Constraining neutron star properties with QCD

...or constraining QCD properties with neutron stars

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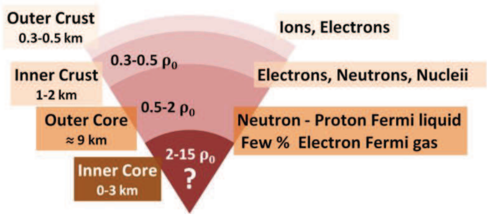
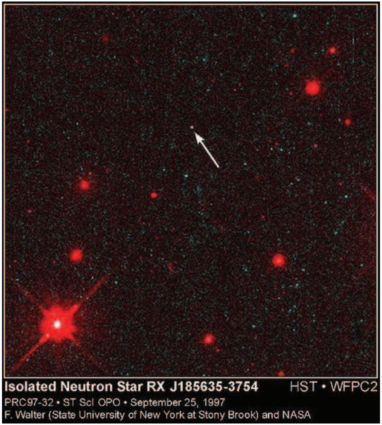
0912.1856, 1006.4062, 1311.5154,1402.6618

21 March 2014, CERN

Compact stars

- Masses $\lesssim 2.0 M_\odot$
- Radii $\sim$ 15km
- $T \lesssim$ KeV
- $n \lesssim 15\rho_0 \quad (\rho_0 = 0.16\mathrm{fm}^{-3})$





Can we understand these objects from 1st principles = QCD?

Structure

Two competing forces:

- Gravity tries to pull the star into a black hole
- Pressure of strong nuclear force resists the gravity

Neutron stars in hydrostatic equilibrium: rotation can be included

- Gravity: Tolman-Oppenheimer-Volkov (TOV)

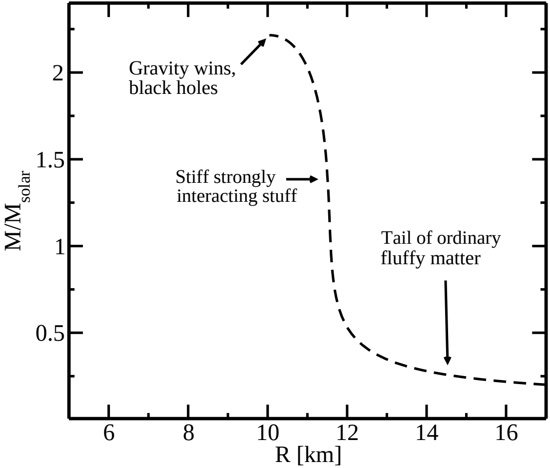
$$
\begin{aligned}
\frac{dP}{dr} &= -\frac{G\epsilon(r)M(r)}{r^2}\left[1+\frac{P(r)}{\epsilon(r)}\right]\left[1+\frac{4\pi r^3 P(r)}{M(r)}\right]\left[1-\frac{2GM(r)}{r}\right]^{-1} \\
\frac{dM}{dr} &= 4\pi r^2 \epsilon(r)
\end{aligned}
$$

- Strong nuclear force: Equation of state

$$\epsilon(P) \quad (\text{or} \quad P(\mu)\ldots)$$

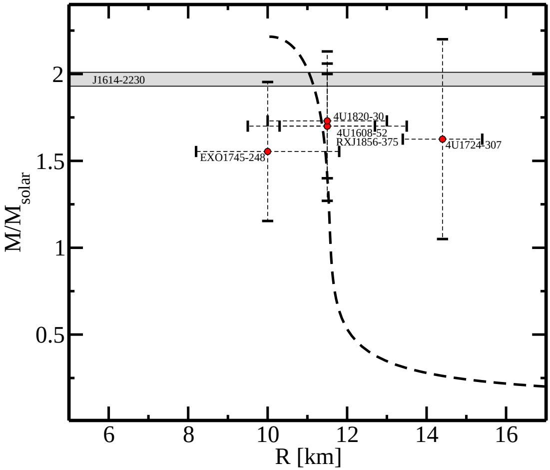
M-R plots: A map from micro to macro

For each EoS, there is a unique M-R relation



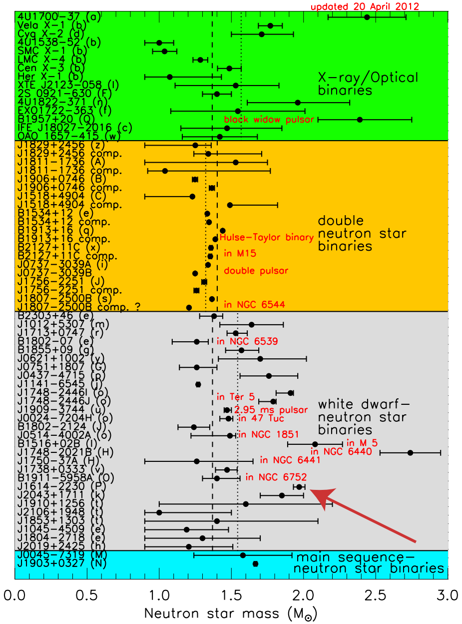
M-R plots: A map from micro to macro

Can be compared to observations!



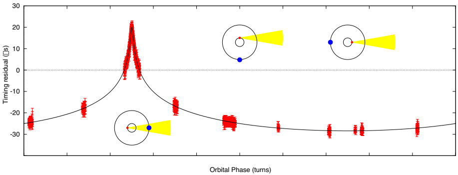
M-R: direct information about EoS. Just M: must be able to support all $M_{\max}$

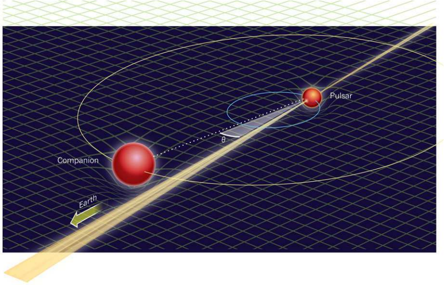
Mass measurements:



Lattimer Annu. Rev. Nucl. Part. Sci. 62, 485 (2012)

Mass measurements, PSR J1614-2230





Shapiro delay:

- Binary pulsar

  period: 3.1508076534271(6)ms
  orbital period: 8.6866194196(2)d

- Nearly edge-on orbits

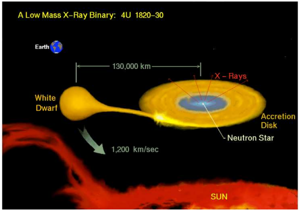
  sin $i = 0.999894(5)$

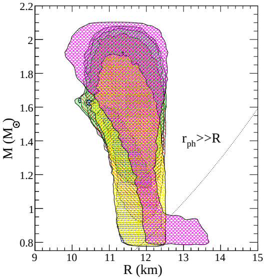
- increase in light travel time through the curved space-time near a wight dwarf.

$$\boxed{M_{\max} > \begin{cases} 1.97 \pm 0.04 & \text{J1614-2230} \\ 2.01 \pm 0.04 & \text{J0348+0432} \end{cases}}$$

Demorest et al. Nature 467, 1081-1083 (2010) and Antoniadis et al., Science 240, 448 (2013)

Radius measurements





Steiner et al. Astrophys.J. 722 (2010) 33-54

Type I X-ray burst:

- NS accretes matter from a companion
- Ignition of the envelope generates thermonuclear explosion
- During explosion, NS radiate thermally
- Measurement of T and flux F_∞ gives R

$$A = \frac{F_\infty}{\sigma T_{bb}^4} = f_c^{-4} \left(\frac{R}{D}\right)^2 (1-2\beta)^{-1}$$

- Lots of subtleties: PRE, distance, interstellar absorption...

Outline

- Model of neutron stars from nuclear physics
- Approaching neutrons stars from pQCD
- Filling out the missing pieces
- Summary

Idealized EoS of stellar matter

Nuclear matter in ground state:

- $T \ll k_F$: T=0 for all practical purposes
- Electrically neutral:

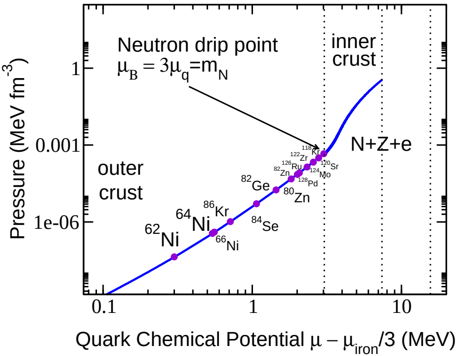
$$n_e = 2/3 n_u - n_d/3$$

- Only one ground state: no "chemical composition":
  - Neutrinos not bound, chemical β-equilibrium: $(d \to u + e + \bar{\nu}_e)$

$$\mu_B/3 = \mu_s = \mu_d = \mu_u + \mu_e$$

  - EoS unique: $P(\mu_B)$

Structure of a neutron star: Crust



Outer crust:

- Lattice of nuclei in electron sea:

$$P = P_e + \frac{1}{3} W_L n_N$$

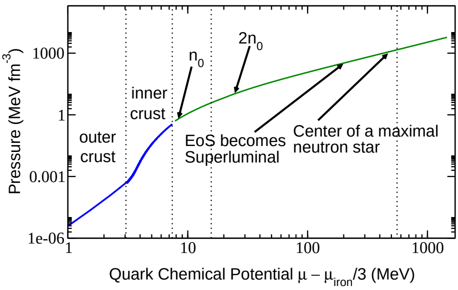
- Nuclear ground state at zero P: ^{56}Fe with $\mu_{\mathrm{iron}} = 310\mathrm{MeV}$
- As μ increases, more neutron rich elements become favourable

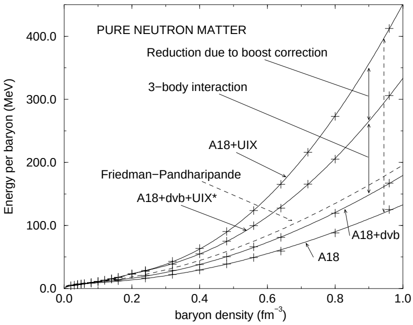
Inner crust:

- Neutron gas + Z + e
- NN interactions become important:
  - Typically modelled by phenom. pot. models (AV18)

Negele & Vautherin Nucl.Phys. A207 (1973) 298-320; Baym et al. Nucl.Phys. A207 (1973) 298-320;

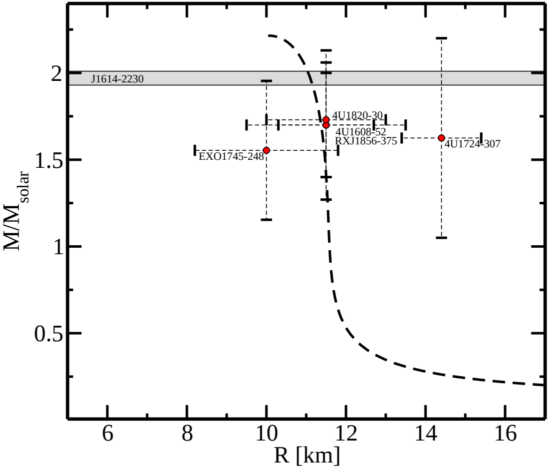
Extrapolating further: APR





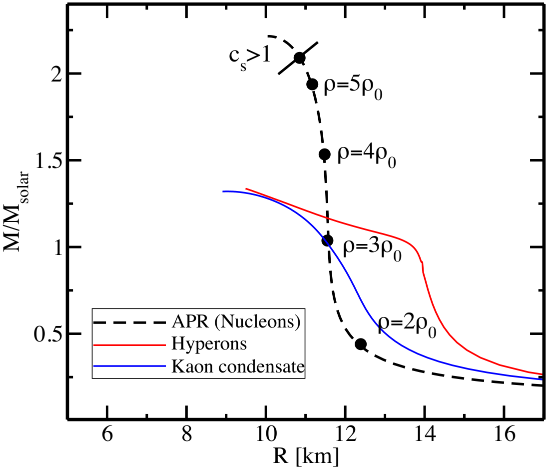
- Try to extrapolate to higher densities by introducing 3N, boost corrections, etc.
- corrections become large soon: Not weakly interacting neutrons: something more complicated...

Akmal, Pandharipande & Ravenhall Phys.Rev. C58 (1998) 1804-1828



Observation:

- APR seems to agree with observations
- Easily accommodates for $M > 2M_{\odot}$



Observation:

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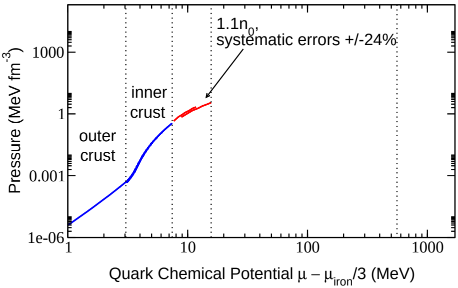
- Easily accommodates for $M > 2M_\odot$

Theory:

- Central densities very high
  - At $\rho \sim 3\rho_0$, nucleons occupy total volume.
- APR superluminal at $\rho \sim 5.3\rho_0$
- Inclusion of exotics makes EoS softer

State of the art in nuclear EoS: Chiral EFT at N^3LO



- Based on relativistic Weinberg EFT, includes 3N, 4N
- Breakdown scale: $\Lambda_b \sim 500 MeV < M_\rho$
- Systematic errors from variation of resolution scale.
- Uncertainties from the low-energy constants dominate
- Errors $\pm 24\%$ at $n = 1.1 n_0$.

Tews et al. Phys.Rev.Lett. 110 (2013) 032504; Hebeler et al. Astrophys.J. 773 (2013) 11

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Approaching from the high density side:

We know the theory of strong interactions: QCD

$$\begin{aligned}\Omega(\mu_u, \mu_d, \mu_s, \mu_e, m_s) &= \log \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu e^{-\int d^4x \mathcal{L}_{QCD}} \\ \mathcal{L}_{QCD} &= \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{\psi}_i(\gamma_\mu D_\mu + m_i - \mu_i\gamma_0)\psi_i.\end{aligned}$$

But that's one hard integral to evaluate. . .

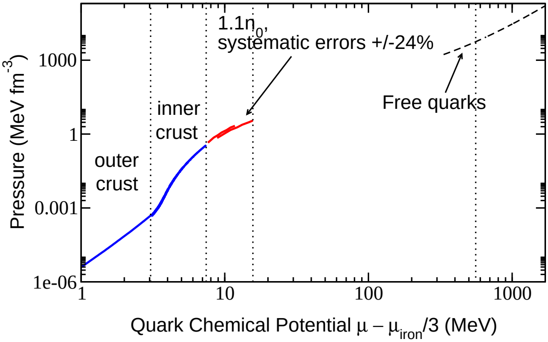
- At $T \neq 0$ and $\mu \lesssim T$: Lattice is the definitive non-perturbative method for QCD thermodynamics.
- At $\mu \gtrsim T$ simulations become unfeasible due to sign problem.
- At (asymptotically) high densities $\alpha_s(\mu) \sim 1/\log(\mu^2)$

At high densities: Degenerate quark gas

First approximation $\alpha_s(\mu_B) = 0$:

- Thermodynamics of free massless fermi gas:

$$P_{\rm free}(\mu_B) = \sum_{\rm d.o.f} \int \frac{d^3p}{(2\pi)^4} E(p)\theta(\mu_q - E(p)) = \frac{1}{4\pi^2}(\mu_u^4 + \mu_d^4 + \mu_s^4)$$



- Eventually EoS must coincide with free quarks, but when?
- Need to go to higher orders for errorbars

Higher order corrections:

$$P(\mu_B) = \int \frac{d^3p}{(2\pi)^4} E(p)\theta(\mu_q - E(p))$$

NLO:

- Interactions cause corrections to disp. rel.:

$$E^2(p) \sim p^2 + g^2\mu^2$$
$$P(\mu_B) \sim P_{\rm free}(1 + c_1 g^2)$$

NNLO:

- Corrections to $E^2(p) \sim p^2 + g^2\mu^2 + g^4\mu^2$.
- $p \sim g\mu$ contribute at $\int d^3pp \sim g^4\mu^4$.
  - Order-1 mod. to disp. rel. $\Rightarrow$ non.pert.
  - Resummation of ring diagrams. Integral over scales gives a log:

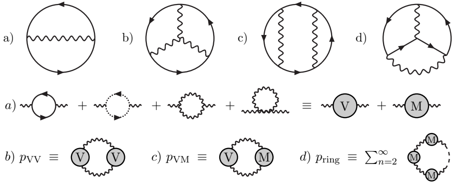
$$P(\mu_B) \sim P_{\rm free}(1 + c_1 g^2 + c_2 g^4 + c_2' g^4 \log\left[\frac{g\mu}{\mu}\right])$$

Freedman & McLerran Phys.Rev. D16 (1977) 1169, Baluni Phys.Rev. D17 (1978) 2092

Full NNLO with mass dependece: AK et al. Phys.Rev. D81 (2010) 105021

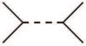
Higher order corrections:

In practice, effective potential though vacuum graphs:



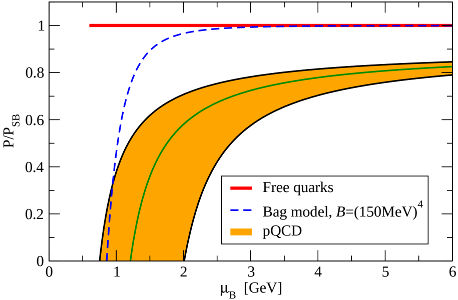
- Finite μ causes a shift in (euclidean) frequencies $p_0 \to p_0 \pm +i\mu$
- Our strategy: perform p_0 intergrals first to get vacuum feynman diagrams with μ-dependent phase space integrals.
- Automatized integral reduction of vacuum diagrams (FIRE).

 $\rightarrow$  $- 2 \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{\theta(\mu - E(\vec{p}))}{2E(\vec{p})}$ 

$+ \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{\theta(\mu - E(\vec{p}))}{2E(\vec{p})} \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{\theta(\mu - E(\vec{q}))}{2E(\vec{q})}$ 

AK et al. Phys.Rev. D81 (2010) 105021

Reliability on N^2LO pQCD:



Coefficients depend on renormalization scale $\bar{\Lambda}$

$$P(\mu_B)/P_{\rm free} \sim 1 + c_1 g^2[\bar{\Lambda}] + c_2[\bar{\Lambda}]g^4[\bar{\Lambda}] + c_2'[\bar{\Lambda}]g^4[\bar{\Lambda}]\log[g]$$

- Uncertainties estimated though scale variation $\bar{\Lambda} = X\mu_q$ with $X = \{1, 2, 4\}$

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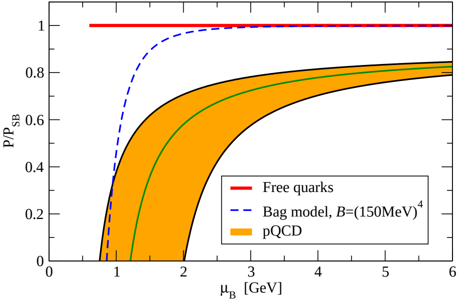

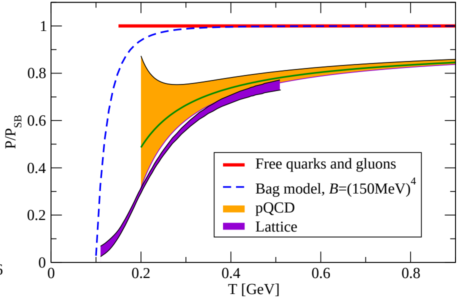

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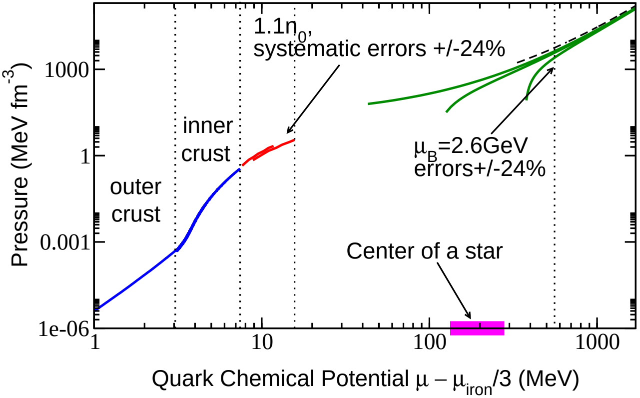
- Uncertainties estimated though scale variation $\bar{\Lambda} = X\mu_q$ with $X = \{1, 2, 4\}$
- At $T \neq 0$, $\mu_B = 0$, lattice simulations provide nonpert. tool to address the reliability of uncertainties.

DR resummation by Laine & Schröder JHEP 1202 (2012) 068, lattice:Borsanyi et al. Phys. Lett. B 370 (2014) 99-104

State of the art in pQCD: N^2LO with m_s



- Relative uncertainty $\pm 24\%$ at $\mu_B = 2.6\text{GeV}$.
- Centers of stars almost "perturbative"

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Interpolating: Polytropic EoS

Strategy:

- Interpolate where EoS not reliably known.

Require:

- Smoothness: $P(\mu_B)$ and $\partial_{\mu_B} P = n(\mu_B)$ continuous
- Thermodynamical consistency: match with nucleonic and pQCD EoS
- Subluminal: $c_s^2 < 1$ everywhere

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Method:

- Piecewise polytropes:

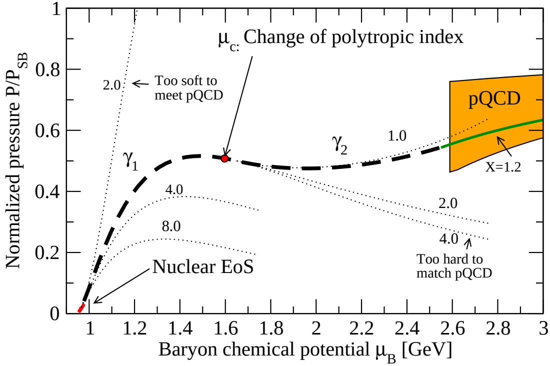
$$P_i(n) = \kappa_i n^{\gamma_i}, \text{ for } \mu_i < \mu_B < \mu_{i+1}$$

  - The larger γ, the stiffer the EoS is
  - For $\gamma > 2$, the EoS becomes superluminal above some μ

γ	EoS
∞	incompressible matter
2	asymptotically $c_s(\mu_B) = 1$
5/3	Non-relativistic degenerate fermi gas
4/3	Ultrarelativistic fermi gas
1	Ideal gas

Example interpolation:

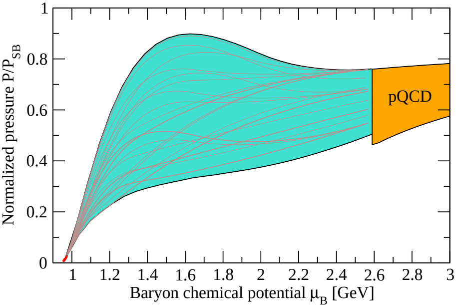
AK et al. 1402.6618



$$P_i(\mu_B) \quad = \quad \kappa_i \left(n_i^{\gamma_i - 1} + \frac{\gamma_i - 1}{\kappa_i \gamma_i} (\mu_B - \mu_{B,i}) \right)^{\frac{\gamma_i}{\gamma_i - 1}},$$

- Need at least two indices for interpolation
- If γ_1 is too small (soft), effective d.o.f becomes larger than QCD
- If γ_2 is too large, pressure too small to match with soft QCD

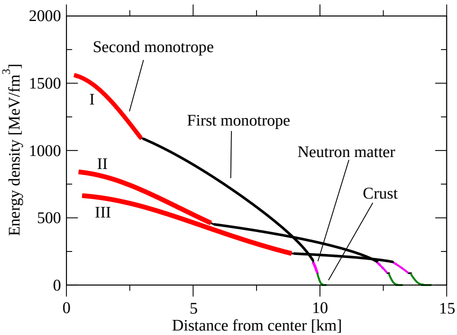
Complete set of bitropic EoSs, no phase transition



- Solutions for:
  - $\mu_c \in [1.08, 2.05]\,GeV$
  - $\gamma_1 \in [2.23, 9.2]$
  - $\gamma_2 \in [1.0, 1.5]$

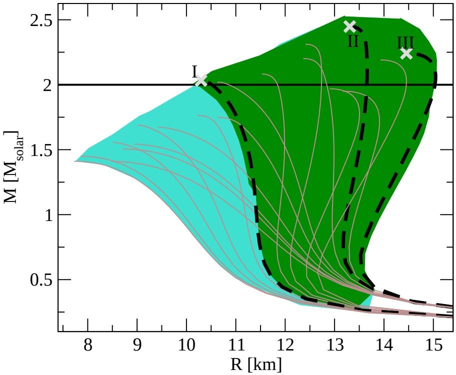
- The softness of pQCD EoS forces γ_2 to be small.
- The first monotrope always stiffer than the second, maximum c_s^2 just below μ_c.

Building stars



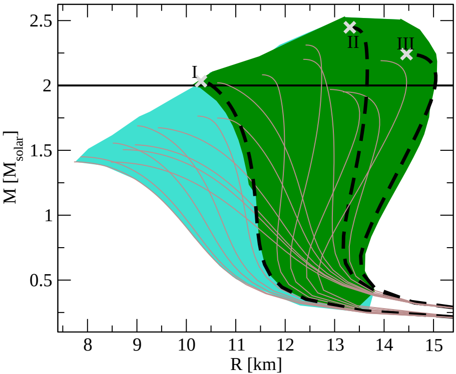
- Nuclear EoS present only as thin crust:
  - Important as a constraint to polytropes, not the EoS itself!
- Softening visible by increase near the center of the star

Constraining neutron star properties using QCD



- Maximal masses: $M_{\max} \in [1.4, 2.5]$, Radii $R \in [8, 16]$km.
- Central $\mu_B \in [1.33, 1.84]$ corresponding to $n \in [3.7, 14.3]n_0$.
- So far consistent with observation

Constraining QCD properties using neutron stars



- EoS can be constrained by using $M_{\max} > 2$
- Strong constraint due tension: Need stiff for $M_{\max}$, soft for pQCD

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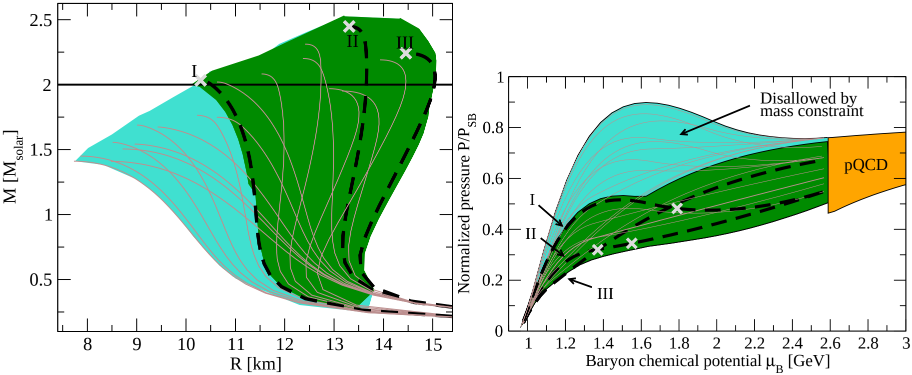


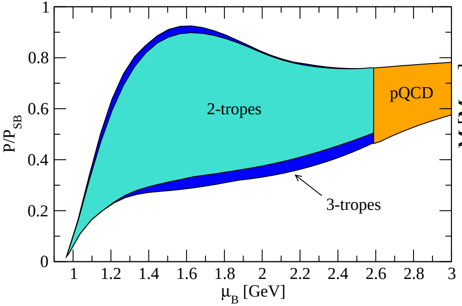
- EoS can be constrained by using $M_{\max} > 2$
- Strong constraint due tension: Need stiff for $M_{\max}$, soft for pQCD

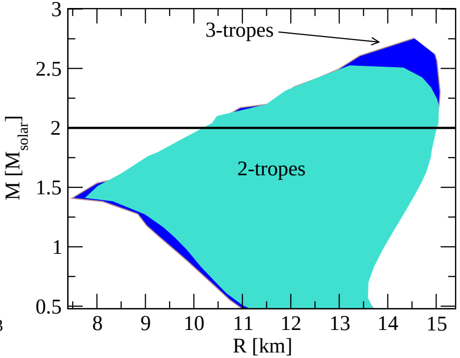
$$\boxed{\text{Uncertainty reduced to less than } \pm 30\% \text{ at all densities}}$$

but model assumptions...

Robustness of the result:

More sophisticated interpolation:

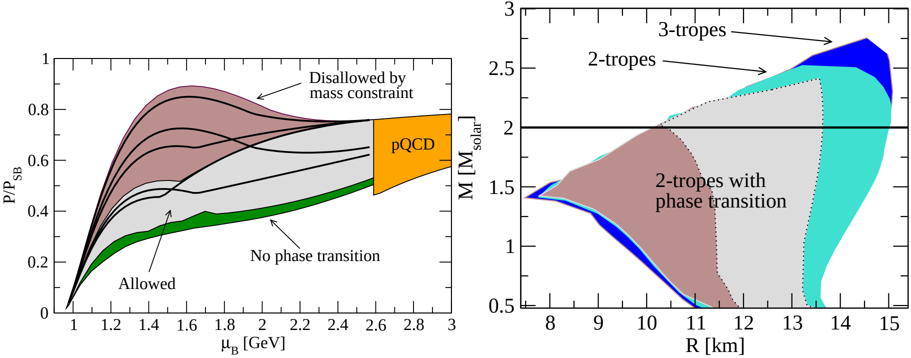




- Adding a third monotrope does not change qualitatively, quantitatively small effect.

Robustness of the result:

Relax smoothness assumption:

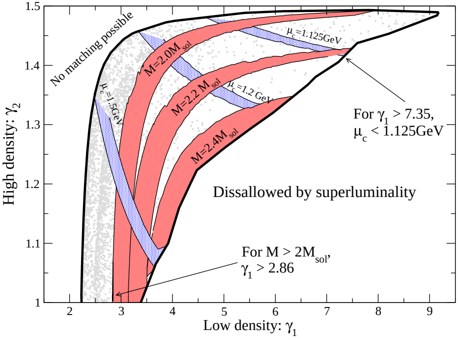


Allow for 1st order phase transition of latent heat of $\Delta Q^{1/4} \in [0, 250] MeV$:

- a discontinuity in derivative $\Delta n = \Delta Q/n$ at μ_c
- Requires *even softer* EoS at high densities for Maxwell construction

Constraints on indices

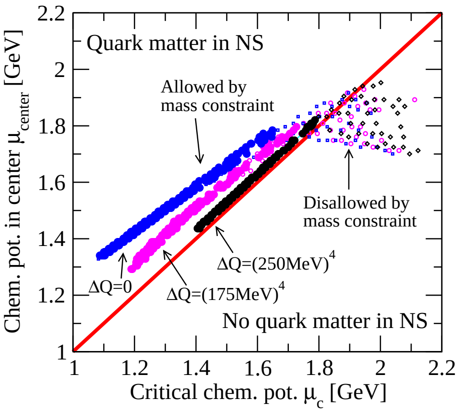
Solving maximum mass and μ_c as functions of γ_i:



- To get high maximal mass, the first monotrope needs to be large enough
  - For $M_{\max} > 2.0$, $\gamma_1 > 2.86$
  - For $M_{\max} > 2.0$, $\gamma_1 > 2.86$
- EoSs with $\gamma > 2$ become eventually superluminal
  - for $\gamma_1 = 7.35$, $\mu_c < 1.125$
  - for $\gamma_1 = 3.65$, $\mu_c < 1.5$

Quark matter in neutron stars?

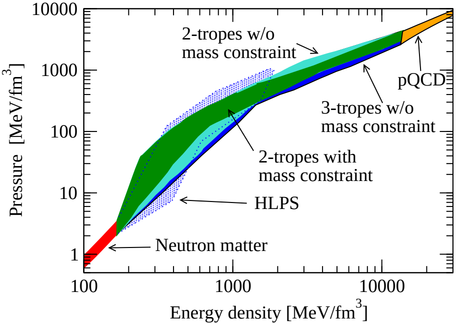
Interpreting the second polytrope as a partonic phase:



Collapse when center becomes soft:

- Maximal central chemical potentials strongly correlated with μ_c
- If clear distinction between phases, no quarks present.
- More likely: smooth transition, large amounts of strongly coupled matter described neither with weakly coupled partonic nor nucleonic d.o.f.s

Effect of imposing pQCD constraint



- HLPS: Tritropes without pQCD constraints but assumptions on γ's

Hebel et al, Astrophys.J. 773 (2013) 11

$$\text{HLPS}: \quad 1 < \gamma_1 < 4.5, \quad 0 < \gamma_2 < 8.5, \quad 0.5 < \gamma_3 < 8.5$$

- HLPS's assumption compatible with our determination:

$$\text{KFVS}: \quad 2.86 < \gamma_1 < 9.2, \quad 1 < \gamma_2 < 1.5$$

- EoS considerably constrained at high densities

Summary

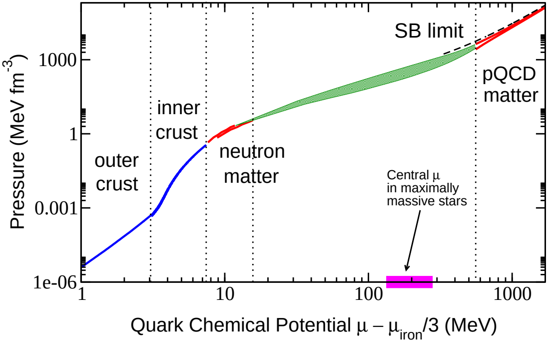
Observation:

- Significant observational advancements neutron stars $M_{\max} > 2M_\odot$, $M_{\max} \sim 2.4$ possible
- Reliable radial information possible in the near future.

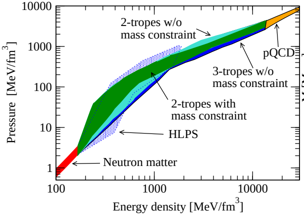
Theory:

- pQCD is relevant near densities at NS cores
- Neutron matter vs. quark matter? Strongly interacting quark matter?
- Tension between $M_{\max}$ and pQCD helpful
- Interpolating is easier than extrapolating:

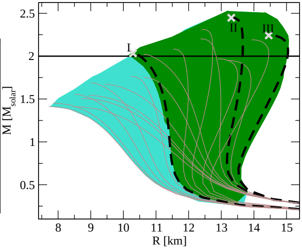
$$\boxed{\text{pQCD constrains the EoS even if there is no ``quark matter''}}$$



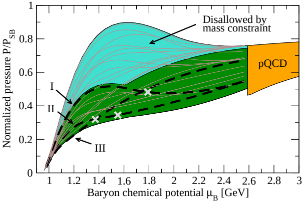
Pressure (MeV fm^{-3})
1000
1
0.001
1e-06
1
10
100
1000
Quark Chemical Potential $\mu - \mu_{iron}/3$ (MeV)
outer crust
inner crust
neutron matter
SB limit
pQCD matter
Central μ in maximally massive stars



Pressure [MeV/fm^3]
Energy density [MeV/fm^3]
10000
1000
100
10
1
100
1000
10000
2-tropes w/o mass constraint
pQCD
3-tropes w/o mass constraint
2-tropes with mass constraint
HLPS
Neutron matter



M [M$_{solar}$]
R [km]
2.5
2
1.5
1
0.5
8
9
10
11
12
13
14
15
I
II
III



Normalized pressure P/P$_{SB}$
Baryon chemical potential μ_B [GeV]
1
0.8
0.6
0.4
0.2
0
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
2.8
3
Disallowed by mass constraint
pQCD
I
II
III

Extra slides

"MIT Bag model"

The vacuum of QCD is not the perturbative one:

- χSB, confinement, etc.
- The physical vacuum has lower free energy ($\sim$ higher pressure) than the perturbative one

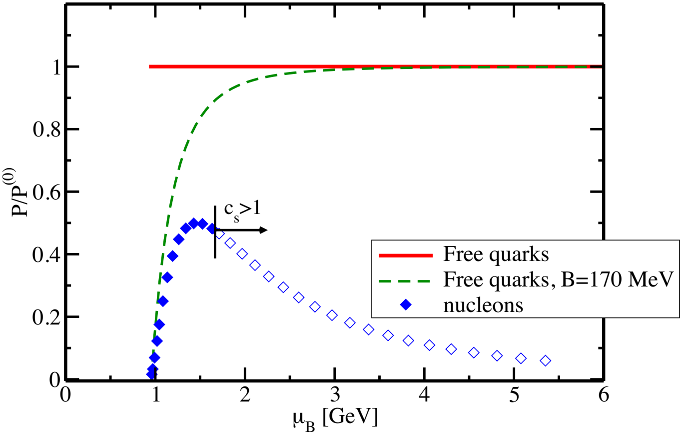
$$P_{\chi\mathrm{SB}}(\mu_B = 0) = B, \quad P_{\mathrm{free}}(\mu_B = 0) = 0$$

- The physical vacuum popularly modelled by introducing B to the perturbative EoS:

$$P_{\mathrm{MIT}}(\mu_B) = P_{\mathrm{free}}(\mu_B) - B$$

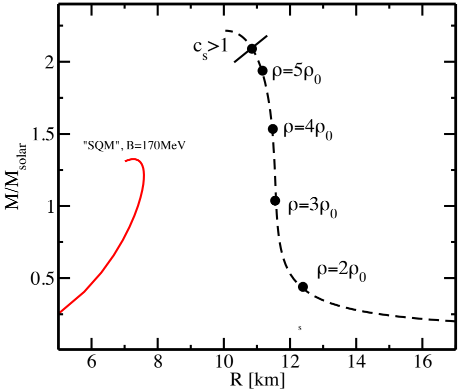
    - Bag constant as the simplest *nucleonic* EoS
- Bag model qualitatively describes hadrons, $B^{1/4} \sim 150 MeV$

"MIT Bag model"



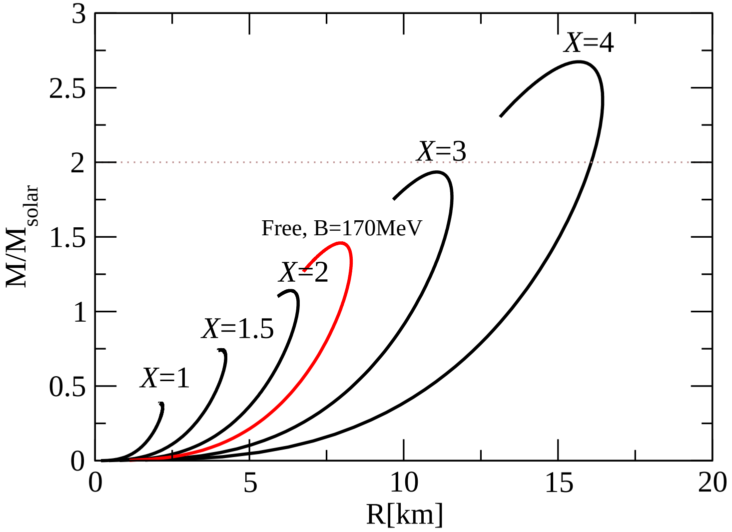
Rudimentary connection with the hadronic EoS's

"MIT Bag model"



Common wisdom: Stars with QM are (too) small.
(Note: No ordinary atmosphere, no tail)

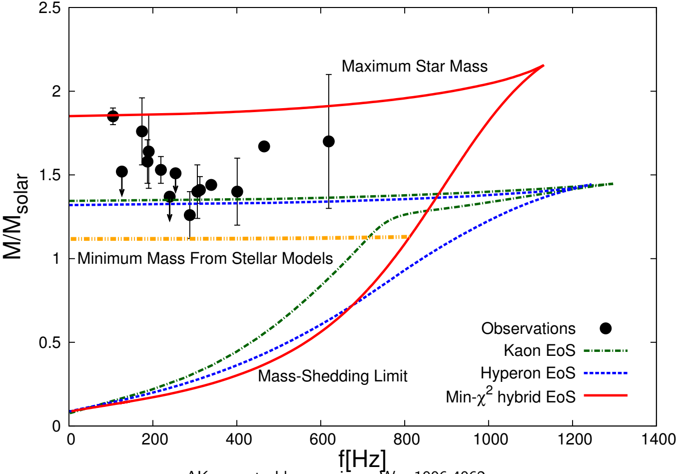
Quark stars from pQCD:



- High sensitivity to scale variation reflects the level of our ignorance
- $M > 2M_{\odot}$ not excluded

Rotating stars:

Upper and lower bounds on masses of rotating stars:



AK, romatschke, vuorinen, Wu, 1006.4062

Strange quark matter hypothesis (or $\mu_{pt} < 0.31\text{GeV}$)

If the energy per baryon in quark matter is less than

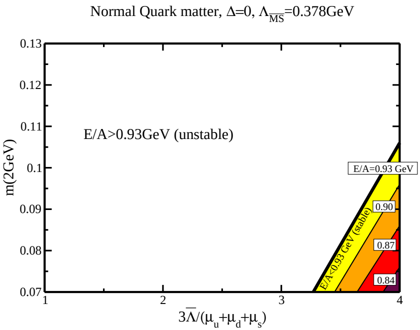
$$E/A = 3\mu_c = 0.93\text{GeV} \qquad {}^{56}\text{Fe}$$

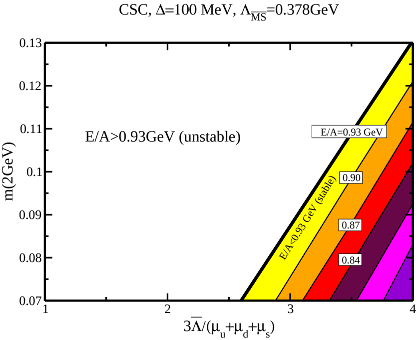
then quark matter is the true ground state $\rightarrow$ Nuclear matter metastable.

- Lifetime:
  - Nucleons $\rightarrow$ 2 Flavor quark matter:
    - Equilibration through Strong interaction $\longrightarrow t_{relax} \sim 1/\Lambda_{QCD}$
    - Short-lived. Ruled out by experiment!
  - Nucleons $\rightarrow$ 3 Flavor quark matter:
    - Equilibration through Weak Interactions (10^{60} years for $A > 6$)
    - Adding d.o.fs increases pressure $\rightarrow$ more likely to be stable
    - Experimentally plausible, let's find out what the theory says!

Strategy: Find out if SQM stable in the parameter space $(m_s, B, \bar{\Lambda})$ with

- $n_s > 0$ (quark mass extremely important!)
- $\mu_c < 0.31\text{GeV}$





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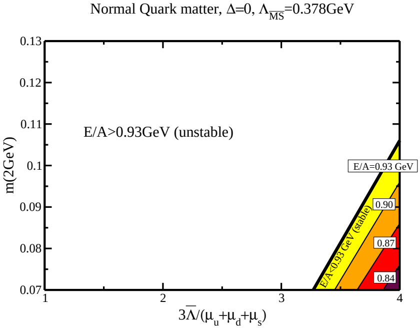
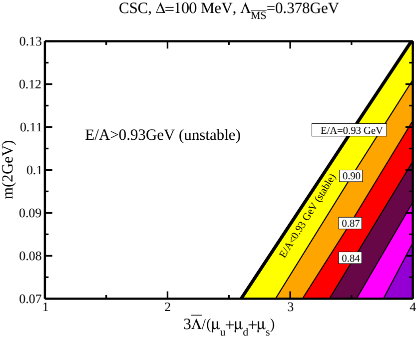
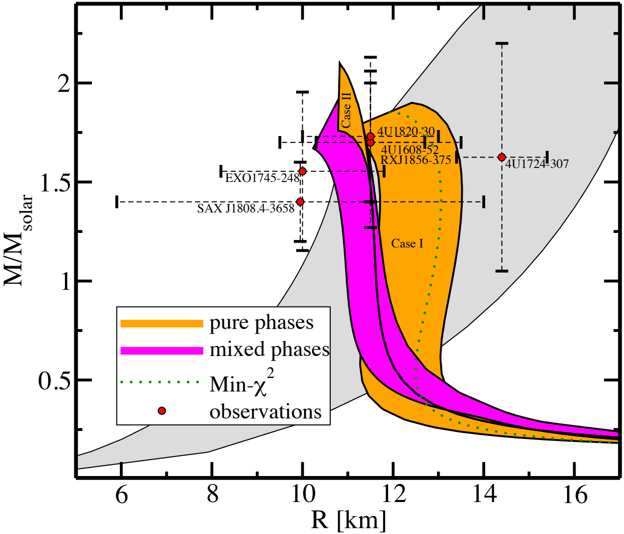


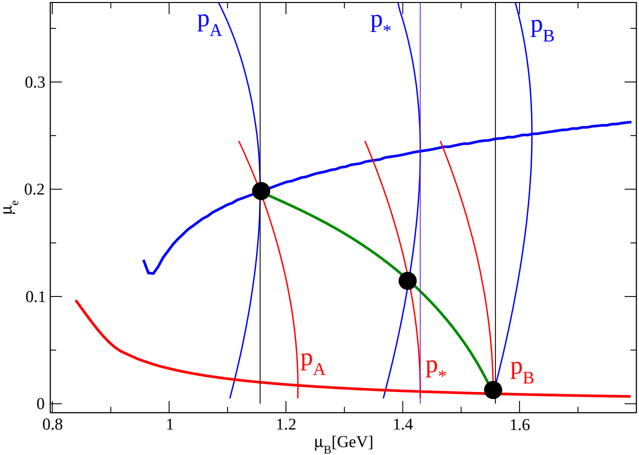


- Parameter space very hostile for $(\Delta = 0)$. Including CSC makes SQM more plausible
  $\longrightarrow$ Stable SQM disfavored but not ruled out.

Quarks stars to $\mathcal{O}(a_s^2)$



Glendenning construction:



In the case of two conserved charges (electric and baryonic), there is a possibility of a 2-phase admixture: