Decomposing colour structures into multiplet bases

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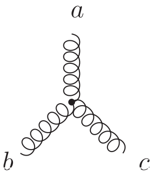
Introduction

Introduction

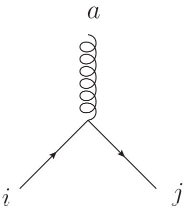
- The high energy of the LHC gives events with many coloured partons in the perturbative regime.
- It is a challenge to deal with the colour structure exactly for many partons, due to the non-Abelian structure of QCD.

Local symmetry of QCD

- The symmetry of QCD is $SU(3)$
- It enters the calculations from the vertices:



$\propto i\, f^{abc}$



$\propto (T^a)^{ij}$

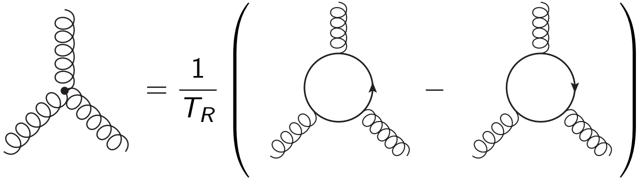
Section 2

Colour space bases

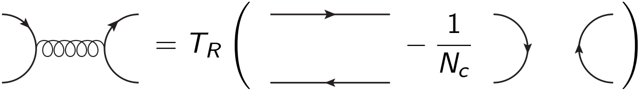
Trace bases

Simple algorithm:

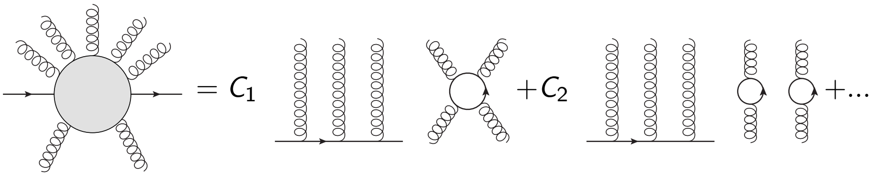
- 4-gluon vertices are rewritten as sums over 3-gluon vertices.
- The antisymmetric 3-gluon vertices are replaced by:



- Internal gluons are removed by:



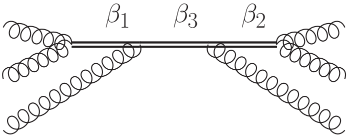
Trace bases



- Straightforward to apply to reduce any colour structure.
- Non-orthogonal and overcomplete!
- Number of vectors grows factorially in the number of external gluons, N_g, plus external $q\bar{q}$-pairs, $N_{q\bar{q}}$: $\sim (N_g + N_{q\bar{q}})!$
- Squaring the amplitude gives factorial square scaling due to the non-orthogonality.

Multiplet bases

- Orthogonal and minimal basis!
  Keppeler and Sjödahl, JHEP 1209.
- Number of vectors grow exponentially, not factorially.
- Orthogonality makes squaring easier!
- Downside is that decomposing a colour structure is not as straightforward as it is with the trace basis.



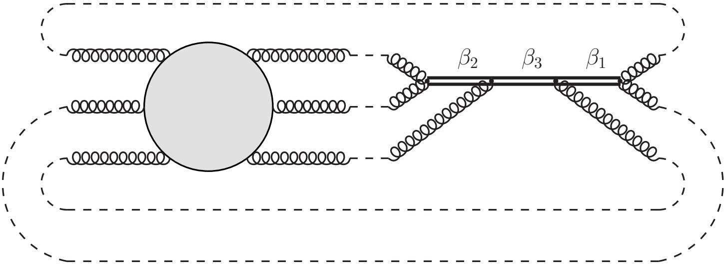
Quarks can be handled by combining the external quarks and antiquarks into pairs, which can then be in either a singlet or an octet.

Section 3

Decomposition into multiplet bases

Decomposition

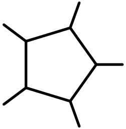
- The decomposition is just an evaluation of the scalar product between the basis vectors and the colour structure:



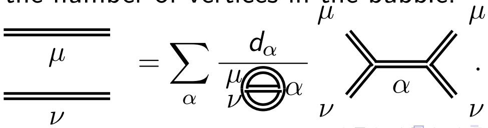
- This is just the evaluation of vacuum bubbles.

Vacuum bubbles

- Any vacuum bubble can rewritten as sums over factors of simpler vacuum bubbles.
- This is achieved by finding loops in the bubble:

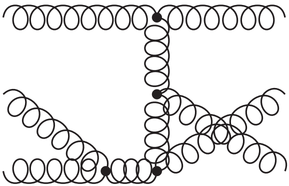


and repeatedly applying completeness relations, resulting in a reduction of the number of vertices in the bubble:



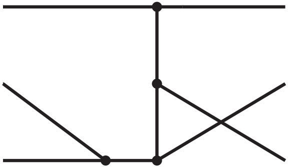
Example for 6 external gluons

One of the colour structures for $3g \rightarrow 3g$ is:



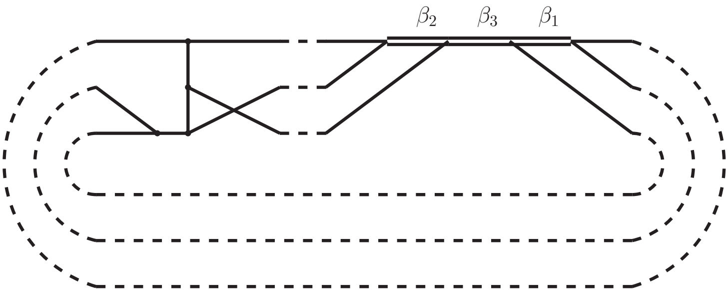
Example for 6 external gluons

Using straight lines instead of curly lines for gluons gives:



Example for 6 external gluons

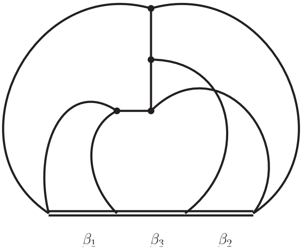
Decomposition into the basis vectors is equivalent to determining the scalar product between the basis vector and the colour structure:



Example for 6 external gluons

In a more compact form:

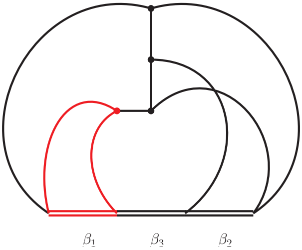
$$A(\beta_1, \beta_3, \beta_2) \quad =$$



Example for 6 external gluons

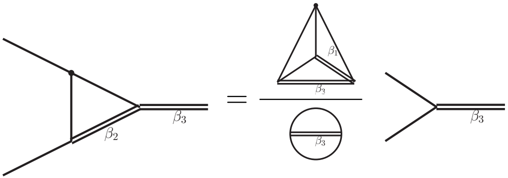
Highlighting the smallest loop:

$$A(\beta_1, \beta_3, \beta_2) \quad =$$

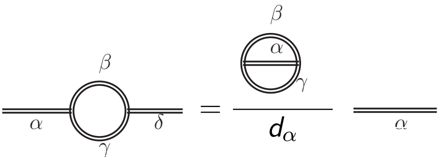


Reducing the loop

A vertex correction only gives a factor (given by small bubbles):



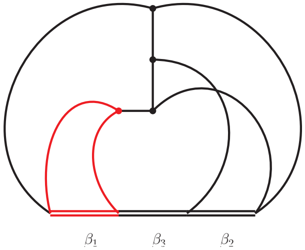
and so do two-vertex loops:



Example for 6 external gluons

Using the vertex correction result on:

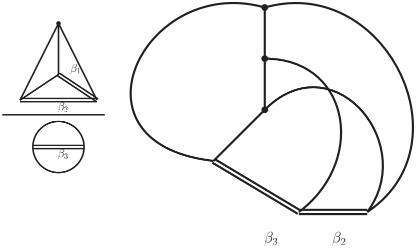
$$A(\beta_1, \beta_3, \beta_2) \quad =$$



Example for 6 external gluons

Gives:

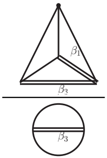
$$A(\beta_1, \beta_3, \beta_2) \quad =$$

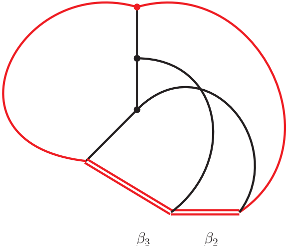


Example for 6 external gluons

Now we must pick a 4-vertex loop:

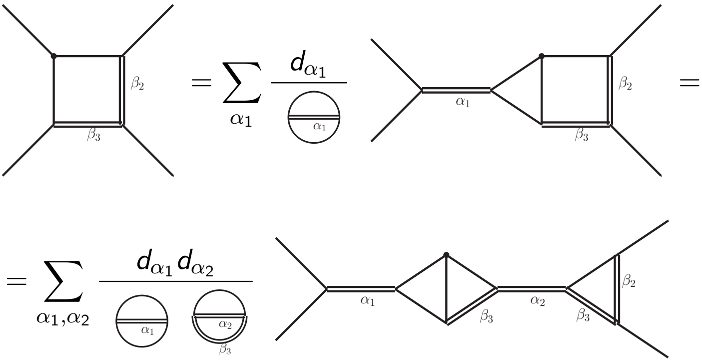
$$A(\beta_1, \beta_3, \beta_2) \quad = \quad \frac{}{}$$





Reducing the second loop

A 4-vertex loop is less trivial than a vertex correction:



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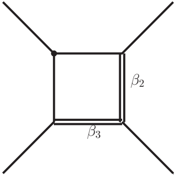
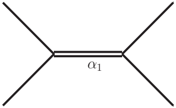


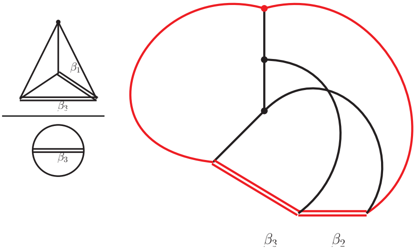
$$= \sum_{\alpha_1} C(\alpha_1, \beta_2, \beta_3)$$



Example for 6 external gluons

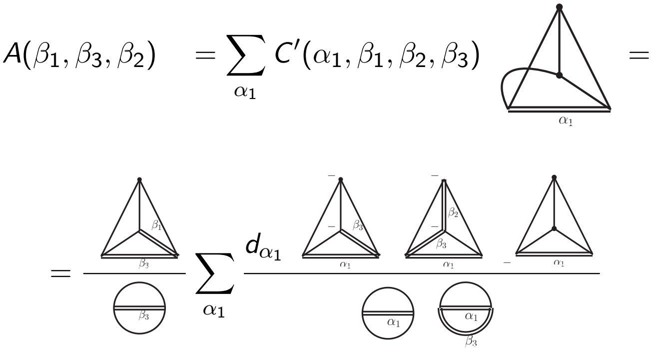
Now we are to remove the 4-vertex loop:

$$A(\beta_1, \beta_3, \beta_2) =$$



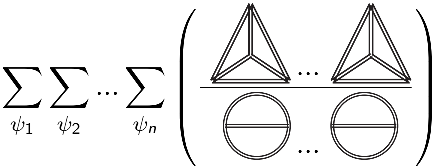
Example for 6 external gluons

Giving us the final expression:



Strategy

- Any vacuum bubble can be rewritten as a sum of factors of smaller vacuum bubbles, called Wigner 3j and 6j coefficients.



- Rewriting into smaller bubbles can be done without specifying for which basis vector it is.
- These smaller bubbles can be calculated once with the trace basis (which would take time) and then be stored.
- Rewriting a colour structure into these coefficients scales as N_g^3.

Wigner coefficients

Requiring the vertices to be non-zero and counting how many different possibilities there are for the Wigner coefficients gives the result:

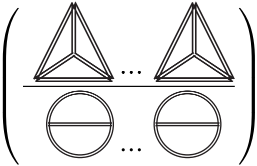
N_g	4	6	8	10	12
$N_c \geq N_g$	52	396	2 126	9 059	32 702
$N_c = 3$	38	130	277	479	736

It is known how to calculate these, and I have calculated them for $N_g = 6$.

Summations

- There might still be a problem in that there are many terms in:

$$\sum_{\psi_1}\sum_{\psi_2}\cdots\sum_{\psi_n}$$



- This has to do with how small loops can be found in the vacuum bubbles.

Summations

- The most difficult colour structures to handle cannot have a number of summations scaling worse than N_g^2
- For a given N_c each sum can be over, at most, $N_c^2 - 1$ reps.
  - This then gives an exponential in N_g^2 as an upper limit for the scaling for the hardest colour structures.
- By rewriting all colour structures for 6, 8 and 10 external gluons it seems that both the average number of summations and the number of summations required for the worst cases scale linearly, rather than quadratically.
- It is also very likely that assuming that every summation is over as many representations as possible is a bad overestimate.

Section 4

Conclusions and outlook

Conclusions and outlook

Conclusions:

- The multiplet bases are minimal and orthogonal.
- Decomposing a colour structure into it is non-trivial.
- Rewriting a vacuum bubble scales as N_g^3.
- A manageable number of vacuum bubbles has to be calculated and stored (even for general N_c).
- In the worst cases the number of terms that have to be added grow exponentially with the number of gluons.

Outlook:

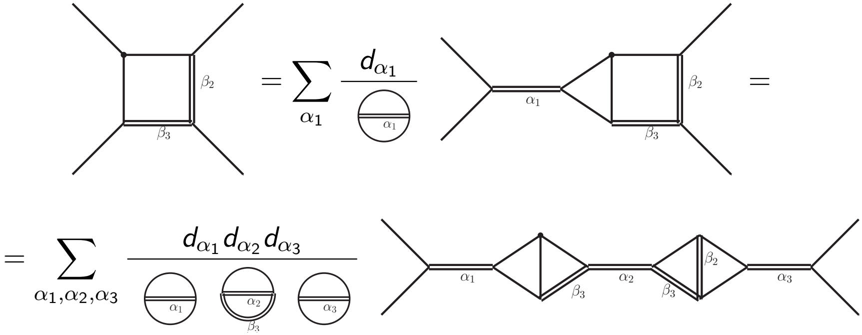
- Currently working together with M. Sjödahl and Y. Du with recursion relations for maximally helicity-violating (MHV) amplitudes.

Section 5

Backup slides

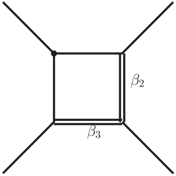
Reduction of 4-vertex loop

A 4-vertex loop is less trivial than a vertex correction:

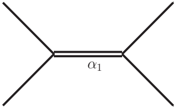


Reduction of 4-vertex loop

Resulting in:



$$= \sum_{\alpha_1} C(\alpha_1, \beta_2, \beta_3)$$



Reducing vertex corrections with a completeness relation

A vertex correction is simple to deal with: