Kinematical constraint and non-linear effects and the CCFM equation

Michal Deák
IFIC/CSIC, Universitat de Valencia

in collaboration with Krzysztof Kutak

Origin of the kinematical constraint

- The kinematical constraint is assumed to hold in derivation of the BFKL equation
  - *t*-channel gluon momentum $$k^2 \simeq -\mathbf{k}^2$$ (next slide)

Kwiecinski, Martin, Sutton Z.Phys. C71 (1996) 585-594

- In the final result it is omitted because:
  - "should" cause subleading/minor effects
  - makes things complicated for analytical studies

- How big is the effect of enforcement of the kinematical constraint?

Origin of the kinematical constraint

- Kinematical constraint motivation and derivation

$$k^2 \simeq -\mathbf{k}^2$$ (BFKL derivation)

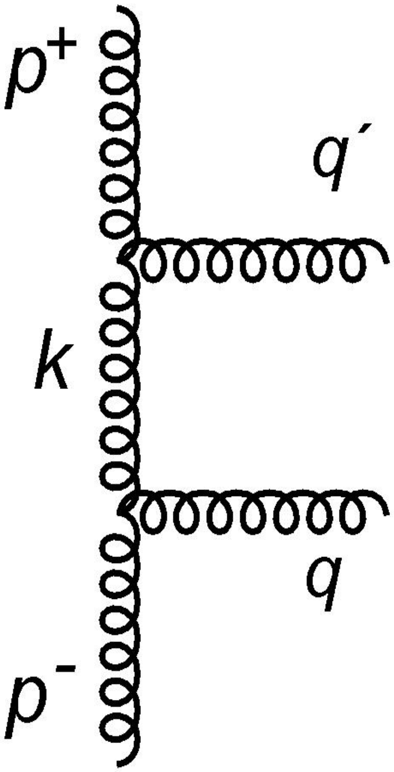
$$k = z\,p^+ + \bar{z}p^- + k_\perp$$ (Sudakov decomposition)

$$k^2 = -z\,\bar{z}\,\hat{s} - \mathbf{k}^2$$

$$q^2 = \bar{z}(1-z)\hat{s} - \mathbf{q}^2 = 0$$

$$\mathbf{k}^2 > \frac{z\,\mathbf{q}^2}{1-z}$$

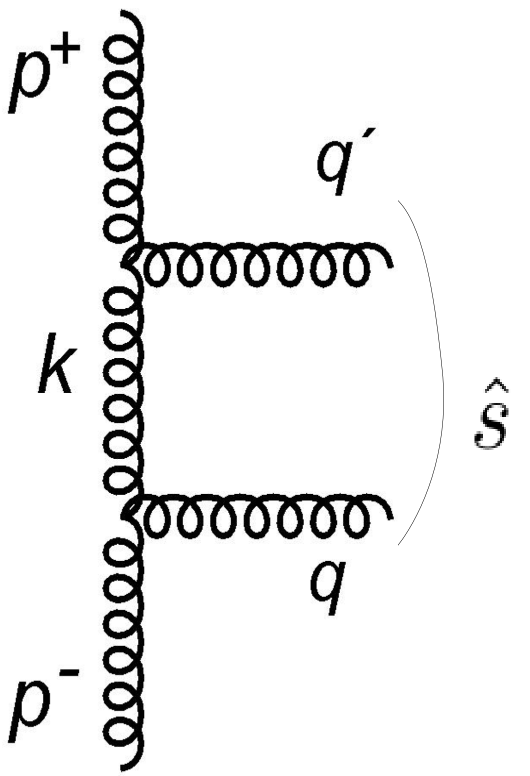
$$\boxed{\mathbf{k}^2 > z\,\mathbf{q}^2}$$



Origin of the kinematical constraint

- Kinematical constraint motivation and derivation

$$\mathbf{k}^2 > z\,\mathbf{q}^2$$
$$z > x$$
$$\mathbf{q}^2 < \mathbf{k}^2/x \simeq \hat{s}$$



- Local “energy conservation” condition

Kinematical constraint in the BFKL equation

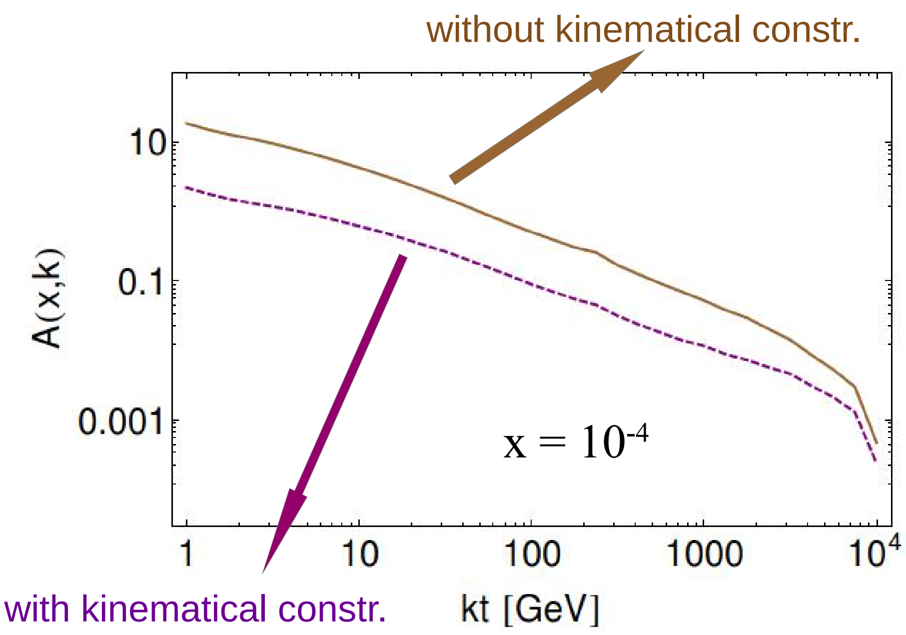
- The equation

$$\phi(x,\mathbf{k}^2) = \tilde{\phi}_0(\mathbf{k}^2) + \bar{\alpha}_S \int_x^1 \frac{dz}{z} \Delta_R(z,\mathbf{k}^2,\mu) \Big[\int \frac{d^2\mathbf{q}}{\pi \mathbf{q}^2} \theta(\mathbf{q}^2 - \mu^2) \phi(x/z, |\mathbf{k}+\mathbf{q}|^2) \Big)$$

Regge form factor:

$$\Delta_R(z,\mathbf{k}^2) = \exp\left(-\bar{\alpha}_s \ln 1/z \ln \mathbf{k}^2/\mu^2\right)$$

enforcing $\mathbf{k}^2 > z\,\mathbf{q}^2$



The CCFM equation - linear

- The equation
- Angular ordering
- Kinematical constraint

Ciafaloni, Nucl. Phys. B296 (1988) 49
Catani, Fioranni, MarchesiniPhys. Lett. B234 (1990) 339
Marchesini, Nucl. Phys. B336 (1990) 18

H. Jung, G. P. Salam, Eur. Phys. J. C19 (2001) 351–360
H. Jung, Comput. Phys. Commun. 143
(2002) 100–111

$$\mathcal{E}(x,\mathbf{k},p) = \mathcal{E}_0(\mathbf{k}) + \bar{\alpha}_S \int \frac{d^2\bar{\mathbf{q}}}{\bar{\mathbf{q}}^2} \int_x^{1-\frac{Q_0}{|\bar{\mathbf{q}}|}} dz\, \theta\left(\frac{\mathbf{k}^2}{(1-z)\bar{\mathbf{q}}^2} - z\right) \mathcal{E}(x/z,\mathbf{k}',|\bar{\mathbf{q}}|)$$
$$\times\ \theta(p - z|\bar{\mathbf{q}}|)\, \mathcal{P}(z,\mathbf{k},\mathbf{q})\, \Delta_S(p, z|\bar{\mathbf{q}}|, Q_0)\ ,$$

With the splitting function:

$$\mathcal{P}(z,\mathbf{k},\mathbf{q}) \quad = \quad \frac{1}{1-z} + \Delta_{NS}(z,\mathbf{k}^2,|\mathbf{q}|)\,\frac{1}{z}$$

Sudakov:

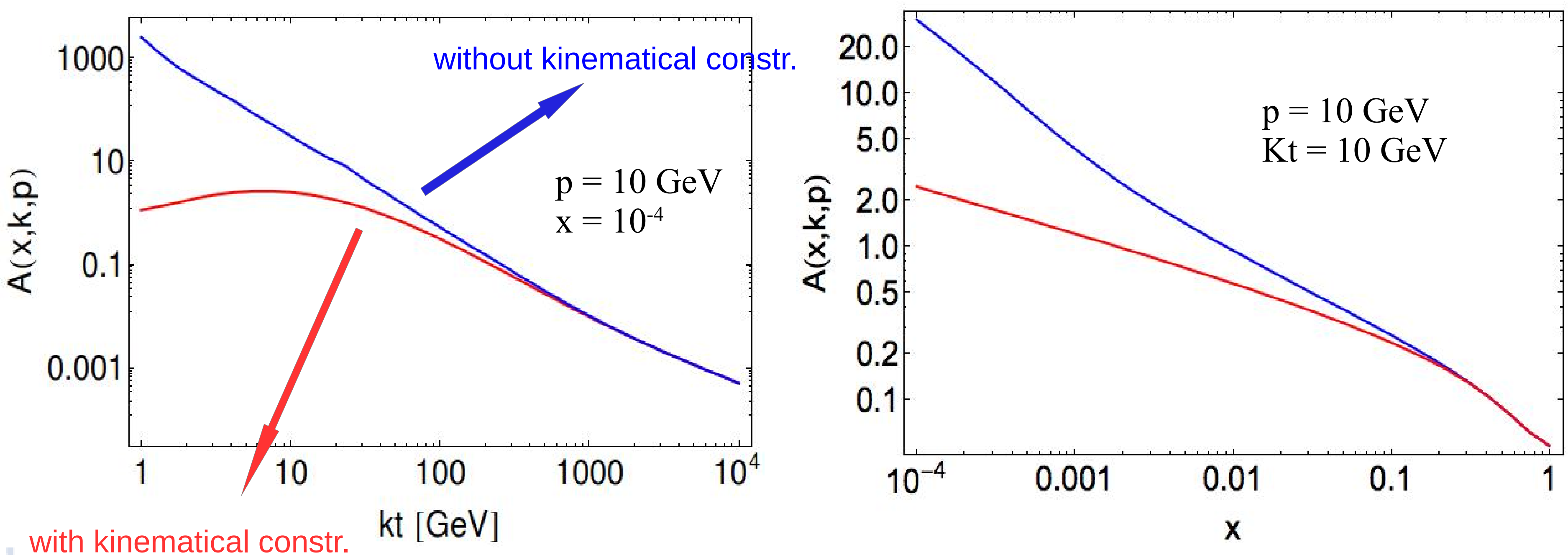
$$\Delta_S(p, z|\bar{\mathbf{q}}|, Q_0) = \exp\left(- \int_{(z\bar{\mathbf{q}})^2}^{p^2} \frac{d^2\mathbf{q}'}{\pi\mathbf{q}'^2} \int_0^{1-\frac{Q_0}{|\mathbf{q}'|}} dz' \frac{\bar{\alpha}_S}{1-z'} \right)$$

Non-Sudakov:

$$\Delta_{NS}(z,\mathbf{k},\mathbf{q},\mu^2) \quad = \quad \exp\left\{ -\bar{\alpha}_S \int_z^1 \frac{dz'}{z'}\, \Theta\left(\frac{(1-z')\mathbf{k}^2}{\mathbf{q}^2} - z'\right) \int \frac{d\mathbf{q}'^2}{\mathbf{q}'^2}\, \Theta(\mathbf{k}^2 - \mathbf{q}'^2)\, \Theta(\mathbf{q}'^2 - \mu^2) \right\}$$

The CCFM equation - linear

$$\mathcal{E}(x,\mathbf{k},p) = \mathcal{E}_0(\mathbf{k}) + \bar{\alpha}_S \int \frac{d^2\bar{\mathbf{q}}}{\bar{\mathbf{q}}^2} \int_x^{1-\frac{Q_0}{|\bar{\mathbf{q}}|}} dz\, \theta\left(\frac{\mathbf{k}^2}{(1-z)\bar{\mathbf{q}}^2} - z\right) \mathcal{E}(x/z,\mathbf{k}',|\bar{\mathbf{q}}|)$$
$$\times\ \theta(p - z|\bar{\mathbf{q}}|)\, \mathcal{P}(z,\mathbf{k},\mathbf{q})\, \Delta_S(p, z|\bar{\mathbf{q}}|, Q_0)\ ,$$



The CCFM equation - simplified

- The (1-z) pole removed
- The Sudakov formfactor integrated out

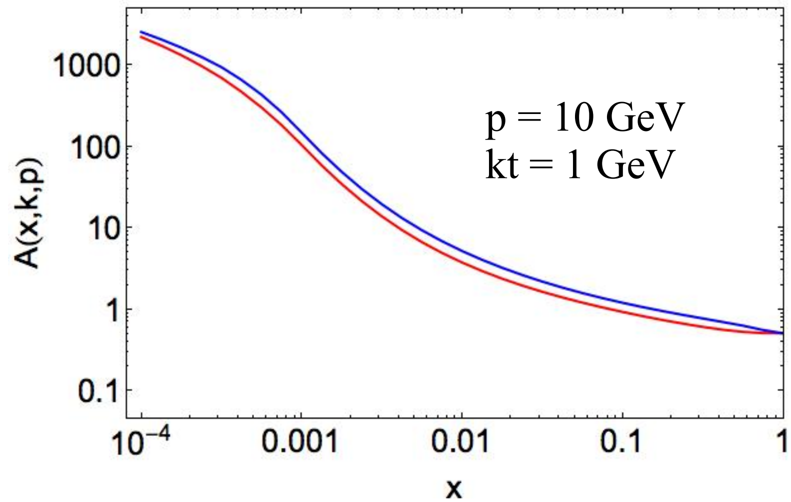
$$\mathcal{E}(x, \mathbf{k}, p) = \mathcal{E}_0(\mathbf{k}) + \bar{\alpha}_S \int \frac{d^2\mathbf{q}}{\mathbf{q}^2} \int_x^1 \frac{dz}{z} \, \mathcal{E}(x/z, \mathbf{k}', |\mathbf{q}|)$$
$$\times \; \theta(p - z|\mathbf{q}|) \, \Delta_{NS}(z, \mathbf{k}^2, |\mathbf{q}|) \; .$$

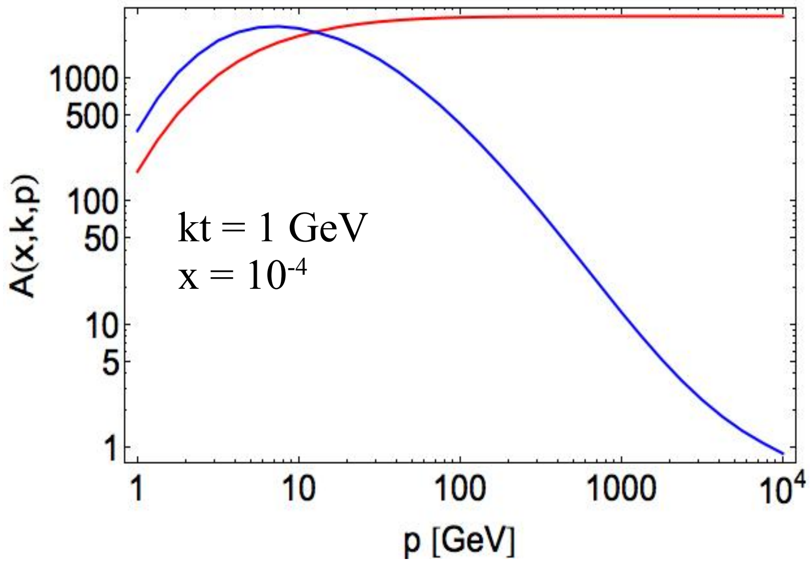
- Introduced to study low-x/angular ordering effects in the CCFM
- Main difference: growth → plateau for large p

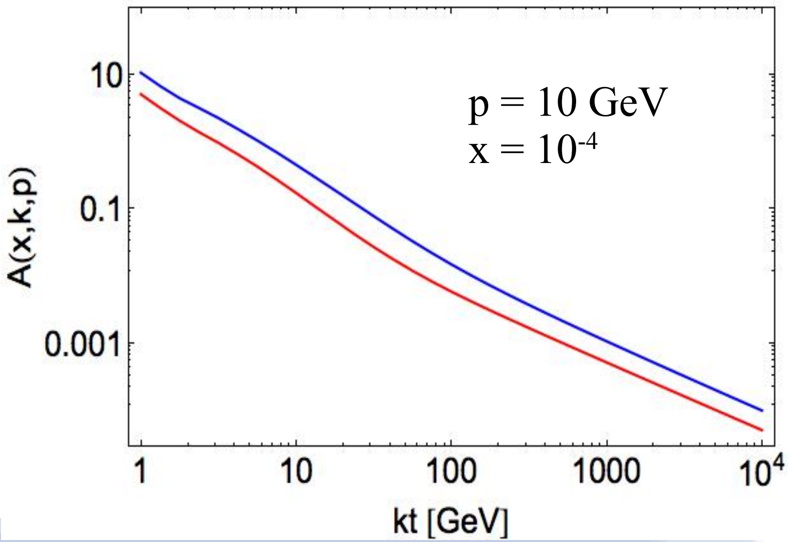
CCFM & simplified CCFM compared

$$\mathcal{E}(x,\mathbf{k},p) = \mathcal{E}_0(\mathbf{k}) + \bar{\alpha}_S \int \frac{d^2\mathbf{q}}{\mathbf{q}^2} \int_x^1 \frac{dz}{z}\, \mathcal{E}(x/z,\mathbf{k}',|\mathbf{q}|)$$
$$\times\ \theta(p - z|\mathbf{q}|)\, \Delta_{NS}(z,\mathbf{k}^2,|\mathbf{q}|)\ .$$

full CCFM   simplified



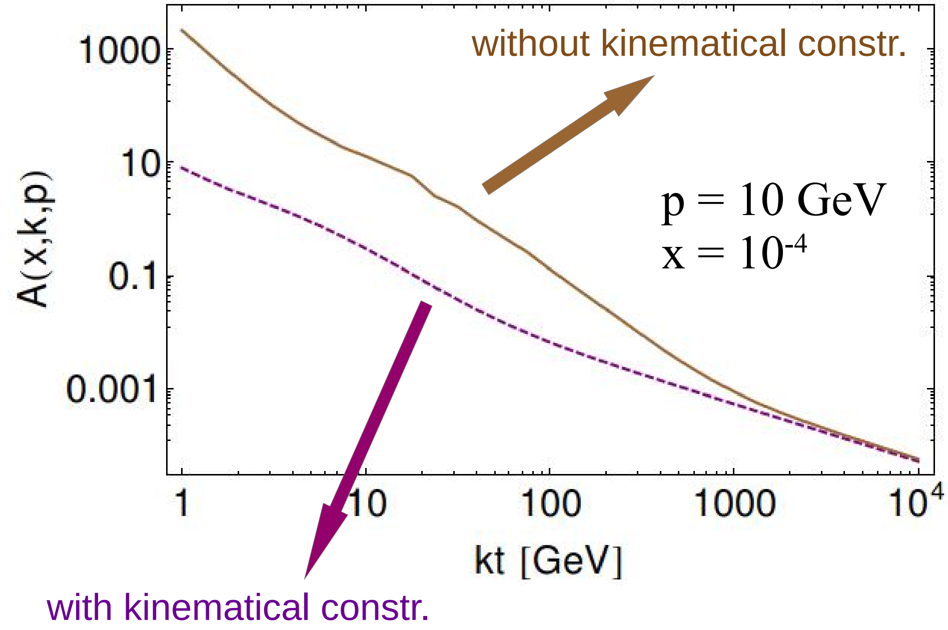




CCFM & simplified kinematical constraint

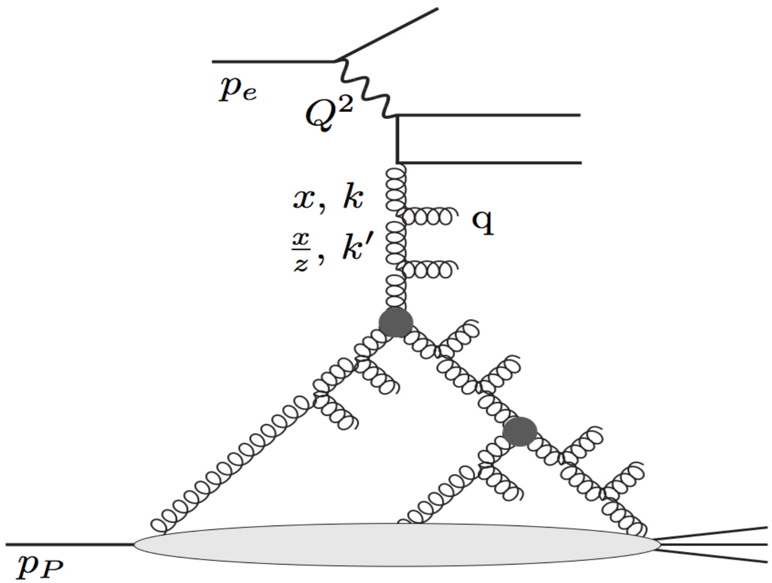
$$\mathcal{E}(x, \mathbf{k}, p) = \mathcal{E}_0(\mathbf{k}) + \bar{\alpha}_S \int \frac{d^2\mathbf{q}}{\mathbf{q}^2} \int_x^1 \frac{dz}{z}\, \mathcal{E}(x/z, \mathbf{k}', |\mathbf{q}|)$$

$$\times \;\; \theta(p - z|\mathbf{q}|)\, \Delta_{NS}(z, \mathbf{k}^2, |\mathbf{q}|)\ .$$



The non-linear CCFM equation – KGBJS equation

- The equation
- Angular ordering
- Kinematical constraint



$$\mathcal{E}(x,\mathbf{k},p) = \mathcal{E}_0(\mathbf{k}) + \bar{\alpha}_S \int \frac{d^2\bar{\mathbf{q}}}{\bar{\mathbf{q}}^2} \int_x^{1-\frac{Q_0}{|\bar{\mathbf{q}}|}} dz\ \theta(p - z|\bar{\mathbf{q}}|)\ \mathcal{P}(z,\mathbf{k},\mathbf{q})\ \Delta_S(p, z|\bar{\mathbf{q}}|, Q_0)$$

$$\times \left(\mathcal{E}(x/z, \mathbf{k}', |\bar{\mathbf{q}}|) - \frac{1}{\pi R^2} \delta\left(\bar{\mathbf{q}}^2 - \frac{\mathbf{k}^2}{(1-z)^2}\right) \bar{\mathbf{q}}^2\ \mathcal{E}^2(x/z, \bar{\mathbf{q}}, |\bar{\mathbf{q}}|) \right)$$

K. Kutak et al., JHEP 1202 (2012) 117, arXiv:1111.6928
M. D., JHEP 1307 (2013) 087, arXiv:1209.6092
K. Kutak, D. Toton: JHEP 1311 (2013) 082, arxiv:1306.3369

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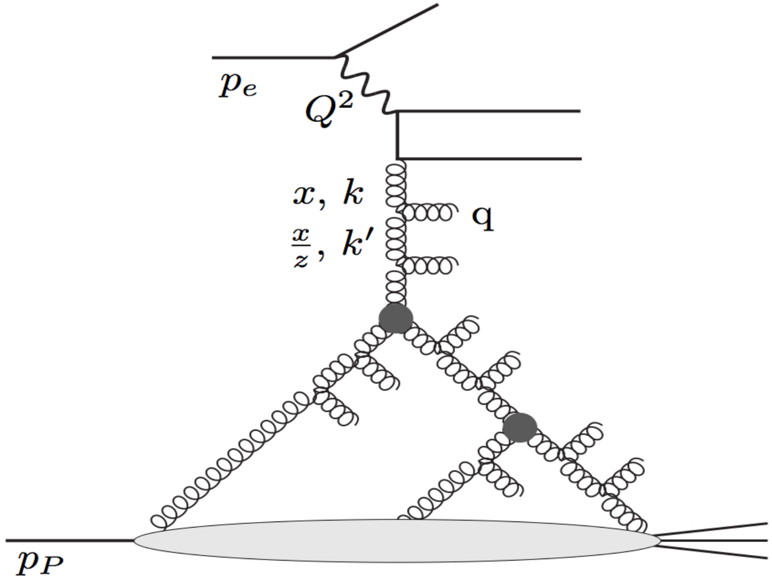


$$\mathcal{E}(x,\mathbf{k},p) = \mathcal{E}_0(\mathbf{k}) + \bar{\alpha}_S \int \frac{d^2\bar{\mathbf{q}}}{\bar{\mathbf{q}}^2} \int_x^{1-\frac{Q_0}{|\bar{\mathbf{q}}|}} dz\, \theta(p - z|\bar{\mathbf{q}}|)\, \mathcal{P}(z,\mathbf{k},\mathbf{q})\, \Delta_S(p, z|\bar{\mathbf{q}}|, Q_0)$$

$$\times \left(\mathcal{E}(x/z, \mathbf{k}', |\bar{\mathbf{q}}|) - \boxed{\frac{1}{\pi R^2} \delta\left(\bar{\mathbf{q}}^2 - \frac{\mathbf{k}^2}{(1-z)^2} \right) \bar{\mathbf{q}}^2\, \mathcal{E}^2(x/z, \bar{\mathbf{q}}, |\bar{\mathbf{q}}|)} \right)$$

related to the hadronic size
connection to soft MPI

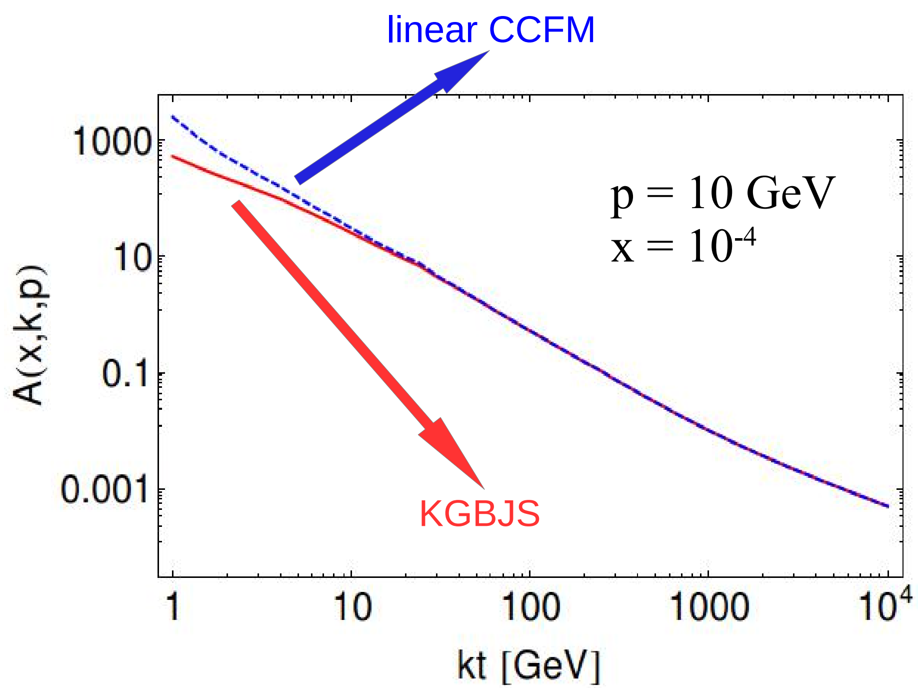
K. Kutak et al., JHEP 1202 (2012) 117, arXiv:1111.6928
M. D., JHEP 1307 (2013) 087, arXiv:1209.6092
K. Kutak, D. Toton: JHEP 1311 (2013) 082, arxiv:1306.3369

The CCFM equation – non-linear KGBJS

$$\mathcal{E}(x,\mathbf{k},p) = \mathcal{E}_0(\mathbf{k}) + \bar{\alpha}_S \int \frac{d^2\bar{\mathbf{q}}}{\bar{\mathbf{q}}^2} \int_x^{1-\frac{Q_0}{|\bar{\mathbf{q}}|}} dz\, \theta(p - z|\bar{\mathbf{q}}|)\, \mathcal{P}(z,\mathbf{k},\mathbf{q})\, \Delta_S(p, z|\bar{\mathbf{q}}|, Q_0)$$

$$\times \left(\mathcal{E}(x/z, \mathbf{k}', |\bar{\mathbf{q}}|) - \frac{1}{\pi R^2} \delta\left(\bar{\mathbf{q}}^2 - \frac{\mathbf{k}^2}{(1-z)^2}\right) \bar{\mathbf{q}}^2\, \mathcal{E}^2(x/z, \bar{\mathbf{q}}, |\bar{\mathbf{q}}|) \right)$$

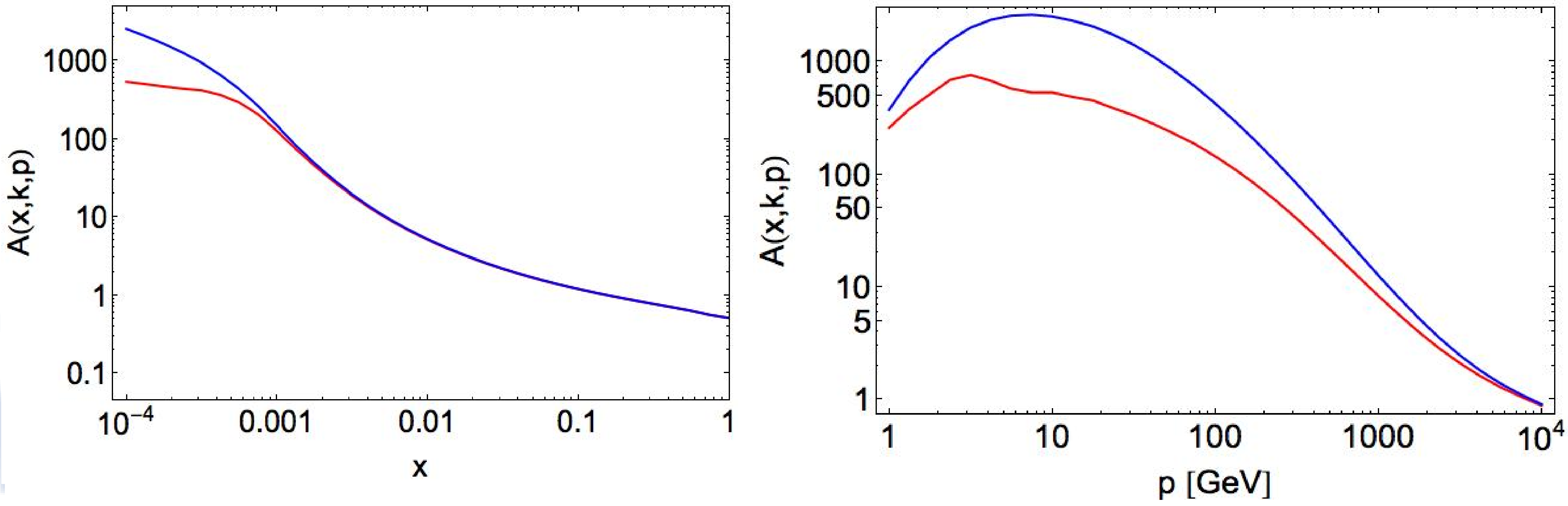
Suppression for small transverse Momentum – large amplitudes



The CCFM equation – non-linear KGBJS

$$\mathcal{E}(x,\mathbf{k},p) = \mathcal{E}_0(\mathbf{k}) + \bar{\alpha}_S \int \frac{d^2\bar{\mathbf{q}}}{\bar{\mathbf{q}}^2} \int_x^{1-\frac{Q_0}{|\bar{\mathbf{q}}|}} dz\, \theta(p - z|\bar{\mathbf{q}}|)\, \mathcal{P}(z,\mathbf{k},\mathbf{q})\, \Delta_S(p, z|\bar{\mathbf{q}}|, Q_0)$$

$$\times \left(\mathcal{E}(x/z, \mathbf{k}', |\bar{\mathbf{q}}|) - \frac{1}{\pi R^2} \delta\left(\bar{\mathbf{q}}^2 - \frac{\mathbf{k}^2}{(1-z)^2}\right) \bar{\mathbf{q}}^2\, \mathcal{E}^2(x/z, \bar{\mathbf{q}}, |\bar{\mathbf{q}}|) \right)$$



Summary

- The kinematical constraint
  - is required by consistency of derivation of the BFKL equation
  - locally induces energy conservation condition
- Numerical results show that
  - represents a big correction – has a big effect on the solution of given evolution equation
- Suppression of the amplitude at low transversal momentum by the non-linear term in the CCFM equation

Prospects:

- Fit to F_2 data