau level contributes to the hadronic part

$$\frac{1}{\pi}\int_0^{s_0} ds\, p(s)\, \mathrm{Im}\Pi_0^{\mathrm{Had}}(s+i\epsilon) = f_\pi^2\, p(eB)$$

Proper time integration – QCD sector

For s in the circle s_0

$$\frac{-1}{2\pi i}\oint_{|s|=s_0} ds\, p(s)\, \Pi_0^{\rm QCD}(s) \quad \rightarrow \quad eB/s \text{ expansion}$$

The expansion in the QCD sector generates the extended Wilson coefficients

$$\begin{aligned} C_0(s,\mu,eB) &= C_0^{(0)} \ln(-s/\mu^2) + \sum_{m=1}^{\infty} C_0^{(m)} \left(\frac{eB}{s}\right)^m \\ C_{2n}(s,eB) &= \sum_{m=0}^{\infty} C_{2n}^{(m)} \left(\frac{eB}{s}\right)^m \end{aligned}$$

FESR at $B \neq 0$

for $p(s) = s^{N-1}$ the FESR at $B \neq 0$ mixes the condensates

$$f_\pi^2 (eB)^{N-1} = -C_0^{(0)} \frac{s_0^N}{N} - C_0^{(N)} (eB)^N - \sum_{m=0}^{N-1} C_{2N-2m}^{(m)} (eB)^m \langle O_{2N-2m} \rangle$$

We will take the sum rules with $N = 1$ and $N = 2$

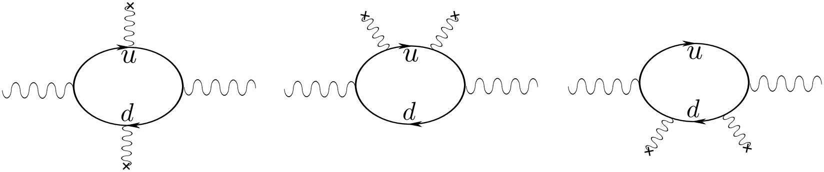
$$f_\pi^2 = -C_0^{(0)} s_0 - C_2^{(0)} \langle O_2 \rangle - C_0^{(1)} eB$$

$$f_\pi^2 eB = -C_0^{(0)} \frac{s_0^2}{2} - C_4^{(0)} \langle O_4 \rangle - C_2^{(1)} \langle O_2 \rangle eB - C_0^{(2)} (eB)^2$$

Results

First 2 FESR

The new QCD diagrams that contribute to Π_0 in the FESR for $N = 1$ and $N = 2$ corresponds to $C_0^{(2)}$



$$N = 1: \qquad f_\pi^2 = -C_0^{(0)} s_0$$

$$N = 2: \qquad f_\pi^2\, eB = -\frac{1}{2} C_0^{(0)} s_0^2 - C_4^{(0)} \langle O_4 \rangle - C_0^{(2)} (eB)^2$$

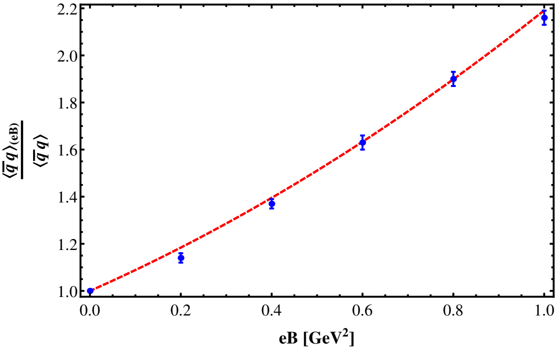
$N = 1$ sum rule: $s_0(B) = 8\pi f_\pi^2(B)$

Using the Gell-Man–Oakes–Renner relation and assuming that, in chiral limit,

$$\frac{m_\pi^2}{m_u + m_d} \longrightarrow \text{constant} \quad \Rightarrow \quad \frac{f_\pi^2(B)}{f_\pi^2(0)} = \frac{\langle \bar{q}q \rangle(B)}{\langle \bar{q}q \rangle(0)}$$

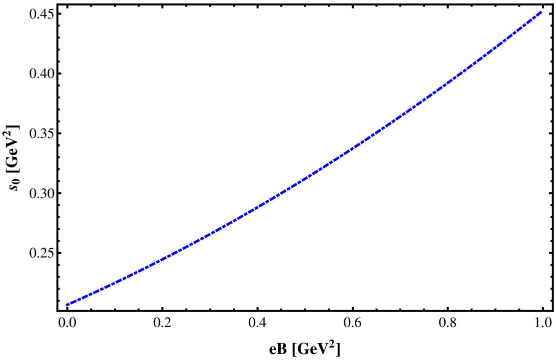
Fitting Lattice results

Bali, Bruckmann, Endrodi, Fodor, Katz, Schäfer, Phys. Rev. D 86, 071502 (2012)



Hadronic continuum threshold at $B \neq 0$

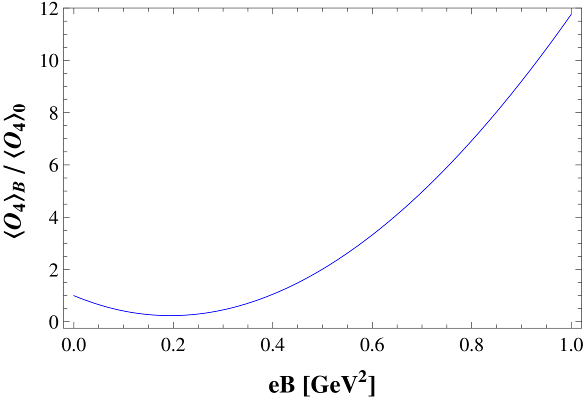
$$s_0(B) \approx f_\pi^2(0)\left[1 + a(eB) + b(eB)^2\right]$$



$N = 2$ sum rule: $\langle \alpha_s GG \rangle (B)$

Once obtained $s_0(B)$ we can obtain the Gluon condensate $\langle O_4 \rangle \propto \langle \alpha_s GG \rangle$

$$C_4^{(0)} \langle O_4 \rangle = -\frac{1}{2} C_0^{(0)} s_0^2(B) - C_0^{(2)} (eB)^2$$



Conclusions and outlook

- Systematic procedure for FESR at finite B
- Gluon condensate prediction at finite $B \to$ (anti)catalisis
- Finite temperature and density effects
- mass and radiative corrections effects $\to$ lower energy radiative techniques (Analytic QCD, Schwinger-Dyson eqs., ...)

THANKS!