Lattice Gluon Propagator as Input for a Potential Model

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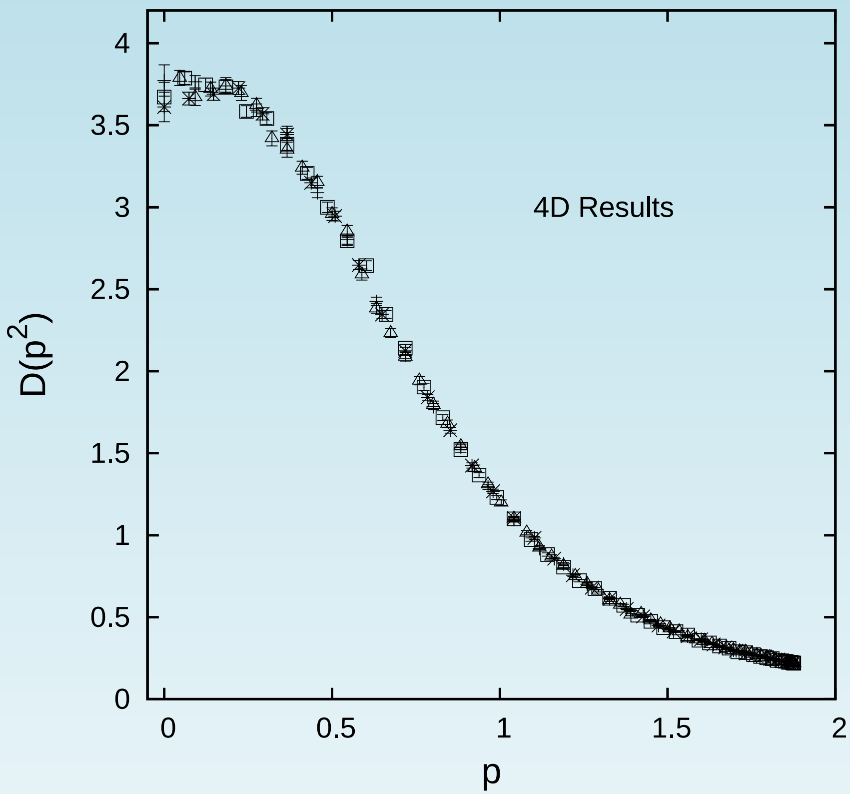
- Compare results to Cornell-potential case

Present-Day Gluon Propagator



Euclidean gluon propagator $D(p^2)$ in Landau gauge versus lattice momentum p (both in physical units) for the pure-$SU(2)$ case considering volumes of up to 128^4 (lattice extent ~ 27 fm)

A. Cucchieri, T.M., PRL2008

Fit: Gribov-Stingl Form for IR Gluon

Gribov-type propagator

$$D(p^2) \; = \; C \, \frac{p^2}{p^4 \; + \; b^2}$$

has purely-imaginary complex-conjugate poles

(vanishes at $p = 0$)

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Poles at masses $m^2 \;=\; a \pm ib$

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In general: pairs of (complex-conjugate) poles $+$ real poles,
starting from p^6 in the denominator, p^4 in numerator

Fit Results

4-parameter fit

$$D(p^2) = C \frac{p^2 + d}{p^4 + u^2 p^2 + t^2}$$

yields

$C = 0.784(5)$, $u = 0.768(17)$ GeV,

$t = 0.720(9)$ GeV^2, $d = 2.508(78)$ GeV^2

A. Cucchieri et al., PRD2012

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Equivalent to

$$D(p^2) \; = \; \frac{\alpha_+}{p^2 + \omega_+^2} + \frac{\alpha_-}{p^2 + \omega_-^2}$$

where $\omega_+^2 = (\omega_-^2)^*$, $\alpha_+ = \alpha_-^*$

Pole masses are $\omega_\pm^2 \; = \; 0.29(2) \, \pm \, i \, 0.66(1) \;\; GeV^2$

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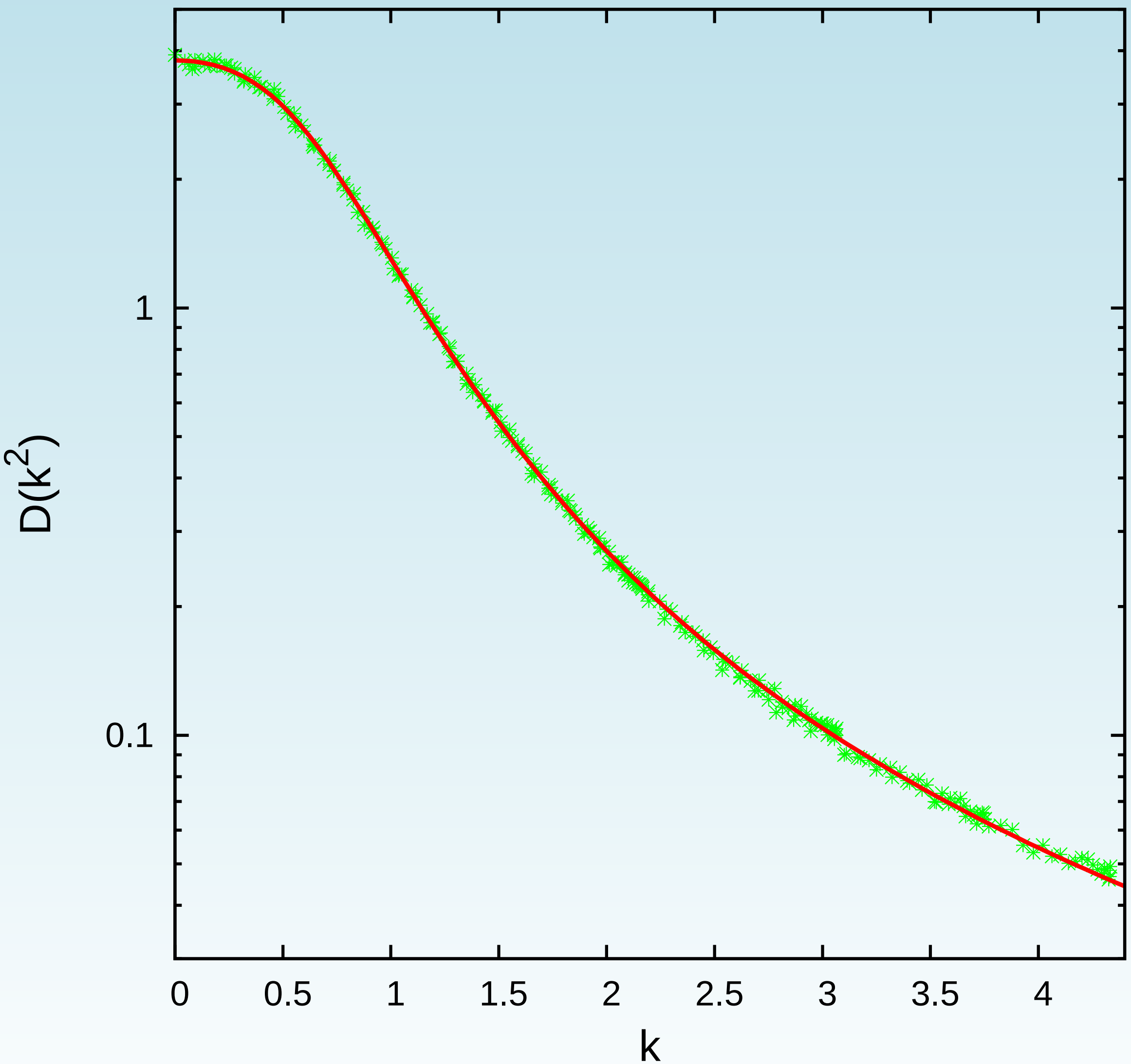
Note: agreement with SU(3) case (Dudal et al., PRD2010)
consistent with RGZ scenario (Dudal et al., PRD2008)

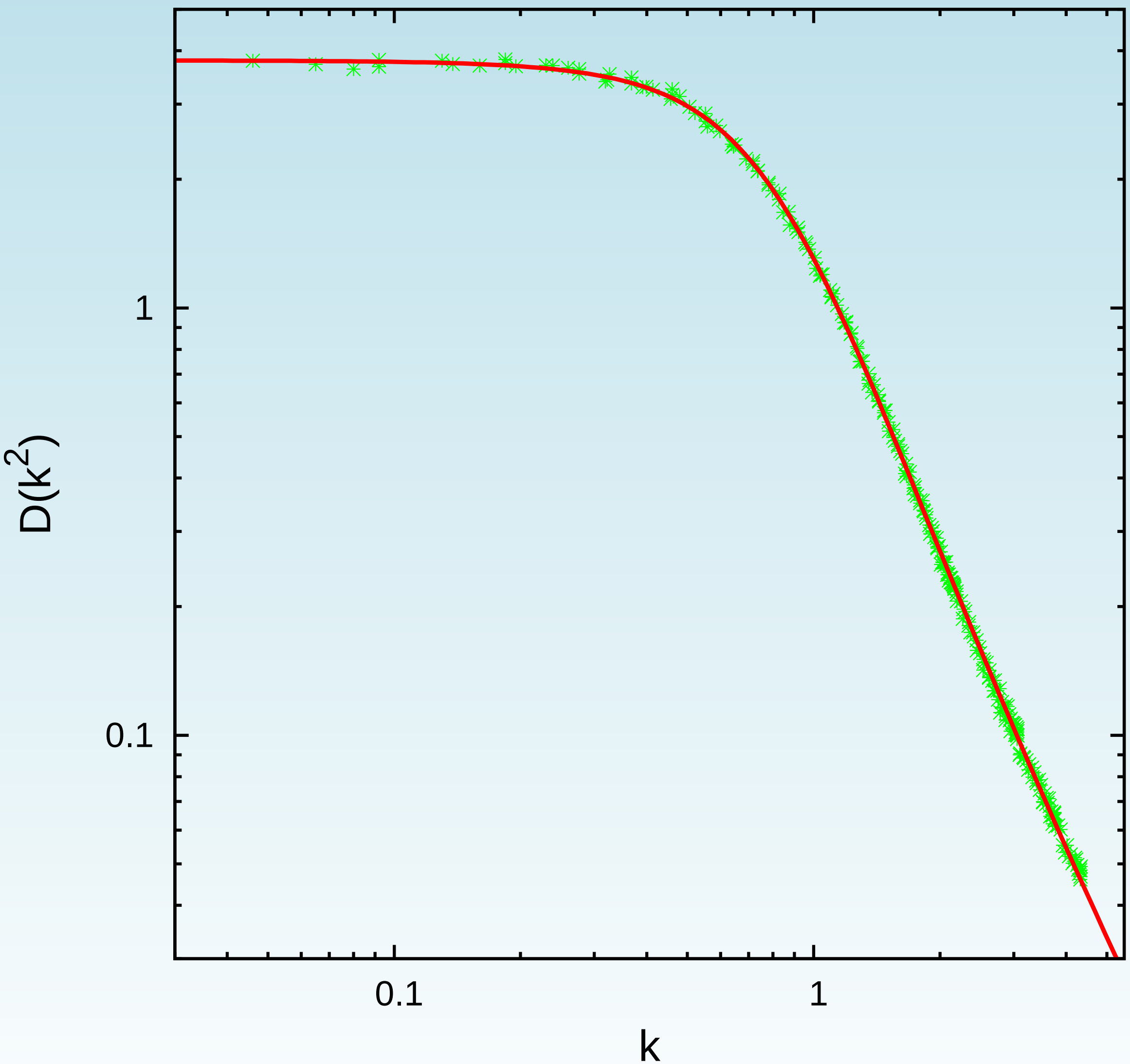
Data + Fit for Gluon Propagator

Fit using above form



Data + Fit for Gluon Propagator

Same, using log-log scale



One-Gluon-Exchange Potential

Nonrelativistic limit of $e^- - e^+$ scattering in QED yields amplitude

$$T_{fi} = \frac{-e^2}{(2\pi)^6} D_{00}(0, \mathbf{k}) \, \delta_{\sigma_1 \tau_1} \delta_{\sigma_2 \tau_2} \quad (t\text{-channel only})$$

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Coulomb potential obtained from $D_{\mu\nu}(k) = -g_{\mu\nu}/k^2$ and

$$V(r) \;=\; -(2\pi)^3 \int \exp(-i\mathbf{k}\cdot\mathbf{r})\, T_{fi}(k)\, d^3k \;=\; -\frac{e^2}{4\pi r}$$

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Analogously, for QCD, inserting color (Casimir) factor

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$$V(r) \;=\; -\frac{4}{3}\,\frac{\alpha_s}{r}$$

Cornell potential is then obtained by adding linear term

$$V(\mathbf{r}) \;=\; -\frac{4}{3}\,\frac{\alpha_s}{r} + \sigma\, r$$

OGE Potential from Lattice Propagator

Recall tensor structure for Landau-gauge propagator

$$D^{ab}_{\mu\nu}(k) \ = \ \frac{C\,(s+k^2)}{t^2+u^2k^2+k^4}\left(\delta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)\delta^{ab}$$

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$+$ take $\delta_{\mu\nu}\rightarrow -g_{\mu\nu}$ to invert the Wick rotation

We get

$$T_{fi} \;=\; \frac{4}{3}\,\frac{g_0^2}{(2\pi)^6}\,\frac{C\,(s+\mathbf{k}^2)}{t^2+u^2\mathbf{k}^2+\mathbf{k}^4}$$

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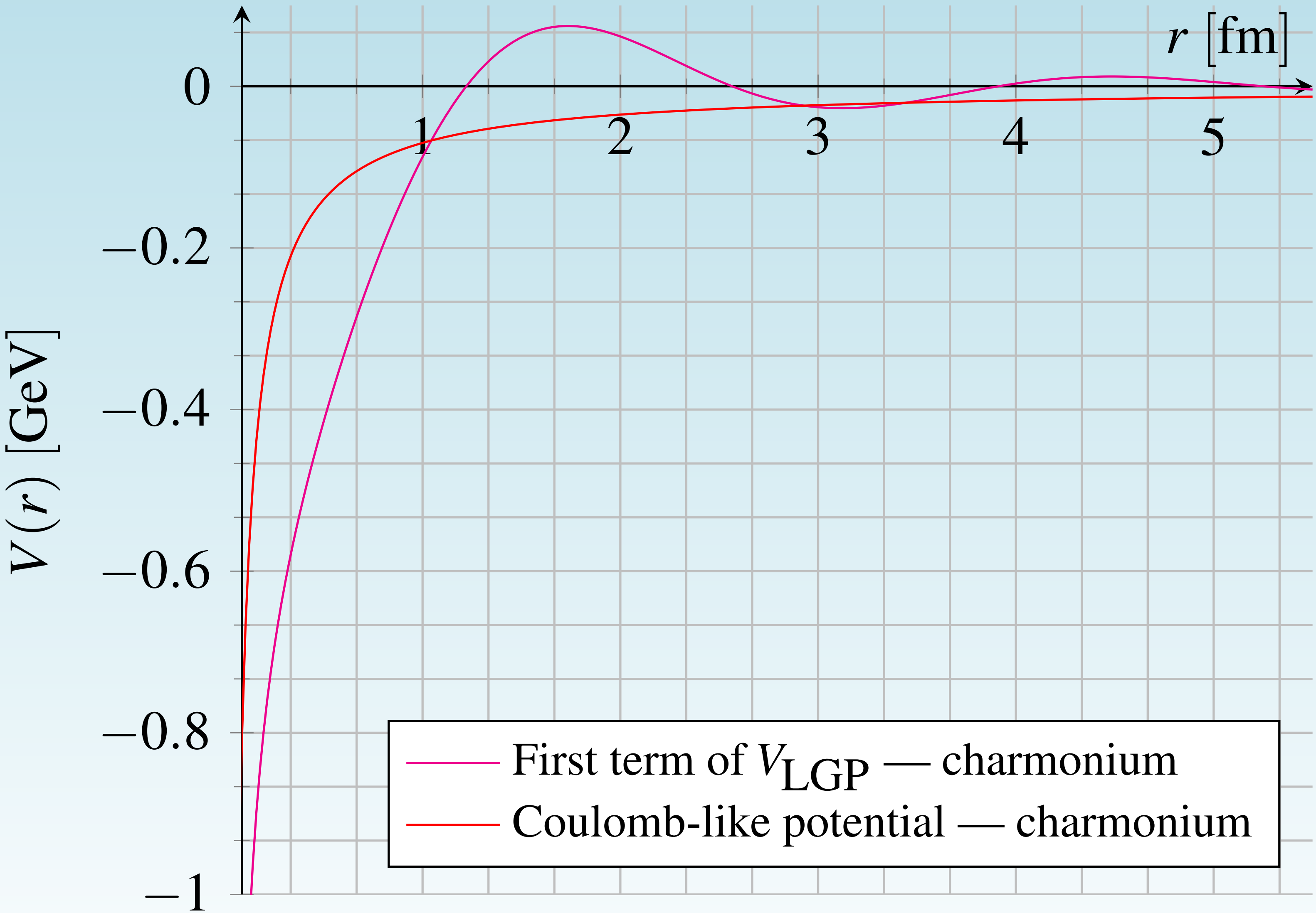
Fourier integral to get potential can still be done analytically (four poles in complex plane)

OGE Potential Plot



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Consider parameter values as a (3d) grid, look for minimum of residual, i.e. deviation with respect to spin-averaged input values

Results

Charmonium Spectrum				
	LGP+L Potential		Cornell Potential	
State	Found Mass (GeV)	Deviation from input	Found Mass (GeV)	Deviation from input
1S	$2.96 \pm .11$	-0.10	$2.93 \pm .17$	-0.14
2S	$3.69 \pm .13$	0.01	$3.69 \pm .20$	0.01
3S	$4.22 \pm .15$	-	$4.28 \pm .24$	-
1P	$3.46 \pm .12$	-0.07	$3.42 \pm .19$	-0.11
2P	$4.02 \pm .14$	0.09	$4.04 \pm .22$	0.12
3P	$4.50 \pm .16$	-	$4.58 \pm .25$	-
1D	$3.81 \pm .14$	-	$3.80 \pm .21$	-
2D	$4.31 \pm .15$	-	$4.36 \pm .24$	-

Results

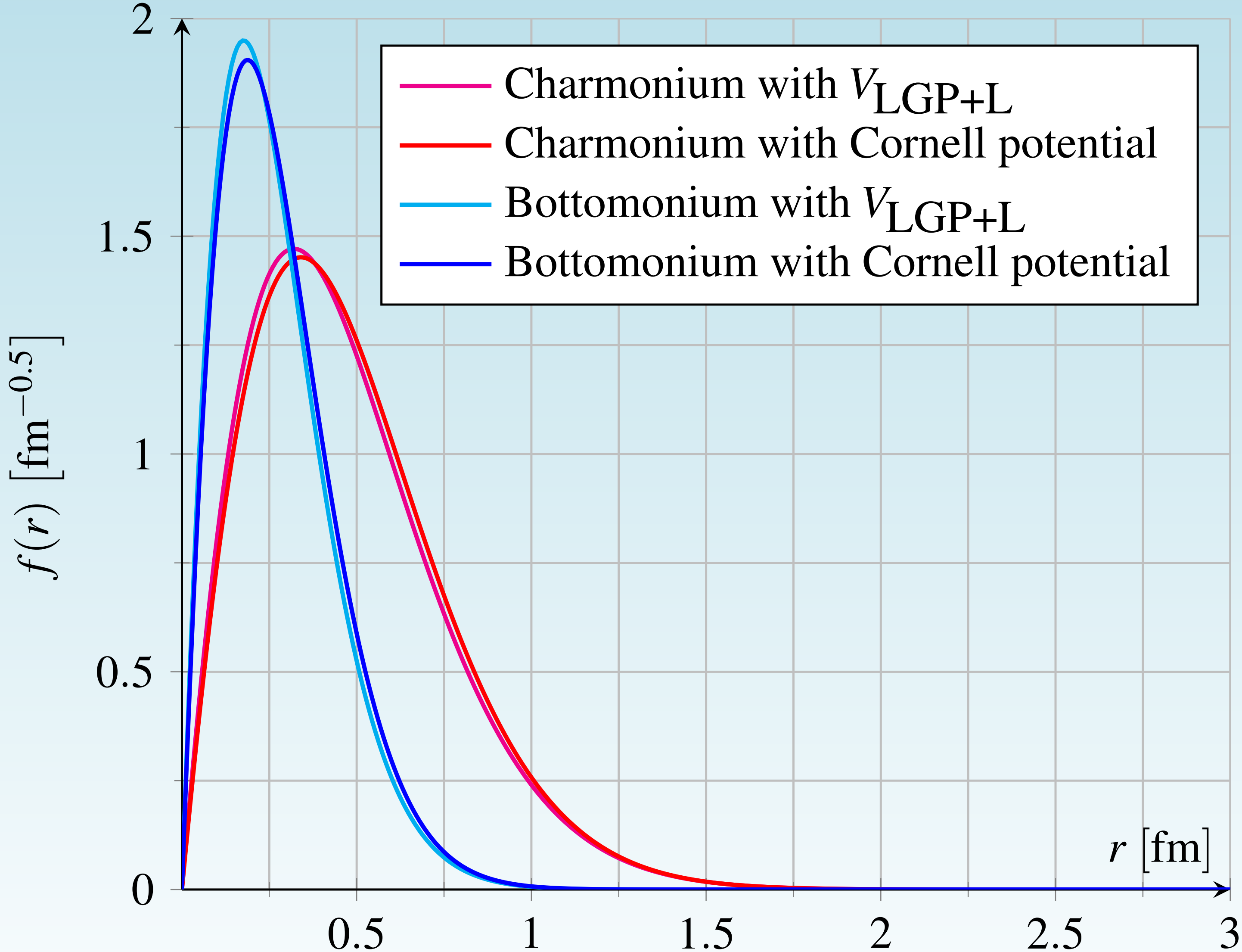
Bottomonium Spectrum				
	LGP Potential		Cornell Potential	
State	Found Mass (GeV)	Deviation from input	Found Mass (GeV)	Deviation from input
1S	$9.47 \pm .39$	0.04	$9.49 \pm .31$	0.06
2S	$10.00 \pm .33$	-0.01	$10.00 \pm .33$	-0.01
3S	$10.37 \pm .69$	0.01	$10.39 \pm .35$	0.03
4S	$10.67 \pm .81$	0.10	$10.72 \pm .37$	0.14
5S	$10.95 \pm .86$	-	$11.01 \pm .39$	$-$
1P	$9.86 \pm .48$	-0.03	$9.84 \pm .33$	-0.04
2P	$10.24 \pm .63$	-0.01	$10.25 \pm .35$	0.00
3P	$10.56 \pm .76$	0.03	$10.59 \pm .36$	0.05
4P	$10.84 \pm .83$	-	$10.89 \pm .38$	-
1D	$10.11 \pm .57$	-0.05	$10.10 \pm .34$	-0.06
2D	$10.44 \pm .71$	-	$10.45 \pm .36$	-
3D	$10.73 \pm .82$	-	$10.77 \pm .37$	-

Results

Parameter vaues

Lattice Propagator + Linear	Cornell Potential
$m_c = 1.16 \pm 0.03$ GeV	$m_c = 1.11\,^{+0.08}_{-0.02}$ GeV
$m_b = 4.61\,^{+0.02}_{-0.01}$ GeV	$m_b = 4.58\,^{+0.04}_{-0.01}$ GeV
$\sigma = 0.23 \pm 0.01$ GeV	$\sigma = 0.26\,^{+0.01}_{-0.03}$ GeV
$\chi^2 = 6.2$	$\chi^2 = 12.1$

Wave Function



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- description of (spin-independent) spectrum similar to Cornell-potential case
- look for other ways to inject NP information into nonrelativistic potential