LEADING LOGARITHMS FOR THE NUCLEON MASS

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Overview

Main question: can we get information on high orders?

1. Leading logarithms (LL)
2. Weinberg's argument
3. $O(N+1)/O(N)$
   - Masses, decay
   - large N
   - Other work
4. $SU(N) \times SU(N)/SU(N)$
5. Nucleon
6. Conclusions

Work done with Alexey A. Vladimirov (nucleon) and Lisa Carloni, Stefan Lanz and Karol Kampf (various mesonic)

References

- JB, L. Carloni, Nucl.Phys. B827 (2010) 237-255 [arXiv:0909.5086] Leading Logarithms in the Massive O(N) Nonlinear Sigma Model
- JB, L. Carloni, Nucl.Phys. B843 (2011) 55-83 [arXiv:1008.3499] The Massive O(N) Non-linear Sigma Model at High Orders
- JB, K. Kampf, S. Lanz, Nucl.Phys. B860 (2012) 245-266 [arXiv:1201.2608] Leading logarithms in the anomalous sector of two-flavour QCD
- JB, K. Kampf, S. Lanz, Nucl.Phys. B873 (2013) 137-164 [arXiv:1303.3125] Leading logarithms in N-flavour mesonic Chiral Perturbation Theory
- JB, A.A. Vladimirov, Nucl. Phys. B91 (2015) 700-719 [arXiv:1409.6127], Leading logarithms for the nucleon mass.

Leading logarithms (LL)

BEWARE: leading logarithms can mean very different things

- Take a quantity with a single scale: $F(M)$
- Subtraction scale in QFT (in dim. reg.) is logarithmic
- $L = \log(\mu/M)$
- $F = F_0 + \left(F_1^1 L + F_0^1\right) + \left(F_2^2 L^2 + F_1^2 L + F_0^2\right) + \left(F_3^3 L^3 + \cdots\right) + \cdots$
- Leading Logarithms: The terms $F_m^m L^m$

The F_m^m can be more easily calculated than the full result

- $\mu\left(dF/d\mu\right) \equiv 0$
- Ultraviolet divergences in Quantum Field Theory are always local

Renormalizable theories

- Loop expansion $\equiv \alpha$ expansion
- $F = \alpha + \left(f_1^1\alpha^2 L + f_0^1\alpha^2\right) + \left(f_2^2\alpha^3 L^2 + f_1^2\alpha^3 L + f_0^2\alpha^3\right) + \left(f_3^3\alpha^4 L^3 + \cdots\right) + \cdots$
- f_i^j are pure numbers
- $\mu\frac{dF}{d\mu} \equiv F',\ \mu\frac{d\alpha}{d\mu} \equiv \alpha',\ \mu\frac{dL}{d\mu} = 1$
- $F' = \alpha' + \left(f_1^1\alpha^2 + f_1^1 2\alpha'\alpha L + f_0^1 2\alpha\alpha'\right) + \left(f_2^2\alpha^3 2L + f_2^2 3\alpha'\alpha^2 L^2 + f_1^2\alpha^3 + 3f_1^2\alpha'\alpha^2 L + 3f_0^2\alpha'\alpha^2\right) + \left(f_3^3\alpha^3 3L^2 + f_3^3 4\alpha'\alpha^3 L^3 + \cdots\right) + \cdots$
- $\alpha' = \beta_1\alpha^2 + \beta_2\alpha^3 + \cdots$
- $0 = F' = \left(\beta_1 + f_1^1\right)\alpha^2 + \left(2\beta_1 f_1^1 + 2f_2^2\right)\alpha^3 L + \left(\beta_2 + 2\beta_1 f_0^1 + f_1^2\right)\alpha^3 + \left(3\beta_1 f_2^2 + 3f_3^3\right)\alpha^4 L^2 + \cdots$
- $\boxed{f_1^1 = -\beta_1,\ f_2^2 = \beta_1^2,\ f_3^3 = -\beta_1^3, \ldots}$

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- $F = \alpha + \left(f_1^1\alpha^2 L + f_0^1\alpha^2\right) + \left(f_2^2\alpha^3 L^2 + f_1^2\alpha^3 L + f_0^2\alpha^3\right)$
  $+ \left(f_3^3\alpha^4 L^3 + \cdots\right) + \cdots$
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  $+ \left(f_3^3\alpha^3 3L^2 + f_3^3 4\alpha'\alpha^3 L^3 + \cdots\right) + \cdots$
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- $\boxed{f_1^1 = -\beta_1,\ f_2^2 = \beta_1^2,\ f_3^3 = -\beta_1^3, \ldots}$

Renormalizable theories

- Leading logs known as soon as β_1 is known
- $F(M) = \alpha\left(1 - \alpha\beta_1 L + (\alpha\beta_1 L)^2 + (\alpha\beta_1 L)^3 + \cdots\right) + \cdots$
  $= \dfrac{\alpha}{1 + \alpha\beta_1 L} + \cdots$
- running coupling constant
- Generalizes to lower leading logarithms as well
- Multiscale problems: many other terms possible

Renormalization Group

- Can be extended to other operators as well
- Underlying argument always $\mu\frac{dF}{d\mu} = 0$.
- Gell-Mann–Low, Callan–Symanzik, Weinberg–'t Hooft
- In great detail: J.C. Collins, *Renormalization*
- Relies on the α the same in all orders
- LL one-loop β_0
- NLL two-loop β_1

- In effective field theories: different Lagrangian at each order
- The recursive argument does not work

Weinberg's argument for leading logarithms

- Weinberg, Physica A96 (1979) 327
- Two-loop leading logarithms can be calculated using only one-loop: Weinberg consistency conditions
- $\pi\pi$ at 2-loop: Colangelo, hep-ph/9502285
- General at 2 loop: JB, Colangelo, Ecker, hep-ph/9808421
- Proof at all orders
  - using β-functions: Büchler, Colangelo, hep-ph/0309049
  - with diagrams: JB, Carloni, arXiv:0909.5086
  - Extension to nucleons: JB, Vladimirov, arXiv:1409.6127
- First mesonic case where

  loop-order = power-counting (or $\hbar$) order
- Later nucleon case where

  loop-order $\neq$ power-counting (or $\hbar$) order

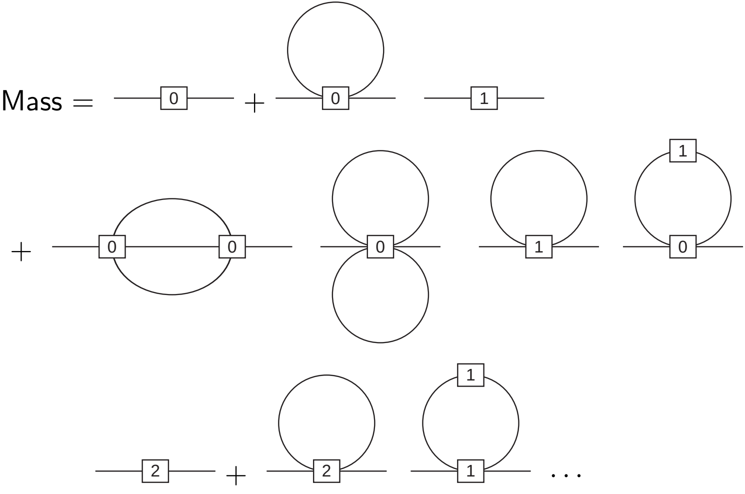
Weinberg's argument

- μ: dimensional regularization scale
- $d = 4 - w$
- loop-expansion $\equiv \hbar$-expansion
- $\mathcal{L}^{\text{bare}} = \sum_{n \geq 0} \hbar^n \mu^{-nw} \mathcal{L}^{(n)}$
- $\mathcal{L}^{(n)} = \sum_i c_i^{(n)} \mathcal{O}_i$
- $c_i^{(n)} = \sum_{k=0,n} \frac{c_{ki}^{(n)}}{w^k}$
- $c_{0i}^{(n)}$ have a direct μ-dependence
- $c_{ki}^{(n)}$ $k \geq 1$ only depend on μ through $c_{0i}^{(m<n)}$

Weinberg's argument

- L_ℓ^n ℓ-loop contribution at order $\hbar^n$
- Expand in divergences from the loops (not from the counterterms) $L_\ell^n = \sum_{k=0,l} \frac{1}{w^k} L_{k\ell}^n$
- Neglected positive powers: not relevant here, but should be kept in general
- $\{c\}_\ell^n$ all combinations $c_{k_1 j_1}^{(m_1)} c_{k_2 j_2}^{(m_2)} \dots c_{k_r j_r}^{(m_r)}$ with $m_i \geq 1$, such that $\sum_{i=1,r} m_i = n$ and $\sum_{i=1,r} k_i = \ell$.
- $\{c_n^n\} \equiv \{c_{ni}^{(n)}\}$, $\{c\}_2^2 = \{c_{2i}^{(2)}, c_{1i}^{(1)} c_{1k}^{(1)}\}$
- $\mathcal{L}^{(n)} =$ $\boxed{\text{n}}$

Weinberg's argument



LL

Weinberg's argument

$O(N+1)$ / $O(N)$

$SU(N)\times SU(N)$ / $SU(N)$

Nucleon

Conclusions

Weinberg's argument

- $\hbar^0$: L_0^0
- $\hbar^1$: $\frac{1}{w}\left(\mu^{-w}L_{00}^1(\{c\}_1^1) + L_{11}^1\right) + \mu^{-w}L_{00}^1(\{c\}_0^1) + L_{10}^1$
- Expand $\mu^{-w} = 1 - w\log\mu + \frac{1}{2}w^2\log^2\mu + \cdots$
- $1/w$ must cancel: $L_{00}^1(\{c\}_1^1) + L_{11}^1 = 0$
  this determines the c_{1i}^1
- Explicit $\log\mu$: $-\log\mu\, L_{00}^1(\{c\}_1^1) \equiv \log\mu\, L_{11}^1$

Weinberg's argument

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Weinberg's argument

- $\hbar^2$:
  $$\frac{1}{w^2}\left(\mu^{-2w}L^2_{00}(\{c\}^2_2)+\mu^{-w}L^2_{11}(\{c\}^1_1)+L^2_{22}\right)$$
  $$+\frac{1}{w}\left(\mu^{-2w}L^2_{00}(\{c\}^2_1)+\mu^{-w}L^2_{11}(\{c\}^1_0)+\mu^{-w}L^2_{10}(\{c\}^1_1)+L^2_{21}\right)$$
  $$+\left(\mu^{-2w}L^2_{00}(\{c\}^2_0)+\mu^{-w}L^2_{10}(\{c\}^1_0)+L^2_{20}\right)$$
- $1/w^2$ and $\log\mu/w$ must cancel
  $$L^2_{00}(\{c\}^2_2)+L^2_{11}(\{c\}^1_1)+L^2_{22}=0$$
  $$2L^2_{00}(\{c\}^2_2)+L^2_{11}(\{c\}^1_1)=0$$
- Solution: $L^2_{00}(\{c\}^2_2)=-\frac{1}{2}L^2_{11}(\{c\}^1_1)$
  $$L^2_{11}(\{c\}^1_1)=-2L^2_{22}$$
- Explicit $\log\mu$ dependence (one-loop is enough)
  $$\frac{1}{2}\log^2\mu\left(4L^2_{00}(\{c\}^2_2)+L^2_{11}(\{c\}^1_1)\right)=-\frac{1}{2}L^2_{11}(\{c\}^1_1)\log^2\mu\,.$$

Weinberg's argument

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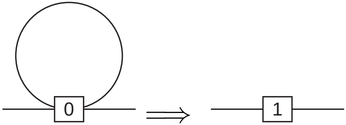
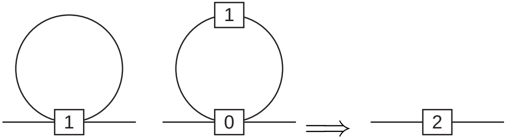
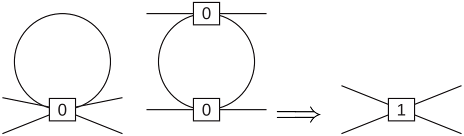
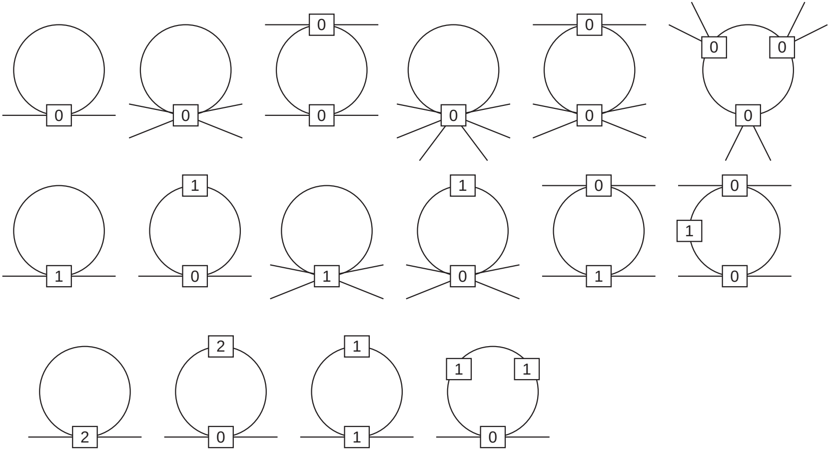
Mass to $\hbar^2$

- $\hbar^1$: 

- $\hbar^2$: 

- but also needs $\hbar^1$: 

Mass to order $\hbar^3$



LL

Weinberg's argument

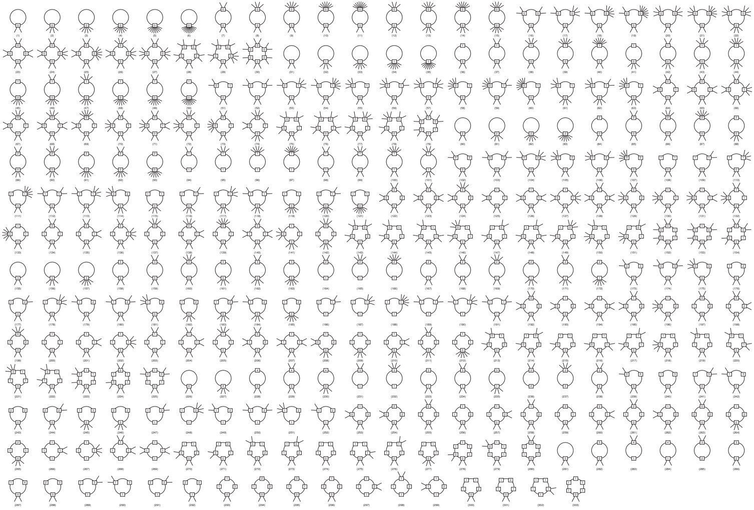
$O(N+1)$ / $O(N)$

$SU(N)\times SU(N)$ / $SU(N)$

Nucleon

Conclusions

Mass to order $\hbar^6$



General

- Calculate the divergence
- rewrite it in terms of a local Lagrangian
  - Luckily: symmetry kept: we know result will be symmetrical, hence do not need to explicitly rewrite the Lagrangians in a nice form
  - Luckily: we do not need to go to a minimal Lagrangian
  - So everything can be computerized
  - Thank Jos Vermaseren for FORM
- We keep all terms to have all 1PI (one particle irreducible) diagrams finite

Massive $O(N)$ sigma model

- $O(N+1)/O(N)$ nonlinear sigma model
- $\mathcal{L}_{n\sigma} = \frac{F^2}{2}\partial_\mu\Phi^T\partial^\mu\Phi + F^2\chi^T\Phi\,.$
- Φ is a real $N+1$ vector; $\Phi \to O\Phi$; $\Phi^T\Phi = 1$.
- Vacuum expectation value $\langle\Phi^T\rangle = (1\;0\ldots0)$
- Explicit symmetry breaking: $\chi^T = \left(M^2\;0\ldots0\right)$
- Both spontaneous and explicit symmetry breaking
- N-vector ϕ
- N (pseudo-)Nambu-Goldstone Bosons
- $N=3$ is two-flavour Chiral Perturbation Theory

Massive $O(N)$ sigma model: Φ vs ϕ

- $$\Phi_1 = \begin{pmatrix} \sqrt{1-\frac{\phi^T\phi}{F^2}} \\ \frac{\phi^1}{F} \\ \vdots \\ \frac{\phi^N}{F} \end{pmatrix} = \begin{pmatrix} \sqrt{1-\frac{\phi^T\phi}{F^2}} \\ \frac{\phi}{F} \end{pmatrix}$$ Gasser, Leutwyler

- $$\Phi_5 = \frac{1}{1+\frac{\phi^T\phi}{4F^2}} \begin{pmatrix} 1-\frac{\phi^T\phi}{2F^2} \\ \frac{\phi}{F} \end{pmatrix} \qquad \Phi_3 = \begin{pmatrix} 1-\frac{1}{2}\frac{\phi^T\phi}{F^2} \\ \sqrt{1-\frac{1}{4}\frac{\phi^T\phi}{F^2}}\,\frac{\phi}{F} \end{pmatrix}$$
  Weinberg — only mass term

- $$\Phi_4 = \begin{pmatrix} \cos\sqrt{\frac{\phi^T\phi}{F^2}} \\ \sin\sqrt{\frac{\phi^T\phi}{F^2}}\,\frac{\phi}{\sqrt{\phi^T\phi}} \end{pmatrix}$$ CCWZ

Massive $O(N)$ sigma model: Checks

Need (many) checks:

- use many different parametrizations
- compare with known results:

$$M^2_{phys} = M^2 \left(1 - \frac{1}{2} L_M + \frac{17}{8} L_M^2 + \cdots \right) ,$$

$$L_M = \frac{M^2}{16\pi^2 F^2} \log \frac{\mu^2}{\mathcal{M}^2}$$

Usual choice $\mathcal{M} = M$.

- large N but massive results more hidden
  Coleman, Jackiw, Politzer 1974
- JB, Carloni, : mass to 5 loops

Results

$$M^2_{\text{phys}} = M^2(1 + a_1 L_M + a_2 L_M^2 + a_3 L_M^3 + \ldots)$$

$$L_M = \frac{M^2}{16\pi^2 F^2} \log \frac{\mu^2}{M^2}$$

i	a_i, $N=3$	a_i for general N
1	$-\frac{1}{2}$	$1 - \frac{N}{2}$
2	$\frac{17}{8}$	$\frac{7}{4} - \frac{7N}{4} + \frac{5\ N^2}{8}$
3	$-\frac{103}{24}$	$\frac{37}{12} - \frac{113N}{24} + \frac{15\ N^2}{4} - N^3$
4	$\frac{24367}{1152}$	$\frac{839}{144} - \frac{1601\ N}{144} + \frac{695\ N^2}{48} - \frac{135\ N^3}{16} + \frac{231\ N^4}{128}$
5	$-\frac{8821}{144}$	$\frac{33661}{2400} - \frac{1151407\ N}{43200} + \frac{197587\ N^2}{4320} - \frac{12709\ N^3}{300} + \frac{6271\ N^4}{320} - \frac{7\ N^5}{2}$
6	$\frac{1922964667}{6220800}$	$158393809/3888000 - 182792131/2592000\ N$ $+1046805817/7776000\ N^2 - 17241967/103680\ N^3$ $+70046633/576000\ N^4 - 23775/512\ N^5 + 7293/1024\ N^6$

Results

$$F_{\text{phys}} = F(1 + a_1 L_M + a_2 L_M^2 + a_3 L_M^3 + \ldots)$$

i	a_i for $N=3$	a_i for general N
1	1	$-1/2 + 1/2\,N$
2	$-5/4$	$-1/2 + 7/8\,N - 3/8\,N^2$
3	$83/24$	$-7/24 + 21/16\,N - 73/48\,N^2 + 1/2\,N^3$
4	$-3013/288$	$47/576 + 1345/864\,N - 14077/3456\,N^2 + 625/192\,N^3$ $-105/128\,N^4$
5	$\frac{2060147}{51840}$	$-23087/64800 + 459413/172800\,N - 189875/20736\,N^2$ $+546941/43200\,N^3 - 1169/160\,N^4 + 3/2\,N^5$
6	$-\frac{69228787}{466560}$	$-277079063/93312000 + 1680071029/186624000\,N$ $-686641633/31104000\,N^2 + 813791909/20736000\,N^3$ $-128643359/3456000\,N^4 + 260399/15360\,N^5 - 3003/1024\,N^6$

Results

$$\langle \bar{q}_i q_i \rangle = -BF^2(1 + c_1 L_M + c_2 L_M^2 + c_3 L_M^3 + \ldots)$$

$$M^2 = 2B\hat{m} \qquad \chi^T = 2B(s\ 0\ \ldots 0)$$

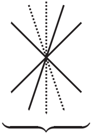
s corresponds to $\bar{u}u + \bar{d}d$ current

i	c_i for $N=3$	c_i for general N
1	$\frac{3}{2}$	$\frac{N}{2}$
2	$-\frac{9}{8}$	$\frac{3N}{4} - \frac{3N^2}{8}$
3	$\frac{9}{2}$	$\frac{3N}{2} - \frac{3N^2}{2} + \frac{N^3}{2}$
4	$-\frac{1285}{128}$	$\frac{145N}{48} - \frac{55N^2}{12} + \frac{105N^3}{32} - \frac{105N^4}{128}$
5	46	$\frac{3007N}{480} - \frac{1471N^2}{120} + \frac{557\ N^3}{40} - \frac{1191\ N^4}{160} + \frac{3\ N^5}{2}$

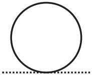
Anyone recognize any funny functions?

Large N

Power counting: pick $\mathcal{L}$ extensive in $N \Rightarrow F^2 \sim N,\ M^2 \sim 1$

- 

$\Leftrightarrow F^{2-2n} \sim \frac{1}{N^{n-1}}$


$\Leftrightarrow N$

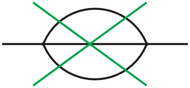
- 1PI diagrams:

$$\left.\begin{array}{l} N_L = N_I - \sum_n N_{2n} + 1 \\ 2N_I + N_E = \sum_n 2nN_{2n} \end{array}\right\} \Rightarrow N_L = \sum_n (n-1) N_{2n} - \frac{1}{2} N_E + 1$$

- diagram suppression factor: $\dfrac{N^{N_L}}{N^{N_E/2-1}}$

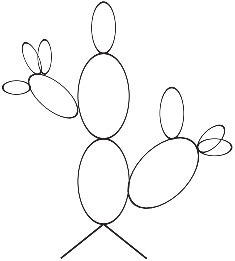
Large N

- diagrams with shared lines are suppressed



each new loop needs also a new flavour loop

- in the large N limit only "*cactus*" diagrams survive:



LL

Weinberg's argument

$O(N+1)/O(N)$

Masses, decay

large N

Other work

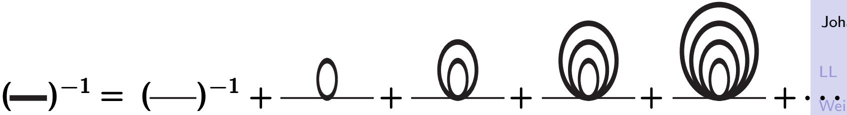
$SU(N)\times SU(N)/SU(N)$

Nucleon

Conclusions

large N: propagator

Generate recursively via a Gap equation



$\Rightarrow$ resum the series and look for the pole

$$M^2 = M^2_{\text{phys}}\sqrt{1+\frac{N}{F^2}\overline{A}(M^2_{\text{phys}})}$$

$$\overline{A}(m^2) = \frac{m^2}{16\pi^2}\log\frac{\mu^2}{m^2}.$$

Solve recursively, agrees with other result

Note: can be done for all parametrizations

LL
Weinberg's argument
$O(N+1)/O(N)$
Masses, decay
large N
Other work
$SU(N)\times SU(N)/SU(N)$
Nucleon
Conclusions

Other results

- Bissegger, Fuhrer, hep-ph/0612096 Dispersive methods, massless Π_S to five loops
- Kivel, Polyakov, Vladimirov, 0809.3236, 0904.3008, 1004.2197, 1105.4990
  - In the massless case tadpoles vanish
  - hence the number of external legs needed does not grow
  - All 4-meson vertices via Legendre polynomials
  - can do divergence of all one-loop diagrams analytically
  - algebraic (but quadratic) recursion relations
  - massless $\pi\pi$, F_V and F_S to arbitrarily high order
  - large N agrees with Coleman, Wess, Zumino
  - large N is not a good approximation

$SU(N) \times SU(N)/SU(N)$

- $SU(N) \times SU(N)/SU(N)$ (vector and scalar)
- Mass, Decay constants, Form-factors
- Meson-Meson, $\gamma\gamma \to \pi\pi$
- No luck with guess for general N-dependence either
- Four different parametrizations of a unitary matrix used

Mass

i	a_i for $N=2$	a_i for $N=3$	a_i for general N
1	$-1/2$	$-1/3$	$-N^{-1}$
2	$17/8$	$27/8$	$9/2\,N^{-2} - 1/2 + 3/8\,N^2$
3	$-103/24$	$-3799/648$	$-89/3\,N^{-3} + 19/3\,N^{-1} - 37/24\,N - 1/12\,N^3$
4	$24367/1152$	$146657/2592$	$2015/8\,N^{-4} - 773/12\,N^{-2} + 193/18 + 121/288\,N^2$ $+41/72\,N^4$
5	$-8821/144$	$-\frac{27470059}{186624}$	$-38684/15\,N^{-5} + 6633/10\,N^{-3} - 59303/1080\,N^{-1}$ $-5077/1440\,N - 11327/4320\,N^3 - 8743/34560\,N^5$
6*	$\frac{1922964667}{6220800}$	$\frac{12902773163}{9331200}$	$7329919/240\,N^{-6} - 1652293/240\,N^{-4}$ $-4910303/15552\,N^{-2} + 205365409/972000$ $-69368761/7776000\,N^2 + 14222209/2592000\,N^4$ $+3778133/3110400\,N^6$

Nucleon Lagrangian

- We use the heavy-baryon approach, explicit powercounting
- LO Lagrangian is order p (mesons p^2):
  $\mathcal{L}^{(0)}_{N\pi} = \bar{N}\left(iv^\mu D_\mu + g_A S^\mu u_\mu\right) N$
- Partly checked with other schemes (IR-regularization)
- Propagator is order $1/p$ (mesons $1/p^2$)
- Loops add p^2 just as for mesons
- Different parametrizations for mesons

Nucleon Lagrangian

Two different p^2 Lagrangians:

- Bernard-Kaiser-Meißner

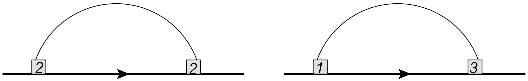
$$\mathcal{L}^{(1)}_{\pi N} = \bar{N}_v \Big[\frac{(v\cdot D)^2 - D\cdot D - i g_A\{S\cdot D, v\cdot u\}}{2M} + c_1 \mathrm{tr}\left(\chi_+\right) + \left(c_2 - \frac{g_A^2}{8M}\right)(v\cdot u)^2 + c_3 u\cdot u + \left(c_4 + \frac{1}{4M}\right) i\epsilon^{\mu\nu\rho\sigma} u_\mu u_\nu v_\rho S_\sigma \Big] N_v$$

- Ecker-Mojžiš

$$\mathcal{L}^{(1)}_{N\pi} = \frac{1}{M}\bar{N}\Big[-\frac{1}{2}\left(D_\mu D^\mu + i g_A\{S_\mu D^\mu, v_\nu u^\nu\}\right) + A_1 \mathrm{tr}\left(u_\mu u^\mu\right) + A_2 \mathrm{tr}\left((v_\mu u^\mu)^2\right) + A_3 \mathrm{tr}\left(\chi_+\right) + A_5 i\epsilon^{\mu\nu\rho\sigma} v_\mu S_\nu u_\rho u_\sigma \Big] N$$

Nucleon loops

- set $\hbar^n \sim p^{n+1}$ for meson-nucleon
- set $\hbar^n \sim p^{n+2}$ for mesons
- Introduce a RGO renormalization group order $\approx$ max power of $1/w$
- same p-order can be different RGO, e.g.



both p^5, left RGO 1, right RGO 2

- Note: same equations, if no tree level contribution next-to-leading log also calculable
- For nucleon can have fractional powers of quark masses

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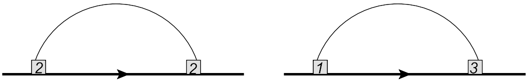

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Results

M: nucleon mass, m: pion mass, $L = \frac{m^2}{(4\pi F)^2} \log \frac{\mu^2}{m^2}$

$$
\begin{aligned}
M_{\text{phys}} &= M + k_2 \frac{m^2}{M} + k_3 \frac{\pi m^3}{(4\pi F)^2} + k_4 \frac{m^4}{(4\pi F)^2 M} \ln \frac{\mu^2}{m^2} \\
&\quad + k_5 \frac{\pi m^5}{(4\pi F)^4} \ln \frac{\mu^2}{m^2} + \ldots \\
&= M + \frac{m^2}{M} \sum_{n=1}^{\infty} k_{2n} L^{n-1} + \pi m \frac{m^2}{(4\pi F)^2} \sum_{n=1}^{\infty} k_{2n+1} L^{n-1},
\end{aligned}
$$

Results

k_2	$-4c_1 M$
k_3	$-\frac{3}{2}g_A^2$
k_4	$\frac{3}{4}\left(g_A^2+(c_2+4c_3-4c_1)M\right)-3c_1 M$
k_5	$\frac{3g_A^2}{8}\left(3-16g_A^2\right)$
k_6	$-\frac{3}{4}\left(g_A^2+(c_2+4c_3-4c_1)M\right)+\frac{3}{2}c_1 M$
k_7	$g_A^2\left(-18g_A^4+\frac{35g_A^2}{4}-\frac{443}{64}\right)$
k_8	$\frac{27}{8}\left(g_A^2+(c_2+4c_3-4c_1)M\right)-\frac{9}{2}c_1 M$
k_9	$\frac{g_A^2}{3}\left(-116g_A^6+\frac{2537g_A^4}{20}-\frac{3569g_A^2}{24}+\frac{55609}{1280}\right)$
k_{10}	$-\frac{257}{32}\left(g_A^2+(c_2+4c_3-4c_1)M\right)+\frac{257}{32}c_1 M$
k_{11}	$\frac{g_A^2}{2}\left(-95g_A^8+\frac{5187407g_A^6}{20160}-\frac{449039g_A^4}{945}+\frac{16733923g_A^2}{60480}-\frac{298785521}{1935360}\right)$

- $g_A \leftrightarrow -g_A$: only even powers
- k_{2n} peculiar structure
- Drop g_A^3 then can calculate k_{12}

Results

r_2	$-4c_1 M$
r_3	$-\frac{3}{2}g_A^2$
r_4	$\frac{3}{4}\left(g_A^2 + (c_2 + 4c_3 - 4c_1)M\right) - 5c_1 M$
r_5	$-6g_A^2$
r_6	$5c_1 M$
r_7	$\frac{g_A^2}{4}\left(-8 + 5g_A^2 - 72g_A^4\right)$
r_8	$\frac{25}{3}c_1 M$
r_9	$\frac{g_A^2}{3}\left(-116g_A^6 + \frac{647g_A^4}{20} - \frac{457g_A^2}{12} + \frac{17}{40}\right)$
r_{10}	$\frac{725}{36}c_1 M$
r_{11}	$\frac{g_A^2}{2}\left(95g_A^8 - \frac{1679567g_A^6}{20160} + \frac{451799g_A^4}{3780} - \frac{320557g_A^2}{15120} + \frac{896467}{60480}\right)$
r_{12}	$\frac{175}{4}c_1 M$

- everything rewritten in terms of physical pion mass
- Simpler expression

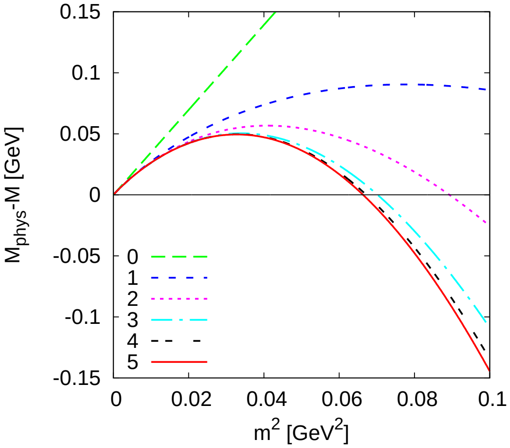
Results

- Conjecture:

$$M = M_{\rm phys} + \frac{3}{4} m^4_{\rm phys} \frac{\log \frac{\mu^2}{m^2_{\rm phys}}}{(4\pi F)^2} \left(\frac{g_A^2}{M_{\rm phys}} - 4c_1 + c_2 + 4c_3 \right)$$
$$- \frac{3c_1}{(4\pi F)^2} \int_{m^2_{\rm phys}}^{\mu^2} m^4_{\rm phys}(\mu') \, \frac{d\mu'^2}{\mu'^2}.$$

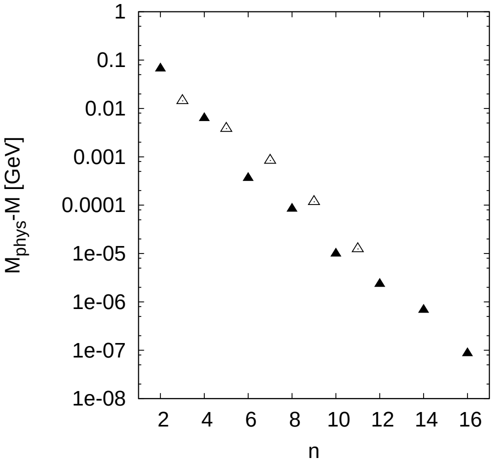
- Take now known result for pion mass, k_{14} and k_{16} calculable

Numerics



$$
\begin{aligned}
M &= 938\ \text{MeV},\\
c_1 &= -0.87\ \text{GeV}^{-1}\\
c_2 &= 3.34\ \text{GeV}^{-1}\\
c_3 &= -5.25\ \text{GeV}^{-1}\\
g_A &= 1.25\\
\mu &= 0.77\ \text{GeV}\\
F &= 92.4\ \text{MeV}
\end{aligned}
$$

Numerics



1
0.1
0.01
0.001
0.0001
1e-05
1e-06
1e-07
1e-08
M_{phys}-M [GeV]
2
4
6
8
10
12
14
16
n

Conclusions

- Leading logarithms can be calculated using only one-loop diagrams
- Results for a large number of quantities for mesons
- Look at the (very) many tables, we would be very interested in all-order conjectures
- Nucleonmass as the first result in the nucleon sector