On the longitudinal structure double parton distributions

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(M.C.Escher, Circle Limit III, however with $N_C = 4$...)

Based on F.A.Ceccopieri, Physics Letters B734 (2014) 79

Outline

- Phenomenological considerations: UE and DPS
- Theoretical considerations : TMD approach
- Definition and modelisation of double parton distributions and σ_{eff}
- Evolution and momentum some rule for DPDs
- Double counting problem
- Summary

Associated production of vector boson and one particle (1)

- One might expect that the UE has an hard component : DPS $\in$ UE
  $\rightarrow$ consider UE associated with Z boson production (first hard trigger)

- At low transverse momentum, the p_t^Z-spectrum is build out of soft parton emissions (double logs) or collinear vectorially compensating (single logs) emissions.

- Selecting lower and lower p_t^Z is an effective way to control and/or minimise ISR

- Look for secondary hard parton-parton scattering in the associated N_{ch} vs p_t, at large transverse momentum.

- Variation on the theme based on the resummation of hadronic transverse energy associated with Z bosons (Webber *et al.* '10): such observable is particularly sensitive to hadronisation and UE

Associated production of vector boson and one particle (2)

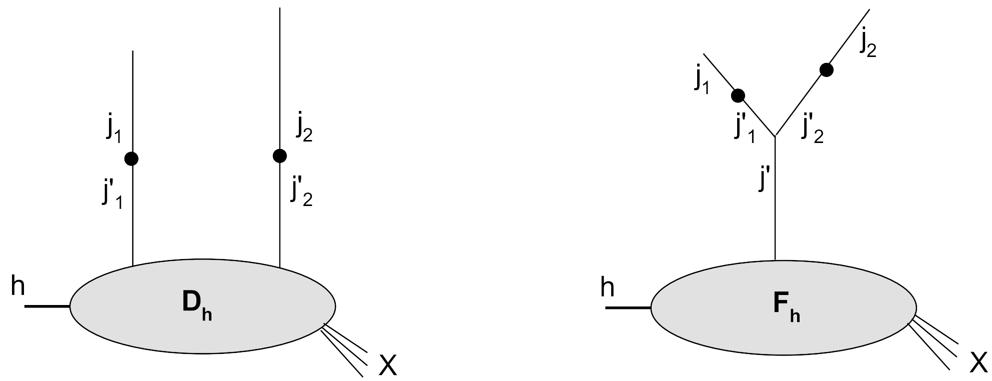
- This strategy has been adopted in the Z+D analyses by LHCb '14 (however no restriction on p_t^Z was imposed)
- In this case, candidate secondary (semi)hard scattering is provided by $ij \to c\bar{c}$
- Such an analysis relies on fragmentation of c-quarks into D-mesons, less known and constrained w.r.t fragmentation of light quarks into mesons (use pions, access very low pt!)
- LHCb analysis reports a significant excess over SPS background, claiming a large contributions from DPS
- Motivation for the derivation of unequal scales evolutions equations for DPDFs (Ceccopieri '11)

TMD approach

- DPS rates are expected to increase as one moves into relatively low p_T region
- Moreover special final state configurations, for example small dijet imbalances in 4 jets production (Blok *et al.* '11) could enhance DPS signal.
- Such regime is, in fact, conveniently treated within the "TMD approach", which in the last few year is converging on theoretical aspects of QCD evolution of TMD PDFs (Collins, Aybat and Rogers, Echevarria *et al.*, Cherednikov and Stefanis, Ceccopieri)
- It appears therefore natural to import in the DPS business many of these technologies (Manohar and Waalewwijn '12, Diehl and Schafer '11) at the price of loss of some precision
- Krakow group has already imported this formalism for the study of four charm quark final state (DPS fully (y, p_t)-differential 4-charms final state within a LO calculation)

The evolution of longitudinal DPDFs

- Within a fully collinear approach, DPDs evolution equations where originally obtained by Snigirev '03



$$Q^2 \frac{\partial D_h^{j_1,j_2}(x_1,x_2,Q^2)}{\partial Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_{\frac{x_1}{1-x_2}}^{1} \frac{du}{u} P_k^{j_1}(u) D_h^{j_2,k}(x_1/u, x_2, Q^2) +$$

$$+\frac{\alpha_s(Q^2)}{2\pi}\int_{\frac{x_2}{1-x_1}}^{1} \frac{du}{u} P_k^{j_2}(u) D_h^{j_1,k}(x_1, x_2/u, Q^2) +$$

$$+\frac{\alpha_s(Q^2)}{2\pi} \frac{F_h^{j'}(x_1+x_2,Q^2)}{x_1+x_2} \widehat{P}_{j'}^{j_1,j_2}\left(\frac{x_1}{x_1+x_2}\right)$$

Definitions of double PDFs

- However these distributions do not enter physical cross sections. Rather one needs to introduce generalised two parton distribution functions (Blok *et al.*, Treleani) which depend additionally on the relative transverse momentum Δ (or relative transverse distance b).

- In momentum space, such transverse momentum imbalance arises because difference of transverse momenta of the interacting partons is not conserved in the scattering.

- This dependence encodes correlations in the hadronic transverse structure, quantified, although with simplifying assumptions, by σ_{eff}

- In the valence sector, we can indeed gain insight on this dependence from hadronic models (Rinaldi *et al.* '14). No clue on such dependence for gluonic modes, D_{gg}.

Sum rules for DPDs

- For the reminder of this talk I will consider longitudinal DPDs in the equal scale case

$$D_h^{j_1,j_2}(x_1,x_2,Q^2) = {}_2D_h^{j_1,j_2}(x_1,x_2,Q^2,\Delta=0) \stackrel{FT}{=} \int d^2b D_h^{j_1,j_2}(x_1,x_2,Q^2,b)$$

- Given scarce informations on DPDs we must relie on sum rule in order to build a sensible ansatz at Q_0^2.

- A number of these sum rules have been proposed and implemente first by Gaunt and Stirling '09

- A consistency check of the momentum sum rule has been given by Blok *et al.* '13 and later independently checked by Ceccopieri '14

- Role of inhomogenous term is debated in the literature :

$\rightarrow$ bad behaviour at small b at CS level

$\rightarrow$ Double counting problem

Brute force method

- Strategy : assume it is valid at Q_0^2

$$\sum_{j_1} \int_0^{1-x_2} dx_1 x_1 D_h^{j_1,j_2}(x_1, x_2, Q_0^2) = (1 - x_2) F_h^{j_2}(x_2, Q_0^2)$$

- Verify that IDPDs fulfil the momentum sum rule at any other Q^2

$$\Downarrow$$

$$\sum_{j_1} \int_0^{1-x_2} dx_1 x_1 D_h^{j_1,j_2}(x_1, x_2, Q^2) = (1 - x_2) F_h^{j_2}(x_2, Q^2)$$

- Such procedure is cumbersome (deals with Green functions) and it is not really transparent (Blok *et al.* '13, Ceccopieri'14)

Fast method (1)

- One can apply the momentum sum rule to IDPDs evolution equations and try to recover single parton DGLAP evolution.
- On the left hand side on simply obtains

$$(1 - x_2) Q^2 \frac{\partial}{\partial Q^2} F_h^{j_2}(x_2, Q^2)$$

- Appling the sum rule to the first term in the evolution equation (parton 1 evolution) and reordering the integrals on gets

$$\frac{\alpha_s(Q^2)}{2\pi} \underbrace{\sum_{j_1} \int_0^1 du u P_k^{j_1}(u)}_{0} \int_0^{1-x_2} dy y D_h^{k,j_2}(y, x_2, Q^2)$$

$\Rightarrow$ overall momentum carried by parton 1 conserved by evolution

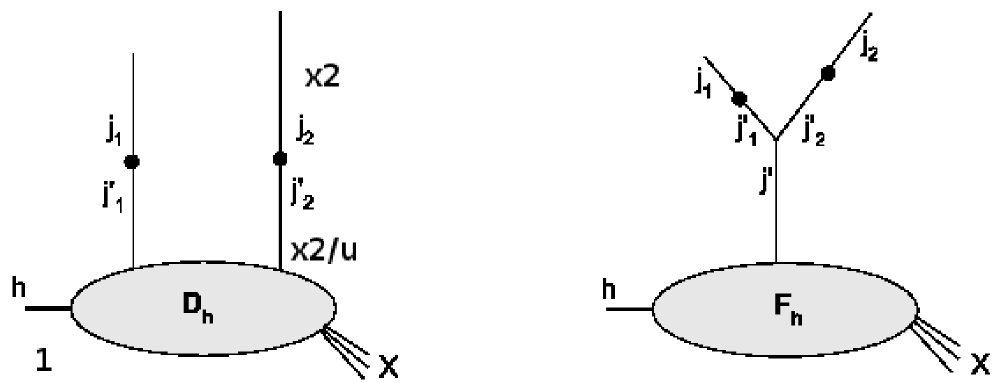
Fast method (2)

- Applying the sum rule to the second term (parton 2 evolution) we get

$$\frac{\alpha_s(Q^2)}{2\pi}\int_{x_2}^{1}\frac{du}{u}P_k^{j_2}(u)\left[\sum_{j_1}\int_0^{1-x_2/u} dx_1 x_1 D_h^{j_1,k}(x_1,x_2/u,Q^2)\right]$$

- In square brackets we recognise the momentum sum rule written for x_2/u so

$$\frac{\alpha_s(Q^2)}{2\pi}\int_{x_2}^{1}\frac{du}{u}P_k^{j_2}(u)\left[1-\frac{x_2}{u}\right]F_h^k\left(\frac{x_2}{u},Q^2\right)$$



Fast method (3)

- We finally handle the correlated term in the evolution equations

$$\sum_{j_1} \int_0^{1-x_2} dx_1 x_1 \frac{\alpha_s(Q^2)}{2\pi} \frac{F_h^{j'}(x_1+x_2, Q^2)}{x_1+x_2} \widehat{P}_{j'}^{j_1,j_2}\left(\frac{x_1}{x_1+x_2}\right)$$

- Changing to $u = x_2/(x_1+x_2)$

$$\frac{\alpha_s(Q^2)}{2\pi} \int_{x_2}^1 \frac{du}{u} x_2 \frac{1-u}{u} F_h^{j'}\left(\frac{x_2}{u}, Q^2\right) \sum_{j_1} \widehat{P}_{j'}^{j_1,j_2}(1-u)$$

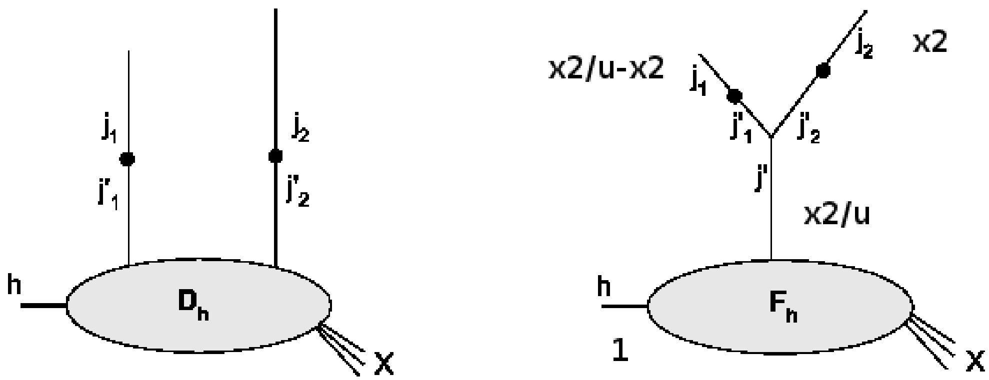
- By exploiting the following integral relation between real and regularised splitting functions

$$\int_x^1 du(1-u)g(u) \sum_{j_1} \widehat{P}_{j'}^{j_1,j_2}(1-u) = \int_x^1 du(1-u)g(u) P_{j'}^{j_2}(u)$$

Fast method (4)

- We then get

$$\frac{\alpha_s(Q^2)}{2\pi}\int_{x_2}^{1}\frac{du}{u}\left[\frac{x_2}{u}-x_2\right]F_h^{j'}\left(\frac{x_2}{u},Q^2\right)P_{j'}^{j_2}(u)$$



- Parton 2 prior to the branching has x_2/u. After the branching it has x_2. The quantity in square brackets is simply the fractional momentum of the radiated parton 1.

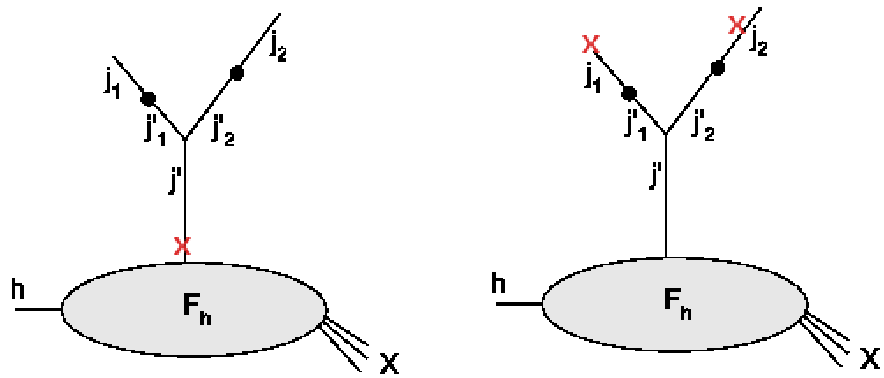
⇒ Adding all pieces together, the x_2/u-terms drop and we get DGLAP for parton 2

Double counting problem

- The inhomogeneous term is essential to preserve longitudinal momentum correlations during evolution
- However its presence induces a double counting problem with higher order SPS loop corrections $gg \to \gamma^* \gamma^*$ (Gaunt and Stirling '11)
- Analogy with photoproduction of dijets in ep collisions : separation of direct and resolved component ambigous when higher order are included (contribute to same final state)
- $d\sigma = d\sigma_{\gamma j}(\mu_\gamma, \ldots) + d\sigma_{ij}(\mu_\gamma, \ldots)$ (Frixione and Ridolfi '97)
- Dependence on arbitrary μ_γ compensates in the two terms $\to$ photon PDFs (inh evo!)
- Apply same strategy in our case: $d\sigma^{4j} = d\sigma^{SPS}(\mu_s, \ldots) + d\sigma^{DPS}(\mu_s, \ldots) + \ldots$

Double counting problem (2)

- We may temptatively identify μ_s with the relative transverse momentum of the parton pair: varying μ_s one reshuffles collinear terms from DPS to SPS



- This procedure indeed works when factorising into distributions singular part of ME
- However SPS $gg \to \gamma^* \gamma^*$ loop diagram is finite! No collinear subtraction needed..
- Open question : can we still adopt this strategy? More like a scheme change?

Summary

- New (qualitative) proposal for DPS searches in the UE associated with Z boson at low p_t^Z : N_{ch} vs p_t
- Invitation to reconsider double parton distribution in a TMD framework
- splitting term in the DPD framework : virtues and vices
  - it is essential to preserve longitudinal correlation via momentum sum rule
  - it poses double counting problem with SPS, singular behaviour as $b \to 0$ at CS level
- A proposal for the solution of the doubling counting problem has been given in analogy with a similar situation encountered in dijet photoproduction