Vector meson extended sigma model with Polyakov loops at finite temperature

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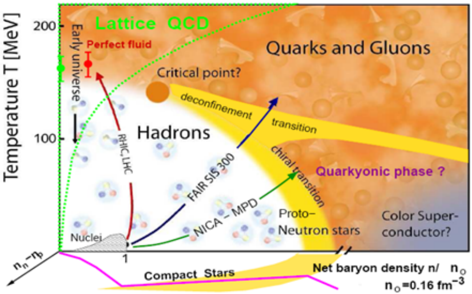
Overview

QCD phase diagram

Phase diagram in the $T - \mu_\mathrm{B} - \mu_\mathrm{I}$ space



- At $\mu_\mathrm{B} = 0$ $T_c = 151$ MeV Y. Aoki,*et al.*, PLB **643**, 46 (2006)
- Is there a CP? (T_{CP}=162 MeV, μ_{CP}=360 MeV, Fodor-Katz)
- At $T = 0$ in μ_B where is the phase boundary?
- Behaviour as a function of $\mu_\mathrm{I}/\mu_\mathrm{S}$?

Details of the phase diagram are heavily studied theoretically (Lattice, EFT), and experimentally (RHIC, LHC, FAIR, NICA)

Addressed problems

- Which scalars are the chiral partners of the pseudoscalar nonet?
- Which parameterizations give phenomenologically good description of the phase transition?
- Which of them predict the existence of the CP?
- What is the order of phase transition on the T=0 line?
- How the order parameters behave at finite temperature/chemical potential?
- How the masses change in medium?

Efective models

Since QCD is very hard to solve $\longrightarrow$ low energy effective models were set up $\longrightarrow$ reflecting the global symmetries of QCD

- Nambu-Jona-Lasinio model (+Kobayashi-Maskawa)
- Chiral perturbation theory
- Linear and nonlinear (it does not contain degrees of freedom relevant at high T) sigma model

Chiral symmetry

If the quark masses are zero (chiral limit) $\Longrightarrow$ QCD invariant under the following global transformation (chiral symmetry):

$U(3)_L \times U(3)_R \simeq U(3)_V \times U(3)_A =$
$SU(3)_V \times SU(3)_A \times U(1)_V \times U(1)_A$

$U(1)_V$ term $\longrightarrow$ baryon number conservation

$U(1)_A$ term $\longrightarrow$ broken through axial anomaly

$SU(3)_A$ term $\longrightarrow$ broken down by any quark mass

$SU(3)_V$ term $\longrightarrow$ broken down to $SU(2)_V$ if $m_u = m_d \neq m_s$
$\longrightarrow$ totally broken if $m_u \neq m_d \neq m_s$ (realized in nature)

Meson fields - pseudoscalar and scalar meson nonets

$$\Phi_{PS} = \sum_{i=0}^{8} \pi_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\eta_N+\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N-\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix} (\sim \bar{q}_i \gamma_5 q_j)$$

$$\Phi_S = \sum_{i=0}^{8} \sigma_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\sigma_N+a_0^0}{\sqrt{2}} & a_0^+ & K_S^+ \\ a_0^- & \frac{\sigma_N-a_0^0}{\sqrt{2}} & K_S^0 \\ K_S^- & \bar{K}_S^0 & \sigma_S \end{pmatrix} (\sim \bar{q}_i q_j)$$

Particle content:

Pseudoscalars: $\pi(138), K(495), \eta(548), \eta'(958)$

Scalars: $a_0(980 \text{ or } 1450), K_0^\star(800 \text{ or } 1430)$,

(σ_N, σ_S) : 2 of $f_0(500, 980, 1370, 1500, 1710)$

Structure of scalar mesons

	Mass (MeV)	width (MeV)	decays
$A_0(980)$	980 ± 20	$50 - 100$	$\pi\pi$ dominant
$A_0(1450)$	1474 ± 19	265 ± 13	$\pi\eta$, $\pi\eta'$, $K\bar{K}$
$K_s(800) = \kappa$	682 ± 29	547 ± 24	$K\pi$
$K_s(1430)$	1425 ± 50	270 ± 80	$K\pi$ dominant
$f_0(500) = \sigma$	400–550	$400 - 700$	$\pi\pi$ dominant
$f_0(980)$	980 ± 20	$40 - 100$	$\pi\pi$ dominant
$f_0(1370)$	1200–1500	$200 - 500$	$\pi\pi \approx 250$, $K\bar{K} \approx 150$
$f_0(1500)$	1505 ± 6	109 ± 7	$\pi\pi \approx 38$, $K\bar{K} \approx 9.4$
$f_0(1710)$	1722 ± 6	135 ± 7	$\pi\pi \approx 30$, $K\bar{K} \approx 71$

Possible scalar states: $\bar{q}q$, $\bar{q}q\bar{q}q$, meson-meson molecules, glueballs

pseudoscalar nonet: π, K, η, η', scalar nonet: A_0, K_0, 2 f_0

multiquark states: $f_0(980)$,$A_0(980)$ $f_0(600)$, $K_0(800)$???

meson-meson bound state $(K\bar{K})$: $f_0(980)$???

glueballs: $f_0(1500)$ (weak coupling to $\gamma\gamma$), $f_0(1710)$???

Included fields - vector meson nonets

$$V^\mu = \sum_{i=0}^{8} \rho_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N+\rho^0}{\sqrt{2}} & \rho^+ & K^{\star+} \\ \rho^- & \frac{\omega_N-\rho^0}{\sqrt{2}} & K^{\star 0} \\ K^{\star-} & \bar{K}^{\star 0} & \omega_S \end{pmatrix}^\mu$$

$$A^\mu = \sum_{i=0}^{8} b_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{f_{1N}+a_1^0}{\sqrt{2}} & a_1^+ & K_1^+ \\ a_1^- & \frac{f_{1N}-a_1^0}{\sqrt{2}} & K_1^0 \\ K_1^- & \bar{K}_1^0 & f_{1S} \end{pmatrix}^\mu$$

Particle content:

Vector mesons: $\rho(770), K^\star(894), \omega_N = \omega(782), \omega_S = \phi(1020)$

Axial vectors: $a_1(1230), K_1(1270), f_{1N}(1280), f_{1S}(1426)$

Lagrangian (2/1)

$$\begin{aligned}
\mathcal{L}_{\text{Tot}} &= \text{Tr}[(D_\mu\Phi)^\dagger(D_\mu\Phi)] - m_0^2\text{Tr}(\Phi^\dagger\Phi) - \lambda_1[\text{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2\text{Tr}(\Phi^\dagger\Phi)^2 \\
&- \frac{1}{4}\text{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) + \text{Tr}\left[\left(\frac{m_1^2}{2} + \Delta\right)(L_\mu^2 + R_\mu^2)\right] + \text{Tr}[H(\Phi + \Phi^\dagger)] \\
&+ c_1(\det\Phi + \det\Phi^\dagger) + i\frac{g_2}{2}(\text{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \text{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\
&+ \frac{h_1}{2}\text{Tr}(\Phi^\dagger\Phi)\text{Tr}(L_\mu^2 + R_\mu^2) + h_2\text{Tr}[(L_\mu\Phi)^2 + (\Phi R_\mu)^2] + 2h_3\text{Tr}(L_\mu\Phi R^\mu\Phi^\dagger). \\
&+ g_3[\text{Tr}(L_\mu L_\nu L^\mu L^\nu) + \text{Tr}(R_\mu R_\nu R^\mu R^\nu)] + g_4[\text{Tr}\left(L_\mu L^\mu L_\nu L^\nu\right) \\
&+ \text{Tr}\left(R_\mu R^\mu R_\nu R^\nu\right)] + g_5\text{Tr}\left(L_\mu L^\mu\right)\text{Tr}\left(R_\nu R^\nu\right) + g_6[\text{Tr}(L_\mu L^\mu)\text{Tr}(L_\nu L^\nu) \\
&+ \text{Tr}(R_\mu R^\mu)\text{Tr}(R_\nu R^\nu)] \\
&+ \bar{\Psi}i\partial\!\!\!/\Psi - g_F\bar{\Psi}\left(\Phi_S + i\gamma_5\Phi_{PS}\right)\Psi + g_V\bar{\Psi}\gamma^\mu\left(V_\mu + \frac{g_A}{g_V}\gamma_5 A_\mu\right)\Psi
\end{aligned}$$

+ Polyakov loops

D. Parganlija, P. Kovacs, Gy. Wolf, F. Giacosa, D.H. Rischke, Phys. Rev. D87 (2013) 014011

Lagrangian (2/2)

where

$$D^\mu\Phi = \partial^\mu\Phi - ig_1(L^\mu\Phi - \Phi R^\mu) - ieA_e^\mu[T_3,\Phi]$$

$$\Phi = \sum_{i=0}^{8}(\sigma_i + i\pi_i)T_i, \quad H = \sum_{i=0}^{8} h_i T_i \qquad T_i : U(3) \text{ generators}$$

$$R^\mu = \sum_{i=0}^{8}(\rho_i^\mu - b_i^\mu)T_i, \quad L^\mu = \sum_{i=0}^{8}(\rho_i^\mu + b_i^\mu)T_i$$

$$L^{\mu\nu} = \partial^\mu L^\nu - ieA_e^\mu[T_3, L^\nu] - \{\partial^\nu L^\mu - ieA_e^\nu[T_3, L^\mu]\}$$

$$R^{\mu\nu} = \partial^\mu R^\nu - ieA_e^\mu[T_3, R^\nu] - \{\partial^\nu R^\mu - ieA_e^\nu[T_3, R^\mu]\}$$

$$\bar{\Psi} = (\bar{u}, \bar{d}, \bar{s})$$

non strange – strange base:

$$\varphi_N = \sqrt{2/3}\varphi_0 + \sqrt{1/3}\varphi_8,$$

$$\varphi_S = \sqrt{1/3}\varphi_0 - \sqrt{2/3}\varphi_8, \qquad \varphi \in (\sigma_i, \pi_i, \rho_i^\mu, b_i^\mu, h_i)$$

broken symmetry: non-zero condensates $\langle\sigma_N\rangle, \langle\sigma_S\rangle \longleftrightarrow \phi_N, \phi_S$

Symmetry properties of the model

Global $U(3)_L \times U(3)_R$ transformation:

$$\begin{aligned} \Phi &\rightarrow U_L \Phi U_R^\dagger \\ L^\mu &\rightarrow U_L L^\mu U_L^\dagger \\ R^\mu &\rightarrow U_R R^\mu U_R^\dagger \end{aligned}$$

Consequences (using the unitarity of U's):

$$\begin{aligned} D^\mu \Phi &\rightarrow U_L D^\mu \Phi U_R^\dagger \\ L^{\mu\nu} &\rightarrow U_L L^{\mu\nu} U_L^\dagger \\ R^{\mu\nu} &\rightarrow U_R R^{\mu\nu} U_R^\dagger \end{aligned}$$

$$(\mathrm{Tr}(\Phi^\dagger \Phi))' = \mathrm{Tr}(U_R \Phi^\dagger U_L^\dagger U_L \Phi U_R^\dagger) = \mathrm{Tr}(U_R \Phi^\dagger \Phi U_R^\dagger) = \mathrm{Tr}(\Phi^\dagger \Phi U_R^\dagger U_R) =$$

All terms are invariant except the determinant and the explicit symmetry breaking term.

Spontaneous symmetry breaking

Interaction is approximately chiral symmetric, spectra not, so SSB:

$$\sigma_{N/S} \to \sigma_{N/S} + \phi_{N/S} \qquad \phi_{N/S} \equiv <\sigma_{N/S}>$$

For tree level masses we have to select all terms quadratic in the new fields. Some of the terms include mixings arising from terms like $\mathrm{Tr}[(D_\mu \Phi)^\dagger (D_\mu \Phi)]$:

$$\begin{aligned}
\pi_N - a^\mu_{1N} &: \quad -g_1 \phi_N a^\mu_{1N} \partial_\mu \pi_N, \\
\pi - a^\mu_1 &: \quad -g_1 \phi_N (a^{\mu +}_1 \partial_\mu \pi^- + a^{\mu 0}_1 \partial_\mu \pi^0) + \mathrm{h.c.}, \\
\pi_S - a^\mu_{1S} &: \quad -\sqrt{2} g_1 \phi_S a^\mu_{1S} \partial_\mu \pi_S, \\
K_S - K^\star_\mu &: \quad \frac{i g_1}{2} (\sqrt{2}\phi_S - \phi_N)(\bar{K}^{\star 0}_\mu \partial^\mu K^0_S + K^{\star -}_\mu \partial^\mu K^+_S) + \mathrm{h.c.}, \\
K - K^\mu_1 &: \quad -\frac{g_1}{2} (\phi_N + \sqrt{2}\phi_S)(K^{\mu 0}_1 \partial_\mu \bar{K}^0 + K^{\mu +}_1 \partial_\mu K^-) + \mathrm{h.c.}.
\end{aligned}$$

Determination of the parameters of the Lagrangian

16 unknown parameters $(m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_S, \Phi_N, \Phi_S, g_F, g_V, g_A) \longrightarrow$ Determined by the min. of χ^2:

$$\chi^2(x_1, \ldots, x_N) = \sum_{i=1}^{M} \left[\frac{Q_i(x_1, \ldots, x_N) - Q_i^{\mathrm{exp}}}{\delta Q_i} \right]^2 ,$$

where $(x_1, \ldots, x_N) = (m_0, \lambda_1, \lambda_2, \ldots)$, $Q_i(x_1, \ldots, x_N)$ calculated from the model, while Q_i^{exp} taken from the PDG

multiparametric minimalization $\longrightarrow$ MINUIT

- PCAC $\rightarrow$ 2 physical quantities: f_π, f_K
- Tree-level masses $\rightarrow$ 16 physical quantities: $m_{u/d}, m_s, m_\pi, m_\eta, m_{\eta'}, m_K, m_\rho, m_\Phi, m_{K^\star}, m_{a_1}, m_{f_1^H}, m_{K_1}, m_{a_0}, m_{K_s}, m_{f_0^L}, m_{f_0^H}$
- Decay widths $\rightarrow$ 12 physical quantities: $\Gamma_{\rho\to\pi\pi}, \Gamma_{\Phi\to KK}, \Gamma_{K^\star\to K\pi}, \Gamma_{a_1\to\pi\gamma}, \Gamma_{a_1\to\rho\pi}, \Gamma_{f_1\to KK^\star}, \Gamma_{a_0}, \Gamma_{K_S\to K\pi}, \Gamma_{f_0^L\to\pi\pi}, \Gamma_{f_0^L\to KK}, \Gamma_{f_0^H\to\pi\pi}, \Gamma_{f_0^H\to KK}$

Results

	Cal(GeV)	Mass		Cal(GeV)	Width
m_π	0.1383	0.1380	f_π	0.0923	0.0922
m_K	0.5060	0.4956	f_K	0.1099	0.1100
m_η	0.5280	0.5479	$\Gamma_{f_0 L \to KK}$	0.1437	0.150
$m_{\eta'}$	0.9651	0.9578	$\Gamma_{f_0 H \to KK}$	0.158	0.0714
m_ρ	0.7672	0.7755	Γ_ρ	0.1694	0.149
m_ϕ	1.0152	1.0195	Γ_ϕ	0.001535	0.00177
m_{K*}	0.9006	0.8938	Γ_{K*}	0.044194	0.0462
$m_{f_1 H}$	1.4083	1.4264	$\Gamma_{f_1 \to KK}$	0.0438	0.0439
m_{a_1}	1.1829	1.2300	$\Gamma_{A_1 \to \rho\pi}$	0.656	0.425
m_{K_1}	1.2999	1.2720	$\Gamma_{A_1 \to \gamma\pi}$	0.000705	0.000640
m_{a_0}	1.4467	1.4740	Γ_{A_0}	0.254	0.265
m_{K_s}	1.5390	1.4250	Γ_{K_s}	0.363	0.270
$m_{f_0 L}$	1.303	1.3700	$\Gamma_{f_0 L \to \pi\pi}$	0.172	0.250
$m_{f_0 H}$	1.5865	1.7200	$\Gamma_{f_0 H \to \pi\pi}$	0.032744	0.0297
m_{ud}	0.292	0.314	m_s	0.314	0.500

Parameters

Parameter	Value
ϕ_N [GeV]	0.1622
ϕ_S [GeV]	0.1262
C_1 [GeV^2]	-0.7537
C_2 [GeV^2]	0.3953
λ_1	undetermined
λ_2	65.3221
h_1	undetermined
h_2	11.6586
h_3	4.7028
δ_S [GeV^2]	0.1534
c_1 [GeV]	1.12
g_1	-5.8943
g_2	-2.9960
g_F	4.9429

- with this set $f_0^L = 1303$ GeV
- by setting $\lambda_1 \to f_0^L$ mass can be lowered

Polyakov loops in Polyakov gauge

Polyakov loop variables: $\Phi(\vec{x}) = \frac{\mathrm{Tr}_c L(\vec{x})}{N_c}$ and $\bar{\Phi}(\vec{x}) = \frac{\mathrm{Tr}_c \bar{L}(\vec{x})}{N_c}$ with

$$L(x) = \mathcal{P} \exp \left[i \int_0^\beta d\tau A_4(\vec{x}, \tau) \right]$$

$\longrightarrow$ signals center symmetry ($\mathbb{Z}_3$) breaking at the deconfinement

low T: confined phase, $\langle \Phi(\vec{x}) \rangle, \langle \bar{\Phi}(\vec{x}) \rangle = 0$
high T: deconfined phase, $\langle \Phi(\vec{x}) \rangle, \langle \bar{\Phi}(\vec{x}) \rangle \neq 0$

Polyakov gauge: the temporal component of the gauge field is time independent and can be gauge rotated to a diagonal form in the color space

Effects of the gauge fields:

- In this gauge the effect of the gauge field on the quarks acts like an imaginary chemical potential
  $\longrightarrow$ modified quark distribution function.
- Polyakov potential: $\mathcal{U}\left(\Phi, \bar{\Phi}\right)$ models the free energy of a pure gauge theory, parameters are fitted to the pure gauge lattice data

T/μ_B dependence of the Polyakov-loops

$$\frac{\partial\Omega}{\partial\Phi} = \frac{\partial\Omega}{\partial\bar{\Phi}}\Big|_{\varphi_N=\phi_N,\varphi_S=\phi_S} = 0, \qquad \Omega : \text{grand canonical potential}$$

$$-\quad \frac{d}{d\Phi}\left(\frac{U(\Phi,\bar{\Phi})}{T^4}\right) + \frac{2N_c}{T^3}\sum_{q=u,d,s}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\left(\frac{e^{-\beta E_q^-(p)}}{g_q^-(p)} + \frac{e^{-2\beta E_q^+(p)}}{g_q^+(p)}\right) = 0$$

$$-\quad \frac{d}{d\bar{\Phi}}\left(\frac{U(\Phi,\bar{\Phi})}{T^4}\right) + \frac{2N_c}{T^3}\sum_{q=u,d,s}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\left(\frac{e^{-\beta E_q^+(p)}}{g_q^+(p)} + \frac{e^{-2\beta E_q^-(p)}}{g_q^-(p)}\right) = 0$$

$$g_q^+(p) \quad = \quad 1 + 3\left(\bar{\Phi} + \Phi e^{-\beta E_q^+(p)}\right)e^{-\beta E_q^+(p)} + e^{-3\beta E_q^+(p)}$$

$$g_q^-(p) \quad = \quad 1 + 3\left(\Phi + \bar{\Phi} e^{-\beta E_q^-(p)}\right)e^{-\beta E_q^-(p)} + e^{-3\beta E_q^-(p)}$$

$$E_q^\pm(p) = E_q(p) \mp \mu_B/3, \;\; E_{u/d}(p) = \sqrt{p^2 + m_{u/d}^2}, E_s(p) = \sqrt{p^2 + m_s^2}$$

T/μ_B dependence of the condensates ($\phi_{N/S}$)

$$\frac{\partial\Omega}{\partial\phi_N} = \frac{\partial\Omega}{\partial\phi_S}\Big|_{\Phi,\bar\Phi} = 0, \qquad \text{(after the SSB)}$$

Hybrid approach: fermions at one-loop, mesons at tree-level (their effects are much smaller)

$$m_0^2\phi_N + \left(\lambda_1 + \frac{1}{2}\lambda_2\right)\phi_N^3 + \lambda_1\phi_N\phi_S^2 - h_N + \frac{g_F}{2}N_c\left(\langle u\bar u\rangle_T + \langle d\bar d\rangle_T\right) = 0$$
$$m_0^2\phi_S + (\lambda_1 + \lambda_2)\phi_S^3 + \lambda_1\phi_N^2\phi_S - h_S + \frac{g_F}{\sqrt{2}}N_c\langle s\bar s\rangle_T = 0$$

$$\langle q\bar q\rangle_T = -4m_q\int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{2E_q(p)}\left(1 - f_\Phi^-(E_q(p)) - f_\Phi^+(E_q(p))\right)$$

Masses

$$M^2_{i,ab} = \frac{\partial^2 \Omega(T,\mu_f)}{\partial \varphi_{i,a} \partial \varphi_{i,b}}\bigg|_{\min} = m^2_{i,ab} + \Delta_0 m^2_{i,ab} + \Delta_T m^2_{i,ab},$$

$m^2_{i,ab} \longrightarrow$ tree-level mass matrix,
$\Delta_{0/T} m^2_{i,ab} \longrightarrow$ fermion vacuum/thermal fluctuation,

$$\Delta_0 m^2_{i,ab} = \frac{\partial^2 \Omega^{\mathrm{vac}}_{q\bar{q}}}{\partial \varphi_{i,a} \partial \varphi_{i,b}}\bigg|_{\min} = -\frac{3}{8\pi^2} \sum_{f=u,d,s} \left[\left(\frac{3}{2} + \log \frac{m_f^2}{M^2} \right) m^{2(i)}_{f,a} m^{2(i)}_{f,b} + m_f^2 \left(\frac{1}{2} + \log \frac{m_f^2}{M^2} \right) m^{2(i)}_{f,ab} \right],$$

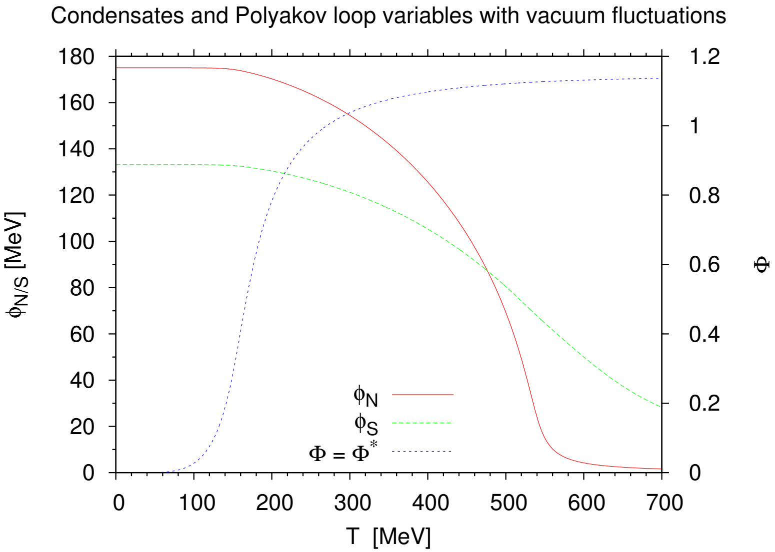
$$\Delta_T m^2_{i,ab} = \frac{\partial^2 \Omega^{\mathrm{th}}_{q\bar{q}}}{\partial \varphi_{i,a} \partial \varphi_{i,b}}\bigg|_{\min} = 6 \sum_{f=u,d,s} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_f(p)} \left[(f_f^+(p) + f_f^-(p)) \left(m^{2(i)}_{f,ab} - \frac{m^{2(i)}_{f,a} m^{2(i)}_{f,b}}{2E_f^2(p)} \right) + (B_f^+(p) + B_f^-(p)) \frac{m^{2(i)}_{f,a} m^{2(i)}_{f,b}}{2TE_f(p)} \right],$$

where $m^{2(i)}_{f,a} \equiv \partial m_f^2/\partial \varphi_{i,a}$, $m^{2(i)}_{f,ab} \equiv \partial^2 m_f^2/\partial \varphi_{i,a} \partial \varphi_{i,b}$

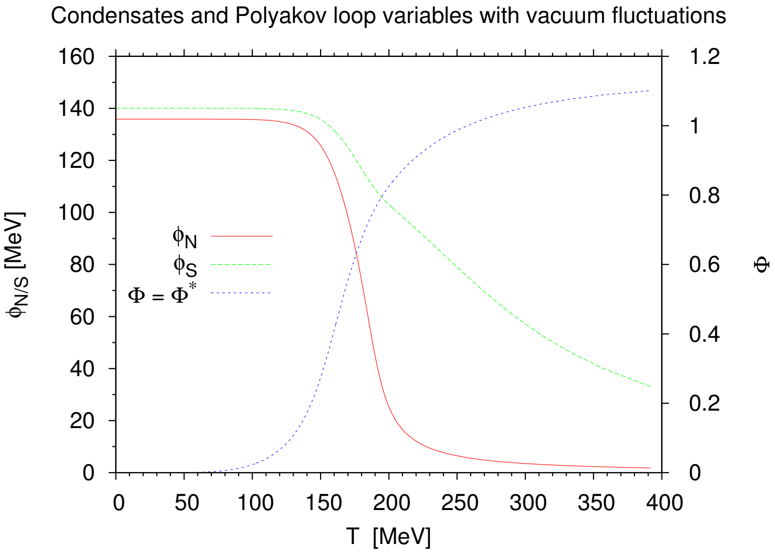
Features of our approach

- D.O.F's: scalar, pseudoscalar, vector, axial vector nonets,
- Polyakov loop variables,
- constituent quarks
- Four order parameters $(\phi_N, \phi_S, \Phi, \bar{\Phi}) \rightarrow$ four T/μ-dependent equations
- Fermion vacuum fluctuations
- Fermion thermal fluctuations

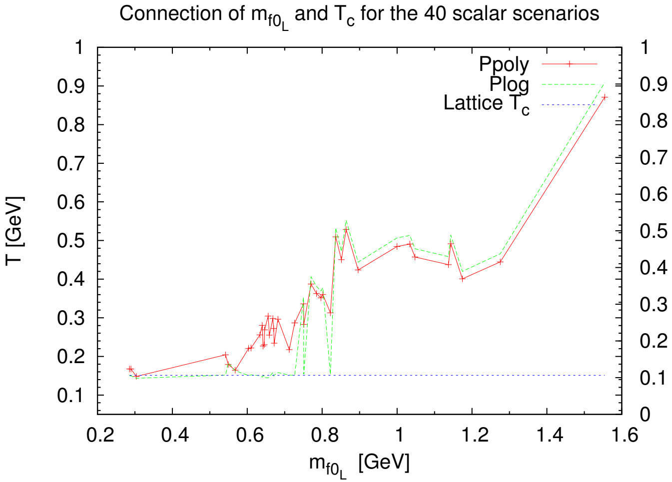
$\phi_{N/S}(T), \Phi/\bar{\Phi}(T)$ *with* Polyakov loop $m_{f_0^L} = 1326$ MeV



With low mass scalars, $m_{f_0^L} = 402$ MeV



T_c at $\mu_B = 0$ for various parameterizations

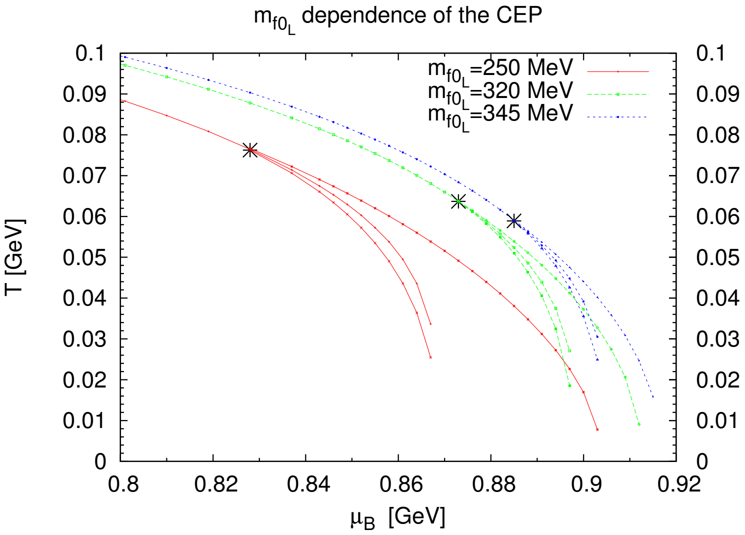


40 parameterizations $\longrightarrow$ all the possible combinations for the masses of the scalar sector

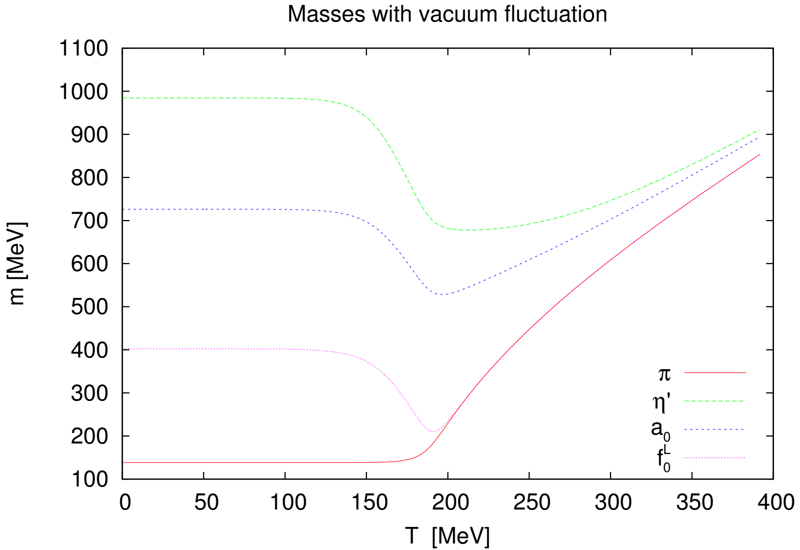
Important remarks

- In all 40 cases the best χ^2 solution was chosen
- Only parameterizations, which produced $m_{f_0^L} \lesssim 800$ MeV can have $T_c \approx 151$ MeV (lattice data)
- Only parameterizations, which produced $m_{f_0^L} \lesssim 400$ MeV can have 1st order transition in $\mu_B \implies$ there is CEP
- If $T_c \approx 150$ MeV and the CEP exists $\implies m_{a_0}$ and m_{K_S} is also below 1 GeV

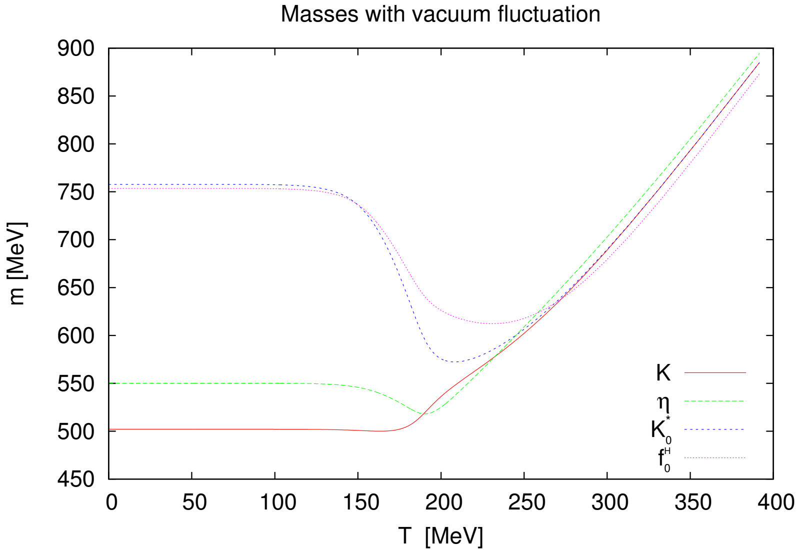
CEP for different f_0^L masses



π, η', a_0, f_0^L masses



K, η, $K^\star$, f_0^H masses



Summary

- An extended linear σ - model was shown with constituent quarks and Polyakov - loops
- The meson phenomenology was very well described by scalars above 1 GeV
- We used hybrid approach at $T = 0$: only fermion loops, since it has the largest contribution
- At finite T/μ_B there was 4 coupled equations for the 4 order parameters
- The phase transition temperature requires low mass f_0

→ To do . . .

→ Improve the vacuum phenomenology by tetraquarks (and glueballs)

Thank you for your attention!