High statistics analysis of nucleon form factor in lattice QCD

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OUTLINE

- Introduction
- Error reduction technique in Wilson-clover fermion
- Lattice result
  - Plateau and summation method for axial charge
  - (iso-)scalar and tensor charge
  - Isovector form factor
- Summary

What is our motivation ?

- Lattice QCD is able to determine the nucleon structure function from the first principle of QCD.
- Compared to experiment, the current precision is not enough.

  axial charge: Δg_A lattice ~ 10% ⇔ experiment ~ 0.1 %

  electric charge radius: $\Delta\langle r_E\rangle$ lattice~ 20% ⇔ experiment < 0.5%
- Reducing the uncertainties is essential task for consistency test
  - Monte-Carlo study is rigorous, but there is systematic deviation due to using **unphysical** parameter

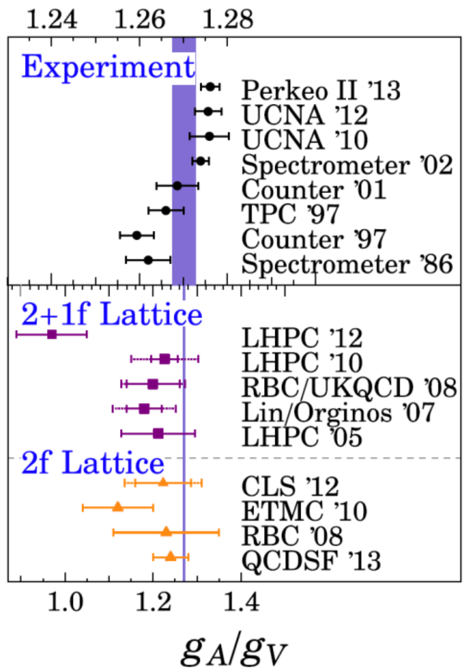
    $O_{\rm phys} - O^{\rm MC}_{\rm lat} = \Delta_{\rm sys}$ $\Delta_{sys} = \Delta(m-m_{phys}, V\neq\infty, \psi_N + \psi_{other\ state}$ etc)
  - Because of stochastic, O^{MC}_{lat} itself has statistical uncertainty.
  - Size of Δ_{sys} and statistical precision are correlated,

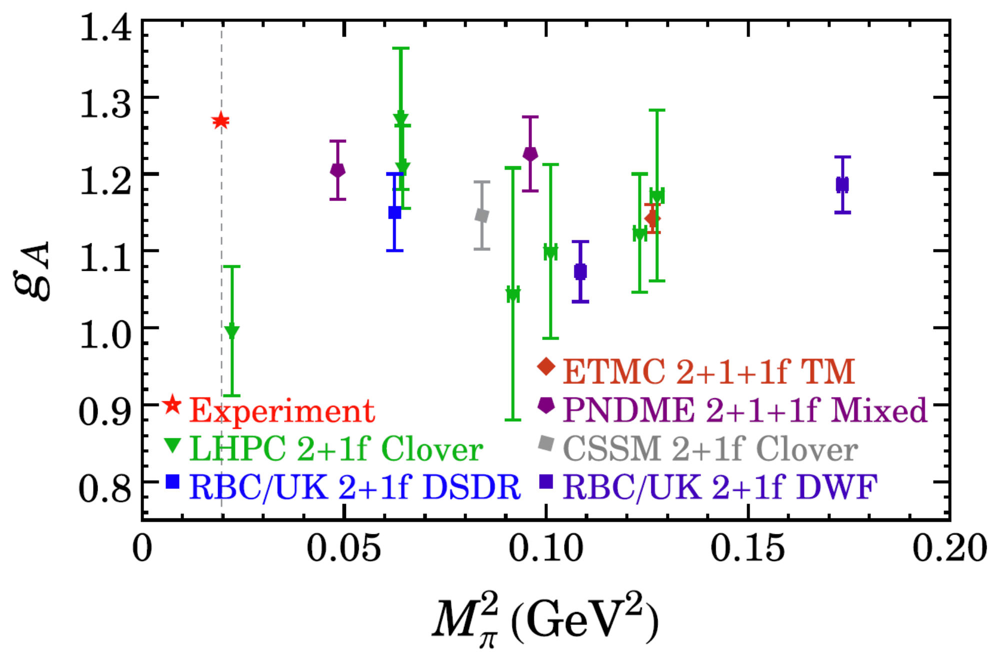
    ⇒ increasing statistics in MC make a reduction in the size of Δ_{sys}

"Puzzle" of axial charge

- Is g_A determinable from lattice QCD ?

PNDME, PRD89(2014)





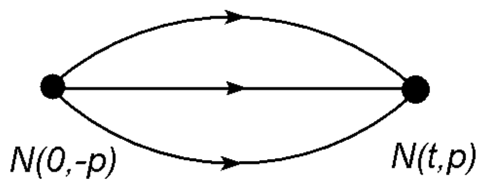
- Although there have been many lattice efforts in N_f=2, 2+1(also 2+1+1) with several kinds of lattice action, those values have not been fixed yet.
- There is slight tension from experiment.
- The careful estimate of systematic errors is necessary.

1. Introduction

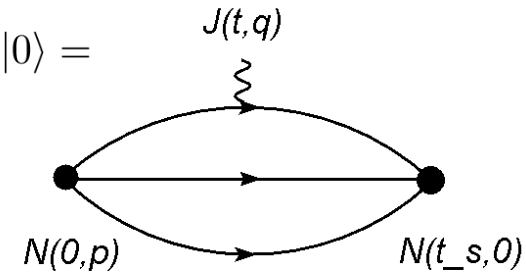
Computation of matrix element

- 2pt, 3pt function

$\langle 0|\mathcal{N}(t)\mathcal{N}^\dagger(0)|0\rangle =$



$\langle 0|\mathcal{N}(t_s,0)J(t,q)\mathcal{N}^\dagger(0,p)|0\rangle =$



$$\langle 0|\mathcal{N}(t)\mathcal{N}^\dagger(0)|0\rangle = |\langle 0|\mathcal{N}|N\rangle|^2 e^{-m_N t} + |\langle 0|\mathcal{N}|N'\rangle|^2 e^{-m'_N t} + \cdots$$

$$\begin{aligned}&\langle 0|T\{\mathcal{N}(t_s,0)J_\mu(t,q)\mathcal{N}^\dagger(0,p)|0\rangle \\ &= \langle 0|\mathcal{N}|N\rangle\langle N|J_\mu|N\rangle\langle N|\mathcal{N}^\dagger|0\rangle e^{-E_N t - m_N(t_s-t)} + \langle 0|\mathcal{N}|N'\rangle\langle N'|J_\mu|N'\rangle\langle N'|\mathcal{N}^\dagger|0\rangle e^{-E'_N t - m'_N(t_s-t)} + \cdots \\ &\simeq Z_N(0)Z_N(p)e^{-E_N t - m_N(t_{\text{sep}}-t)} \times \left[\{G_X, g_A\} + c_1 e^{-\Delta(t_{\text{sep}}-t)} + c_2 e^{-\Delta' t}\right]\end{aligned}$$

$\{G_X, g_A\}$: Matrix element of ground state

$c_1 e^{-\Delta(t_{\text{sep}}-t)} + c_2 e^{-\Delta' t}$: First excited state contamination $\Delta = m'_N - m_N > 0, \Delta' = E'_N - E_N > 0$

- Matrix element is extracted from ratio of 3pt and 2pt function after removing exponent and amplitude.

What is problem ?

- Signal-to-noise ratio problem
  - Noise of nucleon propagator at time-slice t behaves like

$$\mathrm{S/N} \sim \sqrt{N}\exp[-(m_N - 3m_\pi/2)t]$$

    it means statistics N ~ $\exp[(2m_N-3m_\pi)t]$ are needed for same precision.
- Excited state contamination
  - Excited state rapidly decays at large t, because of $m_N < m_{excited}$
  - To evaluate ground state mass by fitting with finite t range, precision of nucleon propagator at large t is needed.

Our strategy:

- To reduce statistical error at large t, the all-mode-averaging is efficient way.
- Systematic study of excited state contamination is performed in light pion mass and large volume, $m_\pi L > 4$.

2. Error reduction technique

All-mode-averaging

Blum, Izubuchi, ES (2013)

- Effective to reduce statistical error of correlation function without additional computational cost.

$$O^{(\mathrm{imp})} = O^{(\mathrm{rest})} + \frac{1}{N_G}\sum_{g\in G} O^{(\mathrm{appx}),g}, \quad O^{(\mathrm{rest})} = O - O^{(\mathrm{appx})}$$

- O: 2pt, 3pt function with high precision solver (10^{-10} residue) ⇒ expensive
- $O^{(\mathrm{appx})}$: 2pt, 3pt function with low precision solver (~10^{-2} residue) ⇒ cheap
- perform N_g computation of $O^{(\mathrm{appx}),g}$ (g : translational shift).

AMA estimator $O^{(\mathrm{imp})}$ whose error supposes to be

$$\frac{\sigma^{\mathrm{imp}}}{\sigma} \simeq \sqrt{\frac{1}{N_G} + 2(1-r) + \frac{1}{N_g^2}\sum_{g\neq g'} r_{gg'}}$$

$$r = \frac{\langle \Delta O \Delta O^{(\mathrm{appx})}\rangle}{\sigma\sigma^{(\mathrm{appx})}} \qquad r_{gg'} = \frac{\langle \Delta O^{(\mathrm{appx}),g}\Delta O^{(\mathrm{appx}),g'}\rangle}{\sigma^{(\mathrm{appx}),g}\sigma^{(\mathrm{appx}),g'}}$$

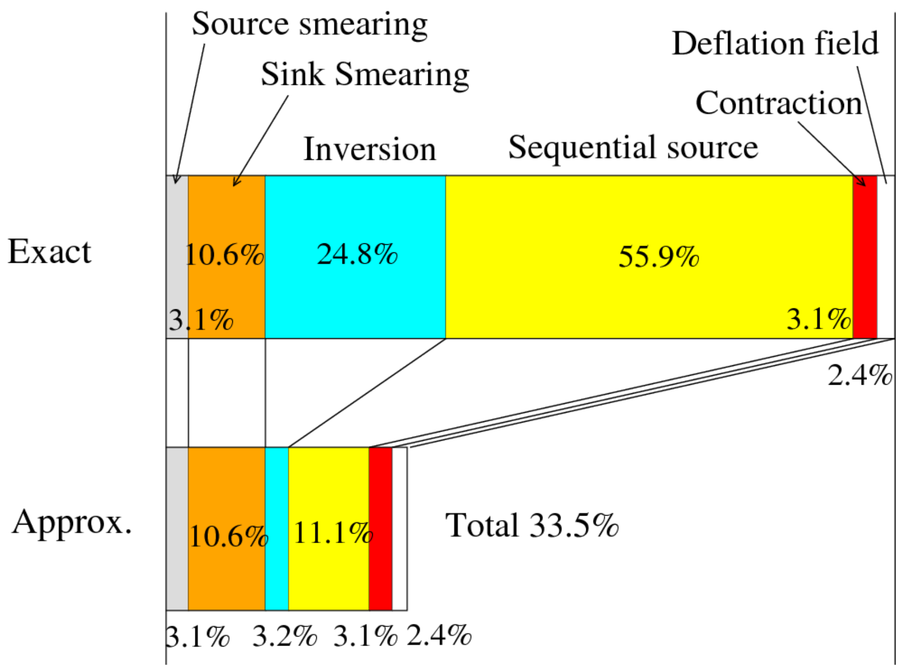
r: correlation between O and $O^{(\mathrm{appx})}$

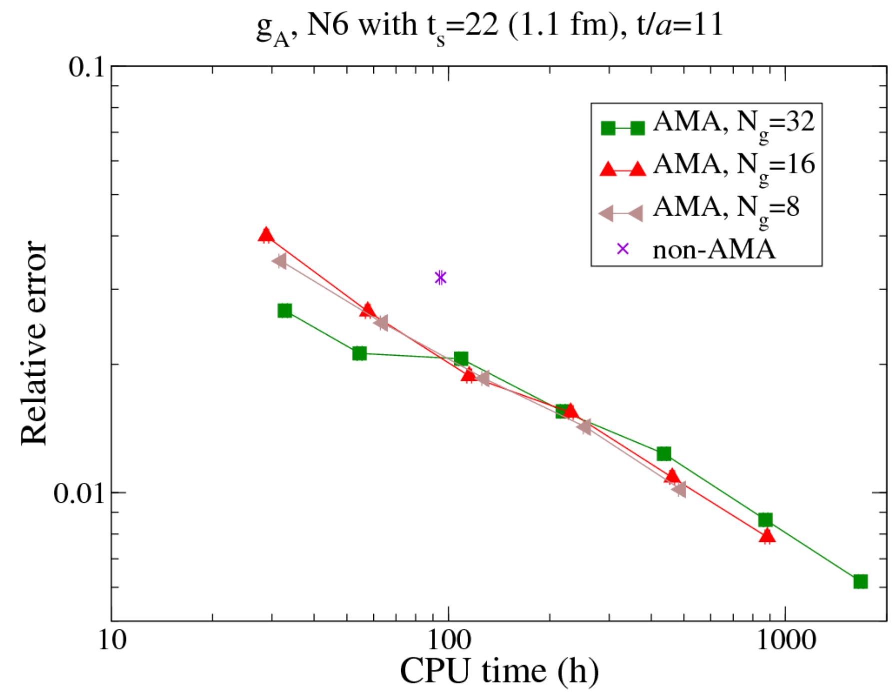
$r_{gg'}$: correlation between $O^{(\mathrm{appx}),g}$ and $O^{(\mathrm{appx}),g'}$

- $O^{(\mathrm{appx})}$ has several tuning parameters to control of r and $r_{gg'}$
  e.g. stopping condition, deflation field, source location

Performance test of AMA

- Reduction of computational cost





- Cost of computing quark propagator is reduced to 1/5 and less.
- Total speed-up is about factor 2 and more. (depending on lattice size and pion mass)

3. Lattice results (preliminary)

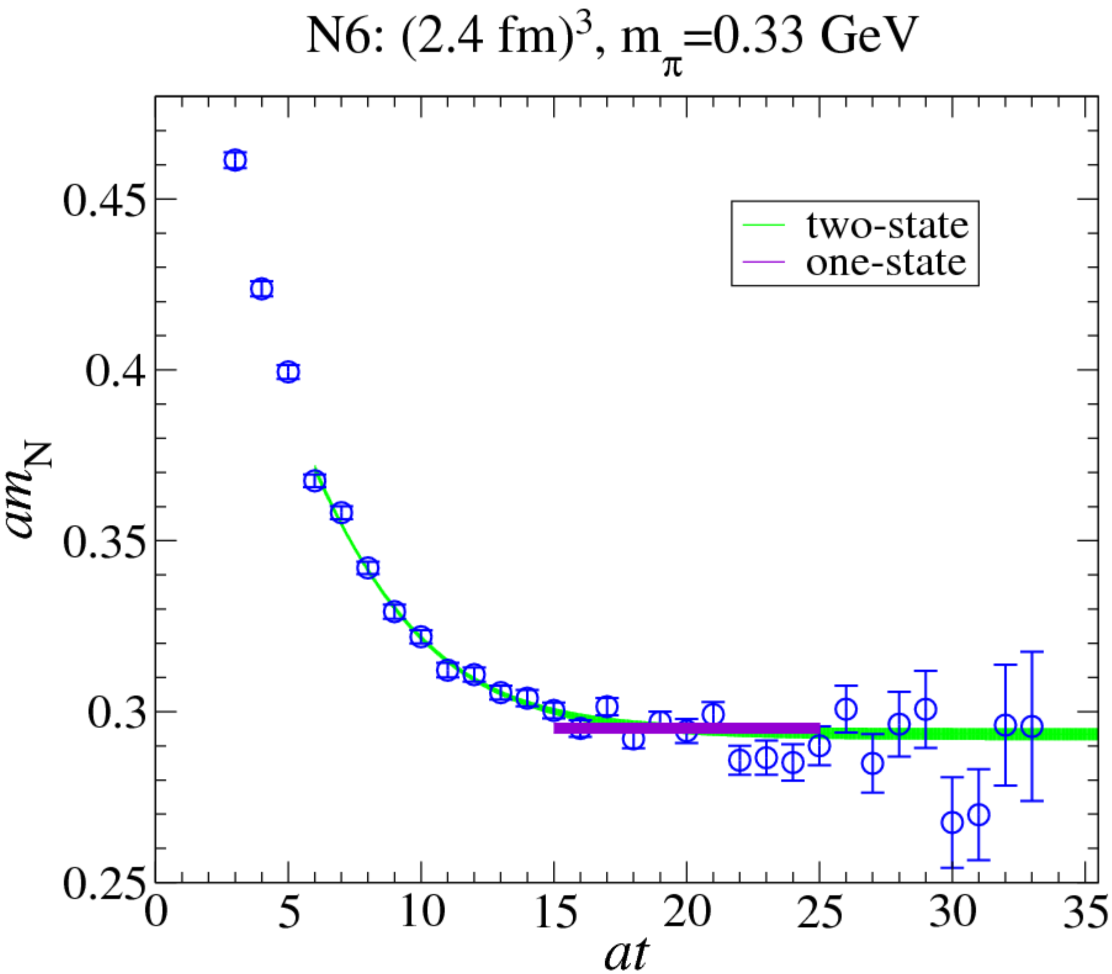
CLS config, $N_f = 2$ Wilson-clover fermion

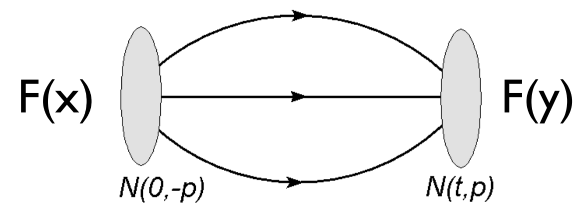
	Lattice	*a* (fm)	m_π (GeV)	N_G	t_s (fm)	#conf	#meas(*)
E5	64×32^3	0.063	0.456	64	0.82, 0.95, 1.13	~480	~30,000
	$(2.0\text{ fm})^3$		($m_\pi L$=4.7)		1.32	~1000	~64,000
					1.51	~1600	~102,400
F7	96×48^3	0.063	0.277	64	0.82, 0.95, 1.07	~250	~16,000
	$(3.0\text{ fm})^3$		($m_\pi L$=4.2)	128	1.20, 1.32	~250	~32,000
				192	1.51	243	~48,000
N6	96×48^3	0.05	0.332	32	0.9	110	3,520
	$(2.4\text{ fm})^3$		($m_\pi L$=4.1)	32	1.1,1.3	803	25,696
				32	1.5, 1.7	~930	~30,000
G8	128×64^3	0.063	0.193	80	0.88	103	8,240
	$(4.0\text{ fm})^3$		($m_\pi L$=4.0)	112	1.07	94	10,528
				160	1.26	101	16,160
				64	1.51	170	10,880

* Effective statistics : #mes = $N_G \times$ #conf

3. Lattice results (preliminary)

Nucleon mass and its excited state





F(x): Jacobian function with APE smearing link.

- The ground-state dominant, t/a = 15 → t= 0.75 fm.
- Fitting function
  One-state : Ze^{-mt},
  Two-state : $Z\, e^{-m\, t} + Z' e^{-m'\, t}$
- Good χ^2 value
- Precision, $\Delta m_N/m_N < 1$ %

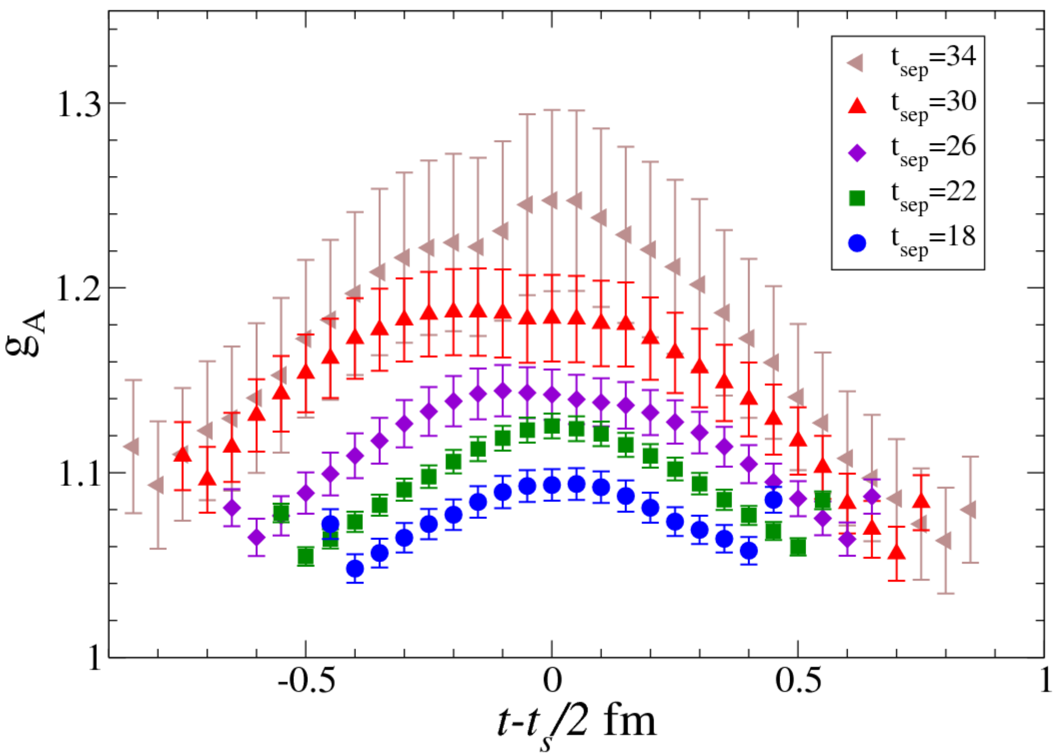
3. Lattice results (preliminary)

Axial charge

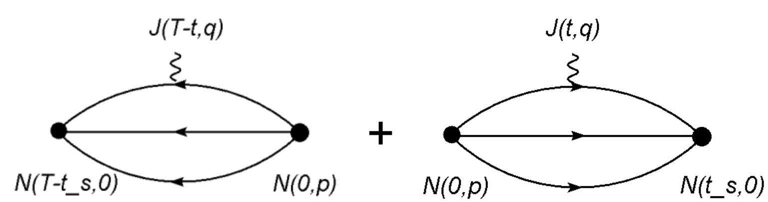
- Single ratio of 2pt and 3pt with fixed t_s

$$R_A(t,t_s) = Z\frac{\mathcal{P}\langle 0|\mathcal{N}(t_s,0)J_3(t,q)\mathcal{N}^\dagger(0,0)|0\rangle}{\mathcal{P}\langle 0|\mathcal{N}(t_s,0)\mathcal{N}^\dagger(0,0)|0\rangle} \simeq g_A + c_1 e^{-\Delta t_s} + c_2 e^{-\Delta'(t_s-t)}$$

N6: $(2.4\ \mathrm{fm})^3$, a^{-1}=3.95 GeV, m_π=0.332 GeV



- Computation of 3pt and 2pt function at zero momentum with spin projection P.
- Signal around t-t_s/2 >> 1.
- But the size of excited state (2nd and 3rd terms) are still unknown ! → significant uncertainty
- Forward and backward averaging



3. Lattice results (preliminary)

Extraction of g_A

- Ground and excited state ansatz
  - Ground state dominance (Plateau method)

$$R_A(t,t_s) = Z\frac{\mathcal{P}\langle 0|\mathcal{N}(t_s,0)J_3(t,q)\mathcal{N}^\dagger(0,0)|0\rangle}{\mathcal{P}\langle 0|\mathcal{N}(t_s,0)\mathcal{N}^\dagger(0,0)|0\rangle} \quad \simeq \quad g_A, \; (t_s, t_s - t \gg 1)$$

    - Evaluation from constant fitting for t with fixed t_s.
    - To suppress the excited state contamination, measurement at large t_s is needed.
  - First excited state ansatz PNDME(2014), RQCD(2014), …

$$R_A(t,t_s) \quad \simeq \quad g_A + c\left(e^{-\Delta t_s} + e^{-\Delta'(t_s-t)}\right)$$

    - Δ is mass difference between ground and 1st excited state.
- Summation method Capitani et al. PRD86 (2012)

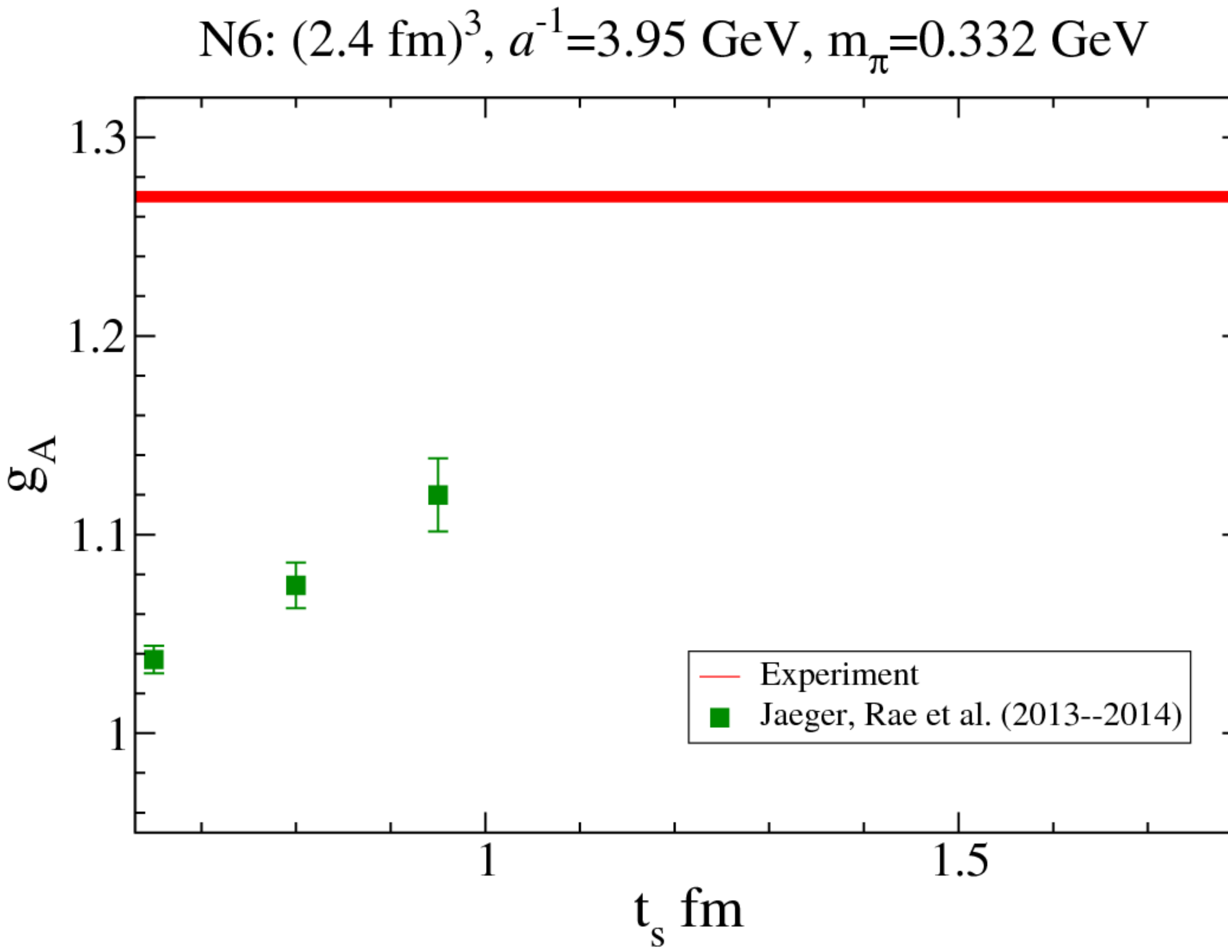
$$R_A^{\mathrm{sum}}(t_s) = \sum_{t=t_{\mathrm{cut}}}^{t_s-t_{\mathrm{cut}}} R_A(t,t_s) \simeq a_0 + t_s(g_A + c_1' e^{-\Delta t_s} + c_2 e^{-\Delta' t_s})$$

- Using summation in $[t_{cut}, t_s\text{-}t_{cut}]$ at fixed t_s , the excited state effect is ~ $O(e^{-\Delta t_s})$
- g_A is given from t_s linear part at t_s >> 1.

3. Lattice results (preliminary)

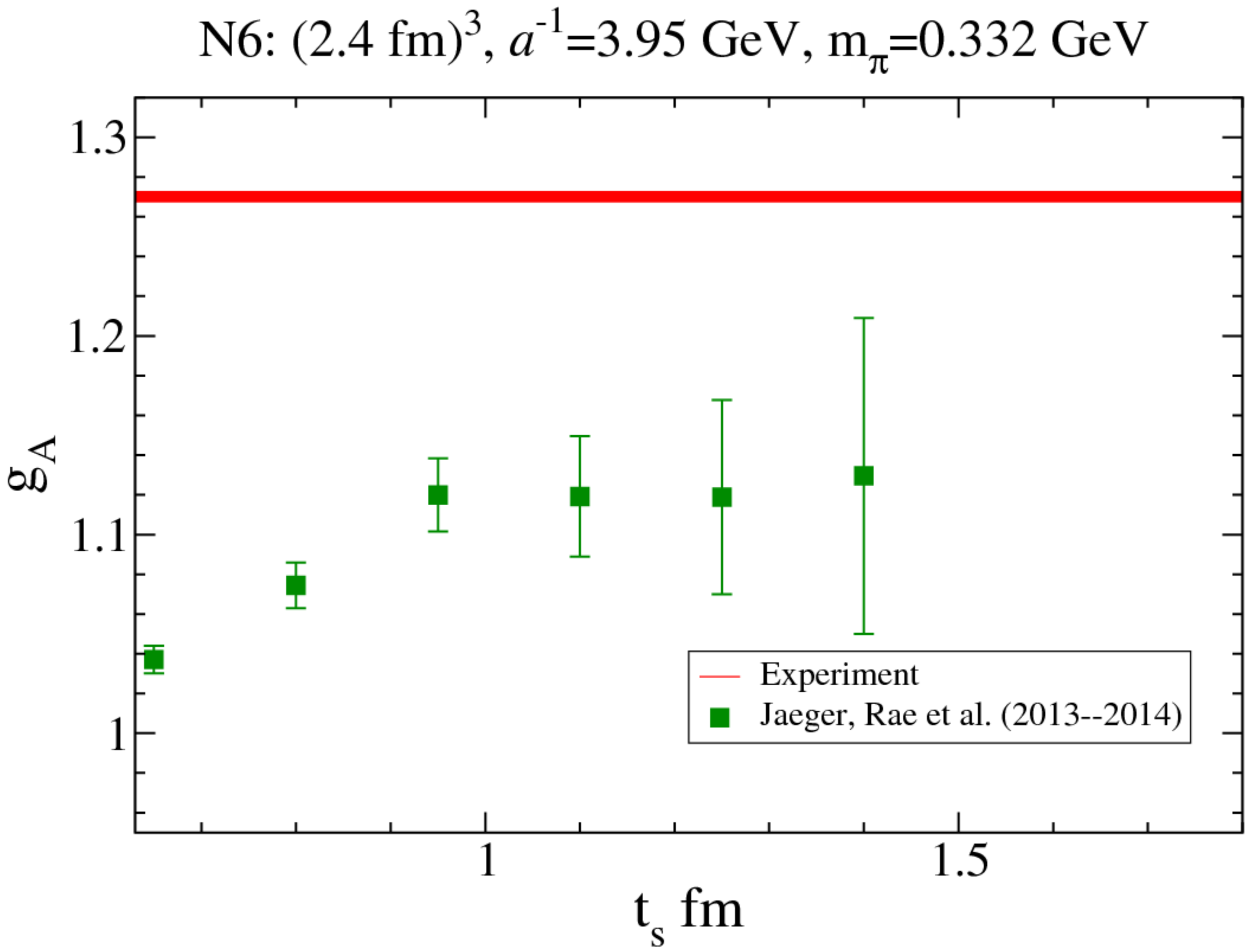
Plateau method

- Non-AMA results at t_s <1 fm



Plateau method

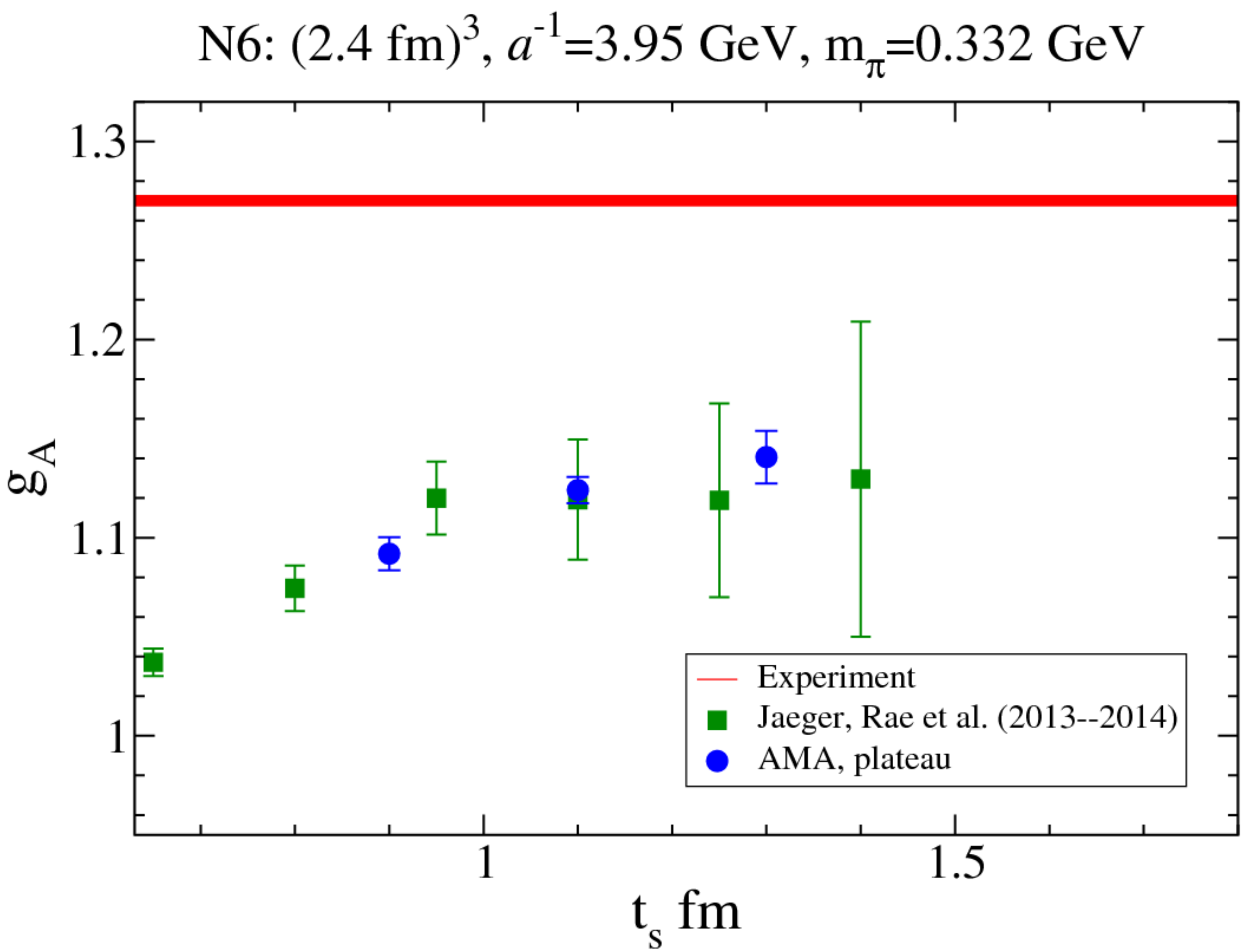
- Non-AMA results at t_s <1.5 fm



3. Lattice results (preliminary)

Plateau method

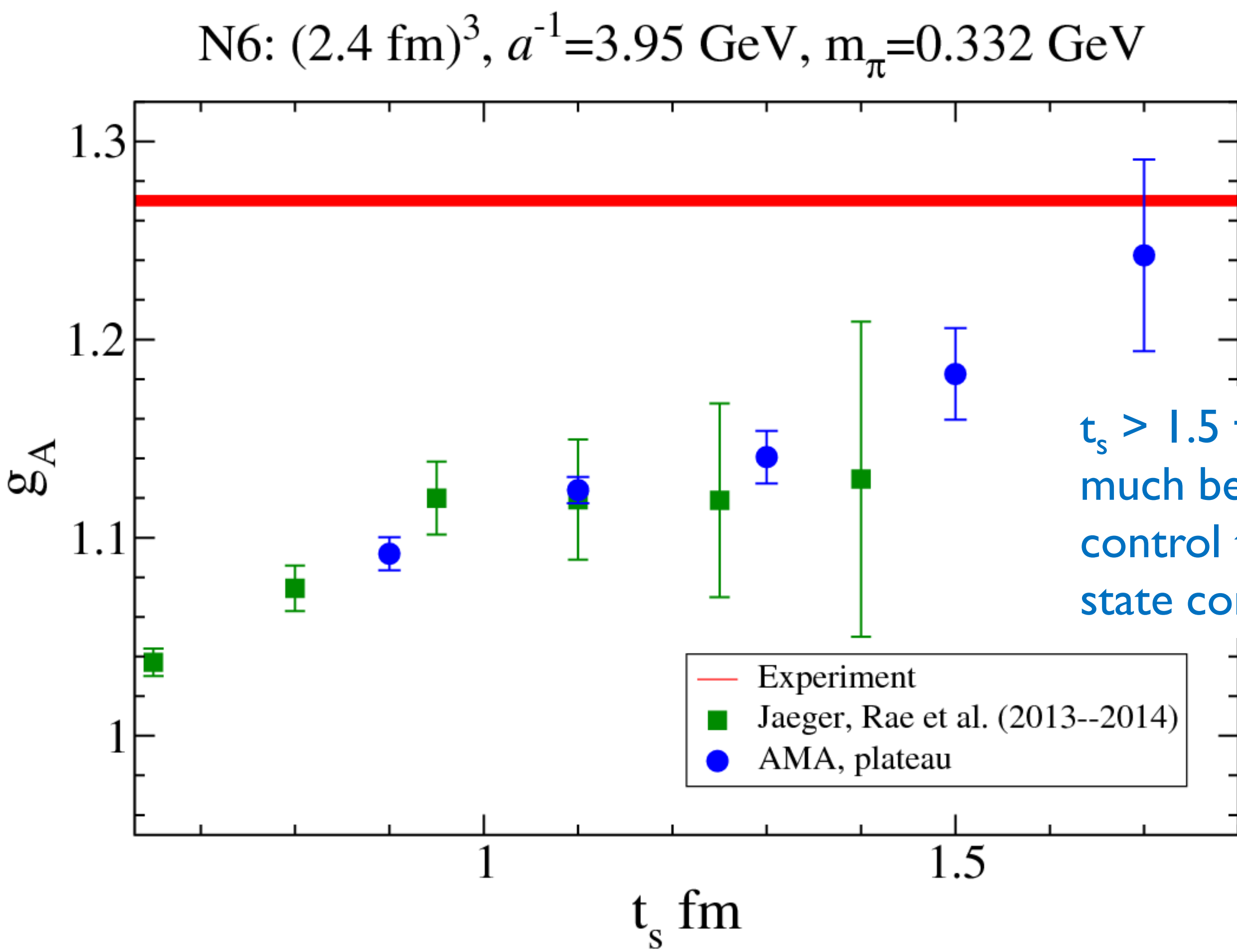
- AMA results at t_s <1.5 fm



3. Lattice results (preliminary)

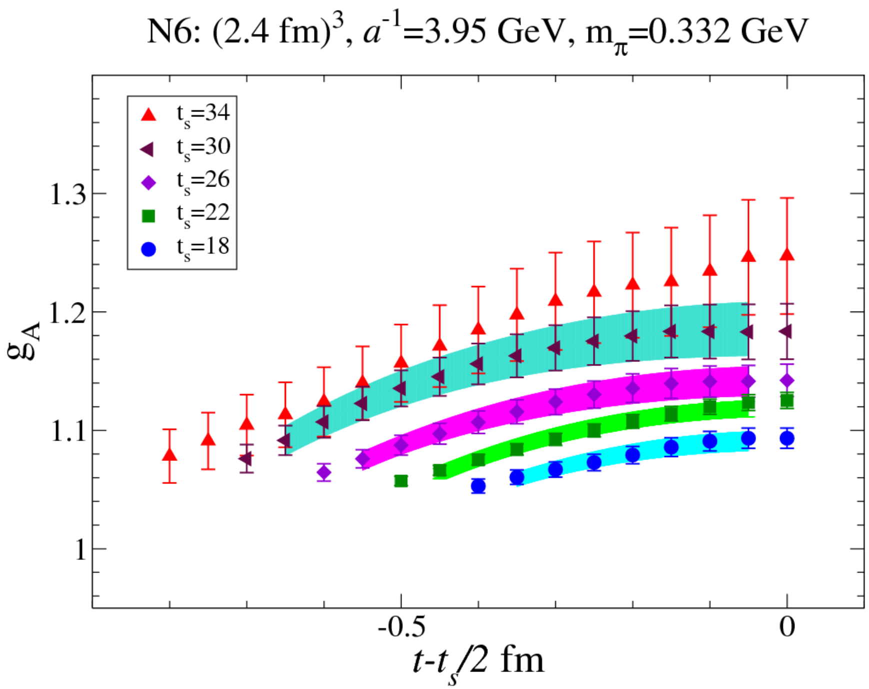
Plateau method

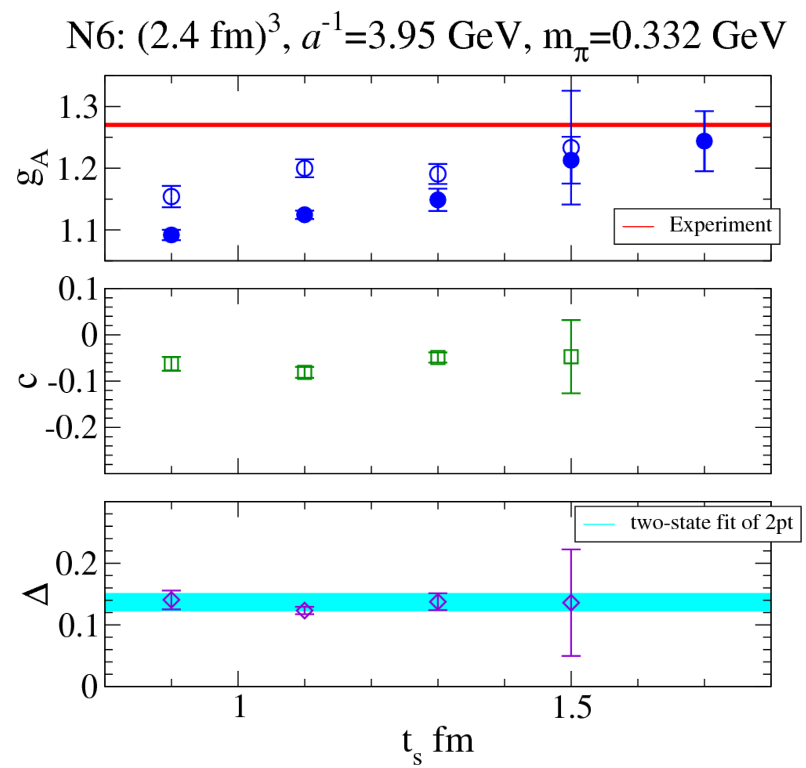
- AMA results at t_s >1.5 fm



t_s > 1.5 fm region is much better to control the excited state contribution.

Excited state ansatz





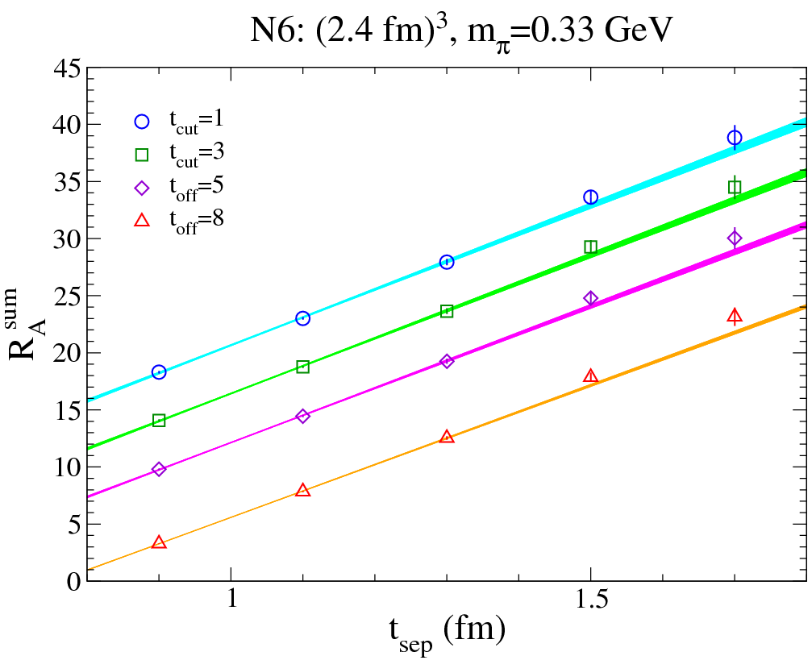
- Holding in middle point of source-sink separation ⇒ additional averaging
- Fitting the function

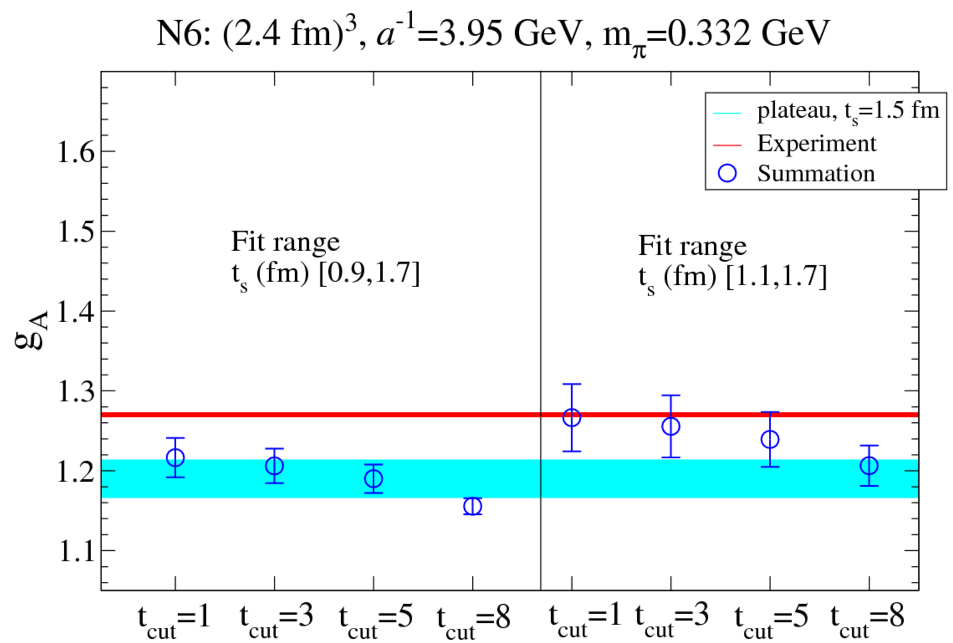
$$f(t, t_s) = g_A + c\left(e^{-\Delta t_s} + e^{-\Delta(t_s - t)}\right)$$

- After correction to excited state, g_A increases at t_s ~ 1fm, and in agreement with plateau method in t_s > 1.5 fm.
- Mass difference Δ has consistency with two state fit.

3. Lattice results (preliminary)

Summation method

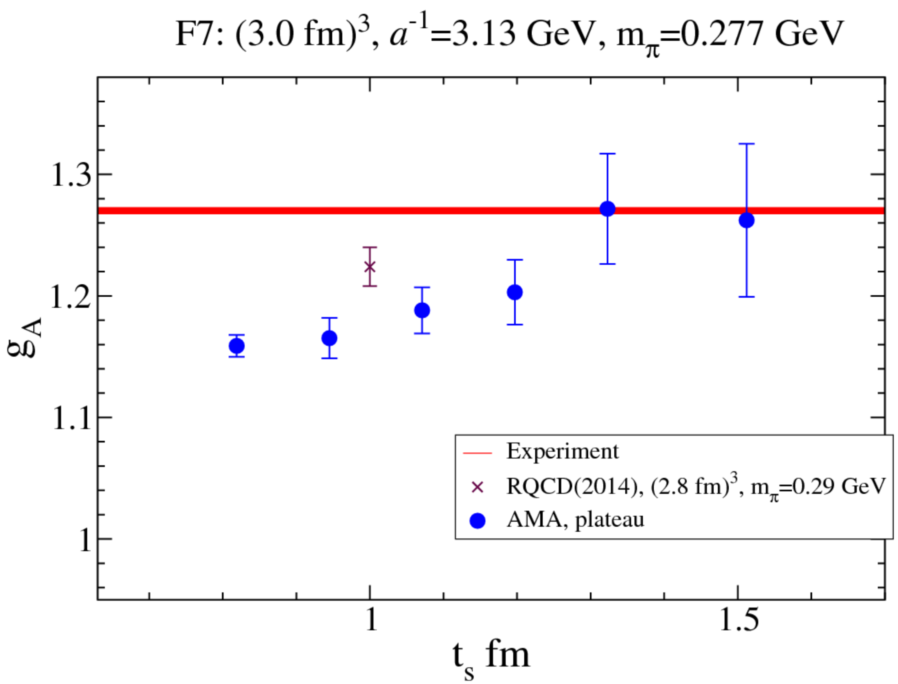


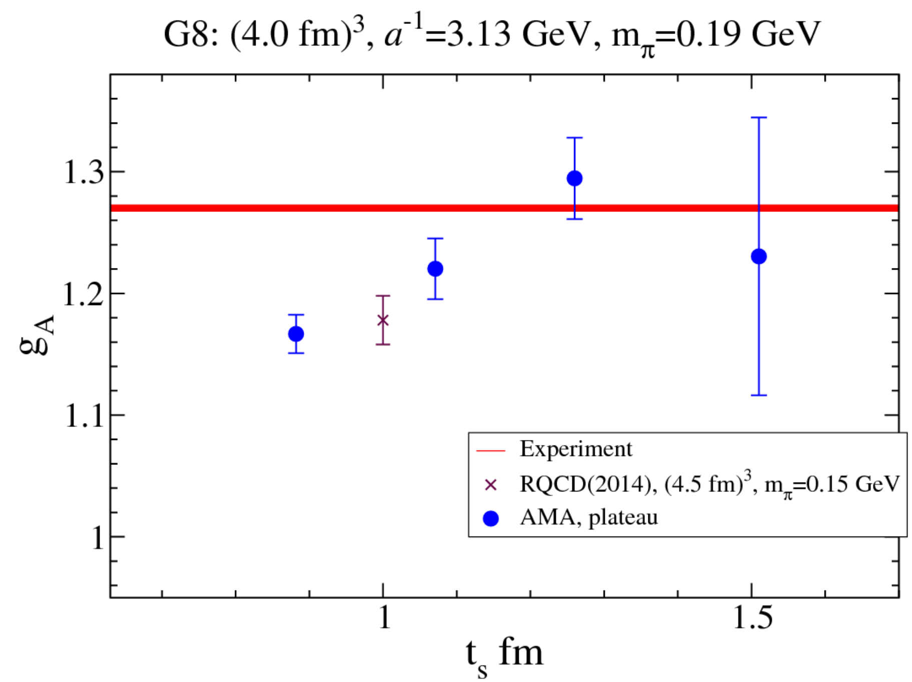


- Fitting with linear function using t_{cut} = 1, 3, 5, 8 ⇒stability check
- Fitting function is in good agreement with lattice data.
- becomes stable using minimum fit range t_s > 1 fm.
- The excited state contribution $O(\exp(-\Delta' t_s))$ is not significant in t_s > 1.5 fm.

3. Lattice results (preliminary)

Comparison in $m_\pi = 0.19$ and 0.28 GeV





- RQCD collaboration uses $N_f = 2$ Wilson-clover fermion.
- In good agreement with RQCD result in $t_s \sim 1$ fm
- It seems large excited state contamination in $m_\pi = 0.19$ GeV rather than 0.28 GeV

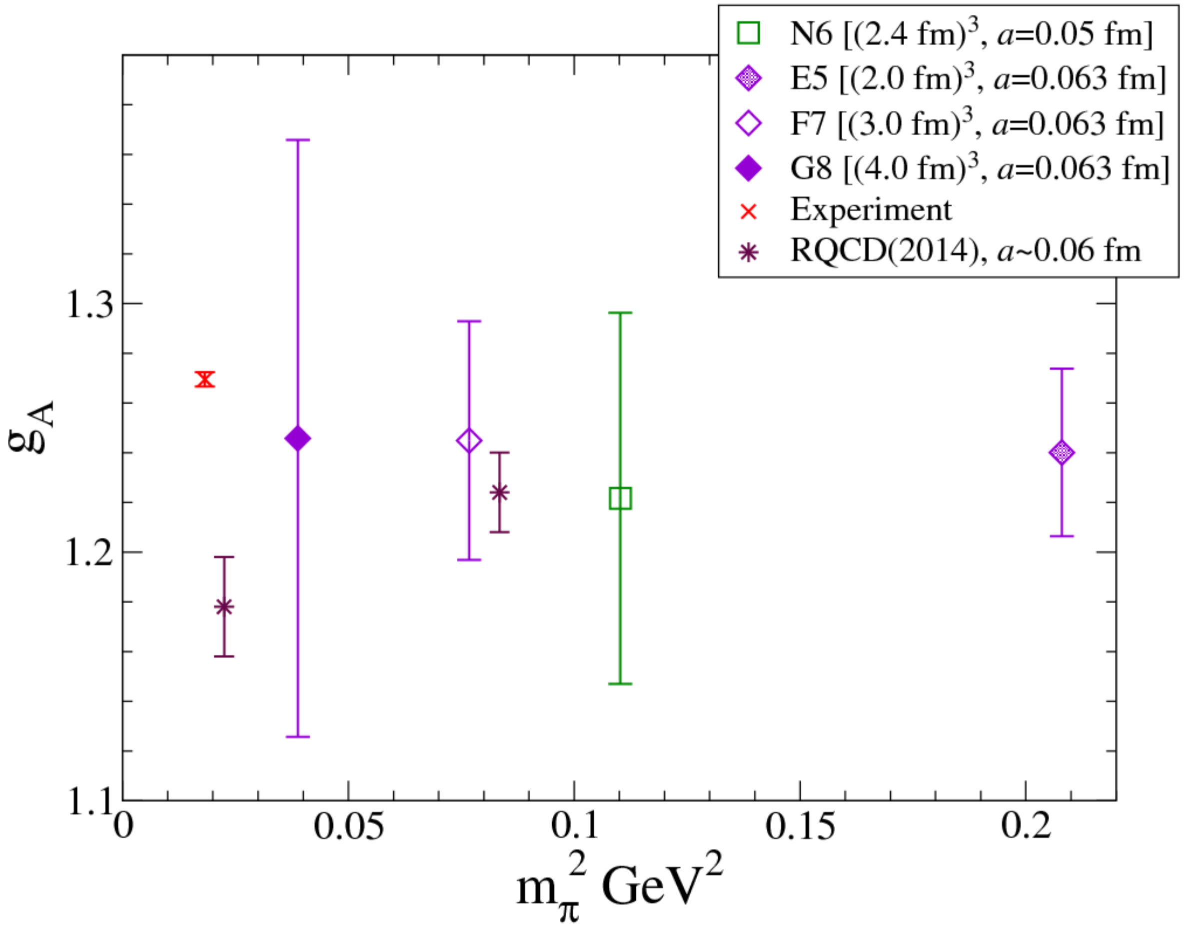
Systematic error

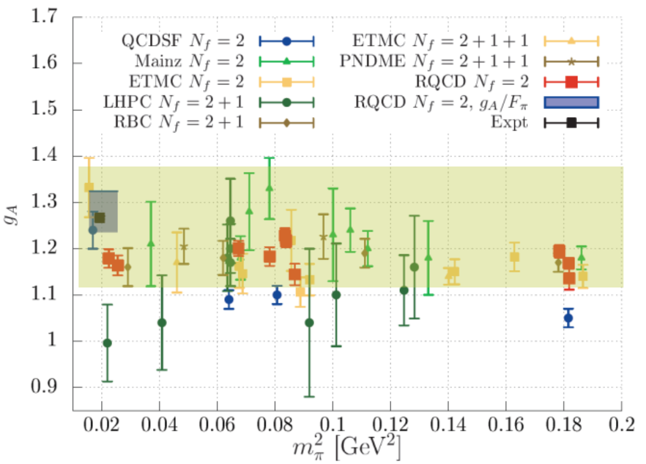
- Central value
  - Summation method in t_s > 1 fm, and maximum value of t_{cut} in t_s > 1 fm
- Systematic error
  - A) Difference from summation method in t_s^{min} < 1 fm
  - B) Difference from fitting with the first excited state in t_s > 1 fm

lattice	g_A	Statistical	Systematic A)	Systematics B)
E5	1.240	0.021(1.7%)	0..021(1.7%)	0.022(1.7%)
F7	1.245	0.033(2.7%)	0.003(0.2%)	0.035(2.8%)
N6	1.222	0.024(2.0%)	0.045(3.6%)	0.054(4.4%)
G8	1.246	0.103(8.3%)	0.003(0.2%)	0.062(5%)

3. Lattice results (preliminary)

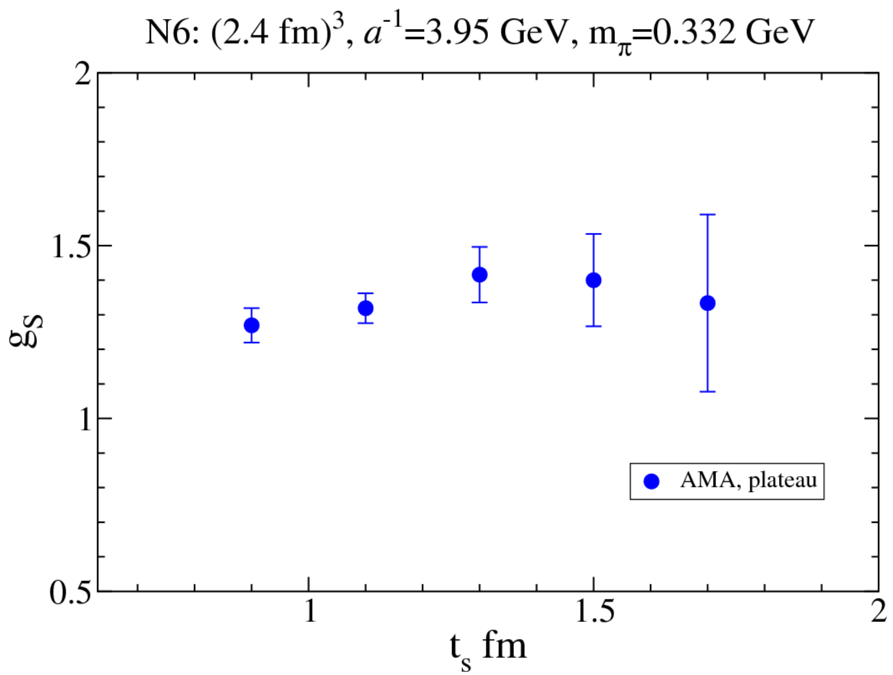
m_π dependence



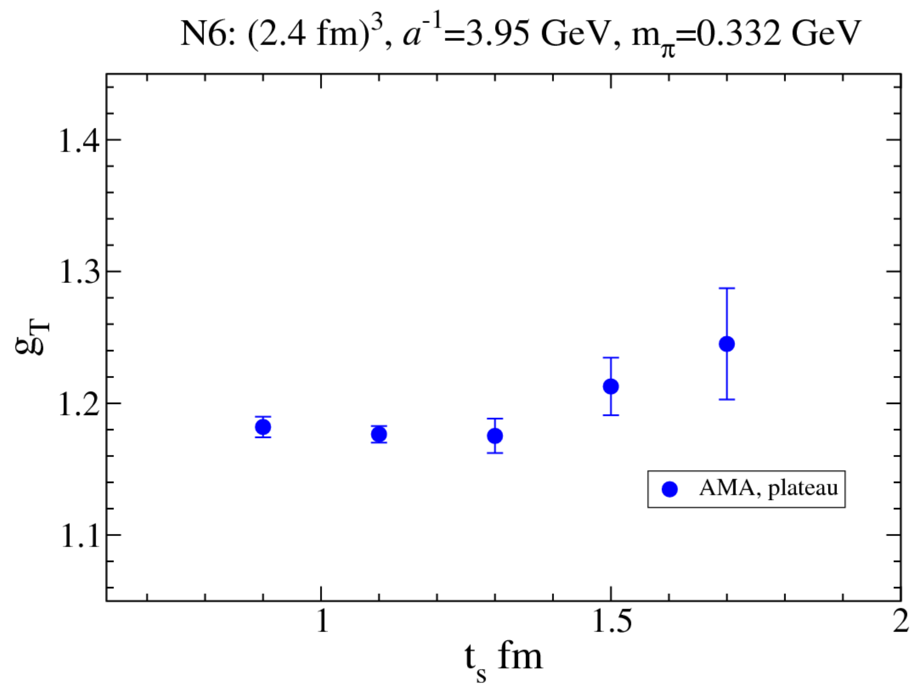


Scalar and tensor charge

Scalar (lattice)



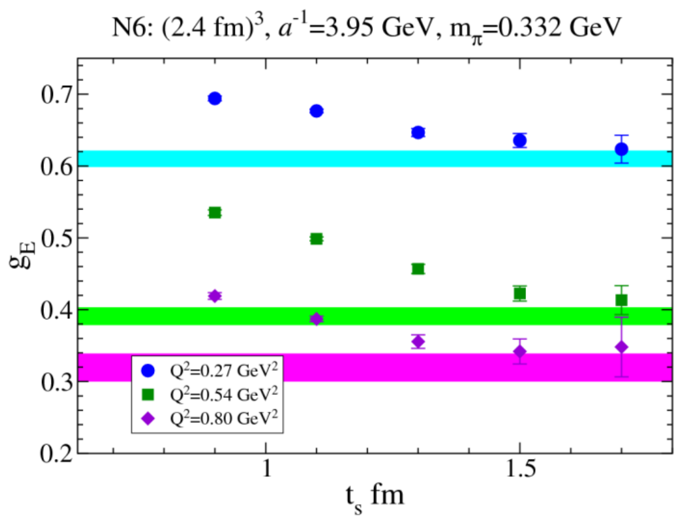
Tensor (lattice)



- There does not appear significant effect of excited state.

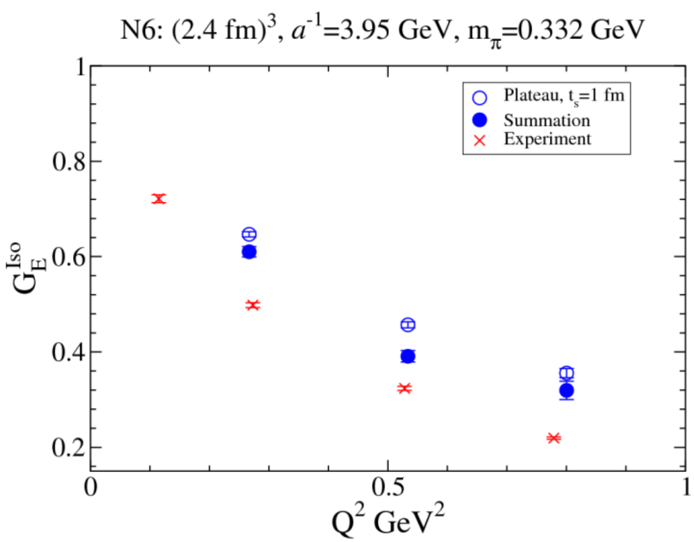
3. Preliminary results

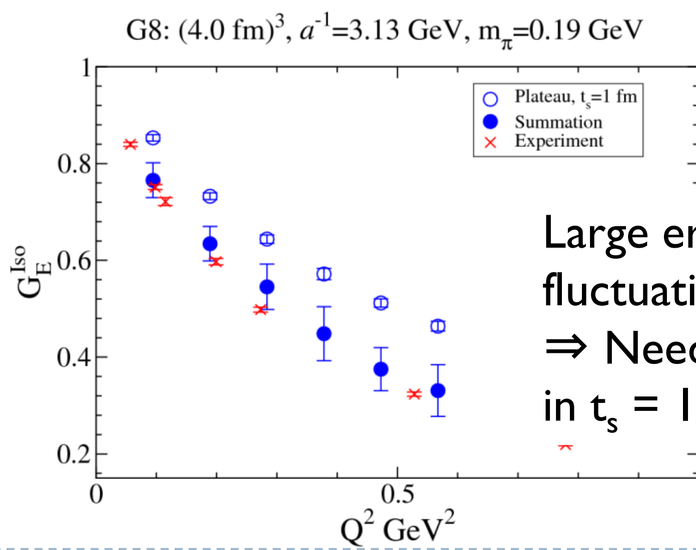
Isovector form factor



Summation method

- Good consistency with plateau and summation method at t_s > 1.5 fm.
- Excited state contamination effect is observed, which has large impact in light quark mass.





Large error comes from fluctuation in t_s = 1.5 fm.
⇒ Need error reduction in t_s = 1.5 fm

Summary

- High statistics calculation of nucleon form factor is performed in N_f=2 Wilson-clover at $Lm_\pi > 4$ with m_π = 0.19--0.46 GeV.
- All-mode-averaging technique is working well for reduction of statistical error in Wilson-clover fermion.
- t_s > 1.5 fm is required for small contribution of excited state contamination in axial charge, rather than scalar and tensor charge.
- Axial charge is now close to experiment.
- Feasible study for application to N_f = 2+1 CLS configurations with open boundary condition.

Thank you for your attention.

3. Preliminary results

Isovector form factor

- Ratio with momentum transition

$$R_G(t,t_s) = Z\frac{\mathcal{P}\langle 0|\mathcal{N}(t_s,p_1)J_\mu(t,q)\mathcal{N}^\dagger(0,p_0)|0\rangle}{\mathcal{P}\langle 0|\mathcal{N}(t_s,p_0)\mathcal{N}^\dagger(0,p_0)|0\rangle}K(p_1,p_0) \simeq G_X + d_1 e^{-\Delta t_s} + d_2 e^{-\Delta'(t_s-t)}$$

$$K(p_1,p_0) = \sqrt{\frac{C_{\rm 2pt}^{\rm lc}(p_1,t_s-t)C_{\rm 2pt}^{\rm sm}(p_0,t)C_{\rm 2pt}^{\rm lc}(p_0,t_s)}{C_{\rm 2pt}^{\rm lc}(p_0,t_s-t)C_{\rm 2pt}^{\rm sm}(p_1,t)C_{\rm 2pt}^{\rm lc}(p_1,t_s)}},$$

- The ratio consists of 3pt and 2pt, with combination of local "lc" and smeared "sm" sink.
- Matrix element with Sachs form factor

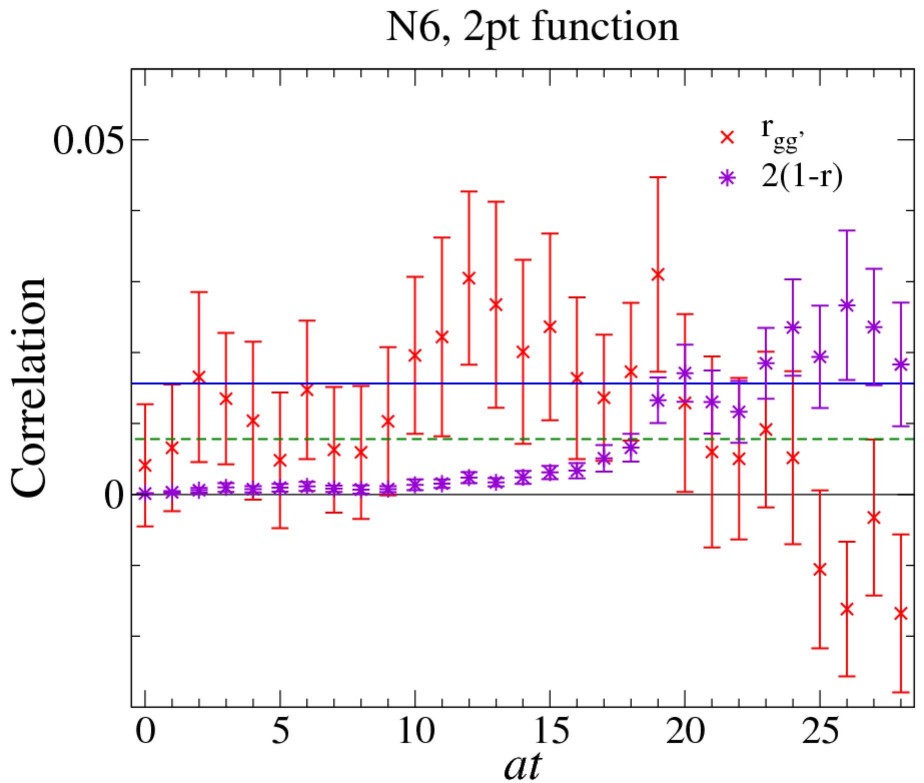
$$\langle N(\vec{p}_1)|J_\mu|N(\vec{p}_0)\rangle = \bar{u}(p_1)\left[F_1^v(q^2)\gamma_\mu + F_2 q_\nu\sigma_{\mu\nu}/2m_N\right]u_N(p_0)$$

$$G_E = F_1 - \frac{q^2}{4m_N^2}F_2,\ G_M = F_1 + F_2$$

- Form factor G_X as a function of q^2, $q = p_1 - p_0$, in which $p_1 = (0,m_N)$ $p_0 = (p,E)$ are used.
- Systematic study of excited state contamination with plateau and summation method is necessary.

Performance test of AMA

- Correlation



Expected error reduction in AMA:

$$\frac{\sigma^{\mathrm{imp}}}{\sigma} \simeq \sqrt{\frac{1}{N_G} + 2(1-r) + \frac{1}{N_g^2}\sum_{g\neq g'} r_{gg'}}$$

$1/N_G = 1/64$

$1/N_G = 1/128$

- $r_{gg'}$: correlation between $O^{(\mathrm{appx})}$ with g and g' transformation.
- $2(1\text{-}r)$: correlation between $O^{(\mathrm{appx})}$ and O.
- At t ~ 24, size of correlation is similar to $1/N_G$, ⇒maximum point to reduce error