QNP 2015

Valparaiso, Chile: March 2-6, 2015.

TRANSVERSE MOMENTUM DISTRIBUTIONS

Hrayr Matevosyan

SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

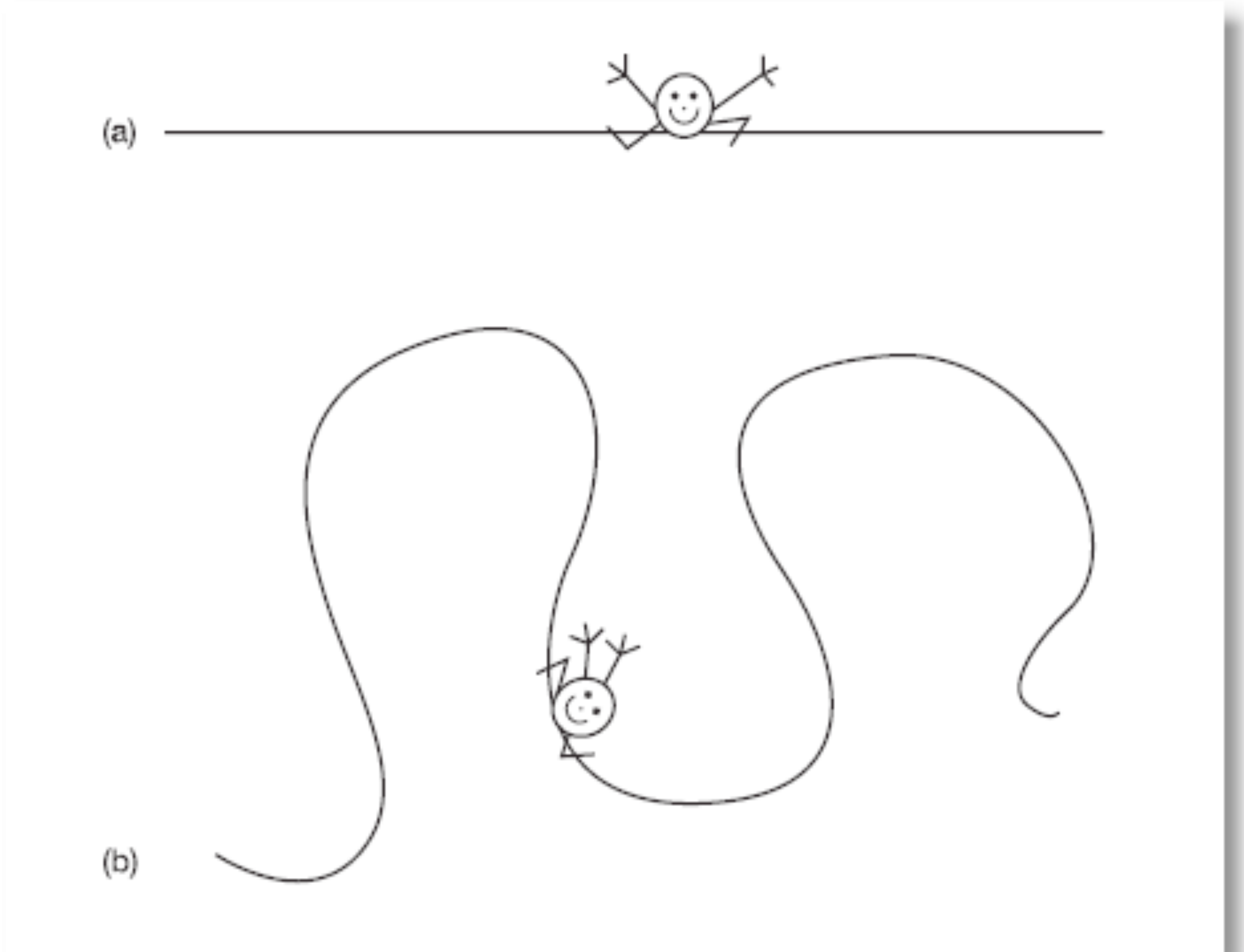
CoEPP
ARC Centre of Excellence for Particle Physics at the Terascale

THE UNIVERSITY of ADELAIDE
SUB CRUCE LUMEN

OUTLOOK

- *What are TMD PDFs?*
- *How to Access Them Experimentally?*
- *Empirical Extractions.*
- *Sivers Effect in Two Hadron SIDIS and Modified Full Event Generators.*
- *Conclusions.*

Covers only a small fraction of topics! Apologies for not mentioning a particular work, e.g. **Talk by A. Szabelski**

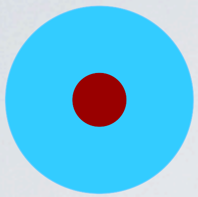


COLLINEAR PDFs

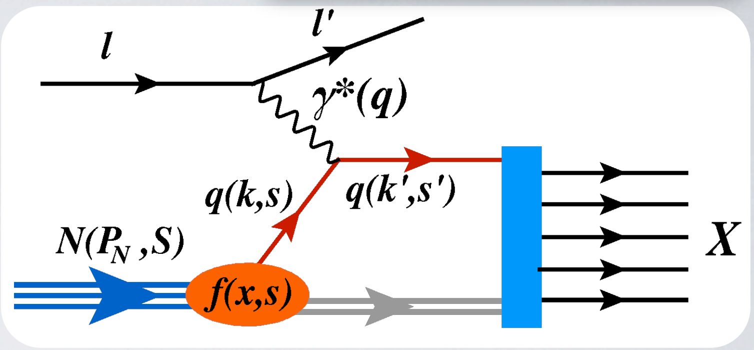
NUCLEON PARTON DISTRIBUTION FUNCTIONS

Talk by E. Nocera

- *Unpolarized* quark in *Unpolarized* nucleon.

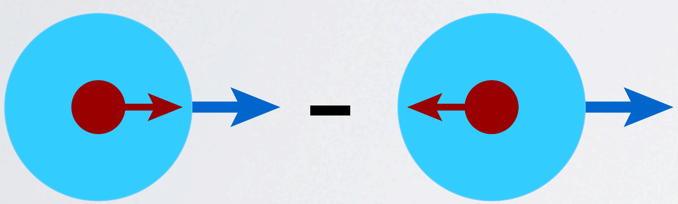


$$f_1^q(x, Q^2)$$



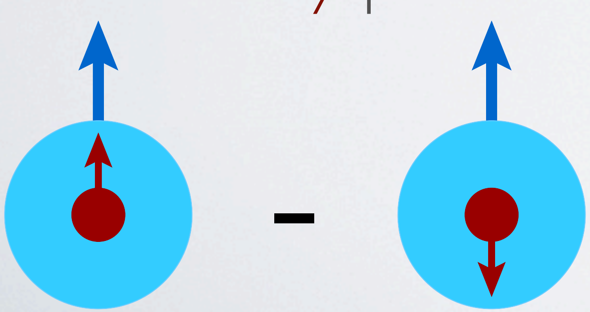
- ***The momentum and the spin of the partons are correlated with the polarization of the nucleon!***

- *Longitudinally* polarized quark in *Longitudinally* polarized nucleon.



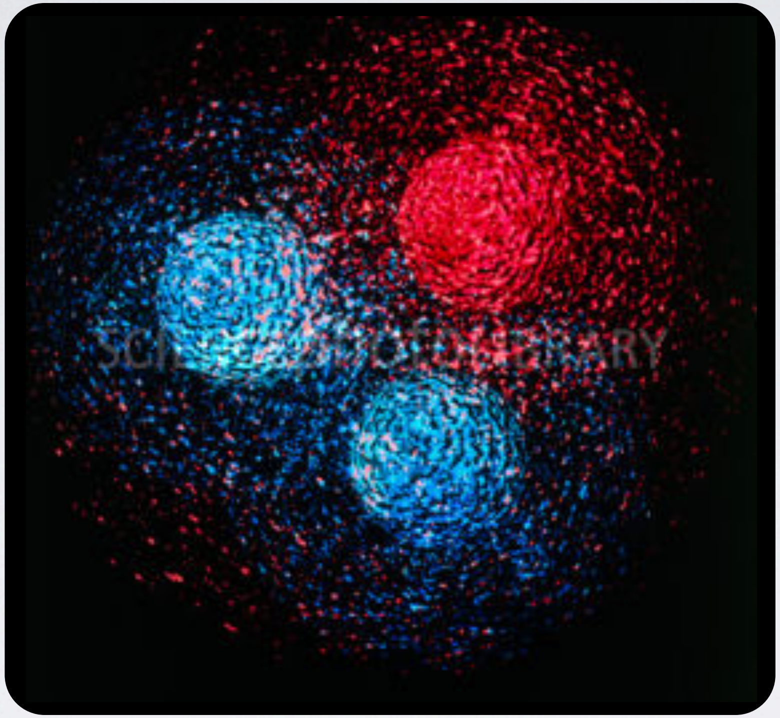
$$g_{1L}^q(x, Q^2)$$

- *Transversely* polarized quark in *Transversely* polarized nucleon.



$$h_{1T}^q(x, Q^2)$$

Chiral-odd: Suppressed in Inclusive DIS



3-D PICTURE OF NUCLEON:

TRANSVERSE MOMENTUM DEPENDENCE

PDFS WITH TRANSVERSE MOMENTUM DEPENDENCE

Talk by F.-X. Girod

- ***TMDs: Momentum Space***
- ***GPDs: Impact Parameter***
- ***The transverse momentum (TM) of the parton can couple with both its own spin and the spin of the nucleon!***
- ***Leading Order TMD PDFs***

N/q	U	L	T
U	f_1		$h_1^{\perp}$
L		g_{1L}	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	$g_{1T}^{\perp}$	$h_1 \, h_{1T}^{\perp}$

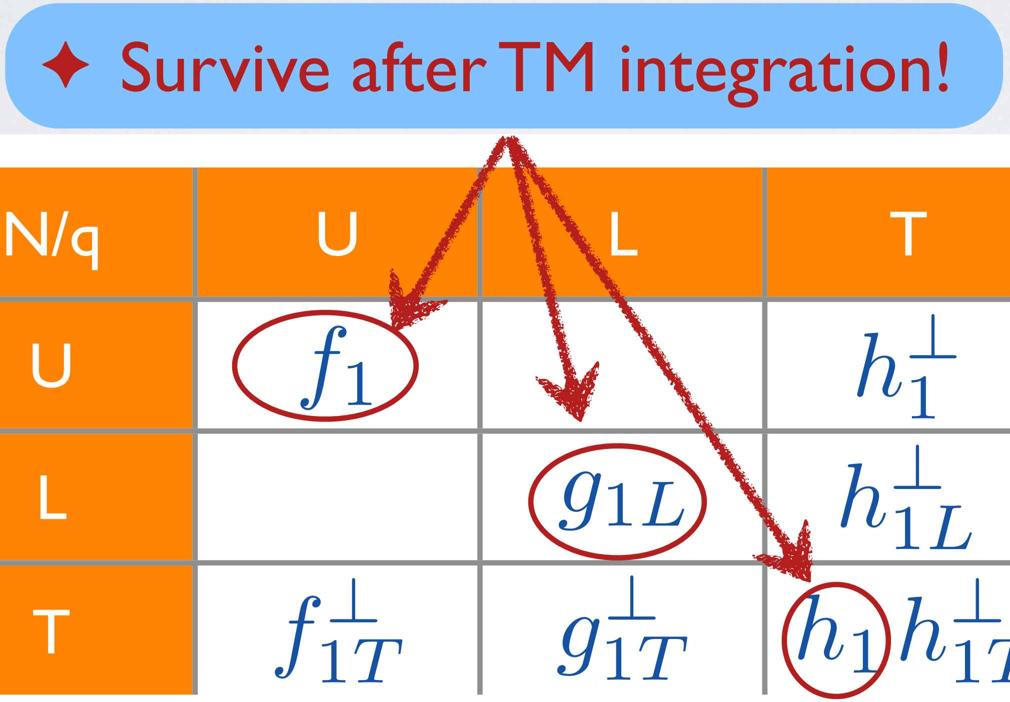
PDFS WITH TRANSVERSE MOMENTUM DEPENDENCE

Talk by F.-X. Girod

❖ ***TMDs: Momentum Space*** ❖ ***GPDs: Impact Parameter***

❖ ***The transverse momentum (TM) of the parton can couple with both its own spin and the spin of the nucleon!***

❖ ***Leading Order TMD PDFs***



N/q	U	L	T
U	f_1		$h_1^\perp$
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T	$f_{1T}^\perp$	$g_{1T}^\perp$	h_1 $h_{1T}^\perp$

PDFS WITH TRANSVERSE MOMENTUM DEPENDENCE

Talk by F.-X. Girod

❖ ***TMDs: Momentum Space*** ❖ ***GPDs: Impact Parameter***

❖ ***The transverse momentum (TM) of the parton can couple with both its own spin and the spin of the nucleon!***

❖ ***Leading Order TMD PDFs***

✦ Survive after TM integration!

N/q	U	L	T
U	f_1		$h_1^{\perp}$
L		g_{1L}	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	$g_{1T}^{\perp}$	h_1 $h_{1T}^{\perp}$

Boer-Mulders

Worm Gear

Pretzelocity

Sivers

Worm Gear

PDFs FROM QUARK-QUARK CORRELATOR

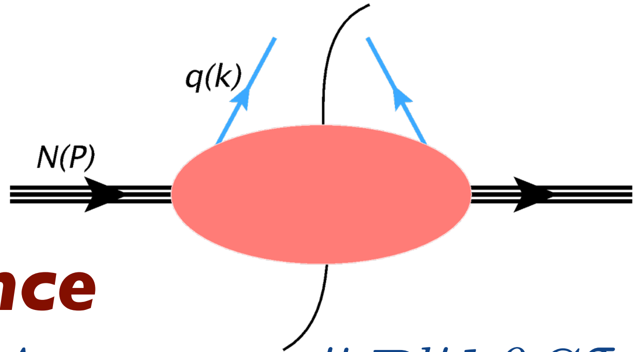
Mulders, Tangerman, NPB 461 197-237 (1996). Bacchetta et al., JHEP 0702, 093 (2007).

- **Quark-Quark Correlation Function**

$$\Phi_{ij}(P,S;k) = \frac{1}{(2\pi)^4}\int d^4x e^{ik\cdot x}\langle P,S|\bar{\psi}_j(0)\mathcal{L}[0,x;path]\psi_i(x)|P,S\rangle$$

- **Gauge Link**

$$\mathcal{L}[0,x;path] = \mathcal{P}exp\left(-ig\int_0^x ds^\mu A_\mu(s)\right)$$



- **Hermiticity of Fields, P and C invariance**

$$\Phi(P,S;k) = A_1 + A_2 \not{P} + A_3 \not{k} + \ldots + A_{12}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu P^\nu k^\rho S^\sigma$$

- **Traces of the correlator with a Dirac operators**

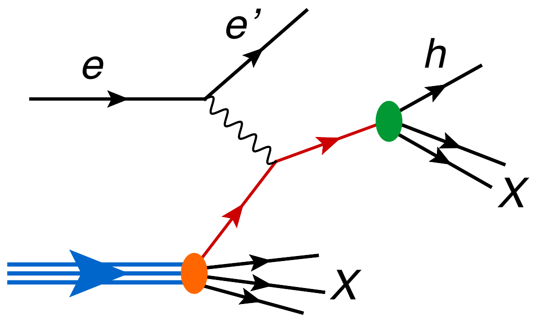
$$\Phi^{[\Gamma]}(x,\vec{k}_T) \equiv \frac{1}{2}\int dk^- \, Tr[\Phi\Gamma]|_{k^+=xP^+}$$

- **Leading-order in M/P^+ (twist two) expansion**

$$\Phi^{[\gamma^+]} = f_1 - \frac{\epsilon_T^{\rho\sigma}k_{T\rho}S_{T\sigma}}{M}f_{1T}^\perp \qquad \Phi^{[\gamma^+\gamma^5]} = S_L g_{1L} - \frac{\vec{k}_T\cdot\vec{S}_T}{M}g_{1T}$$

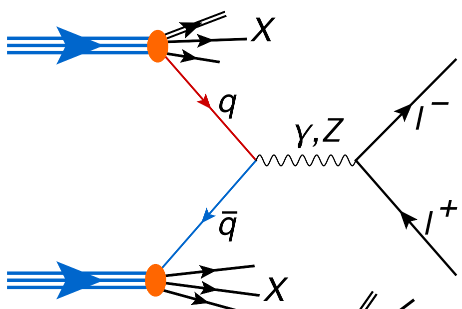
$$\Phi^{[i\sigma^{\alpha+}\gamma^5]} = S_T^\alpha h_1 + S_L\frac{k_T^\alpha}{M}h_{1L}^\perp - \frac{(k_T^\alpha k_T^\rho - \frac{1}{2}k_\perp^2 g_T^{\alpha\rho})S_T^\rho}{M^2}h_{1T}^\perp - \frac{\epsilon_T^{\alpha\rho}k_{T\rho}}{M}h_1^\perp$$

FACTORIZATION AND UNIVERSALITY



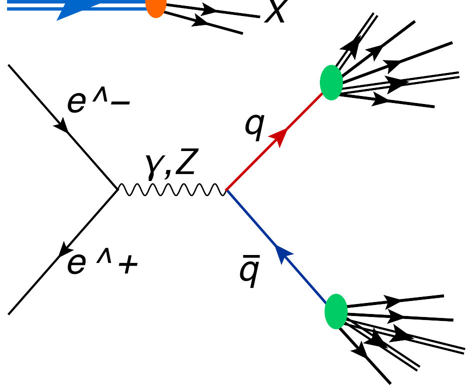
- SEMI INCLUSIVE DIS (SIDIS)

$$\sigma^{eP \to ehX} = \sum_q f_q^P \otimes \sigma^{eq \to eq} \otimes D_q^h$$



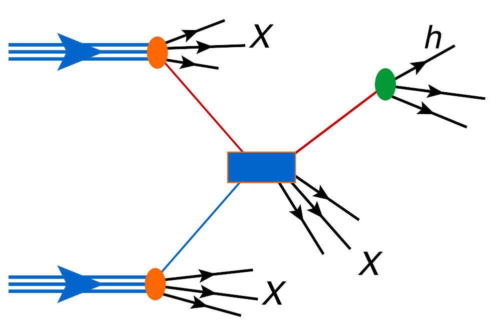
- DRELL-YAN (DY)

$$\sigma^{PP \to l^+ l^- X} = \sum_{q,q'} f_q^P \otimes f_{\bar{q}}^P \otimes \sigma^{q\bar{q} \to l^+ l^-}$$



- e^+e^-

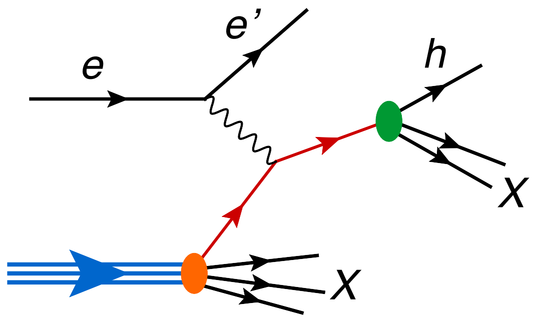
$$\sigma^{e^+e^- \to hX} = \sum_q \sigma^{e^+e^- \to q\bar{q}} \otimes (D_q^h + D_{\bar{q}}^h)$$



- Hadron Production

$$\sigma^{PP \to hX} = \sum_{q,q'} f_q^P \otimes f_{q'}^P \otimes \sigma^{qq' \to qq'} \otimes D_q^h$$

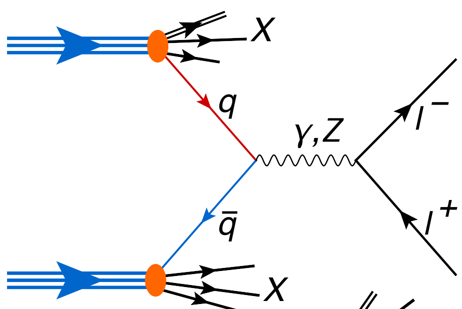
FACTORIZATION AND *UNIVERSALITY*



- SEMI INCLUSIVE DIS (SIDIS)

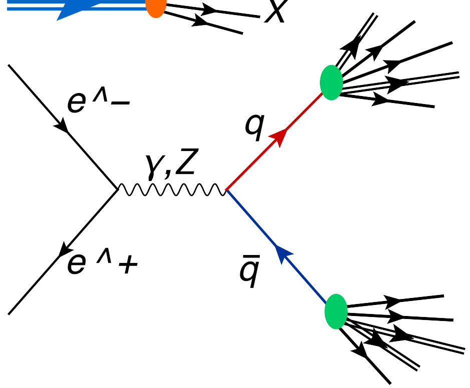
$$\sigma^{eP\to ehX} = \sum_{q} \cdots {}^{eq\to eq} \otimes D_q^h$$

TMD Factorization Proven



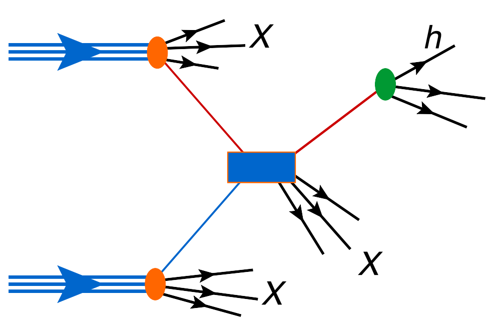
- DRELL-YAN (DY)

$$\sigma^{PP\to l^+l^-X} = \sum_{q,q'} f_q^P \otimes f_{\bar{q}} \cdots {}^{\to l^+l^-}$$



- e^+e^-

$$\sigma^{e^+e^-\to hX} = \sum_{q} \sigma^{e^+e^-\to q\bar{q}} \otimes (D_q^h + D_{\bar{q}}^h)$$



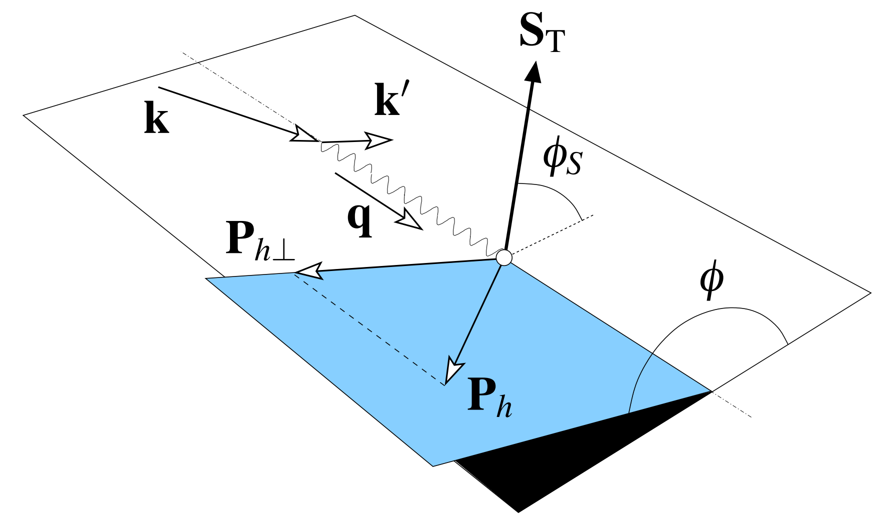
- Hadron Production

$$\sigma^{PP\to hX} = \sum_{q,q'} f_q^P \otimes f_{q'}^P \otimes \sigma^{qq'\to qq'} \otimes D_q^h$$

TMDs from SIDIS $e\,N \rightarrow e\,h\,X$

A. Bacchetta et al., JHEP08 023 (2008).

- **For polarized SIDIS cross-section there are *18 terms* in leading twist expansion:**



$$\frac{d\sigma}{dx\,dy\,dz\,d\phi_S\,d\phi_h\,dP_{h\perp}^2} \sim F_{UU,T} + \varepsilon F_{UU,L} + \ldots$$

$$+ |\boldsymbol{S}_\perp| \left[\sin(\phi_h - \phi_S) \left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\, F_{UT,L}^{\sin(\phi_h-\phi_S)} \right) + \varepsilon\, \sin(\phi_h+\phi_S)\, F_{UT}^{\sin(\phi_h+\phi_S)} + .. \right]$$

Sivers Effect **Collins Effect**

- Access the structure functions via *specific* modulations.
- LO Matching to *convolutions* of PDFs and FFs: $P_T^2 \ll Q^2$

$$F_{UU,T} \sim \mathcal{C}[f_1\ D_1] \qquad F_{UT,T}^{\sin(\phi_h-\phi_S)} \sim \mathcal{C}[f_{1T}^{\perp q}\ D_1]$$

$$F_{UU}^{\cos(2\phi_h)} \sim \mathcal{C}[\mathcal{G}(\vec{k_T},\vec{P}_T)\ h_1^\perp\ H_1^\perp] \qquad F_{UT}^{\sin(\phi_h+\phi_S)} \sim \mathcal{C}[h_1\ H_1^\perp]$$

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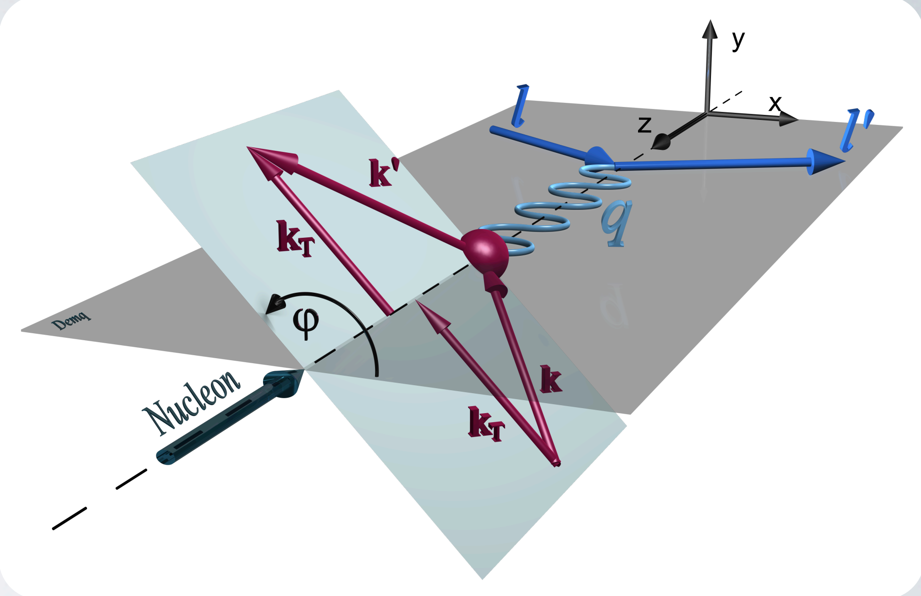

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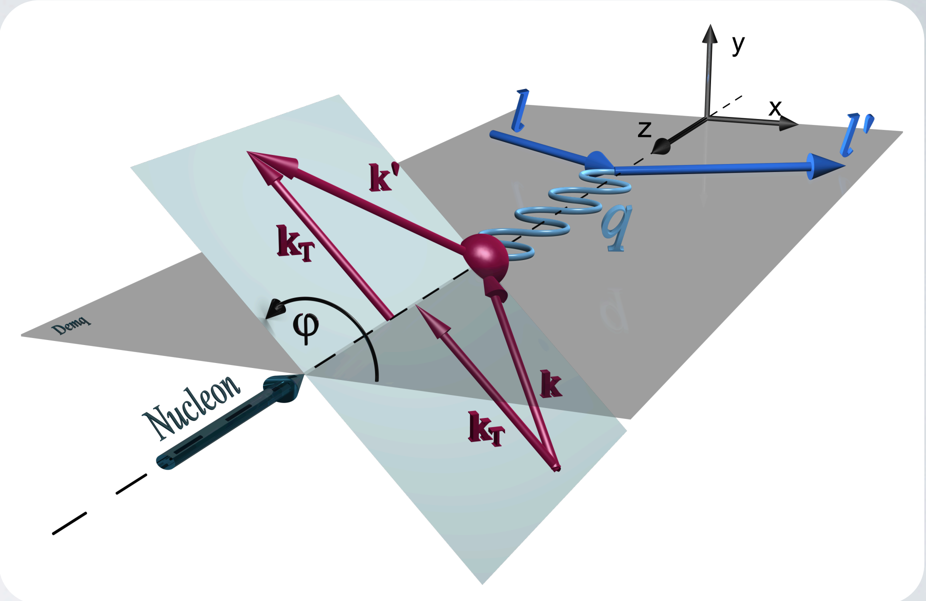
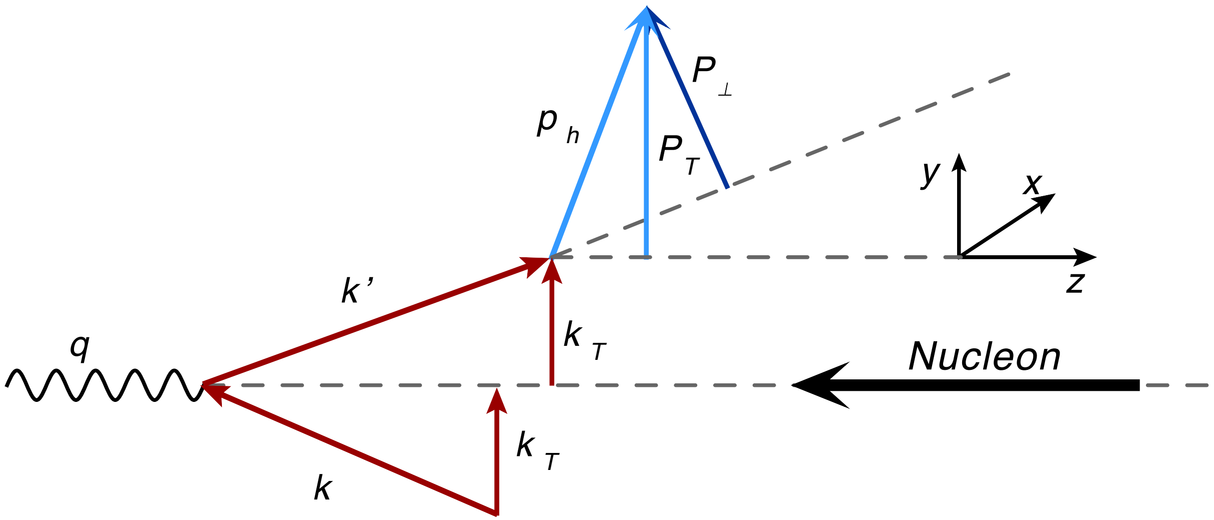


$$\mathcal{C}[f\ g\ \ldots] \equiv \sum_q e_q^2 \int d^2\vec{k}_T\ d^2\vec{P}_\perp\ f\ g\ \ldots\ \delta^2(\vec{P}_T - \vec{P}_\perp - z\vec{k}_T)$$

$$\int d^2\mathbf{k}_\perp f(x, k_\perp^2) = f(x)$$

$$\int d^2\mathbf{P}_\perp D(z, P_\perp^2) = D(z)$$

$$\vec{P}_T = \vec{P}_\perp + z\vec{k}_T$$

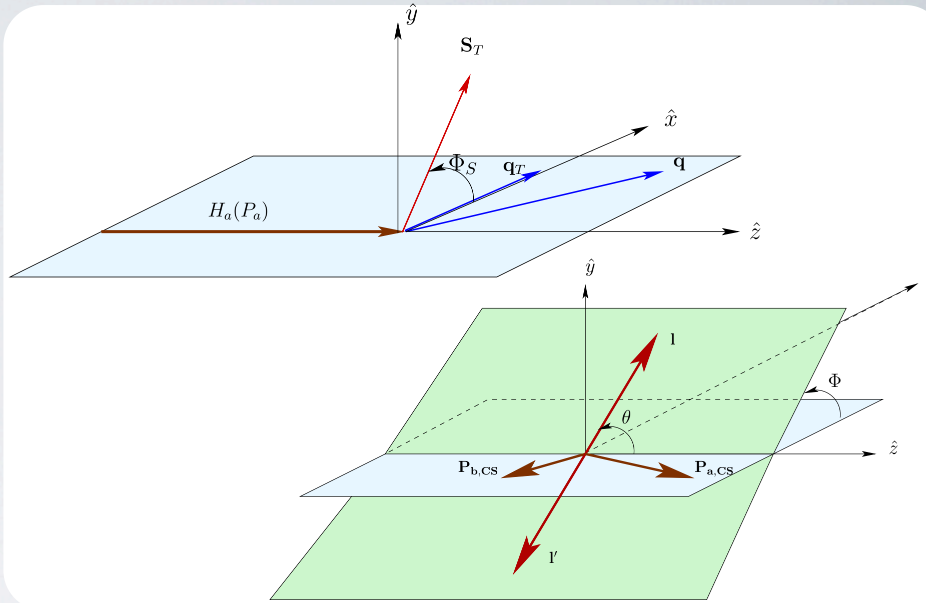


TMDs from DRELL-YAN

S. Arnold et al., PRD 79, 034005 (2009).

$$H_a + H_B \to \gamma^*(q) + X \to l^+ + l^- + X$$

✦**For polarized DY there are *48 terms* in the leading twist expansion.**



$$\frac{d\sigma}{d^4q\, d\Omega} \sim (1+\cos^2\theta)F^1_{UU} + (1-\cos^2\theta)F^2_{UU} + \sin 2\theta F^{\cos\phi}_{UU}\cos\phi + \sin^2\theta F^{\cos 2\phi}_{UU}\cos 2\phi$$
$$+ |\vec{S}_{aT}| \left[\sin\phi_a \left((1+\cos^2\theta)F^1_{TU} + (1-\cos^2\theta)F^2_{TU} + \sin 2\theta F^{\cos\phi}_{TU}\cos\phi + \sin^2\theta F^{\cos 2\phi}_{TU}\cos 2\phi \right) + \ldots \right] + \ldots$$

‣ LO Matching to *convolutions* of PDFs of both hadrons $q_T \ll Q$

$$F^1_{UU} = \mathcal{C}[f_1\ \bar{f}_1] \qquad F^1_{UT} = \mathcal{C}\left[\frac{\vec{q}\cdot\vec{k}_{bT}}{|\vec{q}|M_b} f_1\ \bar{f}^{\perp}_{1T} \right]$$

$$F^{\cos 2\phi}_{UU} = \mathcal{C}\left[\frac{2(\vec{q}\cdot\vec{k}_{aT})(\vec{q}\cdot\vec{k}_{aT})/q^2 - \vec{k}_{aT}\cdot\vec{k}_{bT}}{M_a M_b} h^{\perp}_1\ \bar{h}^{\perp}_1 \right] \qquad F^{\sin(2\phi-\phi_b)}_{UT} = -\mathcal{C}\left[\frac{\vec{q}\cdot\vec{k}_{aT}}{|\vec{q}|M_b} h^{\perp}_1\ \bar{h}_1 \right]$$

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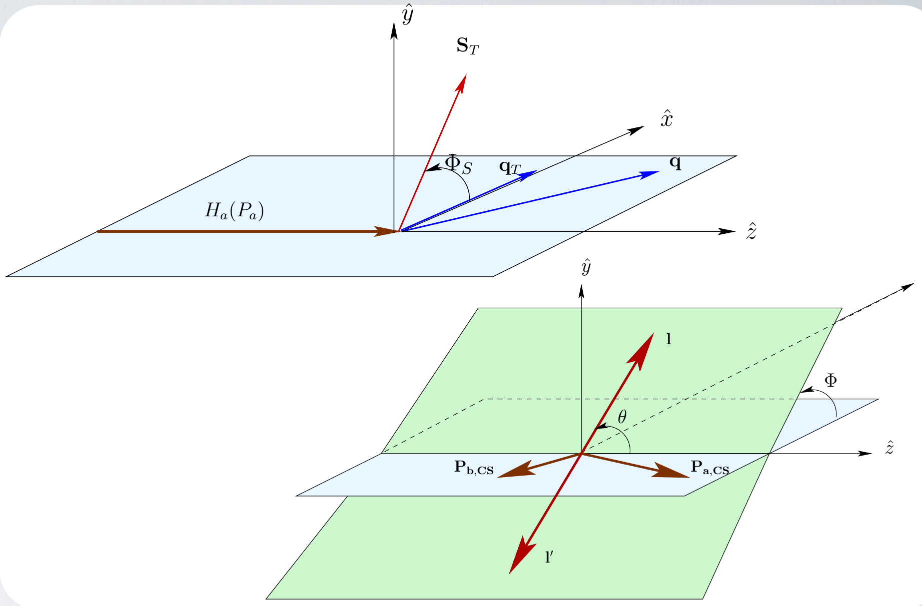
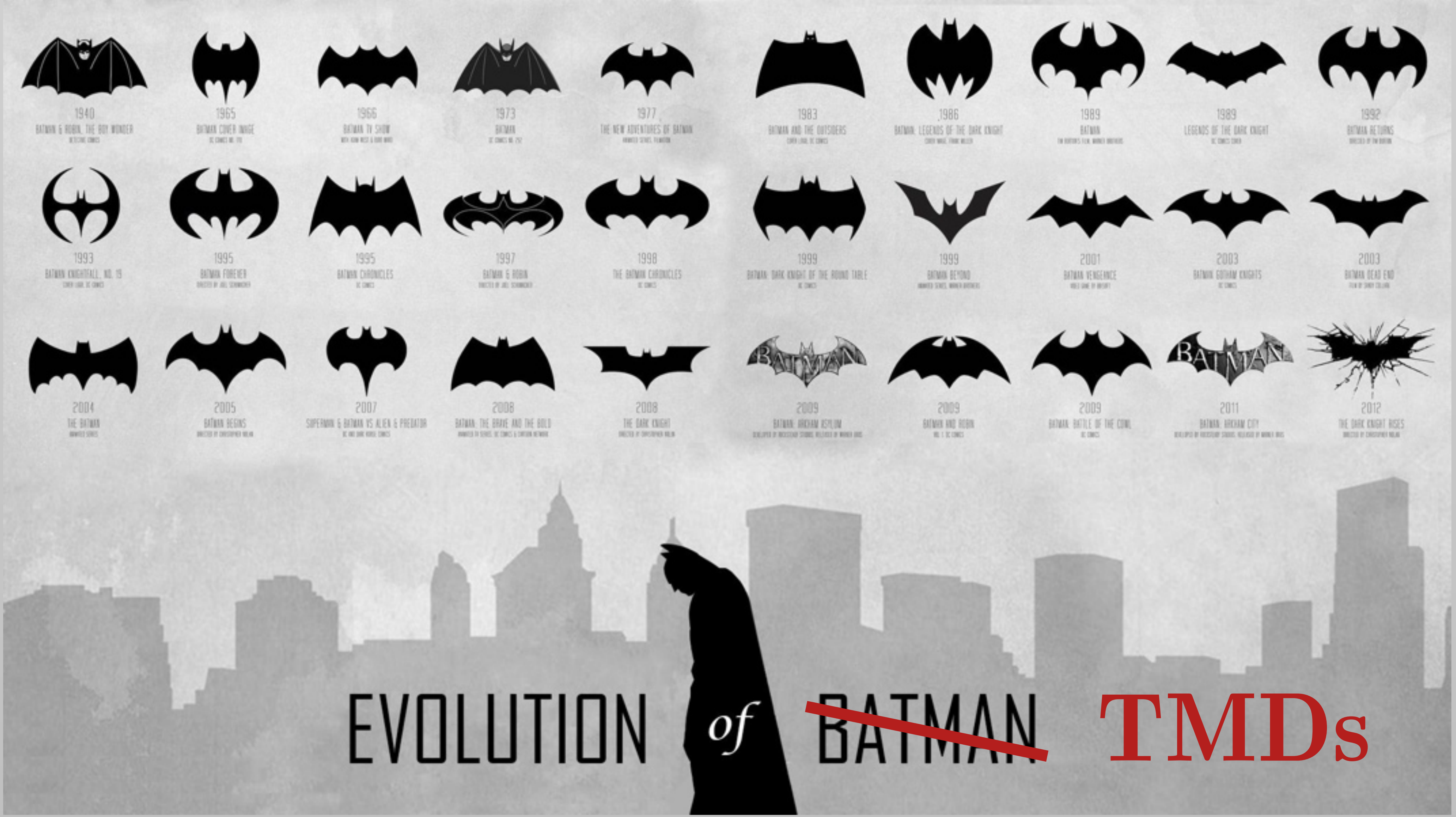


✦**For polarized DY there are *48 terms* in the leading twist expansion.**

$$\frac{d\sigma}{d^4q\, d\Omega} \sim (1+\cos^2\theta)F^1_{UU} + (1-\cos^2\theta)F^2_{UU} + \sin 2\theta F^{\cos\phi}_{UU}\cos\phi + \sin^2\theta F^{\cos 2\phi}_{UU}\cos 2\phi$$
$$+ |\vec{S}_{aT}| \left[\sin\phi_a \left((1+\cos^2\theta)F^1_{TU} + (1-\cos^2\theta)F^2_{TU} + \sin 2\theta F^{\cos\phi}_{TU}\cos\phi + \sin^2\theta F^{\cos 2\phi}_{TU}\cos 2\phi \right) + \ldots \right] + \ldots$$

- LO Matching to *convolutions* of PDFs of both hadrons $q_T \ll Q$

$$\mathcal{C}[f\ \bar{g}\ \ldots] \equiv \frac{1}{N_C}\sum_q e_q^2 \int d^2\vec{k}_{aT}\, d^2\vec{k}_{bT}\, [f^q\ g^{\bar{q}} + f^{\bar{q}}\ g^q] \ldots \delta^2(\vec{q}_T - \vec{k}_{aT} - \vec{k}_{bT})$$


1940
BATMAN & ROBIN, THE BOY WONDER
1965
BATMAN COVER IMAGE
1966
BATMAN TV SHOW
1973
BATMAN
1977
THE NEW ADVENTURES OF BATMAN
1983
BATMAN AND THE OUTSIDERS
1986
BATMAN: LEGENDS OF THE DARK KNIGHT
1989
BATMAN
1989
LEGENDS OF THE DARK KNIGHT
1992
BATMAN RETURNS
1993
BATMAN KNIGHTFALL, NO. 19
1995
BATMAN FOREVER
1995
BATMAN CHRONICLES
1997
BATMAN & ROBIN
1998
THE BATMAN CHRONICLES
1999
BATMAN: DARK KNIGHT OF THE ROUND TABLE
1999
BATMAN BEYOND
2001
BATMAN VENGEANCE
2003
BATMAN GOTHAM KNIGHTS
2003
BATMAN DEAD END
2004
THE BATMAN
2005
BATMAN BEGINS
2007
SUPERMAN & BATMAN VS ALIEN & PREDATOR
2008
BATMAN: THE BRAVE AND THE BOLD
2008
THE DARK KNIGHT
2009
BATMAN: ARKHAM ASYLUM
2009
BATMAN AND ROBIN
2009
BATMAN: BATTLE OF THE COWL
2011
BATMAN: ARKHAM CITY
2012
THE DARK KNIGHT RISES
EVOLUTION of ~~BATMAN~~ TMDs

TMD EVOLUTION

J. Collin, Foundations of Perturbative QCD (2011)

Aybat and Rogers, PRD83 114042 (2011)

✦ CSS Formalism

$$F_{UU,T}(x,z,P_T^2,Q^2) = x\sum_i \mathcal{H}^i_{UU,T}(Q^2;\mu^2)\int d^2\vec{k}_T\; d^2\vec{P}_\perp\; f_1^i(x,k_T^2;\mu^2)D_1^{i\to h}(z,P_\perp^2;\mu^2)\delta^2(\vec{P}_T - z\vec{k}_T\vec{P}_\perp)$$
$$+\, Y_{UU,T}(Q^2,P_T^2) + \mathcal{O}(M^2/Q^2)$$

✦ Fourier Transform of TMD

$$f_1^i(x,k_T^2;\mu^2) = \int d^2\vec{b}_T\; e^{i\vec{B}_T\cdot\vec{k}_\perp}\; \tilde{f}_1^i(x,b_T;\mu^2)$$

✦ TMD Evolution Equation Solution

$$\tilde{f}_1^i(x,b_T;\mu^2) = \sum_j \left(\tilde{C}_{i/j}\otimes f_1^j\right)(x,b_*;\mu_b)\; e^{\tilde{S}(b_*;\mu_b,\mu)}\, e^{g_K(b_T)\ln\frac{\mu}{\mu_0}}\;\; \hat{f}^i_{NP}(x,b_T)$$

Collinear PDF

Perturbative

Non-Perturbative

$$b_* \equiv \frac{b_T}{\sqrt{1+b_T^2/b_{max}^2}} \qquad \mu_b \equiv b_0/b_*$$

✦ Various Prescriptions/Models for NP.

✦ Evolution of polarized TMD PDFs/FFs.

Brock et al., PRD 67 (2013), 073016

Sun, Yuan, PRD88 (2013), 114012

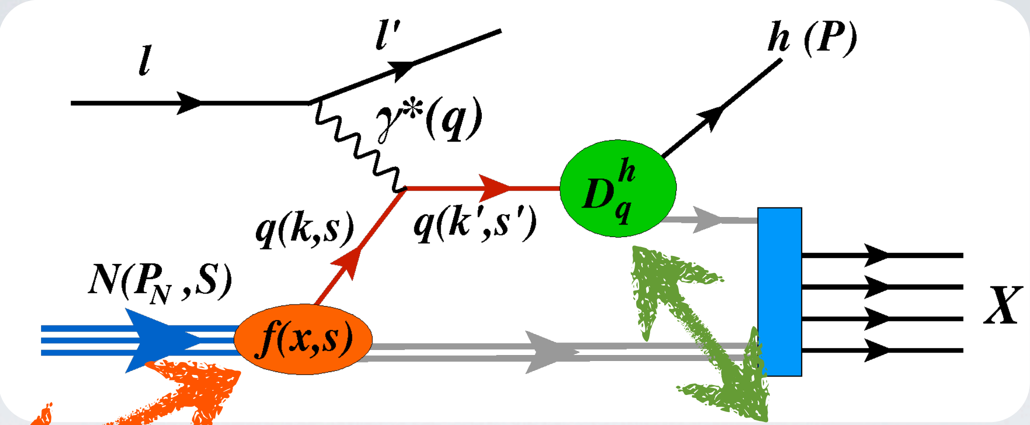
Echevarria et al.: PRD89 (2014), 074013

...

Mantry, Petriello, PRD84 (2011), 014030

EXTRACTION FROM EXPERIMENTAL DATA

EXTRACTING TMDS FROM SIDIS



- ***TMD PDFs***

N/q	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}^\perp$	$h_1\ h_{1T}^\perp$

- ***TMD FFs*** *(unpolarized)*

q/h	U
U	D_1
L	
T	$H_1^\perp$

$$\mathcal{C}[f\ g\ ...] \equiv \sum_q e_q^2 \int d^2\vec{k}_T\ d^2\vec{P}_\perp\ f\ g\ ...\ \delta^2(\vec{P}_T - \vec{P}_\perp - z\vec{k}_T)$$

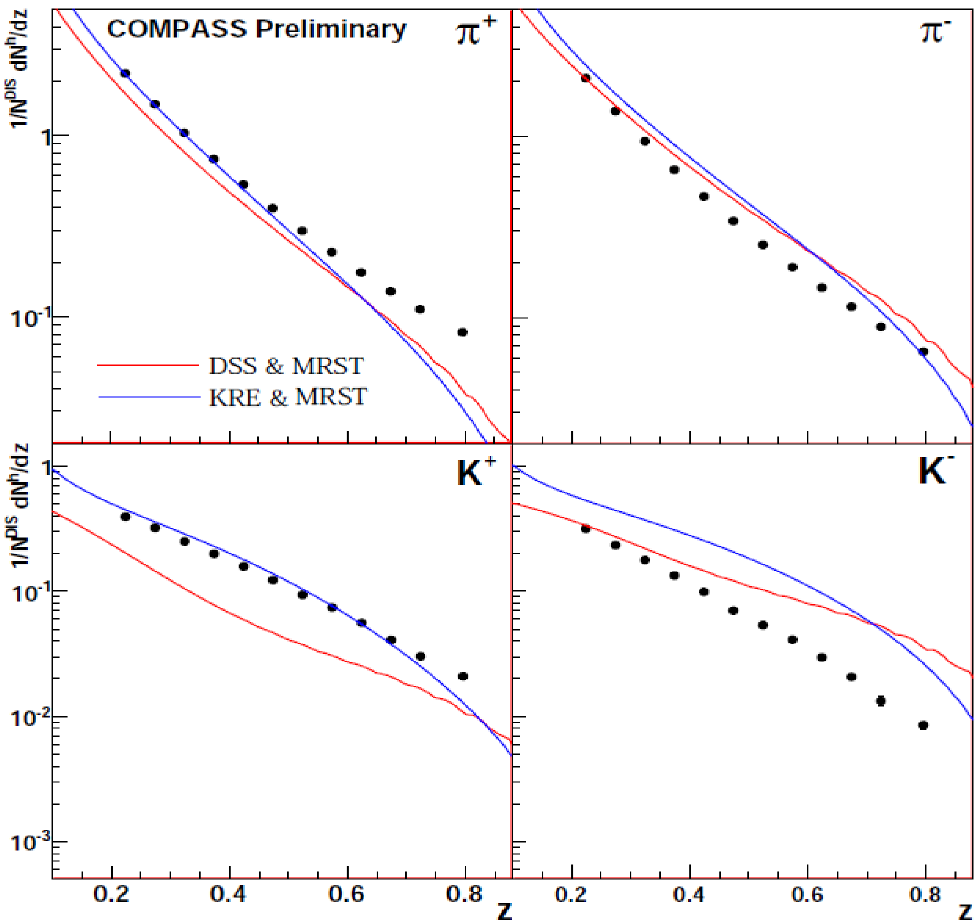
- Need: TM dependent FFs to extract TMDs from SIDIS.
- Many types of final hadrons and different targets.
- *FFs are poorly determined, even in the collinear case.*

Unfavored FFs NOT well known!

Hadron Multiplicities

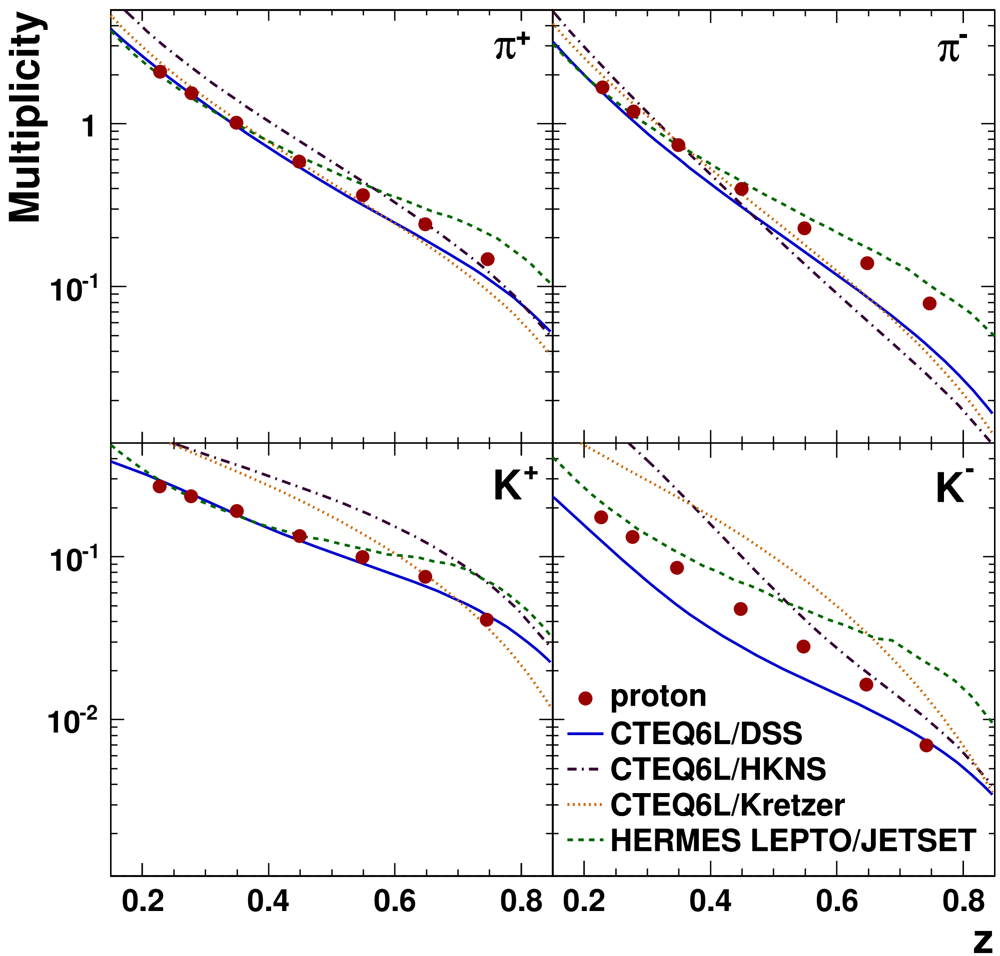
- Preliminary from COMPASS

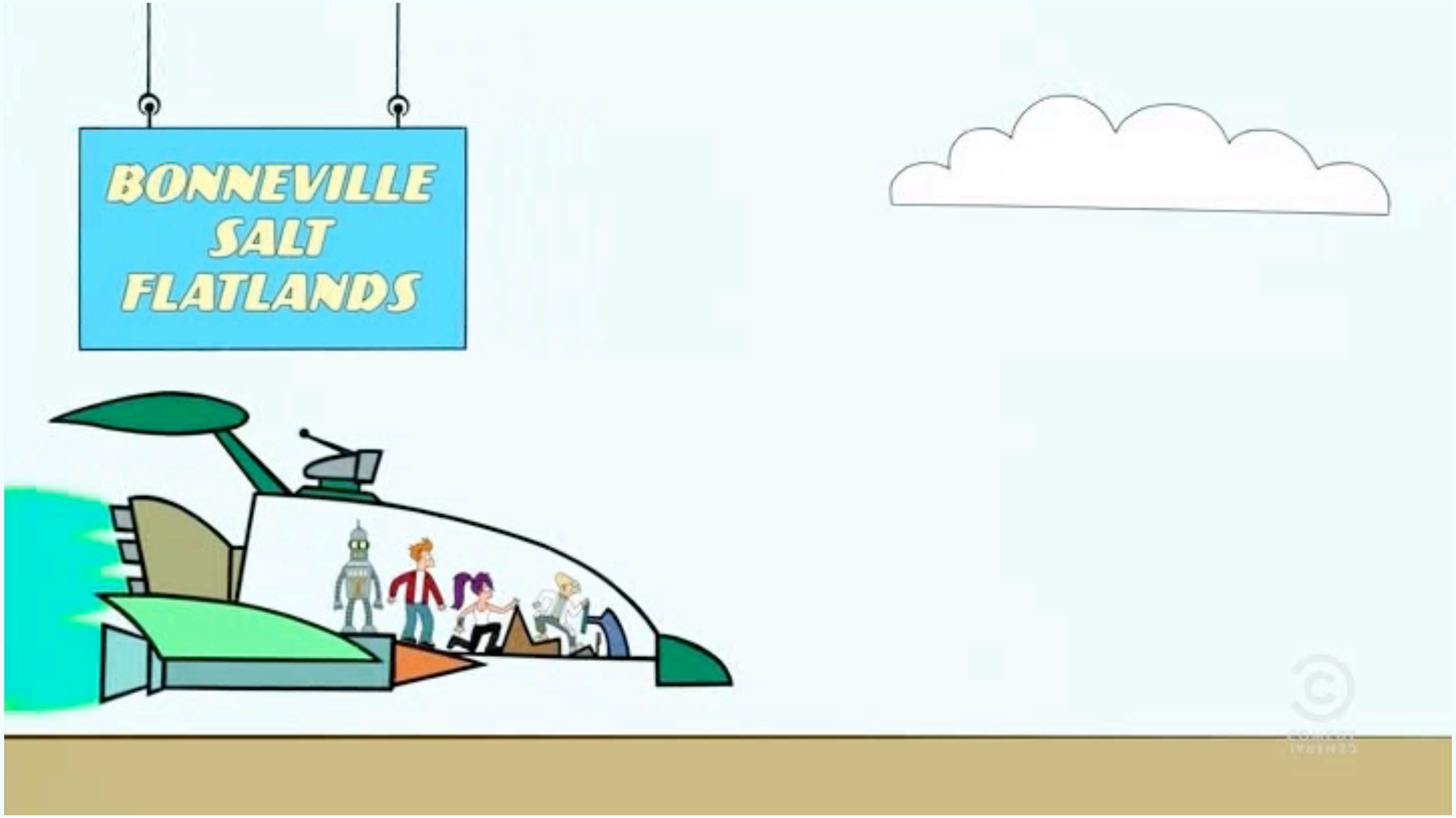
Talk by C.Franco at CIPANP 2012.



- Also results from HERMES

Phys. Rev. D 87, 074029 (2013)





Unpolarized TMDs

UNPOLARIZED MULTIPLICITIES IN SIDIS

HERMES

PRD 87, 074029 (2013)

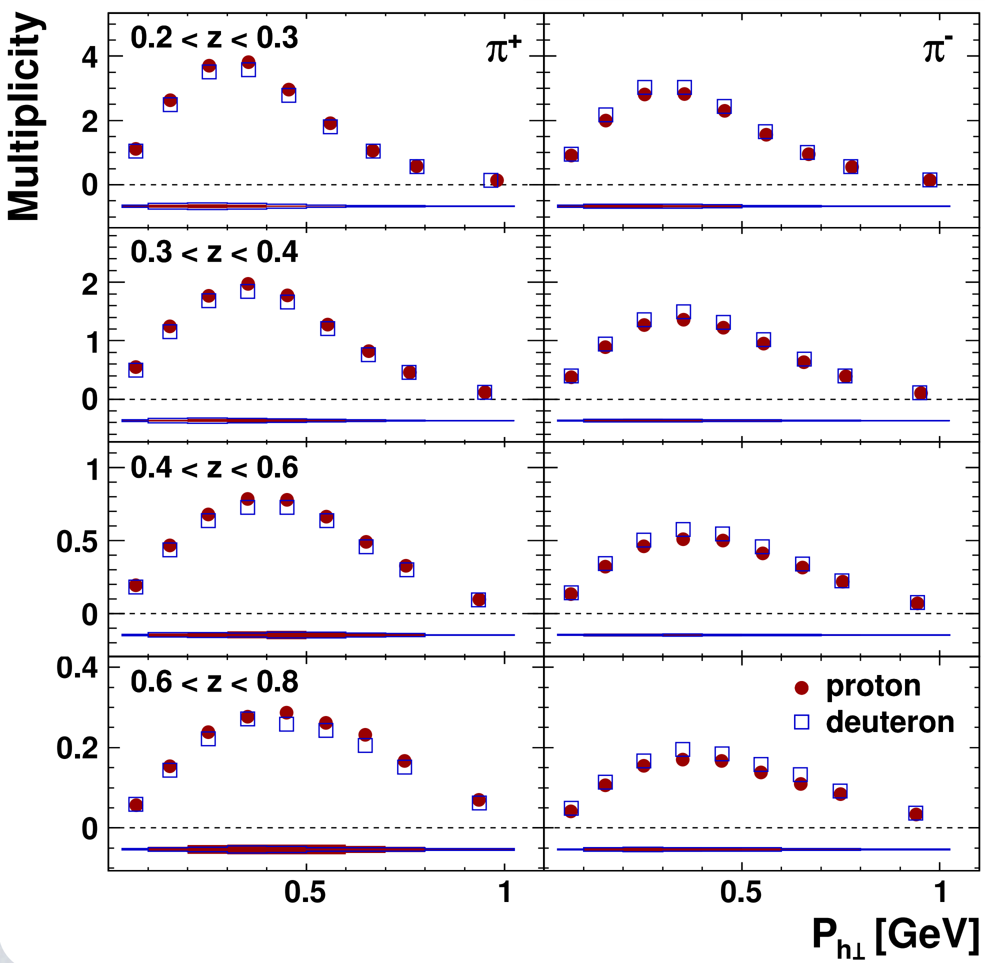
❖**P, D target:** $\pi^{\pm}$ and $K^{\pm}$

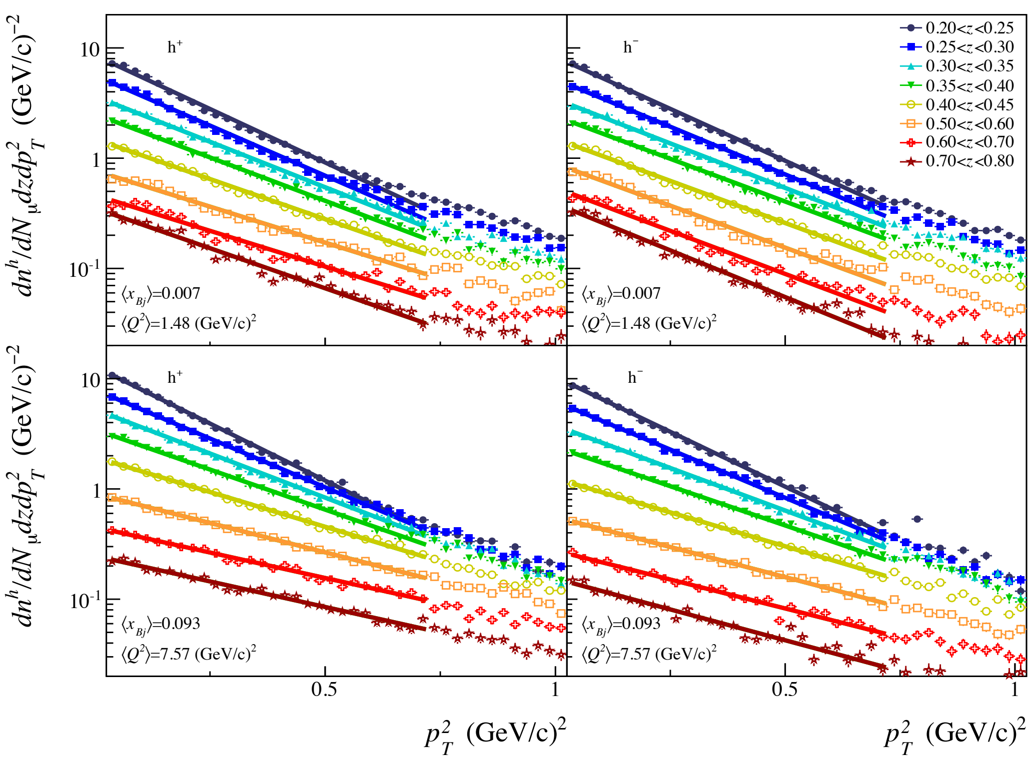
COMPASS

Eur.Phys.J. C73 (2013) 2531 | Err. C75 (2015) 94

❖**D(^{6}LiD) target:** $h^{\pm}$

Multidimensional: Q^2, x, z, P_T^2





GAUSSIAN ANSATZ

Modeling TM dependence...

GAUSSIAN ANSATZ

- Need to calculate convolution PDFs and FFs:

$$F_{UU} = \sum_q e_q^2 \; f_1^q(x, k_T^2, Q^2) \otimes d\sigma^{lq \to lq} \otimes D_q^h(z, P_\perp^2, Q^2)$$

- Using Gaussian Ansatz For TM dependences of PDFs and FFs:

$$f_1^q(x, k_T^2) = f_1^q(x) \frac{e^{-k_T^2/\langle k_{T,q}^2 \rangle}}{\pi \langle k_{T,q}^2 \rangle} \qquad D_q^h(z, P_\perp^2) = D(z)_q^h \frac{e^{-P_\perp^2/\langle P_\perp^{2,q\to h} \rangle}}{\pi \langle P_\perp^{2,q\to h} \rangle}$$

- Only involved collinear PDFs and FFs.

$$F_{UU} = \sum_q e_q^2 \; f_1^q(x, Q^2) D_q^h(z, Q^2) \frac{e^{-P_T^2/\langle P_T^2 \rangle}}{\pi \langle P_T^2 \rangle}$$

$$\boxed{\langle P_T^2 \rangle(z) = \langle P_\perp^2 \rangle + z^2 \langle k_T^2 \rangle}$$

$$\langle k_T^2 \rangle \equiv \frac{\int d^2\mathbf{k_T} \; k_T^2 f(x, k_T^2)}{\int d^2\mathbf{k_T} \; f(x, k_T^2)} \qquad \langle P_\perp^2 \rangle \equiv \frac{\int d^2\mathbf{P}_\perp \; P_\perp^2 D(z, P_\perp^2)}{\int d^2\mathbf{P}_\perp \; D(z, P_\perp^2)}$$

EMPIRICAL EXTRACTIONS OF AVERAGE TM

M. Anselmino et. al.: JHEP 1404 (2014) 005.

- *Fit of both HERMES and COMPASS data.*
- Use Gaussian for TM dependence of PDFs and FFs.
- *CTEQ6L* collinear PDFs and *DSS* collinear FFs.
- Only DGLAP evolution of collinear PDFs and FFs.
- Constant $\langle k_T^2 \rangle$ and $\langle P_\perp^2 \rangle$.

$$\langle P_T^2 \rangle(z) = \langle P_\perp^2 \rangle + z^2 \langle k_T^2 \rangle$$

EMPIRICAL EXTRACTIONS OF AVERAGE TM

M. Anselmino et. al.: JHEP 1404 (2014) 005.

HERMES

Cuts	$\chi^2_{\rm dof}$	n. points	$[\chi^2_{\rm point}]^{\pi^+}$	$[\chi^2_{\rm point}]^{\pi^-}$	Parameters
$Q^2 > 1.69$ GeV^2 $0.2 < P_T < 0.9$ GeV $z < 0.6$	1.69	497	1.93	1.45	$\langle k_\perp^2 \rangle = 0.57 \pm 0.08$ GeV^2 $\langle p_\perp^2 \rangle = 0.12 \pm 0.01$ GeV^2
$Q^2 > 1.69$ GeV^2 $0.2 < P_T < 0.9$ GeV $z < 0.7$	2.62	576	2.56	2.68	$\langle k_\perp^2 \rangle = 0.46 \pm 0.09$ GeV^2 $\langle p_\perp^2 \rangle = 0.13 \pm 0.01$ GeV^2

COMPASS

$Q^2 > 1.69$ GeV^2 $0.2 < P_T < 0.9$ GeV $z < 0.6$ $N_y = A + B\,y$	3.42	5385	3.25	3.60	$\langle k_\perp^2 \rangle = 0.60 \pm 0.14$ GeV^2 $\langle p_\perp^2 \rangle = 0.20 \pm 0.02$ GeV^2 $A = 1.06 \pm 0.06$ $B = -0.43 \pm 0.14$
$Q^2 > 1.69$ GeV^2 $0.2 < P_T < 0.9$ GeV $z < 0.7$ $N_y = A + B\,y$	3.79	6284	3.63	3.96	$\langle k_\perp^2 \rangle = 0.52 \pm 0.14$ GeV^2 $\langle p_\perp^2 \rangle = 0.21 \pm 0.02$ GeV^2 $A = 1.06 \pm 0.07$ $B = -0.46 \pm 0.15$

EMPIRICAL EXTRACTIONS OF AVERAGE TM

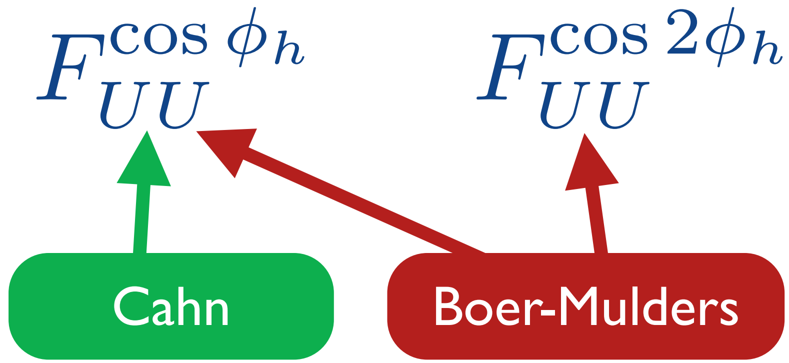
M. Anselmino et. al.: JHEP 1404 (2014) 005.

- Unable to access flavor dependence in TMD.
- Slightly better fit to COMPASS when allowing favored and disfavored TM to be different.
- Need more high precision data with different targets and many detected hadrons to extract flavor dep. Need a wide range of Q^2 for testing TMD evolution.
- Extracted average TMs of PDF and FF highly *correlated*!

EMPIRICAL EXTRACTIONS OF AVERAGE TM

V. Barone et. al.: arXiv:1502.04214 (2015).

- Fit azimuthal asymmetries in *unpolarized* SIDIS from COMPASS and HERMES.



$$F_{UU}^{\cos\phi_h}|_{Cahn} \sim \sum_q e_q^2 x f_1^q(x) D_1^{q\to h}(z) \frac{z\langle k_T^2\rangle}{\langle P_T^2\rangle} \frac{e^{-P_T^2/\langle P_T^2\rangle}}{\pi\langle P_T^2\rangle}$$

- Combined fit with multiplicities yields

$$\langle k_T^2\rangle \simeq 0.03 - 0.04\ \mathrm{GeV}^2$$

- Possible higher-twist effects,etc, *but* demonstrates the lack of constraints on quark TM from multiplicities alone.

EMPIRICAL EXTRACTIONS OF AVERAGE TM II

Signori et al.: JHEP 1311, 194 (2013)

*Use **Gaussian** Ansatz and allow TM dependencies*

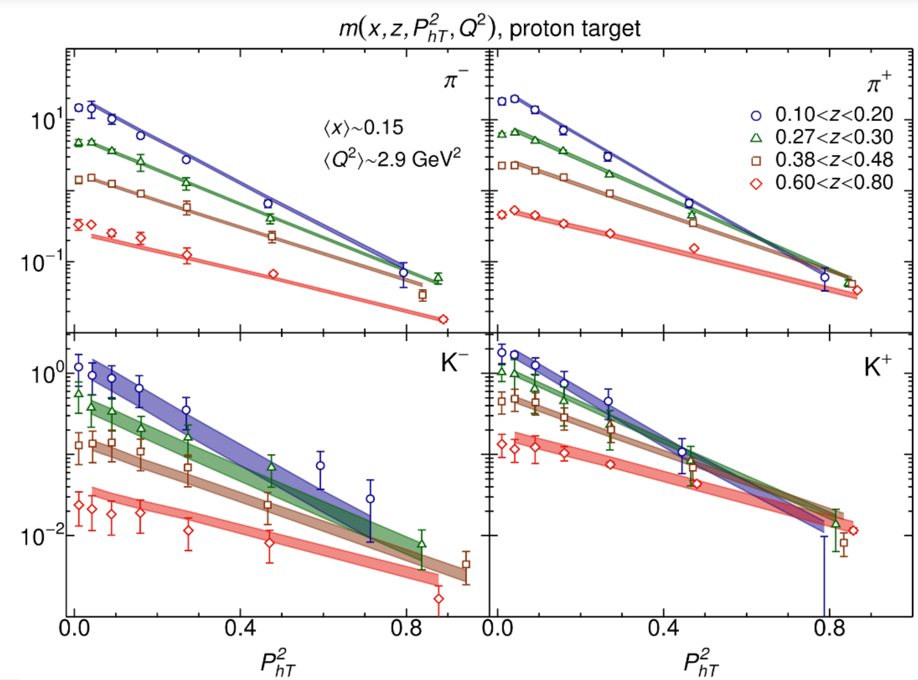
1) Dynamic. ***2) Quark and Hadron type.***

$\langle k_T^2 \rangle (x)$ $\quad$ $\langle k_T^2 \rangle^{u_v} \neq \langle k_T^2 \rangle^{d_v} \neq \langle k_T^2 \rangle^{sea}$

$\langle P_\perp^2 \rangle (z)$ $\quad$ $\langle P_\perp^2 \rangle^{u\to\pi^+} \neq \langle P_\perp^2 \rangle^{u\to K^+} \neq \langle P_\perp^2 \rangle^{s\to K^-} \neq \langle P_\perp^2 \rangle^{unf}$

$$\langle P_T^2 \rangle^{q\to h}(x,z) = \langle P_\perp^2 \rangle^{q\to h}(z) + z^2 \langle k_T^2 \rangle^q (x)$$

- ✦Use HERMES multiplicity data for P and D target.
- ✦MSTW08 LO for PDF.
- ✦DSS LO for FF.
- ✦200 replicas of data by Gaussian smearing are fitted.

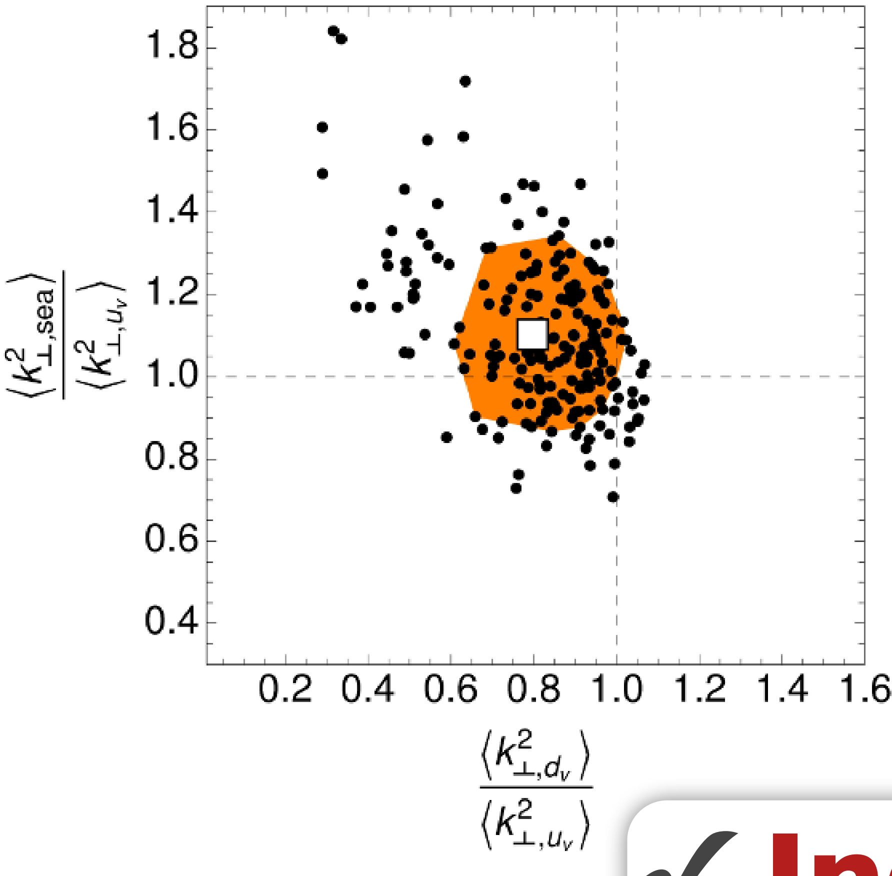


EMPIRICAL EXTRACTIONS OF AVERAGE TM II

Signori et al.: JHEP 1311, 194 (2013)

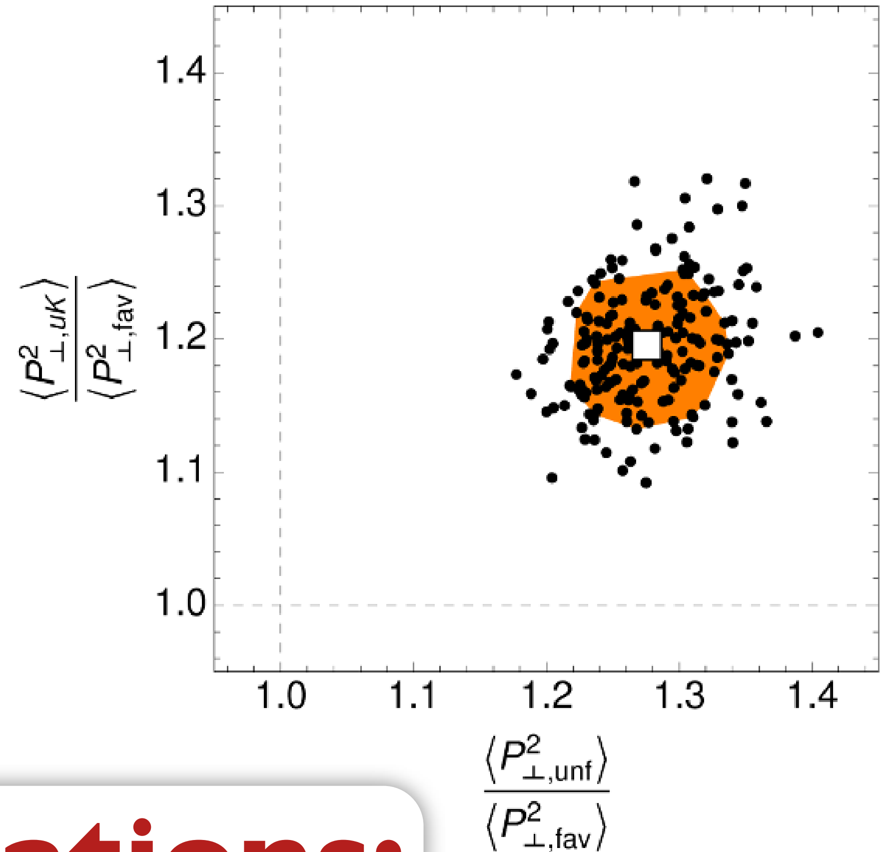
✦TMD: Flavor dep.

$$0.04 \lesssim x \lesssim 0.4$$



✦FF: Quark and Hadron type dep.

$$0.1 < z < 0.8$$



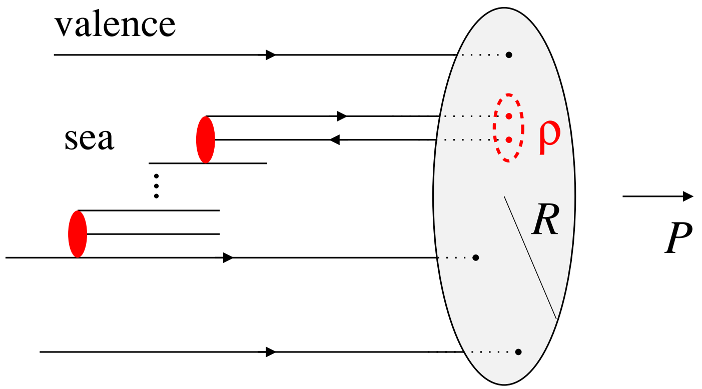
✓ **Indications:**

$$\langle k_T^2\rangle^{d_v} < \langle k_T^2\rangle^{u_v} < \langle k_T^2\rangle^{sea}$$

$$\langle P_\perp^2\rangle^{u\to\pi^+} < \langle P_\perp^2\rangle^{u\to K^+} \sim \langle P_\perp^2\rangle^{unf}$$

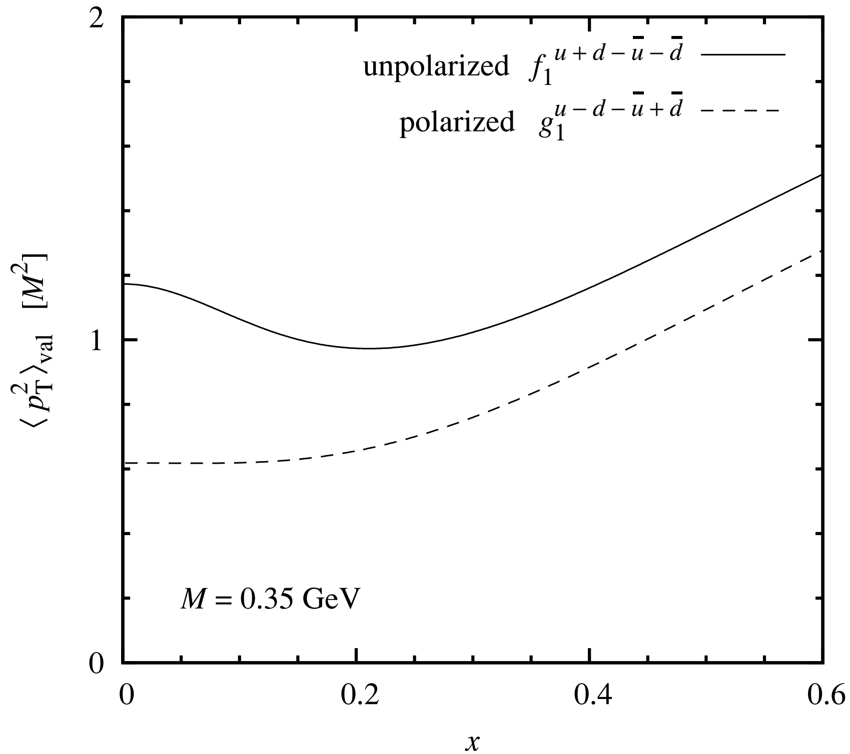
CHIRAL QUARK SOLITON MODEL

Schweitzer et al.: JHEP 1301, 163 (2013)

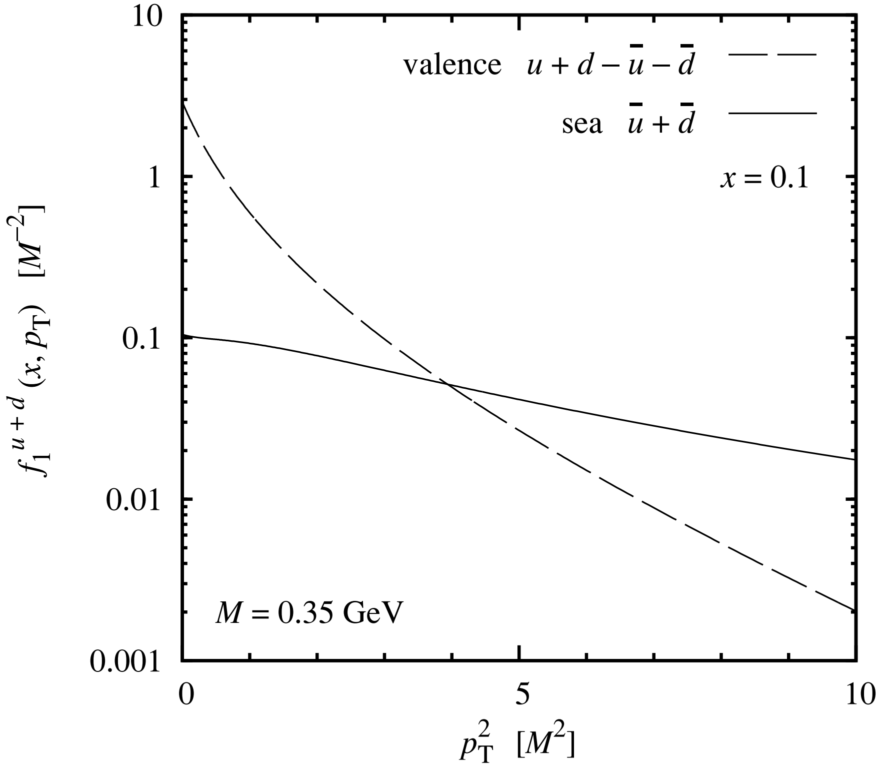


✦ Nucleon as constituent quarks and antiquarks moving in self-consistent chiral field.

✦ Sizable x dependence of average TM for valence quarks.



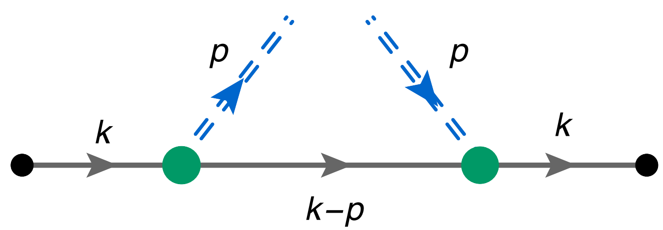
✦ Very different TM dep. of *valence* vs *sea*.

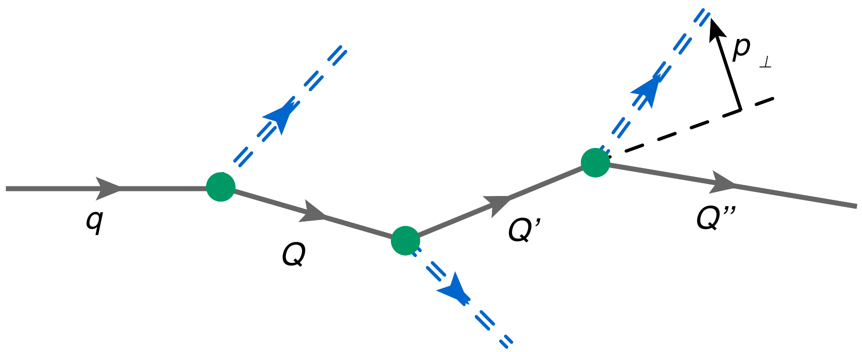


H.M., Bentz, Cloet, Thomas, PRD.85:014021, 2012

NJL - Jet

- *Multi-hadron emission framework with effective quark model input (NJL).*
- *Monte-Carlo framework allows flexibility in including the transverse momentum, spin effects, two-hadron correlations, etc.*

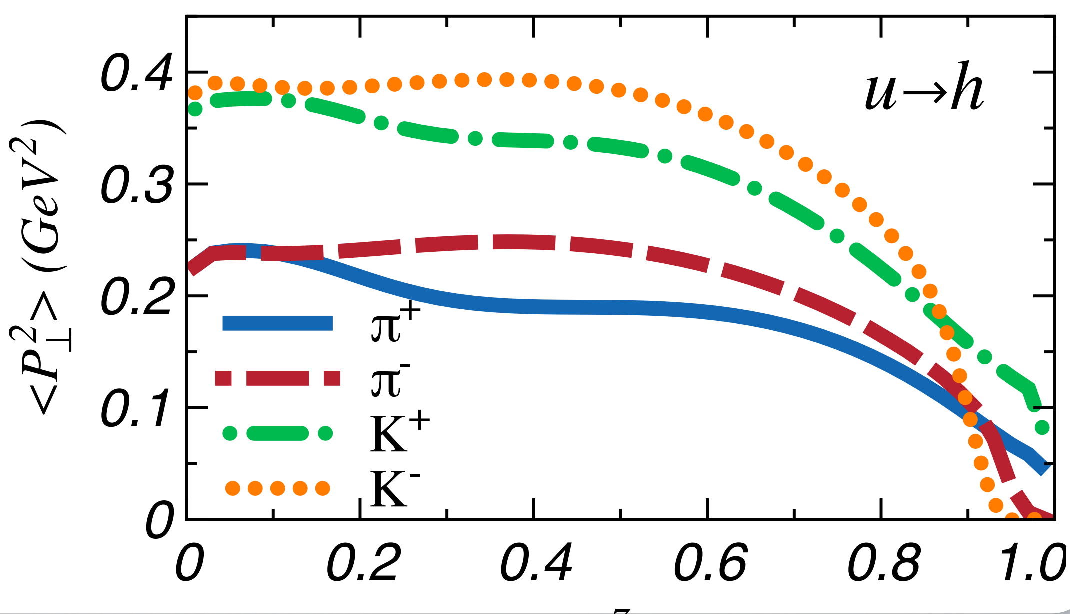




TMD FFs

$$\langle P_\perp^2 \rangle_{unf} > \langle P_\perp^2 \rangle_{fav}$$

$$\langle P_\perp^2 \rangle_K > \langle P_\perp^2 \rangle_\pi$$

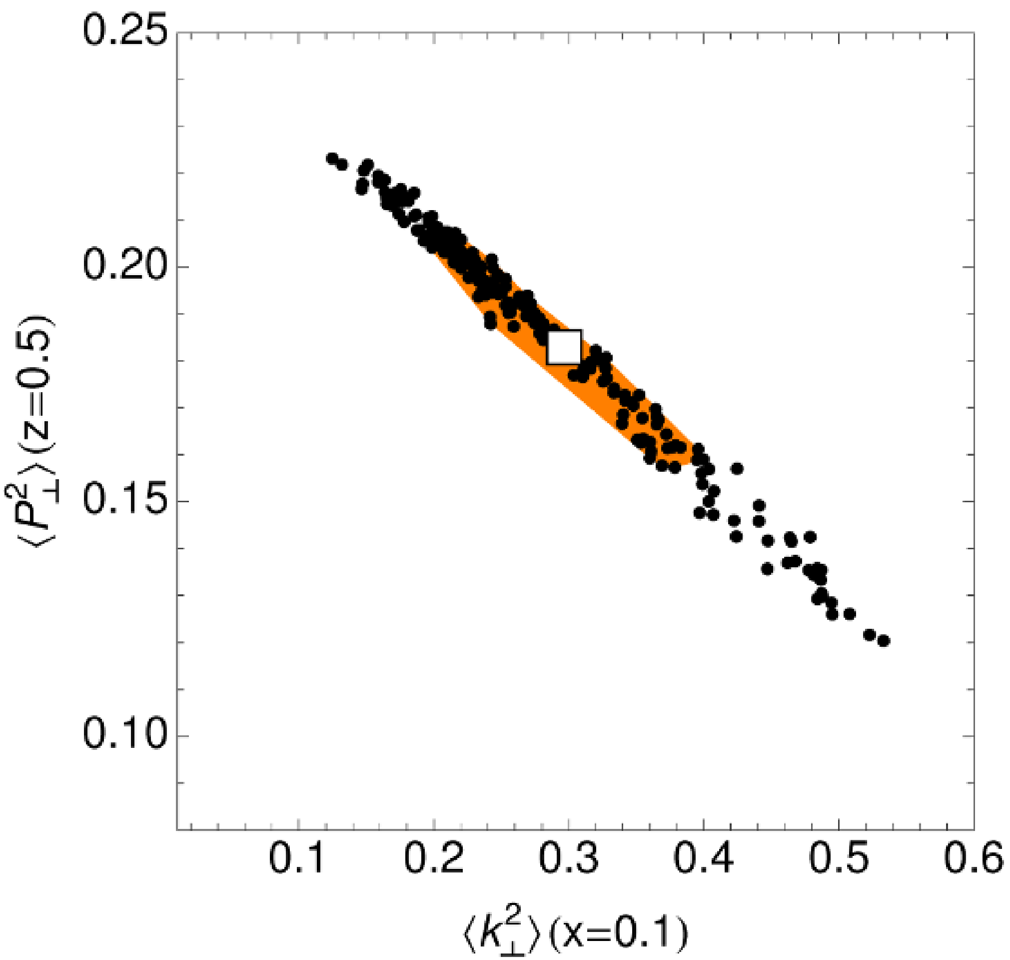


EMPIRICAL EXTRACTIONS OF AVERAGE TM II

Signori et al.: JHEP 1311, 194 (2013)

✦*Correlations* of TM extractions for PDF and FF

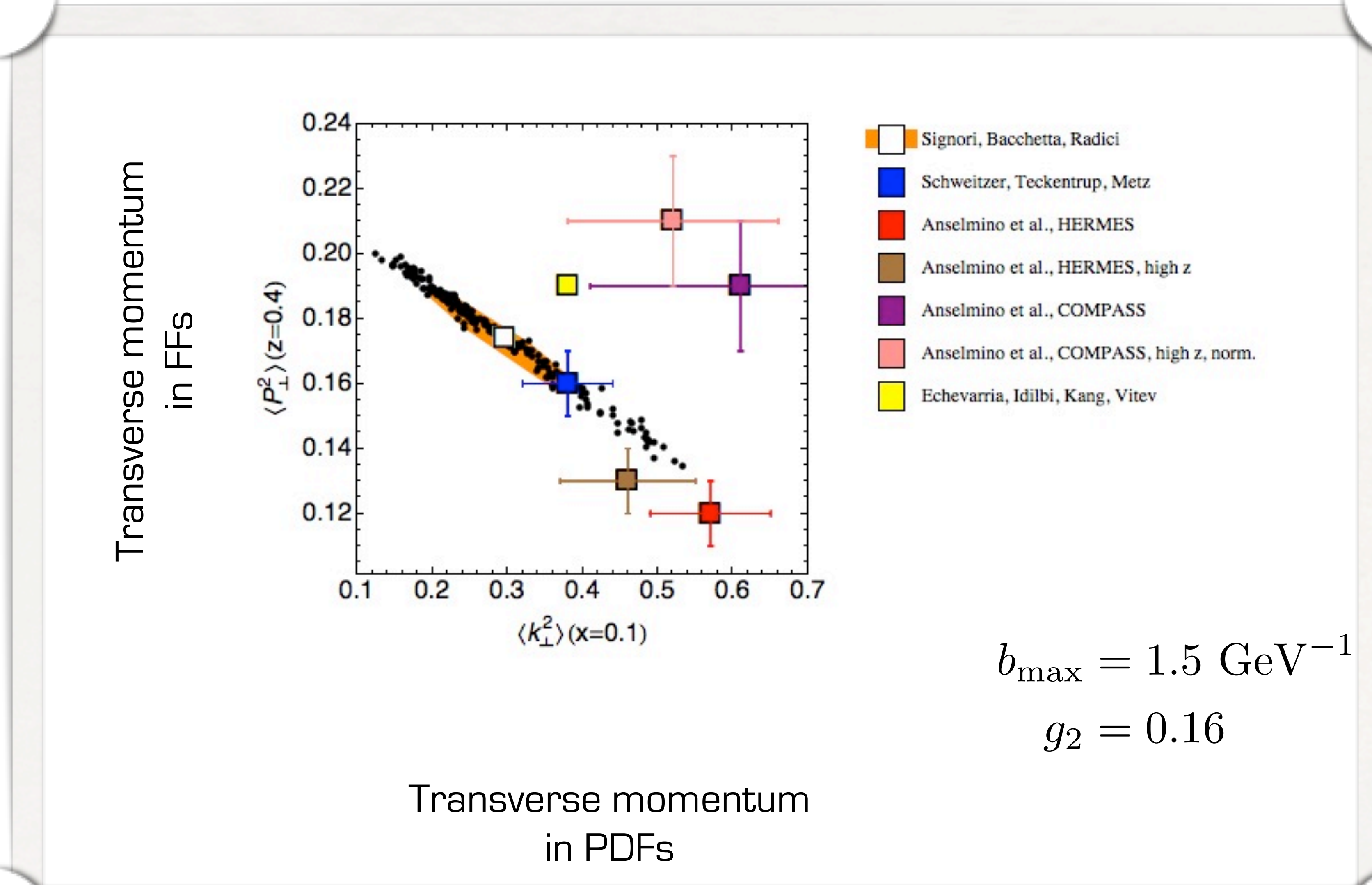
$z = 0.5 \qquad x = 0.1$



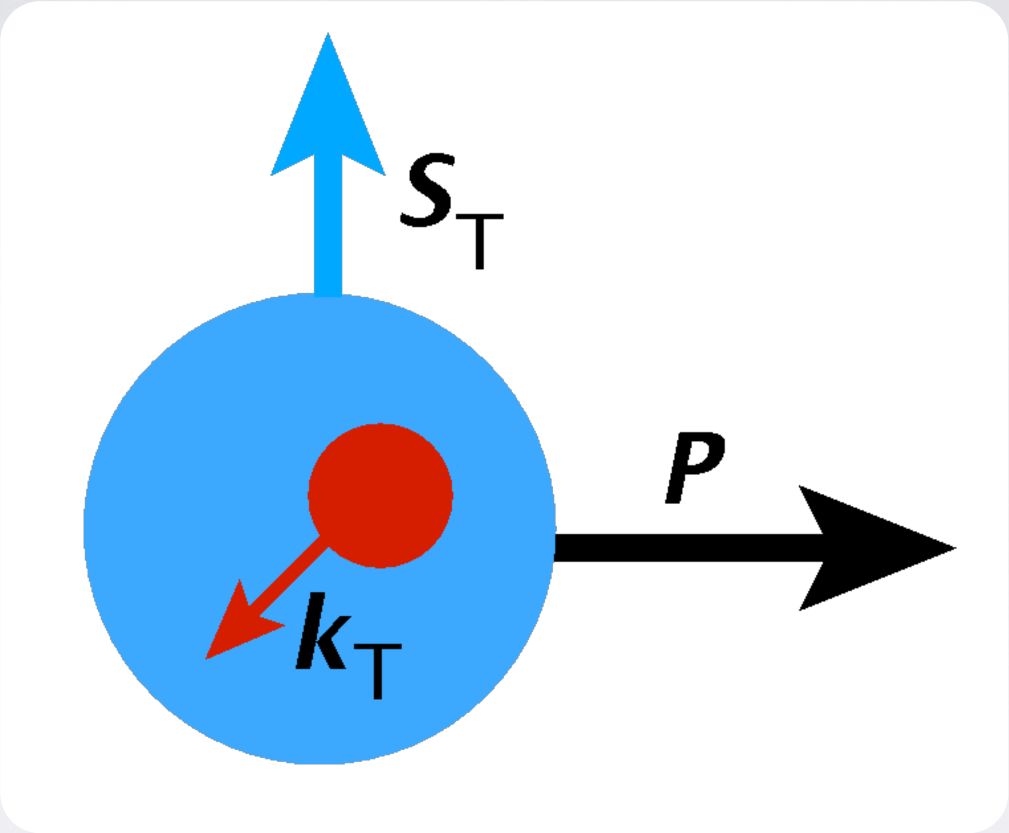
EMPIRICAL EXTRACTIONS OF AVERAGE TM II

Signori et al.: JHEP 1311, 194 (2013)

✦*Correlations* of TM extractions for PDF and FF



From A. Bacchetta's talk at INT Workshop 14-55W, Feb. 2014



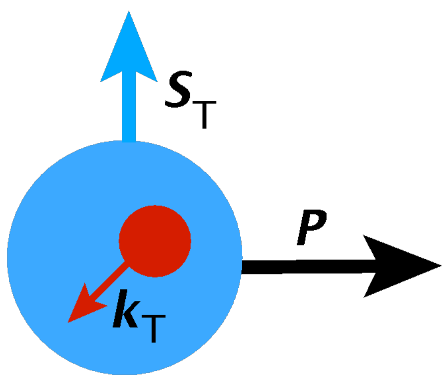
SIVERS PDF

SIVERS PDF

N/q	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}^\perp$	$h_1\ h_{1T}^\perp$

D. Sivers, Phys.Rev. D41 (1990).

- Correlation of k_T and S_T
- Proposed by Dennis Sivers in 1990 to explain the single spin asymmetry in $pp^\uparrow \to \pi + X$.



$S_T k_T \sin(\varphi_k - \varphi_S)$

$$f_\uparrow^q(x, \vec{k}_T) = f_1^q(x, k_T) + \frac{[\vec{S} \times \vec{k}_T]_3}{M} f_{1T}^{\perp q}(x, k_T)$$

- Naively *T-odd,* gauge-link should be included in the definition.
- **Accessible in Polarized SIDIS, Drell-Yan.**

$$f_{1T}^{\perp SIDIS} = -f_{1T}^{\perp DY}$$

SIVERS PDF

N/q	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}^\perp$	$h_1 \; h_{1T}^\perp$

D. Sivers, Phys.Rev. D41 (1990).

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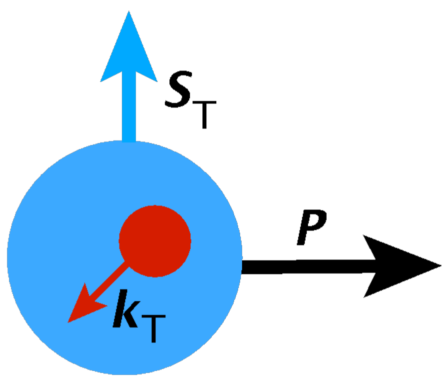
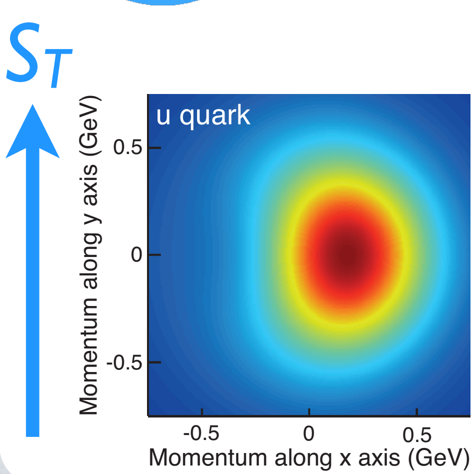
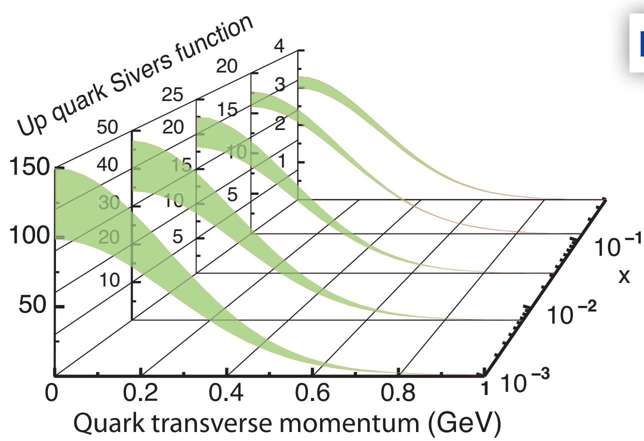


$$f^q_\uparrow(x, \vec{k}_T) = f^q_1(x, k_T) + \frac{[\vec{S} \times \vec{k}_T]_3}{M} f^{\perp q}_{1T}(x, k_T)$$

$S_T k_T \sin(\varphi_k - \varphi_S)$





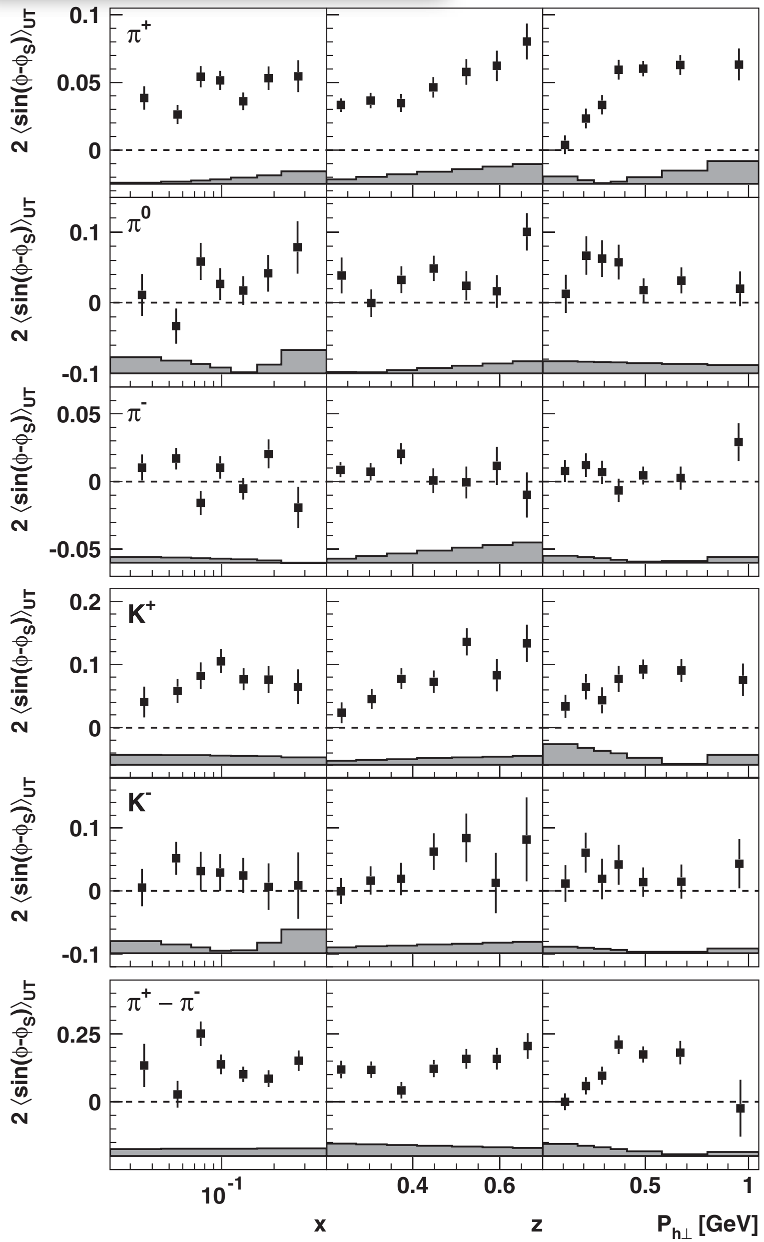
EIC White Paper, arXiv:1212.1701 (2012).

- Preliminary extractions from experimental data and projections for EIC.

SIVERS SSA MEASUREMENTS IN SIDIS

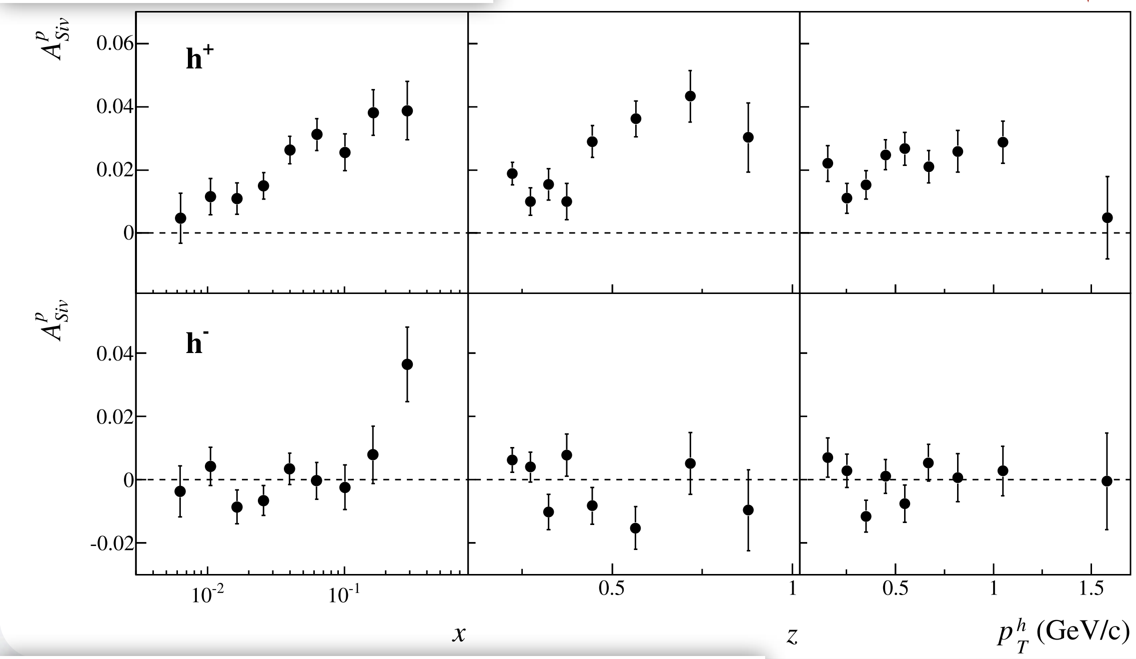
HERMES: P

PRL 103, 152002 (2009).



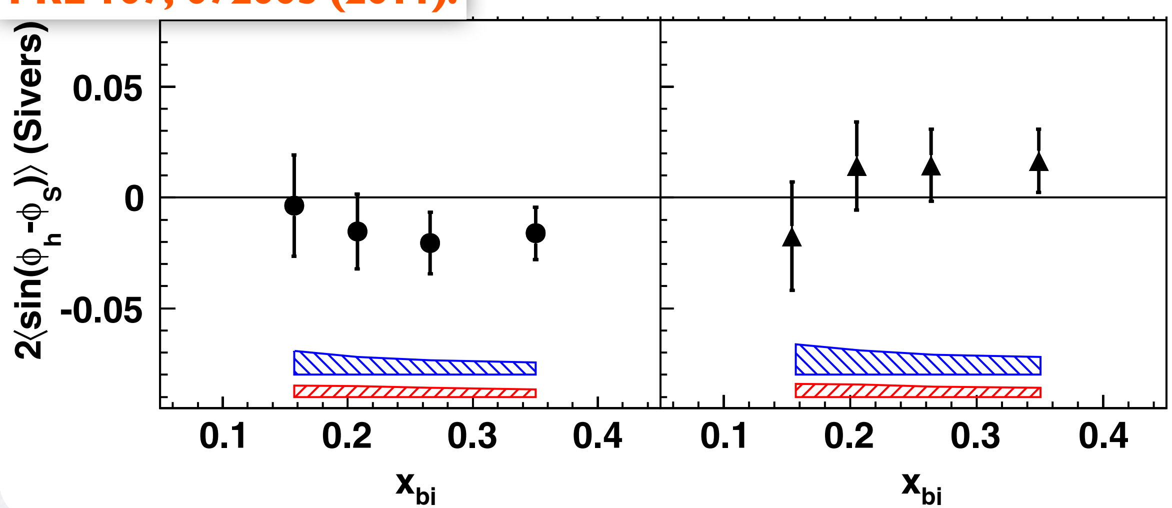
COMPASS: P

PLB 717, 383–389 (2012).



HALL-A: ^3He (access N)

PRL 107, 072003 (2011).



EMPIRICAL EXTRACTIONS OF SIVERS PDF

M. Anselmino et. al.: PRD 72, 094007 (2005). PRD 86, 014028 (2012).

- *Sivers SSAs from SIDIS*
- Use LO expression for factorized cross-section.
- Parametrize PDFs and FFs.
- Use Gaussian TMD dependence.
- Also *TMD evolution* in 2012.

$$A^h_{Siv} \equiv 2\frac{\int d\varphi_S d\varphi_h\ (\sigma^h_\uparrow - \sigma^h_\downarrow)\sin(\varphi_h - \varphi_S)}{\int d\varphi_S d\varphi_h\ (\sigma^h_\uparrow + \sigma^h_\downarrow)}.$$

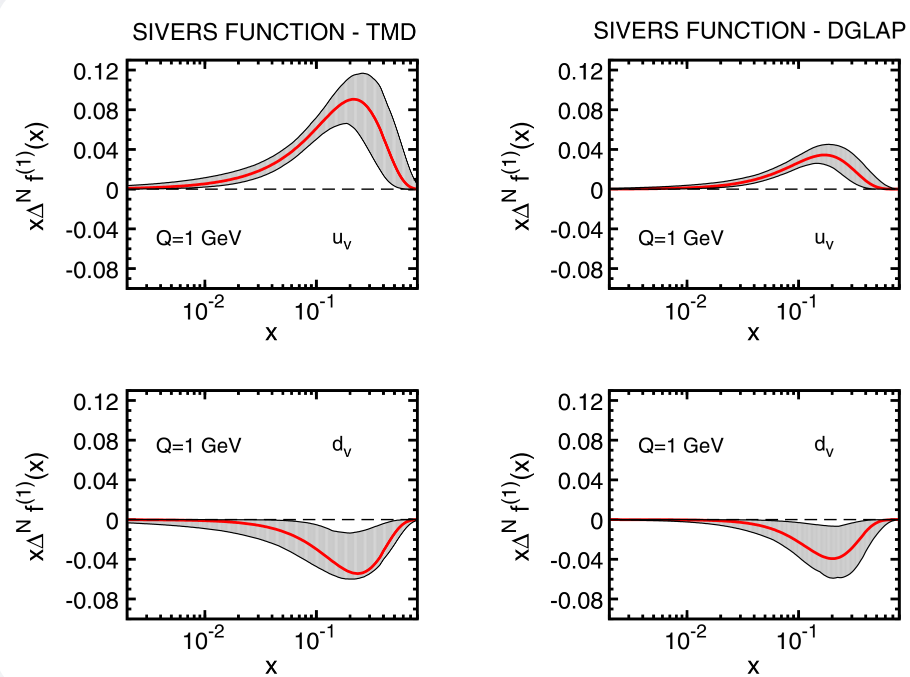
$$A^h_{Siv} \sim \mathcal{C}[k_T f_{1T}^{\perp q}\ D_1]/\mathcal{C}[f_1^q\ D_1^{h/q}]$$

$$\Delta^N f_{q/p^\uparrow}(x,k_T) = \mathcal{N}_q(x) h(k_T) f_1^q(x,k_T)$$

$$\Delta^N f_{q/p^\uparrow} \equiv -\frac{2k_T}{M} f_{1T}^{\perp q}$$

$$f_1^q(x,k_T) = f_q(x)\frac{1}{\pi\mu^2}e^{-k_T^2/\mu^2}$$

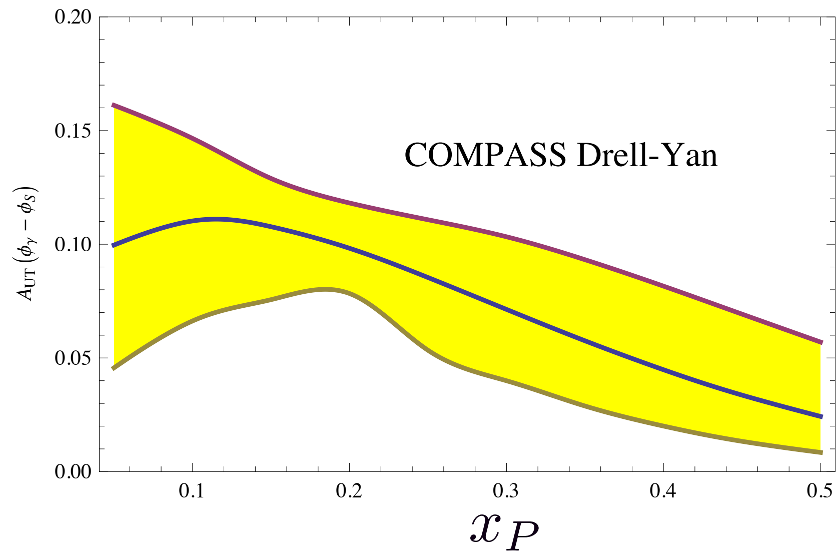
- ***Fits to HERMES and COMPASS:***
- ***Current Data can only afford:***
  - ***Large*** uncertainties, esp. for sea.
  - Approximations: TM and flavor dependence of FF, etc.



EXTRACTIONS WITH TMD EVOLUTION

Sun, Yuan, PRD88 (2013), 114012

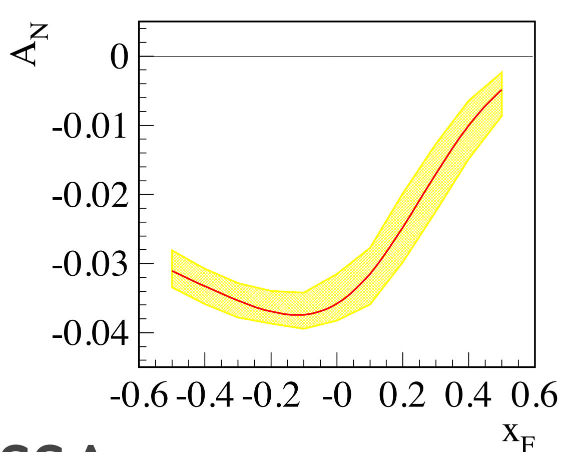
- ✦ Sun-Yuan prescription for TMD evolution.
- ✦ Gaussian TM dependence of NP TMD dependence at initial scale.
- ✦ Fit HERMES & COMPASS multiplicities and Sivers SSAs.
- ✦ Predict Sivers SSA and W production in COMPASS DY and PP.



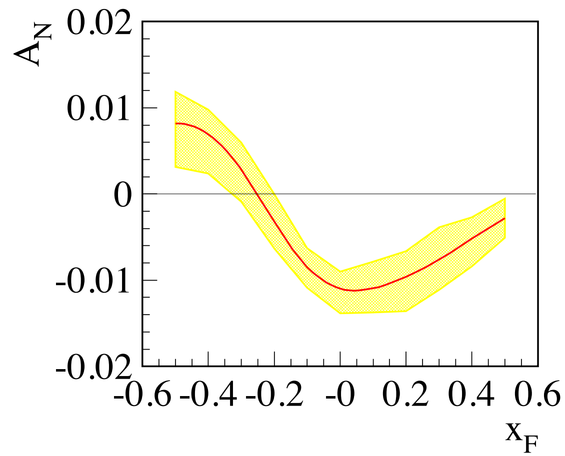
Echevarria et al.: PRD.89 074013, (2014)

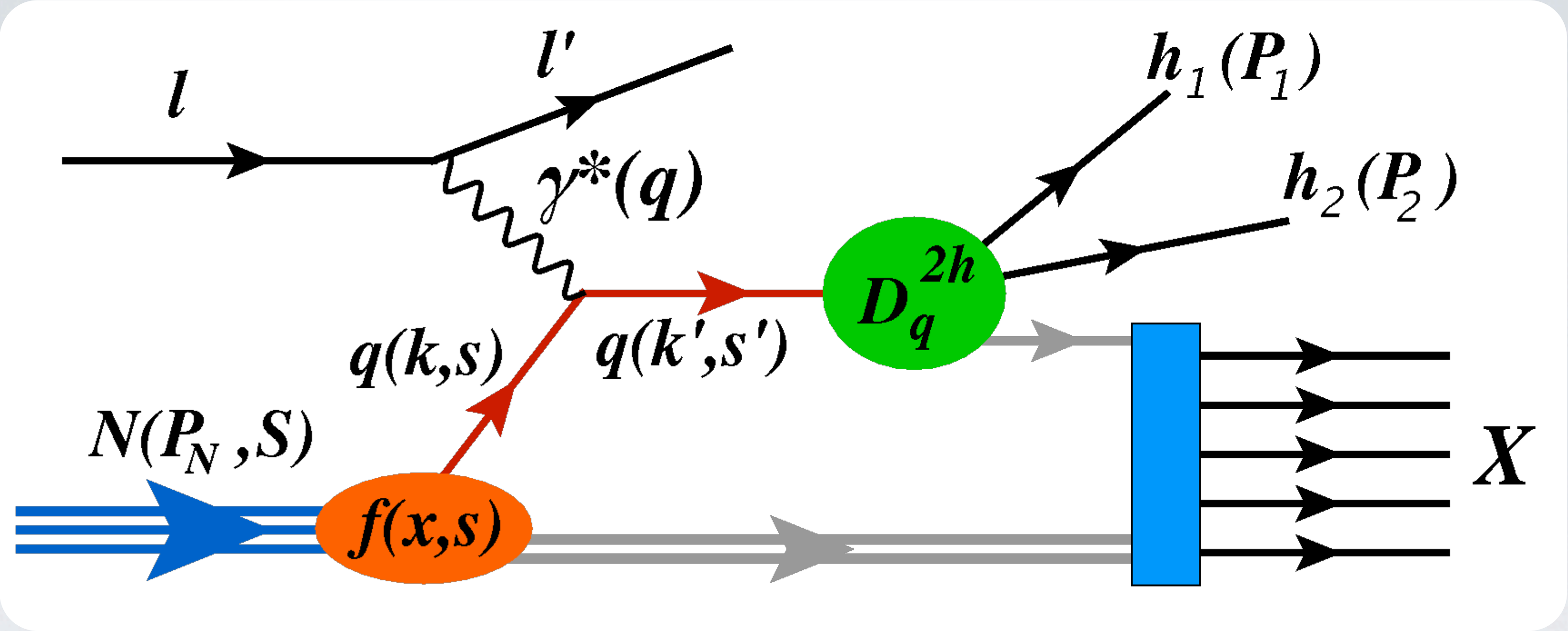
- ✦ Find non-perturbative Sudakov factor that describes W, Z production in $P\bar{P}$ at Fermilab +HERMS & COMPAS.
- ✦ Use it to fit Sivers SSA at HERMES, COMPASS, JLAB.
- ✦ Predict Sivers Effect for DY SSA.

COMPASS



RHIC



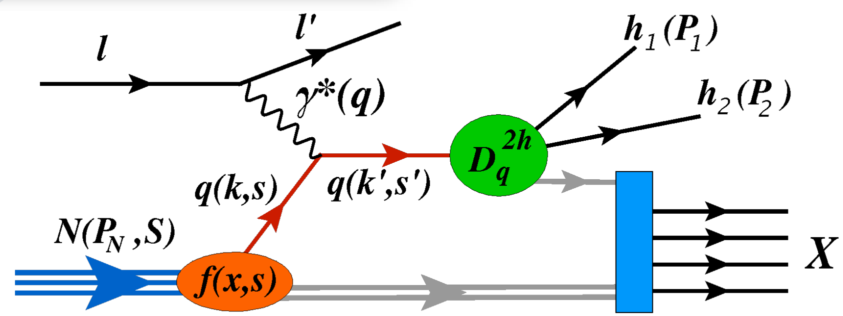


Sivers Effect in Two Hadron SIDIS

TWO-HADRON SIDIS

Kotzinian, H.M., Thomas: PRL.113, 062003 ; PRD.90, 074006 ; 1407.6572 (2014);

- Correlations of quark's TM transferred to **two hadrons.**



$$\frac{d\sigma^{h_1 h_2}}{dz_1\, dz_2\, d^2\boldsymbol{P}_{1T}\, d^2\boldsymbol{P}_{2T}} = C(x,Q^2)\,(\sigma_U + \sigma_S)$$

$$\sigma_U = \sum_q e_q^2 \int d^2\boldsymbol{k}_T\; f_1^q\; D_{1q}^{h_1 h_2} \qquad \sigma_S = \sum_q e_q^2 \int d^2\boldsymbol{k}_T \frac{[\boldsymbol{S}_T \times \boldsymbol{k}_T]_3}{M} f_{1T}^{\perp q}\; D_{1q}^{h_1,h_2}$$

- Unpolarized fully unintegrated dihadron Fragmentation Function

✦ **Single hadron** FF.

$$D_{1q}^{h}(z, P_\perp)$$

✦ **Dihadron** FF.

$$D_{1q}^{h_1,h_2}(z_1, z_2, P_{1\perp}, P_{2\perp}, \boldsymbol{P}_{1\perp}\cdot\boldsymbol{P}_{2\perp})$$

TWO-HADRON SIDIS

Kotzinian, H.M., Thomas: PRL.113, 062003 ; PRD.90, 074006 ; 1407.6572 (2014);

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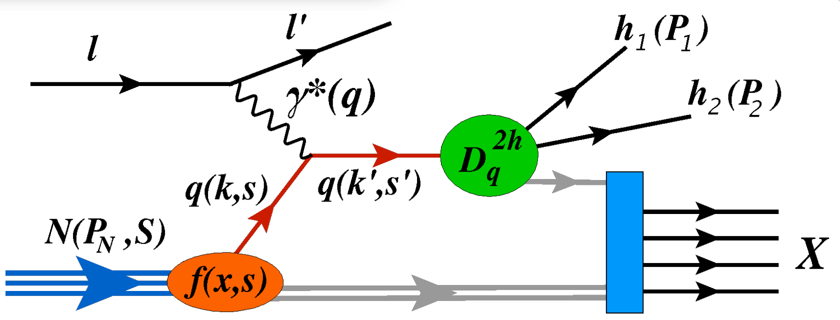


$$\frac{d\sigma^{h_1 h_2}}{dz_1\, dz_2\, d^2\boldsymbol{P}_{1T}\, d^2\,\boldsymbol{P}_{2T}} = C(x,Q^2)\,(\sigma_U + \sigma_S)$$

$$\sigma_U = \sum_q e_q^2 \int d^2\boldsymbol{k}_T\; f_1^q\; D_{1q}^{h_1 h_2} \qquad \sigma_S = \sum_q e_q^2 \int d^2\boldsymbol{k}_T \frac{[\boldsymbol{S}_T \times \boldsymbol{k}_T]_3}{M} f_{1T}^{\perp q}\; D_{1q}^{h_1,h_2}$$

- Unpolarized fully unintegrated dihadron Fragmentation Function

✦ **Single hadron** FF.

$$D_{1q}^{h}(z, P_\perp)$$

✦ **Dihadron** FF.

$$D_{1q}^{h_1,h_2}(z_1, z_2, P_{1\perp}, P_{2\perp}, \boldsymbol{P}_{1\perp}\cdot\boldsymbol{P}_{2\perp})$$

two-hadron correlations

TWO-HADRON SIDIS

- Cross Section in terms of **Total and Relative Momenta**

$$\boldsymbol{P}_h = \boldsymbol{P}_1 + \boldsymbol{P}_2 \qquad \boldsymbol{R} = \frac{1}{2}(\boldsymbol{P}_1 - \boldsymbol{P}_2)$$

- The Sivers term:

$$\sigma_S = S_T \left(\sigma_T \frac{P_{hT}}{M} \sin(\varphi_T - \varphi_S) + \sigma_R \frac{R_T}{M} \sin(\varphi_R - \varphi_S) \right)$$

$$\int d\varphi_R \; \sigma_S = S_T \left(\sigma_{T,0} \frac{P_{hT}}{M} + \sigma_{R,1} \frac{R}{2M} \right) \sin(\varphi_T - \varphi_S)$$

$$\int d\varphi_T \; \sigma_S = S_T \left(\sigma_{T,1} \frac{P_{hT}}{2M} + \sigma_{R,0} \frac{R}{M} \right) \sin(\varphi_R - \varphi_S)$$

✦ **Non-vanishing σ_R is new! Contradiction with earlier results** **Bianconi: PRD62, 034008 (2000)** **? No:** **Kotzinian: EPJConf. 85 02026 (2015)**

$$\boldsymbol{R}^P \equiv \boldsymbol{R} - (\boldsymbol{R} \cdot \hat{\boldsymbol{P}}_h)\hat{\boldsymbol{P}}_h \qquad \boldsymbol{R}^P \simeq \xi_2 \boldsymbol{P}_1 - \xi_1 \boldsymbol{P}_2$$

$$\boldsymbol{R}_T^P \simeq \xi_2 \boldsymbol{P}_{1\perp} - \xi_1 \boldsymbol{P}_{2\perp} \qquad \xi_i \equiv z_i/(z_1 + z_2)$$

No k_T dependence at LO! No contradiction, different R !

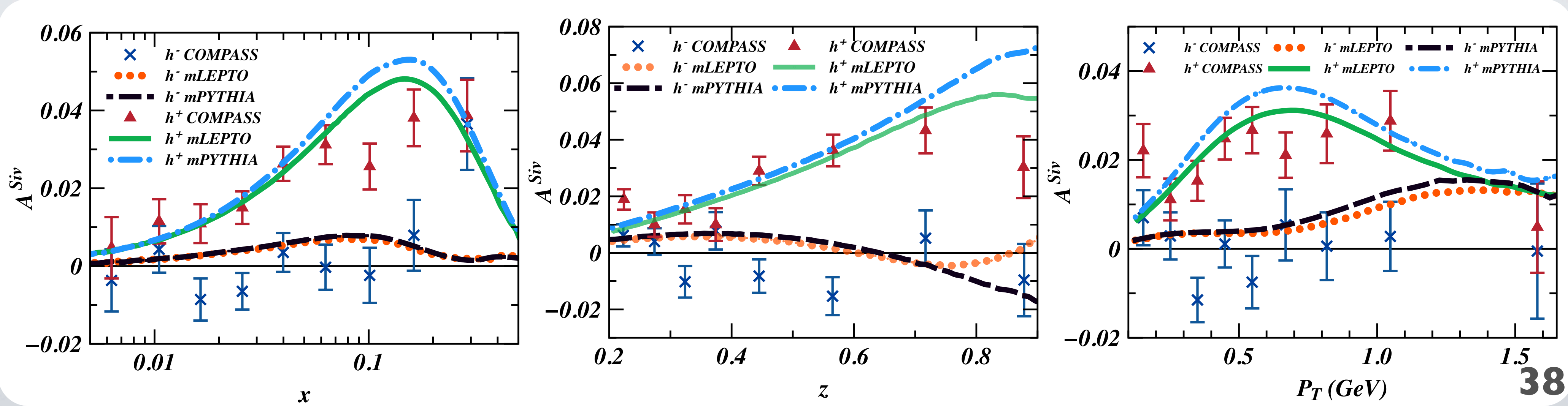
mPYTHIA 6.4

Sivers Effect in PYTHIA and Simulations for CLAS12 and EIC

EVENT GENERATORS + SIVERS EFFECT

Kotzinian, H.M., Thomas: PRL.113, 062003 ; PRD.90, 074006 ; 1407.6572 (2014);

- Two-hadron Sivers SSA *need* dihadron FF: yet unknown.
- Event generators allow to study *exp. kinematics effects*.
- Sivers effect modulates quark's azimuthal angle: *relatively easy* to include in MC generators.
- Use Sivers PDF extraction from *Torino group*.
- mLPETO used for COMPASS. Earlier studies + Cahn effect, also for CLAS.
- mLEPTO and mPYTHIA agree pretty well.



LO APPROXIMATION FOR SSA

- Fits for *Sivers PDF* from HERMES and COMPASS data utilize *LO DIS-only* expressions for *SSAs*.

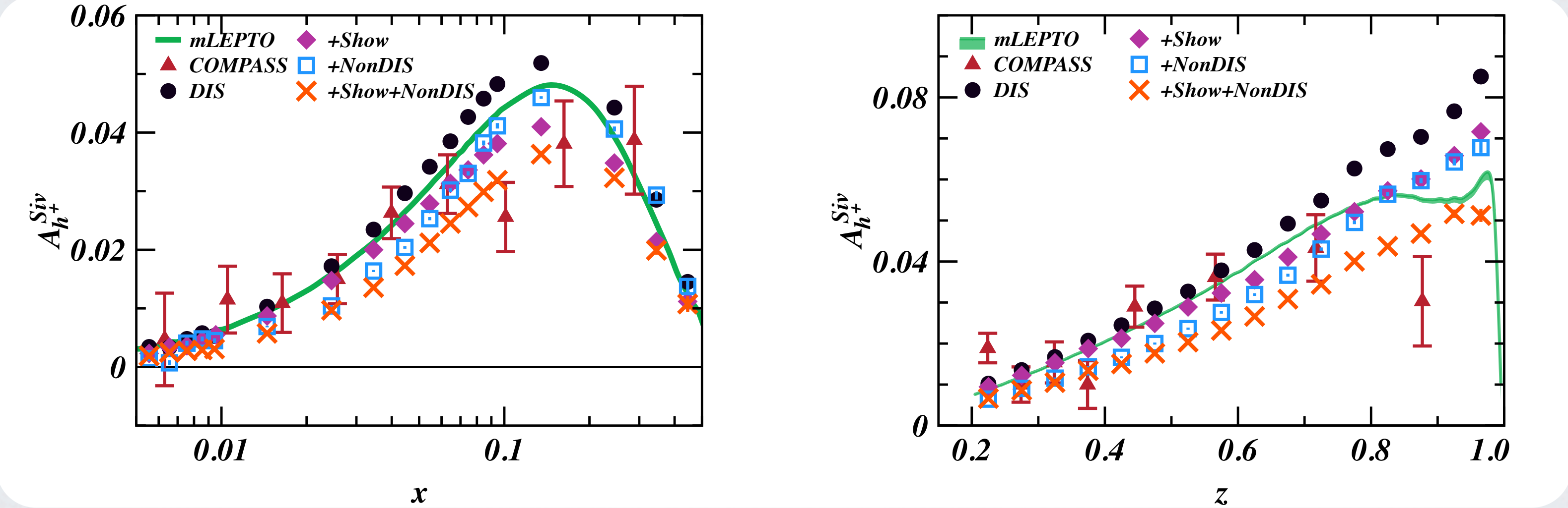
M. Anselmino et. al.: PRD 86, 014028 (2012).

$$A_{UT}^{\sin(\phi_h-\phi_S)} = \frac{\sum_q \int d\phi_S d\phi_h d^2\boldsymbol{k}_\perp \Delta^N \hat{f}_{q/p^\uparrow}(x,k_\perp,Q)\sin(\varphi-\phi_S)\frac{d\hat{\sigma}^{\ell q\to\ell q}}{dQ^2}\hat{D}_q^h(z,p_\perp,Q)\sin(\phi_h-\phi_S)}{\sum_q \int d\phi_S d\phi_h d^2\boldsymbol{k}_\perp \hat{f}_{q/p}(x,k_\perp,Q)\frac{d\hat{\sigma}^{\ell q\to\ell q}}{dQ^2}\hat{D}_q^h(z,p_\perp,Q)}.$$

- **Is this justified at COMPASS energies?**

H.M et al., arXiv:1502.02669 (2015).

- ***Test using mPYTHIA: turn on non-DIS effects (VMD, GVMD, "direct") and parton showering (QCD+QED).***



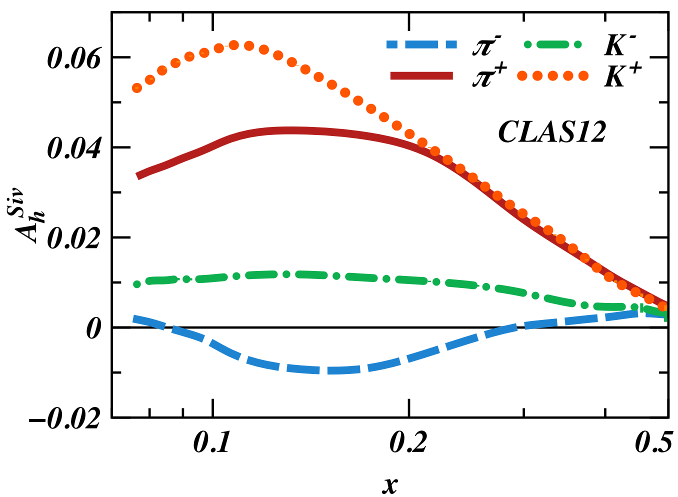
- **Significant** effects, but still agrees with data**!**
- Current Sivers PDF extractions *may* be underestimated.

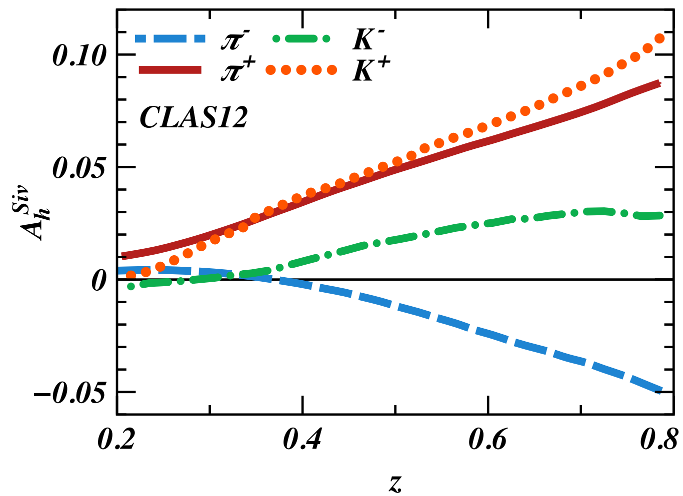
Sivers SSAs at CLAS12

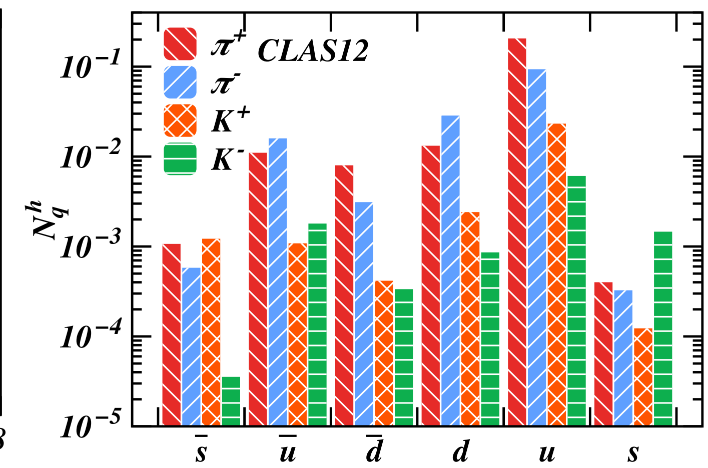
H.M et al., arXiv:1502.02669 (2015).

❖ ***Exploring the large x region.***

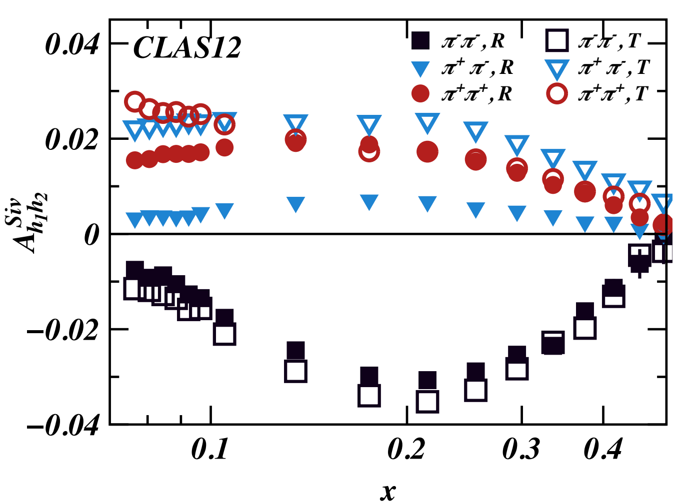
✦ **Single hadron** SSAs.

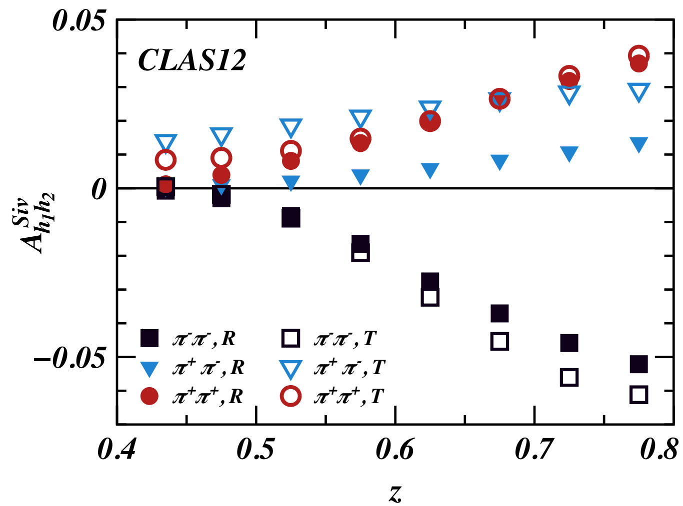


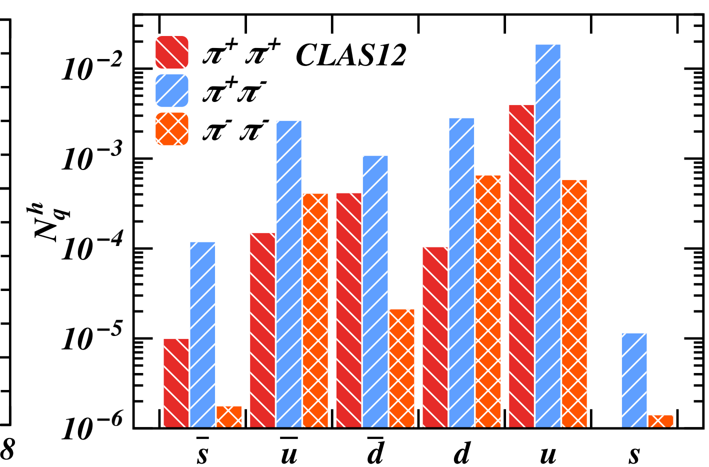




✦ **Dihadron** SSAs for pion pairs: identical pairs via z-ordering $z_1 \geq z_2$





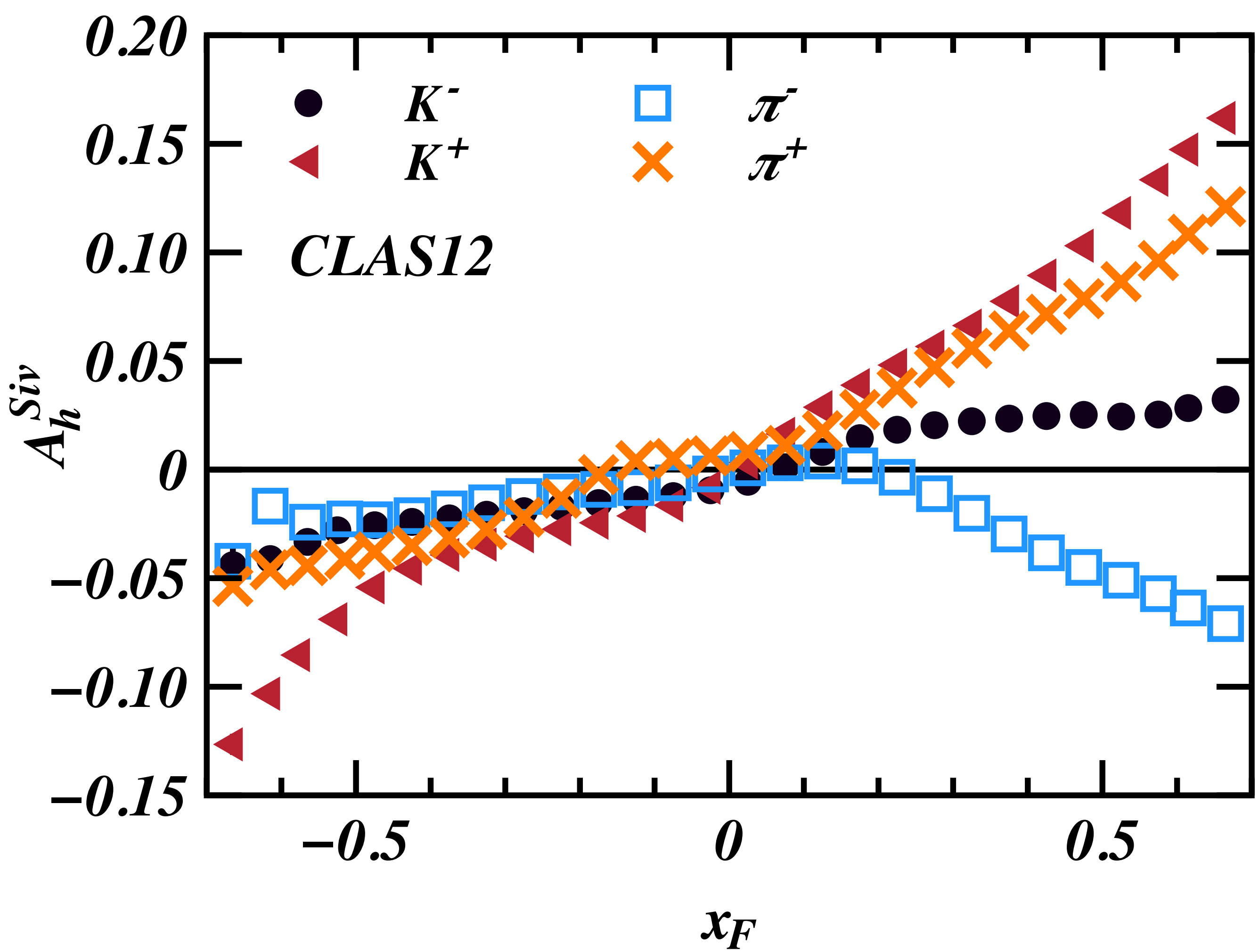


✦ Both Single and Dihadron SSAs are comparable in size!

Sivers SSAs at CLAS12

H.M et al., arXiv:1502.02669 (2015).

- ***Explore Target Fragmentation Regions*** $x_F < 0$.

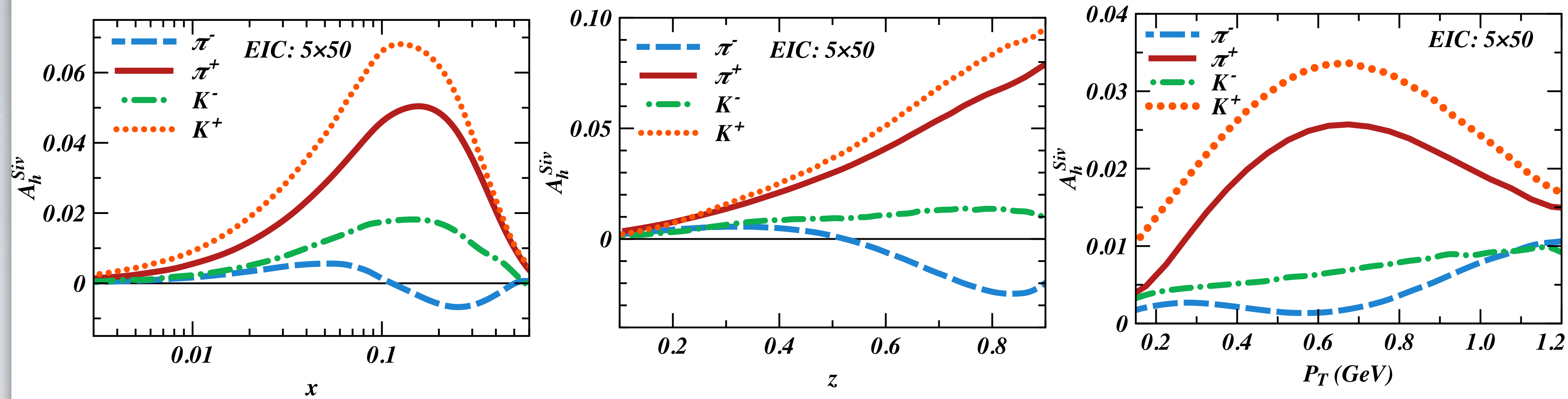


- Sivers SSA changes sign in some channels, fragmentation of nucleon remnant (recoil TM)!

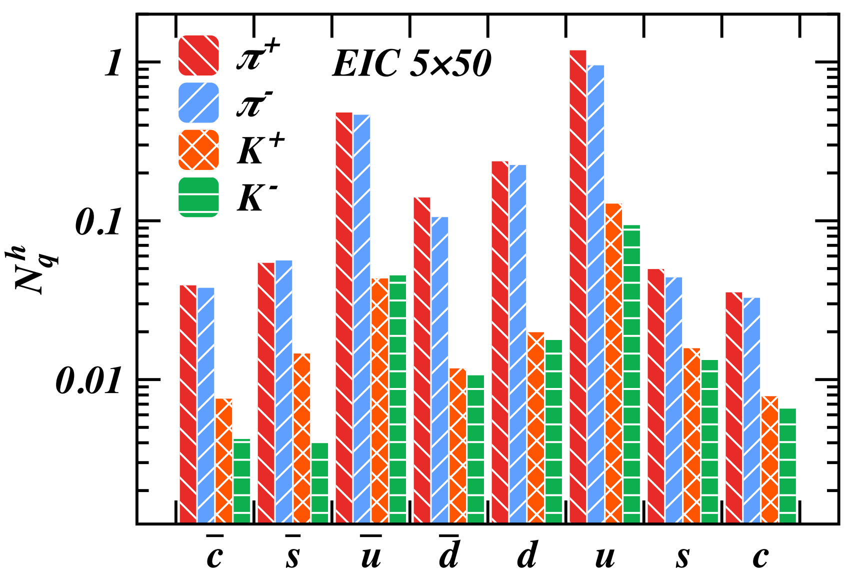
mPYTHIA RESULTS FOR EIC: ONE H

H.M et al., arXiv:1502.02669 (2015).

✦ SSAs for charged pions and kaons from ***proton*** target - low x region.



✦ **Average** number of hadrons by struck quark flavor.

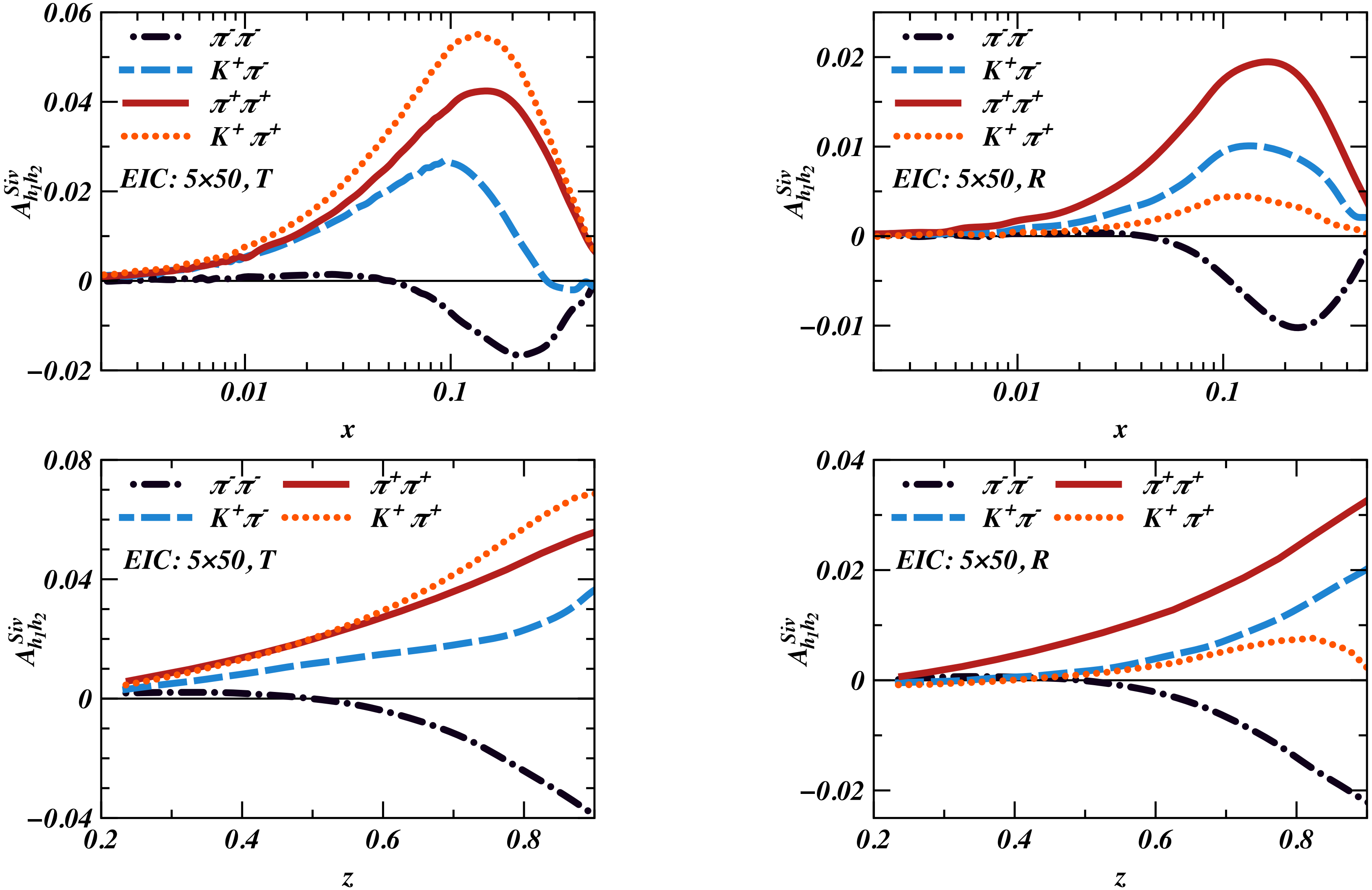


✦ π^+ multiplicities larger than K^+, but kaon SSAs are larger. Up quark dominates the multiplicities.

Dihadron Sivers SSAs for EIC

H.M et al., arXiv:1502.02669 (2015).

✦ Identical pairs via z-ordering: $z_1 \geq z_2$ (so $\sigma_R \neq 0$)



- Dihadron SSAs are *comparable* to single hadron ones!

(the one- and two-hadron FFs should mostly cancel in the ratios)

CONCLUSIONS

- ***TMDs*** describe the spin and momentum correlations of partons inside of the nucleon.

- They are essentially ***non-perturbative*** objects that should be extracted from experiments such as ***SIDIS*** and ***Drell-Yan***.

- A lot of effort in various areas: Theory (Factorization, Universality, Evolution), Experiment (Unfolded multiplicity data, SSAs), Phenomenology (Models, Empirical Extractions), Lattice QCD (TM widths, Sivers and Boer-Mulders PDFs).

- Current empirical extractions ***suffer from both*** sizable experimental errors, large uncertainties in knowledge of FFs, and lack of detailed understanding of some theoretical aspects (NP input in evolution, etc).

CONCLUSIONS II

- Precise data from future experiments: SIDIS (JLab 12GeV, EIC), DY(COMPASS II), e^+e^-(BELLE II) crucial for reliable extraction of both TMD PDFs and FFs in a global fit.

- Using new methods, such as ***Two-Hadron SIDIS*** will provide an additional information for mapping the TM and flavor dependences of TMDs.

- Development of full Event Generators that *incorporate TMD physics* (spin-orbit correlations, polarized parton fragmentation, evolution?, etc) will provide an important tool to both phenomenology and experiment for a detailed understanding of the experimental results and a reliable extraction of TMDs.


Thanks!
Photo by Jun Zhang

BACKUP SLIDES

SIVERS SSA MEASUREMENTS IN SIDIS

- **Sivers Single Spin Asymmetry:**

$$\langle \sin(\phi - \phi_S) \rangle^h_{UT} \equiv \frac{\int d\phi_h d\phi_S \sin(\phi_h - \phi_S)[d\sigma(\phi_h, \phi_S) - d\sigma(\phi_h, \phi_S + \pi)]}{\int d\phi_h d\phi_S [d\sigma(\phi_h, \phi_S) + d\sigma(\phi_h, \phi_S + \pi)]}$$

$$A^P_{Siv} \equiv 2\langle \sin(\phi - \phi_S) \rangle^h_{UT}$$

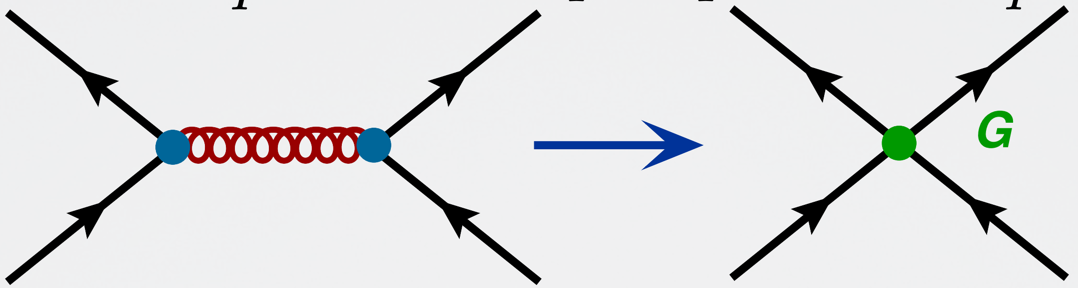
$$\langle \sin(\phi - \phi_S) \rangle^h_{UT} \sim \frac{\mathcal{C}[f_{1T}^{\perp,q} \;\; D_1^{h/q}]}{\mathcal{C}[f_1^q \; D_1^{h/q}]}$$

NAMBU--JONA-LASINIO MODEL

Effective Quark model of QCD

- Effective Quark Lagrangian

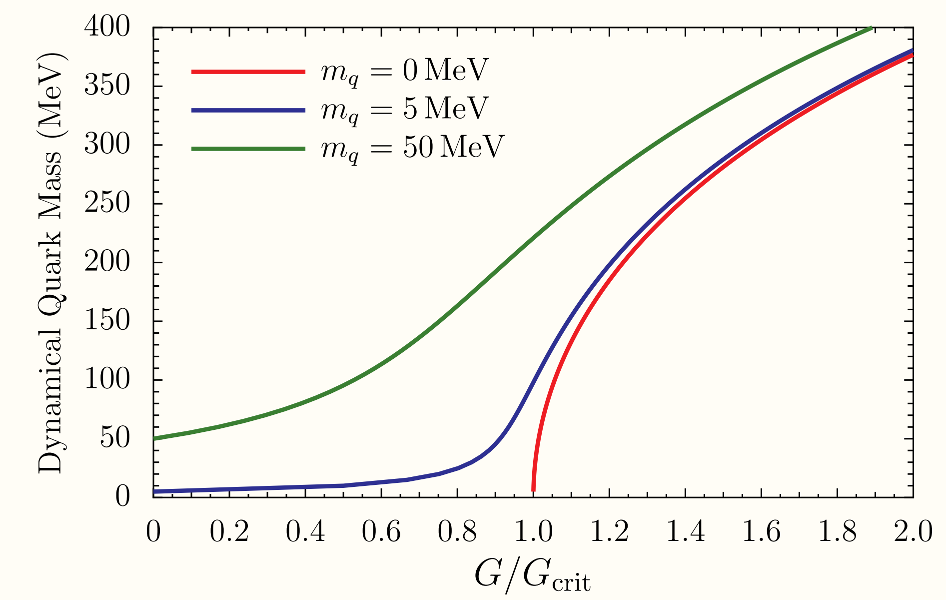
$$\mathcal{L}_{NJL} = \overline{\psi}_q(i\partial\!\!\!/ - m_q)\psi_q + G(\overline{\psi}_q\Gamma\psi_q)^2$$



- Low energy chiral effective theory of QCD.

- Covariant, has the same flavor symmetries as QCD.

- Dynamically Generated Quark Mass from GAP Eqn.



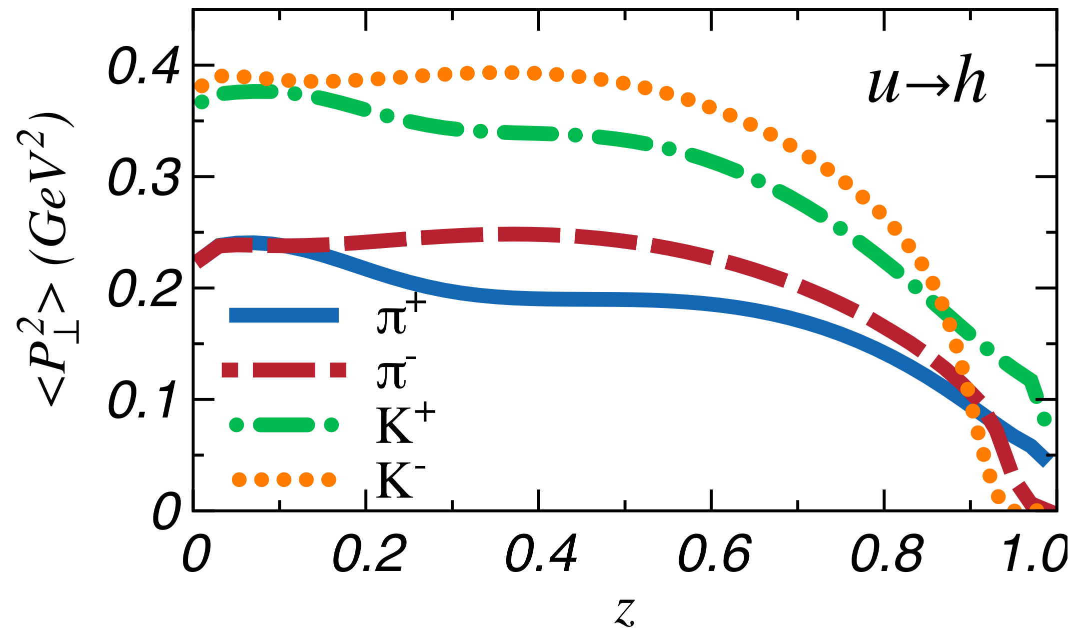
AVERAGE TRANSVERSE MOMENTA VS Z

FRAGMENTATION

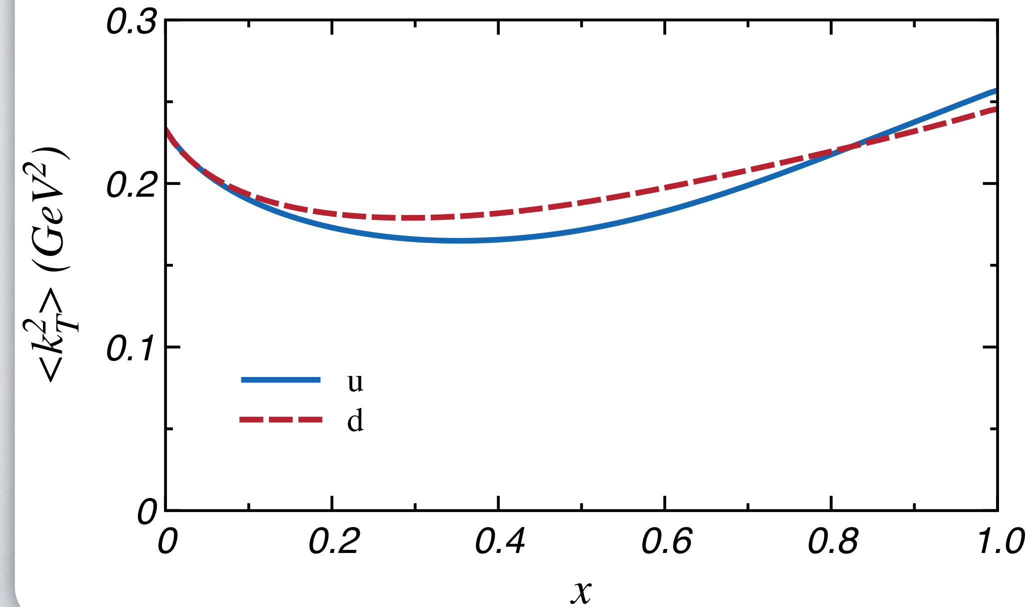
$$\langle P_\perp^2 \rangle_{unf} > \langle P_\perp^2 \rangle_f$$

- Indications from HERMES data:

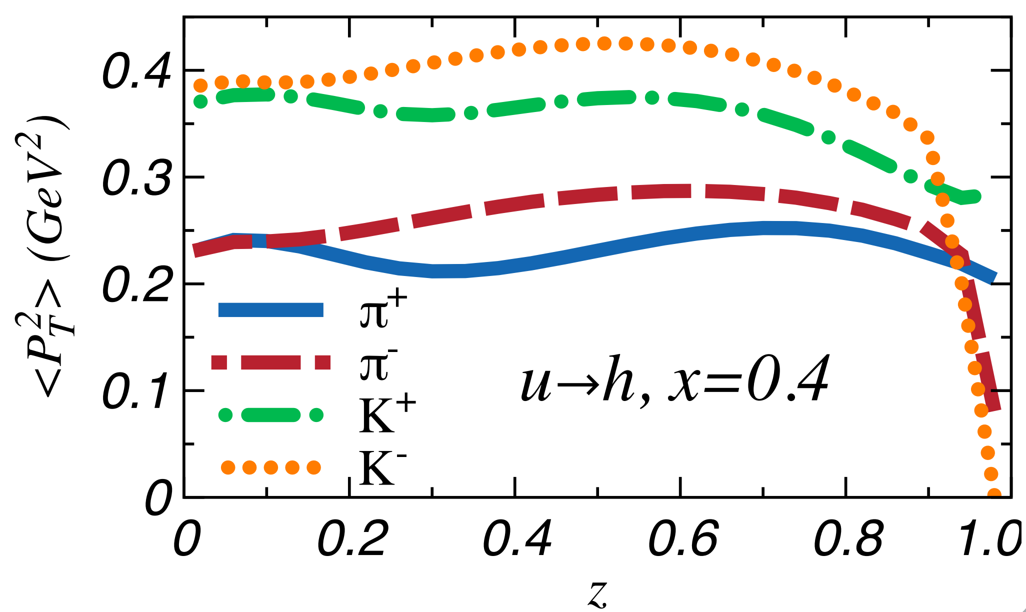
A. Signori, et al: JHEP 1311, 194 (2013)



PDF



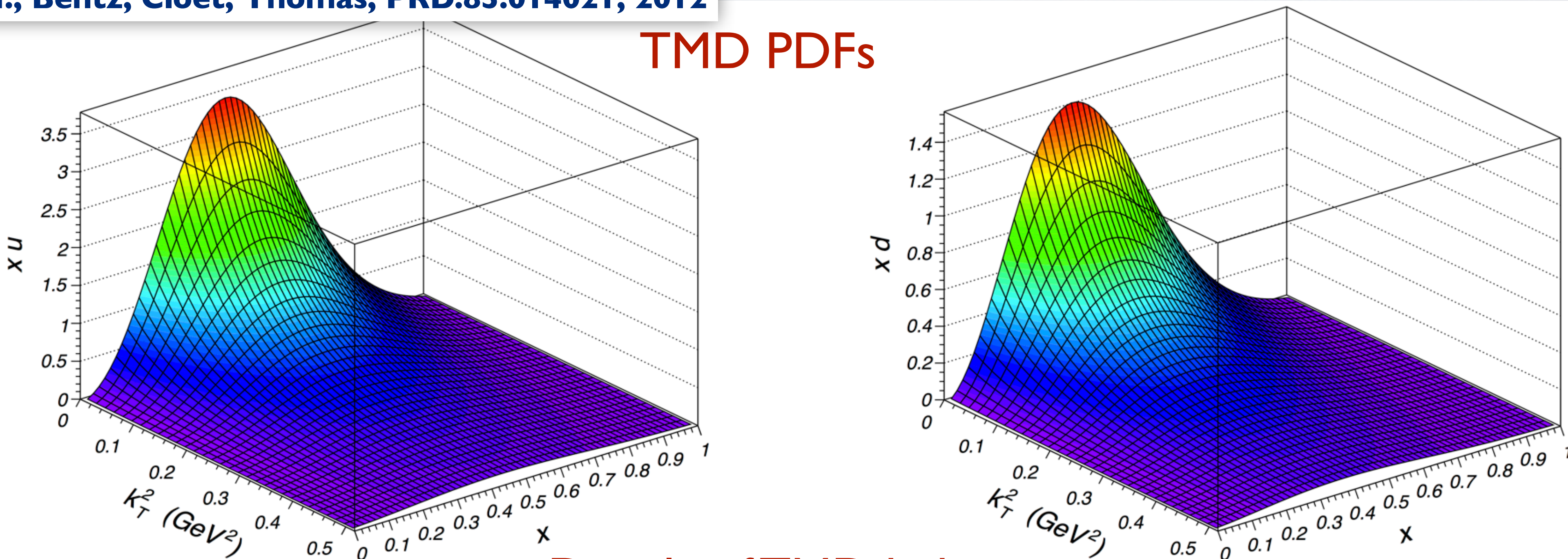
SIDIS



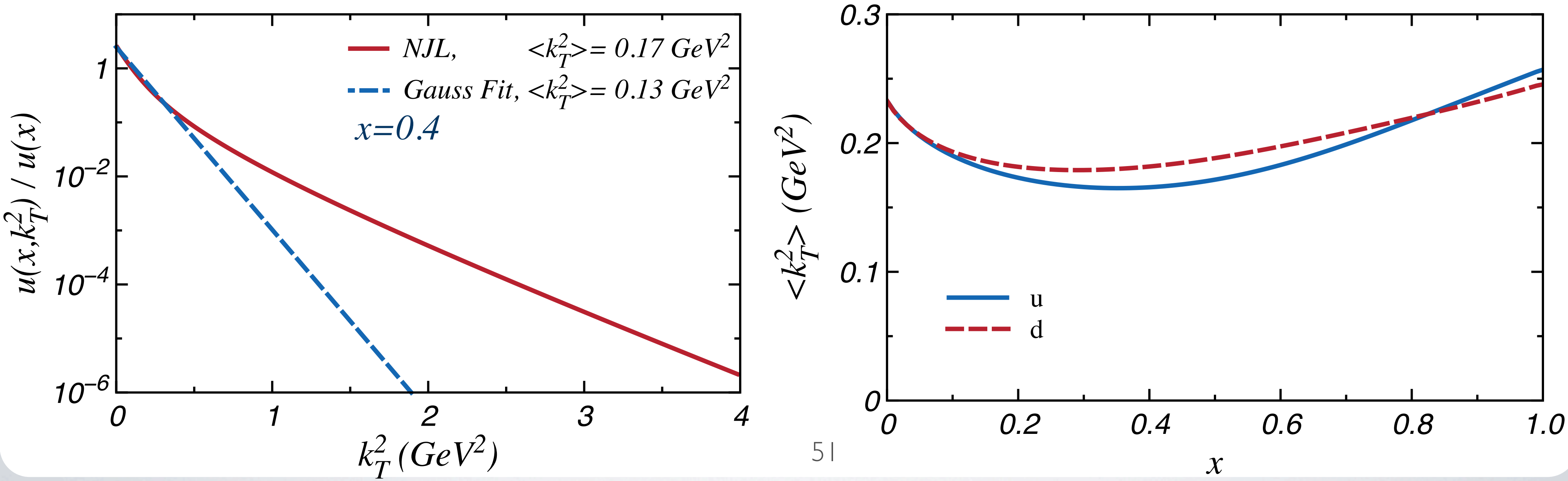
NJL: NUCLEON PDFS - TMD RESULTS

H.M., Bentz, Cloet, Thomas, PRD.85:014021, 2012

TMD PDFs



Details of TMD behavior



Link to GPDs and Quark $\vec{J}$

Bacchetta, Radici: PRL 107, 212001 (2011).

- Model assumption: Moment of *Sivers PDF* relates to GPD E via "lensing" function.

$$f_{1T}^{\perp(0)q}(x;Q_L^2) = -L(x)E^q(x,0,0;Q_L^2)$$

- Lensing Function from spectator model calculation.
- Fit Sivers Function parametrization to experimental data.
- Use magnetic moment constraints for GPD E. GPD H from PDF.

$$\kappa^p = \int_0^1 \frac{dx}{3}\left(2E^{u_v}(x,0,0) - E^{d_v}(x,0,0) - E^{s_v}(x,0,0)\right)$$

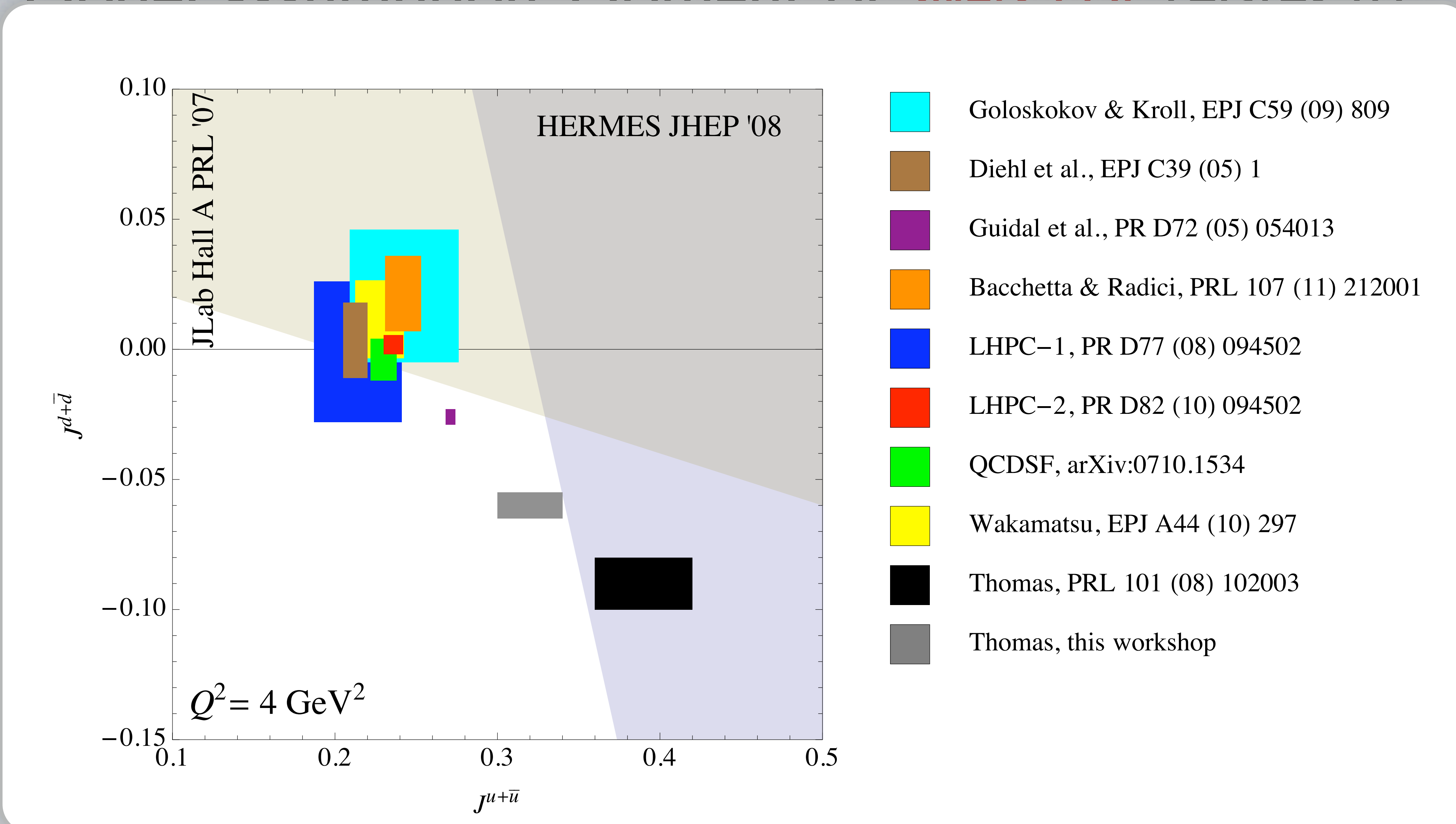
$$\kappa^n = \int_0^1 \frac{dx}{3}\left(2E^{d_v}(x,0,0) - E^{u_v}(x,0,0) - E^{s_v}(x,0,0)\right)$$

$$J^q = \frac{1}{2}\int_0^1 dx\, x\left(H^q(x,0,0) + E^q(x,0,0)\right)$$

Link to GPDs and Quark $\vec{J}$

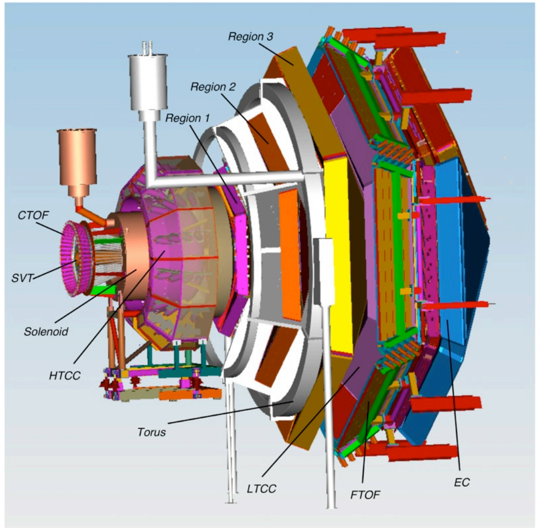
Bacchetta, Radici: PRL 107, 212001 (2011).

- Model assumption: Moment of *Sivers PDF* relates to



$$J^q = \frac{1}{2}\int_0^1 dx\,x\,(H^q(x,0,0) + E^q(x,0,0))$$

CLAS12 @ JLAB 12GeV



- Upcoming SIDIS experiment, 1H and 2H
- 11 GeV electron off polarized proton target.
- Access to large x region of nucleon structure.
- We use mPYTHIA for SIDIS predictions.

- Include the kinematical cuts on $x, Q^2, W, \theta_{e'}, \theta_h, M_{Mis}, z, \ldots$

$$0.075 \leq x \leq 0.532$$

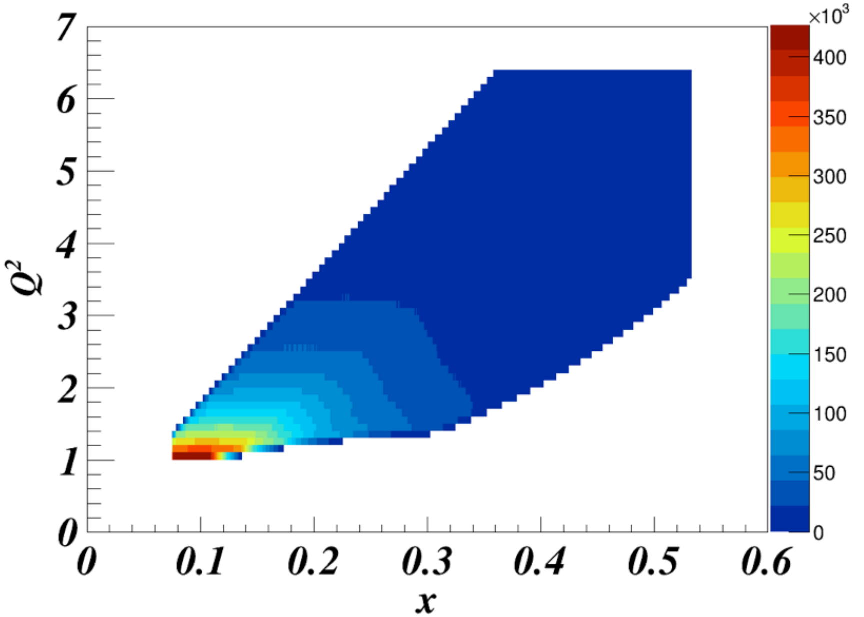
$$1 \text{ GeV} \leq Q^2 \leq 6.3 \text{ GeV}$$

$$W \geq 2 \text{ GeV}$$

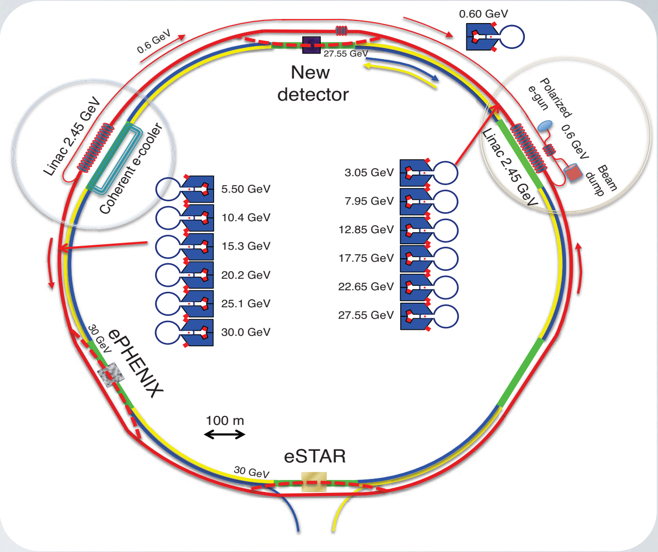
$$MMis_{(ep)-(e'hX)} \geq 1.5 \text{ GeV}$$

$$MMis_{(ep)-(e'h_1h_2X)} \geq 1.5 \text{ GeV}$$

DIS kinematics

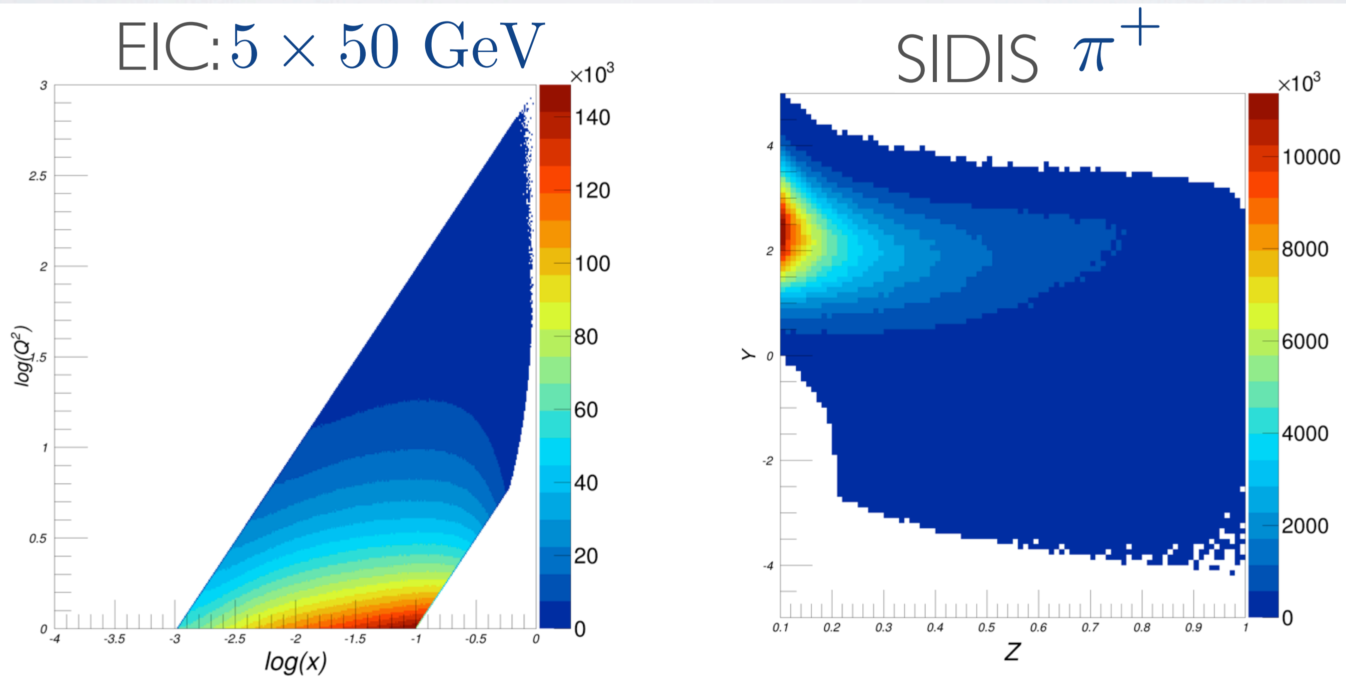


EIC: eRHIC



White Paper -- Accardi et. al. : 1212.1701(2012).

- EIC using RHIC + electron ring.
- Various proposed beam momenta: $l_e \times P_N$
- We use mPYTHIA for SIDIS predictions.

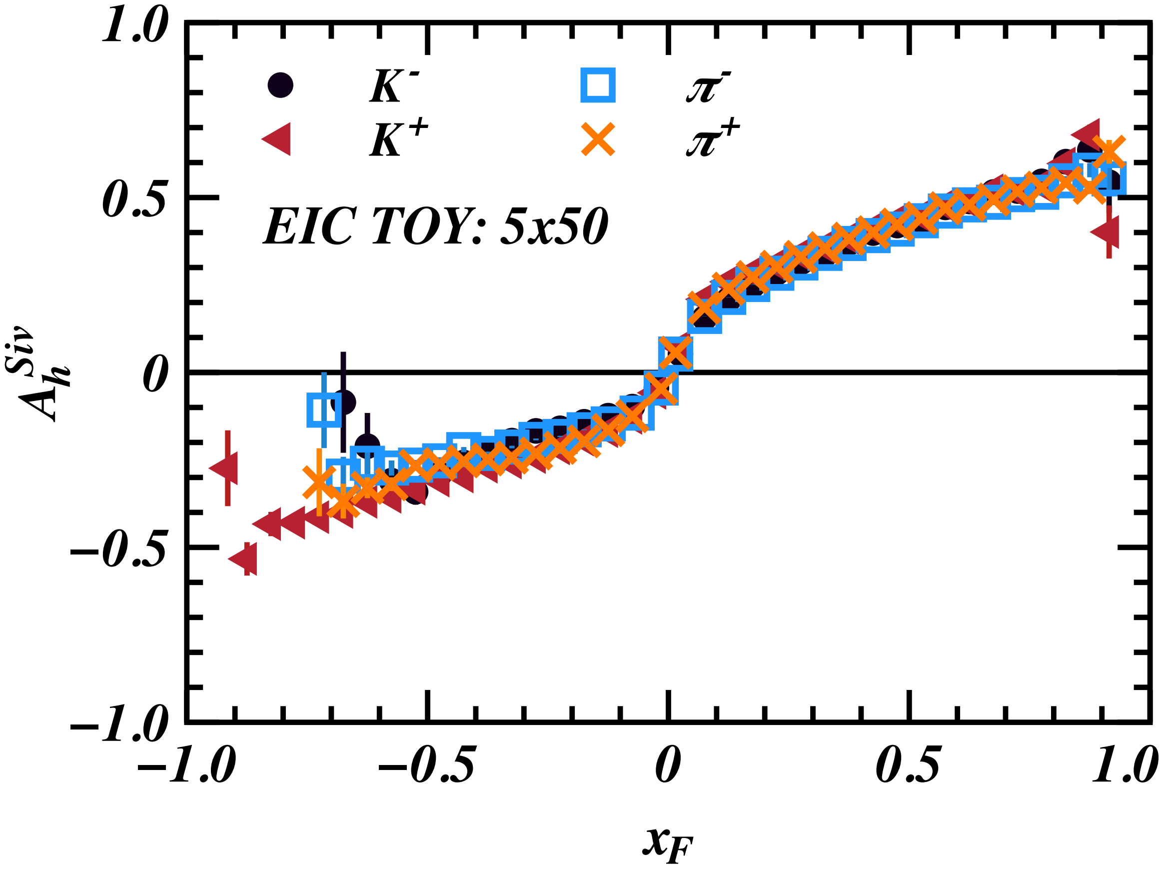


TOY MODEL :TARGET FRAGMENTATION

- What can we learn about Sivers PDF at EIC?
- Use a TOY model for Sivers PDF to explore.

$$f_{\uparrow}^{q,Toy}(x, \boldsymbol{k}_T) = f_1^q(x, k_T)[1 + 0.9\sin(\varphi_q - \varphi_S)]$$

- ***Explore Target Fragmentation Regions*** $x_F < 0$.



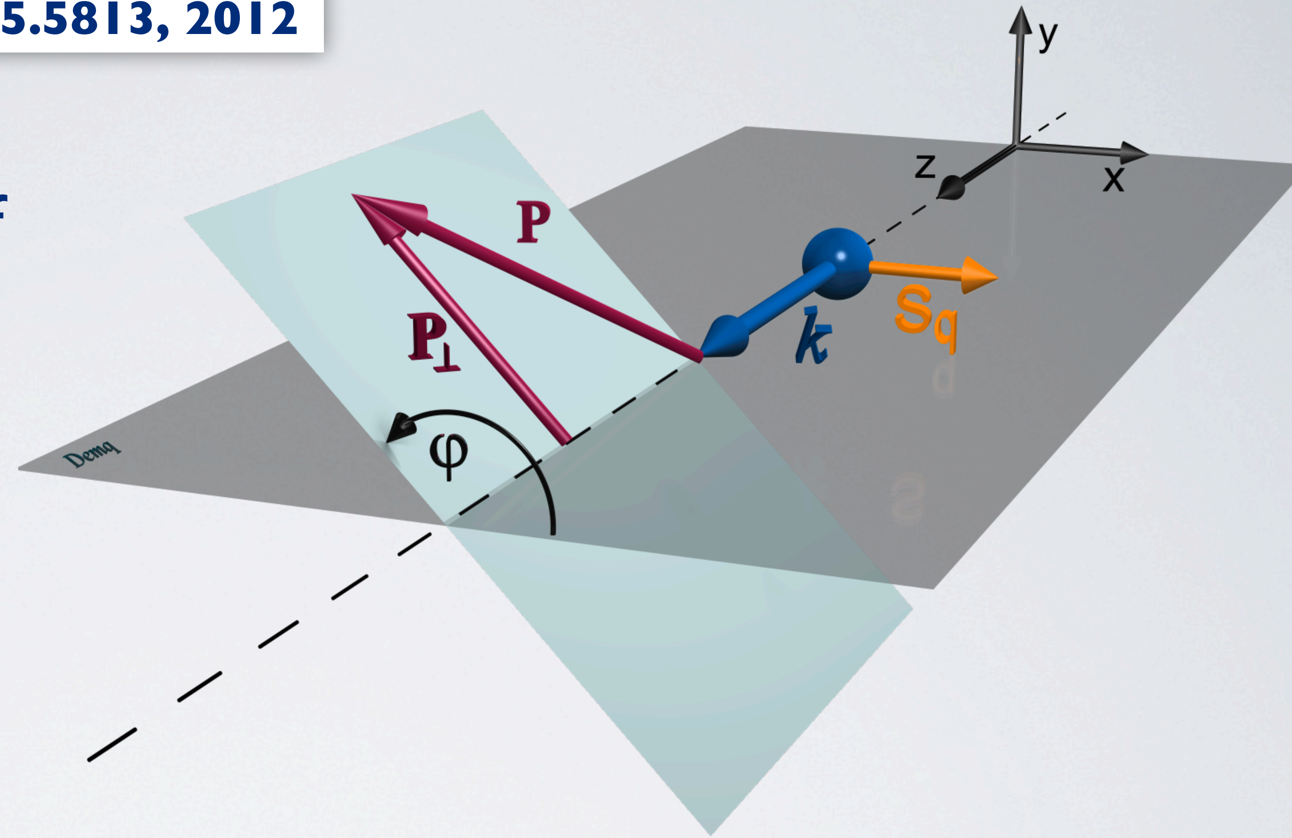
- Sivers SSA changes sign, fragmentation of nucleon remnant!

COLLINS FRAGMENTATION FUNCTION

H.M., Thomas, Bentz, arXiv:1205.5813, 2012

- **Collins Effect:**

  Azimuthal Modulation of Transversely Polarized Quark's Fragmentation Function.

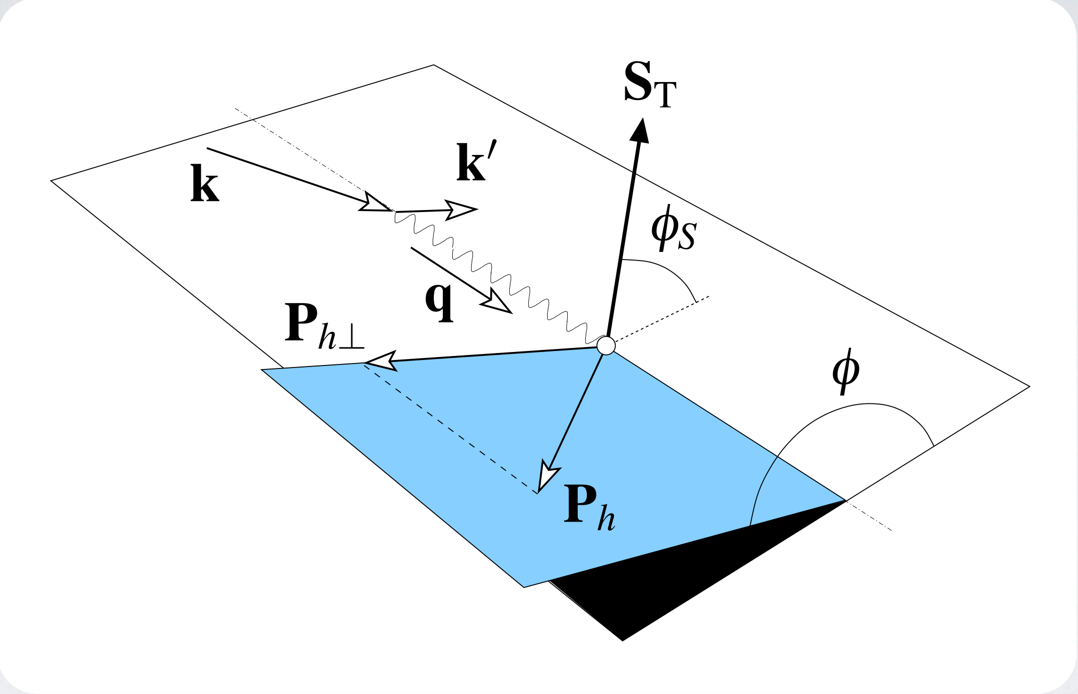


Unpolarized

$$D_{h/q^\uparrow}(z, P_\perp^2, \varphi) = D_1^{h/q}(z, P_\perp^2) - H_1^{\perp h/q}(z, P_\perp^2)\frac{P_\perp S_q}{z m_h}\sin(\varphi)$$

Collins

- **Chiral-ODD: Needs to be coupled with another chiral-odd quantity to be observed.**

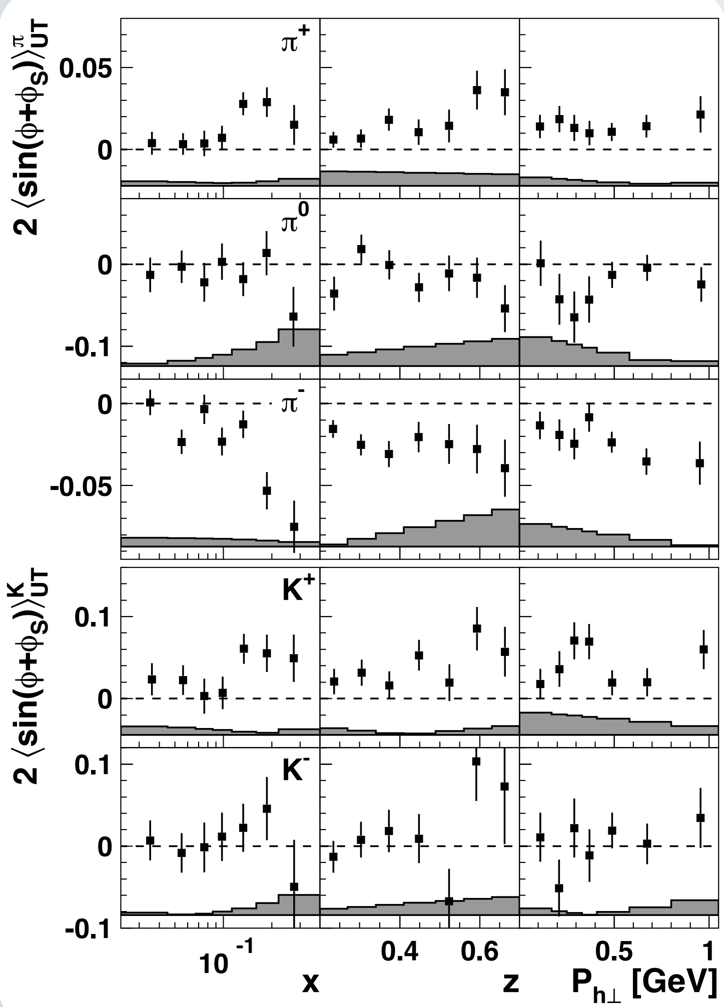


TRANSVERSITY I:
ONE HADRON SIDIS AND COLLINS FRAGMENTATION FUNCTION

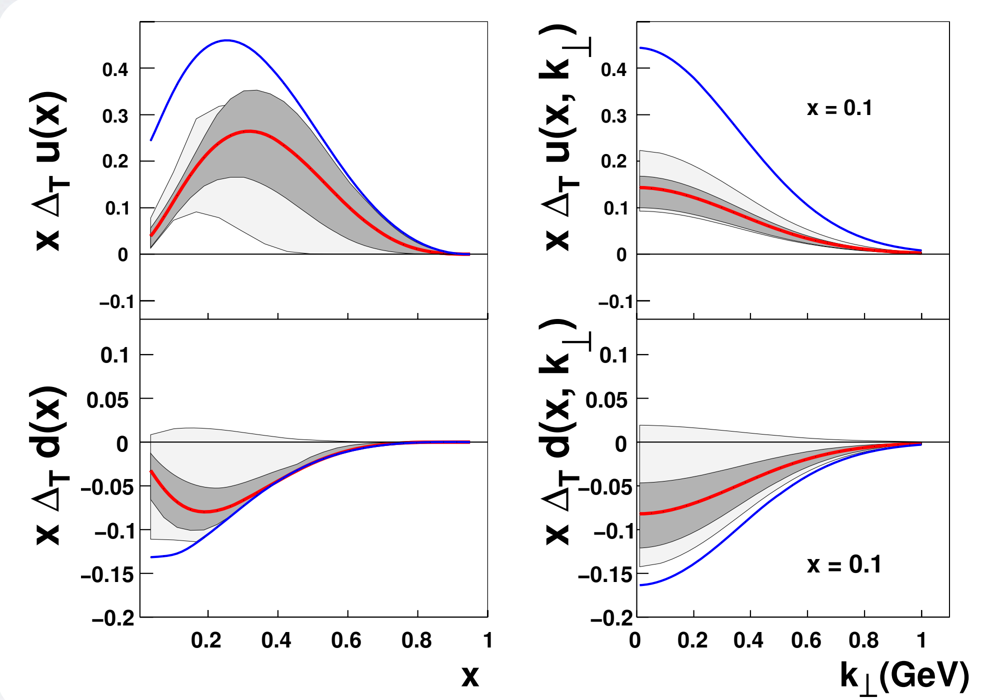
EMPIRICAL EXTRACTIONS OF TRANSVERSITY

- ***SIDIS at HERMES***
  PLB693 (2010) 11-16.

$$\langle \sin(\phi+\phi_S)\rangle^h_{UT} \sim \frac{\mathcal{C}[h_1^q\ H_{1q}^{\perp h/q}]}{\mathcal{C}[f_1^q\ D_1^{h/q}]}$$



- Opposite sign for the charged pions.
- Large positive signal for K^+.
- Consistent with 0 for π^0 and K^-.
- ***Fits to HERMES, COMPASS and BELLE:*** **NPB (Proc. Suppl.) 191 (2009) 98-107.**

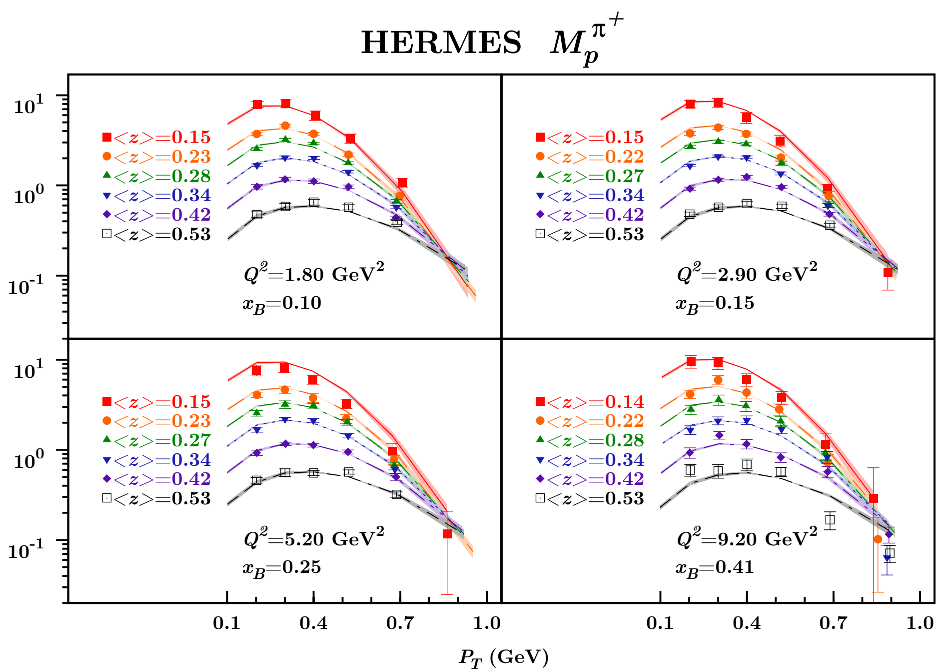


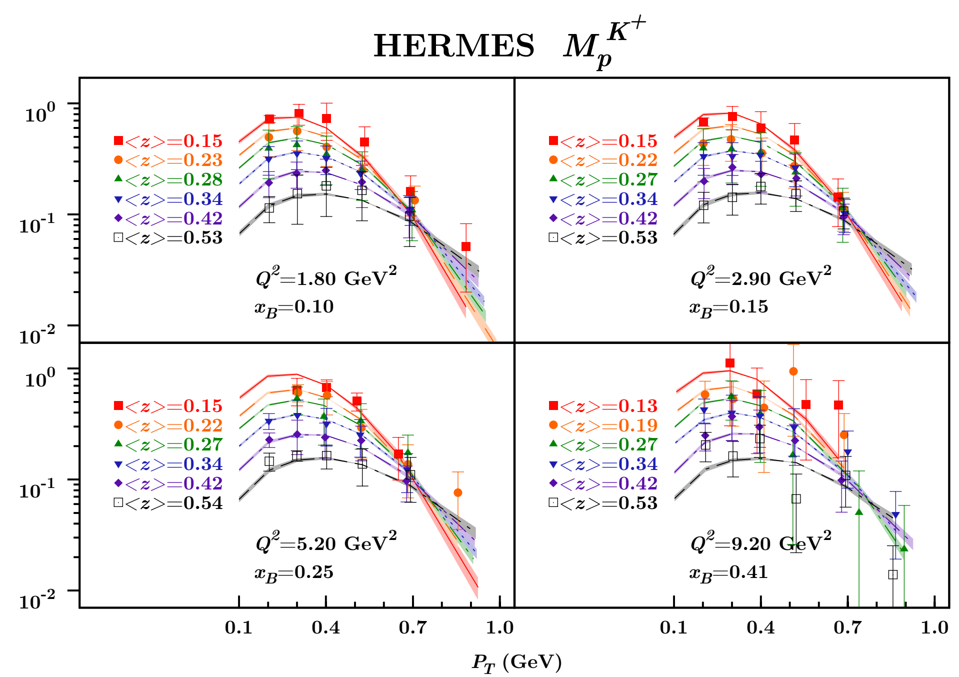
- **Large Uncertainties!**
- **Simplistic Approximations !**

EMPIRICAL EXTRACTIONS OF AVERAGE TM

M. Anselmino et. al.: JHEP 1404 (2014) 005.

✓ *HERMES fit.*





EMPIRICAL EXTRACTIONS OF AVERAGE TM

M. Anselmino et. al.: JHEP 1404 (2014) 005.

✓ *COMPASS fit.*