Leptonic Decay Constants of the Pion and its Excitations in Holographic QCD

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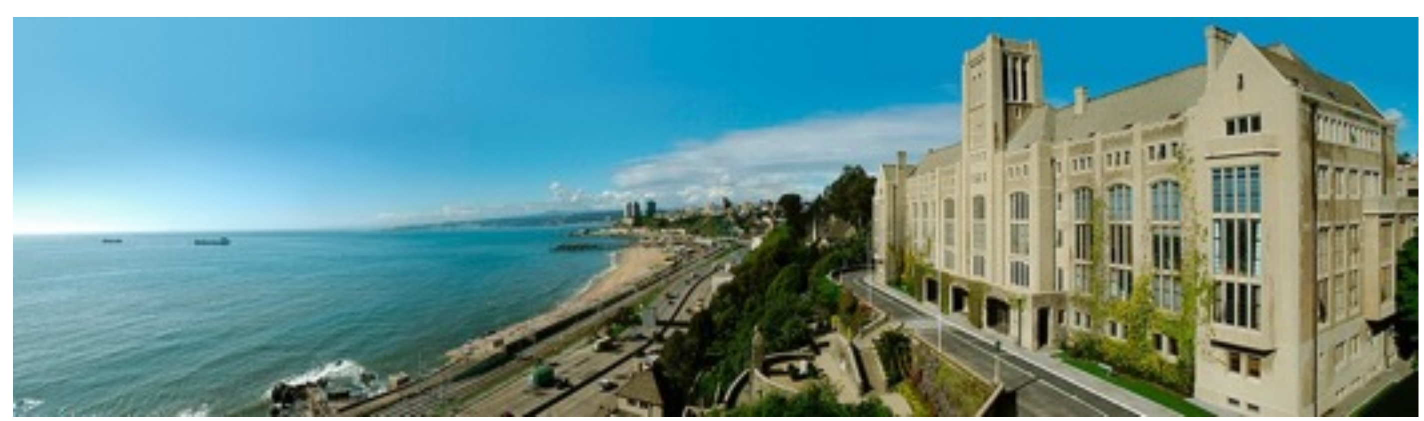
QNP 2015

7th International Conference on Quarks and Nuclear Physics



A. Ballon-Bayona, GK and C. Miller

PRD (2015), arXiv:1412.7505 [hep-ph]

Prediction of QCD*

The leptonic decay constants of the excited states

of the pion vanish in the chiral limit when

chiral symmetry is dynamically broken

* A. Holl, A. Krassnigg and C. D. Roberts, Phys. Rev. C 70, 042203 (2004)

Prediction*

Based on an exact, generalized Gell-Mann-Oakes-Renner relationship:

$$f_{\pi^n} m_{\pi^n}^2 = 2\, m_q\, \rho_{\pi^n}$$

ρ_{π^n} — gauge-invariant residues at the pole $P^2 = -m_{\pi^n}^2$ in the axial-vector and pseudoscalar vertex functions

f_{π^n} — given in terms of the Bethe-Salpeter wavefunction $\chi_{\pi^n}(P,q)$

m_q & ρ_{π^n}: each is scale dependent, but product is RG invariant

* A. Holl, A. Krassnigg and C. D. Roberts, Phys. Rev. C 70, 042203 (2004)

Key role of Ward-Takahashi identity

$$\Gamma^j_{5\mu}(k;P)|_{P^2+m^2_{\pi_n}\approx 0} = \frac{f_{\pi_n} P_\mu}{P^2+m^2_{\pi_n}} \Gamma^j_{\pi_n}(k;P)$$

$$i\Gamma^j_5(k;P)|_{P^2+m^2_{\pi_n}\approx 0} = \frac{\rho_{\pi_n}}{P^2+m^2_{\pi_n}} \Gamma^j_{\pi_n}(k;P)$$

Ward-Takahashi identity:

$$P_\mu \Gamma^j_{5\mu}(k;P) = S^{-1}(k_+) i\gamma_5 \frac{\tau^j}{2} + i\gamma_5 \frac{\tau^j}{2} S^{-1}(k_-) - 2im(\zeta)\Gamma^j_5(k;P)$$

$$f_{\pi_n} m^2_{\pi_n} = 2m(\zeta)\rho_{\pi_n}(\zeta)$$

$$f_{\pi_n} \delta^{ij} P_\mu = Z_2 \,\mathrm{tr} \int_q^\Lambda \frac{1}{2}\tau^i \gamma_5 \gamma_\mu \chi^j_{\pi_n}(q;P)$$

$$i\rho_{\pi_n}(\zeta)\delta^{ij} = Z_4 \,\mathrm{tr} \int_q^\Lambda \frac{1}{2}\tau^i \gamma_5 \chi^j_{\pi_n}(q;P)$$

U.V. behavior QCD, integrals are cutoff independent and therefore

$$\rho^0_{\pi_n}(\zeta) := \lim_{\hat{m}\to 0} \rho_{\pi_n}(\zeta) < \infty, \forall\, n$$

$$f_{\pi^n}\, m_{\pi^n}^2 = 2\, m_q\, \rho_{\pi^n}$$

For the ground-state pion, DCSB implies

$$\rho_{\pi^0} = -\frac{1}{f_{\pi^0}} \langle \bar{q} q \rangle$$

GOR relationship

$$f_{\pi^0}^2\, m_{\pi^0}^2 = 2 m_q\, |\langle \bar{q} q \rangle|$$

For the excited states $n > 0$

$$f_{\pi^n} = \frac{2\, m_q\, \rho_{\pi^n}}{m_{\pi^n}^2}$$

Chiral limit

$m_{\pi^n}^2 > 0$

$$f_{\pi^n}^0 := \lim_{m_q \to 0} f_{\pi^n} = 0$$

Physics Letters B 642 (2006) 244–247

The decay constant of the first excited pion from lattice QCD

UKQCD Collaboration

C. McNeile [a,*], C. Michael [b]

1st excited state

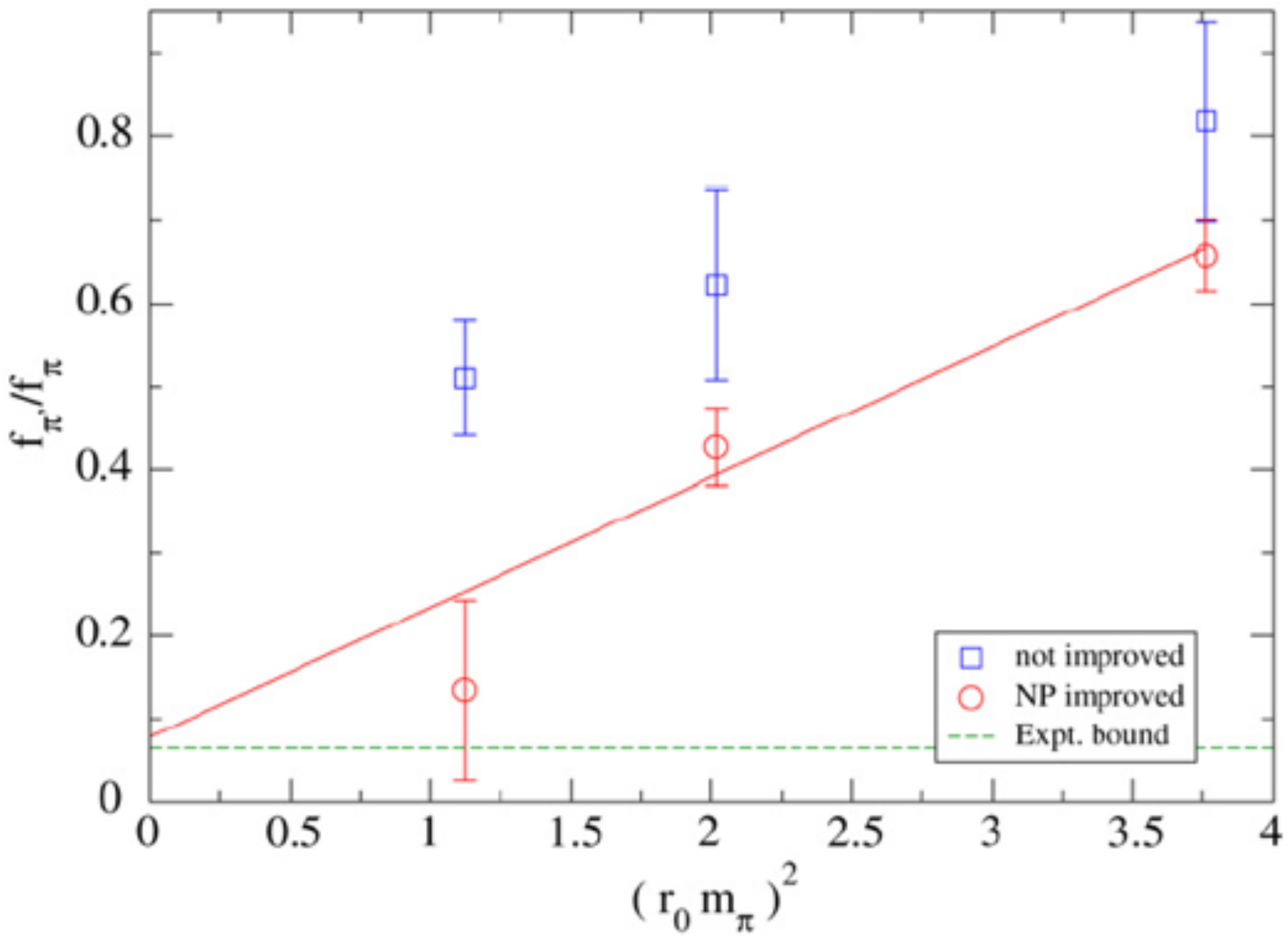


Fig. 2. Ratio of the decay constants of the first excited to ground state light pseudo-scalar meson as a function of the pion mass squared. The horizontal line is the experimental upper limit from Diehl and Hiller [8].

Progress of Theoretical Physics, Vol. 119, No. 4, April 2008

η' Meson from Two Flavor Dynamical Domain Wall Fermions

Koichi HASHIMOTO[1,2] and Taku IZUBUCHI[1,3]
(for the RBC Collaboration)

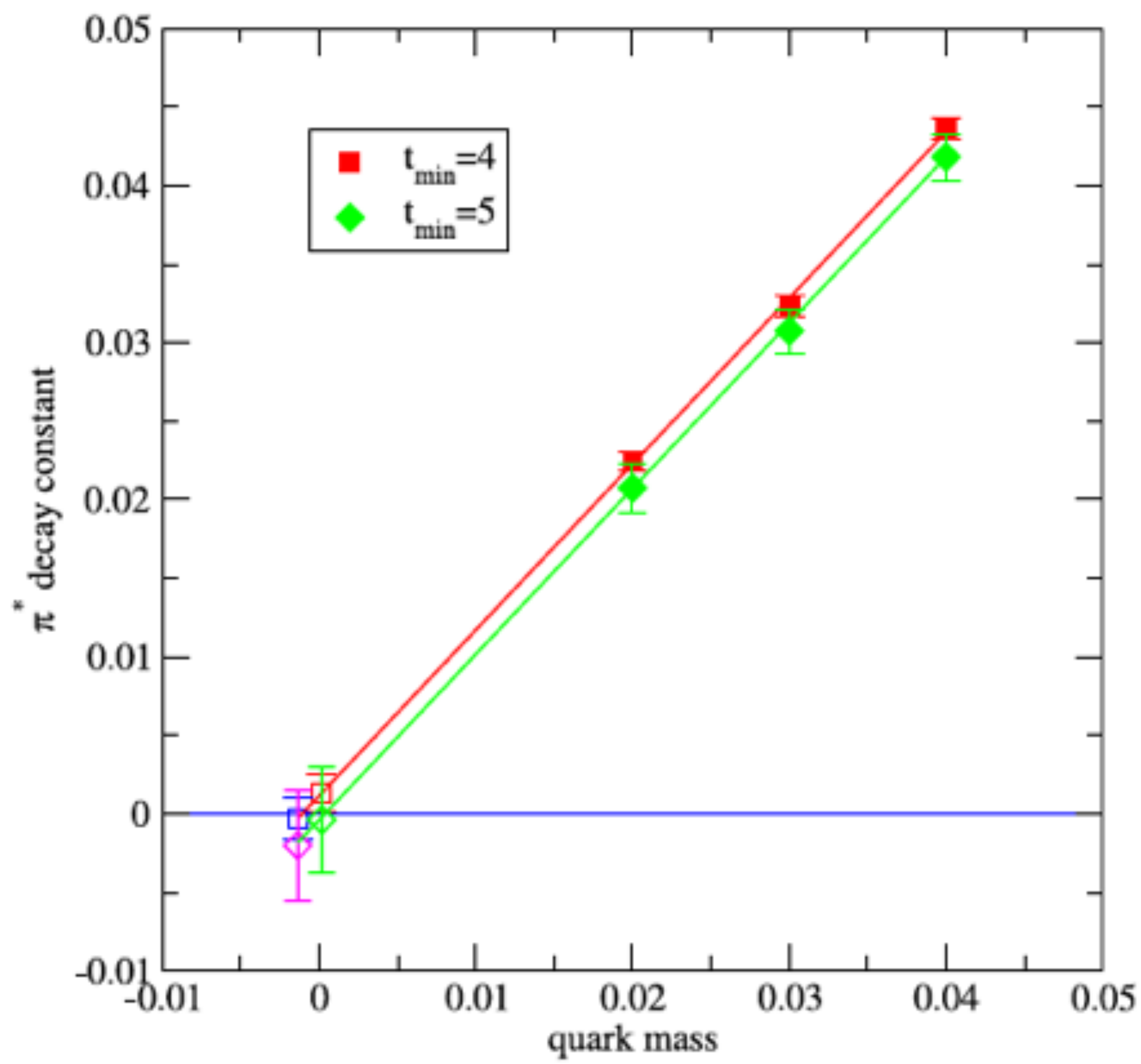


Fig. 26. f_{π^*} vs m_f.

PHYSICAL REVIEW D **90**, 014511 (2014)

Decay constants of the pion and its excitations on the lattice

Ekaterina V. Mastropas,[1] and David G. Richards[2]
(Hadron Spectrum Collaboration)

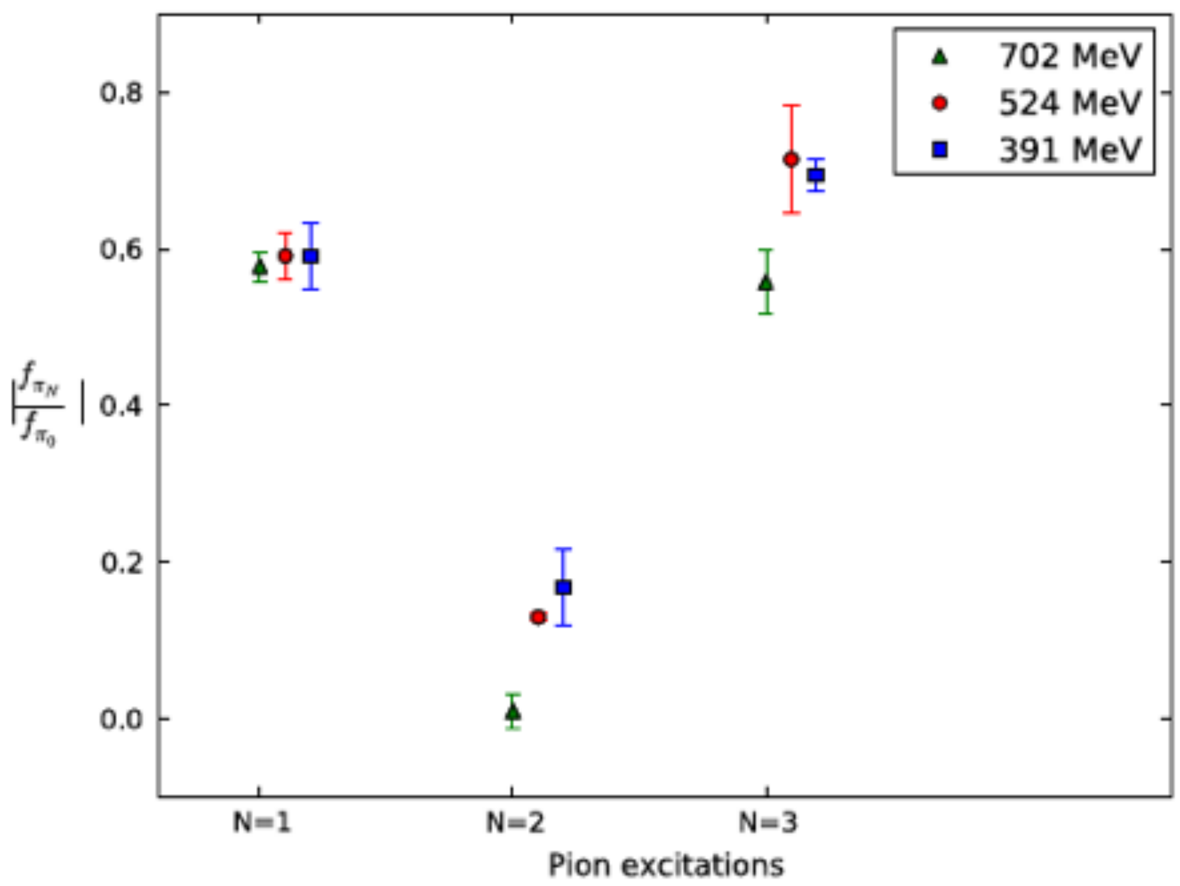


FIG. 5 (color online). Ratios of the excited-state decay constants f_{π_N} to the ground-state decay constant f_{π_0} for the first three pion excitations ($N = 1, 2, 3$), using the unimproved current.

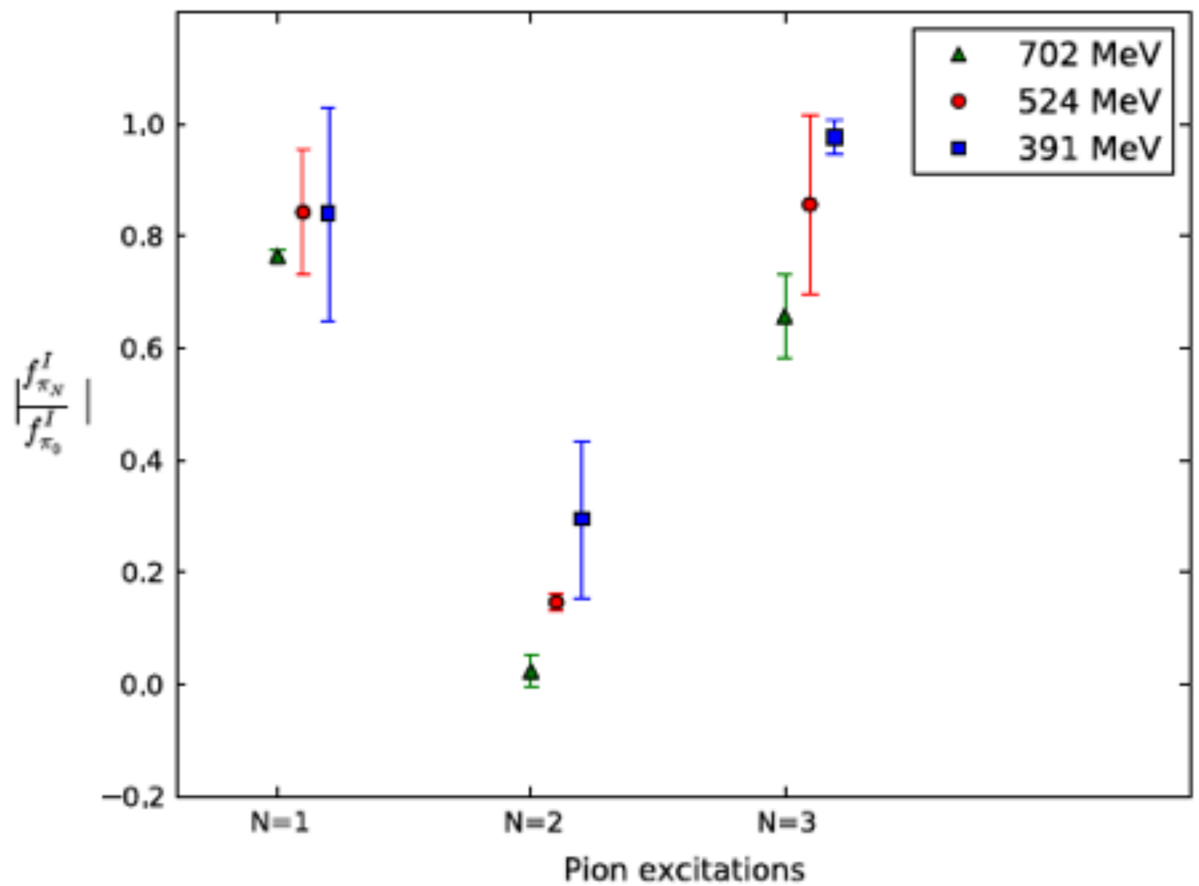


FIG. 6 (color online). Ratios of the excited-state decay constants f_{π_N} to the ground-state decay constant f_{π_0} for the first three pion excitations ($N = 1, 2, 3$), using the improved current.

Light-front holography*

Mass spectrum

$$M_{nJ}^2 = 4\kappa^2\left(n + \frac{L+J}{2}\right) + \int_0^1 dx\left(\frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right) f^2(x, m_1, m_2)$$

Leptonic decay constants

$$f_M = \kappa\frac{\sqrt{6}}{\pi}\int_0^1 dx\sqrt{x(1-x)}\, f(x, m_1, m_2)$$

$$f(x, m_1, m_2) = N\, x^{\alpha_1}\,(1-x)^{\alpha_2}$$

α_1, α_2 parameters fixed to get consistency with QCD

$$m_1 = m_2 = 0$$
$$n = L = J = 0$$

$\longrightarrow$ $M_\pi = 0$

* Brodsky, de Teramond, Dosh
Brantz, Gutsche, Lyubovitskij, Schmidt, Vega

PHYSICAL REVIEW D **82**, 074022 (2010)

Light and heavy mesons in a soft-wall holographic approach

Tanja Branz,[1] Thomas Gutsche,[1] Valery E. Lyubovitskij,[1,*] Ivan Schmidt,[2] and Alfredo Vega[2]

Masses of light mesons.

Meson	n	L	S	Mass [MeV]			
π	0	0,1,2,3	0	$M_{\pi(140)} = 140$	$M_{b_1(1235)} = 1355$	$M_{\pi_2(1670)} = 1777$	$M_{\pi_4} = 2099$
π	0,1,2,3	0	0	$M_{\pi(140)} = 140$	$M_{\pi(1300)} = 1355$	$M_{\pi(1800)} = 1777$	$M_{\pi(4s)} = 2099$
K	0	0,1,2,3	0	$M_K = 495$	$M_{K_1(1270)} = 1505$	$M_{K_2(1770)} = 1901$	$M_{K_3} = 2207$
η	0,1,2,3	0	0	$M_{\eta(1s)} = 544$	$M_{\eta(2s)} = 1552$	$M_{\eta(3s)} = 1946$	$M_{\eta(4s)} = 2248$
$f_0[\bar{n}n]$	0,1,2,3	1	1	$M_{f_0(1p)} = 1114$	$M_{f_0(2p)} = 1600$	$M_{f_0(3p)} = 1952$	$M_{f_0(4p)} = 2244$
$f_0[\bar{s}s]$	0,1,2,3	1	1	$M_{f_0(1p)} = 1304$	$M_{f_0(2p)} = 1762$	$M_{f_0(3p)} = 2093$	$M_{f_0(4p)} = 2372$
$a_0(980)$	0,1,2,3	1	1	$M_{a_0(1p)} = 1114$	$M_{a_0(2p)} = 1600$	$M_{a_0(3p)} = 1952$	$M_{a_0(4p)} = 2372$
$\rho(770)$	0,1,2,3	0	1	$M_{\rho(770)} = 804$	$M_{\rho(1450)} = 1565$	$M_{\rho(1700)} = 1942$	$M_{\rho(4s)} = 2240$
$\rho(770)$	0	0,1,2,3	1	$M_{\rho(770)} = 804$	$M_{a_2(1320)} = 1565$	$M_{\rho_3(1690)} = 1942$	$M_{a_4(2040)} = 2240$
$\omega(782)$	0,1,2,3	0	1	$M_{\omega(782)} = 804$	$M_{\omega(1420)} = 1565$	$M_{\omega(1650)} = 1942$	$M_{\omega(4s)} = 2240$
$\omega(782)$	0	0,1,2,3	1	$M_{\omega(782)} = 804$	$M_{f_2(1270)} = 1565$	$M_{\omega_3(1670)} = 1942$	$M_{f_4(2050)} = 2240$
$\phi(1020)$	0,1,2,3	0	1	$M_{\phi(1s)} = 1019$	$M_{\phi(2s)} = 1818$	$M_{\phi(3s)} = 2170$	$M_{\phi(4s)} = 2447$
$a_1(1260)$	0,1,2,3	1	1	$M_{a_1(1p)} = 1358$	$M_{a_1(2p)} = 1779$	$M_{a_1(3p)} = 2101$	$M_{a_1(4p)} = 2375$

$$m_1 = m_2 = 0 \longrightarrow f_{\pi^n} \neq 0, \quad n > 0$$

Our calculation

— hard-wall holographic model*

AdS spacetime: $(x, z) = (x^0, x^1, x^2, x^3, z)$

$$ds^2 = \frac{1}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2\right), \quad 0 < z \leq z_0\,, \quad z_0 \sim \frac{1}{\Lambda_{\rm QCD}}$$

AdS/CFT: correspondence operators $\mathcal{O}(x)$ of 4-d field theory
and fields $\phi(x,z)$ 5-d AdS space

*J. Erlich, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. Lett. 95, 261602 (2005)
L. Da Rold and A. Pomarol, Nucl. Phys. B 721, 79 (2005)
H. R. Grigoryan and A. V. Radyushkin, Phys. Rev. D 76, 115007 (2007)

QCD:

$SU(2)_L \times SU(2)_R$ Flavor symmetry + DCSB

$$\mathcal{O}(x) = \begin{cases} J^a_{L\mu} = \bar{q}_L \gamma_\mu t^a q_L, \quad J^a_{R\mu} = \bar{q}_R \gamma_\mu t^a q_R \\ \bar{q}_R q_L \end{cases} \qquad t^a = \frac{1}{2}\tau^a$$

Gauge fields + scalar fields

$$\phi(x,z) = \begin{cases} L^a_m(x,z), \quad R^a_m(x,z) \\ X(x,z) \end{cases}$$

Lagrangian density

$$S = \int d^5x \sqrt{|g|}\, \mathrm{Tr}\Big[(D^m X)^\dagger (D_m X) + 3|X|^2 - \frac{1}{4g_5^2}\left(L^{mn}L_{mn} + R^{mn}R_{mn}\right)$$

$$\begin{aligned} D_m X &= \partial_m X - iL_m X + iXR_m \\ L_{mn} &= \partial_m L_n - \partial_n L_m - i\,[L_m, L_n] \\ R_{mn} &= \partial_m R_n - \partial_n R_m - i\,[R_m, R_n] \end{aligned}$$

$$X = e^{i\pi^a t^a} X_0\, e^{i\pi^a t^a}$$

$$X_0 = \zeta M z + \frac{\Sigma}{\zeta} z^3 \qquad M = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}, \quad \Sigma = \begin{pmatrix} \sigma_u & 0 \\ 0 & \sigma_d \end{pmatrix} \qquad \zeta = \frac{\sqrt{N_c}}{2\pi}$$

Free parameters:

z_0

$m_q = m_u = m_d$

$\sigma_q = \sigma_u = \sigma_d$

Spectrum of mesons and Holographic currents

— Expand action up to quadratic terms in fields

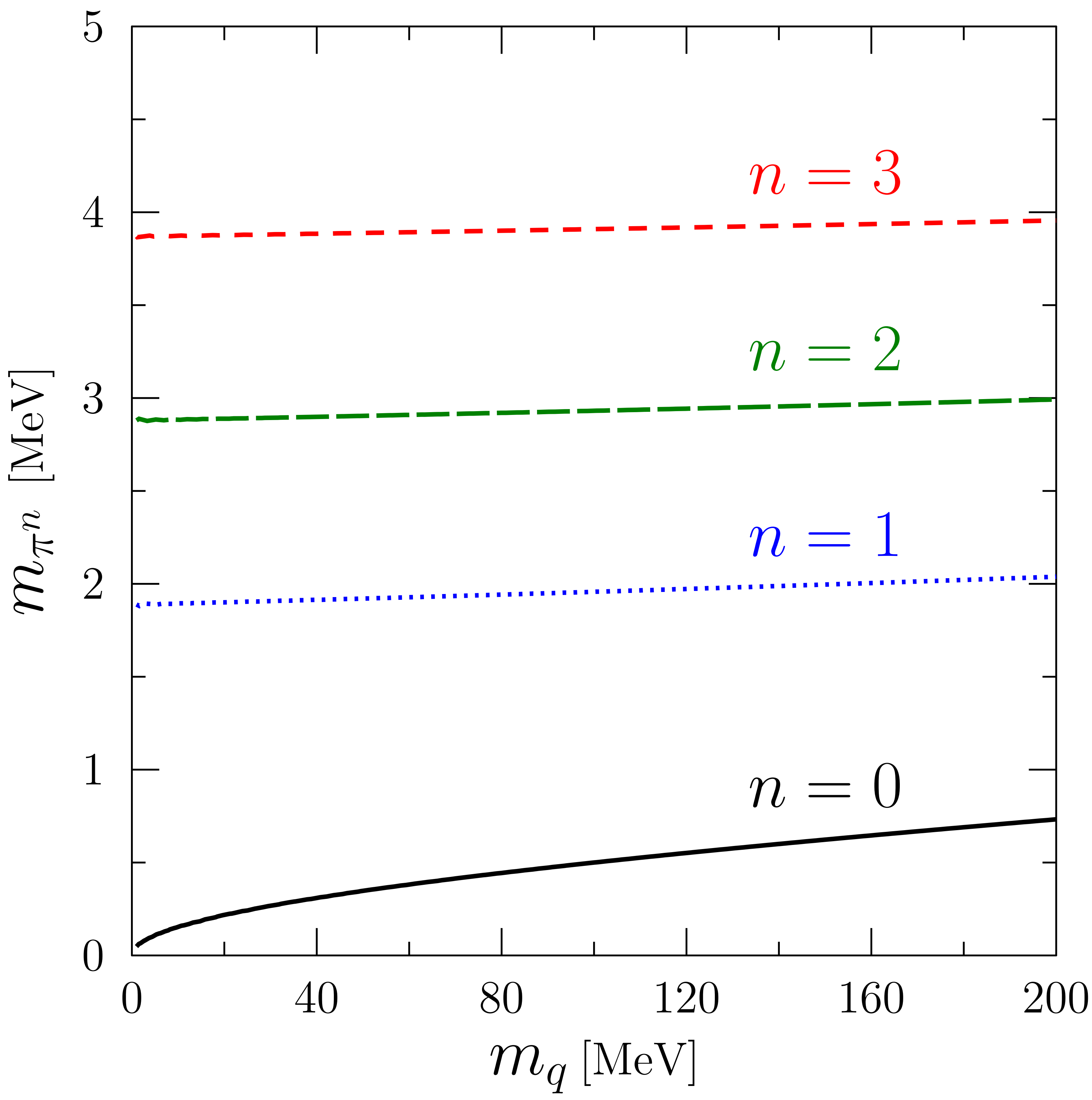
— Make Kaluza-Klein expansion of fields

— Vary action to obtain

- equations of motion
- holographic currents

— Solve equations of motion imposing appropriate b.c.

Quark mass dependence of the pion masses



Holographic currents

$$\langle J_V^{\hat{\mu}}(x)\rangle = \sum_{n=0}^{\infty}\left[\frac{1}{g_5 z}\partial_z v^n(z)\right]_{z=\epsilon}\hat{V}_n^{\hat{\mu}}(x)$$

$$\langle J_A^{\hat{\mu}}(x)\rangle = \sum_{n=0}^{\infty}\left[\frac{1}{g_5 z}\partial_z a^n(z)\right]_{z=\epsilon}\hat{A}_n^{\hat{\mu}}(x) + \sum_{n=0}^{\infty}\left[\frac{1}{g_5 z}\partial_z \phi^n(z)\right]_{z=\epsilon}\partial^{\hat{\mu}}\hat{\pi}^n(x)$$

$$\partial_{\hat{\mu}}\langle J_A^{\hat{\mu}}(x)\rangle = -\sum_{n=0}^{\infty}\left[\frac{\beta(z)}{g_5 z}\partial_z \pi^n(z)\right]_{z=\epsilon}\hat{\pi}_n(x)$$

$$\beta(z) = g_5^2\left(\zeta m_q + \frac{\sigma_q}{\zeta}z^2\right)^2$$

$$g_5^2 = \frac{12\pi}{N_c}$$

From these, one can identify

— the leptonic decay constants

— a generalized GOR relationship

Leptonic Decay Constants

$$g_{V^n} = \left[\frac{1}{g_5 z}\partial_z v^n(z)\right]_{z=\epsilon}$$

$$g_{A^n} = \left[\frac{1}{g_5 z}\partial_z a^n(z)\right]_{z=\epsilon}$$

$$f_{\pi^n} = \left[-\frac{1}{g_5 z}\partial_z \phi^n(z)\right]_{z=\epsilon}$$

$\Leftrightarrow$

$$\langle 0|J_V^{\hat{\mu}}(0)|V_n(p,\lambda)\rangle = \epsilon^{\hat{\mu}}(p,\lambda) g_{V^n}$$

$$\langle 0|J_A^{\hat{\mu}}(0)|A_n(p,\lambda)\rangle = \epsilon^{\hat{\mu}}(p,\lambda) g_{A^n}$$

$$\langle 0|J_A^{\hat{\mu}}(0)|\pi_n(p)\rangle = i p^{\hat{\mu}} f_{\pi^n}$$

Generalized GOR relationship

From

$$(1) \qquad \langle J_A^{\hat{\mu}}(x)\rangle = \sum_{n=0}^{\infty}\left[\frac{1}{g_5 z}\partial_z a^n(z)\right]_{z=\epsilon}\hat{A}_n^{\hat{\mu}}(x) + \sum_{n=0}^{\infty}\left[\frac{1}{g_5 z}\partial_z \phi^n(z)\right]_{z=\epsilon}\partial^{\hat{\mu}}\hat{\pi}^n(x)$$

$$(2) \qquad \partial_{\hat{\mu}}\langle J_A^{\hat{\mu}}(x)\rangle = -\sum_{n=0}^{\infty}\left[\frac{\beta(z)}{g_5 z}\partial_z \pi^n(z)\right]_{z=\epsilon}\hat{\pi}_n(x)$$

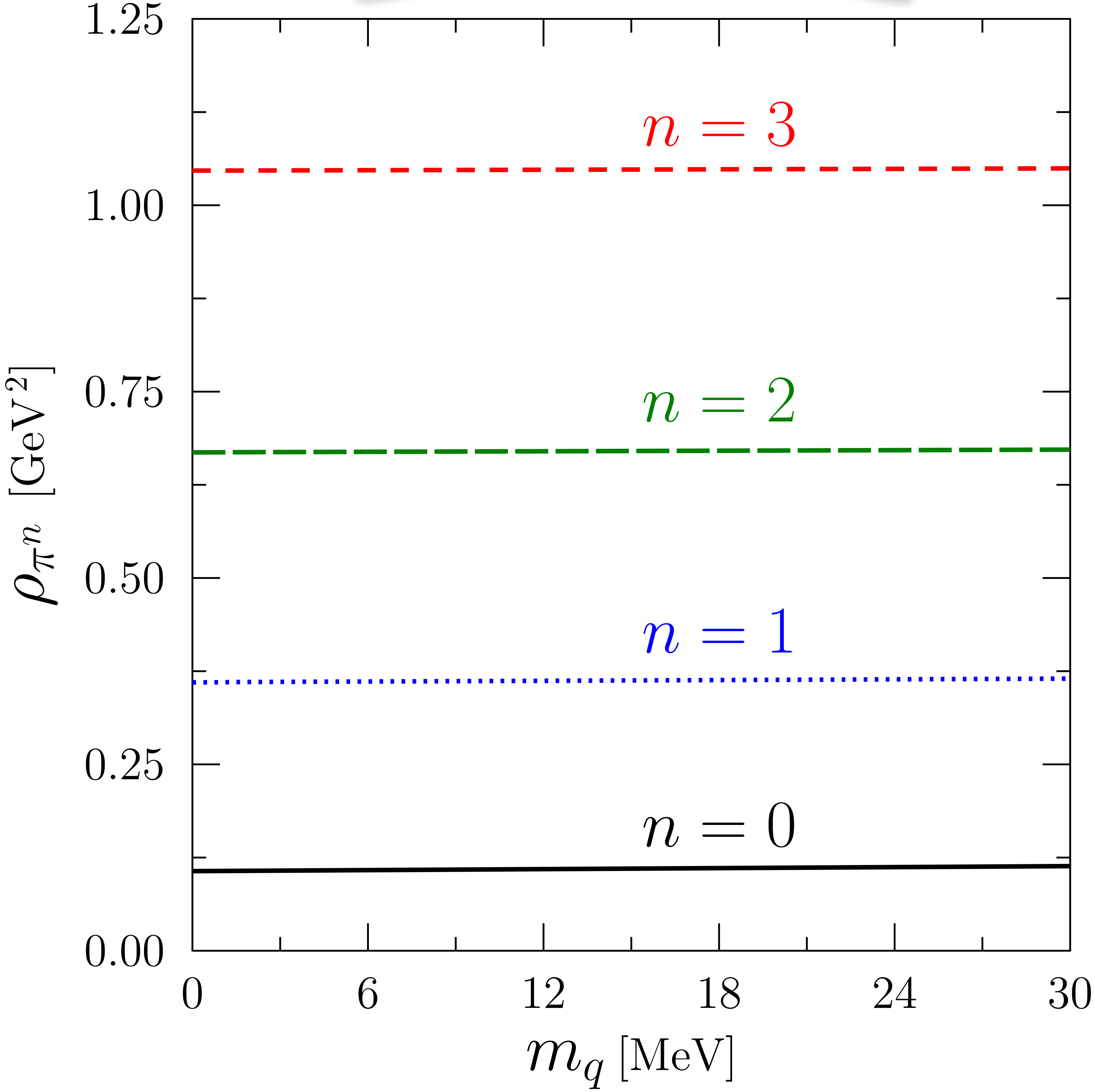
taking divergence of the (1) and using e.o.m.

$$\partial^2\hat{\pi}^n(x) + m_{\pi^n}^2\hat{\pi}^n(x) = 0$$

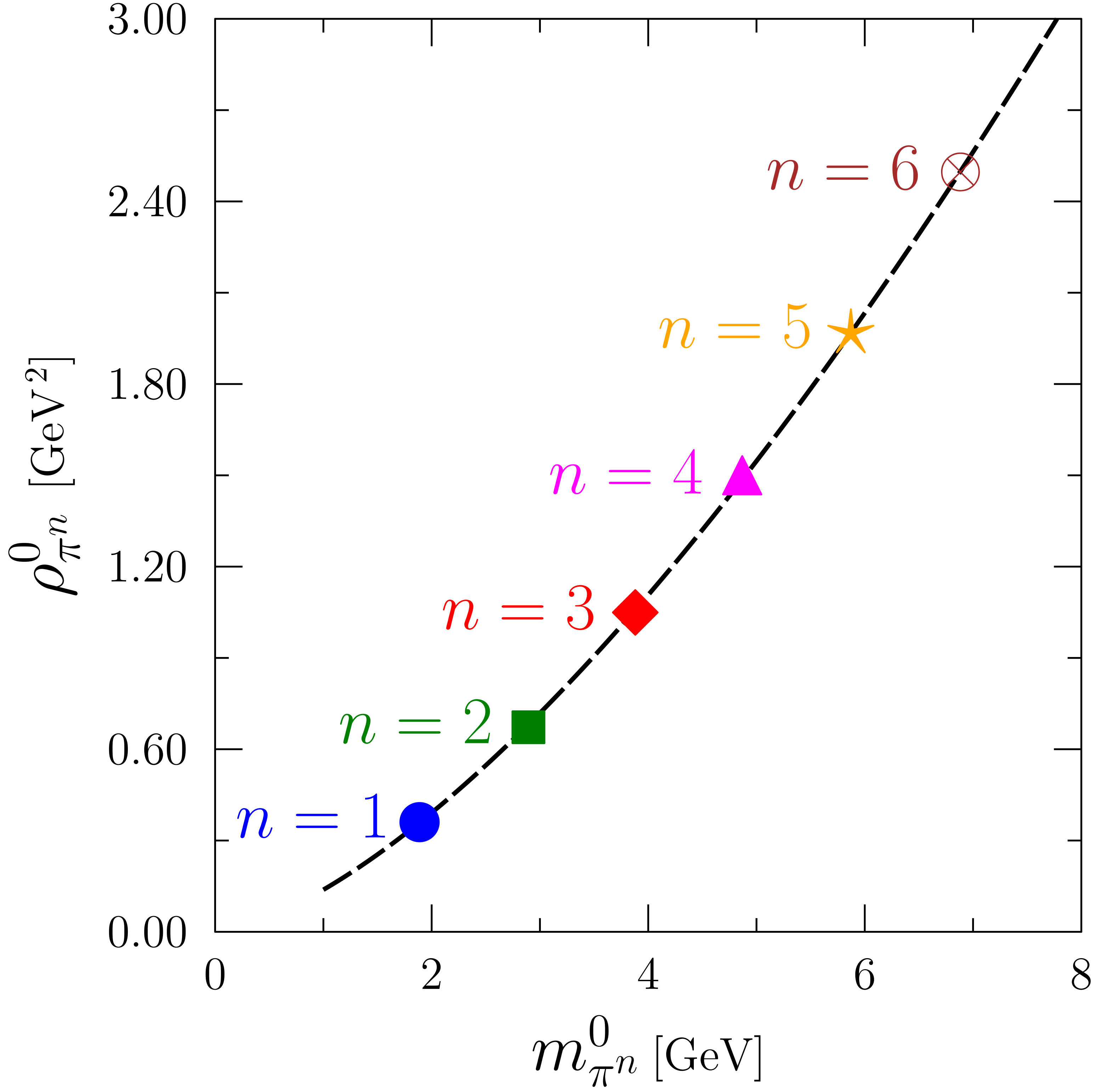
and comparing with (2), obtain

$$f_{\pi^n} m_{\pi^n}^2 = -\frac{1}{g_5}\left[\frac{\beta(z)}{z}\partial_z \pi^n(z)\right]_{z=\epsilon} \equiv 2\, m_q\, \rho_{\pi^n}$$

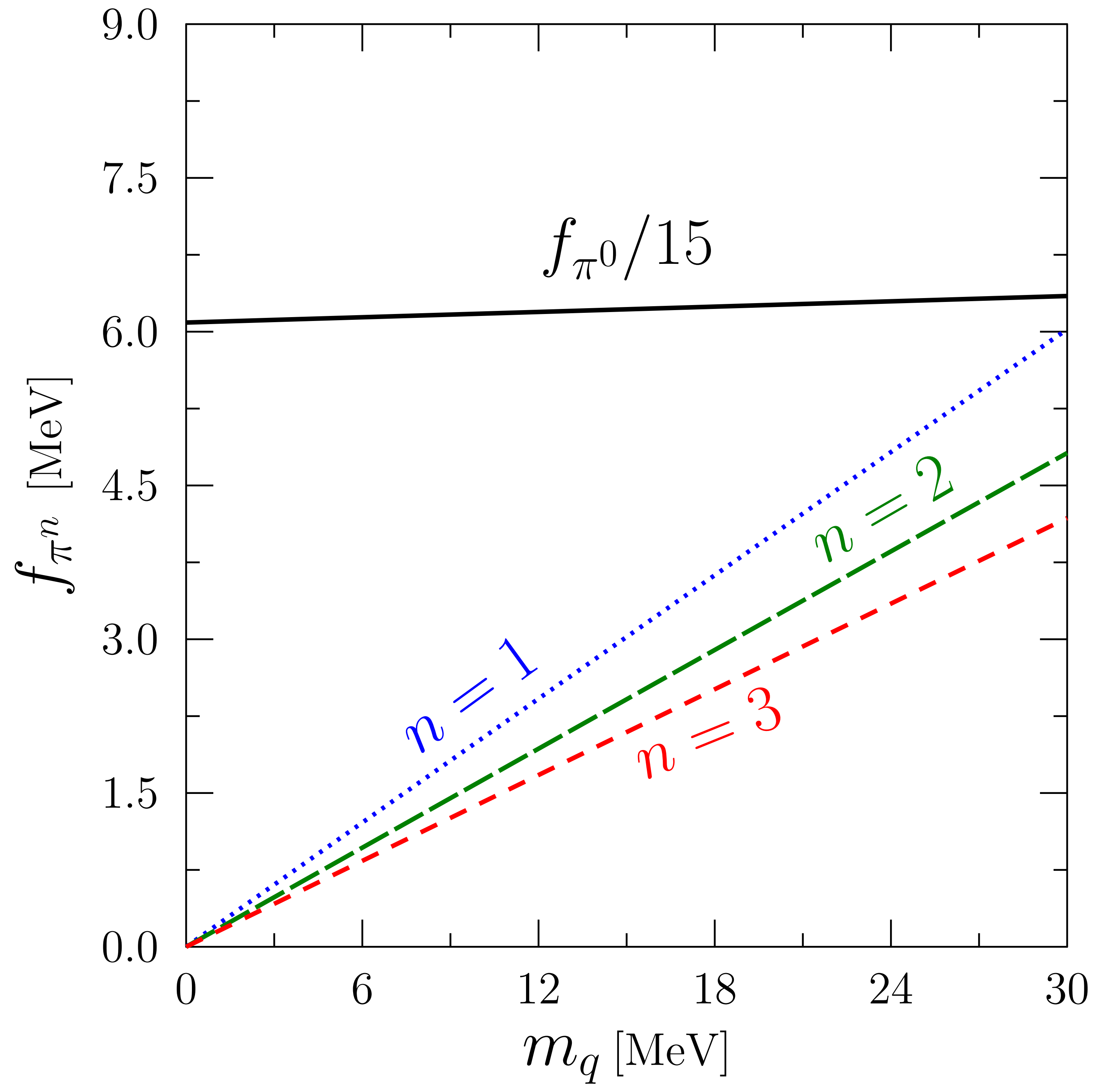
$$f_{\pi^n} = \frac{2\, m_q\, \rho_{\pi^n}}{m_{\pi^n}^2}$$



$$\rho^0_{\pi^n} := \lim_{m_q \to 0} \rho_{\pi^n} = \text{finite}$$



The leptonic decay constants of pion's excited states vanish in the chiral limit when DCSB



Comparison with Experiment

Our results

n	0	1	2	3
f_{π^n} (MeV)	92.4	1.68	1.34	1.16

Experiment

$$f_{\pi^1} < 0.064\, f_{\pi^0} = 5.91\,\mathrm{MeV}$$

Why does light-front holography fail in this case?

Light-front holography — pion wave function

$$|\Psi_h(P^+, \vec{P}_\perp)\rangle = \sum_{n,\lambda_i} \int [dx_i\, d^2\vec{k}_{\perp i}]\psi_{n/h}(x_i, \vec{k}_{\perp i}, \lambda_i) \times |n : x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}, \lambda_i\rangle$$

From Zee's book

Fock-space state of the pion truncated to a quark-antiquark pair — but Goldstone boson is a very collective state

Conclusions & Perspectives

— obtained agreement with QCD

— consistent with experimental bound for f_{π^1}

— but need to improve mass spectrum

Use of a soft-wall model