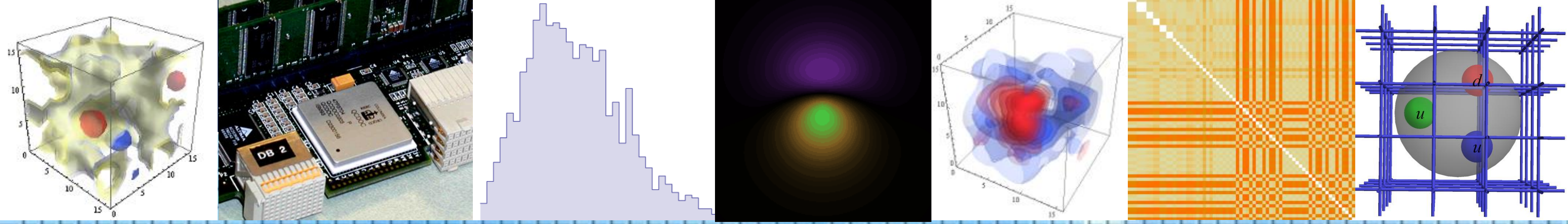


Frontiers in Lattice Nucleon Structure

Huey-Wen Lin

Outline

§ Lattice QCD for Nucleon Structure

- A brief introduction and selected results near physical pion mass

§ Spotlight on New Calculations

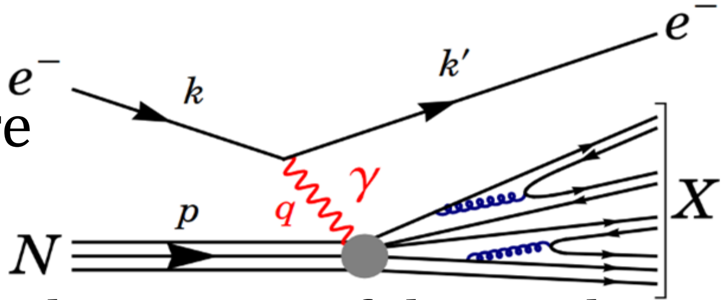
- Work in progress with great future prospects

§ Applications to New-Physics Searches

- nEDM, neutron beta decays, ...

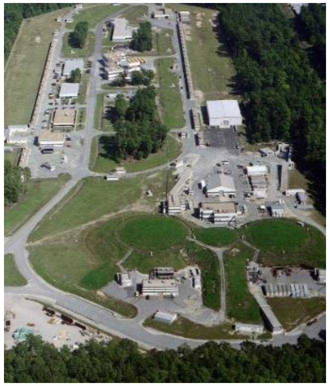
Nucleon Structure

- § Study nucleon structure since '60s
  - Deep inelastic scattering @ SLAC, more



- § Fundamental QCD property
  - Exploration of the valence and sea-quark content of the nucleon
- § Important for BSM searches
  - Provides SM cross-section prediction for LHC new-physics search
- § Still limited knowledge
  - Many on-going/planned experiments: (Jlab, J-PARC, GSI, EIC, LHeC, ...)

This conference:
McKeown, Nagae, Foka, ...



How Can LQCD Help?

§ Lattice QCD is an ideal theoretical tool for investigating strong-coupling regime of quantum field theories

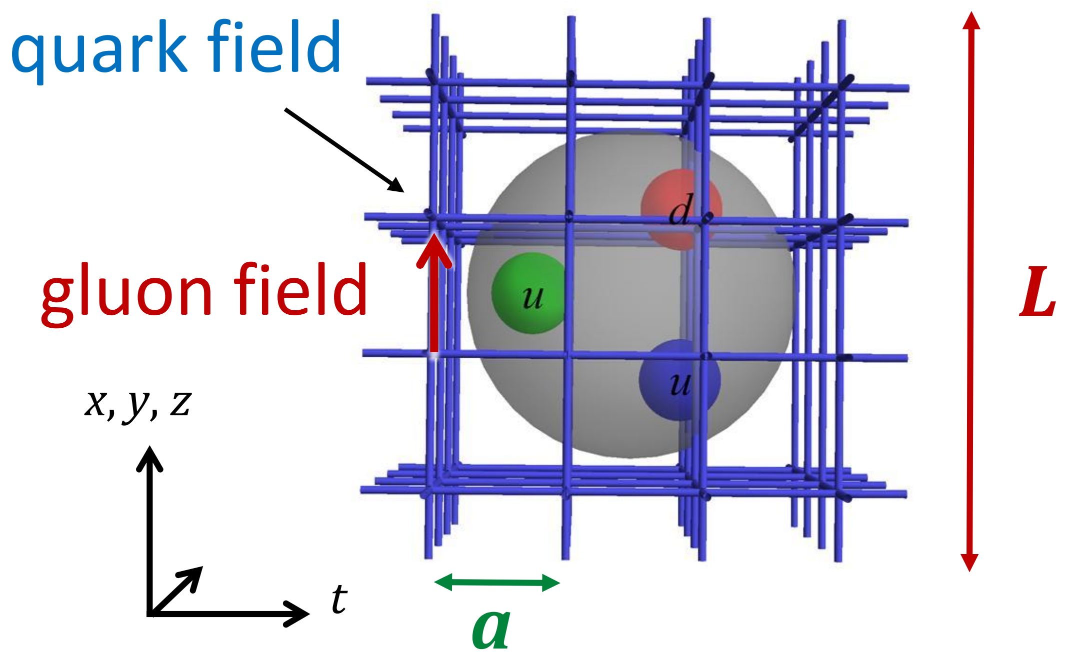
- Great for studying nonperturbative hadron structure

§ Physical observables are calculated from the path integral

$$\langle 0|O(\bar{\psi},\psi,A)|0\rangle = \frac{1}{Z}\int \mathcal{D}A\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi\, e^{iS(\bar{\psi},\psi,A)}\,O\,(\bar{\psi},\psi,A)$$

in **Euclidian** space

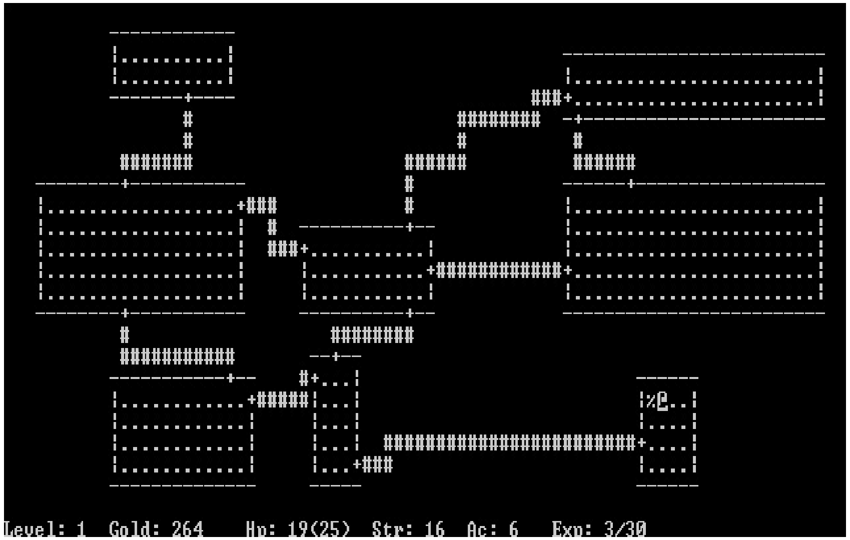
- Impose a UV cutoff
  discretize spacetime
- Impose an infrared cutoff
  finite volume



Are We There Yet?

- Lattice gauge theory was proposed in the 1970s by Wilson
  - Why haven't we solved QCD yet?
- Progress is limited by computational resources
  - Evolution of technology

80's

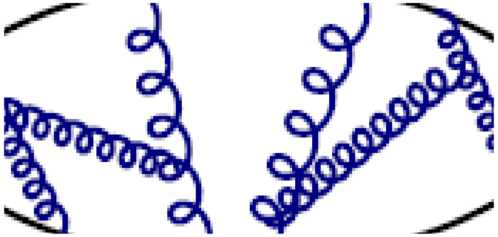


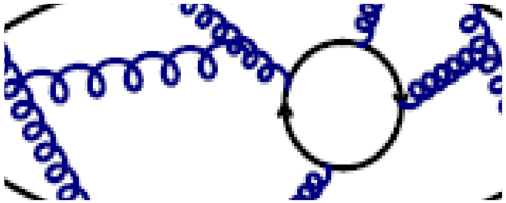
Today

- Greatly assisted by advances in algorithms

Are We There Yet?

§ From "quenched" to "dynamical" QCD vacuum





- Around the year 2000, **pure Yang-Mills** theory starts to become obsolete as a QCD approximation

- Include quark loops in vacuum
- Degenerate up/down quarks 2f
- Add strange (2+1f)
- Add charm 2+1+1f

  MILC (HISQ), ETMC (TMW), …
- Some now include isospin breaking (1+1+…) with QED

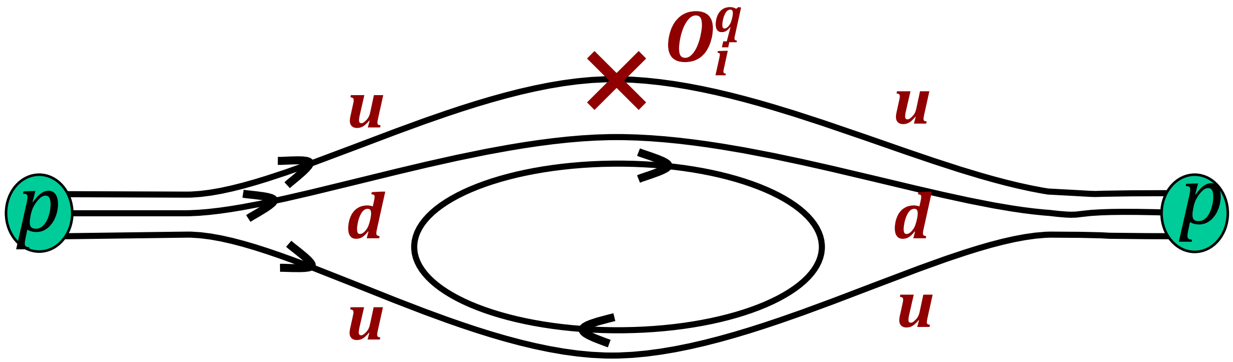
  PACS-CS, BMWc, RBC/UKQCD

§ Physical pion-mass ensembles are not uncommon!

§ Generate QCD vacuum closer to real world

Nucleon Matrix Elements

Lattice-QCD calculation of $\langle N|\bar{q}\Gamma q|N\rangle$



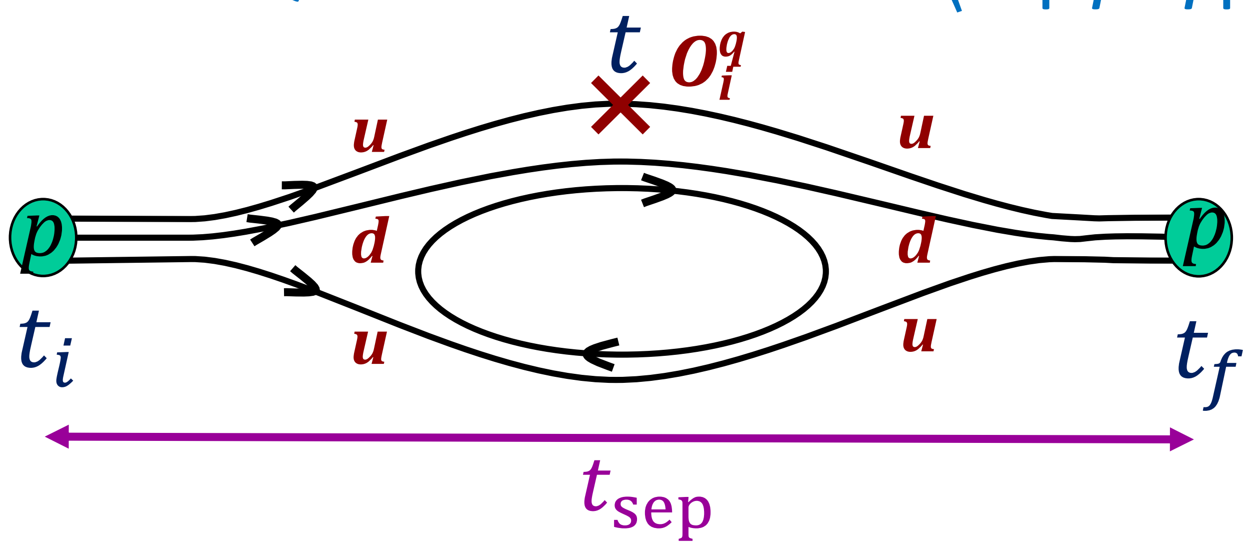
§ Construct correlators (hadronic observables)

- Requires "quark propagator"

  Invert Dirac-operator matrix (rank $O(10^{12})$)

Nucleon Matrix Elements

Lattice-QCD calculation of $\langle N|\bar{q}\Gamma q|N\rangle$



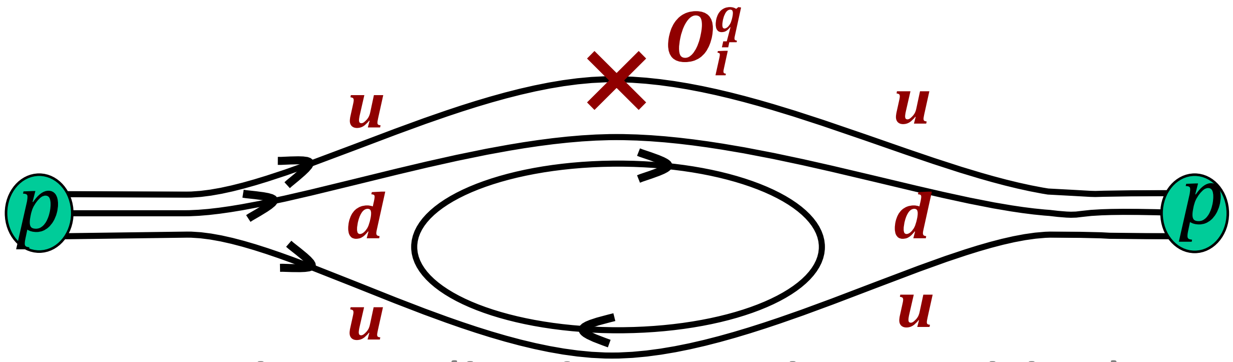
§ Analysis (extract couplings)

$$C^{\text{2pt}}(t_f,t_i)=|\mathcal{A}_0|^2 e^{-M_0(t_f-t_i)}+|\mathcal{A}_1|^2 e^{-M_1(t_f-t_i)}+\ldots$$

$$C^{\text{3pt}}(t_f,t,t_i)=|\mathcal{A}_0|^2\langle 0|\mathcal{O}_\Gamma|0\rangle e^{-M_0(t_f-t_i)}$$
$$+\mathcal{A}_0\mathcal{A}_1^*\langle 0|\mathcal{O}_\Gamma|1\rangle e^{-M_0(t-t_i)}e^{-M_1(t_f-t)}$$
$$+\mathcal{A}_0^*\mathcal{A}_1\langle 1|\mathcal{O}_\Gamma|0\rangle e^{-M_1(t-t_i)}e^{-M_0(t_f-t)}$$
$$+|\mathcal{A}_1|^2\langle 1|\mathcal{O}_\Gamma|1\rangle e^{-M_1(t_f-t_i)}$$

Nucleon Matrix Elements

Lattice-QCD calculation of $\langle N|\bar{q}\Gamma q|N\rangle$



- § Construct correlators (hadronic observables)
  - Invert Dirac-operator matrix (rank $O(10^{12})$)
- § Analysis (extract couplings)
- § Extrapolate to continuum limit
  - Take $m_\pi \to m_\pi^{\text{phys}}, a \to 0, L \to \infty$
- § Control all systematic errors:
  - Choice of extrapolation (e.g. dipole for form factors), renormalization to 2 GeV $\overline{\text{MS}}$ scheme, etc.

Progress

§ Much effort has been devoted to controlling systematics

Characteristics

§ Lighter pion masses

- More measurements at the physical pion mass!

§ More intense systematics study

- Remove assumptions in the simulation (a, L, m_q)
- 10^4 measurements not uncommon

§ Investigate previously difficult calculations

- Such as disconnected diagrams, etc.

Progress

- Much effort has been devoted to controlling systematics
- For example, PNDME's calculations

a (fm)	V	$M_\pi L$	M_π (MeV)	t_{sep}	# Meas.
0.12	$24^3 \times 64$	4.54	310	8,9,10,11,12	8104
0.12	$24^3 \times 64$	3.21	220	8,10,12	24000
0.12	$32^3 \times 64$	4.29	220	8,10,12	7664
0.12	$40^3 \times 64$	5.36	220	10	8080
0.09	$32^3 \times 96$	4.50	310	10,12,14	7048
0.09	$48^3 \times 96$	4.73	220	10,12,14	7120
0.09	$64^3 \times 96$	3.80	130	10,12,14	7064
0.06	$48^3 \times 144$	4.53	310	16,20,22,24	4000
0.06	$64^3 \times 144$	4.28	220	16,20,22,24	2400
0.06	$96^3 \times 192$	3.80	130		0

Example

§ Much effort has been devoted to controlling systematics

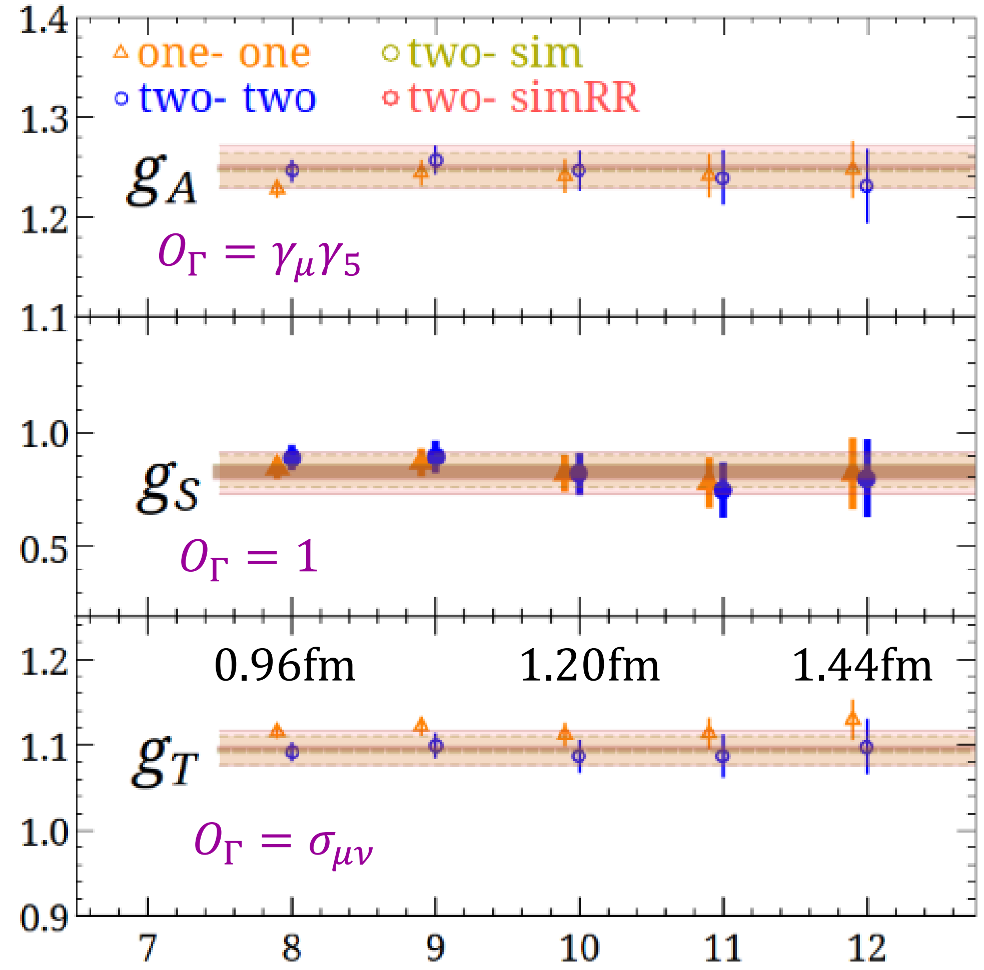
§ For example, PNDME's calculations

- Move the excited-state systematic into the statistical error

$$C^{\text{3pt}}(t_f, t, t_i) = |\mathcal{A}_0|^2 \langle 0|\mathcal{O}_\Gamma|0\rangle e^{-M_0(t_f - t_i)}$$
$$+\mathcal{A}_0\mathcal{A}_1^* \langle 0|\mathcal{O}_\Gamma|1\rangle e^{-M_0(t-t_i)} e^{-M_1(t_f - t)}$$
$$+\mathcal{A}_0^*\mathcal{A}_1 \langle 1|\mathcal{O}_\Gamma|0\rangle e^{-M_1(t-t_i)} e^{-M_0(t_f - t)}$$
$$+|\mathcal{A}_1|^2 \langle 1|\mathcal{O}_\Gamma|1\rangle e^{-M_1(t_f - t_i)}$$

- No obvious contamination between 0.96 and 1.44 fm separation

a = 0.12 fm, 310-MeV pion

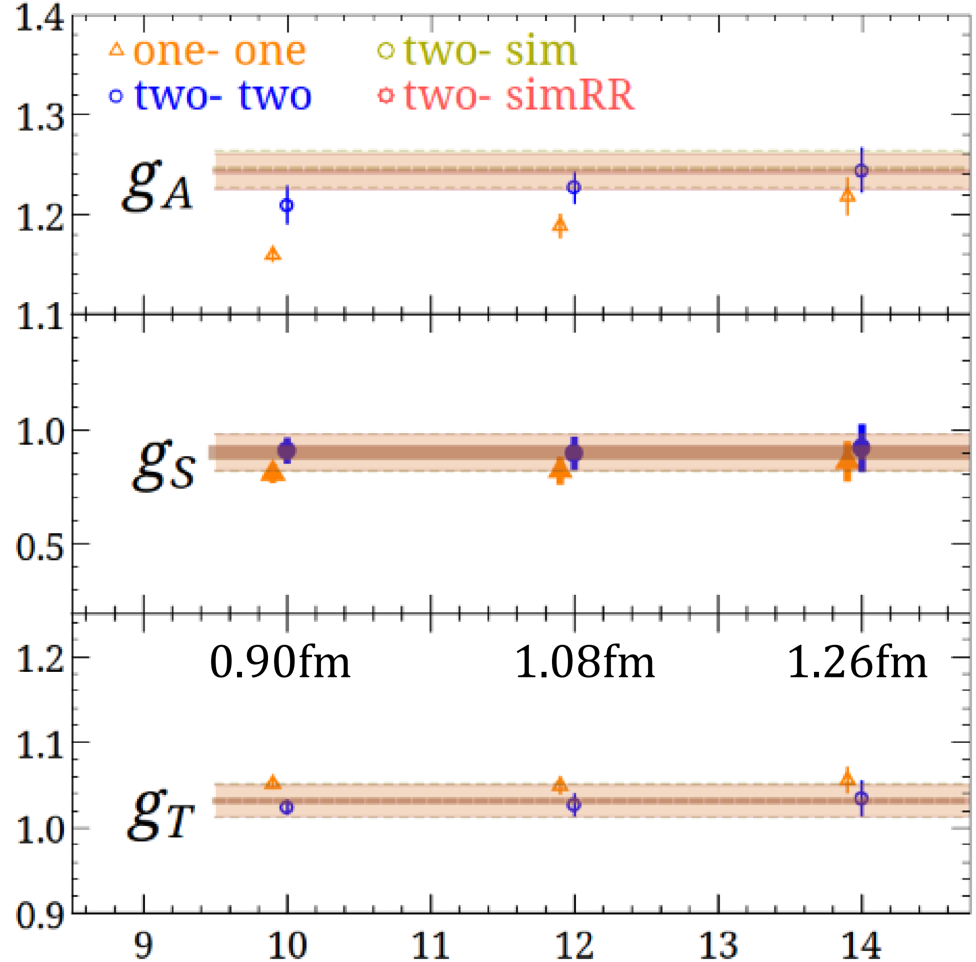


Example

§ Much effort has been devoted to controlling systematics

§ For example, PNDME's calculations

$a = 0.09$ fm, 310-MeV pion



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$$C^{3\text{pt}}(t_f, t, t_i) = |\mathcal{A}_0|^2 \langle 0|\mathcal{O}_\Gamma|0\rangle e^{-M_0(t_f-t_i)}$$
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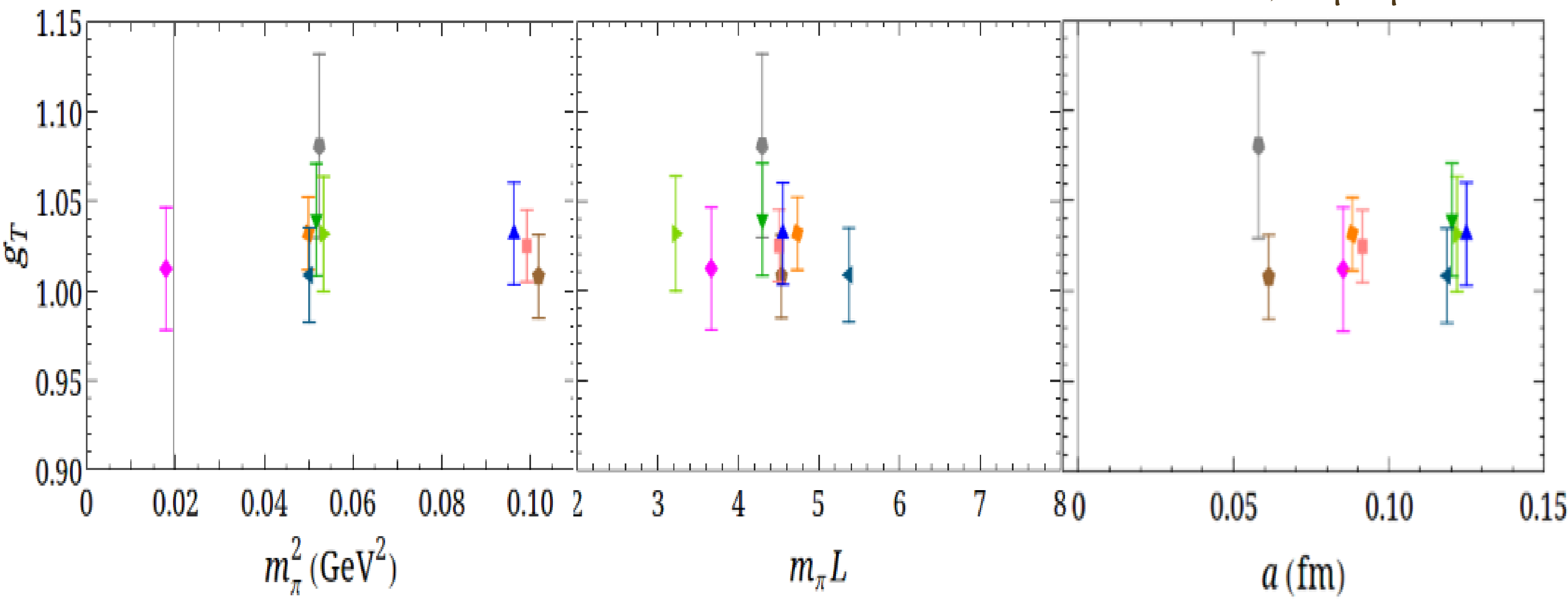
$$+|\mathcal{A}_1|^2 \langle 1|\mathcal{O}_\Gamma|1\rangle e^{-M_1(t_f-t_i)}$$

- Much stronger effect at finer lattice spacing!
- Needs to be studied case by case

Example

- § Much effort has been devoted to controlling systematics
- § For example, PNDME's g_T calculations
  - ☙ Study the pion-mass, volume and lattice spacing dependences (sometimes they are mild)

PNDME, in preparation

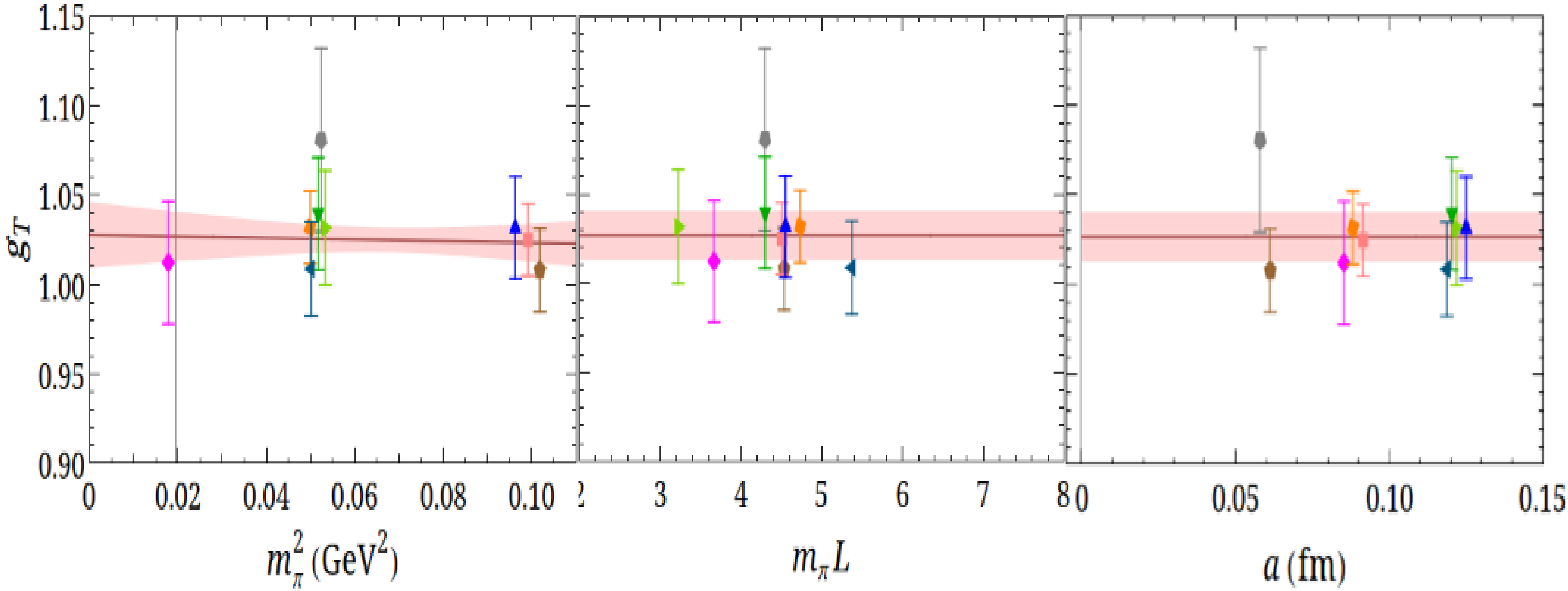


Example

- § Much effort has been devoted to controlling systematics
- § For example, PNDME's g_T calculations
  - Extrapolate to physical limit
    (assume no dependence on L or a)

g_T = 1.026(14)

PNDME, in preparation



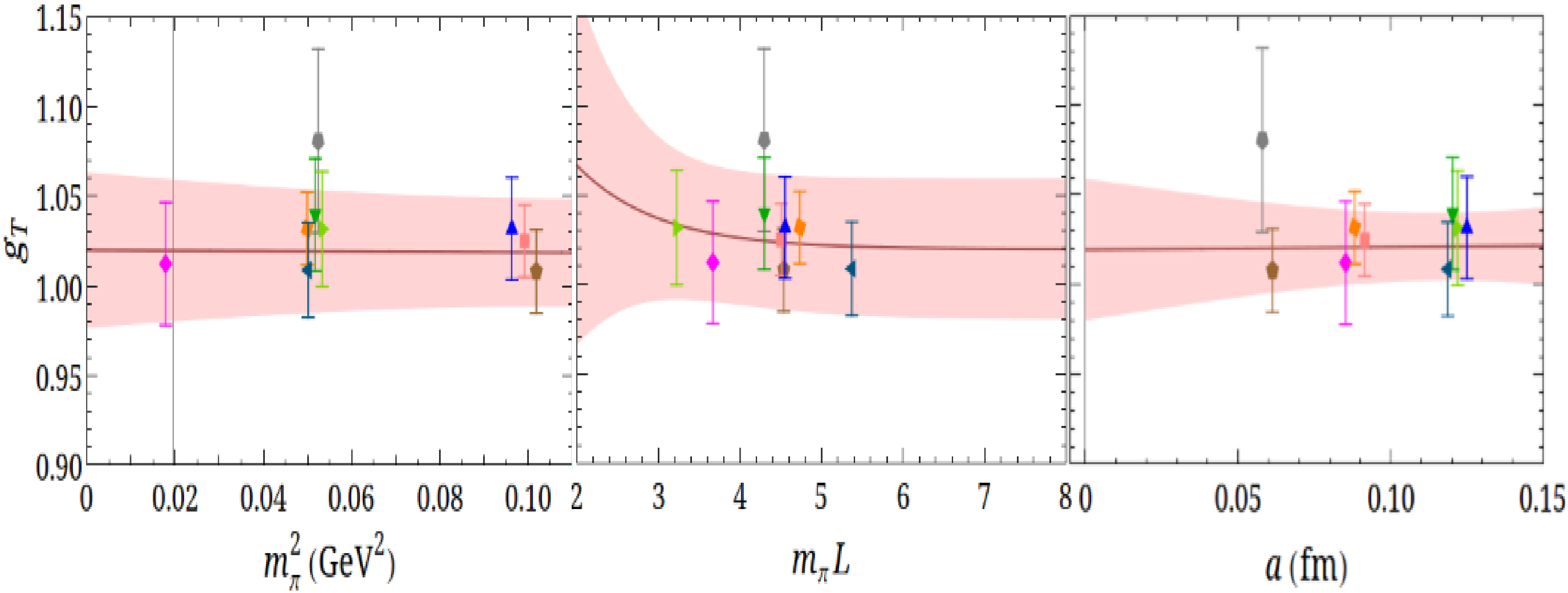
Example

- Much effort has been devoted to controlling systematics
- For example, PNDME's g_T calculations
  - Extrapolate to physical limit

$$g_T(a, m_\pi, L) = c_1 + c_2 a + c_3 m_\pi^2 + c_4 e^{-m_\pi L}$$

g_T = 1.018(42)

PNDME, in preparation



Example

- Much effort has been devoted to controlling systematics
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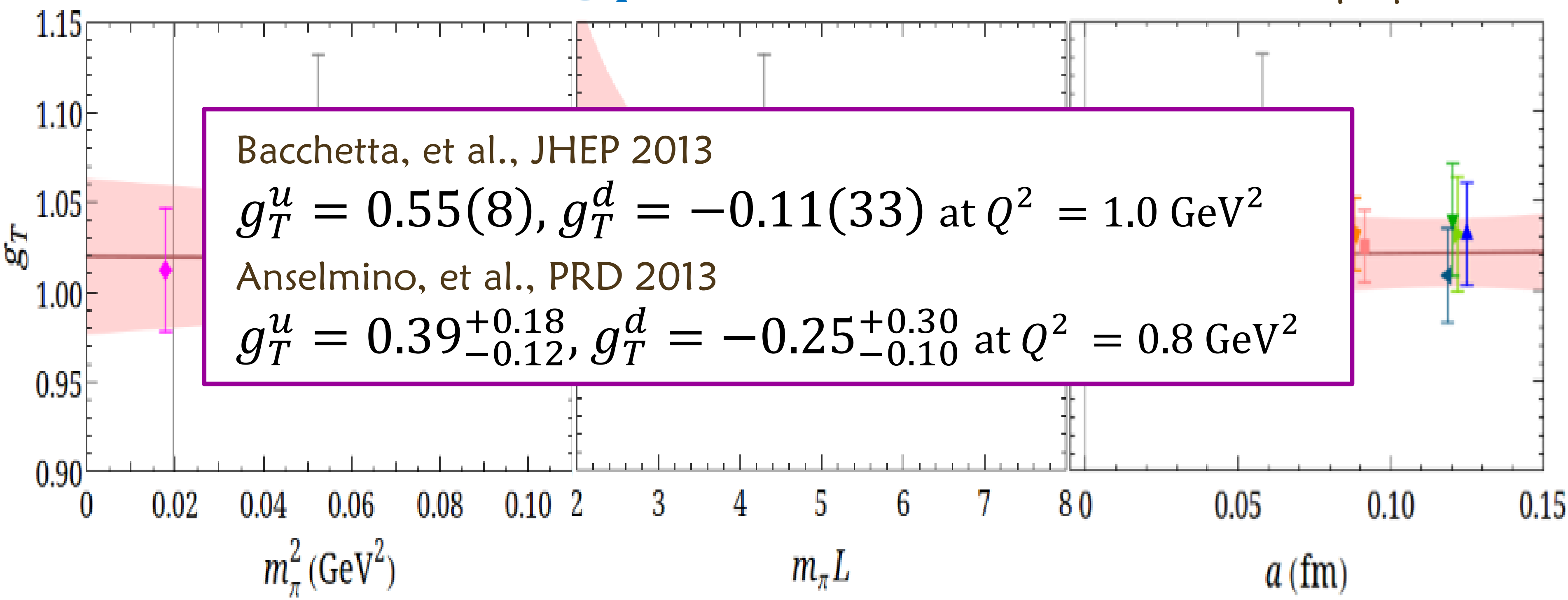
$$\mathbf{g_T = 1.018(42)}$$

PNDME, in preparation



Bacchetta, et al., JHEP 2013

$g_T^u = 0.55(8)$, $g_T^d = -0.11(33)$ at $Q^2 = 1.0$ GeV^2

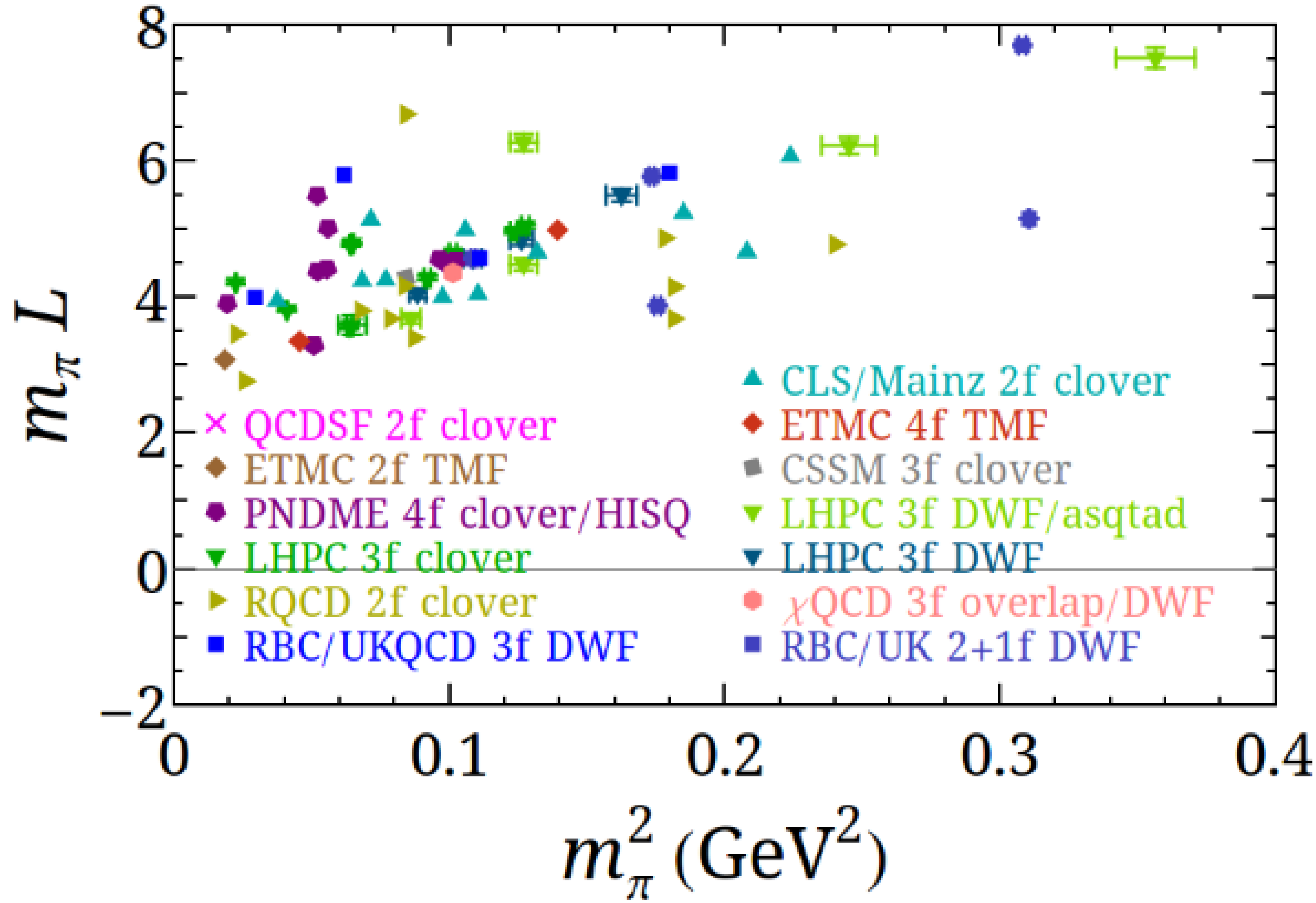
Anselmino, et al., PRD 2013

$g_T^u = 0.39^{+0.18}_{-0.12}$, $g_T^d = -0.25^{+0.30}_{-0.10}$ at $Q^2 = 0.8$ GeV^2

Progress

§ World players in LQCD nucleon structure

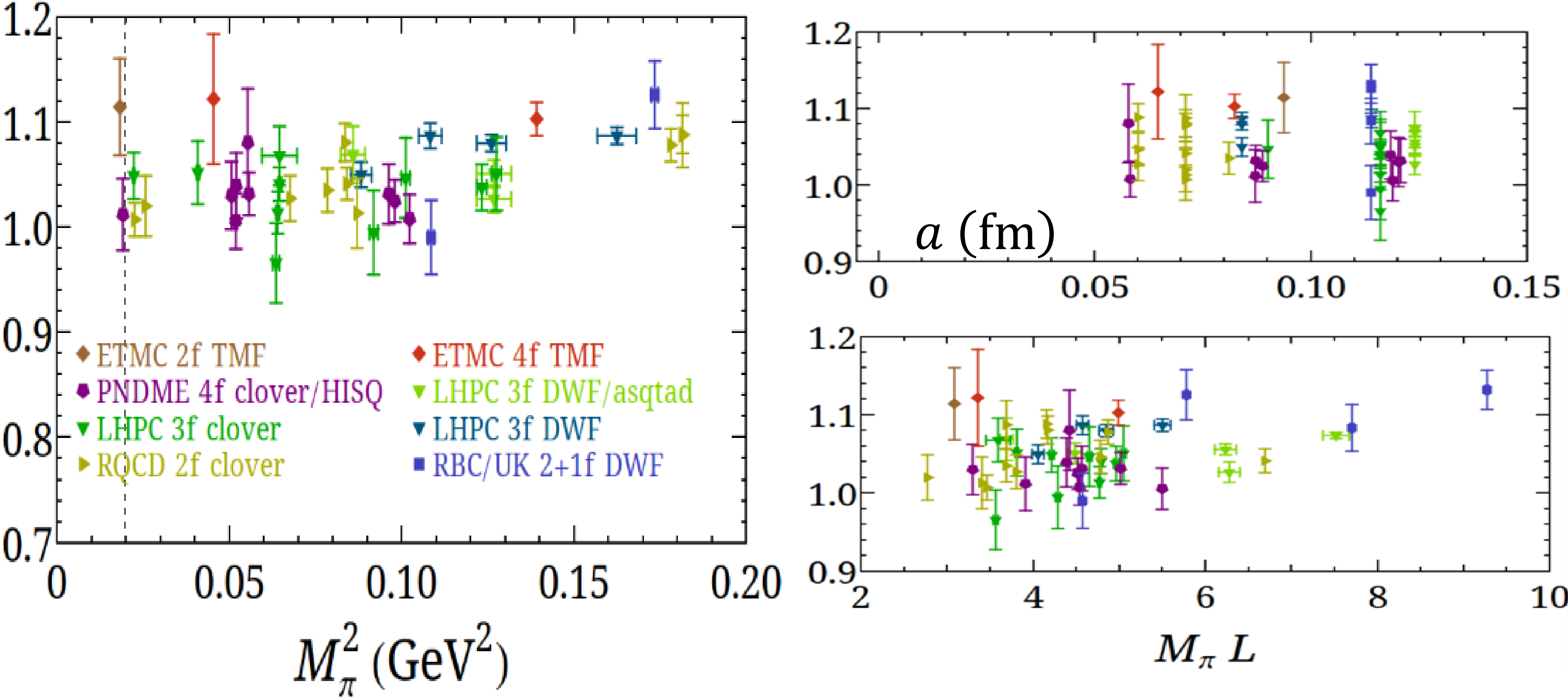
- Majority use $a < 0.1$fm



Progress

§ Tensor charge: the zeroth moment of transversity

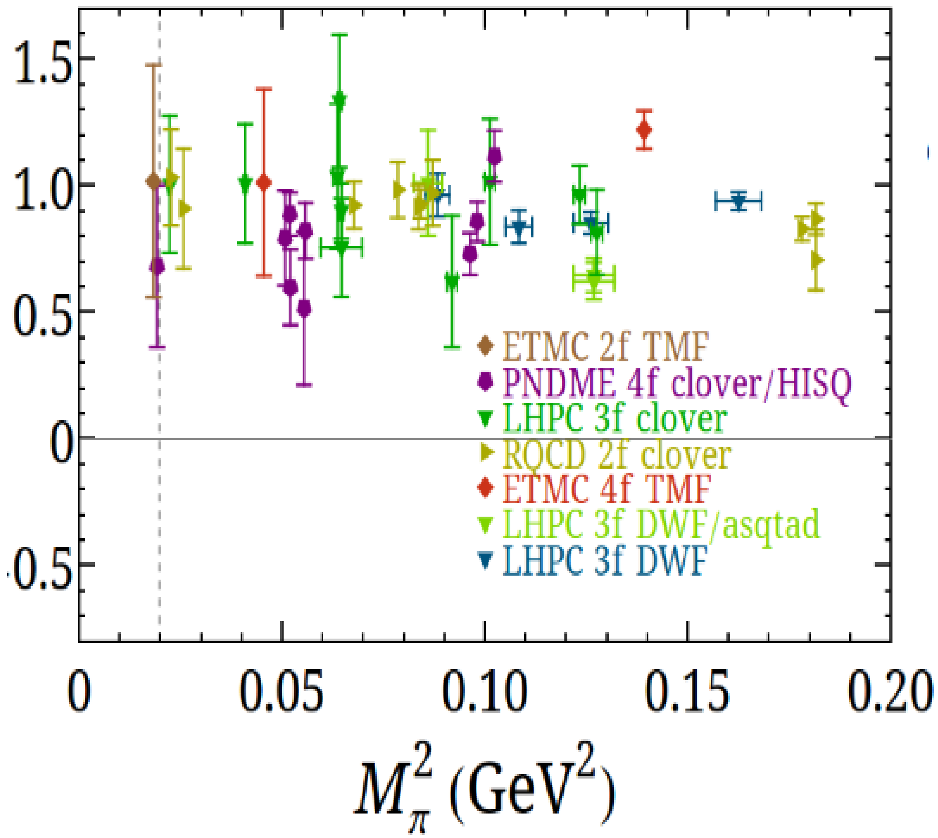
- Experimentally, probed through SIDIS, …
- Poor determination so far

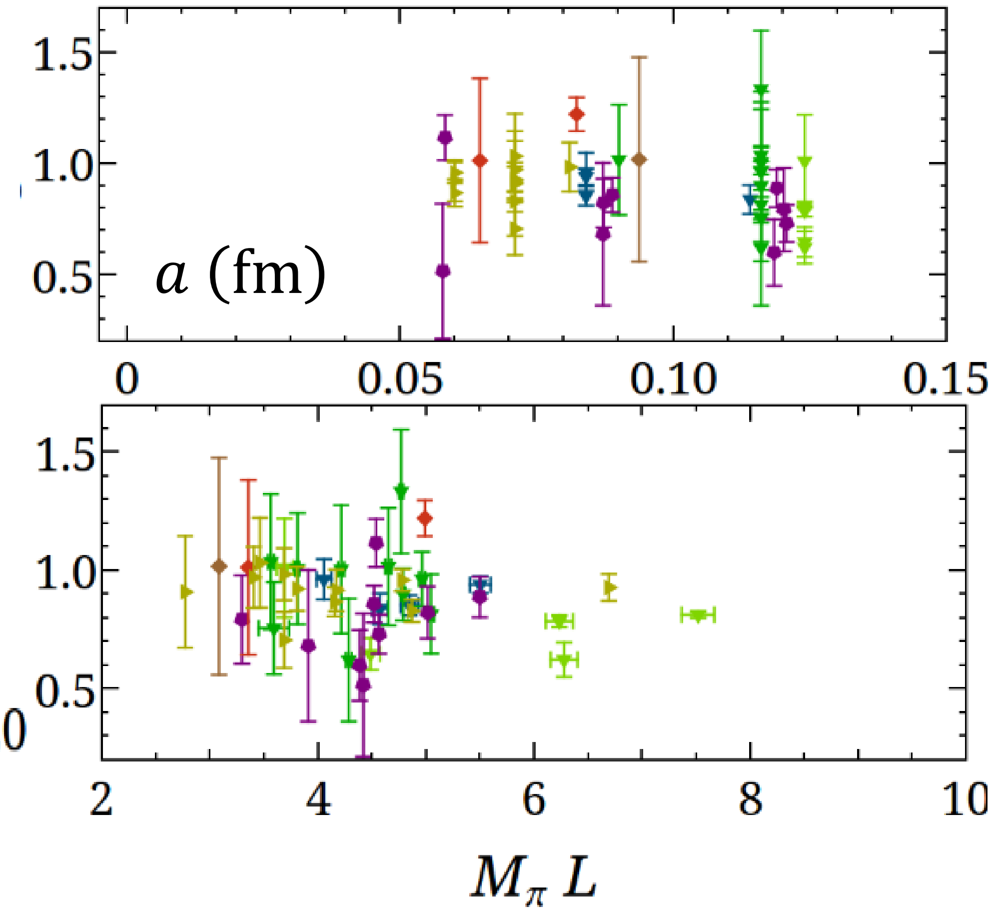


Progress

§ Scalar charge

- Hadronic inputs in nucleon(nuclear) beta decay BSM search
- Related CP-violating pion-nucleon coupling in nEDM EFT

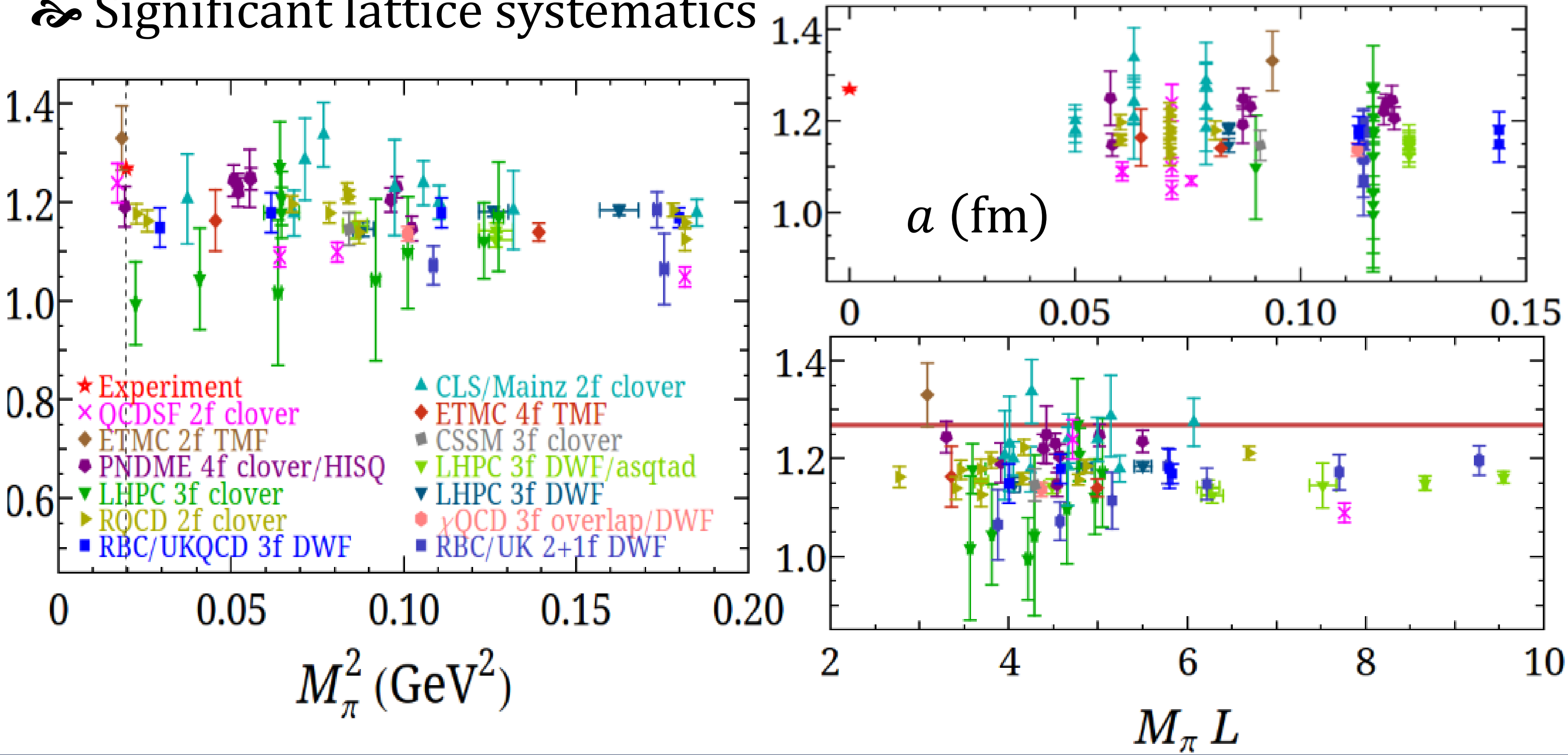




Progress

§ Axial charge

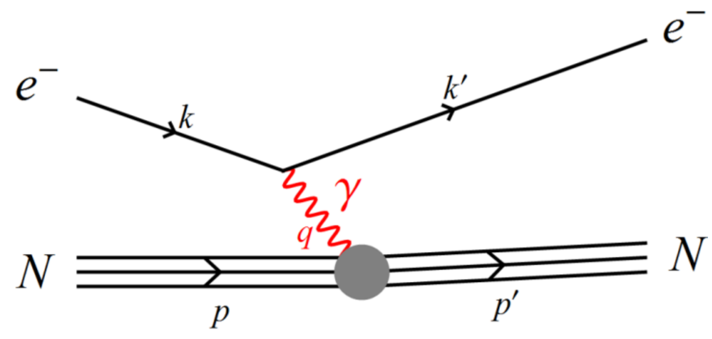
- Most well-known coupling with subpercent errors
- Important to the rate of *pp* fusion, *n*-lifetime, ...
- Significant lattice systematics



Form Factors

§ Form factors

- Elastic scattering
- $F_1(Q^2), F_2(Q^2), G_A(Q^2), G_P(Q^2)$
- For example, octet baryons



$$\langle B|V_\mu|B\rangle(q) = \bar{u}_B(p')\left[\gamma_\mu F_1(q^2) + \sigma_{\mu\nu}q_\nu \frac{F_2(q^2)}{2M_B}\right]u_B(p)$$

Dirac Pauli

$$\langle B|A_\mu|B\rangle(q) = \bar{u}_B(p')\left[\gamma_\mu\gamma_5 G_A(q^2) + \gamma_5 q_\nu \frac{G_P(q^2)}{2M_B}\right]u_B(p)$$

Axial Induced Pseudoscalar

Sachs:

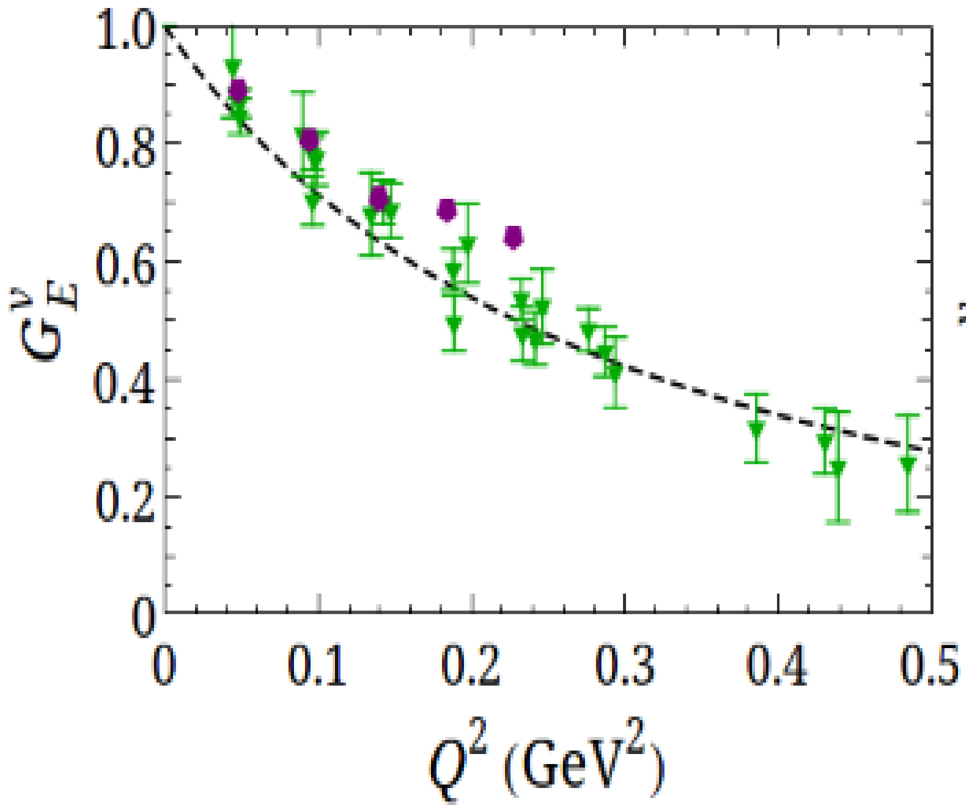
$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M_B^2}F_2(Q^2),\ G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

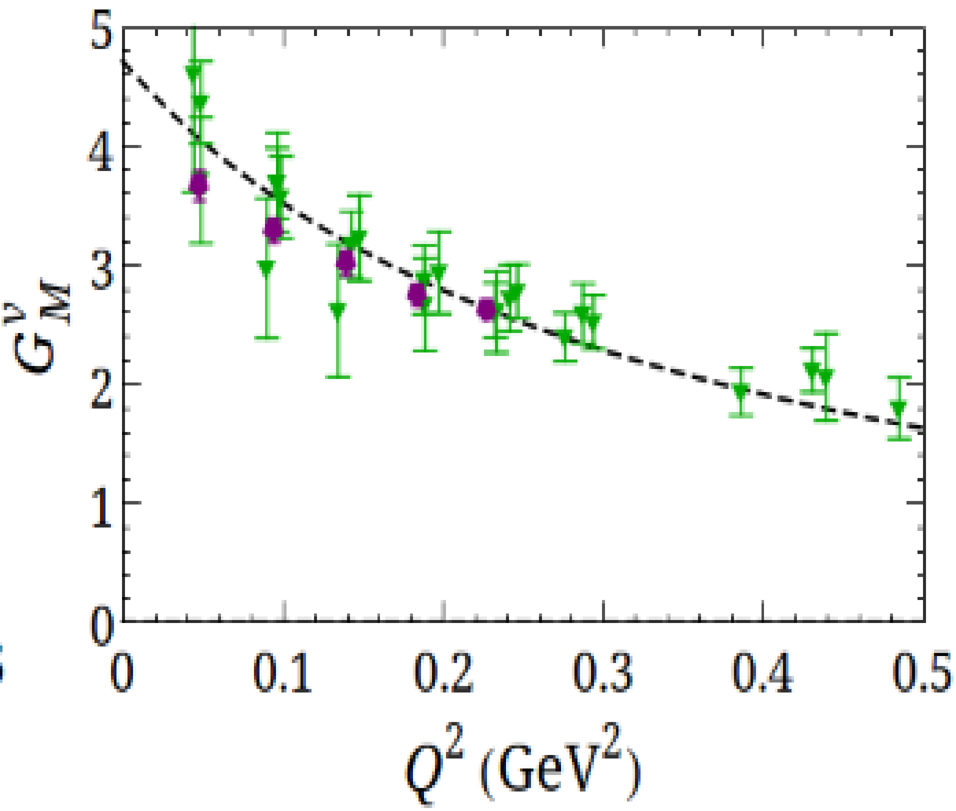
Electric Magnetic

Form Factors

- EM form factors very close to physical pion mass
  - Examples from LHPC and PNDME

J. Green et al, 1404.4029
PNDME, in preparation



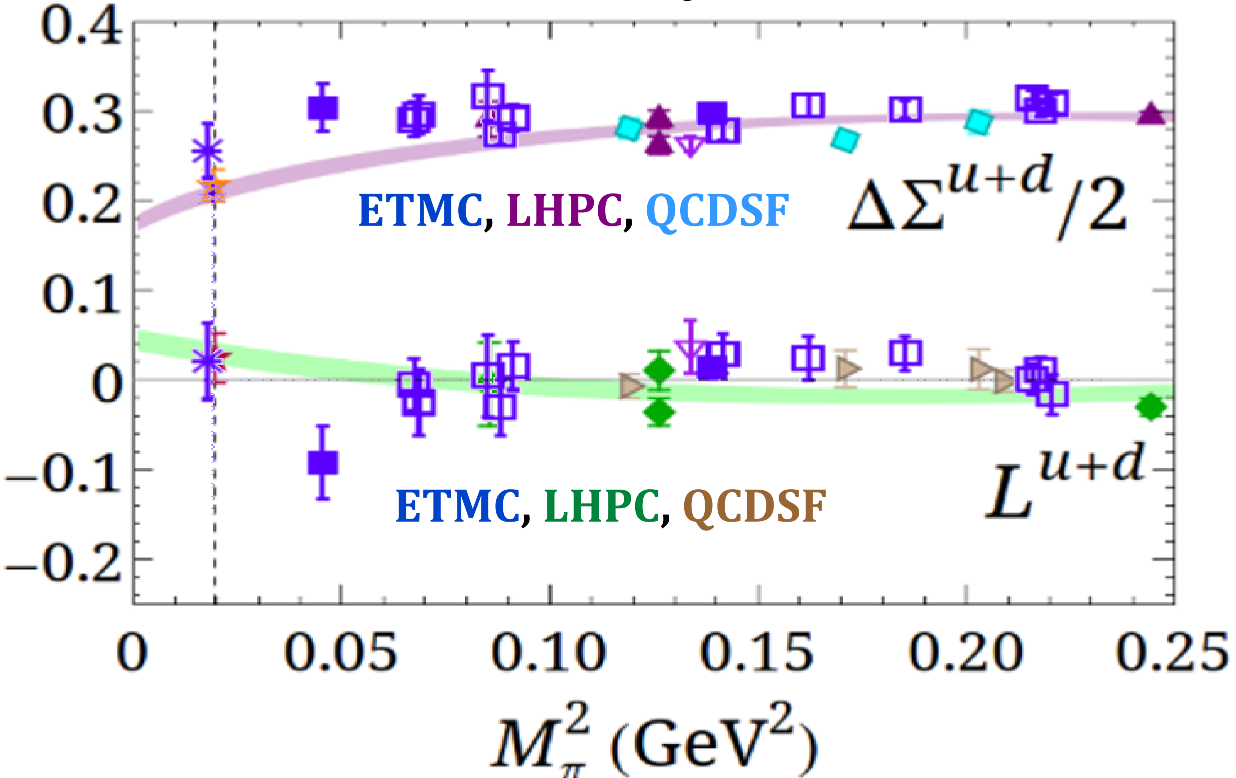


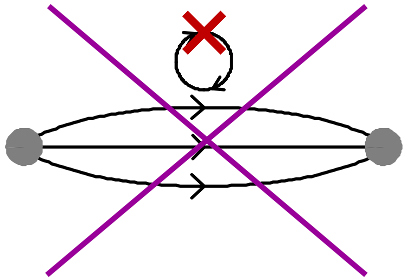
- Expecting more precise results in the next couple years

Origin of Proton Spin

§ What is the makeup of the nucleon?

- The origin of the nucleon's spin (the "spin crisis")
- Results from LHPC, QCDSF, ETMC





ΔΣ: quark spin

L: orbital angular momentum

§ Ignore disconnected diagram

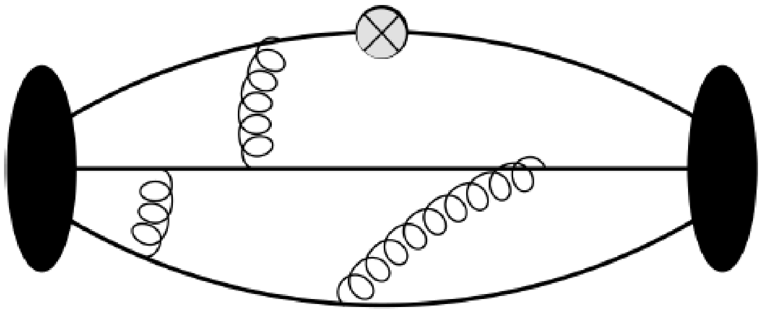
§ Gluon contribution estimated from sum rule

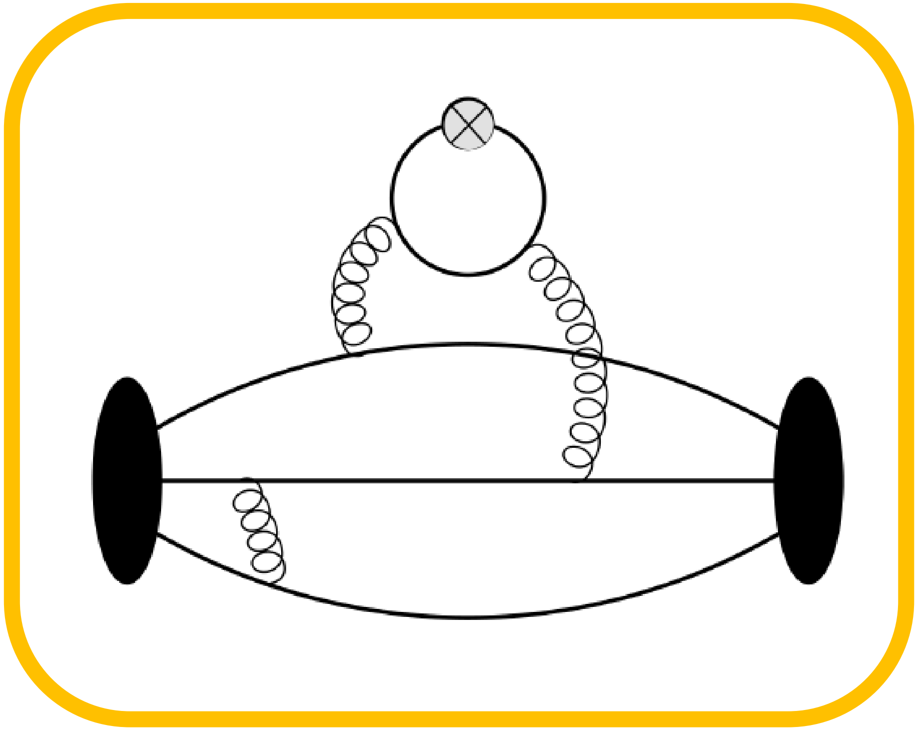
Spotlight on New Calculations

Disconnected Diagrams

§ Disconnected diagram

- Multiple ways to calculate this notorious contribution…
- Truncated solver, hopping-parameter expansion, hierarchical probing, …

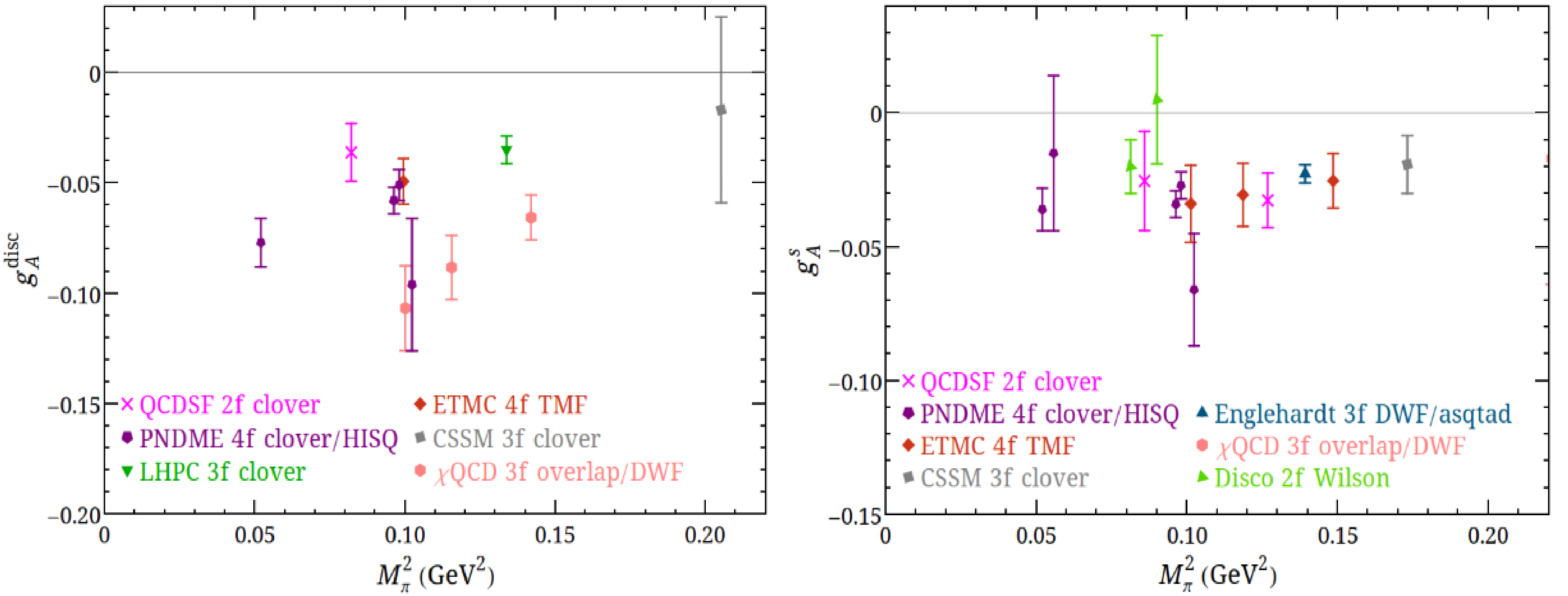




Disconnected Diagrams

§ High statistics reveals the previously neglected contribution

- Reduces total quark contribution to nucleon spin by $O(0.1)$

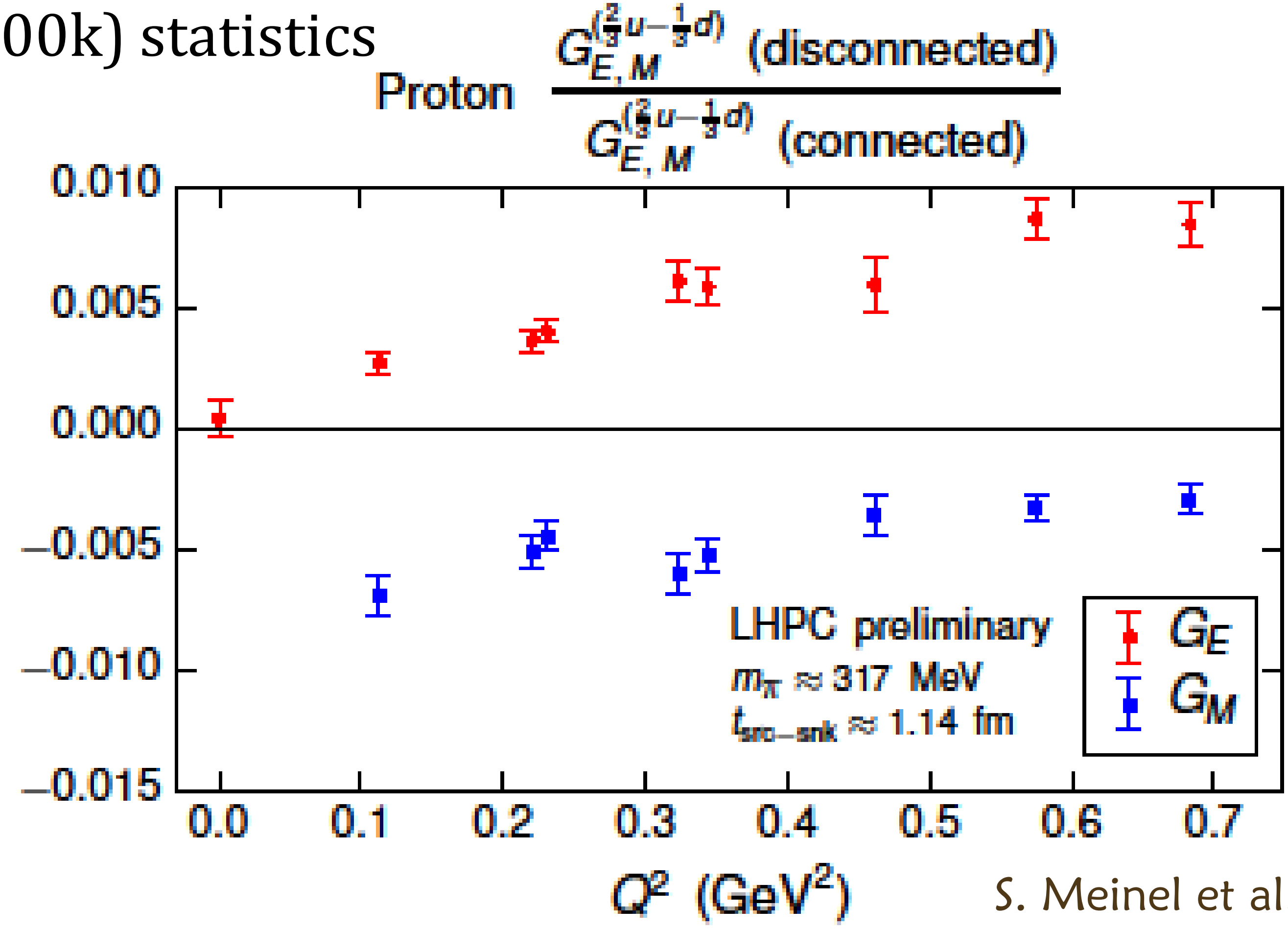


- More work needs to be done on disconnected OAM

Disconnected Diagrams

§ Disconnected diagram in form factors

- For example, LHPC work using clover fermions
- $O(100\text{k})$ statistics

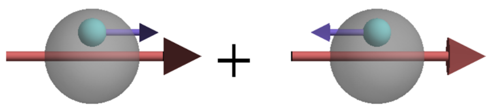


S. Meinel et al. (LHPC) 2014

PDFs on the Lattice

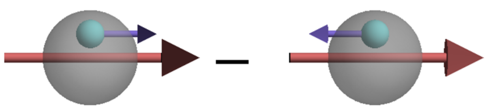
§ Lattice calculations rely on operator product expansion, only provide moments

most well known



spin-averaged/unpolarized

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$



spin-dependent longitudinally polarized

$$\langle x^{n-1}\rangle_{\Delta q} = \int_{-1}^{1} dx\, x^{n-1} \Delta q(x)$$

spin-dependent transversely polarized

$$\langle x^{n-1}\rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1} \delta q(x)$$

very poorly known

§ True distribution can only be recovered with all moments

PDFs on the Lattice

§ Limited to the lowest few moments

- For higher moments, all ops mix with lower-dimension ops
- No practical proposal yet to overcome this problem

§ Relative error grows in higher moments

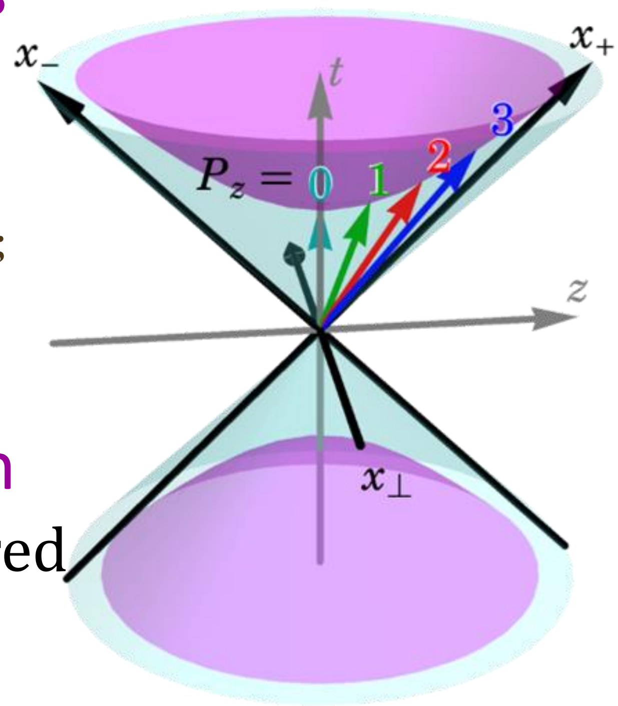
- Calculation would be costly
- Cannot separate valence contrib. from sea

PDFs on the Lattice

- § Limited to the lowest few moments
  - For higher moments, all ops mix with lower-dimension ops
  - No practical proposal yet to overcome this problem
- § Relative error grows in higher moments
  - Calculation would be costly
  - Cannot separate valence contrib. from sea

New Strategy: Xiangdong Ji, PRL 111, 039103 (2013); this conference

- § Adopt lightcone description for PDFs
- § Calculate finite-boost quark distribution
  - In $P_z \to \infty$ limit, parton distribution recovered
  - For finite P_z, corrections are applied through effective theory
- § Demonstration: Feasible with today's resources!

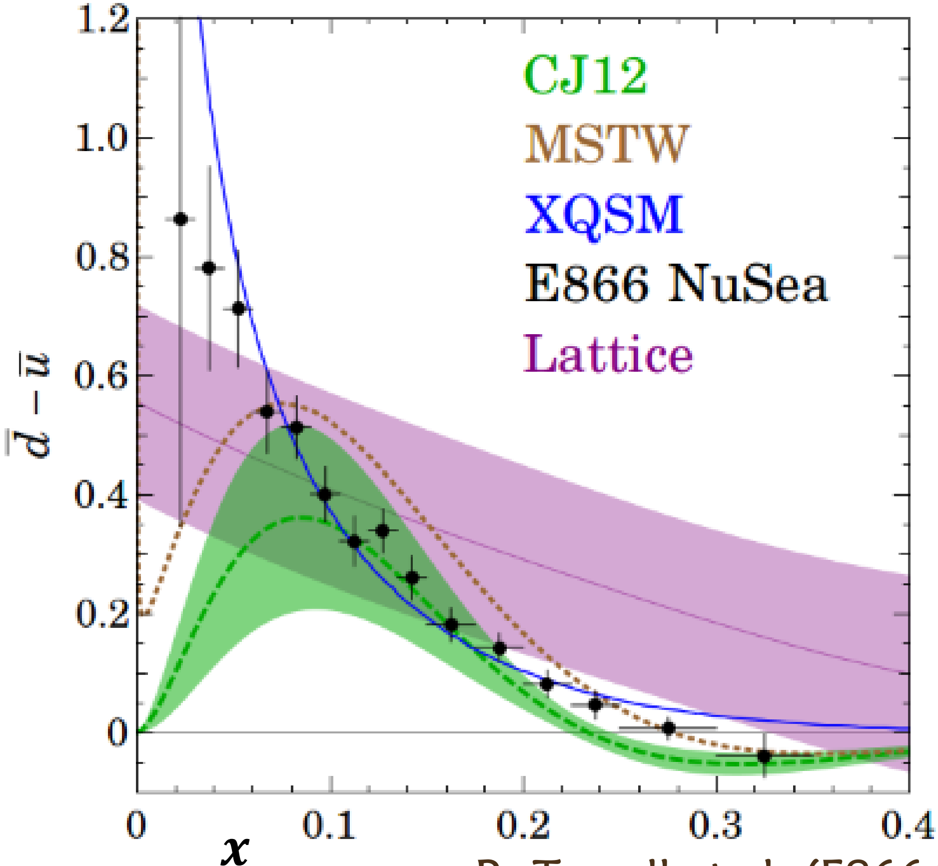


Sea Flavor Asymmetry

§ Lattice exploratory study

- $M_\pi \approx 310$ MeV

HWL et al 1402.1462; in preparation



Compared with E866

Too good to be true?

Lost resolution in small-*x* region

Future improvement to have larger lattice volume

$$\int dx\, (\bar{u}(x) - \bar{d}(x)) \approx -0.16(7)$$

Experiment	x range	$\int_0^1 [\bar{d}(x) - \bar{u}(x)]dx$
E866	$0.015<x<0.35$	0.118 ± 0.012
NMC	$0.004<x<0.80$	0.148 ± 0.039
HERMES	$0.020<x<0.30$	0.16 ± 0.03

R. Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)

Sea Flavor Asymmetry

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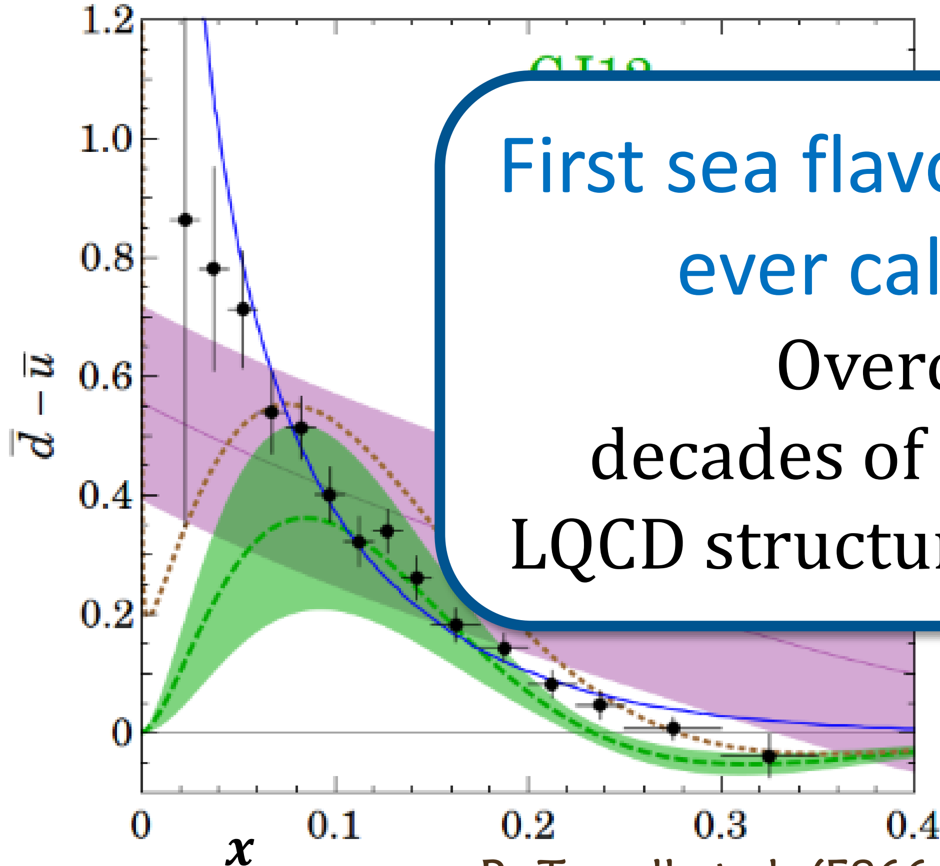

Compared with E866

Too good to be true?

Lost resolution in

ment to

ce volume

$) \approx -0.16(7)$

First sea flavor asymmetry ever calculated!

Overcomes
decades of obstacles in
LQCD structure calculations

		$\int_0^1[\bar{d}(x)-\bar{u}(x)]dx$
E866	$0.015<x<0.35$	0.118 ± 0.012
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R. Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)

Gluon Helicity

- Gluon helicity in long. pol. proton $\Delta G = \int_0^1 dx\, \Delta g(x)$

$$\Delta g(x) = \frac{i}{4\pi x P_+^2} \int d\lambda\, e^{ix\lambda} \left\langle PS \middle| F_+^{\alpha}(\lambda n) \mathcal{P} e^{ig\int_0^{\lambda} A_+(\eta n) d\eta} \tilde{F}_{+\alpha}(0) \middle| PS \right\rangle$$

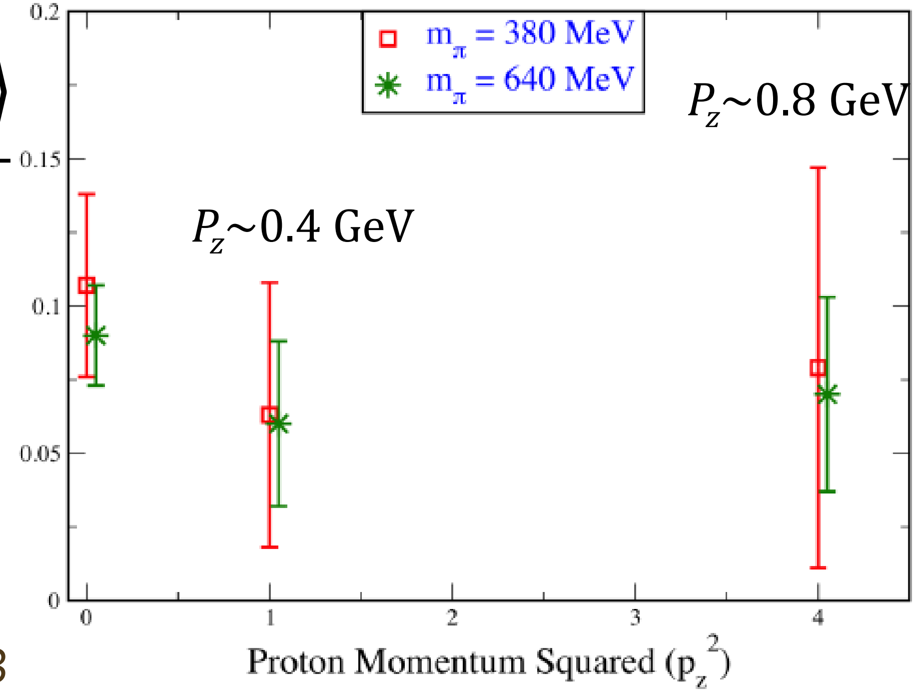
- It can be calculated through large-momentum frame

X. Ji et al., PRL. 111 (2013) 112002; 110 (2013) 262002; PRD 89, 085030 (2014)

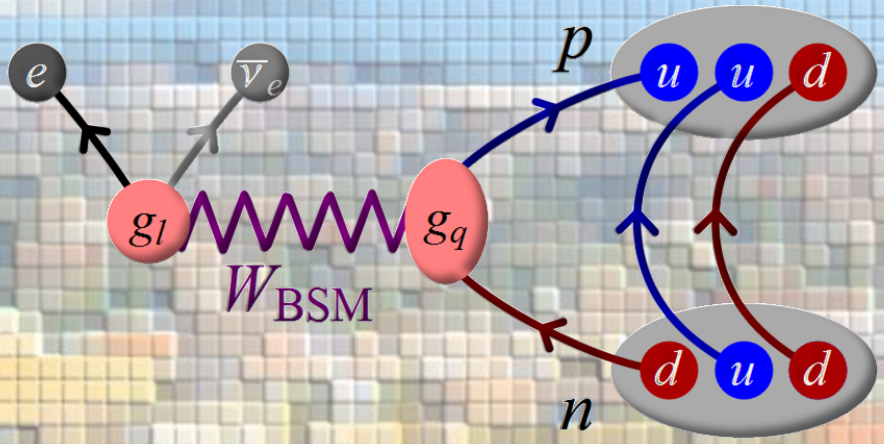
$$\Delta G\, S_z = \frac{\left\langle PS \middle| \int d^3x \left(\vec{E} \times \vec{A}_{\text{phys}}\right)_z \middle| PS \right\rangle}{2E_P}$$

- Preliminary study by χQCD
  - Overlap/DWF
  - Mass corrections and large-P_z extrapolation needed

R. Sufian et al., 1412.7168



Applications beyond QCD



Low-Precision Experiments

Many opportunities to probe BSM with LQCD

§ Dark-Matter Searches
Andre Walker-Loud, this conference

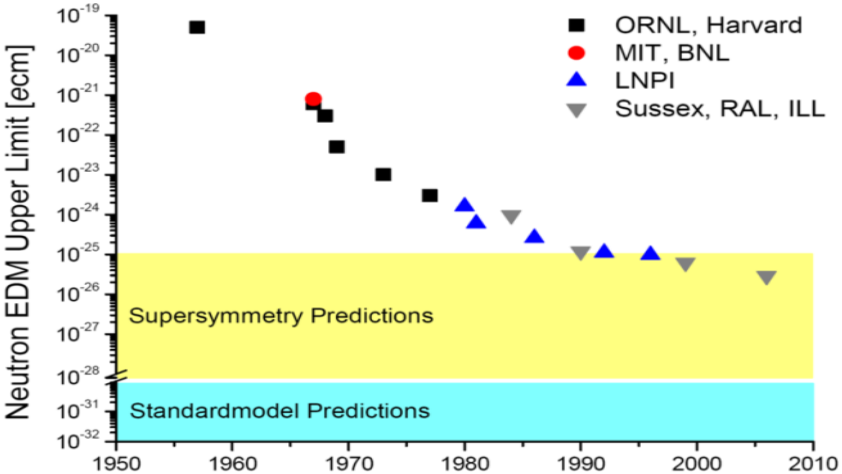
- Nucleon (nuclear) sigma terms, strangeness

§ Nucleon beta decay
Susan Gardner, this conference

- Probing non-V–A (e.g. isovector scalar and tensor) interactions

§ Electric dipole moment

- CP-violating effect
- Extremely small in SM: $\approx 10^{-30}$ e-cm
- Best SUSY model killer



BSM Interactions

§ Neutron beta decay could be related to new interactions: the scalar and tensor

$$H_{\text{eff}} = G_F \left(J^{\text{lept}}_{V-A} \times J^{\text{quark}}_{V-A} + \sum_i \varepsilon_i^{\text{BSM}} \hat{O}_i^{\text{lept}} \times \hat{O}_i^{\text{quark}} \right)$$

$$\hat{O}_S = \overline{u}d \times \overline{e}(1-\gamma_5)\nu_e \quad \rightarrow \quad g_S = \langle n|\overline{u}d|p\rangle$$

$$\hat{O}_T = \overline{u}\sigma_{\mu\nu}d \times \overline{e}\sigma^{\mu\nu}(1-\gamma_5)\nu_e \quad \rightarrow \quad g_T = \langle n|\overline{u}\sigma_{\mu\nu}d|p\rangle$$

- ε_S and ε_T are related to the masses of the new TeV-scale particles
- ... but the unknown coupling constants $g_{S,T}$ are needed

§ Given precision $g_{S,T}$ and O_{BSM}, predict new-physics scales

Experiment → $O_{\text{BSM}} = f_O(\varepsilon_{S,T}\, g_{S,T})$ ← Precision LQCD input ($m_\pi \approx 140$ MeV, $a \rightarrow 0$)

$$\varepsilon_{S,T} \propto \Lambda_{S,T}^{-2}$$

Beta Decays & BSM

§ Given precision $g_{S,T}$ and O_{BSM}, predict new-physics scales

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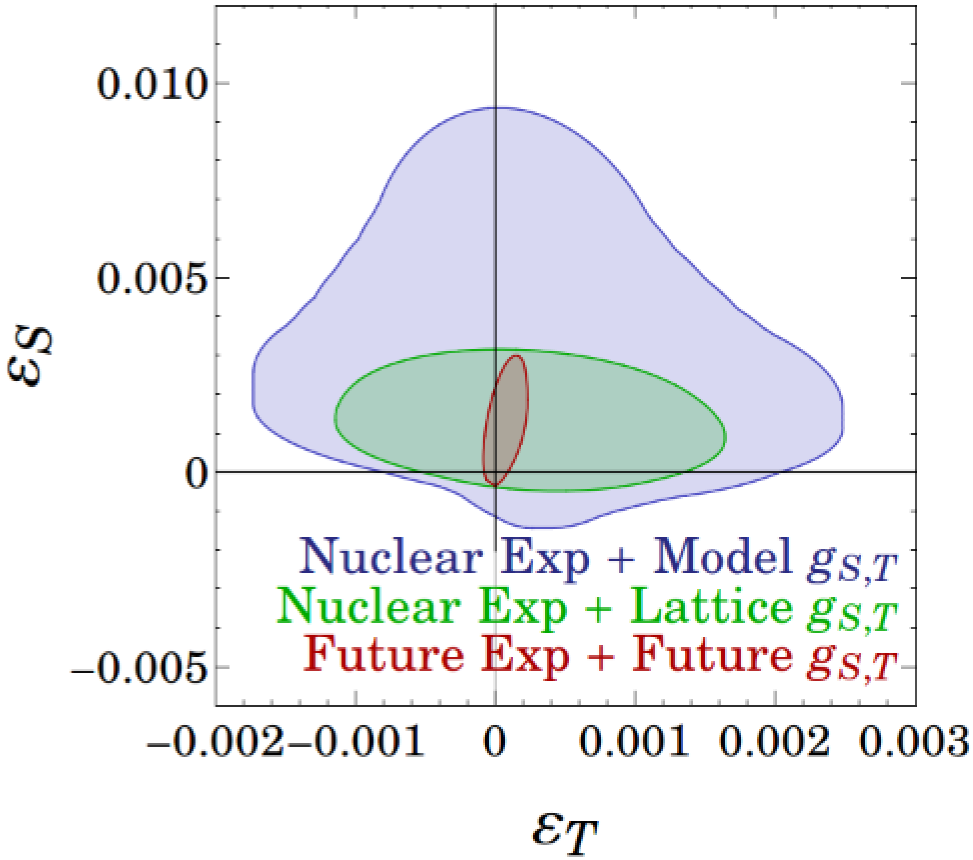

$\boldsymbol{\varepsilon_{S,T} \propto \Lambda_{S,T}^{-2}}$

LANL UCN neutron decay exp't

Expect by 2013:

$|B_1 - b|_{\text{BSM}} < 10^{-3}$

$|b|_{\text{BSM}} < 10^{-3}$

Proposal at ORNL by 2015

Expect at CENPA: $^6\text{He}(b_{\text{GT}})$ at 10^{-3}

PNDME, PRD85 054512 (2012); 1306.5435

Beta Decays & BSM

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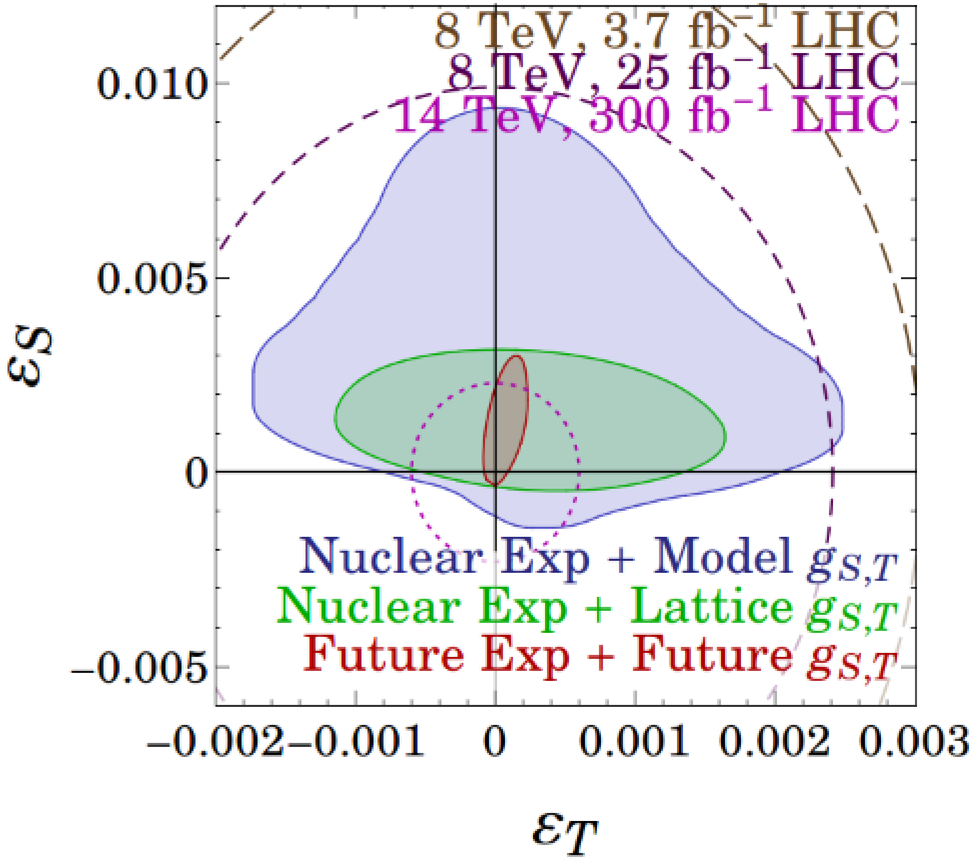

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PNDME, PRD85 054512 (2012); 1306.5435

$nEDM$

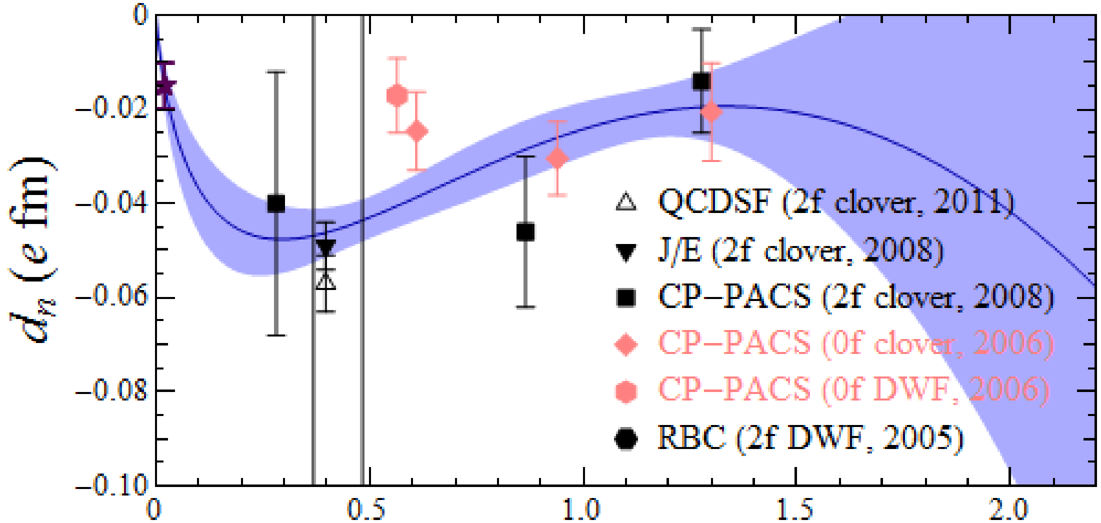
§ Lagrangian $L = L_{\text{QCD}}^{CP\text{ Even}} + L_\Theta + \cdots$

$$i\Theta \frac{g^2}{16\pi^2}\int d^4x\, G^{\mu\nu}\tilde{G}_{\mu\nu}$$

§ Leading contribution

- Using CP-even QCD vacuum with θ-term expansion

  RBC, J/E, CP-PACS(2005), CP-PACS(2006, 2010), QCDSF(2011), …



Chiral extrapolation
K. Ottnad et al., 2010

−0.015(5) θ e·fm

HWL, 1112.2435

§ Plenty of room for improvement

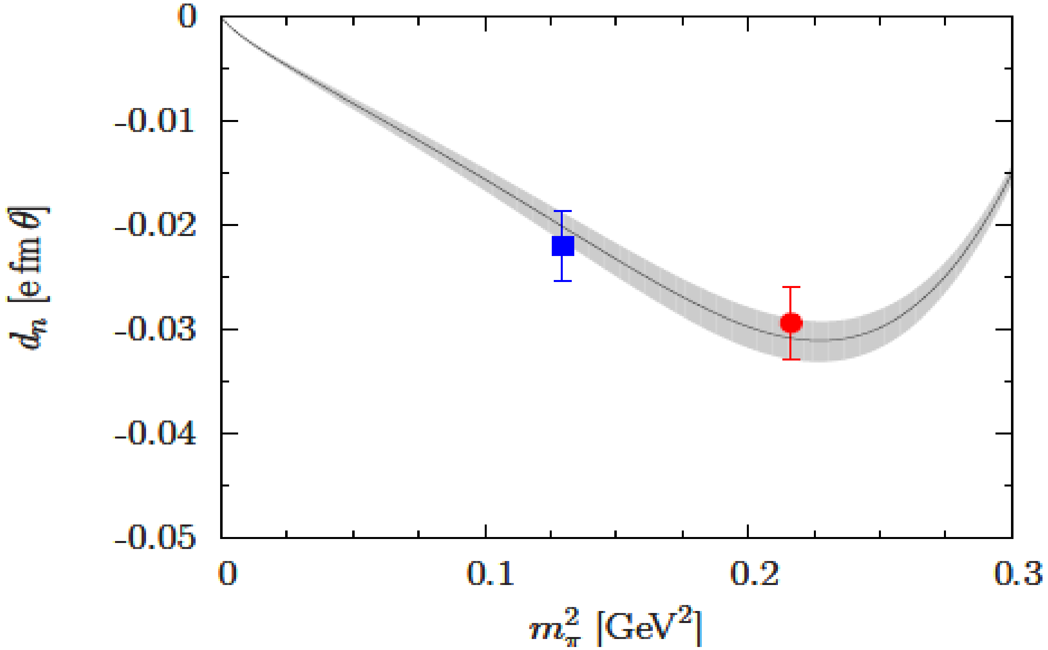
nEDM

§ Lagrangian $L = L_{\text{QCD}}^{CP\ \text{Even}} + L_{\Theta} + \cdots$

$$i\Theta \frac{g^2}{16\pi^2}\int d^4x\, G^{\mu\nu}\tilde{G}_{\mu\nu}$$

§ Leading contribution

- *CP*-odd QCD vacuum in dynamical quarks by QCDSF
- Demonstrated of concept at 3f clover at $M_\pi \approx 360$ and 465 MeV



−0.0038(2)(9) *θ e*·fm

QCDSF 1502.02295

§ Stay-tuned for lighter pion mass results

nEDM

- § Lagrangian $L = L_{\text{QCD}}^{CP\ \text{Even}} + L_{\Theta} + L_{\text{quark}}^{\text{dim}-5} + \cdots$

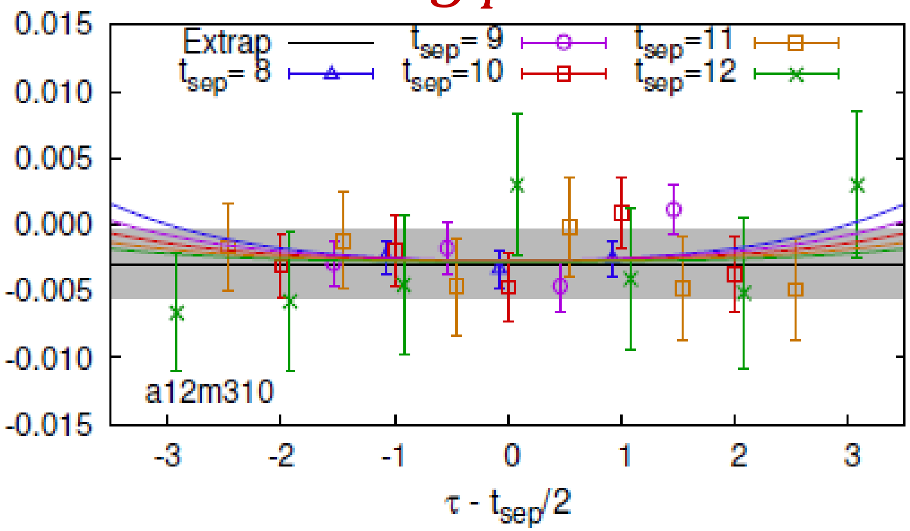
$$-\frac{i}{2}\bar{q}\sigma_{\mu\nu}\gamma_5 q F^{\mu\nu}$$

- § Hadronic contribution: $\langle N|\bar{q}\sigma_{\mu\nu}q|N\rangle$
- § Quark EDM

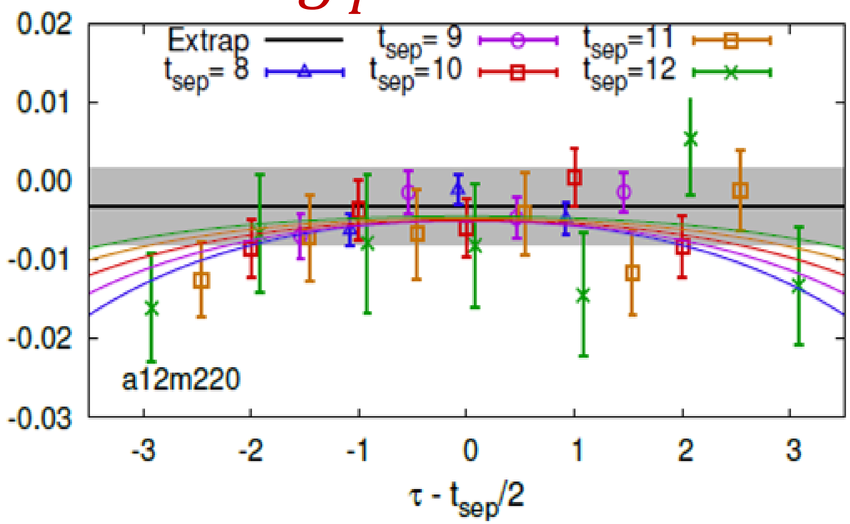
PNDME, in preparation

- Strange/light disconnected contributions consistent with zero

g_T^s



$g_T^{u,d\ \text{disc}}$



nEDM

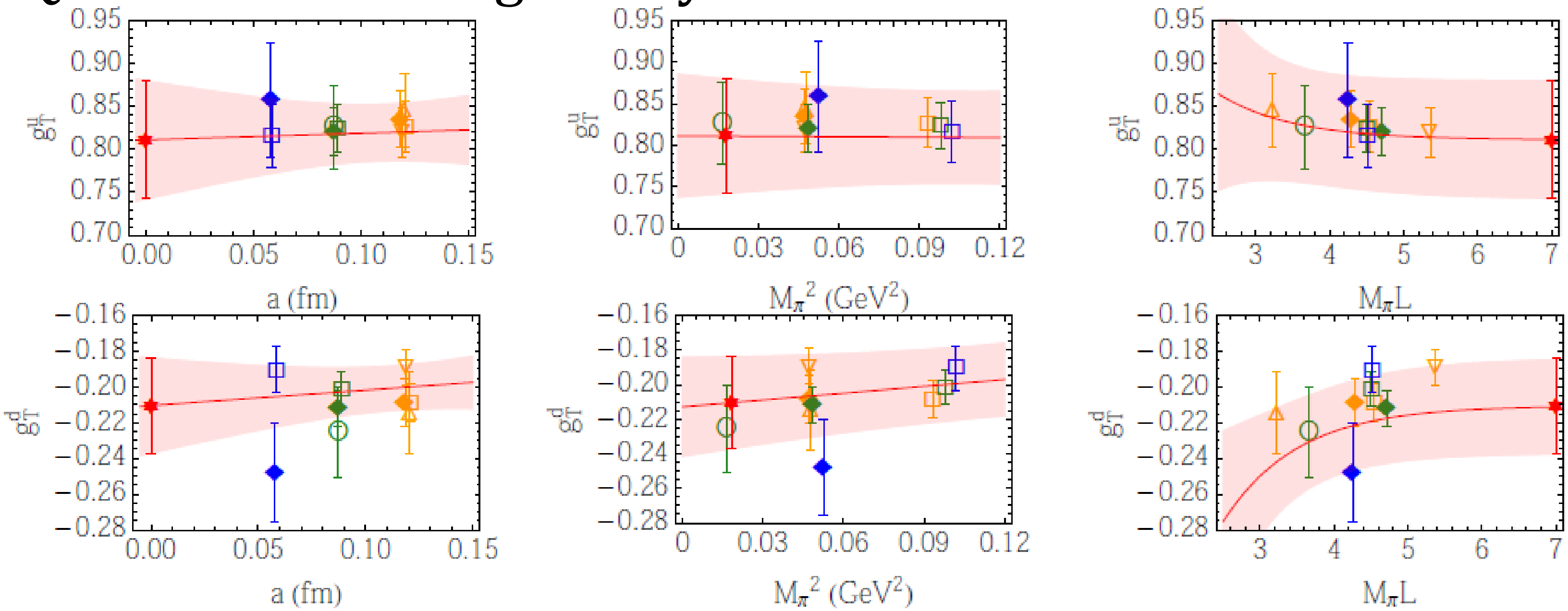
§ Lagrangian $L = L_{\text{QCD}}^{CP\ \text{Even}} + L_{\Theta} + L_{\text{quark}}^{\text{dim}-5} + \cdots$

$$-\frac{i}{2}\bar{q}\sigma_{\mu\nu}\gamma_5 q F^{\mu\nu}$$

§ Quark EDM

PNDME, in preparation

- Quark tensor charge analysis



$nEDM$

§ Lagrangian $L = L_{\text{QCD}}^{CP\ \text{Even}} + L_{\Theta} + L_{\text{quark}}^{\text{dim}-5} + \cdots$

$$-\frac{i}{2}\bar{q}\sigma_{\mu\nu}\gamma_5 q F^{\mu\nu}$$

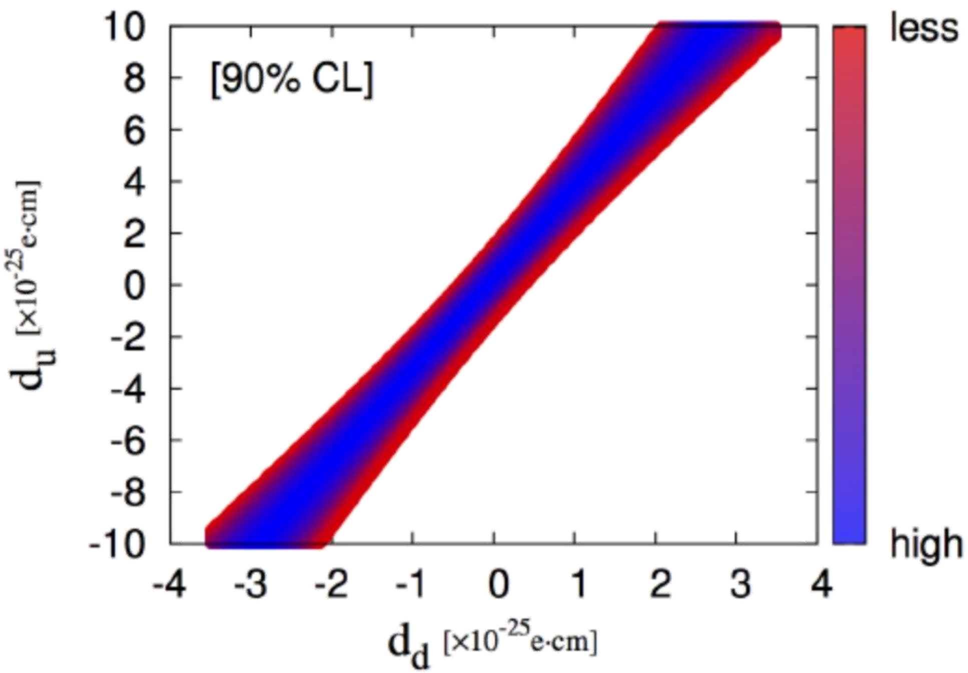
§ Quark EDM

- Quark tensor charge analysis (at 2 GeV, $\overline{\text{MS}}$ scheme)

$$g_T^u = 0.788(75)$$
$$g_T^d = -0.223(25)$$

- Assuming this is only source, gives constraint

PNDME, in preparation



nEDM

§ Lagrangian $L = L_{\text{QCD}}^{CP\ \text{Even}} + L_{\Theta} + L_{\text{quark}}^{\text{dim}-5} + L_{\text{chromo}-\text{quark}}^{\text{dim}-5} + \cdots$

$\bar{q}\sigma_{\mu\nu}\gamma_5\lambda^A G^{\mu\nu A} q$

§ Chromo-quark operators

- Complicated quark/gluon structure, non-trivial mixings at non-zero lattice spacing, symmetry breaking, and further problems
- Work progressing in understanding these issues and preliminary numerical simulations are on the way

T. Bhattacharya et al, 1403.2445

Summary

§ Exciting era using LQCD to study nucleon structure

- Well-studied systematics → precision inputs
- Ensembles with physical pion masses

§ Overcoming longstanding obstacles

- Address neglected contributions to many quantities
- Bjorken-*x* dependence of parton distribution functions
- First lattice study on gluon helicity

§ Precision low-*E* exp'ts to probe BSM physics

- Probe high-energy (TeV) physics at low energy (GeV)
- Combined effort from experiment and theory sides
- More work devoted to the intensity frontier, e.g. nEDM

§ Fill gaps in experimental coverage, provide SM backgrounds

- LQCD necessary when experiment is limited (e.g. g_S)
- Many topics omitted due to time; see Lattice Conference for more

Backup Slides

Gluonic Momentum Fraction

§ Exploratory stage: quenched results

$$O_{\mu\nu} = -\mathrm{tr}_c F_{\mu\alpha} F_{\nu\alpha}$$

- Quenched, heavy pion masses, linear chiral extrapolation

§ QCDSF ('97) and LHPC ('07)

- Direct matrix-element calculation: $\langle x \rangle_g = 0.53(23)$ QCDSF
- HYP-smearing, study pion: $\langle x \rangle_g = 0.6(2)(1)$

§ χQCD K.F. Liu et al., 1203.6388

- Rewrite gluonic observables with massless overlap kernel

$$\mathrm{tr}_s(\sigma_{\mu\nu} D_{\mathrm{ov}}) \propto a^2 F_{\mu\nu}$$

§ QCDSF R. Horseley et al., 1205.6410

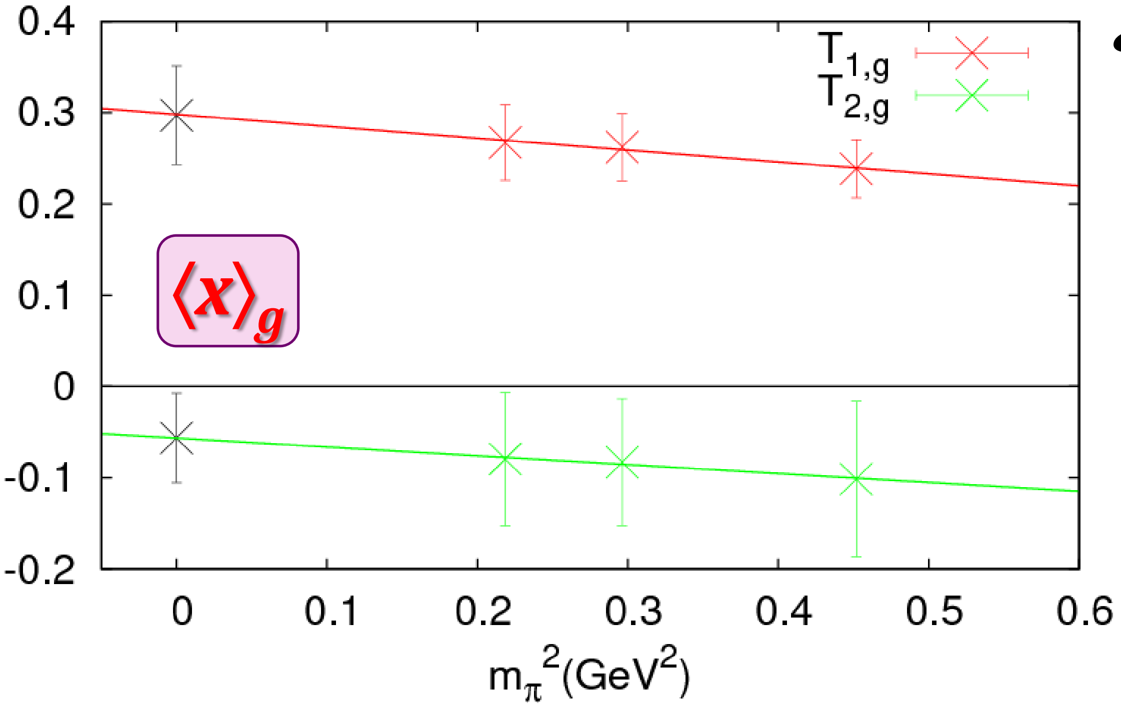
- Feynman-Hellmann theorem with modification of the action

$$S \to S + \lambda S_{\mathcal{O}} \longrightarrow \beta\lambda \frac{1}{3}\left(\sum_{\vec{x},i} \mathrm{Re}\,\mathrm{tr}_c[1 - P_{i4}(\vec{x})] - \sum_{\vec{x},i<j} \mathrm{Re}\,\mathrm{tr}_c[1 - P_{ij}(\vec{x})]\right)$$

Gluonic Momentum Fraction

§ χQCD K.F. Liu et al., 1203.6388 & private communication

- Rewrite gluonic observables as $\mathrm{tr}_s(\sigma_{\mu\nu} D_{\mathrm{ov}}) \propto a^2 F_{\mu\nu}$
- Use $\mathbf{Z}_4$ noise sources to estimate D_{ov} stochastically
- $a \approx 0.1$ fm, 16^3 Wilson + Wilson, $M_\pi \approx 480$–650 MeV, 500 confs, $\langle x \rangle_g = 0.313(56)$

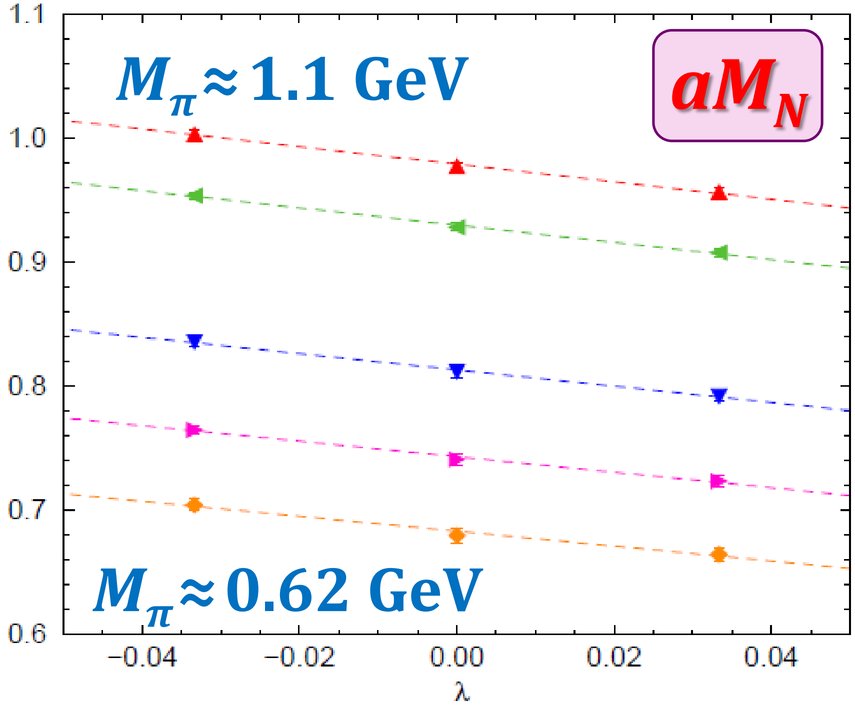


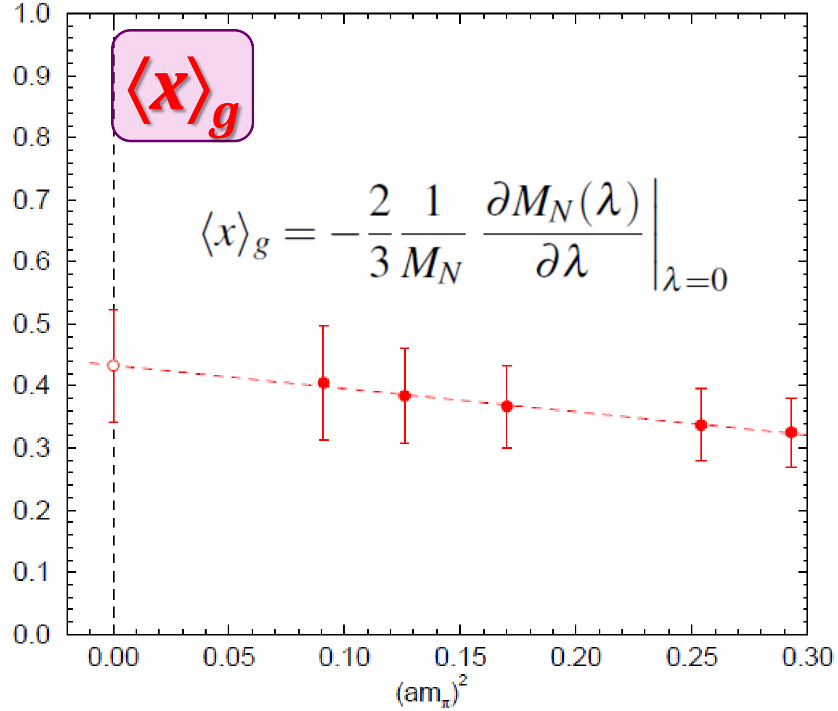
- Expensive!
  (but D_{ov} reusable for disconnected contribution?)

Gluonic Momentum Fraction

§ QCDSF   R. Horseley et al., 1205.6410

- Feynman-Hellmann theorem (absorbed operators in the action)
- $a \approx 0.1$ fm, 24^3 Wilson + NP clover, $M_\pi \approx 1100–600$ MeV
  $O(500)$ confs, $\langle x\rangle_g = 0.43(7)$
- Cheap, operator by operator; reweighting for dynamical lattices

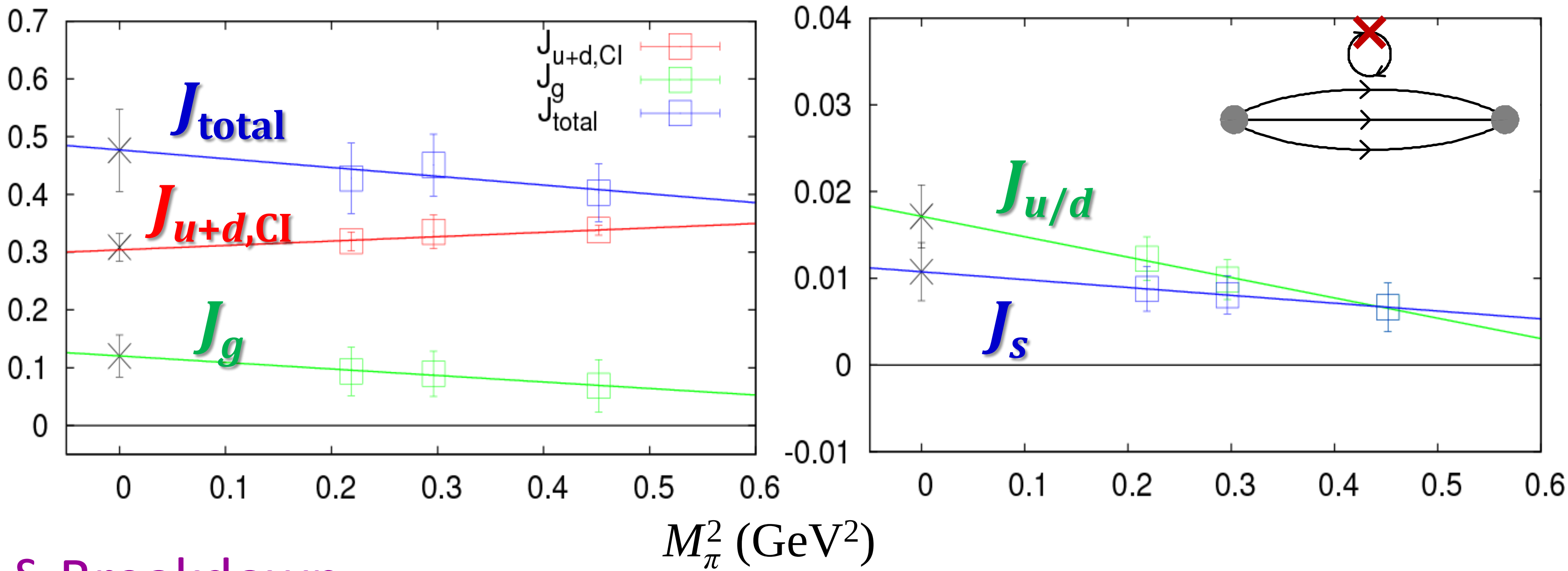




Origin of Proton Spin

§ What is the makeup of the nucleon?

χQCD, 1203.6388 [hep-ph] and private communication w/ Y. Yang



§ Breakdown:

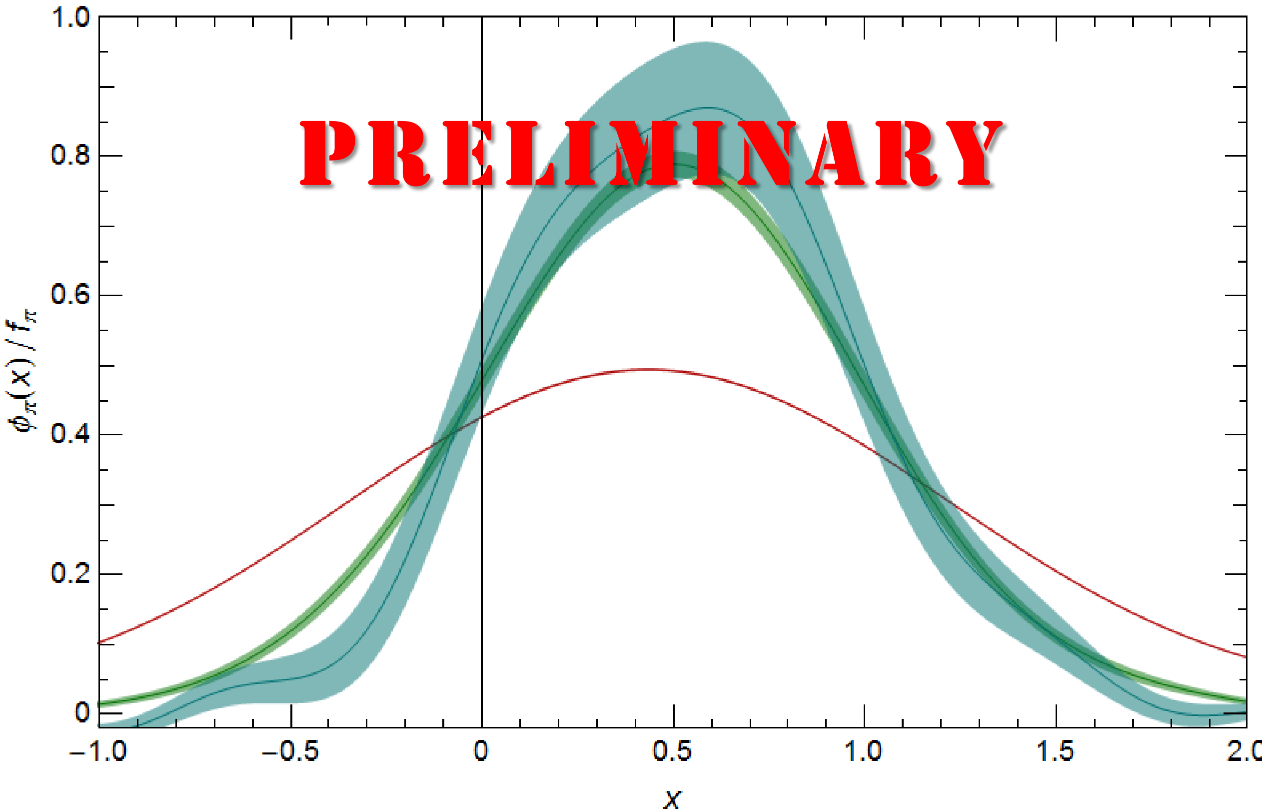
- $\Delta\Sigma_q = 50(2)\%$, $L_q = 25(12)\%$ (mostly DI), $J_g = 25(8)\%$

§ Looking forward to χQCD (overlap/DWF), QCDSF (clover)

Pion Distribution Amplitude

§ Exploratory study

$$\int \frac{dz}{2\pi} e^{-izk_z} \left\langle 0 \middle| \bar{d}(z)\, \gamma_z\, \gamma_5 \exp\left(-ig\int_0^z dz'\, A_z(z')\right) u(0) \middle| \pi^+(P) \right\rangle$$



Only leading mass correction applied

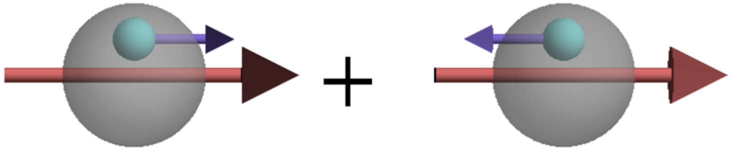
Dominated by $O(\Lambda_{QCD}^2/P_z^2)$ errors

P_z □ {1, 2, 3} $^{2\pi}/_L$

Parton Distribution Functions

§ Quark distribution

- Processes: DIS (F_2, σ), Drell-Yan, W-asymmetry, Z-rapidity, (γ+) jet, …
- Experiments: BCDMS, NMC, SLAC, JLab, HERA, E866, CDF, DØ,…



spin-averaged/unpolarized

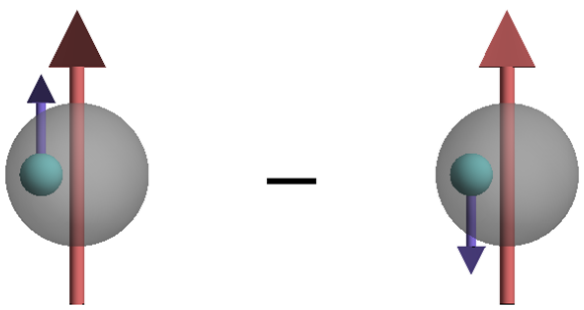
§ Helicity distribution

- Processes: polarized DIS, semi-inclusive DIS, photo- and electroproduction of hadrons and charm, *pp* collisions
- Experiments: EMC, HERMES, Hall A, CLAS, COMPASS, STAR, PHENIX, …

spin-dep./long. polarized

§ Transversity distribution

- Process: single-spin asymmetry in SIDIS, …
- Experiments: HERMES, COMPASS, Belle…

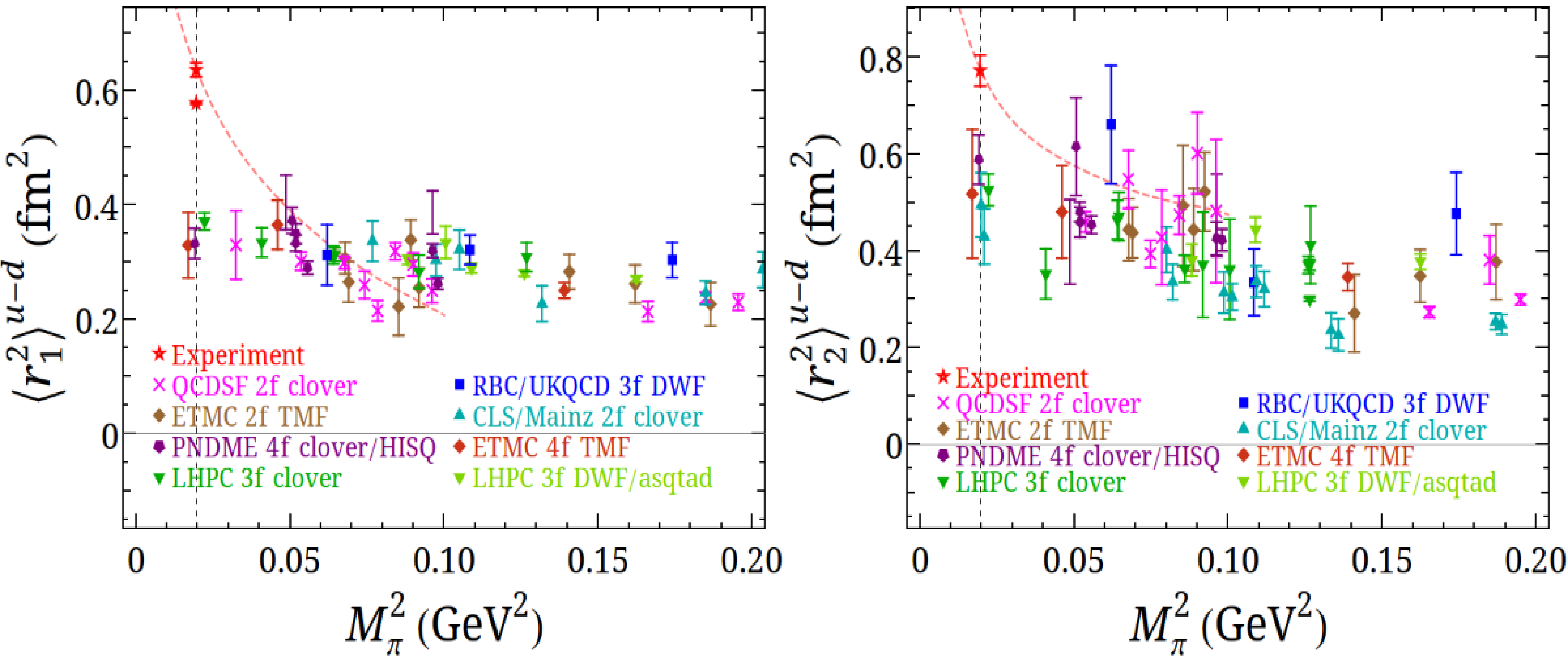


transversely polarized

Form Factors

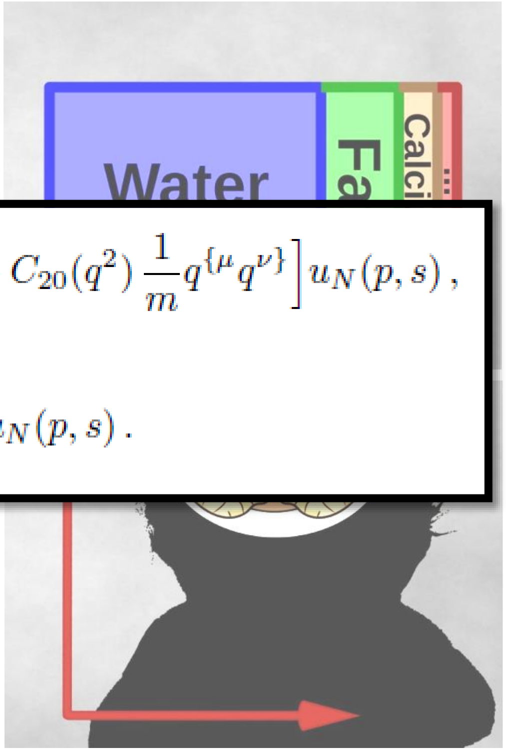
§ Charge radii

- Lattice data way too low
- No help for the proton-radius puzzle



Generalized Parton Distribution

$$\mathcal{O}_V^{\mu_1\cdots\mu_n} = \bar\psi\gamma^{\{\mu_1} i \overleftrightarrow{D}^{\mu_2}\ldots i\overleftrightarrow{D}^{\mu_n\}}\psi$$
$$\mathcal{O}_A^{\mu_1\cdots\mu_n} = \bar\psi\gamma^{\{\mu_1} i \overleftrightarrow{D}^{\mu_2}\ldots i\overleftrightarrow{D}^{\mu_n\}}\gamma_5\psi$$



$$\langle N(p',s')|\mathcal{O}^{\mu\nu}_{\not{q}}|N(p,s)\rangle = \bar u_N(p',s')\left[A_{20}(q^2)\,\gamma^{\{\mu}P^{\nu\}} + B_{20}(q^2)\,\frac{i\sigma^{\{\mu\alpha}q_\alpha P^{\nu\}}}{2m} + C_{20}(q^2)\,\frac{1}{m}q^{\{\mu}q^{\nu\}}\right]u_N(p,s)\,,$$

$$\langle N(p',s')|\mathcal{O}^{\mu\nu}_{\not{q}\gamma_5}|N(p,s)\rangle = \bar u_N(p',s')\left[\tilde A_{20}(q^2)\,\gamma^{\{\mu}P^{\nu\}}\gamma^5 + \tilde B_{20}(q^2)\,\frac{q^{\{\mu}P^{\nu\}}}{2m}\gamma^5\right]u_N(p,s)\,.$$

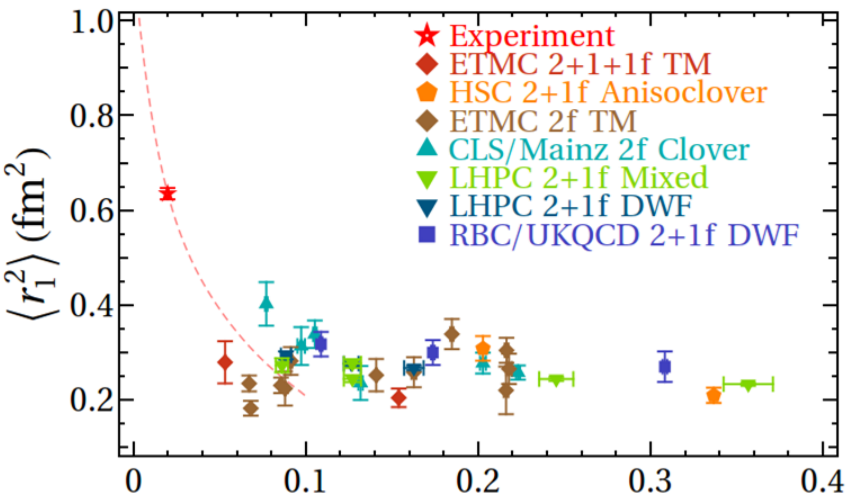
§ Generalized Parton Distribution

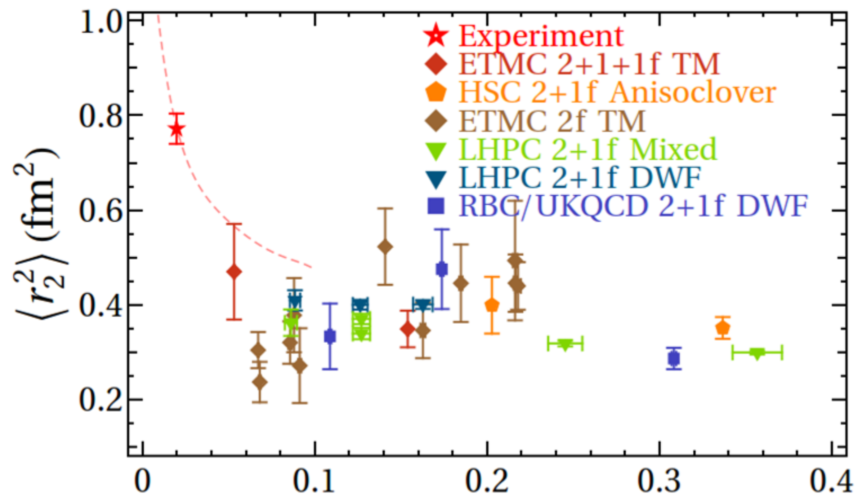
- Deeply virtual Compton scattering (DVCS)
- $\langle x^{n-1}\rangle_q = A_{n0}(0)$, $\langle x^{n-1}\rangle_{\Delta q} = \tilde A_{n0}(0)$, $\langle x^n\rangle_{\delta q} = A_{Tn0}(0)$
- $F_1(Q^2) = A_{10}(Q^2)$, $F_2(Q^2) = B_{10}(Q^2)$, $G_A(Q^2) = \tilde A_{10}(Q^2)$, $G_P(Q^2) = \tilde B_{10}(Q^2)$
- Nucleon spin $A_{20}(0)$, $B_{20}(0)$

Form Factors

§ Charge radii

- Lattice data way too low; no help for the proton-radius puzzle





§ Induced-pseudoscalar

$$g^*_P = [m_\mu G_P(0.88\, m_\mu^2) \,/\, 2\, m_N]$$

- Poor constraints (DWF numbers so far)
- Important for muon physics