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Recent results
in chiral
meson physics

Johan Bijnens

Chiral
Perturbation
Theory

Determination
of LECs in the
continuum

Charged Pion
Polarizabilities

Finite volume

Beyond QCD

A mesonic
ChPT
program
framework

Leading
logarithms

Conclusions

RECENT RESULTS IN CHIRAL MESON PHYSICS

QNP
2015

7th International Conference on

**Quarks and Nuclear
Physics**

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Exploring the consequences of
the chiral symmetry of QCD
and its spontaneous breaking
using effective field theory techniques

Derivation from QCD:

H. Leutwyler,

On The Foundations Of Chiral Perturbation Theory,
Ann. Phys. 235 (1994) 165 [hep-ph/9311274]

For references to lectures see:

<http://www.thep.lu.se/~bijnens/chpt.html>

A general Effective Field Theory:

- Relevant degrees of freedom
- A powercounting principle (predictivity)
- Has a certain range of validity

Chiral Perturbation Theory:

- **Degrees of freedom:** Goldstone Bosons from spontaneous breaking of chiral symmetry
- **Powercounting:** Dimensional counting in momenta/masses
- **Breakdown scale:** Resonances, so about M_ρ .



A general Effective Field Theory:

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Chiral Symmetry

QCD: N_f light quarks: equal mass: interchange: $SU(N_f)_V$

But
$$\mathcal{L}_{QCD} = \sum_{q=u,d,s} [i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q (\bar{q}_R q_L + \bar{q}_L q_R)]$$

So if $m_q = 0$ then $SU(3)_L \times SU(3)_R$.

Spontaneous breakdown

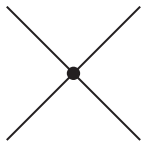
- $\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$
- $SU(3)_L \times SU(3)_R$ broken spontaneously to $SU(3)_V$
- 8 generators broken \implies 8 massless degrees of freedom
and interaction vanishes at zero momentum

Goldstone Bosons



Power counting in momenta: Meson loops, Weinberg powercounting

rules



$$p^2$$

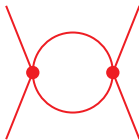


$$1/p^2$$

$$\int d^4p$$

$$p^4$$

one loop example



$$(p^2)^2 (1/p^2)^2 p^4 = p^4$$



$$(p^2)(1/p^2)p^4 = p^4$$



- Which chiral symmetry: $SU(N_f)_L \times SU(N_f)_R$, for $N_f = 2, 3, \dots$ and extensions to (partially) quenched
- Or beyond QCD
- Space-time symmetry: Continuum or broken on the lattice: Wilson, staggered, mixed action
- Volume: Infinite, finite in space, finite T
- Which interactions to include beyond the strong one
- Which particles included as non Goldstone Bosons
- My general belief: if it involves soft pions (or soft K, η) some version of ChPT exists



Lagrangians: Lowest order

$U(\phi) = \exp(i\sqrt{2}\Phi/F_0)$ parametrizes Goldstone Bosons

$$\Phi(x) = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} \end{pmatrix}.$$

LO Lagrangian: $\mathcal{L}_2 = \frac{F_0^2}{4} \{ \langle D_\mu U^\dagger D^\mu U \rangle + \langle \chi^\dagger U + \chi U^\dagger \rangle \},$

$$D_\mu U = \partial_\mu U - ir_\mu U + iU l_\mu,$$

left and right external currents: $r(l)_\mu = v_\mu + (-)a_\mu$

Scalar and pseudoscalar external densities: $\chi = 2B_0(s + ip)$ quark masses via
scalar density: $s = \mathcal{M} + \dots$

$$\langle A \rangle = Tr_F(A)$$

The main predictions of ChPT:

- Relates processes with different numbers of pseudoscalars
- Chiral logarithms
- includes Isospin and the eightfold way ($SU(3)_V$)
- Unitarity included perturbatively

$$m_\pi^2 = 2B\hat{m} + \left(\frac{2B\hat{m}}{F}\right)^2 \left[\frac{1}{32\pi^2} \log \frac{(2B\hat{m})}{\mu^2} + 2l_3^r(\mu) \right] + \dots$$

$$M^2 = 2B\hat{m}$$



Let's go over to the next point: dealing with the parameters

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(Partial) History/References

- Original determination at p^4 : Gasser, Leutwyler, *Annals Phys.*158 (1984) 142, *Nucl. Phys.* B250 (1985) 465
- p^6 2 flavour: several papers (see later)
- p^6 3 flavour: Amorós, JB, Talavera, *Nucl. Phys.* B602 (2001) 87 [hep-ph/0101127]
- Review article two-loops:
JB, *Prog. Part. Nucl. Phys.* 58 (2007) 521 [hep-ph/0604043]
- Update of fits + new input:
JB, Jemos, *Nucl. Phys.* B 854 (2012) 631 [arXiv:1103.5945]
- Recent review with more p^6 input: JB, Ecker, *Ann. Rev. Nucl. Part. Sci.* **64** (2014) 149 [arXiv:1405.6488]
- Review Kaon physics: Cirigliano, Ecker, Neufeld, Pich, Portoles, *Rev.Mod.Phys.* 84 (2012) 399 [arXiv:1107.6001]
- Lattice: FLAG reports:
Colangelo et al., *Eur.Phys.J.* C71 (2011) 1695 [arXiv:1011.4408]
Aoki et al., *Eur. Phys. J. C* **74** (2014) 9, 2890 [arXiv:1310.8555]

Two flavour LECs



- \bar{l}_1 to \bar{l}_4 : ChPT at order p^6 and the Roy equation analysis in $\pi\pi$ and F_S Colangelo, Gasser and Leutwyler, *Nucl. Phys. B* 603 (2001) 125 [hep-ph/0103088] Compatible with Rios, Nebrada, Pelaez
- \bar{l}_5 and \bar{l}_6 : from F_V and $\pi \rightarrow \ell\nu\gamma$ JB,(Colangelo,)Talavera and from $\Pi_V - \Pi_A$ González-Alonso, Pich, Prades
- $\bar{l}_1 = -0.4 \pm 0.6$, $\bar{l}_2 = 4.3 \pm 0.1$,
 $\bar{l}_3 = 2.9 \pm 2.4$, $\bar{l}_4 = 4.4 \pm 0.2$,
 $\bar{l}_5 = 12.24 \pm 0.21$, $\bar{l}_6 - \bar{l}_5 = 3.0 \pm 0.3$,
 $\bar{l}_6 = 16.0 \pm 0.5 \pm 0.7$.
- $l_7 \sim 5 \cdot 10^{-3}$ from π^0 - η mixing Gasser, Leutwyler 1984
- guesstimate including lattice: $\bar{l}_3 = 3.0 \pm 0.8$ $\bar{l}_4 = 4.3 \pm 0.3$



Three flavour LECs: uncertainties

- $m_K^2, m_\eta^2 \gg m_\pi^2$
- Contributions from p^6 Lagrangian are larger
- Reliance on estimates of the C_i much larger
- Typically: C_i^r : (terms with)
kinematical dependence \equiv measurable
quark mass dependence \equiv impossible (without lattice)
100% correlated with L_i^r
- How suppressed are the $1/N_c$ -suppressed terms?
- Are we really testing ChPT or just doing a phenomenological fit?

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Testing if ChPT works: relations

Yes: JB, Jemos, *Eur.Phys.J. C64* (2009) 273-282 [arXiv:0906.3118]

Systematic search for relations between observables that do not depend on the C_i^r

Included:

- m_M^2 and F_M for π, K, η .
- 11 $\pi\pi$ threshold parameters
- 14 πK threshold parameters
- 6 $\eta \rightarrow 3\pi$ decay parameters,
- 10 observables in $K_{\ell 4}$
- 18 in the scalar formfactors
- 11 in the vectorformfactors
- Total: 76

We found 35 relations

Relations at NNLO: summary



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- We did numerics for $\pi\pi$ (7), πK (5) and $K_{\ell 4}$ (1)
13 relations
- $\pi\pi$: similar quality in two and three flavour ChPT
The two involving a_3^- significantly did not work well
- πK : relation involving a_3^- not OK
one more has very large NNLO corrections
- The relation with $K_{\ell 4}$ also did not work: related to that
ChPT has trouble with curvature in $K_{\ell 4}$
- **Conclusion: Three flavour ChPT “sort of” works**

Fits: inputs



Amorós, JB, Talavera, Nucl. Phys. B602 (2001) 87 [hep-ph/0101127]

(ABT01)

JB, Jemos, Nucl. Phys. B 854 (2012) 631 [arXiv:1103.5945] (BJ12)

JB, Ecker, arXiv:1405.6488, Ann. Rev. Nucl. Part. Sc.(in press) (BE14)

- $M_\pi, M_K, M_\eta, F_\pi, F_K/F_\pi$
- $\langle r^2 \rangle_S^\pi, c_S^\pi$ slope and curvature of F_S
- $\pi\pi$ and πK scattering lengths $a_0^0, a_0^2, a_0^{1/2}$ and $a_0^{3/2}$.
- Value and slope of F and G in $K_{\ell 4}$
- $\frac{m_s}{\hat{m}} = 27.5$ (lattice)
- $\bar{l}_1, \dots, \bar{l}_4$
- more variation with C_i^r , a penalty for a large p^6 contribution to the masses
- 17+3 inputs and 8 L_i^r +34 C_i^r to fit

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Main fit



	ABT01	BJ12	L_4^r free	BE14
	old data			
$10^3 L_1^r$	0.39(12)	0.88(09)	0.64(06)	0.53(06)
$10^3 L_2^r$	0.73(12)	0.61(20)	0.59(04)	0.81(04)
$10^3 L_3^r$	-2.34(37)	-3.04(43)	-2.80(20)	-3.07(20)
$10^3 L_4^r$	$\equiv 0$	0.75(75)	0.76(18)	$\equiv 0.3$
$10^3 L_5^r$	0.97(11)	0.58(13)	0.50(07)	1.01(06)
$10^3 L_6^r$	$\equiv 0$	0.29(8)	0.49(25)	0.14(05)
$10^3 L_7^r$	-0.30(15)	-0.11(15)	-0.19(08)	-0.34(09)
$10^3 L_8^r$	0.60(20)	0.18(18)	0.17(11)	0.47(10)
χ^2	0.26	1.28	0.48	1.04
dof	1	4	?	?
F_0 [MeV]	87	65	64	71

$$? = (17 + 3) - (8 + 34)$$



Main fit: Comments

- All values of the C_i^r we settled on are “reasonable”
- Leaving L_4^r free ends up with $L_4^r \approx 0.76$
- keeping L_4^r small: also L_6^r and $2L_1^r - L_2^r$ small (large N_c relations)
- Compatible with lattice determinations
- Not too bad with resonance saturation both for L_i^r and C_i^r
- decent convergence (but enforced for masses)
- Many prejudices went in: large N_c , resonance model, quark model estimates, . . .

Some results of this fit



Mass:

$$m_{\pi}^2/m_{\pi phys}^2 = 1.055(p^2) - 0.005(p^4) - 0.050(p^6),$$

$$m_K^2/m_{K phys}^2 = 1.112(p^2) - 0.069(p^4) - 0.043(p^6),$$

$$m_{\eta}^2/m_{\eta phys}^2 = 1.197(p^2) - 0.214(p^4) + 0.017(p^6),$$

Decay constants:

$$F_{\pi}/F_0 = 1.000(p^2) + 0.208(p^4) + 0.088(p^6),$$

$$F_K/F_{\pi} = 1.000(p^2) + 0.176(p^4) + 0.023(p^6).$$

Scattering:

$$a_0^0 = 0.160(p^2) + 0.044(p^4) + 0.012(p^6),$$

$$a_0^{1/2} = 0.142(p^2) + 0.031(p^4) + 0.051(p^6).$$



An example where ChPT triumphed

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Charged pion polarizabilities: experiment



Review: Holstein, Scherer, *Ann. Rev. Nucl. Part. Sci.* **64** (2014) 51 [1401.0140]

- Expand $\gamma\pi^\pm \rightarrow \gamma\pi^\pm$ near threshold: ($z_\pm = 1 \pm \cos\theta_{\text{cm}}$)

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega_{\text{Born}}} - \frac{\alpha m_\pi^3 ((s - m_\pi^2)^2)}{4s^2 (sz_+ + m_\pi^2 z_-)} \left(z_-^2 (\alpha - \beta) + \frac{s^2}{m_\pi^4} z_+^2 (\alpha + \beta) \right)$$

- Three ways to measure: (all assume $\alpha + \beta = 0$)
 - $\pi\gamma \rightarrow \pi\gamma$ (Primakoff, high energy pion beam)
Dubna (1985) $\alpha = (6.8 \pm 1.4) 10^{-4} \text{ fm}^3$
Compass (CERN, 2015) $\alpha = (2.0 \pm 0.6 \pm 0.7) 10^{-4} \text{ fm}^3$
 - $\gamma\pi \rightarrow \pi\gamma$ (via one-pion exchange)
Lebedev (1986) $\alpha = (20 \pm 12) 10^{-4} \text{ fm}^3$
Mainz (2005) $\alpha = (5.8 \pm 0.75 \pm 1.5 \pm 0.25) 10^{-4} \text{ fm}^3$
 - $\gamma\gamma \rightarrow \pi\pi$ (in $e^+e^- \rightarrow e^+e^-\pi^+\pi^-$)
MarkII data analyzed (1992) $\alpha = (2.2 \pm 1.1) 10^{-4} \text{ fm}^3$
- Extrapolation and subtraction: difficult experiments

Polarizabilities: extrapolations needed



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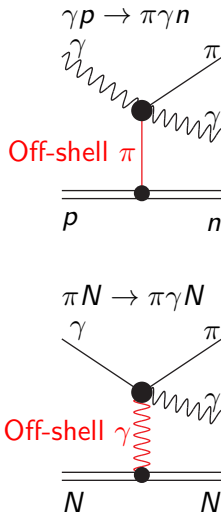
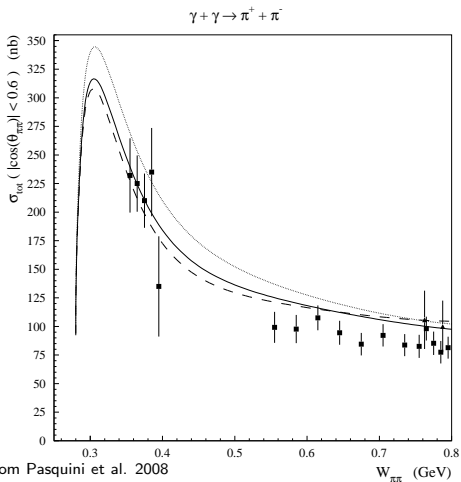
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Charged pion polarizabilities: theory



- ChPT:

- One-loop JB, Cornet, 1986, Donoghue-Holstein 1989

$$\alpha + \beta = 0, \alpha = (2.8 \pm 0.2) \cdot 10^{-4} \text{ fm}^3$$

input $\pi \rightarrow e\nu\gamma$ (error only from this)

- Two-loop Bürigi, 1996, Gasser, Ivanov, Sainio 2006

$$\alpha + \beta = 0.16 \cdot 10^{-4} \text{ fm}^3, \alpha = (2.8 \pm 0.5) \cdot 10^{-4} \text{ fm}^3$$

- Dispersive analysis from $\gamma\gamma \rightarrow \pi\pi$:

- Fil'kov-Kashevarov, 2005 $(\alpha_1 - \beta_1) = (13.0^{+2.6}_{-1.9}) \cdot 10^{-4} \text{ fm}^3$

- Critized by Pasquini-Drechsel-Scherer, 2008

“Large model dependence in their extraction”

“Our calculations... are in reasonable agreement with ChPT for charged pions”

$$(\alpha_1 - \beta_1) = (5.7) \cdot 10^{-4} \text{ fm}^3 \text{ perfectly possible}$$



An example of other effects:

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- Lattice QCD calculates at different quark masses, volumes boundary conditions, . . .
- A general result by Lüscher: relate finite volume effects to scattering (1986)
- Chiral Perturbation Theory is also useful for this
- Start: Gasser and Leutwyler, *Phys. Lett. B*184 (1987) 83, *Nucl. Phys. B* 307 (1988) 763
 $M_\pi, F_\pi, \langle \bar{q}q \rangle$ one-loop equal mass case
- I will stay with ChPT and the p regime ($M_\pi L \gg 1$)
- $1/m_\pi = 1.4$ fm
may need to go beyond leading $e^{-m_\pi L}$ terms
- Convergence of ChPT is given by $1/m_\rho \approx 0.25$ fm

Finite volume: selection of ChPT results



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- masses and decay constants for π, K, η one-loop
Becirevic, Villadoro, Phys. Rev. D 69 (2004) 054010
- M_π at 2-loops (2-flavour)
Colangelo, Haefeli, Nucl.Phys. B744 (2006) 14 [hep-lat/0602017]
- $\langle \bar{q}q \rangle$ at 2 loops (3-flavour)
JB, Ghorbani, Phys. Lett. B636 (2006) 51 [hep-lat/0602019]
- Twisted mass at one-loop
Colangelo, Wenger, Wu, Phys.Rev. D82 (2010) 034502 [arXiv:1003.0847]
- Twisted boundary conditions
Sachrajda, Villadoro, Phys. Lett. B 609 (2005) 73 [hep-lat/0411033]
- This talk:
 - Twisted boundary conditions and some funny effects
 - Some results on masses 3-flavours at two loop order



Twisted boundary conditions

- On a lattice at finite volume $p^i = 2\pi n^i/L$: very few momenta directly accessible
- Put a constraint on certain quark fields in some directions:
 $q(x^i + L) = e^{i\theta^i} q(x^i)$
- Then momenta are $p^i = \theta^i/L + 2\pi n^i/L$. Allows to map out momentum space on the lattice much better Bedaque,...
- But:
 - Box: Rotation invariance \rightarrow cubic invariance
 - Twisting: reduces symmetry further

Consequences:

- $m^2(\vec{p}^2) = E^2 - \vec{p}^2$ is not constant
- There are typically more form-factors
- In general: quantities depend on more (all) components of the momenta
- Charge conjugation involves a change in momentum



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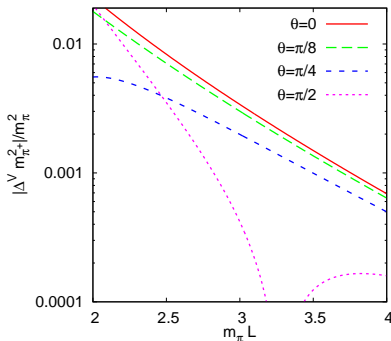
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Twisted boundary conditions: volume correction masses

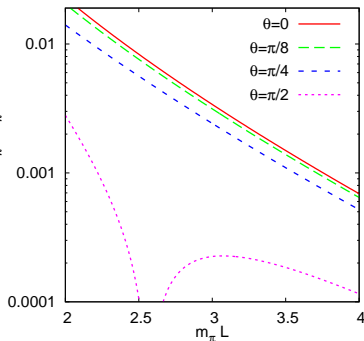


JB, Relefors, JHEP 05 (2014) 015 [arXiv:1402.1385]

$$m_\pi L = 2, \vec{\theta}_u = (\theta, 0, 0), \vec{\theta}_d = \vec{\theta}_s = 0$$



Charged pion mass



Neutral pion mass

$$\Delta^V X = X^V - X^\infty \text{ (dip is going through zero)}$$

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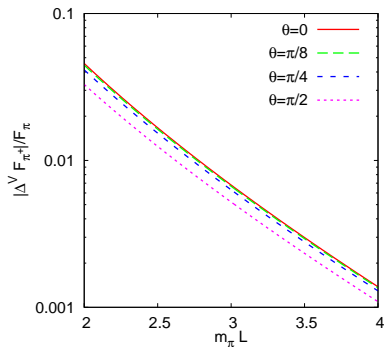
Volume correction decay constants: F_{π^+}



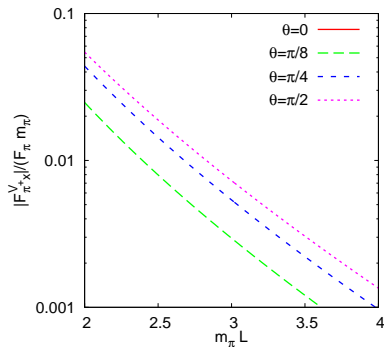
- JB, Relefors, JHEP 05 (2014) 015 [arXiv:1402.1385]

- $\langle 0 | A_{\mu}^M | M(p) \rangle = i\sqrt{2}F_M p_{\mu} + i\sqrt{2}F_{M\mu}^V$

- Extra terms are needed for Ward identities



relative for F_{π}



Extra for $\mu = x$

Volume correction electromagnetic formfactor



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earlier two-flavour work:

Bunton, Jiang, Tiburzi, Phys.Rev. D74 (2006) 034514 [hep-lat/0607001]

- $\langle M'(p') | j_\mu | M(p) \rangle = f_\mu = f_+(p_\mu + p'_\mu) + f_- q_\mu + h_\mu$
- Extra terms are again needed for Ward identities
- Note that masses have finite volume corrections
 - q^2 for fixed \vec{p} and \vec{p}' has corrections
small effect
 - This also affects the ward identities, e.g.
 $q^\mu f_\mu = (p^2 - p'^2) f_+ + q^2 f_- + q^\mu h_\mu = 0$
is satisfied but all effects should be considered



Volume correction electromagnetic formfactor

- JB, Relefors, JHEP 05 (2014) 015 [arXiv:1402.1385]

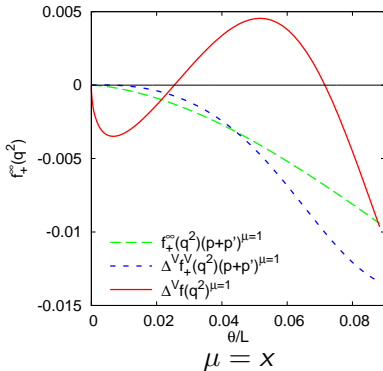
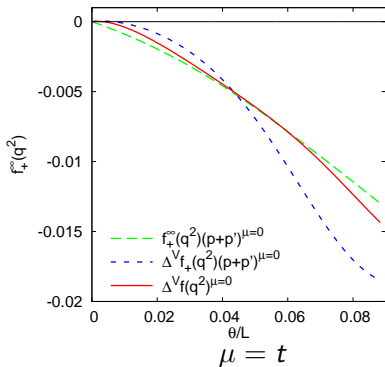
earlier two-flavour work:

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- Note that masses have finite volume corrections
 - q^2 for fixed \vec{p} and \vec{p}' has corrections
small effect
 - This also affects the ward identities, e.g.
 $q^\mu f_\mu = (p^2 - p'^2) f_+ + q^2 f_- + q^\mu h_\mu = 0$
is satisfied but all effects should be considered

Volume correction electromagnetic formfactor

- $f_\mu = -\frac{1}{\sqrt{2}} \langle \pi^0(p') | \bar{d} \gamma_\mu u | \pi^+(p) \rangle$
 $= (1 + f_+^\infty + \Delta^V f_+) (p + p')_\mu + \Delta^V f_- q_\mu + \Delta^V h_\mu$
- Pure loop plotted: $f_+^\infty(p + p')$, $\Delta^V f_+(p + p')$ and $\Delta^V f_\mu$



Finite volume corrections large, different for different μ

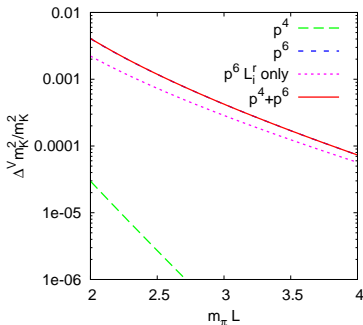
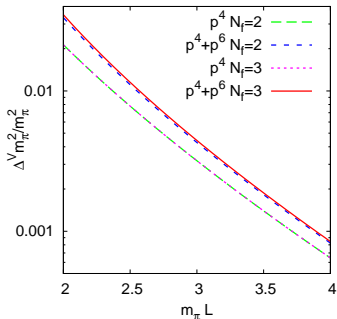
Masses at two-loop order

- Sunset integrals at finite volume done

JB, Boström and Lähde, JHEP 01 (2014) 019 [arXiv:1311.3531]

- Loop calculations:

JB, Rössler, JHEP 1501 (2015) 034 [arXiv:1411.6384]



- Agreement for $N_f = 2, 3$ for pion
- K has no pion loop at LO

Decay constants at two-loop order

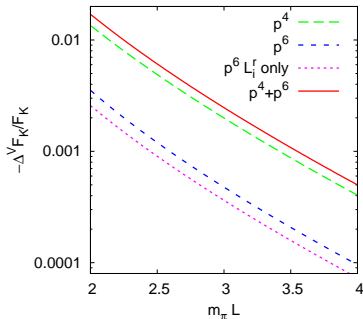
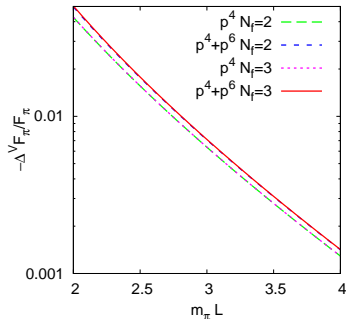


- Sunset integrals at finite volume done

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- Loop calculations:

JB, Rössler, JHEP 1501 (2015) 034 [arXiv:1411.6384]



- Agreement for $N_f = 2, 3$ for pion
- K now has a pion loop at LO

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ChPT for other theories:

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- One can also have different symmetry breaking patterns from underlying fermions
- Three generic cases
 - $SU(N) \times SU(N)/SU(N)$
 - $SU(2N)/SO(2N)$
 - $SU(2N)/Sp(2N)$
- Many one-loop results existed especially for the first case (several discovered only after we published our work)
- Equal mass case pushed to two loops [JB, Lu, 2009-11](#)



N_F fermions in a representation of the gauge group

- complex (QCD):
 - $q^T = (q_1 \ q_2 \ \dots \ q_{N_F})$
 - Global $G = SU(N_F)_L \times SU(N_F)_R$
 $q_L \rightarrow g_L q_L$ and $q_R \rightarrow g_R q_R$
 - Vacuum condensate $\Sigma_{ij} = \langle \bar{q}_j q_i \rangle \propto \delta_{ij}$
 - $g_L = g_R$ then $\Sigma_{ij} \rightarrow \Sigma_{ij} \implies$ conserved $H = SU(N_F)_V$:
- Real (e.g. adjoint): $\hat{q}^T = (q_{R1} \ \dots \ q_{RN_F} \ \tilde{q}_{R1} \ \dots \ \tilde{q}_{RN_F})$
 - $\tilde{q}_{Ri} \equiv C \bar{q}_{Li}^T$ goes under gauge group as q_{Ri}
 - some Goldstone bosons have baryonnumber
 - Global $G = SU(2N_F)$ and $\hat{q} \rightarrow g \hat{q}$
 - $\langle \bar{q}_j q_i \rangle$ is really $\langle (\hat{q}_j)^T C \hat{q}_i \rangle \propto J_{Sij} \ J_S = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$
 - Conserved if $g J_S g^T = J_S \implies H = SO(2N_F)$



N_F fermions in a representation of the gauge group

- complex (QCD):
 - $q^T = (q_1 \ q_2 \ \dots \ q_{N_F})$
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N_F fermions in a representation of the gauge group

- complex (QCD): $q^T = (q_1 \ q_2 \ \dots \ q_{N_F})$
 - Global $G = SU(N_F)_L \times SU(N_F)_R$ $q_L \rightarrow g_L q_L$ and $g_R \rightarrow g_R q_R$
 - Vacuum condensate $\Sigma_{ij} = \langle \bar{q}_j q_i \rangle \propto \delta_{ij}$
 - Conserved $H = SU(N_F)_V$: $g_L = g_R$ then $\Sigma_{ij} \rightarrow \Sigma_{ij}$

- Pseudoreal (e.g. two-colours):

$$\hat{q}^T = (q_{R1} \ \dots \ q_{RN_F} \ \tilde{q}_{R1} \ \dots \ \tilde{q}_{RN_F})$$

- $\tilde{q}_{R\alpha i} \equiv \epsilon_{\alpha\beta} C \bar{q}_{L\beta i}^T$ goes under gauge group as $q_{R\alpha i}$
- some Goldstone bosons have baryonnumber
- Global $G = SU(2N_F)$ and $\hat{q} \rightarrow g \hat{q}$
- $\langle \bar{q}_j q_i \rangle$ is really $\epsilon_{\alpha\beta} \langle (\hat{q}_{\alpha j})^T C \hat{q}_{\beta i} \rangle \propto J_{Aij}$ $J_A = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}$
- Conserved if $g J_A g^T = J_A \implies H = Sp(2N_F)$

JB, Lu, arXiv:0910.5424: 3 cases similar with $u = \exp\left(\frac{i}{\sqrt{2}F}\phi^a X^a\right)$

But the matrices X^a are:

- Complex or $SU(N) \times SU(N)/SU(N)$:
all $SU(N)$ generators
- Real or $SU(2N)/SO(2N)$:
 $SU(2N)$ generators with $X^a J_S = J_S X^{aT}$
- Pseudoreal or $SU(2N)/Sp(2N)$:
 $SU(2N)$ generators with $X^a J_A = J_A X^{aT}$
- Note that the latter are not the usual ways of parametrizing $SO(2N)$ and $Sp(2N)$ matrices

The main useful formulae



Calculating for equal mass case goes through using:

$$\text{Complex :} \quad \langle X^a A X^a B \rangle = \langle A \rangle \langle B \rangle - \frac{1}{N_F} \langle AB \rangle ,$$

$$\langle X^a A \rangle \langle X^a B \rangle = \langle AB \rangle - \frac{1}{N_F} \langle A \rangle \langle B \rangle .$$

$$\text{Real :} \quad \langle X^a A X^a B \rangle = \frac{1}{2} \langle A \rangle \langle B \rangle + \frac{1}{2} \langle A J_S B^T J_S \rangle - \frac{1}{2N_F} \langle AB \rangle ,$$

$$\langle X^a A \rangle \langle X^a B \rangle = \frac{1}{2} \langle AB \rangle + \frac{1}{2} \langle A J_S B^T J_S \rangle - \frac{1}{2N_F} \langle A \rangle \langle B \rangle .$$

$$\text{Pseudoreal :} \quad \langle X^a A X^a B \rangle = \frac{1}{2} \langle A \rangle \langle B \rangle + \frac{1}{2} \langle A J_A B^T J_A \rangle - \frac{1}{2N_F} \langle AB \rangle ,$$

$$\langle X^a A \rangle \langle X^a B \rangle = \frac{1}{2} \langle AB \rangle - \frac{1}{2} \langle A J_A B^T J_A \rangle - \frac{1}{2N_F} \langle A \rangle \langle B \rangle$$

So can do the calculations for all cases



$$\phi\phi \rightarrow \phi\phi$$

- $\pi\pi$ scattering

- Amplitude in terms of $A(s, t, u)$

$$M_{\pi\pi}(s, t, u) = \delta^{ab}\delta^{cd}A(s, t, u) + \delta^{ac}\delta^{bd}A(t, u, s) + \delta^{ad}\delta^{bc}A(u, s, t).$$

- Three intermediate states $I = 0, 1, 2$

- Our three cases

- Two amplitudes needed $B(s, t, u)$ and $C(s, t, u)$

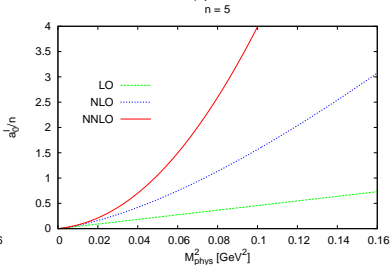
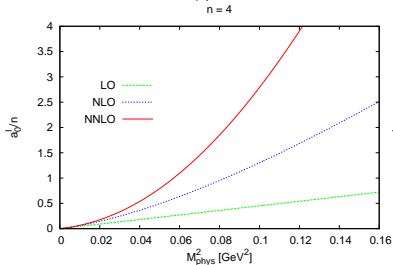
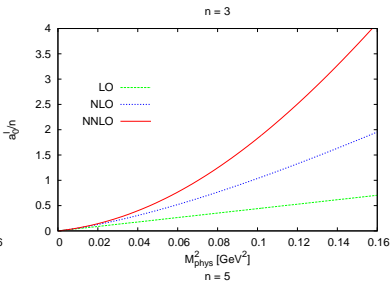
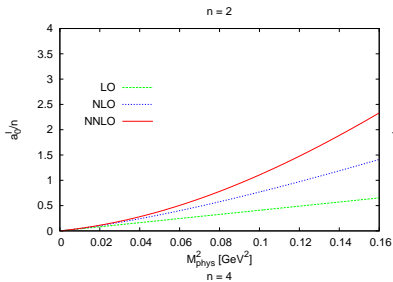
$$\begin{aligned} M(s, t, u) = & \left[\langle X^a X^b X^c X^d \rangle + \langle X^a X^d X^c X^b \rangle \right] B(s, t, u) \\ & + \left[\langle X^a X^c X^d X^b \rangle + \langle X^a X^b X^d X^c \rangle \right] B(t, u, s) \\ & + \left[\langle X^a X^d X^b X^c \rangle + \langle X^a X^c X^b X^d \rangle \right] B(u, s, t) \\ & + \delta^{ab}\delta^{cd}C(s, t, u) + \delta^{ac}\delta^{bd}C(t, u, s) + \delta^{ad}\delta^{bc}C(u, s, t). \end{aligned}$$

$$B(s, t, u) = B(u, t, s) \quad C(s, t, u) = C(s, u, t).$$

- 7, 6 and 6 possible intermediate states

- All formulas similar length to $\pi\pi$ cases but there are so many of them

$$\phi\phi \rightarrow \phi\phi: a_0^I/n$$



Conclusions for “Beyond QCD”



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Calculations done:

- M_{phys}^2
- F_{phys}
- Meson-meson scattering
- Equal mass case: allows to get fully analytical result just as for 2-flavour ChPT
- Two-point functions relevant for S -parameter

To remember:

- Different symmetry patterns can appear for different gaugegroups and fermion representations
- Nonperturbative: lattice needs extrapolation formulae

Making the programs more accessible for others to use:

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Program availability



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Conclusions

- Two-loop results have very long expressions
- Many not published but available from <http://www.thep.lu.se/~bijnens/chpt/>
- Many programs available on request from the authors
- Idea: make a more general framework
- CHIRON:

JB,

“CHIRON: a package for ChPT numerical results
at two loops,”

Eur. Phys. J. C **75** (2015) 27 [arXiv:1412.0887]

<http://www.thep.lu.se/~bijnens/chiron/>



Program availability: CHIRON



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Conclusions

- Present version: 0.52
- Classes to deal with L_i , C_i , $L_i^{(n)}$, K_i , standardized in/output, changing the scale, . . .
- Loop integrals: one-loop and sunset integrals
- Included so far (at two-loop order):
 - Masses, decay constants and $\langle \bar{q}q \rangle$ for the three flavour case
 - Masses and decay constants at finite volume in the three flavour case
 - Masses and decay constants in the partially quenched case for three sea quarks
- A large number of example programs is included



Can we calculate something of high loop orders?

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- Take a quantity with a single scale: $F(M)$
- The dependence on the scale in field theory is typically logarithmic
- $L = \log(\mu/M)$
- $F = F_0 + F_1^1 L + F_0^1 + F_2^2 L^2 + F_1^2 L + F_0^2 + F_3^3 L^3 + \dots$
- Leading Logarithms: The terms $F_m^m L^m$

The F_m^m can be more easily calculated than the full result

- $\mu(dF/d\mu) \equiv 0$
- Ultraviolet divergences in Quantum Field Theory are always local

Weinberg's argument



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- Weinberg, *Physica A96* (1979) 327
- Two-loop leading logarithms can be calculated using only one-loop: **Weinberg consistency conditions**
- Proof at all orders:
 - using β -functions: Büchler, Colangelo, hep-ph/0309049
 - Proof with diagrams: JB, Carloni, arXiv:0909.5086
- Proof relies on
 - μ : dimensional regularization scale
 - $d = 4 - w$
 - at n -loop order (\hbar^n) must cancel:
 - $1/w^n, \log \mu/w^{n-1}, \dots, \log^{n-1} \mu/w$
 - This allows for relations between diagrams
 - All needed for $\log^n \mu$ coefficient can be calculated from one-loop diagrams

Mass to \hbar^2



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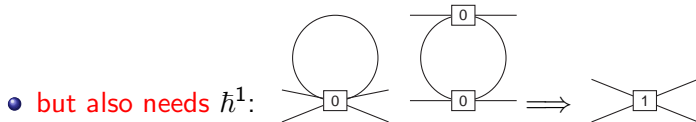
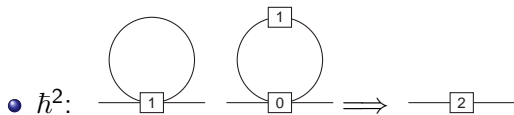
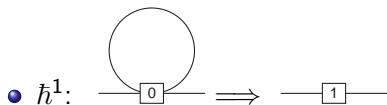
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Conclusions





- Calculate the divergence
- rewrite it in terms of a local Lagrangian
 - Luckily: symmetry kept: we know result will be symmetrical, hence do not need to explicitly rewrite the Lagrangians in a nice form
 - Luckily: we do not need to go to a minimal Lagrangian
 - So everything can be computerized
- We keep all terms to have all 1PI (one particle irreducible) diagrams finite



Massive $O(N)$ sigma model

- $O(N + 1)/O(N)$ nonlinear sigma model
- $\mathcal{L}_{n\sigma} = \frac{F^2}{2} \partial_\mu \Phi^T \partial^\mu \Phi + F^2 \chi^T \Phi$.
 - Φ is a real $N + 1$ vector; $\Phi \rightarrow O\Phi$; $\Phi^T \Phi = 1$.
 - Vacuum expectation value $\langle \Phi^T \rangle = (1 \ 0 \dots 0)$
 - Explicit symmetry breaking: $\chi^T = (M^2 \ 0 \dots 0)$
 - Both spontaneous and explicit symmetry breaking
 - N -vector ϕ
- N (pseudo-)Nambu-Goldstone Bosons
- $N = 3$ is two-flavour Chiral Perturbation Theory

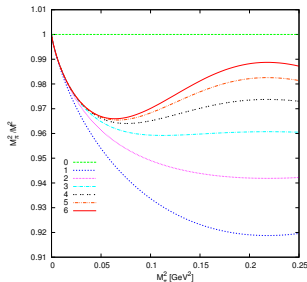
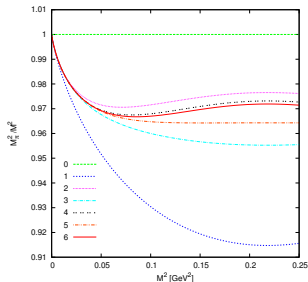
- $M_{\text{phys}}^2 = M^2(1 + a_1 L_M + a_2 L_M^2 + a_3 L_M^3 + \dots)$

$$L_M = \frac{M^2}{16\pi^2 F^2} \log \frac{\mu^2}{M^2}$$

i	$a_i, N = 3$	a_i for general N
1	$-\frac{1}{2}$	$1 - \frac{N}{2}$
2	$\frac{17}{8}$	$\frac{7}{4} - \frac{7N}{4} + \frac{5 N^2}{8}$
3	$-\frac{103}{24}$	$\frac{37}{12} - \frac{113N}{24} + \frac{15 N^2}{4} - N^3$
4	$\frac{24367}{1152}$	$\frac{839}{144} - \frac{1601 N}{144} + \frac{695 N^2}{48} - \frac{135 N^3}{16} + \frac{231 N^4}{128}$
5	$-\frac{8821}{144}$	$\frac{33661}{2400} - \frac{1151407 N}{43200} + \frac{197587 N^2}{4320} - \frac{12709 N^3}{300} + \frac{6271 N^4}{320} - \frac{7 N^5}{2}$

- $F_{\text{phys}}, \langle \bar{q}_i q_i \rangle$ as well done
- Anyone recognize any funny functions?
- Many more and larger tables in the papers

Numerical results: masses



Left: $\frac{M_{\text{phys}}^2}{M^2} = 1 + a_1 L_M + a_2 L_M^2 + a_3 L_M^3 + \dots$

Right: $\frac{M_{\text{phys}}^2}{M^2} = 1 + d_1 L_{M_{\text{phys}}} + d_2 L_{M_{\text{phys}}}^2 + d_3 L_{M_{\text{phys}}}^3 + \dots$

$F = 90 \text{ MeV}$ (left), $\mu = 0.77 \text{ GeV}$, $F_\pi = 93 \text{ MeV}$ (right)

Numerical results: decay constants



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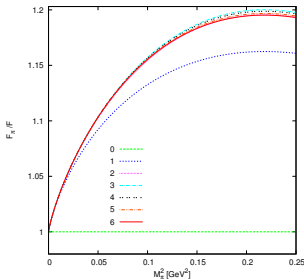
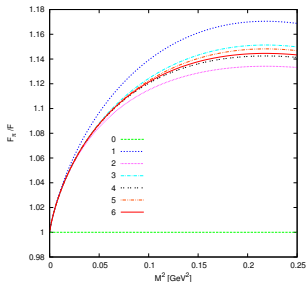
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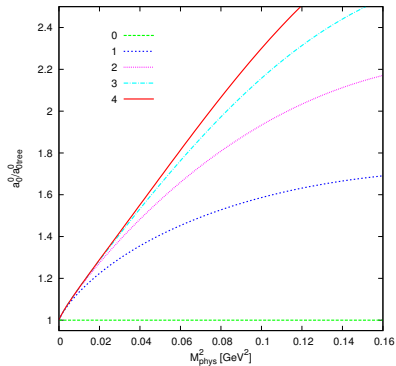


Left: $\frac{F_{\text{phys}}}{F} = 1 + a_1 L_M + a_2 L_M^2 + a_3 L_M^3 + \dots$

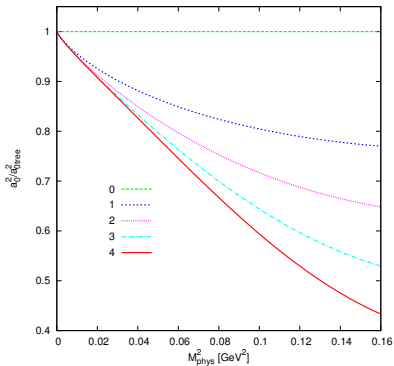
Right: $\frac{F_{\text{phys}}}{F} = 1 + d_1 L_{M_{\text{phys}}} + d_2 L_{M_{\text{phys}}}^2 + d_3 L_{M_{\text{phys}}}^3 + \dots$

$F = 90 \text{ MeV}$ (left), $\mu = 0.77 \text{ GeV}$, $F_\pi = 93 \text{ MeV}$ (right)

Numerical results: $\pi\pi$ scattering



a_0^0



a_0^2

$\mu = 0.77$ GeV, $F_\pi = 93$ MeV (right)
Here the leading logs actually dominate

Anomaly for $O(4)/O(3)$

JB, Kampf, Lanz, arXiv:1201.2608

- $$\mathcal{L}_{WZW} = -\frac{N_c}{8\pi^2} \epsilon^{\mu\nu\rho\sigma} \left\{ \epsilon^{abc} \left(\frac{1}{3} \Phi^0 \partial_\mu \Phi^a \partial_\nu \Phi^b \partial_\rho \Phi^c - \partial_\mu \Phi^0 \partial_\nu \Phi^a \partial_\rho \Phi^b \Phi^c \right) v_\sigma^0 \right. \\ \left. + (\partial_\mu \Phi^0 \Phi^a - \Phi^0 \partial_\mu \Phi^a) v_\nu^a \partial_\rho v_\sigma^0 + \frac{1}{2} \epsilon^{abc} \Phi^0 \Phi^a v_\mu^b v_\nu^c \partial_\rho v_\sigma^0 \right\}.$$

- $$A(\pi^0 \rightarrow \gamma(k_1)\gamma(k_2)) = \epsilon_{\mu\nu\alpha\beta} \epsilon_1^{*\mu}(k_1) \epsilon_2^{*\nu}(k_2) k_1^\alpha k_2^\beta F_{\pi\gamma\gamma}(k_1^2, k_2^2)$$

- $$F_{\pi\gamma\gamma}(k_1^2, k_2^2) = \frac{e^2}{4\pi^2 F_\pi} \hat{F} F_\gamma(k_1^2) F_\gamma(k_2^2) F_{\gamma\gamma}(k_1^2, k_2^2)$$

- \hat{F} : on-shell photon; $F_\gamma(k^2)$: formfactor;
 $F_{\gamma\gamma}$ nonfactorizable part

Anomaly for $O(4)/O(3)$



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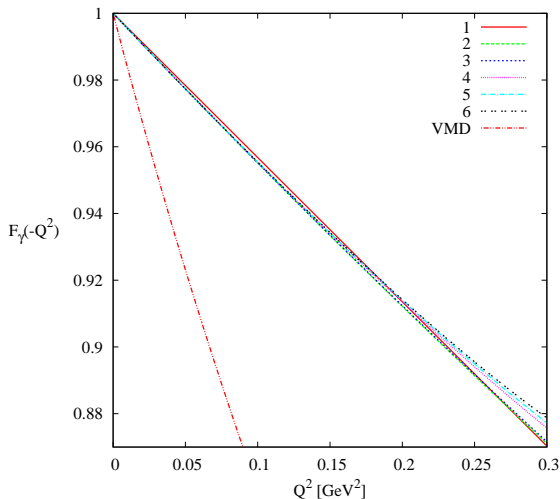
Leading
logarithms

Conclusions

- Done to six-loops
- $\hat{F} = 1 + 0 - 0.000372 + 0.000088 + 0.000036 + 0.000009 + 0.0000002 + \dots$
- Really good convergence
- $F_{\gamma\gamma}$ only starts at three-loop order (could have been two)
- $F_{\gamma\gamma}$ in the chiral limit only starts at four-loops.
- The leading logarithms thus predict this part to be fairly small.
- $F_{\gamma}(k^2)$: plot



Anomaly for $O(4)/O(3)$



Leading logs small, converge fast

Conclusions Leading Logs



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Conclusions

- Many quantities in massive $O(N)$
- Had hoped: recognize the series also for other cases
- Limited essentially by CPU time and size of intermediate files
- Some studies on convergence etc.
- $\pi\pi$, F_V and F_S to four-loop order (F_V higher)
- The technique can be generalized to other models/theories
- More details, nucleon and many more results: talk in parallel session

Conclusions



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Recent results
in chiral
meson physics

Johan Bijnens

Chiral
Perturbation
Theory

Determination
of LECs in the
continuum

Charged Pion
Polarizabilities

Finite volume

Beyond QCD

A mesonic
ChPT
program
framework

Leading
logarithms

Conclusions

ChPT is a tool for many different areas of phenomenology.

I talked about a few of them :

- 1 Chiral Perturbation Theory
- 2 Determination of LECs in the continuum
- 3 Charged Pion Polarizabilities
- 4 Finite volume
- 5 Beyond QCD
- 6 A mesonic ChPT program framework
- 7 Leading logarithms
- 8 Conclusions