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Towards correlations in double parton distributions

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PARTON MODEL

Elastic scattering : electron — proton

——–> proton (hadron) is **NOT point-like**

Deep inelastic scattering : electron — proton

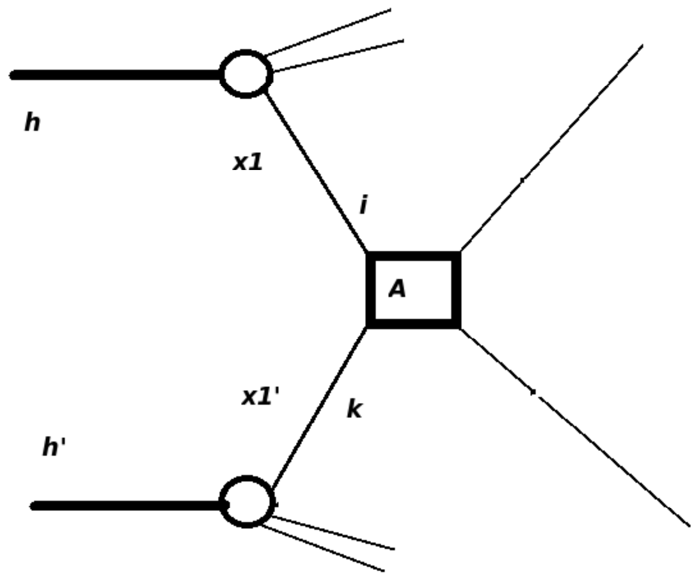
——–> proton (hadron) consists of **point-like particles-partons**

Cross section (hadron) = Σ cross section (parton) $\times$ weights

Weights — probabilities in the system of infinite momentum

(Bjorken, Feynman)

IN QCD weights depend on Q of hard processes
(SCALING VIOLATION, improved PM)



$$\sigma^{A}_{\mathrm{SPS}} = \sum_{i,k} \int D^{i}_{h}(x_1;Q^2_1)\hat{\sigma}^{A}_{ik}(x_1,x^{'}_1)D^{k}_{h'}(x^{'}_1;Q^2_1)dx_1dx^{'}_1$$

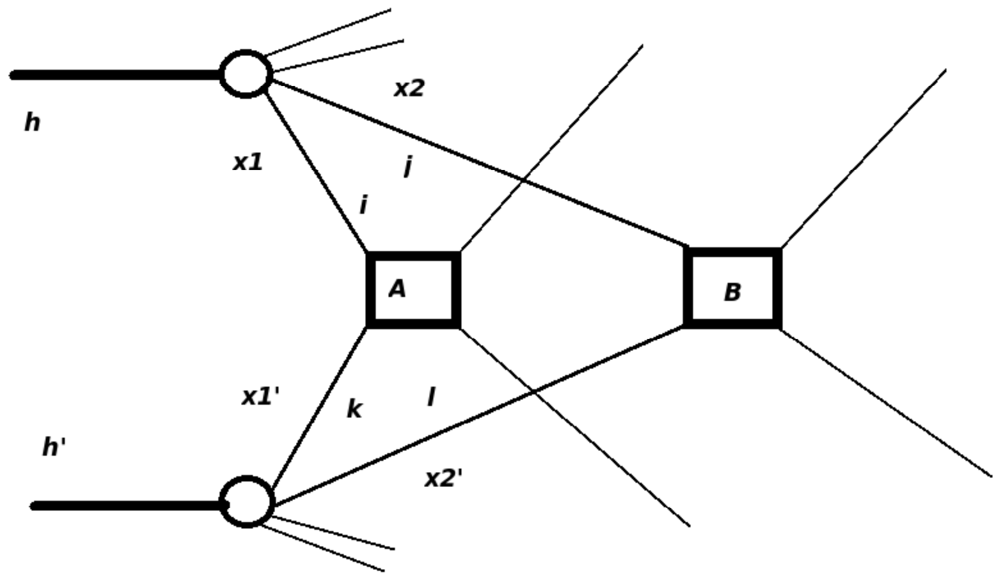
Scaling violation (dependence on Q) from
DGLAP (*Dokshitzer-Gribov-Lipatov-Altarelli-Parisi*) equations:

$$\frac{dD_i^j(x,t)}{dt} = \sum_{j'} \int_x^1 \frac{dx'}{x'} D_i^{j'}(x',t) P_{j' \to j}\left(\frac{x}{x'}\right)$$

$$t = \frac{1}{2\pi b} \ln\left[1 + \frac{g^2(\mu^2)}{4\pi} b \ln\left(\frac{Q^2}{\mu^2}\right)\right] = \frac{1}{2\pi b} \ln\left[\frac{\ln\left(\frac{Q^2}{\Lambda_{QCD}^2}\right)}{\ln\left(\frac{\mu^2}{\Lambda_{QCD}^2}\right)}\right], \quad b = \frac{33 - 2n_f}{12\pi},$$

where $g(\mu^2)$ is the running coupling constant at the reference scale μ^2,
n_f is the number of active flavours,
Λ_{QCD} is the dimensional QCD parameter.

It is possible (BUT very rarely): hard double parton scattering (subprocesses A and B)



The inclusive cross section of a double parton scattering process in a hadron collision is written in the following form (with only the assumption of factorization of the two hard parton subprocesses A and B)
(Paver, Treleani,..., Blok,...., Diehl,...).

$$\sigma^{AB}_{DPS} = \frac{m}{2} \sum_{i,j,k,l} \int \Gamma_{ij}(x_1, x_2; \mathrm{b}_1, \mathrm{b}_2; Q_1^2, Q_2^2) \hat{\sigma}^A_{ik}(x_1, x_1^{'}, Q_1^2) \hat{\sigma}^B_{jl}(x_2, x_2^{'}, Q_2^2)$$

$$\times \Gamma_{kl}(x_1^{'}, x_2^{'}; \mathrm{b}_1 - \mathrm{b}, \mathrm{b}_2 - \mathrm{b}; Q_1^2, Q_2^2) dx_1 dx_2 dx_1^{'} dx_2^{'} d^2b_1 d^2b_2 d^2b,$$

where $\mathbf{b}$ is the impact parameter — the distance between centers of colliding (e.g., the beam and the target) hadrons in transverse plane.

$\Gamma_{ij}(x_1, x_2; \mathbf{b_1}, \mathbf{b_2}; Q_1^2, Q_2^2)$ are the double parton distribution functions, which depend on the longitudinal momentum fractions x_1 and x_2, and on the transverse position $\mathbf{b_1}$ and $\mathbf{b_2}$ of the two parton undergoing hard processes A and B at the scales Q_1 and Q_2.

$\hat{\sigma}^A_{ik}$ and $\hat{\sigma}^B_{jl}$ are the parton-level subprocess cross sections.

The factor $m/2$ appears due to the symmetry of the expression for interchanging parton species i and j. $m = 1$ if $A = B$, and $m = 2$ otherwise.

The double parton distribution functions $\Gamma_{ij}(x_1, x_2; \mathbf{b_1}, \mathbf{b_2}; Q_1^2, Q_2^2)$ are the main object of interest as concerns multiple parton interactions. In fact, these distributions contain all the information when probing the hadron in two different points simultaneously, through the hard processes A and B.

It is typically assumed that the double parton distribution functions may be decomposed in terms of longitudinal and transverse components as follows:

$$\Gamma_{ij}(x_1, x_2; \mathrm{b}_1, \mathrm{b}_2; Q_1^2, Q_2^2) = D_h^{ij}(x_1, x_2; Q_1^2, Q_2^2) f(\mathrm{b}_1) f(\mathrm{b}_2),$$

where $f(\mathbf{b_1})$ is supposed to be a universal function for all kinds of partons with the fixed normalization

$$\int f(\mathrm{b}_1) f(\mathrm{b}_1 - \mathrm{b}) d^2 b_1 d^2 b = \int T(\mathrm{b}) d^2 b = 1,$$

and

$$T(\mathbf{b}) = \int f(\mathbf{b_1}) f(\mathbf{b_1} - \mathbf{b}) d^2 b_1$$

is the overlap function (not calculated in pQCD).

If one makes the further assumption that the longitudinal components $D_h^{ij}(x_1, x_2; Q_1^2, Q_2^2)$ reduce to the product of two independent one parton distributions,

$$D_h^{ij}(x_1, x_2; Q_1^2, Q_2^2) = D_h^i(x_1; Q_1^2) D_h^j(x_2; Q_2^2),$$

the cross section of double parton scattering can be expressed in the simple form

$$\sigma_{\rm DPS}^{\rm AB} = \frac{m}{2} \frac{\sigma_{\rm SPS}^A \sigma_{\rm SPS}^B}{\sigma_{\rm eff}},$$

$$\pi R_{\rm eff}^2 = \sigma_{\rm eff} = [\int d^2b (T({\rm b}))^2]^{-1}$$

is the effective interaction transverse area (effective cross section). $R_{\rm eff}$ is an estimate of the size of the hadron.

The momentum (*instead of the mixed (momentum and coordinate)*) representation is more convenient sometimes:

$$\sigma^{AB}_{DPS} = \frac{m}{2} \sum_{i,j,k,l} \int \Gamma_{ij}(x_1, x_2; \mathrm{q}; Q_1^2, Q_2^2) \hat{\sigma}^A_{ik}(x_1, x_1^{'}) \hat{\sigma}^B_{jl}(x_2, x_2^{'})$$

$$\times \Gamma_{kl}(x_1^{'}, x_2^{'}; -\mathrm{q}; Q_1^2, Q_2^2) dx_1 dx_2 dx_1^{'} dx_2^{'} \frac{d^2q}{(2\pi)^2}.$$

Here the transverse vector q is equal to the difference of the momenta of partons from the wave function of the colliding hadrons in the amplitude and the amplitude conjugated. Such dependence arises because the diffe - rence of parton transverse momenta within the parton pair is not conserved.

The main problems are

* **to make the correct calculation of the two-parton functions** $\Gamma_{ij}(x_1, x_2; \mathbf{q}; Q_1^2, Q_2^2)$ **WITHOUT simplifying factorization assumptions** (*which are not sufficiently justified and should be revised: (Blok, Dokshitzer, Frankfurt, Strikman; Diehl, Schafer; Gaunt, Stirling; Ryskin, Snigirev;...)*)

* **to find (observe) longitudinal momentum parton correlations and deviation from the factorization form of DPS cross section.**

These functions are available in the current literature only for $\mathbf{q} = 0$ **in the collinear approximation. In this approximation the two-parton distribution functions,**$\Gamma_{ij}(x_1, x_2; \mathbf{q} = 0; Q^2, Q^2) = D_h^{ij}(x_1, x_2; Q^2, Q^2)$ **with the two hard scales set equal, satisfy the generalized DGLAP evolution equations** *(Kirshner; Shelest, Snigirev, Zinovjev)*.

$$\frac{dD_i^{j_1 j_2}(x_1, x_2, t)}{dt} = \sum_{j_1'} \int_{x_1}^{1-x_2} \frac{dx_1'}{x_1'} D_i^{j_1' j_2}(x_1', x_2, t) P_{j_1' \to j_1}\left(\frac{x_1}{x_1'}\right)$$

$$+ \sum_{j_2'} \int_{x_2}^{1-x_1} \frac{dx_2'}{x_2'} D_i^{j_1 j_2'}(x_1, x_2', t) P_{j_2' \to j_2}\left(\frac{x_2}{x_2'}\right)$$

$$+ \sum_{j'} D_i^{j'}(x_1 + x_2, t) \frac{1}{x_1 + x_2} P_{j' \to j_1 j_2}\left(\frac{x_1}{x_1 + x_2}\right)$$

The solutions of the generalized DGLAP evolution equations with the given initial conditions at the reference scales $\mu^2(t=0)$ may be written in the form:

$$D_h^{j_1j_2}(x_1,x_2,t) \ = \ D_{h1}^{j_1j_2}(x_1,x_2,t) \ + D_{h(QCD)}^{j_1j_2}(x_1,x_2,t),$$

where

$$D_{h1}^{j_1j_2}(x_1,x_2,t) =$$

$$\sum_{{j_1}'{j_2}'}\int\limits_{x_1}^{1-x_2}\frac{dz_1}{z_1}\int\limits_{x_2}^{1-z_1}\frac{dz_2}{z_2}\ D_h^{{j_1}'{j_2}'}(z_1,z_2,0)D_{{j_1}'}^{j_1}(\frac{x_1}{z_1},t)D_{{j_2}'}^{j_2}(\frac{x_2}{z_2},t)\ ,$$

$$D_{h(QCD)}^{j_1j_2}(x_1,x_2,t) =$$

$$\sum_{j'{j_1}'{j_2}'}\int\limits_{0}^{t}dt'\int\limits_{x_1}^{1-x_2}\frac{dz_1}{z_1}\int\limits_{x_2}^{1-z_1}\frac{dz_2}{z_2}\ D_h^{j'}(z_1+z_2,t')$$

$$\frac{1}{z_1+z_2}P_{j'\to{j_1}'{j_2}'}(\frac{z_1}{z_1+z_2})D_{{j_1}'}^{j_1}(\frac{x_1}{z_1},t-t')D_{{j_2}'}^{j_2}(\frac{x_2}{z_2},t-t').$$

The first term is the solution of homogeneous evolution equation (independent evolution of two branches), where the input two-parton distribution is generally NOT known at the low scale $\mu(t=0)$. For this non-perturbative two-parton function at low z_1, z_2 one may assume the factorization $D_h^{j_1' j_2'}(z_1, z_2, 0) \simeq D_h^{j_1'}(z_1, 0) D_h^{j_2'}(z_2, 0)$ neglecting the constraints due to momentum conservation ($z_1 + z_2 < 1$).

This leads to

$$\boldsymbol{D_{h1}^{ij}(x_1, x_2, t) \simeq D_h^i(x_1, t) D_h^j(x_2, t)}$$

the factorization hypothesis usually used in current estimations.

This MAIN result shows that if the two-parton distributions are factorized at some scale μ^2, then the evolution (second term) violates this factorization *inevitably* at any different scale ($Q^2 \neq \mu^2$), apart from the violation due to the kinematic correlations induced by the momentum conservation.

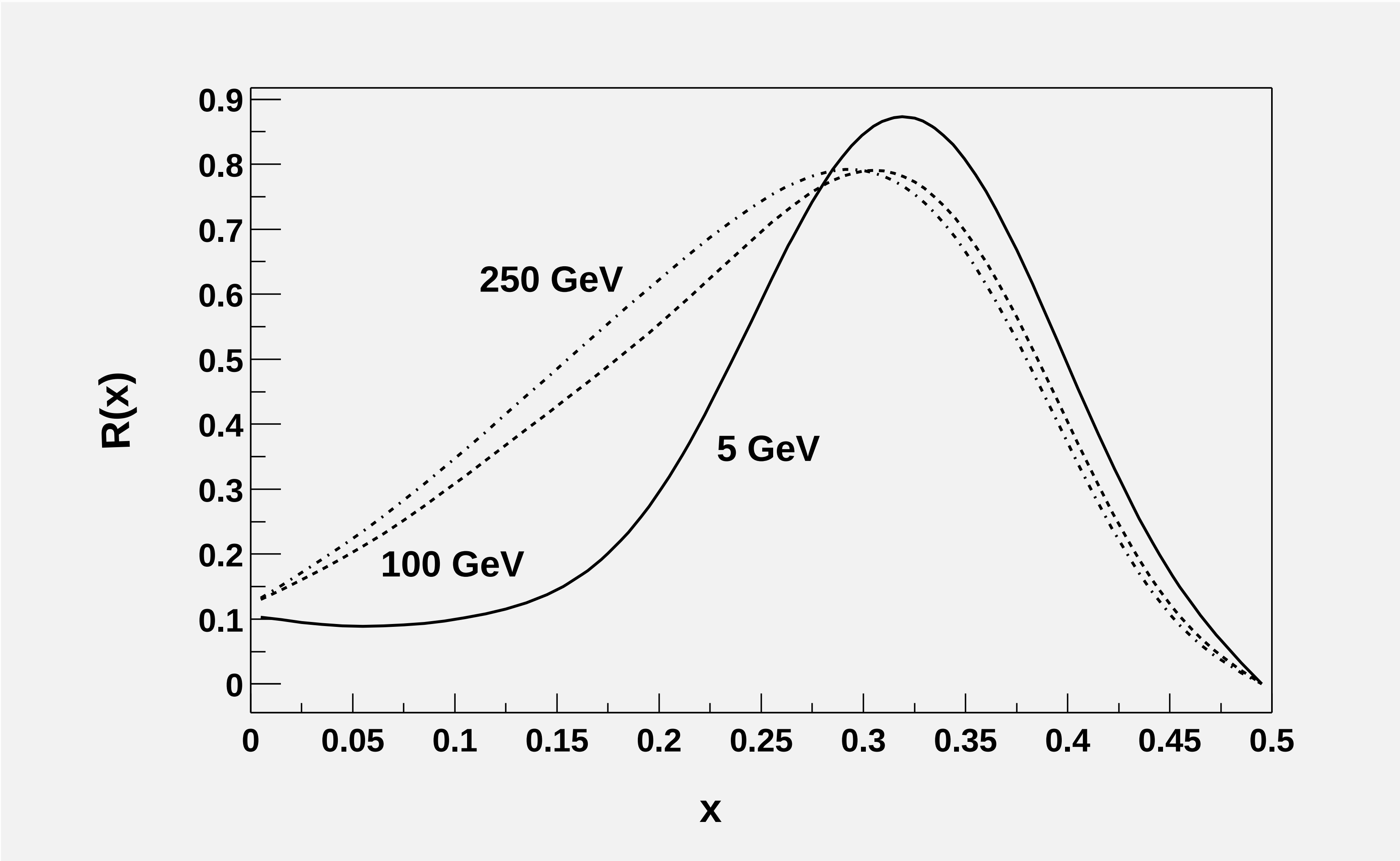
For a practical employment it is interesting to know the degree of this violation. We did *(Korotkikh, Snigirev)* it using the CTEQ fit for single distributions as an input. The nonperturbative initial conditions $D_h^j(x,0)$ are specified in a parametrized form at a fixed low-energy scale $Q_0 = \mu = 1.3$ GeV. The particular function forms and the value of Q_0 are not crucial for the CTEQ global analysis at the flexible enough parametrization, which reads

$$\boldsymbol{xD_p^j(x,0) \; = \; A_0^j x^{A_1^j}(1-x)^{A_2^j} e^{A_3^j x}(1+e^{A_4^j}x)^{A_5^j}.}$$

The independent parameters $A_0^j,\ A_1^j,\ A_2^j,\ A_3^j,\ A_4^j,\ A_5^j$ for parton flavour combinations $u_v \equiv u - \bar{u}$, $d_v \equiv d - \bar{d}$, g and $\bar{u} + \bar{d}$ are given in Appendix A of work: *J.Pamplin, et al., JHEP 0207 (2002) 012.*

The results of numerical calculations are presented in Fig. for the ratio:

$$\boldsymbol{R(x,t) \; = \; (D_{p(QCD)}^{gg}(x_1,x_2,t)/D_p^g(x_1,t)D_p^g(x_2,t)(1-x_1-x_2)^2)|_{x_1=x_2=x}.}$$



R(x)
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
250 GeV
5 GeV
100 GeV
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
x

The evolution effects are getting larger with increasing hard scales. The numerical estimations by integrating **directly** the evolution equations (*Gaunt, Stirling; Diehl, Kasemets, Keane*) confirm also this conclusion.

The particular solutions of non-homogeneous equations contribute to the inclusive cross section of DPS with a **larger weight** (different effective cross section *(Cattaruzza,Del Fabbro,Treleani; Ryskin, Snigirev; Blok, Dokshitzer, Frankfurt, Strikman; Gaunt, Stirling)*)
as compared to the solutions of homogeneous equations
(**the "traditional" factorization component**).

The latter solutions are usually approximated by a factorized form if the initial nonperturbative correlations are absent. These initial correlation conditions are *a priori* unknown yet not quite arbitrary as they obey the nontrivial sum rules which are imposed upon the evolution equations. The problem of specifying the initial correlation conditions for the evolution equations, which would obey exactly these **sum rules** and have the **correct asymptotic behavior near the kinematical boundaries**, has been extensively studied *(Gaunt, Stirling; Snigirev; Ceccopieri; Chang, Manohar, Waalewijn; Rinaldi, Scopetta,Vento; Golec-Biernat, Lewandowska)*.

Fortunately, the explicit form of evolution equation solutions allows us to answer the question: which correlations (perturbative or nonperturbative) are more significant at sufficiently large hard scale.

Indeed, the evolution equations are explicitly solved by introducing the Mellin transformations

$$M_h^j(n,t) = \int_0^1 dx x^n \, D_h^j(x,t),$$

$$M_h^{j_1 j_2}(n_1,n_2,t) = \int_0^1 dx_1 dx_2 \theta(1-x_1-x_2) x_1^{n_1} x_2^{n_2} D_h^{j_1 j_2}(x_1,x_2,t),$$

which lead to a system of ordinary linear differential equations of the first order. In order to obtain the distributions in x representation, an inverse Mellin transformation should be performed. In the general case this can be done only numerically. However, the asymptotic behavior can be estimated in some interesting and particularly simple limits using the same technique as above.

The exact solutions for single distributions in the moment representation can be written symbolically in a matrix form:

$$M_i^j(n,t) = [\exp P(n)t]_i^j,$$

and the solutions of the generalized DGLAP evolution equations with the given initial conditions may be written again as a convolution of single distributions; in the moment representation, they read

$$M_h^{j_1 j_2}(n_1,n_2,t) = \sum_{j_1' j_2'} M_h^{j_1' j_2'}(n_1,n_2,0) M_{j_1'}^{j_1}(n_1,t) M_{j_2'}^{j_2}(n_2,t)$$

$$+ M_{h(\mathrm{QCD})}^{j_1 j_2}(n_1,n_2,t),$$

where

$$M_{h(\mathrm{QCD})}^{j_1 j_2}(n_1,n_2,t) = \sum_i M_h^i(n_1+n_2,0) M_i^{j_1 j_2}(n_1,n_2,t)$$

are the particular solutions of the complete equations with zero initial conditions at the hadron level, and

$$M_i^{j_1 j_2}(n_1,n_2,t)$$

$$= \sum_{j j_1' j_2'} \int_0^t dt' M_i^j(n_1+n_2,t') P_{j\to j_1' j_2'}(n_1,n_2) M_{j_1'}^{j_1}(n_1,t-t') M_{j_2'}^{j_2}(n_2,t-t').$$

The kernels,

$$P_{j'\to j}(n) = \int_0^1 x^n P_{j'\to j}(x)dx,$$

$$P_{j'\to j_1j_2}(n_1,n_2) = \int_0^1 x^{n_1}(1-x)^{n_2} P_{j'\to j_1j_2}(x)dx,$$

are well-known and can be found in the explicit form.

Now we consider the initial condition effects in the asymptotic behavior ($t\to\infty$). In order to better understand the character of this dependence, at first we use a toy model with one type of partons (for instance, QCD theory with gluons only). In this case:

$$M_h^{11}(n_1,n_2,t) = M_h^{11}(n_1,n_2,0)\exp\{[P(n_1)+P(n_2)]t\}+$$

$$\frac{P(n_1,n_2)M_h^1(n_1+n_2,0)}{P(n_1+n_2)-P(n_1)-P(n_2)}\{\exp[P(n_1+n_2)t]-\exp[(P(n_1)+P(n_2))t]\}.$$

Thus, for t large enough, we have two different asymptotic regimes depending on the relation between the kernels $P(n_1+n_2)$ and $P(n_1)+P(n_2)$:

(1) If $P(n_1+n_2) < P(n_1)+P(n_2)$, then

$$\boldsymbol{M_h^{11}(n_1,n_2,t)|_{t\to\infty} = \Big[M_h^{11}(n_1,n_2,0)+ \frac{P(n_1,n_2)M_h^{1}(n_1+n_2,0)}{P(n_1)+P(n_2)-P(n_1+n_2)}\Big] \times \exp\{[P(n_1)+P(n_2)]t\}.}$$

(2) If $P(n_1+n_2) > P(n_1)+P(n_2)$, then

$$\boldsymbol{M_h^{11}(n_1,n_2,t)|_{t\to\infty} = \frac{P(n_1,n_2)M_h^{1}(n_1+n_2,0)}{P(n_1+n_2)-P(n_1)-P(n_2)} \times \exp[P(n_1+n_2)t].}$$

For the second regime, the asymptotic behavior *does not dependent* on the initial correlation conditions $M_h^{11}(n_1,n_2,0)$ at all, and is specified by the correlations perturbatively calculated.

The presence of several parton types does not essentially complicate the analysis of the asymptotic behavior. Indeed, in this case one has to express single parton distributions via the eigenfunctions of corresponding DGLAP equations, put them into solutions above and take the leading contributions into consideration only.

As a result, the relation between maximum eigenvalues $\Lambda(n_1 + n_2)$ and $\Lambda(n_1)+\Lambda(n_2)$ will determine the asymptotic behavior regime of the dPDFs:

(1) If $\Lambda(n_1 + n_2) < \Lambda(n_1) + \Lambda(n_2)$, then the dPDFs are dependent on the initial correlation conditions $M_h^{j_1 j_2}(n_1, n_2, 0)$.

(2) If $\Lambda(n_1 + n_2) > \Lambda(n_1) + \Lambda(n_2)$, then the dPDFs are independent of the initial correlation conditions $M_h^{j_1 j_2}(n_1, n_2, 0)$.

The eigenvalues and the eigenfunctions for the single distributions in QCD have been thoroughly studied. The results of these studies show that in QCD both asymptotic regimes are realized. Therefore, one needs to know the initial correlation conditions (which, generally speaking, are arbitrary and should be extracted from the experiment) to determine even the asymptotic behavior of the dPDFs. However, we come to the relation

$$\boldsymbol{\Lambda(n_1 + n_2) > \Lambda(n_1) + \Lambda(n_2)}$$

for large moments n_1 and n_2 that determines the dPDFs in the region of not parametrically small x_1 and x_2, because $\Lambda(n) \sim -\ln(n), n \gg 1$.

We conclude that the dPDFs "forget" the initial correlation conditions (unknown *a priori*) at not parametrically small longitudinal momentum fractions, and the correlations perturbatively calculated survive only in the limit of large enough hard scales.

Such a dominance is independent of the strength of the initial correlation conditions.