Reggeon Field Theory and Exact Renormalization Group

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Outline

- Motivation Pomerons y Reggeons
- Exact Renormalization Group Ideas
- RFT and ERG
- Results and Discussion
- Outlook

J. Bartels, C. Contreras and G. P Vacca arXiv: 1411.6670

Reggeon Field Theory before QCD

P.D.B. Collins, *An introduction to Regge theory and high energy physics*, Cambridge University Press, Cambridge, 1977.

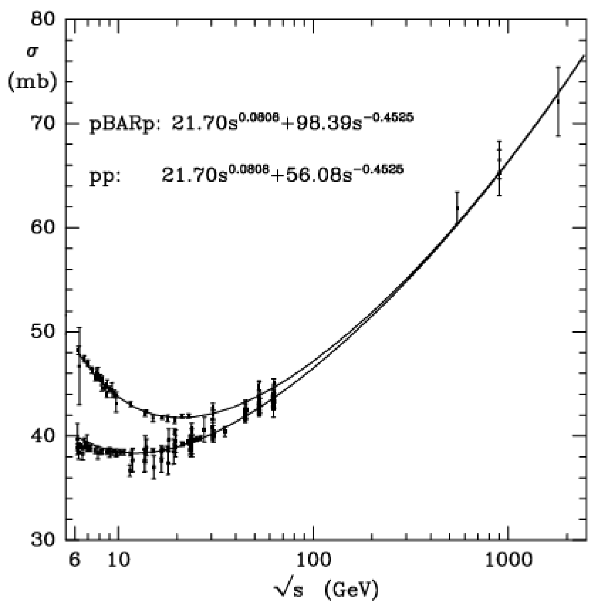
- V. N. Gribov introduce in the 60´s
- The description of scattering amplitud at high energies is according Regge Theory
- The exchange are "quasi particles" characterized by its Regge trajectories : $\alpha_i\,(t)$
- Leading Pole: is Called Pomeron with vaccum quantun numbers $\alpha(t) = \alpha_0 + \alpha' t = 1 + (\alpha_0 - 1) + \alpha' t$
- $\mu = \alpha_0 - 1$ is the pomeron intercept and α' is the slope
- According to the Regge theory the contribution to the total Cross section, is given by:

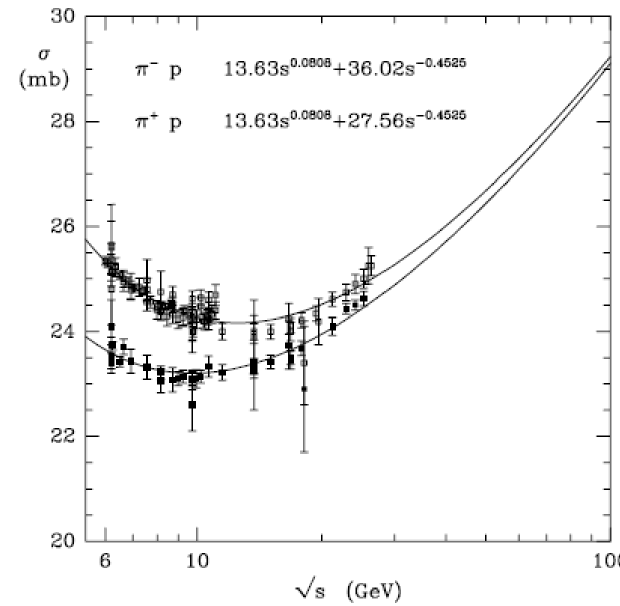
$$\sigma_T = A_i s^{\alpha_i(0)-1}$$

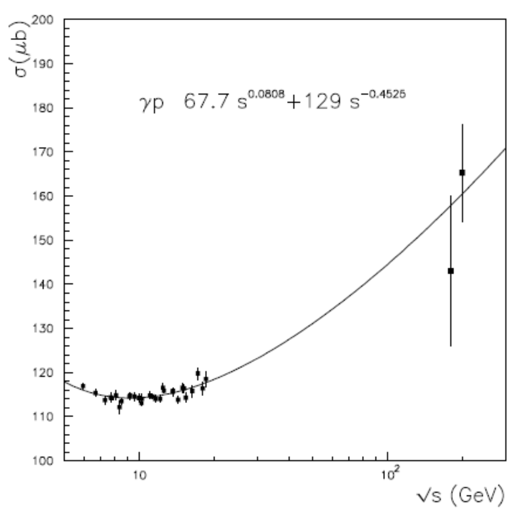
A. Donnachie and Landshoff : arXiv 1309.1292

- $\mu = \alpha_0 - 1 = 0.08$ and $\alpha' = 0.25$ GeV−2

$$\alpha_P(t) = 1.08 + 0.25\,(\mathrm{GeV}^{-2})\,t$$







- Unitarity in the t-channel introduce Multipomerons states and interactions.
- Triple Pomeron exchange term plays a role to describe experimental results
- Local Field Theory for this new fields Reggeons: RFT

$$\psi(t,\boldsymbol{x}) \; y \; \psi^T(t,\boldsymbol{x})$$

in D=2+1 dimensions. (t as rapidity and $\boldsymbol{x}_\perp$ tranverse space)

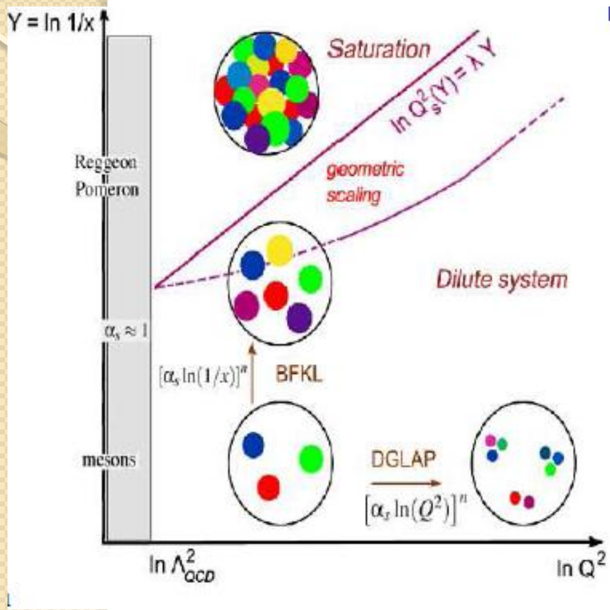
- Study of general properties of the action
- One can consider this as a realization of a PT symmetric QFT (C. Bender).
- Toy models of 0+1 dimensions (in practice a PT symmetric Quantum Mechanics) were studied long ago and also recently
- 1980 Cardy y Sugar found that the RFT is in the same Universality class of " Percolation"
- The action:

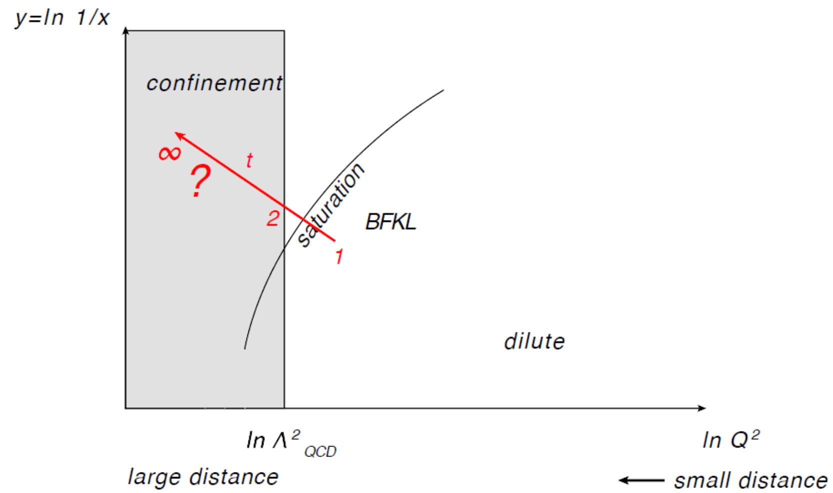
$$\mathcal{L} = (\frac{1}{2}\psi^\dagger \overleftrightarrow{\partial_y} \psi - \alpha' \psi^\dagger \nabla^2 \psi) - \mu \psi^\dagger \psi + i\lambda \psi^\dagger (\psi^\dagger + \psi)\psi + g(\psi^\dagger \psi)^2 + g' \psi^\dagger ({\psi^\dagger}^2 + \psi^2)\psi + \cdots$$

QCD description

- One might hope that QCD in the high energy limit might behave in such a way that scattering amplitudes can describe the Pomeron (RFT).
- The simplest version leads to the BFKL Pomeron which has been studied up to NLO in perturbation theory as composite states of Reggeized gluons. Their interaction, local in rapidity, can be described by an effective action (Lipatov).
- Analysis of QCD diffractive processes in terms of Reggeized gluons lead to the construction of evolution kernels and transition vertices between a different number of Reggeized gluons in the crossed channel (Bartels).
- New triple pomeron vertex. This may be considered the building blocks of a BFKL reggeon field theory which can be seen as a projection in the Reggeized gluon field theory.
- Similar descriptions have been obtained with other approaches (dipole picture) and (Wilson lines, JIMWLK/KLWMIJ). For example a fan structure (without loops) associated to the triple pomeron vertex was encoded in the solution of the BK equation (Balitsky-Kovchegov).

- We concentrate here on the RFT effective description
- The goal is to apply some approximated non perturbative tool to investigate some features of it. Is there a possibility to see consistency with a QCD region?





- If we start from UV energy and we go to IR regime, how we can study this evolutions?
- Pomeron field has internal degrees of freedom (BFKL), nonlocal RFT
- Pomeron field changes as function of scale (rapidity and distance)
- Wilson → RG equation, flow equations
- Running parameters what is running??

Running parameters: what is running?

HERA forward jets

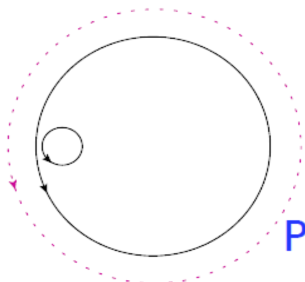
LEP

$\gamma^* \gamma^* \quad \sigma_{tot} \approx s^{\omega_{BFKL}}$

calculable in pQCD: $R^2 \sim 1/Q^2$

$$\varpi_{BFKL} = 4\,\frac{\alpha_s N_c}{\pi} \ln 2$$

HERA



$\gamma^* p \quad \sigma_{tot} \approx (W^2)^{\lambda}$

Partly calculable in pQD

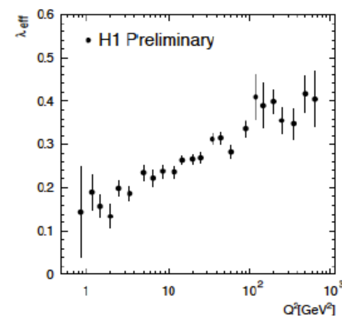
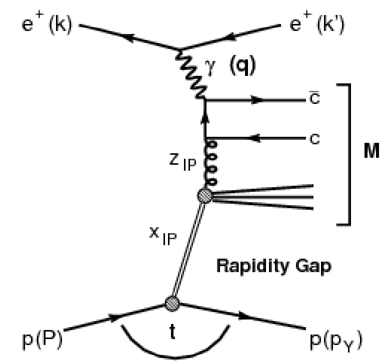
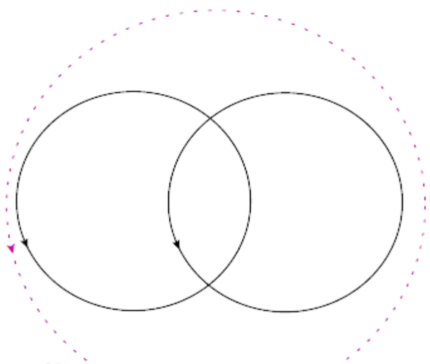


Figure 6: The slope $\lambda_{\rm eff}$ of F_2 as a function of Q^2 .



LHC



$p\,p \quad \sigma_{tot} \approx s^{0.08}$

nonperturbative: $R^2 = R_p^2 + R_p^2 + \alpha' \ln s$

Small: strong rise ---- large: slow rise

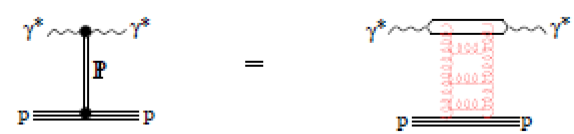


FIG. 4: The virtual photon interacts via its hadronic fluctuations which are $\bar{q}q$ dipoles and more complicated Fock states. The Pomeron exchange is illustrated as a perturbative ladder.

Soft Pomeron vs Hard Pomeron

- $$\alpha_P(0) \approx 1.08 \;\; soft$$

$$\alpha_P(0) \approx 1.4 \;\; Hard\ BFKL/QCD$$

- Soft Pomeron dominates Hadronic total cross section but hard pomeron dominate scattering of very small projectiles at large energies,
- In the dipole picture, hard pomeron dominates for small dipole sizes r

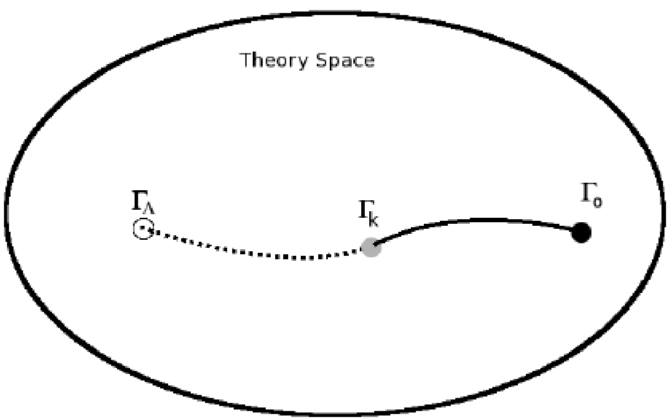
$\alpha_{P,k}(0)$ *can be considered as a variable which depends on the sizes of the projecties.*

How we can connect regions of different sizes and differents sorts of Pomerons

Use: *RFT and Functional Renormalization Group*

ERG vs normal pFT

- The effective Wilson-Action is defined by Integrations of d.o.f in the UV $k < p < \Lambda$
- This iterative approach give at the scale k the action $\Gamma_k(\phi)$
- Quantum action is given when $k \to 0$ for $\Gamma_k(\phi)$



Effective action Γ_k in theory space where ideally $\Lambda \to \infty$ and the dotted line indicates an integration of ϕ_a over momentum degrees of freedom with $p \geq k$.

Berges,Tetradis and Wetterich: hep-ph/0005122

- The evolution of the $\Gamma_k(\phi)$ is given by the FRG/ERG or Wetterich Flow Equation

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr} \left[\left(\Gamma_k^{(2)} + \mathcal{R}_k \right)^{-1} \partial_t \mathcal{R}_k \right] .$$

- where R_k(p) is the IR regulator

  - $\mathcal{R}_k(p^2) > 0$ for $p^2 \ll k^2$
  - $\mathcal{R}_k(p^2) \to 0$ for $p^2 \gg k^2$
  - $\mathcal{R}_k(p^2) \to \infty$ for $k \to \Lambda$ $(\to \infty)$

Optimized cutoff (Litim): $\mathcal{R}_k(p^2) = (k^2 - p^2)\theta(k^2 - p^2)$.

FRG Flows and Steps:

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr}\left[\left(\Gamma_k^{(2)} + \mathcal{R}_k \right)^{-1} \partial_t \mathcal{R}_k \right]$$

- Flow Equations: can be used in different theories {gravity, statistical mechanics, gauge,…}
- $\Gamma_k(\phi, g_i)$ define a **M dimensional Space of Coupling constants**
- **Local Expansion** of $\Gamma_k(\phi, g_i)) = \sum_i g_i(k) O_i(\phi)$ and

$\partial_t \Gamma_k \to \sum_i \partial_t g_i(k) * O_i(\phi) \to \beta_i(k)$ asociadas al $\{O_i\}$ operator basis.

Beta Funtions: $\beta_i(k) = \partial_t g_i(k)$

Fixed Points Conditions $\partial_t \Gamma^*{}_k \cong 0$ $\quad y \quad t = \ln(\frac{k}{\Lambda})$

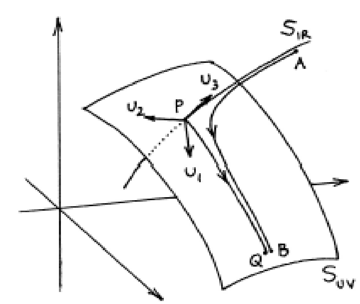
With a local truncations for $\Gamma_k(\phi, g_i)$ and scale invariance behaviour

Critical properties: $g_i \simeq g_i^* + \sum_a c_a e^{\lambda_a t} v_i^a$

Linealizations of the Flow close to a FP

If $\lambda_i > 0$ define a IR – Critical Surfase

With relevant behaviour, where $\lambda_i < 0$ is UV



$k < \Lambda \quad \to \quad IR\ ; \quad k > \Lambda \quad \to \quad UV$

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr}\left[\left(\Gamma_k^{(2)} + \mathcal{R}_k \right)^{-1} \partial_t \mathcal{R}_k \right].$$

- Action RFT $\Gamma_k = \int \mathrm{d}y \, \mathrm{d}^D x \left[Z_k \left(\frac{1}{2} \psi^\dagger \overleftrightarrow{\partial_y} \psi - \alpha'_k \psi^\dagger \nabla^2 \psi \right) + V_k(\psi, \psi^\dagger) \right]$

- Fourier Transformation $\Phi(\omega, q)^T = (\psi(\omega, q) \; \psi^\dagger(-\omega, q))$

- Regulator

$$\Gamma_k^{(2)} + \mathbb{R} = \begin{pmatrix} V_{k\psi\psi} & -iZ_k\omega + Z_k \alpha'_k q^2 + R_k + V_{k\psi\psi^\dagger} \\ iZ_k\omega + Z_k \alpha'_k q^2 + R_k + V_{k\psi^\dagger\psi} & V_{k\psi^\dagger\psi^\dagger} \end{pmatrix}$$

- Flow Equations

$$\dot{V}_k(\psi, \psi^\dagger) = \frac{1}{2} \mathrm{tr} \left\{ \int \frac{\mathrm{d}\omega \, \mathrm{d}^D q}{(2\pi)^{D+1}} \left[\left(\Gamma_k^{(2)} + \mathbb{R} \right)^{-1} \dot{\mathcal{R}}_k \right] \right\}$$

$$R_k(q) = Z_k\alpha'_k(k^2 - q^2)\Theta(k^2 - q^2)$$

$$\dot{R} = 2k^2 Z_k\alpha'_k\theta(k^2 - q^2)\left(1 - \frac{\eta_k + \zeta_k}{2}(1 - \frac{q^2}{k^2})\right)$$

($\eta_k = -\dot{Z}_k/Z_k$ and $\zeta_k = -\dot{\alpha}'_k/\alpha'_k$)

$$\eta_k = -\frac{1}{Z}\partial_t Z$$

$$\zeta_k = -\frac{1}{\alpha'}\partial_t\alpha'.$$

$$\dot{V}_k = 2Z_k\alpha'_k k^2\int\frac{d\omega d^Dq}{(2\pi)^{D+1}}\theta(k^2 - q^2)\frac{\left(Z_k\alpha'_k k^2 + V_{k\psi^\dagger\psi}\right)\left(1 - \frac{\eta_k+\zeta_k}{2}(1 - \frac{q^2}{k^2})\right)}{Z_k^2\omega^2 + (h_k + V_{k\psi\psi^\dagger})^2 - V_{k\psi\psi}V_{k\psi^\dagger\psi^\dagger}}$$

Dimensionless vs Dimensionful:

$$\tilde{\psi} = Z_k^{\frac{1}{2}}k^{-D/2}\psi$$

$$\tilde{\mu}_k = \frac{\mu_k}{Z_k\alpha'_k k^2}$$

$$\tilde{\lambda}_k = \frac{\lambda_k}{Z_k^{\frac{3}{2}}\alpha'_k k^2}k^{D/2}$$

$$\tilde{g}_k = \frac{g_k}{Z_k^2\alpha'_k k^2}k^D$$

$$\tilde{g'}_k = \frac{g'_k}{Z_k^2\alpha'_k k^2}k^D.$$

$$\tilde{V}_k = \frac{V_k}{\alpha'_k k^{D+2}},\ \tilde{V}_{k\tilde{\psi}\tilde{\psi}} = \frac{\delta^2\tilde{V}_k}{\delta\tilde{\psi}\delta\tilde{\psi}} = \frac{V_{k\psi\psi}}{Z_k\alpha'_k k^2}.$$

$$\dot{\tilde{V}}_k[\tilde{\psi}^\dagger, \tilde{\psi}] = (-(D+2) + \zeta)\tilde{V}_k[\tilde{\psi}^\dagger, \tilde{\psi}] + (D/2 + \eta/2)(\tilde{\psi}\frac{\partial \tilde{V}_k}{\tilde{\psi}}|_t + \tilde{\psi}^\dagger \frac{\partial \tilde{V}_k}{\tilde{\psi}^\dagger}|_t) + \frac{\dot{V}_k}{\alpha' k^{D+2}}.$$

After a long calculation….:

$$\begin{aligned}
\dot{\tilde{V}}_k =& \left(-\tilde{\mu}(-2+\zeta+\eta) - 2N_D A_D(\eta_k,\zeta_k)\frac{\tilde{\lambda}^2}{(1-\tilde{\mu})^2}\right)\tilde{\psi}^\dagger\tilde{\psi} \\
&+ i\tilde{\lambda}\left((-2+\zeta+\frac{D}{2}+\frac{3\eta}{2}) + 2N_D A_D(\eta_k,\zeta_k)\left(\frac{4\tilde{\lambda}^2}{(1-\tilde{\mu})^3} + \frac{(\tilde{g}+3\tilde{g}')}{(1-\tilde{\mu})^2}\right)\right)\tilde{\psi}^\dagger(\tilde{\psi}^\dagger + \tilde{\psi})\tilde{\psi} \\
&+ \left(\tilde{g}(-2+D+\zeta+2\eta) + 2N_D A_D(\eta_k,\zeta_k)\left(\frac{27\tilde{\lambda}^4}{(1-\tilde{\mu})^4} + \frac{(16\tilde{g}+24\tilde{g}')\tilde{\lambda}^2}{(1-\tilde{\mu})^3} + \frac{(\tilde{g}^2+9\tilde{g}'^2)}{(1-\tilde{\mu})^2}\right)\right)(\tilde{\psi}^\dagger\tilde{\psi})^2 \\
&+ \left(\tilde{g}'(-2+D+\zeta+2\eta) + 2N_D A_D(\eta_k,\zeta_k)\left(\frac{12\tilde{\lambda}^4}{(1-\tilde{\mu})^4} + \frac{(4\tilde{g}+18\tilde{g}')\tilde{\lambda}^2}{(1-\tilde{\mu})^3} + \frac{3\tilde{g}\tilde{g}'}{(1-\tilde{\mu})^2}\right)\right)\tilde{\psi}^\dagger(\tilde{\psi}^{\dagger 2} + \tilde{\psi}
\end{aligned}$$

$$\dot{\tilde{\mu}} = \tilde{\mu}(-2+\zeta+\eta) + 2N_D A_D(\eta_k,\zeta_k)\frac{\lambda^2}{(1-\tilde{\mu})^2}, \tag{3.16}$$

$$\dot{\tilde{\lambda}} = \tilde{\lambda}\left((-2+\zeta+\frac{D}{2}+\frac{3\eta}{2}) + 2N_D A_D(\eta_k,\zeta_k)\left(\frac{4\tilde{\lambda}^2}{(1-\tilde{\mu})^3} + \frac{(\tilde{g}+3\tilde{g}')}{(1-\tilde{\mu})^2}\right)\right), \tag{3.17}$$

$$\dot{\tilde{g}} = \tilde{g}(-2+D+\zeta+2\eta) + 2N_D A_D(\eta_k,\zeta_k)\left(\frac{27\tilde{\lambda}^4}{(1-\tilde{\mu})^4} + \frac{(16\tilde{g}+24\tilde{g}')\tilde{\lambda}^2}{(1-\tilde{\mu})^3} + \frac{(\tilde{g}^2+9\tilde{g}'^2)}{(1-\tilde{\mu})^2}\right), \tag{3.18}$$

and

$$\dot{\tilde{g}}' = \tilde{g}'(-2+D+\zeta+2\eta) + 2N_D A_D(\eta_k,\zeta_k)\left(\frac{12\tilde{\lambda}^4}{(1-\tilde{\mu})^4} + \frac{(4\tilde{g}+18\tilde{g}')\tilde{\lambda}^2}{(1-\tilde{\mu})^3} + \frac{3\tilde{g}\tilde{g}'}{(1-\tilde{\mu})^2}\right). \tag{3.19}$$

Anomalous Dimension

In order to find the evolution equations for the anomalous dimensions we use:

$$iZ_k = \lim_{\omega\to 0, q\to 0} \frac{\partial}{\partial\omega}\Gamma_k^{(1,1)}(\omega, q) \tag{3.29}$$

and

$$Z_k\alpha_k = \lim_{\omega\to 0, q\to 0} \frac{\partial}{\partial q^2}\Gamma_k^{(1,1)}(\omega, q). \tag{3.30}$$

This leads to

$$-i\eta_k = \frac{1}{Z_k}\lim_{\omega\to 0, q\to 0} \frac{\partial}{\partial\omega}I^{(1,1)}(\omega, q), \tag{3.31}$$

and

$$-\eta_k - \zeta_k = \frac{1}{Z_k\alpha_k}\lim_{\omega\to 0, q\to 0} \frac{\partial}{\partial q^2}I^{(1,1)}(\omega, q), \tag{3.32}$$

$$\eta = -\frac{2N_D}{D(D+2)}\frac{\tilde{\lambda}^2}{(1-\tilde{\mu}_k)^3}\left((D+2) + \frac{N_D}{D}\frac{\tilde{\lambda}^2}{(1-\tilde{\mu}_k)^3}\right)$$

$$\zeta = -\eta - \frac{N_D}{D}\frac{\tilde{\lambda}^2}{(1-\tilde{\mu}_k)^3}.$$

Results for differents truncations:

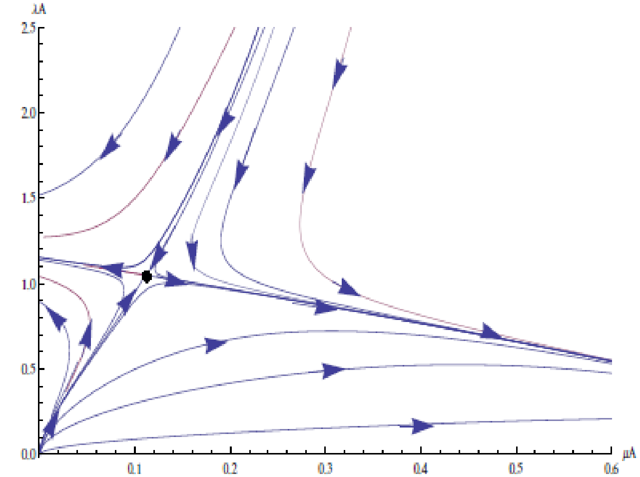


Fig.2: Some trajectories of the flow equation.

- Cubic FP3: Trivial Gaussian Fixed Point and Non Gaussian FP

$(\mu,\ \lambda)^* = (0.11,\ \pm 1.05)$ and

eigenvalues: (2.38, -1.89)

- Quartic FP4

$(\mu,\ \lambda,\ g,\ g')^* = (0.27, \pm 1.35, -2.89, -1.27)$ and

eigenvalues: (19.99, 6.08, 2.51, -1.69)

- Quintic FP5

$(\mu, \lambda, g, g', \lambda_5, \lambda'_5)^* = (0.39, \pm 1.35, -4.10, -1.89, -4.83, -1.34)$ and

eigenvalues: (59.11, 33.12, 16.26, 3,99, 2.12, -1.45)

Percolation and Monte Carlo Simulation:

The critical Exponent **ν = 0.73** with is related with our

ν = - 1/(most negative eigenvalue)

$\nu_3 = 0.52\ ;\ \nu_4 = 0.59\ ;\ \nu_5 = 0.69\ ;\ \nu_6 = 0.78\ ;\ \nu_3 = 0.76\ , \ldots$

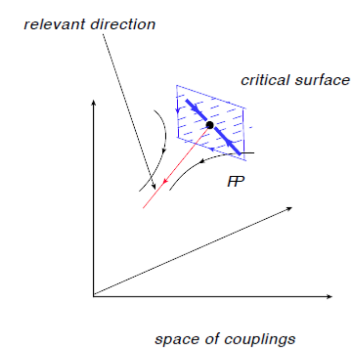
Conclusions:

- We have defined the flow equation for RFT with a specific regulator scheme
- We found for different truncations always one nontrivial fixed point, with one relevant direction. This could define a RFT for the IR region
- We solve and study the flow equations for the running couplings

$$(\mu,\ \lambda,\ g, g', \ldots\ldots)_k$$

- We have made the first attempt to make contact with physical observable: Pomeron Intercept ($\mu_P(0)$)

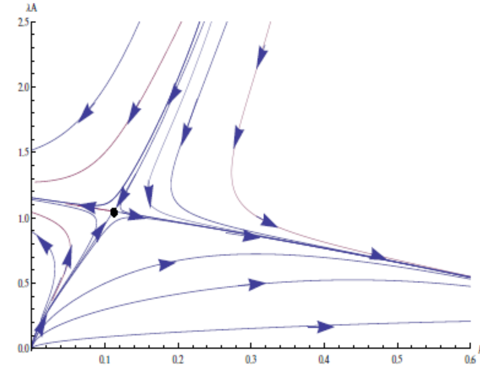
- How important are nonzero field configurations?
- Physical applications:
  - Cut-Off independence, Green functions, Scaling, ...
  - Physical Trajectory and Physical Initial conditions.



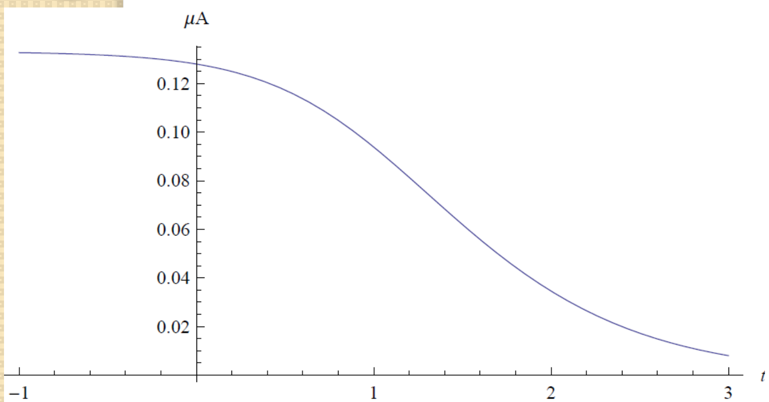
- Study the transition from pQCD to RFT
- Non Local formulations

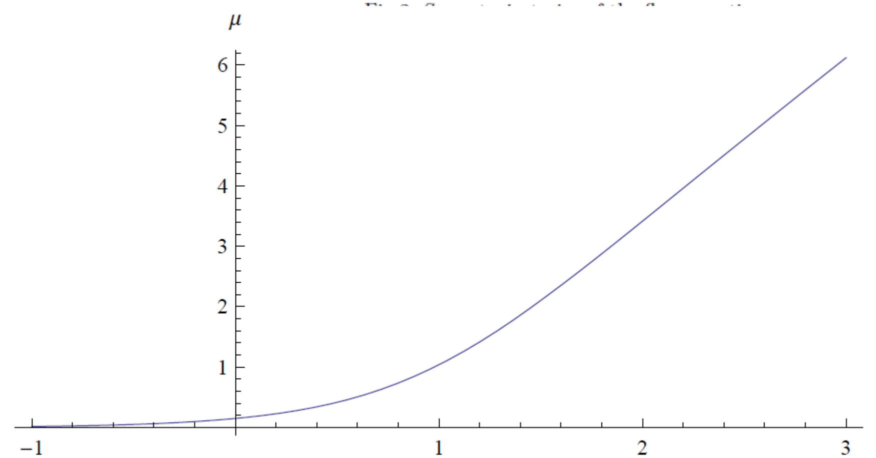
Thank you for your attention

Running couplings

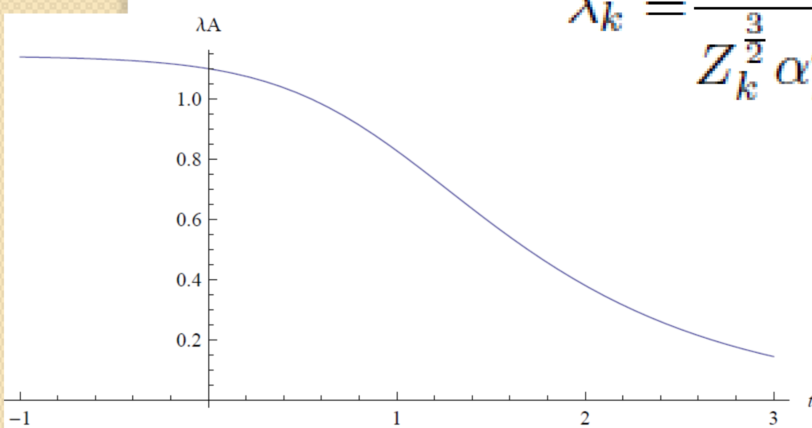


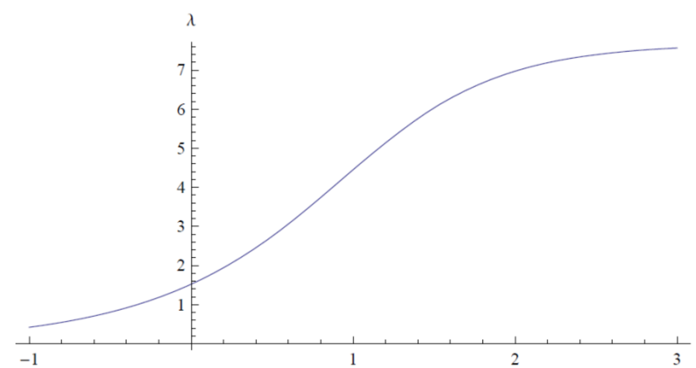
$$\tilde{\mu}_k = \frac{\mu_k}{Z_k \alpha'_k k^2}$$



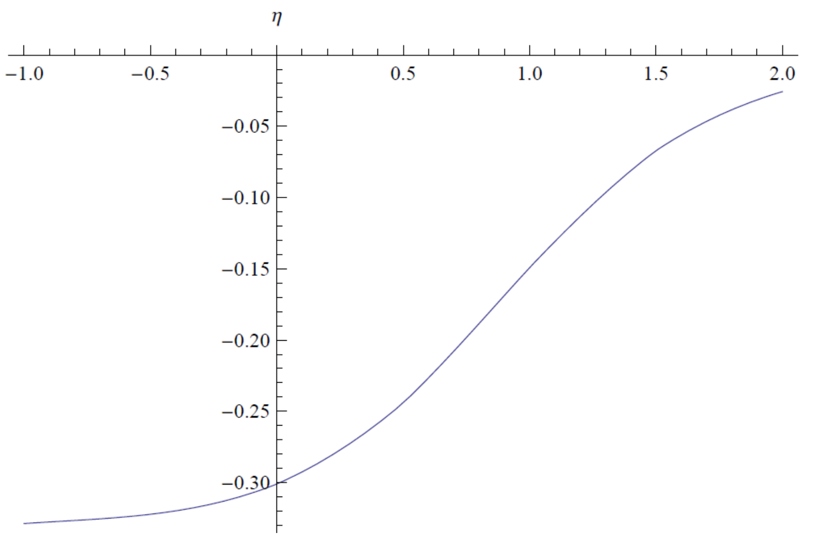


$$\tilde{\lambda}_k = \frac{\lambda_k}{Z_k^{\frac{3}{2}} \alpha'_k k^2} k^{D/2}$$

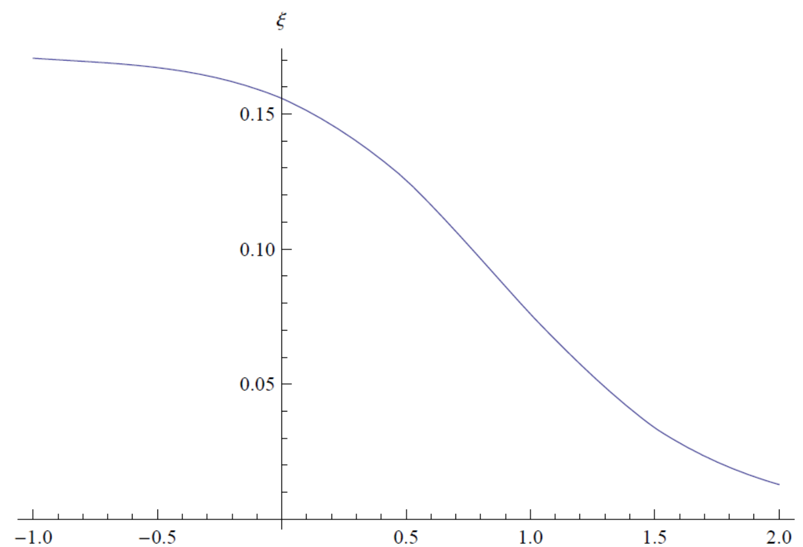


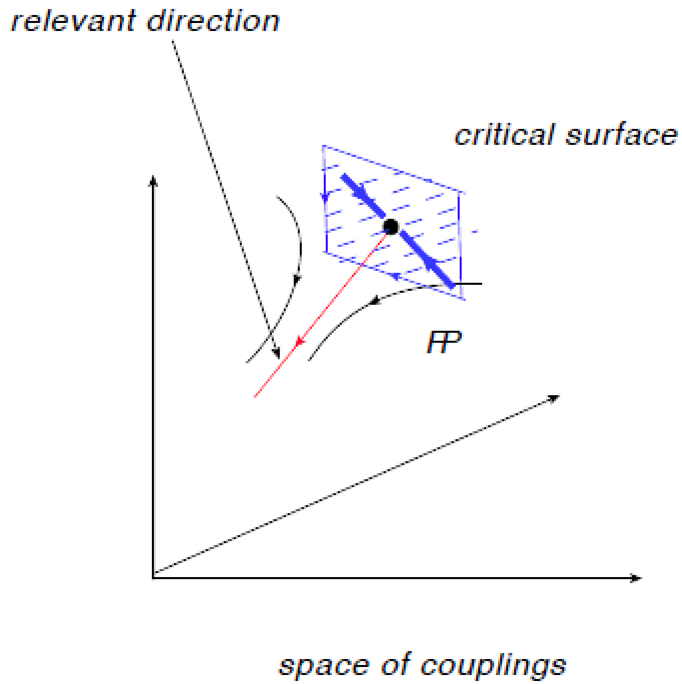


Anomalous Dimensions η and ξ



$$\eta_k = -\frac{1}{Z}\partial_t Z$$
$$\zeta_k = -\frac{1}{\alpha'}\partial_t \alpha'.$$




relevant direction
critical surface
FP
space of couplings

$$\partial_t \Gamma_k^{(1,1)}[1,2] = \frac{1}{2} Tr\left(\partial_t \mathbb{R}_k \frac{1}{[\Gamma_k^{(2)} + \mathbb{R}_k]} \Gamma_{k,1}^{(3)} \frac{1}{[\Gamma_k^{(2)} + \mathbb{R}_k]} \Gamma_{k,2}^{(3)} \frac{1}{[\Gamma_k^{(2)} + \mathbb{R}_k]} + (1 \leftrightarrow 2)\right)$$
$$- \frac{1}{2} Tr\left(\partial_t \mathbb{R}_k \frac{1}{[\Gamma_k^{(2)} + \mathbb{R}_k]} \Gamma_{k,12}^{(4)} \frac{1}{[\Gamma_k^{(2)} + \mathbb{R}_k]}\right). \quad (3.22)$$

This flow equation can be diagrammatically illustrated in the Fig.1.

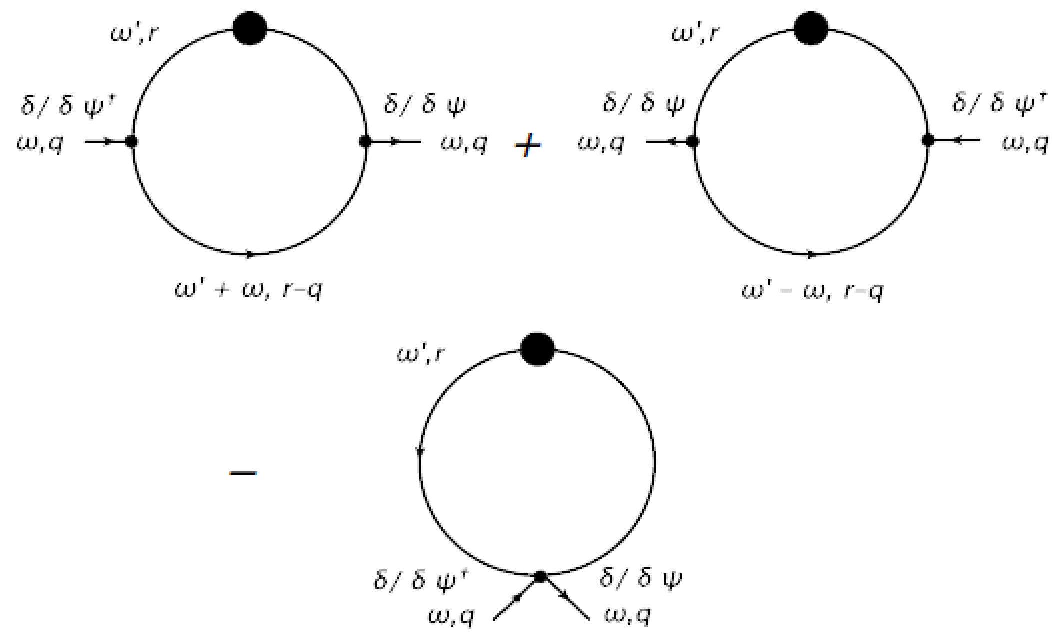


Fig.1: Illustration of the two-pointfunction $\Gamma_k^{(1,1)}$: the big blob represents the regulator insertion, $\partial_t R(k)$, lines the propagator $[\Gamma_k^{(2)} + R(k)]^{-1}$, and small dots the triple or quartic interaction vertices.