Saturation effects on Heavy Quark production in pA collisions

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What is factorization?

- QCD is the fundamental theory of strong interactions among quarks and gluons
- Experiments involve hadrons in their initial and final states, not quarks and gluons
- Hadrons cannot be described perturbatively in QCD
- Scattering amplitudes with time-like on-shell momenta cannot be computed on the lattice

▷ How can we compare theory and experiments?

▷ **Factorization :** separation of short distances (perturbative) and long distance (non perturbative)

What is factorization?

Quantum mechanical expression of a transition amplitude (exact)

$$\text{transition probability from hadrons to X} \equiv \Big|\sum \text{Amplitudes } h_1 h_2 \to X\Big|^2 .$$

Approximation made in the parton model

$$\text{transition probability from hadrons to X} \approx \sum_{\substack{\text{partons}\\ \{q,g\}}} \text{probability to find } \{q,g\} \text{ in } \{h_1,h_2\} \otimes \Big|\sum \text{Amplitudes } \{q,g\} \to X\Big|^2$$

- Some quantum interferences are neglected. Is this legitimate?
- When we compute loop corrections, where should they go?
- Is this *factorization* possible to all orders?

What is factorization?

- Some loop corrections in $\mathcal{O}_{\text{partons}}$ are enhanced by large logarithms, e.g.

$$\alpha_s \ln\left(\frac{M^2}{m_H^2}\right) \quad , \qquad \alpha_s \ln\left(\frac{s}{M^2}\right) \sim \alpha_s \ln\left(\frac{1}{x}\right)$$

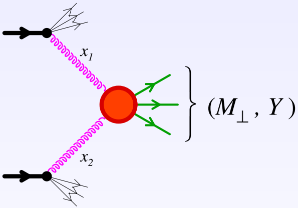
  Note : the log that occurs depends on the details of the kinematics
  - Bjorken limit: $s, M^2 \to +\infty$ with s/M^2 fixed
  - Regge limit: $s \to +\infty$, M^2 fixed

- These logs upset a naive application of perturbation theory when $\alpha_s \ln(\cdot) \sim 1$ ▷ they must be resummed

- This resummation can be performed analytically
  - the result of the resummation is universal
  - all the leading logs can be absorbed in F
    ▷ the factorization formula remains true
    ▷ this summation dictates how F evolves with M^2 or x

What is factorization?

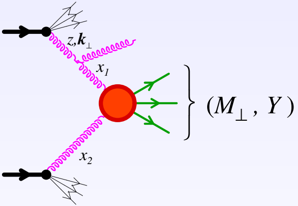
- These logarithms tell us that the relevant parton distributions depend on the resolution scales (in time and in transverse momentum) associated to a given process
- Calculation of some process at LO :



$$\begin{cases} x_1 = M_\perp \ e^{+Y}/\sqrt{s} \\ x_2 = M_\perp \ e^{-Y}/\sqrt{s} \end{cases}$$

What is factorization?

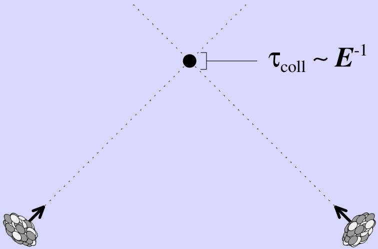
- These logarithms tell us that the relevant parton distributions depend on the resolution scales (in time and in transverse momentum) associated to a given process
- Radiation of an extra gluon :



$$\Longrightarrow \quad \alpha_s \int_{x_1} \frac{dz}{z} \int^{M_\perp} \frac{d^2\vec{\boldsymbol{k}}_\perp}{\boldsymbol{k}_\perp^2}$$

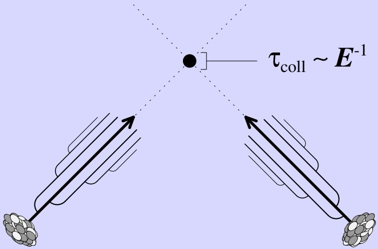
- Practical consequence : pQCD predicts not only the partonic matrix element but also the evolution $\partial_M F$ (or $\partial_x F$)
  - ▷ the only required non-perturbative input is $F(x, M_0)$ or $F(x_0, M)$

Why factorization works



- The duration of the collision is very short: $\tau_{\rm coll} \sim E^{-1}$

Why factorization works



- The duration of the collision is very short: $\tau_{\rm coll} \sim E^{-1}$
- The logarithms we need to resum arise from the radiation of soft gluons, which takes a long time
  ▷ it must happen (long) before the collision

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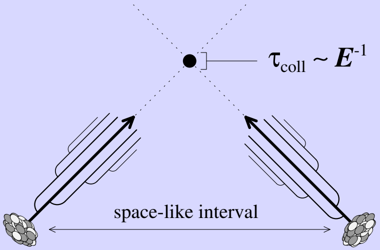


- The duration of the collision is very short: $\tau_{\rm coll} \sim E^{-1}$
- The logarithms we need to resum arise from the radiation of soft gluons, which takes a long time
  ▷ it must happen (long) before the collision
- The projectiles are not in causal contact before the impact
  ▷ the logarithms are intrinsic properties of the projectiles, independent of the measured observable

1. **Generalities on factorization**
   - What is factorization?
   - Causality
   - DGLAP
   - BFKL
   - Limitations
2. **Gluon saturation at small x**
3. **Heavy quark production**

Common forms of factorization: 1. DGLAP

- Logs of $M_\perp \Longrightarrow$ DGLAP. Important when :
  - $M_\perp \gg \Lambda_{QCD}$, while x_1, x_2 are rather large

- Cross-sections read :

$$\frac{d\sigma}{dYd^2\vec{\boldsymbol{P}}_\perp} \propto F(x_1, M_\perp^2)\, F(x_2, M_\perp^2)\, |\mathcal{M}|^2$$

  with $x_{1,2} = M_\perp \exp(\pm Y)/\sqrt{s}$

- Note : there are convolutions in x_1 and x_2 if some particles are integrated out in the final state

- The factorization of logarithms has been proven to all orders for sufficiently inclusive quantities
  (see Collins, Soper, Sterman, 1984–1985)

1 Generalities on factorization

- What is factorization?
- Causality
- DGLAP
- BFKL
- Limitations

2 Gluon saturation at small x

3 Heavy quark production

Common forms of factorization: 2. BFKL

Collins, Ellis (1991), Catani, Ciafaloni, Hautmann (1991)

- Logs of $1/x \Longrightarrow$ BFKL. Important when :
  - $M_\perp$ remains moderate, while x_1 or x_2 (or both) are small
- The BFKL equation is non-local in transverse momentum
  ▷ it applies to non-integrated gluon distributions $\varphi(x, \vec{\boldsymbol{k}}_\perp)$

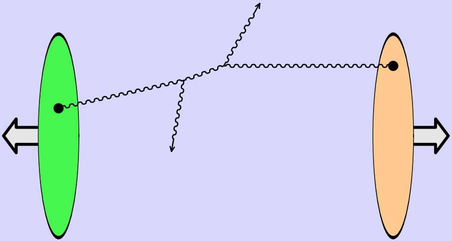
$$xG(x, Q^2) = \int^{Q^2} \frac{d^2\vec{\boldsymbol{k}}_\perp}{(2\pi)^2}\, \varphi(x, \vec{\boldsymbol{k}}_\perp)$$

  ▷ the matrix element must calculated for off-shell gluons with $\vec{\boldsymbol{k}}_\perp \neq \vec{0}$

- In this framework, cross-sections read :

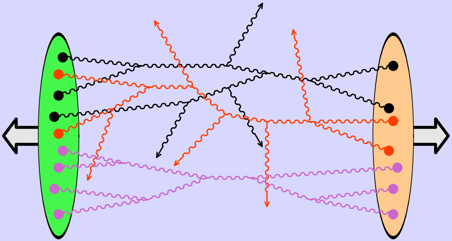
$$\frac{d\sigma}{dYd^2\vec{\boldsymbol{P}}_\perp} \propto \int_{\vec{\boldsymbol{k}}_{1\perp}, \vec{\boldsymbol{k}}_{2\perp}} \delta(\vec{\boldsymbol{k}}_{1\perp} + \vec{\boldsymbol{k}}_{2\perp} - \vec{\boldsymbol{P}}_\perp)\, \varphi_1(x_1, k_{1\perp})\, \varphi_2(x_2, k_{2\perp})\, \frac{|\mathcal{M}|^2}{k_{1\perp}^2 k_{2\perp}^2}$$

Conditions of validity



- Dilute regime : one parton in each projectile interact (what the standard PDFs are made for)

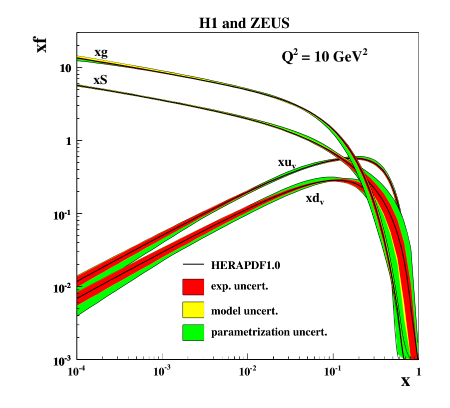
Conditions of validity



- Dilute regime : one parton in each projectile interact (what the standard PDFs are made for)
- Dense regime : multiparton processes become crucial
  - ▷ standard forms of factorization break down
  - ▷ new distributions are required

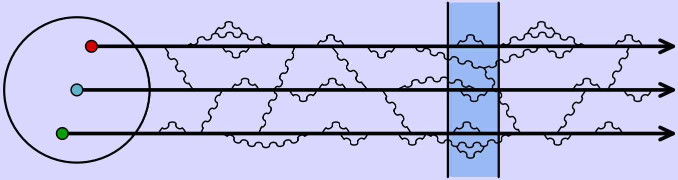
Growth of the gluon distribution at small x

Gluon distribution at small x



- Gluons dominate at any $x \leq 10^{-1}$

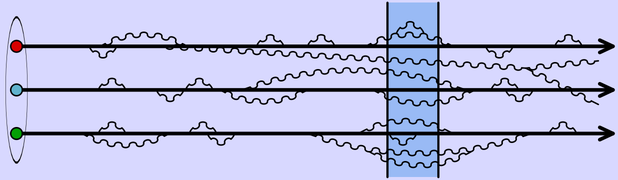
Nucleon partonic structure



At low energy:

- Fluctuations at all space-time scales smaller than its size
- Only the fluctuations that are longer lived than the external probe participate in the interaction process
- Interactions are very complicated if the constituents of the nucleon have a non trivial dynamics over time-scales comparable to those of the probe

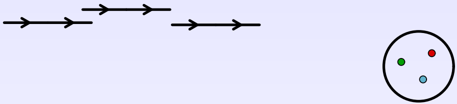
Nucleon partonic structure



At high energy:

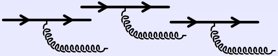
- Dilation of all internal time-scales of the nucleon
- Interactions among constituents now take place over time-scales that are longer than the characteristic time-scale of the probe
  ▷ the constituents behave as if they were free
- Many fluctuations live long enough to be seen by the probe
  ▷ the nucleon appears denser at small x
- Pre-existing fluctuations are frozen over the time-scale of the probe, and act as static sources of new partons

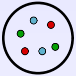
Parton evolution under boosts



▷ at low energy, only valence quarks are present in the hadron wave function

Parton evolution under boosts



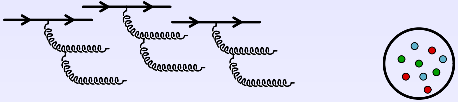


▷ when energy increases, new partons are emitted

▷ the emission probability is $\alpha_s \int \frac{dx}{x} \sim \alpha_s \ln(\frac{1}{x})$, with x the longitudinal momentum fraction of the gluon

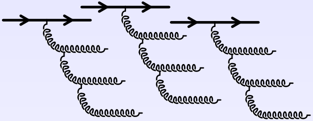
▷ at small-x (i.e. high energy), these logs need to be resummed

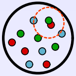
Parton evolution under boosts



▷ as long as the density of constituents remains small, the evolution is linear: the number of partons produced at a given step is proportional to the number of partons at the previous step

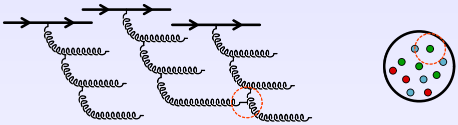
Parton evolution under boosts





▷ eventually, the partons start overlapping in phase-space

Parton evolution under boosts



▷ parton recombination becomes favorable

▷ after this point, the evolution is non-linear:

the number of partons created at a given step depends non-linearly on the number of partons present previously

Balitsky (1996), Kovchegov (1996,2000)

Jalilian-Marian, Kovner, Leonidov, Weigert (1997,1999)

Iancu, Leonidov, McLerran (2001)

- When their occupation number becomes large, gluons can recombine :

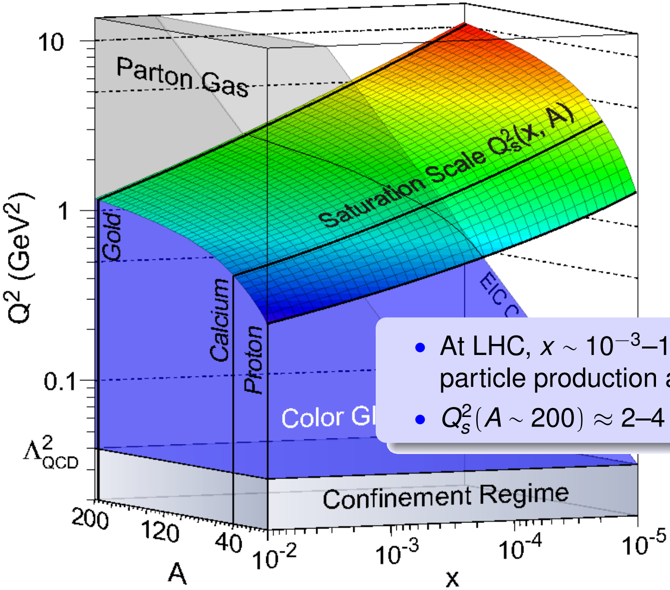
Gluon Saturation

Saturation criterion **[Gribov, Levin, Ryskin (1983)]**

$$\underbrace{\alpha_s Q^{-2}}_{\sigma_{gg\to g}} \times \underbrace{A^{-2/3} xG(x, Q^2)}_{\text{surface density}} \geq 1$$

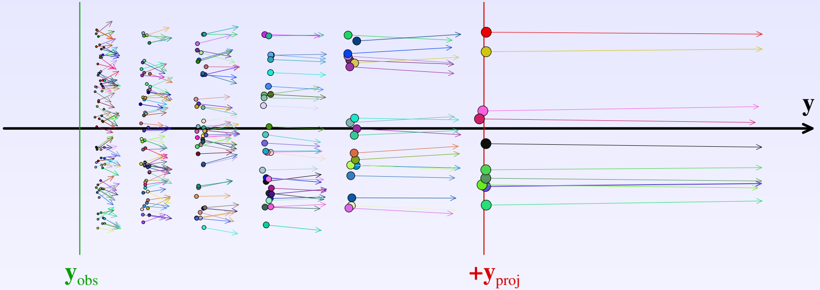
$$Q^2 \leq \underbrace{Q_s^2 \equiv \frac{\alpha_s xG(x, Q_s^2)}{A^{2/3}}}_{(\text{saturation momentum})^2} \sim A^{1/3} x^{-0.3}$$

Saturation domain



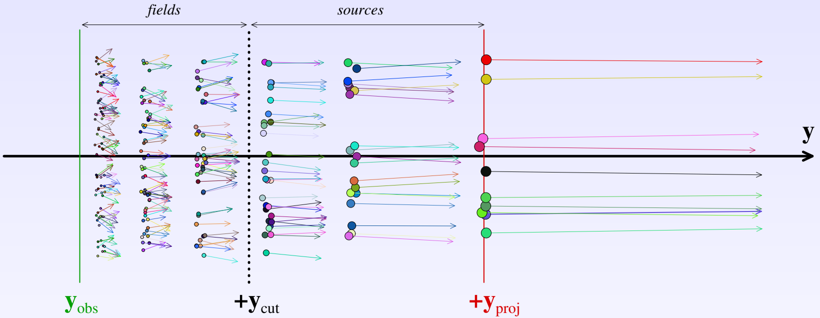
- At LHC, $x \sim 10^{-3}$–10^{-4} (for bulk particle production at mid-rapidity)
- $Q_s^2(A \sim 200) \approx 2$–$4$ GeV^2

Degrees of freedom [McLerran, Venugopalan (1994)]



- $p_\perp^2 \sim Q_s^2 \sim \Lambda_{\rm QCD}\, e^{\lambda(y_{\rm proj}-y)}$, $p_z \sim Q_s\, e^{y-y_{\rm obs}}$
- Fast partons : frozen dynamics, negligible $p_\perp$ $\Rightarrow$ classical sources
- Slow partons : evolve with time $\Rightarrow$ gauge fields

Degrees of freedom [McLerran, Venugopalan (1994)]



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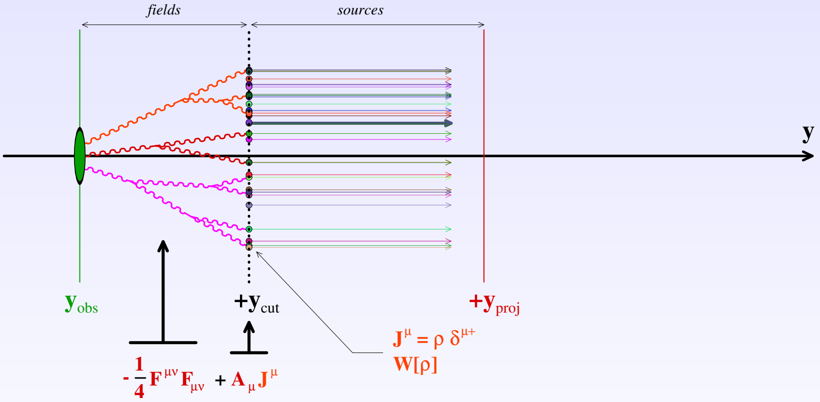

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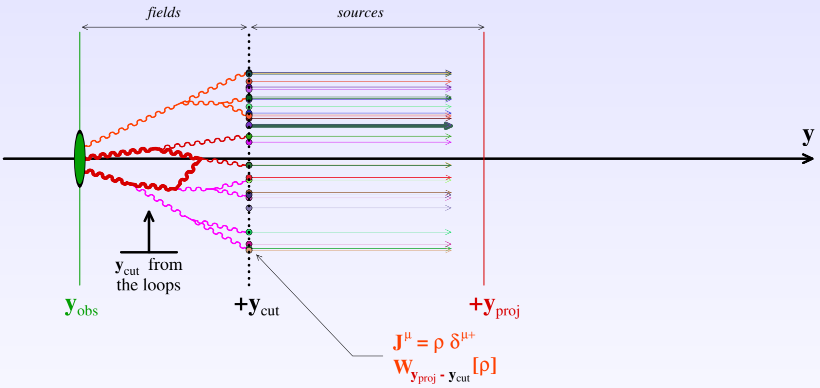
Cancellation of the cutoff dependence



- The cutoff $y_{\rm cut}$ is arbitrary and should not affect the result
- The probability density $W[\rho]$ changes with the cutoff
- Loop corrections cancel the cutoff dependence from $W[\rho]$

B-JIMWLK evolution equation

[Jalilian-Marian, Kovner, Leonidov, Weigert (1998)]
[Balitsky (1996)] [Iancu, Leonidov, McLerran (2001)]

B-JIMWLK equation at Leading Log

$$\frac{\partial W_Y[\rho]}{\partial Y} = \underbrace{\frac{1}{2}\int_{\vec{\boldsymbol{x}}_\perp,\vec{\boldsymbol{y}}_\perp} \frac{\delta}{\delta\rho_a(\vec{\boldsymbol{x}}_\perp)}\chi_{ab}(\vec{\boldsymbol{x}}_\perp,\vec{\boldsymbol{y}}_\perp)\frac{\delta}{\delta\rho_b(\vec{\boldsymbol{y}}_\perp)}}_{\mathcal{H}\quad \text{(JIMWLK Hamiltonian)}} \; W_Y[\rho]$$

- Mean field approx. (BK equation) : **[Kovchegov (1999)]**
- Langevin form of B-JIMWLK : **[Blaizot, Iancu, Weigert (2003)]**
- First numerical solution : **[Rummukainen, Weigert (2004)]**

B-JIMWLK evolution equation

[Jalilian-Marian, Kovner, Leonidov, Weigert (1998)]
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B-JIMWL

Recent developments :

Running coupling correction

[Lappi, Mäntysaari (2012)]

B-JIMWLK equation at Next to Leading Log

[Kovner, Lublinsky, Mulian (2013)]
[Caron-Huot (2013)][Balitsky, Chirilli (2013)]

- Me
- Langevin form of B-JIMWLK : [Blaizot, Iancu, Weigert (2003)]
- First numerical solution : [Rummukainen, Weigert (2004)]

Multiple scatterings

- Power counting :

$$\frac{\text{2 scatterings}}{\text{1 scattering}} \sim \frac{Q_s^2}{M_\perp^2} \quad \text{with} \quad Q_s^2 \sim \alpha_s \frac{xG(x, Q_s^2)}{\pi R^2}$$

- When this ratio becomes ~ 1, all the rescattering corrections become important
  - ▷ one must resum all $\left[Q_s / M_\perp\right]^n$
- These effects are not accounted for in DGLAP or BFKL

DGLAP and saturation

- Question 1: can one define standard PDFs in the saturated regime?

  ▷ yes, but they are insufficient, because they do not provide any information about multi-parton correlations

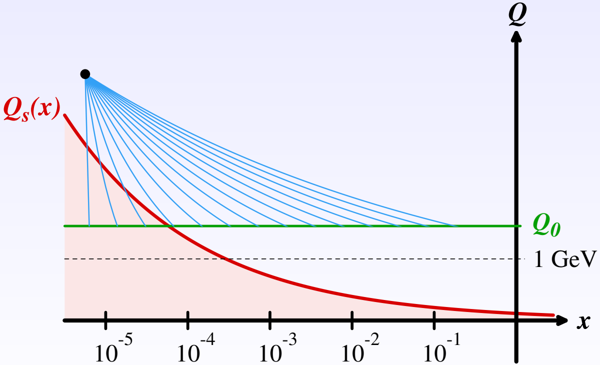
- Question 2: can one define modified PDFs that would encode these correlations?

  ▷ for a given process, maybe. But these functions would not be universal

DGLAP and saturation

- Question 3: my favorite observable sits in the dilute domain: I should be fine with the usual PDFs?

  ▷ maybe not... PDFs may have been contaminated by an improper evolution from a smaller *Q*:



Dense-dilute limit

- Factorization in the dense-dense regime:

$$\langle \mathcal{O} \rangle_{\text{LeadingLog}} = \int \left[D\rho_1\, D\rho_2\right]\; W_1\left[\rho_1\right]\; W_2\left[\rho_2\right]\; \mathcal{O}[\rho_{1,2}]$$

- When ρ_1 is a weak source (projectile 1 is dilute):

$$\mathcal{O}[\rho_{1,2}] = \int_{\vec{\boldsymbol{k}}_{1\perp}} \rho_1^2(\vec{\boldsymbol{k}}_{1\perp})\; \mathcal{O}_2[\vec{\boldsymbol{k}}_{1\perp}, \rho_2] + \mathcal{O}(\rho_1^4)$$

and $\mathcal{O}_2[\vec{\boldsymbol{k}}_{1\perp}, \rho_2]$ has a compact analytical expression

Dense-dilute limit

- For the dilute projectile, one needs only the ordinary (non-integrated) gluon distribution:

$$\int [D\rho_1]\; W_1[\rho_1]\; \rho_1^2(\vec{\boldsymbol{k}}_{1\perp}) \equiv \varphi_1(\vec{\boldsymbol{k}}_{1\perp})$$

- The expectation value of $\mathcal{O}$ can be rewritten as

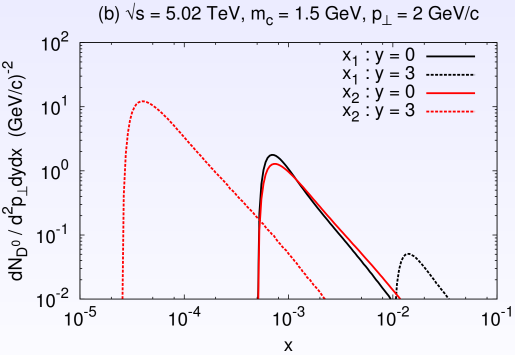
$$\left\langle \mathcal{O} \right\rangle_{\rm LLog} = \int_{\vec{\boldsymbol{k}}_{1\perp}} \varphi_1(\vec{\boldsymbol{k}}_{1\perp}) \int \big[D\rho_2\big]\; W_2\big[\rho_2\big]\; \mathcal{O}_2[\vec{\boldsymbol{k}}_{1\perp}, \rho_2]$$

- This can be further simplified by noting that $\mathcal{O}_2[\vec{\boldsymbol{k}}_{1\perp}, \rho_2]$ contains only simple correlators of Wilson lines

  ▷ one can replace the JIMWLK equation by the much simpler BK equation (mean field approximation)

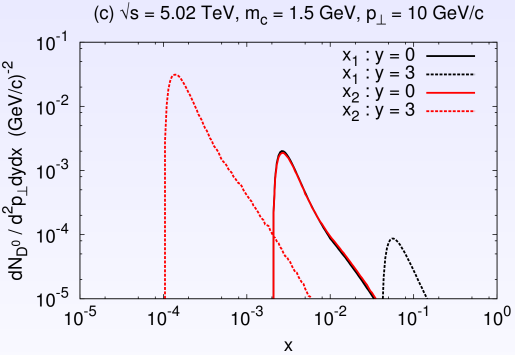
Relevant x range at the LHC

- x coverage for $c\bar{c}$ production at the LHC : $p_\perp = 2$ GeV



Relevant x range at the LHC

- x coverage for $c\bar{c}$ production at the LHC : $p_\perp = 10$ GeV



▷ very small values of x reached in one of the projectiles when produced forward

Classical color field

Blaizot, FG, Venugopalan (2004)

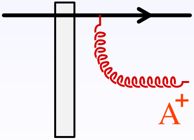
- ρ_p is a weak source, ρ_A is a strong source
  $\Rightarrow$ we want the pair production amplitude to first order in ρ_p and to all orders in ρ_A
- Finding the color field : Yang-Mills equations :

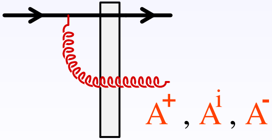
$$[D_\mu, F^{\mu\nu}] = J^\nu \quad , \qquad [D_\nu, J^\nu] = 0$$

$$J^\nu|_{\text{lowest order}} = \delta^{\nu+}\delta(x^-)\rho_p(\boldsymbol{x}_\perp) + \delta^{\nu-}\delta(x^+)\rho_A(\boldsymbol{x}_\perp)$$

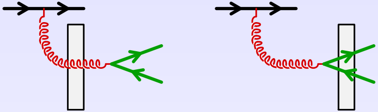
$$\partial_\mu A^\mu = 0 \qquad \text{(Lorenz gauge)}$$

- (Very sketchy) diagrammatic interpretation:





QQbar production amplitude



- Total amplitude:

$$\mathcal{M}_F = g^2 \int\limits_{\vec{\boldsymbol{k}}_{1\perp},\vec{\boldsymbol{k}}_\perp} \frac{\rho_{p,a}(\vec{\boldsymbol{k}}_{1\perp})}{k_{1\perp}^2} \int\limits_{\vec{\boldsymbol{x}}_\perp,\vec{\boldsymbol{y}}_\perp} e^{i\vec{\boldsymbol{k}}_\perp\cdot\vec{\boldsymbol{x}}_\perp} e^{i(\vec{\boldsymbol{p}}_\perp+\vec{\boldsymbol{q}}_\perp-\vec{\boldsymbol{k}}_\perp-\vec{\boldsymbol{k}}_{1\perp})\cdot\vec{\boldsymbol{y}}_\perp}$$

$$\times\ \overline{u}(\vec{\boldsymbol{q}})\Big\{\ [t^b U_{ba}(\vec{\boldsymbol{x}}_\perp)]\not{L}\ +\ [\widetilde{U}(\vec{\boldsymbol{x}}_\perp)t^a\widetilde{U}^\dagger(\vec{\boldsymbol{y}}_\perp)]\,T_{q\bar{q}}(\vec{\boldsymbol{k}}_\perp)\ \Big\}v(\vec{\boldsymbol{p}})$$

with L Lipatov's effective vertex, and

$$T_{q\bar{q}}(\vec{\boldsymbol{k}}_\perp) \equiv \frac{\gamma^+(\not{q}-\not{k}+m)\gamma^-(\not{q}-\not{k}-\not{k}_1+m)\gamma^+}{2p^+[(\vec{\boldsymbol{q}}_\perp-\vec{\boldsymbol{k}}_\perp)^2+m^2]+2q^+[(\vec{\boldsymbol{q}}_\perp-\vec{\boldsymbol{k}}_\perp-\vec{\boldsymbol{k}}_{1\perp})^2+m^2]}$$

- **Notes:** $\widetilde{U}$ = Wilson line for a quark U_{ba} = Wilson line for a gluon

Heavy quark cross-section in pA collisions

Pair production cross-section:

$$\frac{d\sigma_{q\bar{q}}}{d^2\vec{\boldsymbol{p}}_\perp d^2\vec{\boldsymbol{q}}_\perp dy_p dy_q} = \frac{\alpha_s^2 N}{8\pi^4 d_A} \int_{\vec{\boldsymbol{k}}_{1\perp},\vec{\boldsymbol{k}}_{2\perp}} \frac{\delta(\vec{\boldsymbol{p}}_\perp + \vec{\boldsymbol{q}}_\perp - \vec{\boldsymbol{k}}_{1\perp} - \vec{\boldsymbol{k}}_{2\perp})}{\boldsymbol{k}_{1\perp}^2 \boldsymbol{k}_{2\perp}^2}$$

$$\times \Big\{ \int_{\vec{\boldsymbol{k}}_\perp,\vec{\boldsymbol{k}}'_\perp} \mathrm{tr}\Big[(\not{q}+m)\, T_{q\bar{q}}(\vec{\boldsymbol{k}}_\perp)(\not{p}-m)\, T^*_{q\bar{q}}(\vec{\boldsymbol{k}}'_\perp)\Big]\, \phi_A^{(4)}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp, \vec{\boldsymbol{k}}'_\perp)$$

$$+ \int_{\vec{\boldsymbol{k}}_\perp} \mathrm{tr}\Big[(\not{q}+m)\, T_{q\bar{q}}(\vec{\boldsymbol{k}}_\perp)(\not{p}-m)\not{L}^* + \mathrm{h.c.}\Big]\, \phi_A^{(3)}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp)$$

$$+\mathrm{tr}\Big[(\not{q}+m)\not{L}(\not{p}-m)\not{L}^*\Big]\, \phi_A^{(2)}(\vec{\boldsymbol{k}}_{2\perp}) \Big\}\, \varphi_1(\vec{\boldsymbol{k}}_{1\perp})$$

▷ standard factorization broken for the nucleus: one needs three different "distributions" in order to describe the target

Target correlators

- Target "gluon distributions":

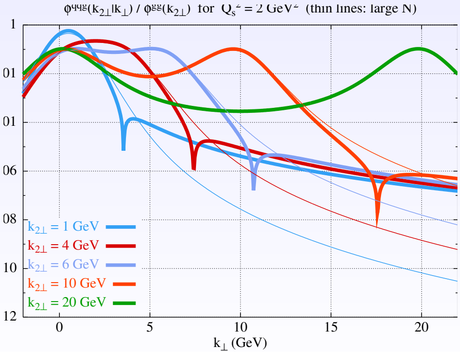
$$\phi_A^{(2)}(\vec{\boldsymbol{k}}_{2\perp}) \propto \int_{\vec{\boldsymbol{x}}_\perp,\vec{\boldsymbol{y}}_\perp} e^{i\vec{\boldsymbol{k}}_{2\perp}\cdot(\vec{\boldsymbol{x}}_\perp-\vec{\boldsymbol{y}}_\perp)}\ \mathrm{tr}\left\langle U(\vec{\boldsymbol{x}}_\perp)U^\dagger(\vec{\boldsymbol{y}}_\perp)\right\rangle$$

$$\phi_A^{(3)}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp) \propto \int_{\vec{\boldsymbol{x}}_\perp,\vec{\boldsymbol{y}}_\perp,\vec{\boldsymbol{z}}_\perp} e^{i\left[\vec{\boldsymbol{k}}_\perp\cdot\vec{\boldsymbol{x}}_\perp+(\vec{\boldsymbol{k}}_{2\perp}-\vec{\boldsymbol{k}}_\perp)\cdot\vec{\boldsymbol{y}}_\perp-\vec{\boldsymbol{k}}_{2\perp}\cdot\vec{\boldsymbol{z}}_\perp\right]} \times\ \mathrm{tr}\left\langle \widetilde{U}(\vec{\boldsymbol{x}}_\perp)t^a\widetilde{U}^\dagger(\vec{\boldsymbol{y}}_\perp)t^b U_{ba}(\vec{\boldsymbol{z}}_\perp)\right\rangle$$

$$\phi_A^{(4)}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp,\vec{\boldsymbol{k}}'_\perp) \propto \int_{\vec{\boldsymbol{x}}_\perp,\vec{\boldsymbol{y}}_\perp,\vec{\boldsymbol{x}}'_\perp,\vec{\boldsymbol{y}}'_\perp} e^{i\left[\vec{\boldsymbol{k}}_\perp\cdot\vec{\boldsymbol{x}}_\perp-\vec{\boldsymbol{k}}'_\perp\cdot\vec{\boldsymbol{x}}'_\perp+(\vec{\boldsymbol{k}}_{2\perp}-\vec{\boldsymbol{k}}_\perp)\cdot\vec{\boldsymbol{y}}_\perp-(\vec{\boldsymbol{k}}_{2\perp}-\vec{\boldsymbol{k}}'_\perp)\cdot\vec{\boldsymbol{y}}'_\perp\right]} \times\ \mathrm{tr}\left\langle \widetilde{U}(\vec{\boldsymbol{x}}_\perp)t^a\widetilde{U}^\dagger(\vec{\boldsymbol{y}}_\perp)\widetilde{U}(\vec{\boldsymbol{y}}'_\perp)t^a\widetilde{U}(\vec{\boldsymbol{x}}'_\perp)\right\rangle$$

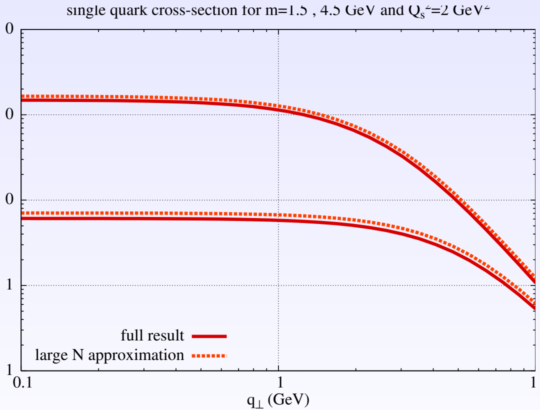
Large Nc limit

- Large *N* approximation :
  - The evaluation of the exact 3-point function is extremely time consuming
  - The 3-point function becomes a product of two 2-point functions in the large *N* limit ▷ much faster numerical evaluation



Large Nc limit

- Quark cross-section : exact vs. large N



▷ From now on, use the large N approximation in order to speed up the computations

Limit of Kt factorization

Fujii, FG, Venugopalan (2005)

- The quark cross-section factorizes if the the 3-point and 2-point functions are related by:

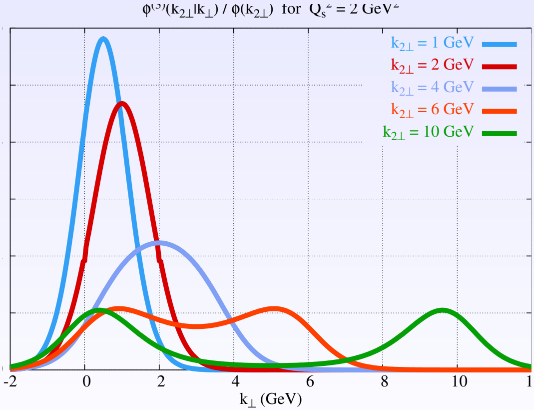
$$\phi_A^{(3)}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp) = (2\pi)^2 \frac{1}{2} \left[\delta(\vec{\boldsymbol{k}}_\perp) + \delta(\vec{\boldsymbol{k}}_\perp - \vec{\boldsymbol{k}}_{2\perp})\right] \phi_A^{(2)}(\vec{\boldsymbol{k}}_{2\perp})$$

- This relation would be satisfied if the $Q\overline{Q}$ pair interacts with the target in such a way that all the momentum exchanged goes to the quark or to the antiquark

- The ratio $\phi_A^{(3)}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp)/\phi_A(\vec{\boldsymbol{k}}_{2\perp})$ should be close to the sum of two delta functions for factorization to be approximately valid

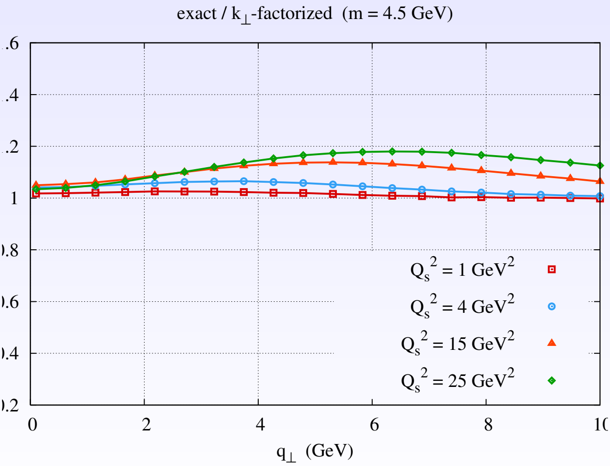
3-point correlator

- 3-point function/2-point function (in the MV model):



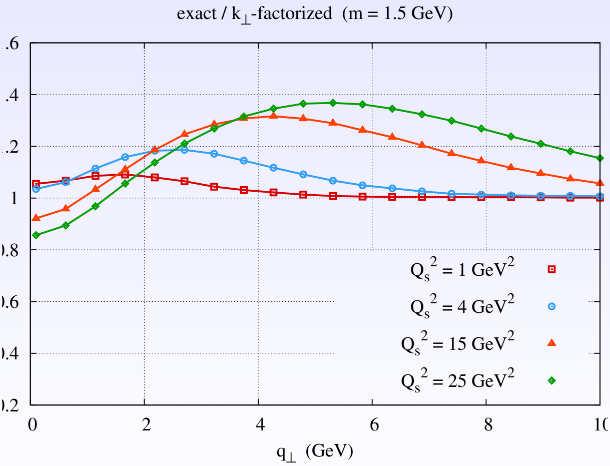
Factorization violation for b quarks

- Single *b*-quark cross-section :



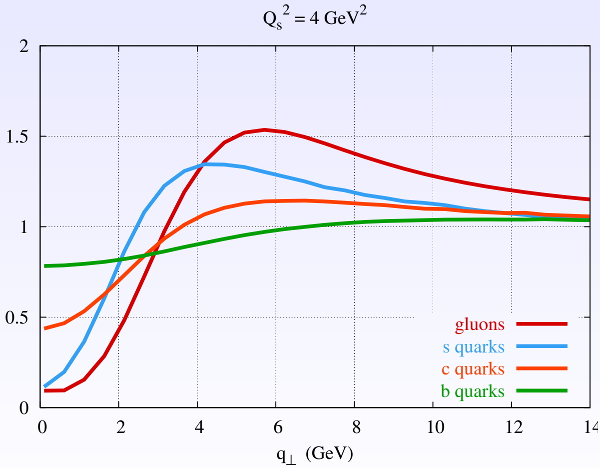
Factorization violation for c quarks

- Single c-quark cross-section :



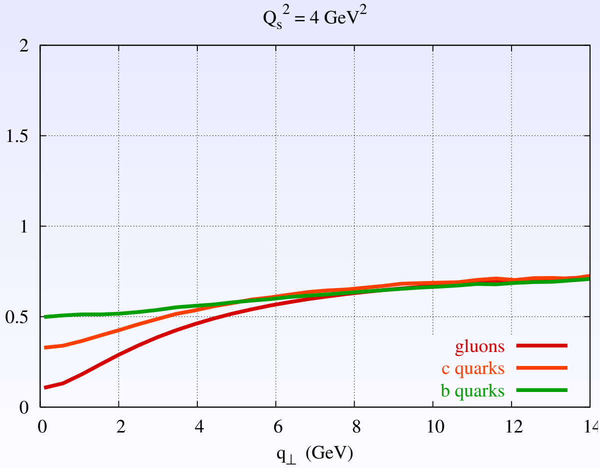
Cronin effect for quark production

- MV model ($Q_s^2 = 4\ \mathrm{GeV}^2$)



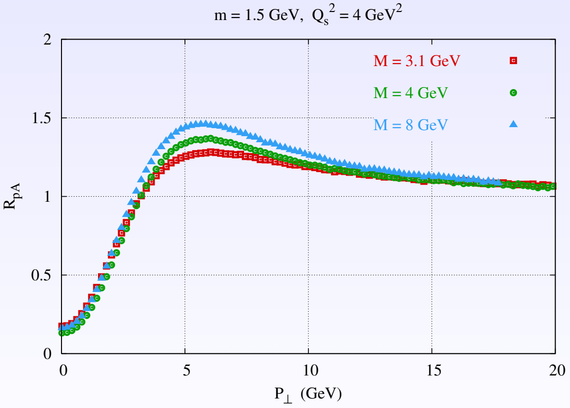
Cronin effect for quark production

- Non-local gaussian model that mimics BK evolution



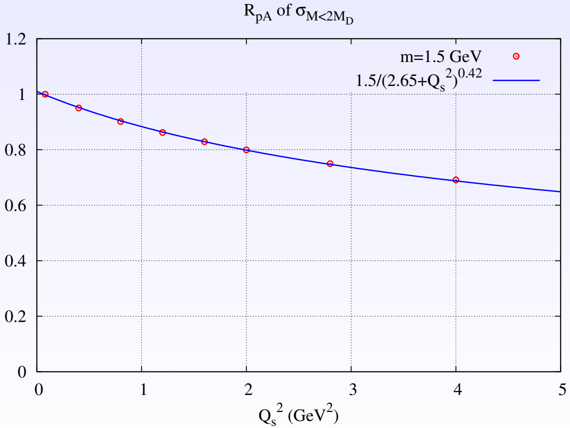
Cronin effect for pair production

- MV model $(m = 1.5 \text{ GeV}, Q_s^2 = 4 \text{ GeV}^2)$



JPsi in the Color Evaporation Model

$$\frac{dN_{J/\psi}}{dYd^2\vec{P}_\perp} = F_{J/\psi}\int_{4m_Q^2}^{4m_D^2} dM^2 \, \frac{dN_{Q\overline{Q}}}{dM^2dYd^2\vec{P}_\perp}$$

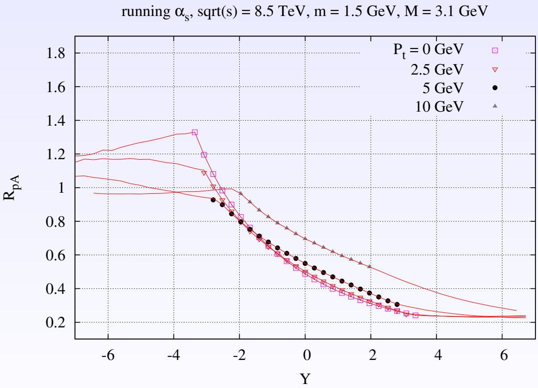


Rapidity dependence

- No rapidity dependence in the MV model
- The Y dependence comes in via the JIMWLK evolution, or in an approximated way via the Balitsky-Kovchegov equation
- Take the MV model with $Q_s^2 = 2$ GeV^2 as the initial condition at $x_0 = 0.01$
- Compute $\left\langle U(0)U^\dagger(\vec{\boldsymbol{x}}_\perp)\right\rangle_Y$ by solving the BK equation
- For the 3- and 4-point functions, use the fact that in the large N_c limit they factorize into products of 2-point functions

Rapidity dependence

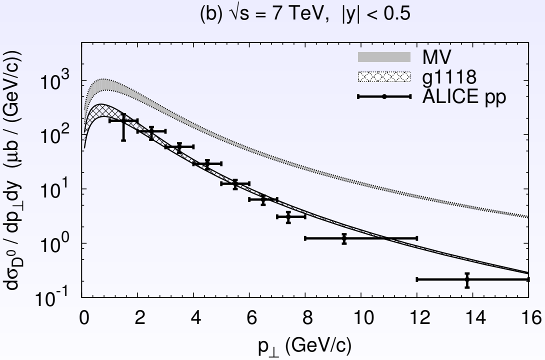
- R_{pA} for pairs at the LHC : $\sqrt{s_{NN}} = 8.5$ TeV



More realistic computations

Fujii, Watanabe (2013)

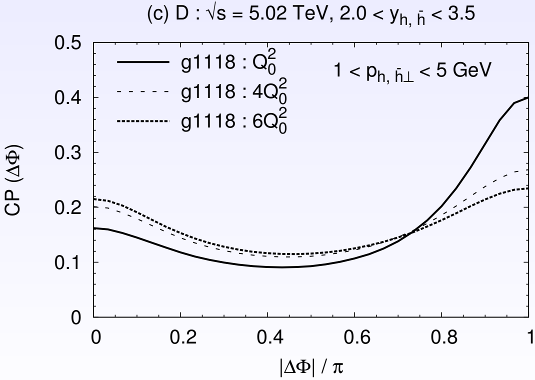
- D_0 meson yield in pp collisions @ LHC :



More realistic computations

Fujii, Watanabe (2013)

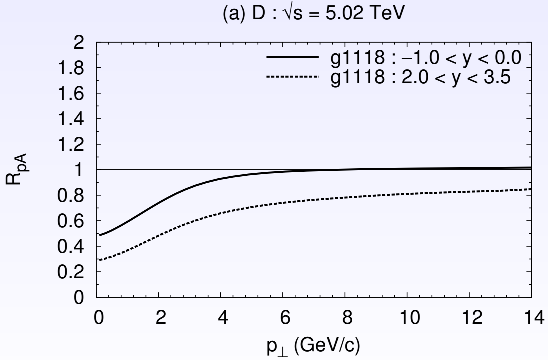
- $D_0 - \overline{D_0}$ azimuthal correlation in pA :



More realistic computations

Fujii, Watanabe (2013)

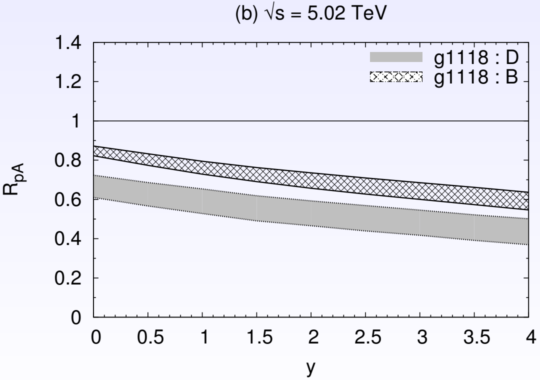
- $p_\perp$-dependence of R_{pA} :



More realistic computations

Fujii, Watanabe (2013)

- y-dependence of R_{pA} :



Summary

- Gluon saturation enhanced in nuclei; reached earlier than in nucleons
- Saturation breaks the standard forms of factorization (DGLAP, kt-factorization)
  - ▷ apparent non universality of parton distributions
- In the saturated non-linear regime, there exist generalized universal distributions $W[\rho]$ that describe the dense projectiles both in DIS, pA and AA collisions
- In pA collisions, forward production (in the proton direction) may be treated by an hybrid factorization scheme where the CGC description is applied only to the nucleus
- Competition between multiple scatterings (Cronin effect) and suppression due to shadowing. Invariant mass increased by rescatterings; reduces the JPsi yield.
- BUT : hadronization not treated in a very satisfactory way...