(Quantum) Dynamical Model of in-Medium Quarkonia

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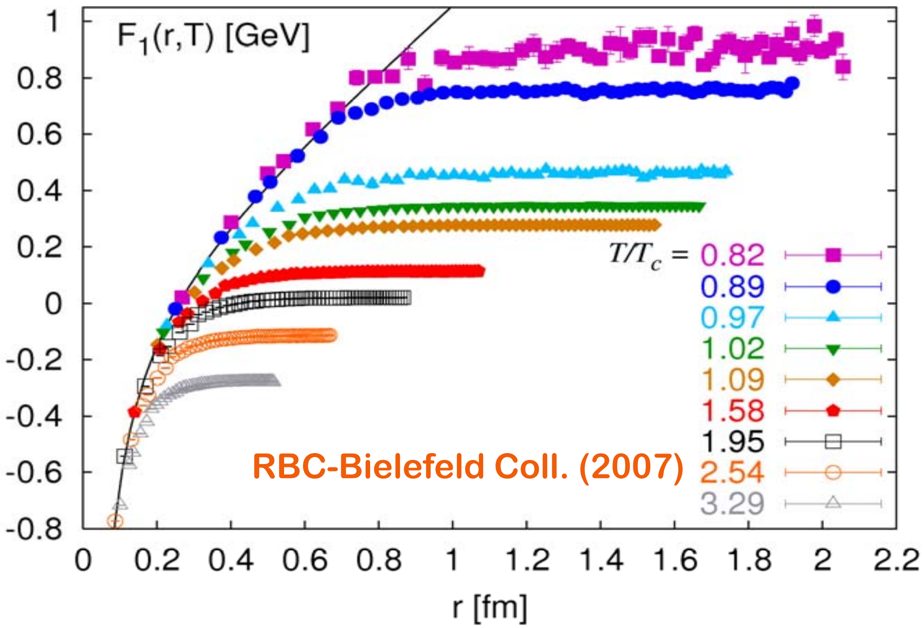
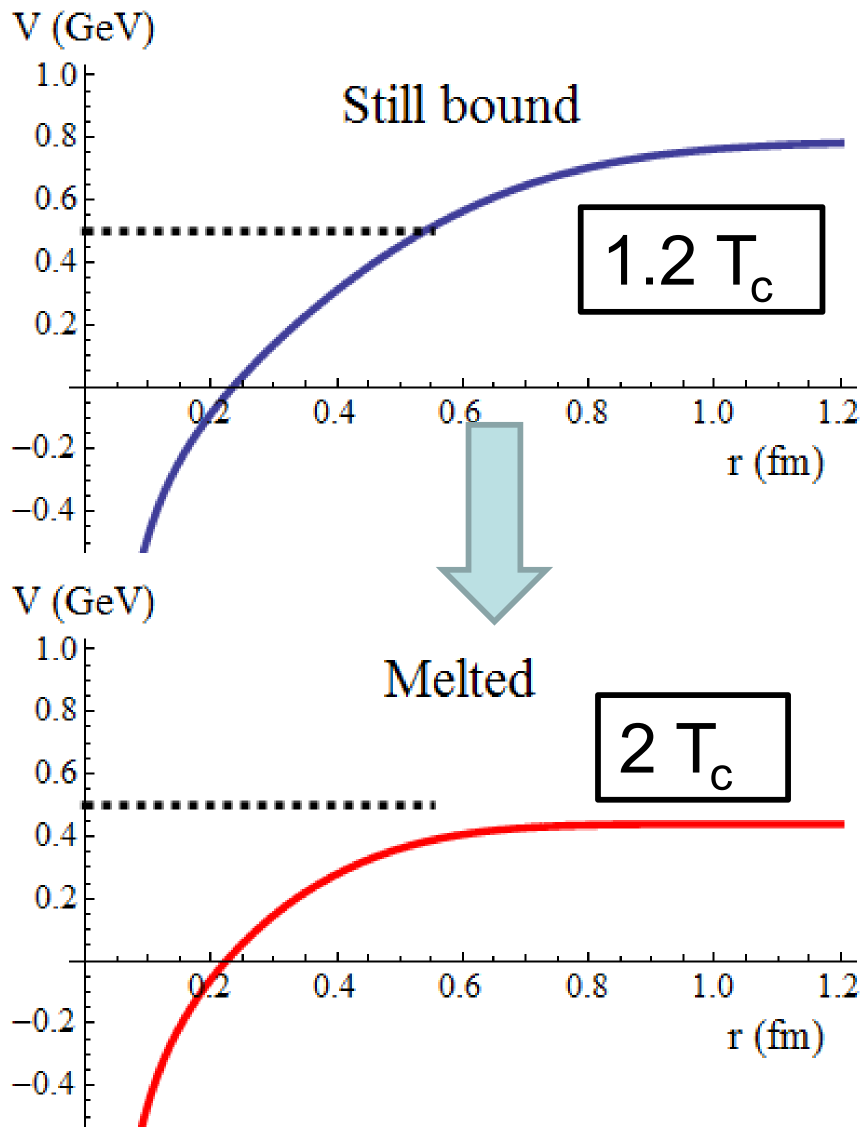
Sapore Gravis 2014 -- Padova

Probing deconfinement in AA collisions ?

QQbar "potential" on the lattice: Increased screening at larger temperatures

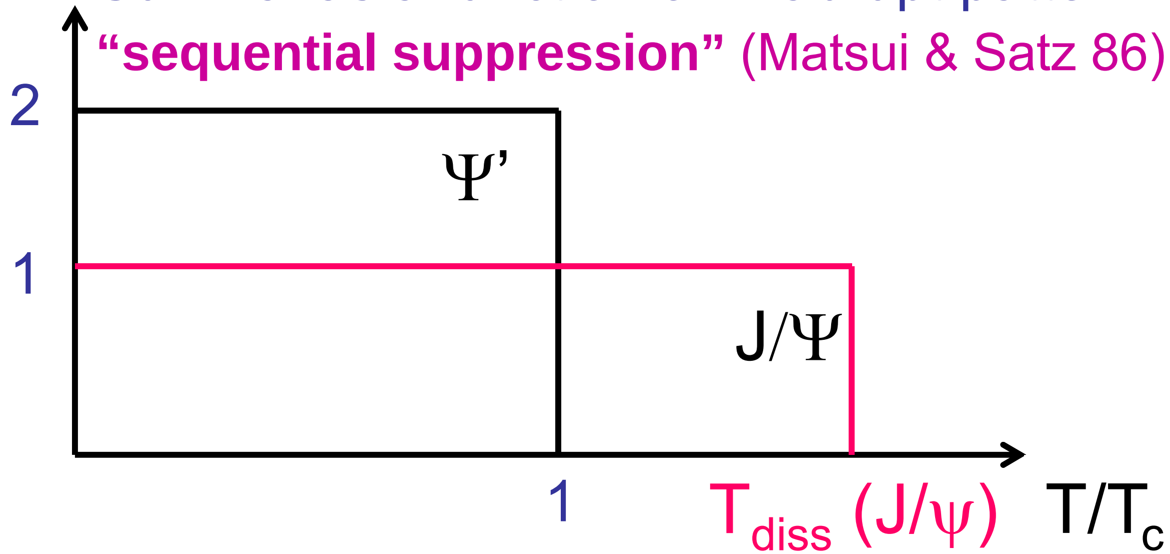


Consequence for Q-Qbar bound states



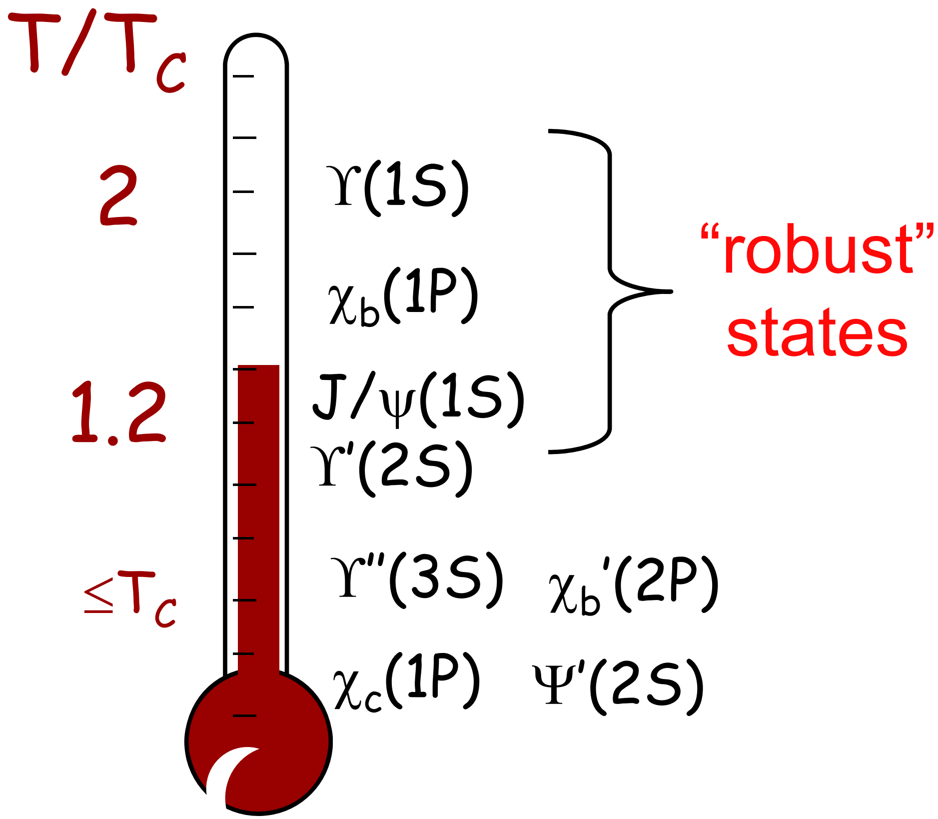
Survival as a function of T: abrupt pattern

"sequential suppression" (Matsui & Satz 86)



Quarkonia in Stationary QGP

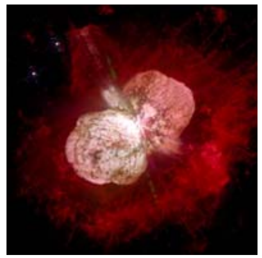
QGP Thermometer



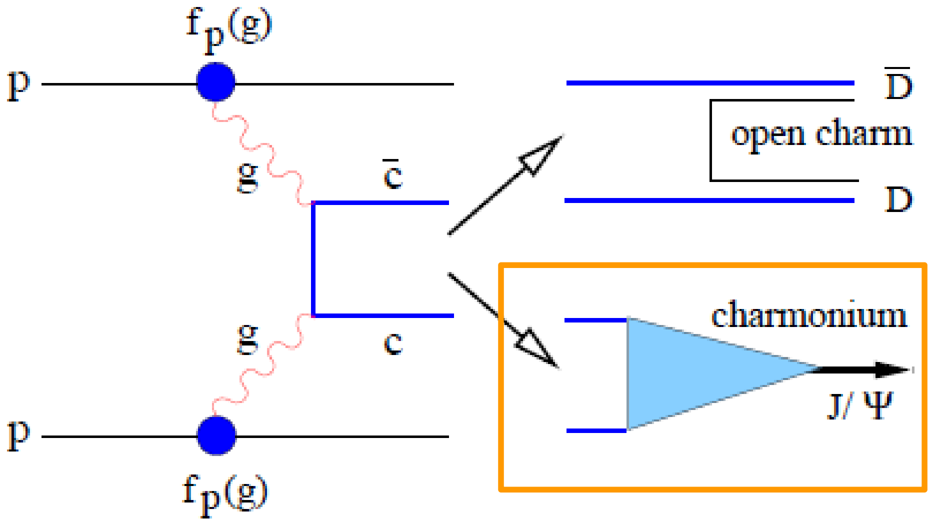
Indeed observed at SPS (CERN) and RHIC (BNL) experiments. However:

- alternative explanations, lots of unknown (also from theory side)
- less suppression at LHC

•Time dependent quarkonia formation in evolving medium ?



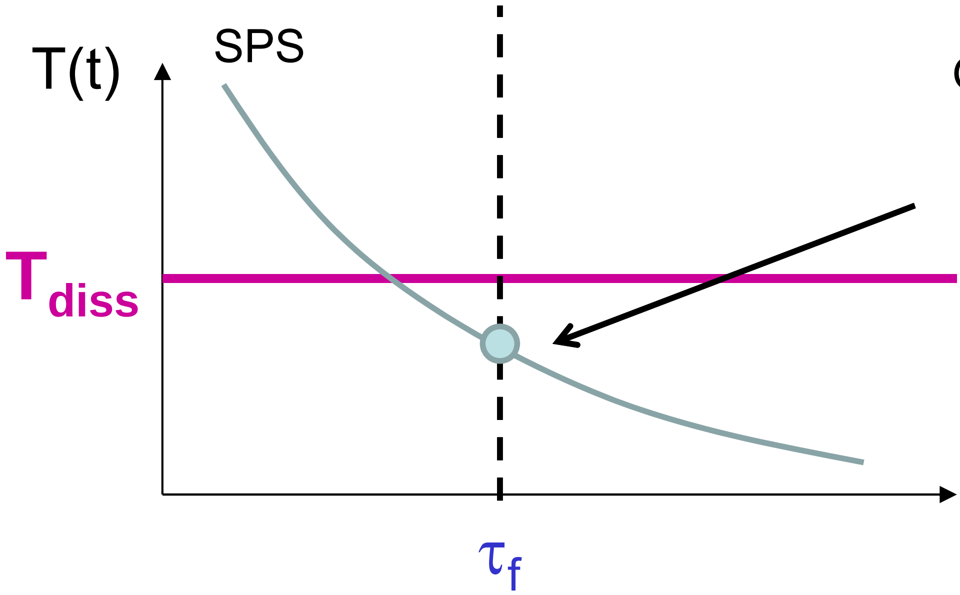
Dynamical version of the sequential suppression scenario



a) In vacuum: Quarkonia are formed after some "formation time" τ_f (typically the Heisenberg time), usually assumed to be independent of the surrounding medium

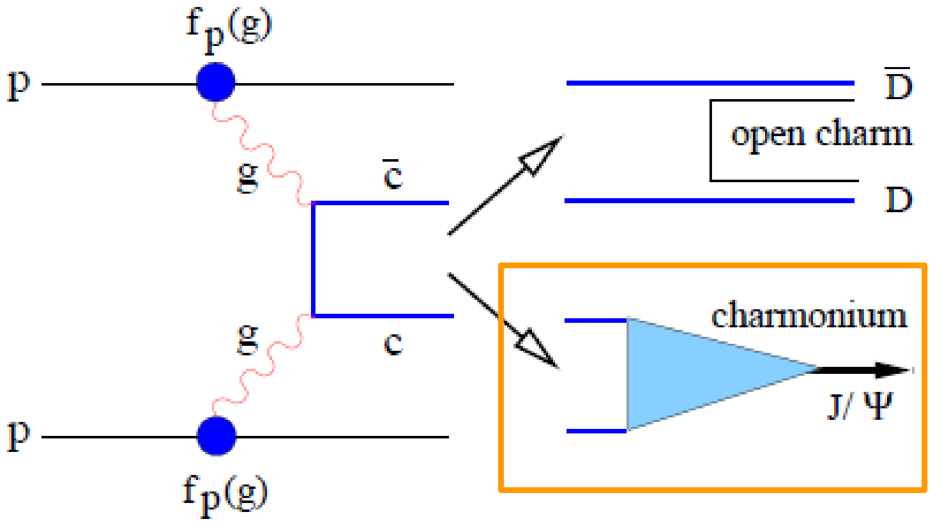
Standard folklore of sequential suppression: b.1) If $T(\tau_f, x_0) < T_{diss}$ the quarkonia is indeed created (as in vacuum)

Local temperature in the medium



Quarkonia state formed as in the vacuum

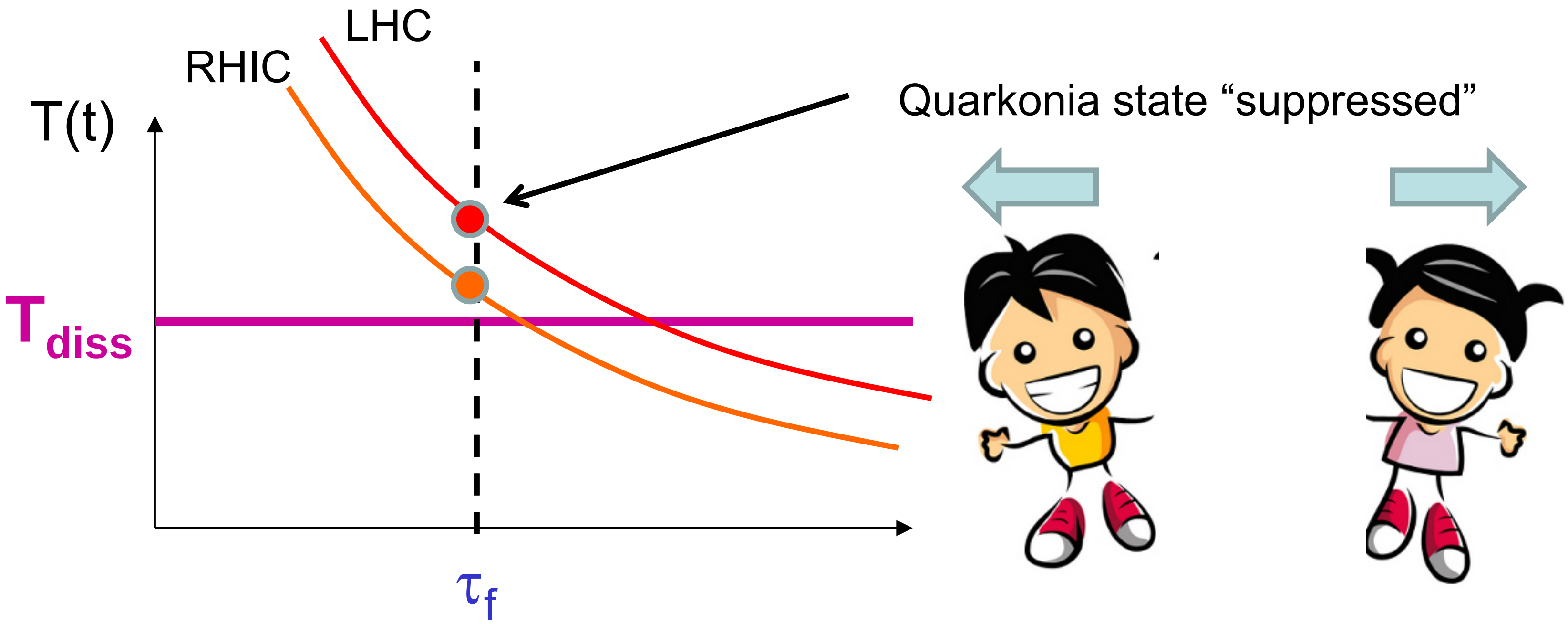
Dynamical version of the sequential suppression scenario



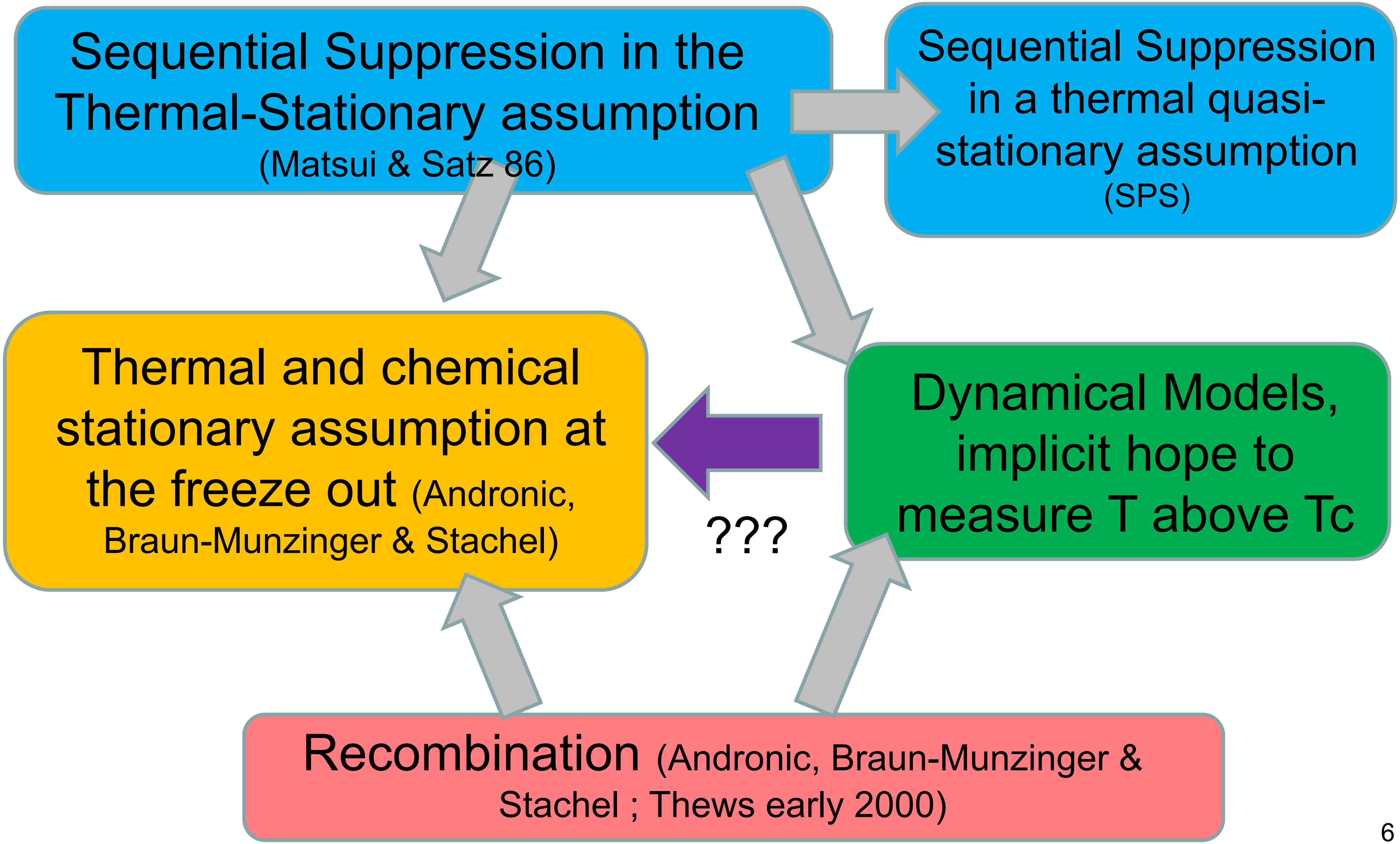
a) In vacuum: Quarkonia are formed after some "formation time" τ_f (typically the Heisenberg time), usually assumed to be independent of the surrounding medium

Standard folklore of sequential suppression: b.2) If $T(\tau_f,x_0)>T_{diss}$ the quarkonia is NOT created (Q-Qbar pair is "lost" for quarkonia production)

Local temperature in the medium



Schematic view of HQ modeling in hot media



Common ingredients in (most of the) state of the art *dynamical* models

Early decoupling btwn various states in the initial stage

Mean field (screening)

- Vetoing at the time of production if $T > T_{dissoc}$
- Evaluation of the wave functions ψ_n at finite T

Fluctuations (dissociation)

- Evaluate dissociation cross sections using transition operators + ψ_n

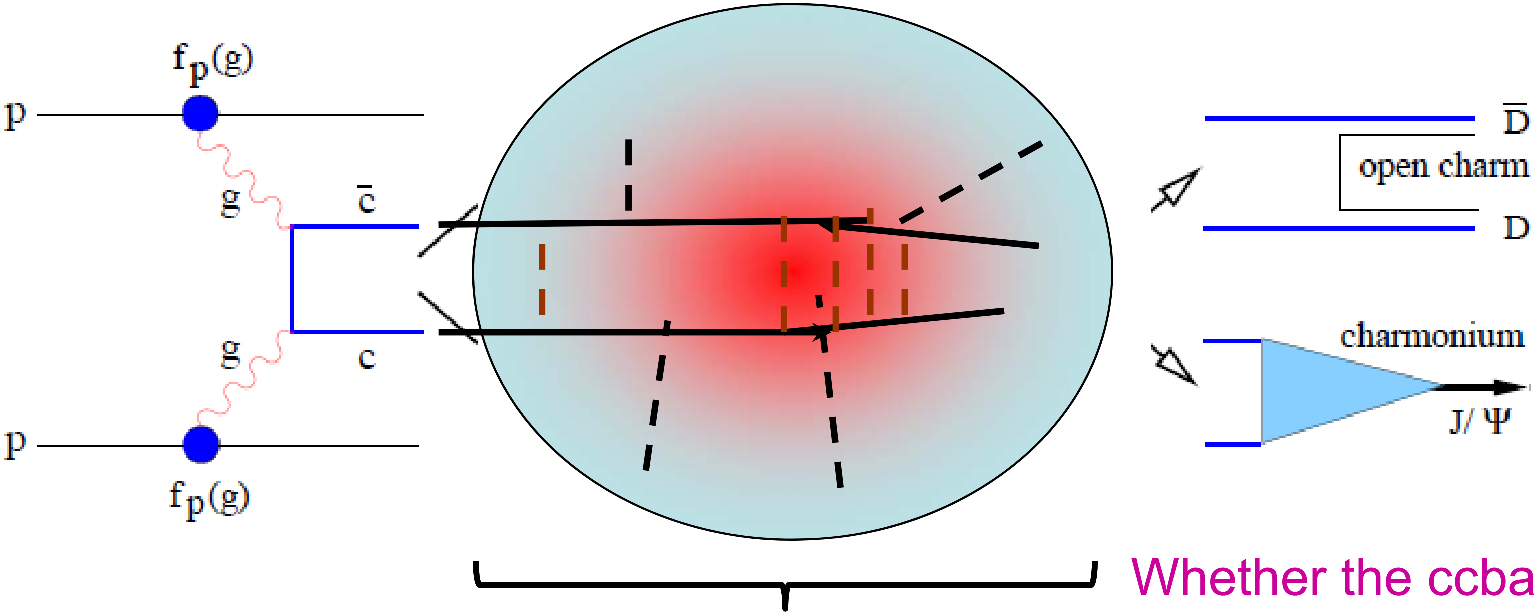
or

- Evaluation of the width Γ using some imaginary potential => survival a $\exp(-\Gamma t)$

+ recombination (using detailed balance of)

Back to the concepts

Reality



Very complicated QFT problem at finiteT(t) !!!

Whether the ccbar pair emerges as a bound quarkonia or as DDbar pair is only resolved at the end of the evolution

Beware of quantum coherence during the evolution

But one should aim at solving it, especially as the quarkonia content of a QQbar quantum state is at most of the order of a few % (continuous transitions under external perturbations)

Need for full quantum treatment

Dating back to Blaizot & Ollitrault, Thews, Cugnon and Gossiaux; early 90's

A case for quantum thermalisation

Background

- RHIC and LHC experimental results => quarkonia thermalise partially in the QGP
- But how to thermalise our wavefunction ? Quantum friction/stochastic effects have been a long standing problem because of their irreversible nature

The open quantum approach: ✗
Considering the whole system, quarkonia and environment, the latter being finally integrating out

Y. Akamatsu [arXiv:1209.5068]
Laine et al. JHEP 0703 (2007) 054

2nd possible approach: ✓
Mock the open quantum approach by using a stochastic operator and a dissipative non-linear potential

A. Rothkopf et al. Phys. Rev. D 85, 105011 (2012)
N. Borghini et al. Eur. Phys. J. C 72 (2012)
S. Garashchuk et al. Jou. of Chem. Phys. 138, 054107 (2013)

- **Stochastic Schrödinger equation**

$$i\hbar\frac{\partial\Psi_{Q\bar{Q}}(\mathbf{r},t)}{\partial t} = \Big(\underbrace{\widehat{H}(\mathbf{r})}_{\text{MF}} - \underbrace{\mathbf{F}(t).\mathbf{r}}_{\text{Fluctuations}} + \underbrace{A\big(S(\mathbf{r},t) - \langle S(\mathbf{r},t)\rangle_{\mathbf{r}}\big)}_{\text{Friction}}\Big)\Psi_{Q\bar{Q}}(\mathbf{r},t)$$

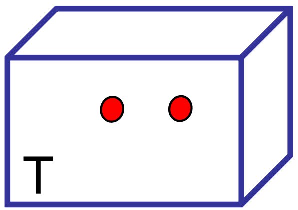
Derived from the Heisenberg-Langevin equation*, in Bohmian mechanics** …

* Kostin The J. of Chem. Phys. 57(9):3589–3590, (1972)
** Garashchuk et al. J. of Chem. Phys. 138, 054107 (2013)

Schrödinger-Langevin (SL) equation

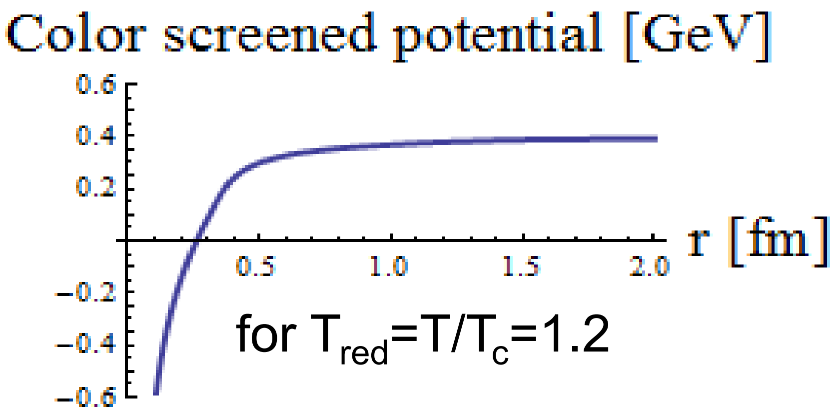
$$i\hbar\frac{\partial\Psi_{Q\bar{Q}}(\mathbf{r},t)}{\partial t} = \Big(\underbrace{\widehat{H}(\mathbf{r})}_{\text{MF}} - \mathbf{F}(t).\mathbf{r} + A\big(S(\mathbf{r},t) - \langle S(\mathbf{r},t)\rangle_{\mathbf{r}}\big)\Big)\Psi_{Q\bar{Q}}(\mathbf{r},t)$$

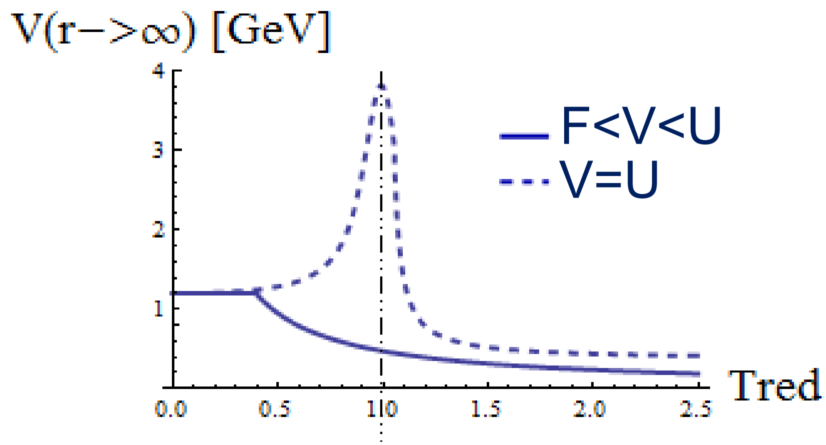
<u>Static lQCD calculations </u>(maximum heat exchange with the medium):



→ { F : free energy
S : entropy } → U=F+TS : internal energy
(no heat exchange)

- "Weak potential" F<V<U * <=> some heat exchange
- "Strong potential" V=U ** <=> adiabatic evolution





Evaluated by Mócsy & Petreczky* and Kaczmarek & Zantow** from lQCD results

* Phys.Rev.D77:014501,2008 **arXiv:hep-lat/0512031

Road map

(1) Results with the mean field only

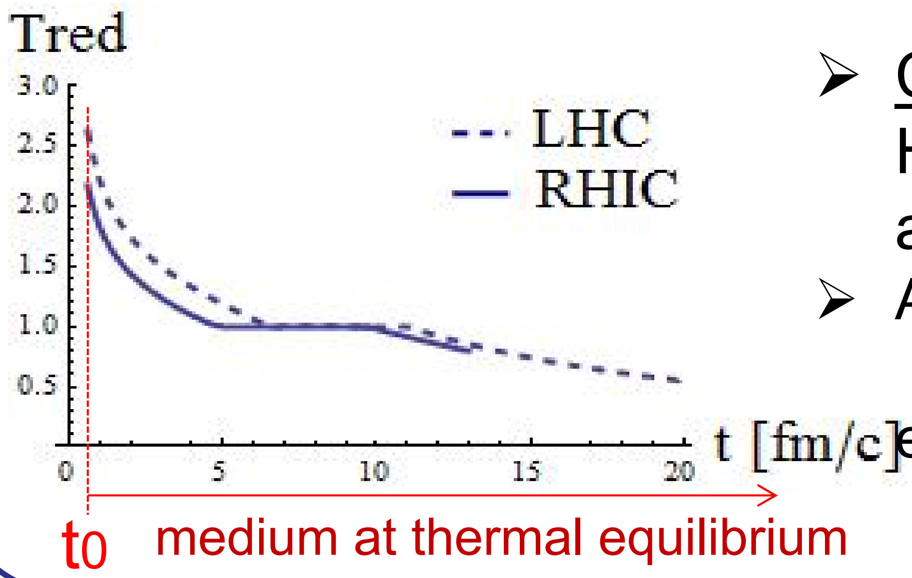
Which one dominates ?

(2) Results with fluctuations and dissipation only

(3) Results with the full SL equation

Quantum evolution in the mean field (alone)

The QGP homogeneous temperature scenarios



- Cooling over time by Kolb and Heinz* (hydrodynamic evolution and entropy conservation)
- At LHC ($\sqrt{s_{NN}} = 2.76\ \mathrm{TeV}$) and RHIC ($\sqrt{s_{NN}} = 200\ \mathrm{GeV}$) energies

* arXiv:nucl-th/0305084v2

Initial $Q\bar{Q}$ pair radial wavefunction

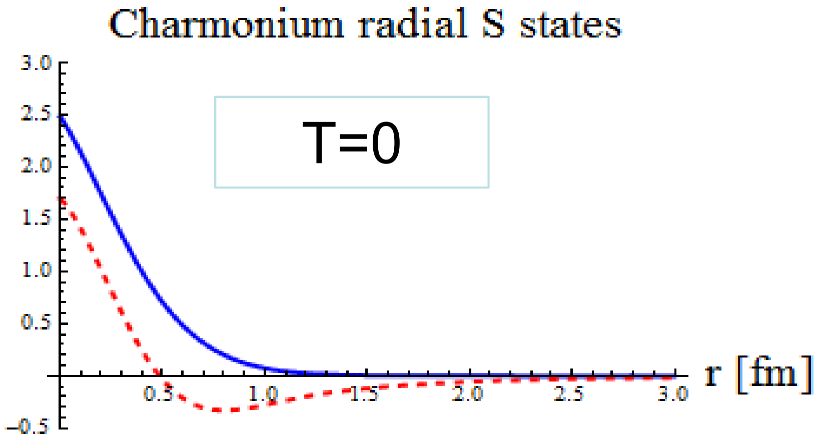
- Assumption: QQ pair created at t_0 in the QGP core
- Gaussian shape with parameters (Heisenberg principle):

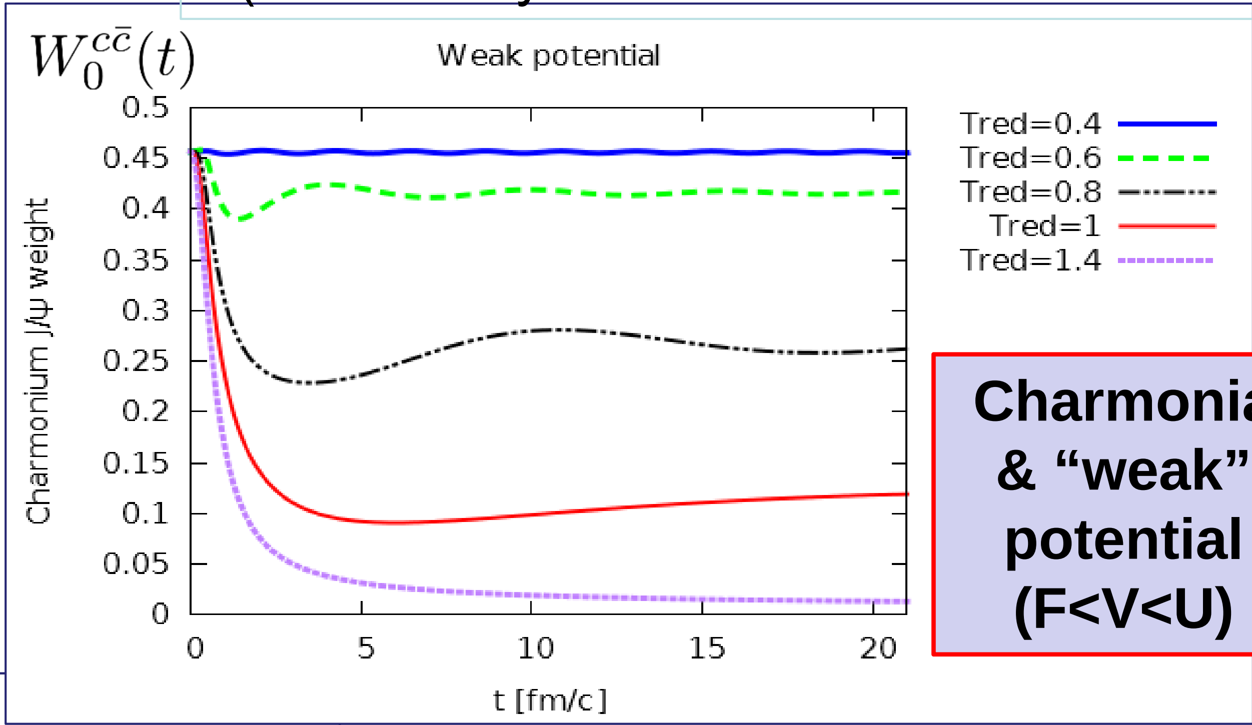
$$a_{c\bar{c}} = 0.165\ \mathrm{fm} \qquad a_{b\bar{b}} = 0.045\ \mathrm{fm}$$

Evolution of the charmonia weights at cst T

(followed by an instantaneous freeze out)

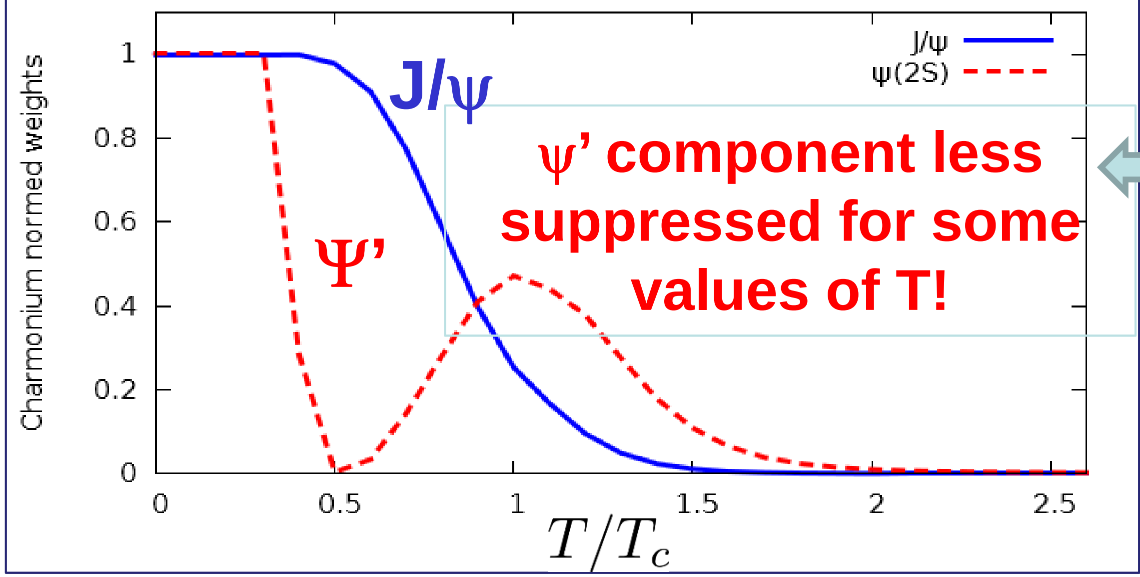
$$W_i(t) = |\langle \psi_i(T=0) | \psi(t) \rangle|^2$$





Charmonia & "weak" potential (F<V<U)

$$S_i(t \to +\infty) = \frac{W_i(t \to +\infty)}{W_i(0)}$$



ψ' component less suppressed for some values of T!

The "suppression" S (normed weights) at t→∞ as function of T

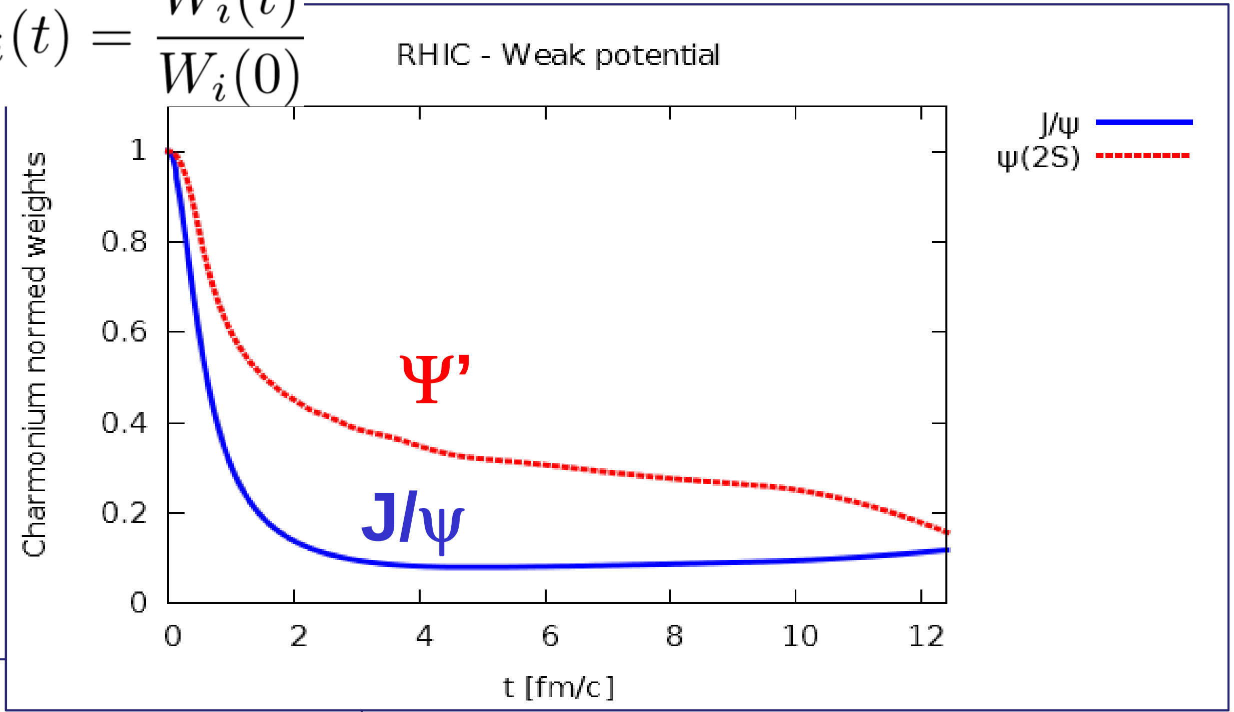
Smooth evolution and no discontinuity in the parameter space

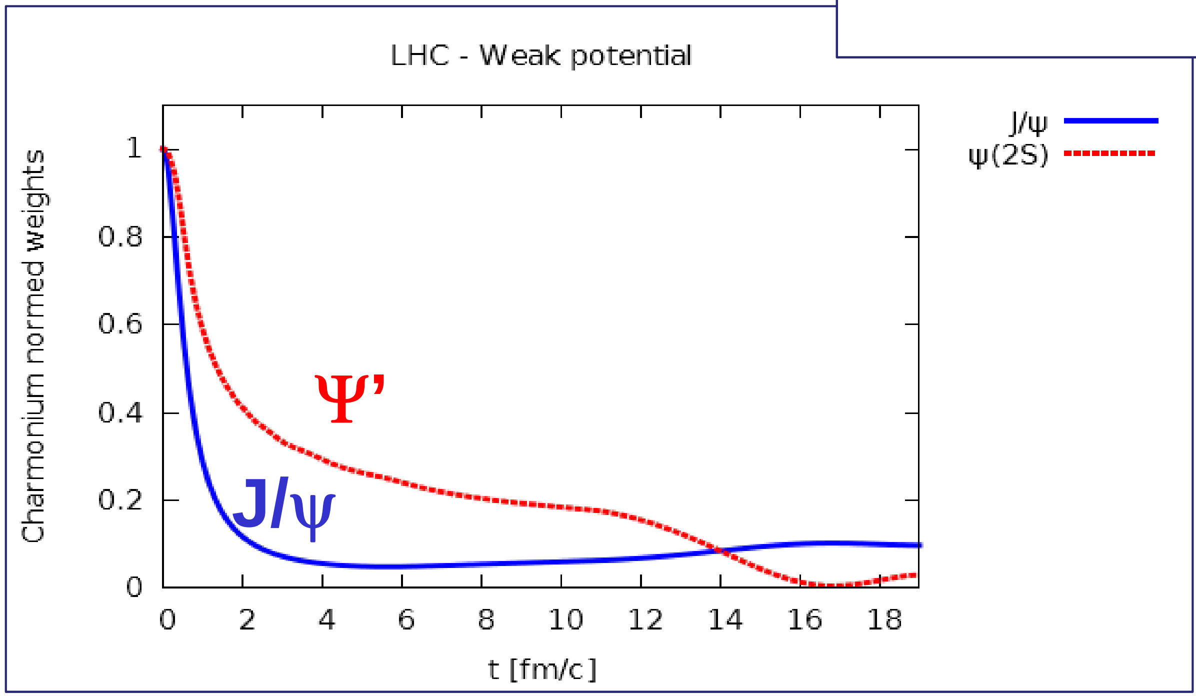
Evolution in realistic T scenarios

Charmonia and weak color potential (F<V<U)

RHIC temperature scenario

$$S_i(t) = \frac{W_i(t)}{W_i(0)}$$

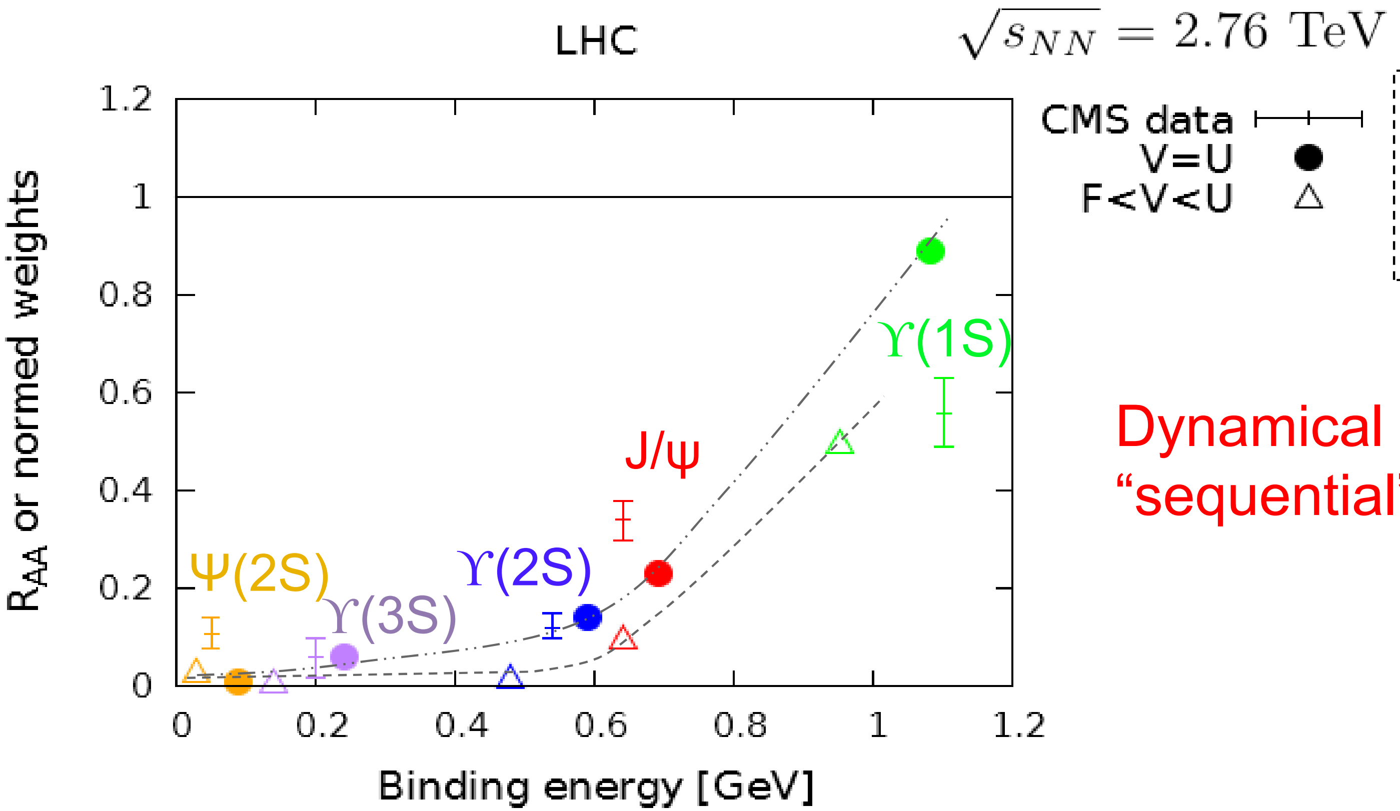




LHC temperature scenario

Inversion of the ψ' vs ψ suppression pattern at longer time

Sum up of LHC results



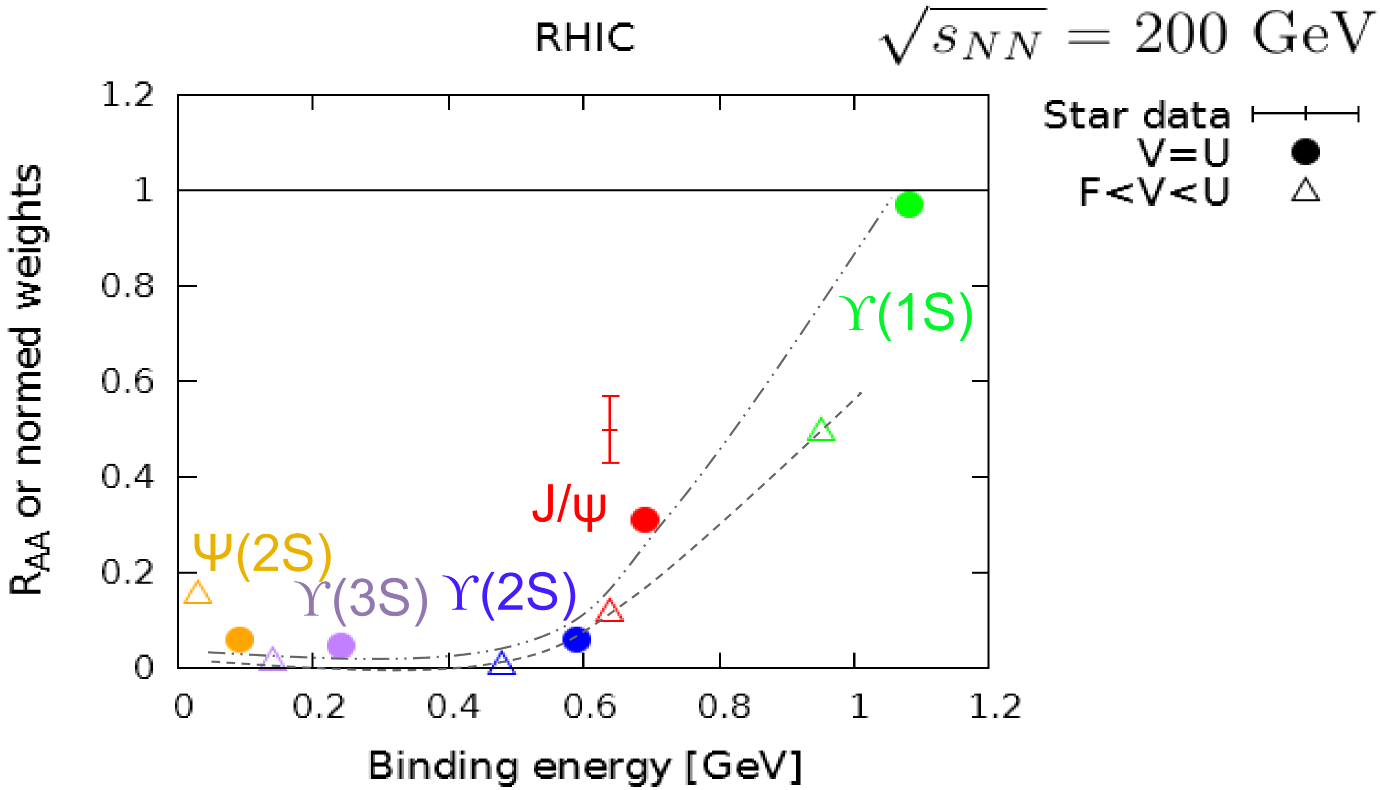
(High p_T and most central)

Dynamical quarkonia "sequential" suppression

- The results are quite encouraging for such a simple scenario !
- J/ψ and ψ(2S) are underestimated (room for regeneration) and $\Upsilon(1S)$ overestimated
- Feed downs from exited states and CNM to be implemented

Central issue: How much of this survives once we consider the fluctuations ?

Sum up of RHIC results



- Similar suppression trends obtained for both RHIC and LHC.
- Less J/ψ suppression at RHIC than at LHC.
- Υ(1S+2S+3S) suppression can be estimated with Star data to ~ 0.55±0.10, we obtain ~ 0.48 for V=U and ~ 0.24 for F<V<U.

Schrödinger-Langevin (SL) equation

$$i\hbar \frac{\partial \Psi_{Q\bar{Q}}(\mathbf{r},t)}{\partial t} = \left(\hat{H}(\mathbf{r}) - \mathbf{F}(t).\mathbf{r} + A\big(S(\mathbf{r},t) - \langle S(\mathbf{r},t)\rangle_{\mathbf{r}}\big) \right) \Psi_{Q\bar{Q}}(\mathbf{r},t)$$

Fluctuations

Stochastic operator; “warming”

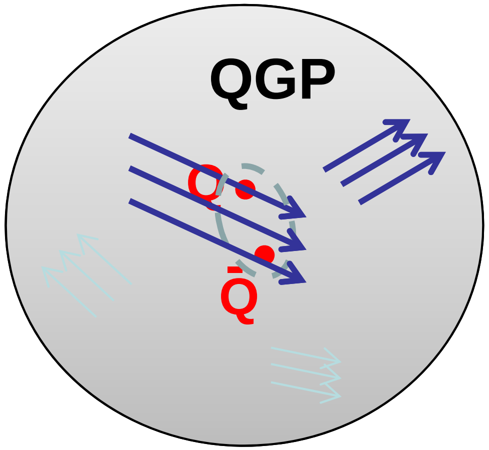
$\langle \mathbf{F}(t) \rangle = 0\,,\ \langle \mathbf{F}(t)\mathbf{F}(t') \rangle = \Gamma(t,t')$?

Brownian hierarchy: $m \gg T \Rightarrow \sigma \ll \tau_{\rm relax}$

✓ σ = autocorrelation time of the gluonic fields

✓ $\tau_{\rm relax}$ = quarkonia relaxation time

$\Gamma(t,t')$: gaussian correlation of parameter σ and norm B



<u>3 parameters</u>: *A* (the drag coef), *B* (the diffusion coef) and *σ* (autocorrelation time)

Schrödinger-Langevin (SL) equation

$$i\hbar\frac{\partial\Psi_{Q\bar{Q}}(\mathbf{r},t)}{\partial t} = \left(\hat{H}(\mathbf{r}) - \mathbf{F}(t).\mathbf{r} + A\big(S(\mathbf{r},t) - \langle S(\mathbf{r},t)\rangle_{\mathbf{r}}\big)\right)\Psi_{Q\bar{Q}}(\mathbf{r},t)$$

dissipative non-linear potential
(wavefunction dependent

where $S(\mathbf{r},t) = \arg\big(\Psi_{Q\bar{Q}}(\mathbf{r},t)\big)$

- ✓ Brings the QQ to the lowest state (0 node)
- ✓ Friction (assumed to be local in time)

- Solution for V=0 (free wave packet): $\psi(\vec{x},t) \propto e^{i\vec{p}_{\mathrm{cl}}(t)\cdot\vec{r}+i\alpha(t)(\vec{r}-\vec{r}_{\mathrm{cl}}(t))^2-i\varphi(t)}$

  where $\vec{p}_{\mathrm{cl}}(t)$ and $\vec{x}_{\mathrm{cl}}(t)$ satisfy the classical laws of motion

- $\vec{p}_{\mathrm{cl}}(t) = \vec{p}_{\mathrm{cl}}(0)e^{-At} \Rightarrow$ A is the drag coefficient (inverse relaxation time)

A can be fixed through the modelling of single heavy quarks observables and comparison with the data OR using lattice QCD calculations

Schrödinger-Langevin (SL) equation

$$i\hbar\frac{\partial\Psi_{Q\bar{Q}}(\mathbf{r},t)}{\partial t} = \left(\widehat{H}(\mathbf{r}) - \mathbf{F}(t).\mathbf{r} + A\left(S(\mathbf{r},t) - \langle S(\mathbf{r},t)\rangle_{\mathbf{r}}\right)\right)\Psi_{Q\bar{Q}}(\mathbf{r},t)$$

dissipative non-linear potential
(wavefunction dependent

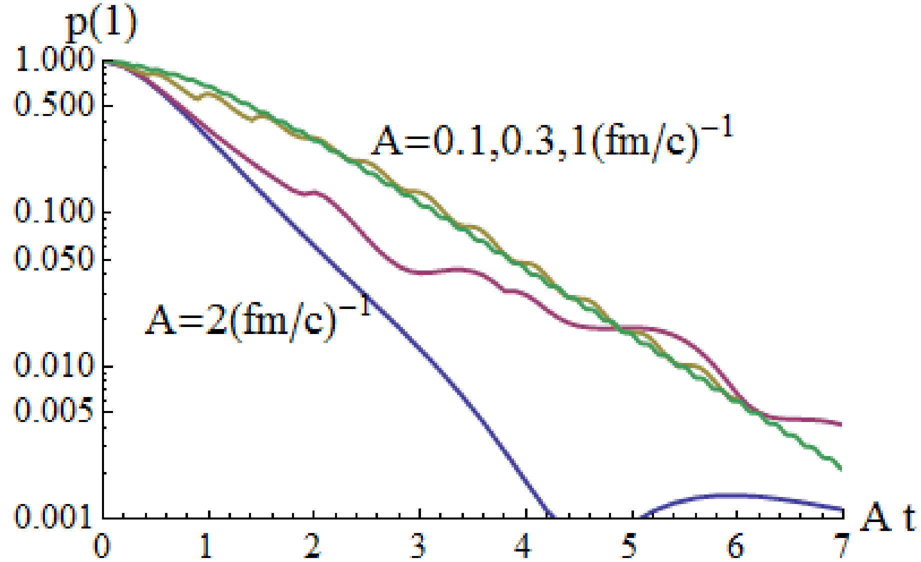
where $S(\mathbf{r},t) = \arg\left(\Psi_{Q\bar{Q}}(\mathbf{r},t)\right)$

✓ Brings the $Q\bar{Q}$ to the lowest state (0 node)
✓ Friction (assumed to be local in time)

➢ Solution for harmonic potential as well: $\psi(\vec{x},t) \propto e^{i\vec{p}_{\rm cl}(t)\cdot\vec{r}+i\alpha(t)(\vec{r}-\vec{r}_{\rm cl})^2-i\varphi(t)}$

Illustration: probability of finding the first excited state in a 1D-harmonic potential, as function of time, for various values of A …

Scaling relation found for A<ω



Properties of the SL equation

- Unitarity (no decay of the norm as with imaginary potential)

- Heisenberg principle satisfied at any T

- Non linear => Violation of the superposition principle (=> decoherence)

- Gradual evolution from pure to mixed states

- Mixed state observables:

$$\left\langle \langle \psi_S(t)|\hat{A}|\psi_S(t)\rangle \right\rangle_{\mathrm{stat}} = \lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} \langle \psi_S^{(r)}(t)|\hat{A}|\psi_S^{(r)}(t)\rangle$$

- « Easy » to implement numerically (especially in Monte-Carlo generator)

Thermalization with the SL equation

Essential feature to make contact with the statistical approaches

- For an harmonic potential:
  - Asymptotic distribution of states proven to be $\propto e^{-\frac{E_n}{kT}}$
  - Fluctuation dissipation theorem:

$$\frac{B}{2m} = A\int_{-\infty}^{+\infty} \frac{(\nabla S)^2}{m}|\psi|^2 dr \Rightarrow B = m\hbar\omega\left(\coth\left(\frac{\hbar\omega}{2kT}\right) - 1\right)A \xrightarrow{kT\gg\hbar\omega} 2mkTA$$

Classical Einstein law

NB: for quantum noise acting on operators in the Heisenberg representation

$$B = m\hbar\omega\left\{\underbrace{\left[\coth\left(\frac{\hbar\omega}{2kT}\right) - 1\right]}_{\text{Same as in SL}}\underbrace{+1}_{}\right\}$$

Same as in SL

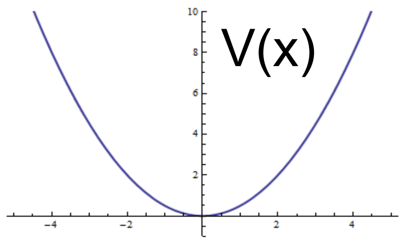
Ground state energy… included in the width of the wave packet in the Schroedinger representation

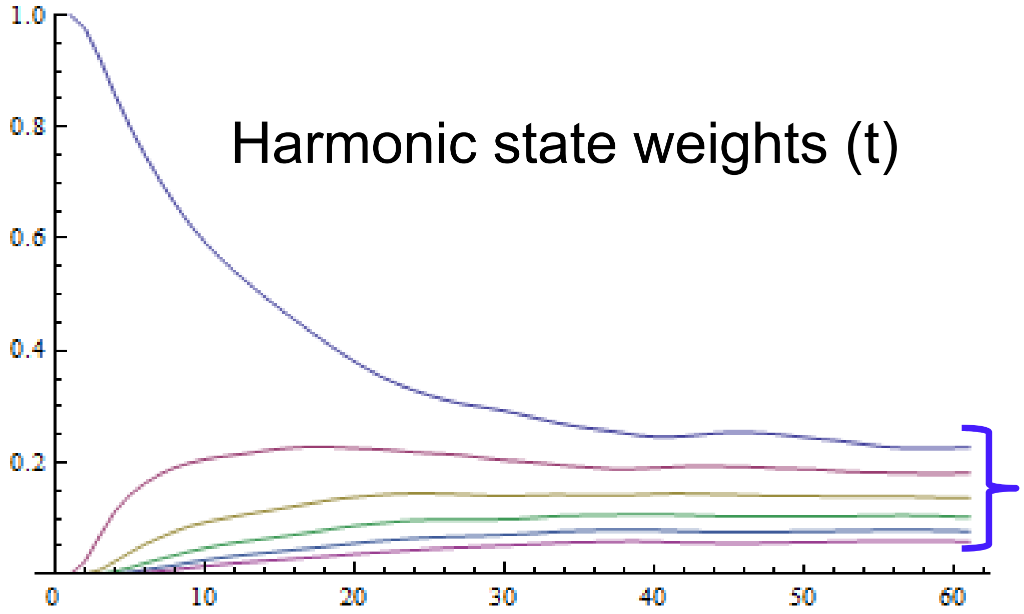
- Asymptotic convergence shown for a wide class of potentials, but distribution of states less understood **=> numerical study**

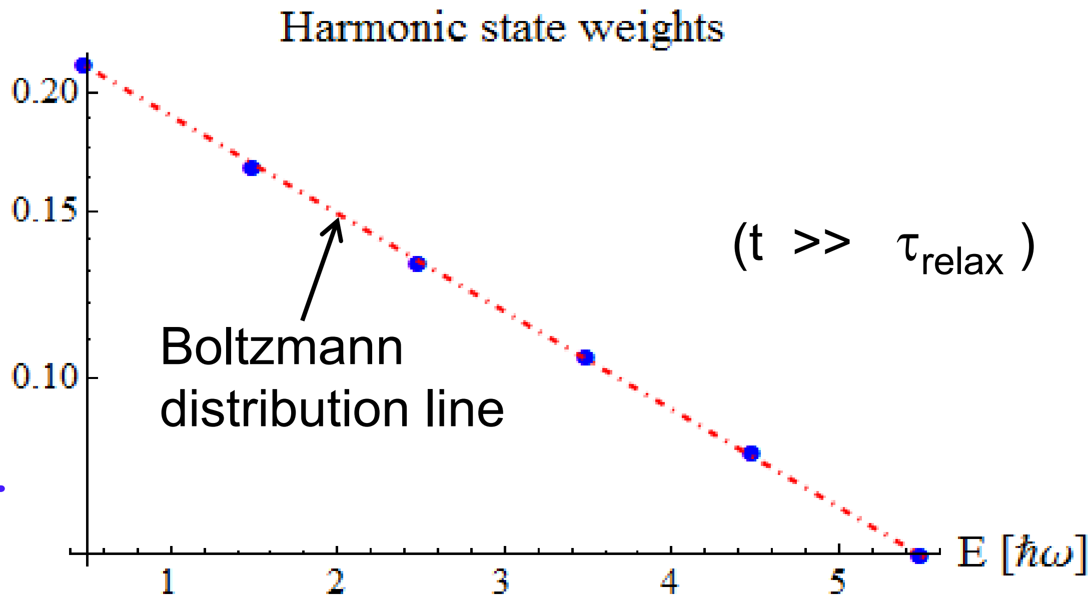
numerical tests of thermalization

Harmonic potential

Asymptotic thermal equilibrium
for any (A,B,σ) and from any initial state



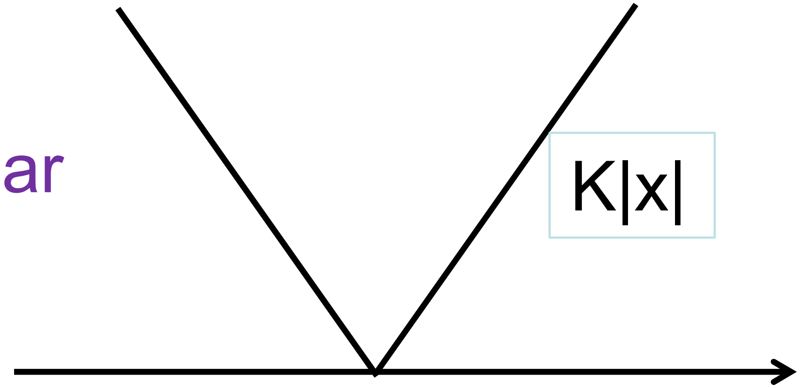




numerical tests of thermalization

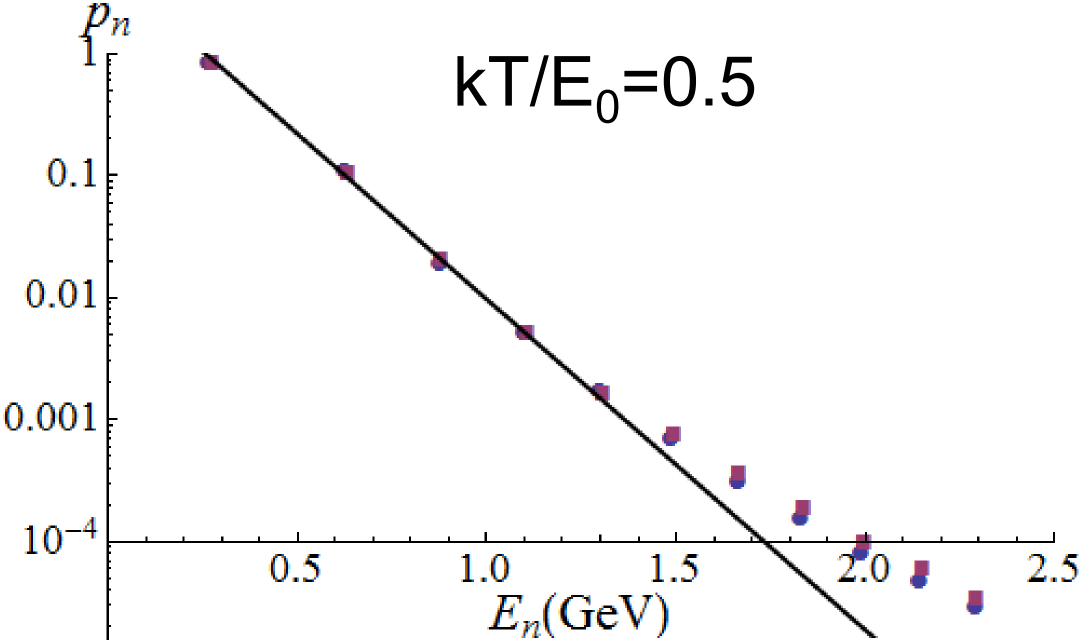
Other potentials

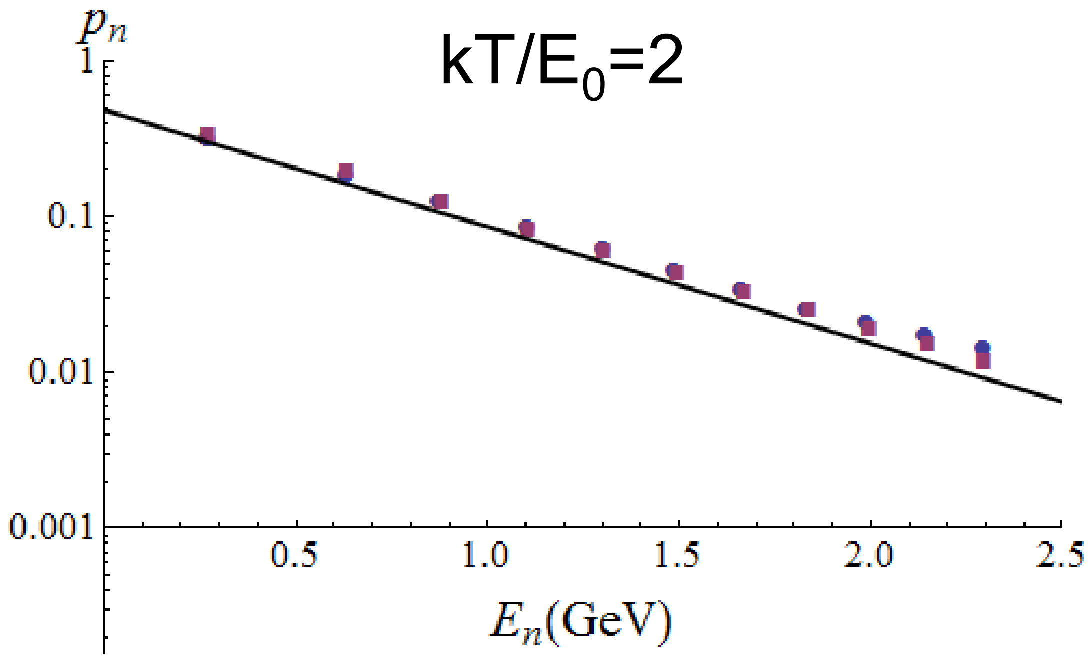
1D Linear



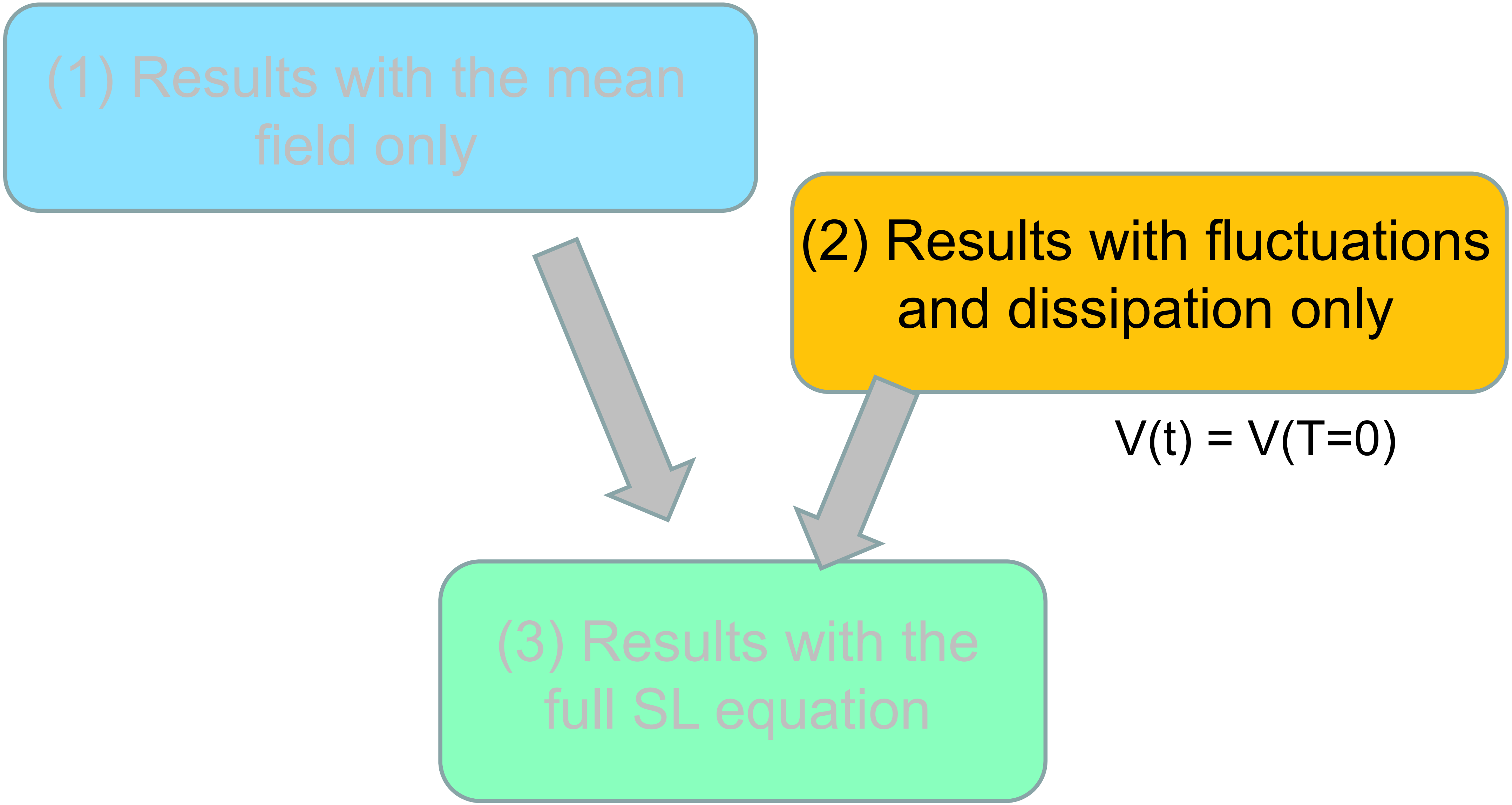
Asymptotic Boltzmann distributions ?

Yes; deviations from Boltzmann seen for higher states for $kT<<E_0$





Road map



(1) Results with the mean field only
(2) Results with fluctuations and dissipation only
V(t) = V(T=0)
(3) Results with the full SL equation

Dynamics of QQbar with SL equation

Aimed as a proof of principle => simplifying assumptions

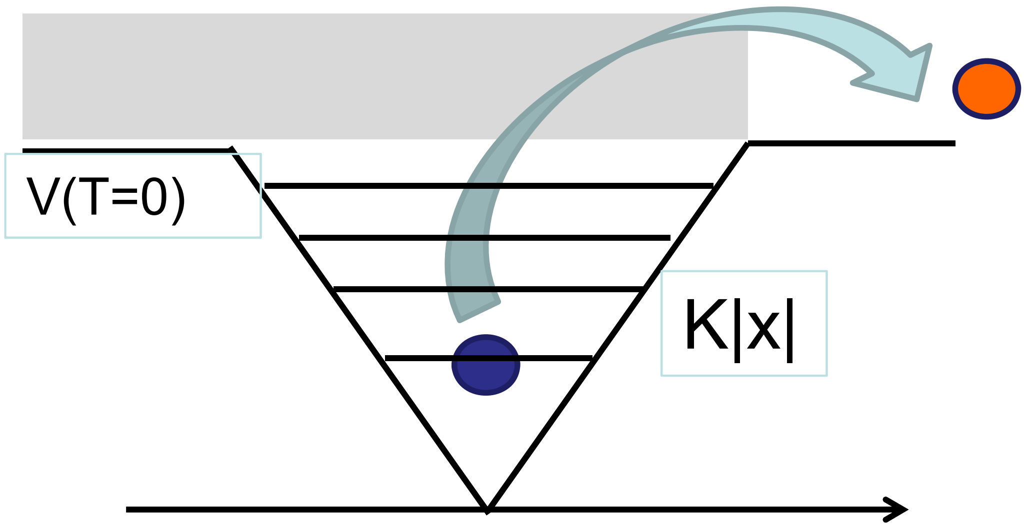
- 3D -> 1D ($\chi\equiv$1rst odd state, $\psi'\equiv$ 1rst excited even state)
- Drag coeff. for c quarks: $A(T)[(\mathrm{fm/c})^{-1}] \cong 3T[\mathrm{GeV}] + 2.5T^2$

Typically T ∈ [0.1 ; 0.43] GeV => A ∈ [0.32 ; 1.75] $(\mathrm{fm/c})^{-1}$

- σ=0

First, considering the effect of the fluctuations-dissipation **only** (neglecting the screening of the potential):

- Potential:

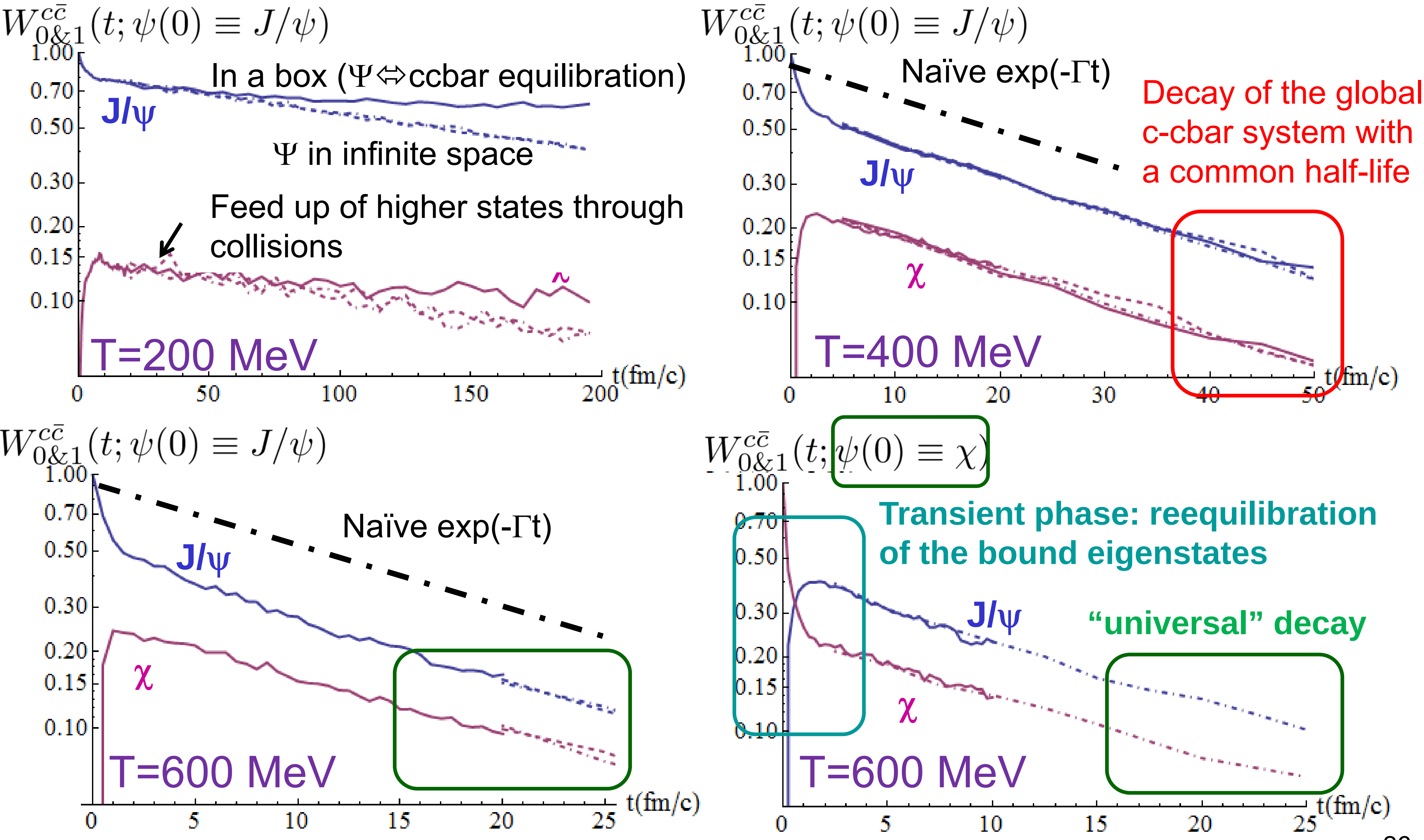


Stochastic forces => leakage to continuum

K chosen such that
E2-E0=E(ψ')- E(J/ψ))=600MeV

4 bound eigenstates

Evolution of the weights with V(T=0) and initial eigenstate



Road map

(1) Results with the mean field only

(2) Results with fluctuations and dissipation only

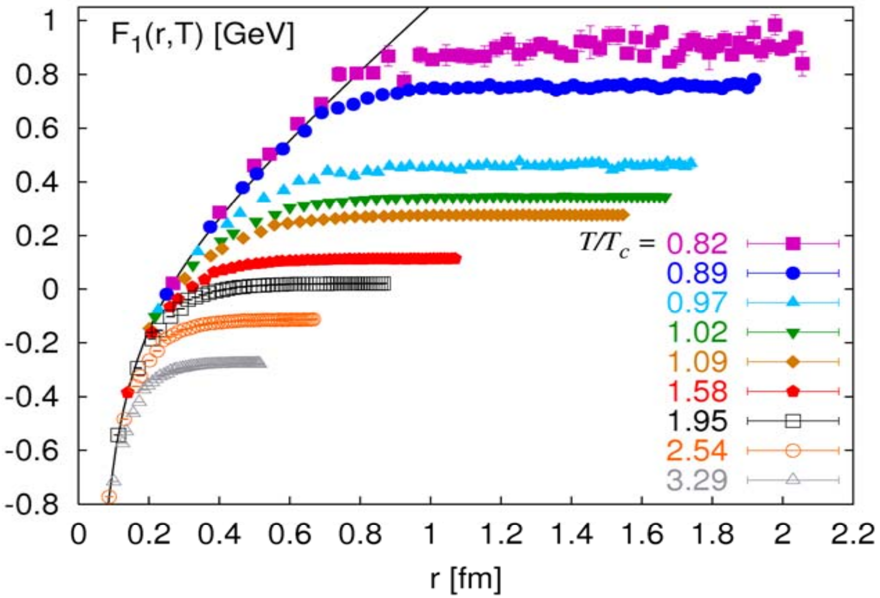
(3) Results with the full SL equation

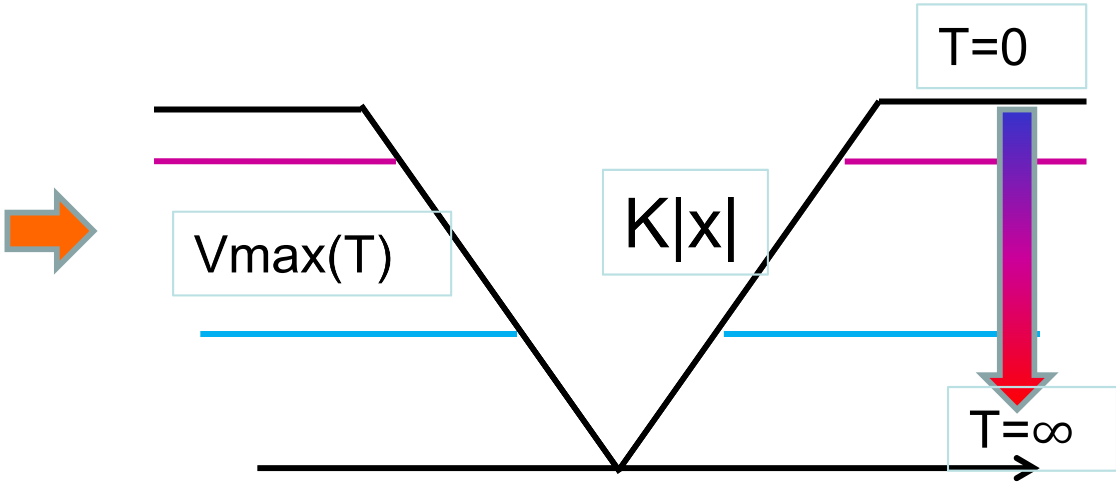
a) V(t) = V(T)

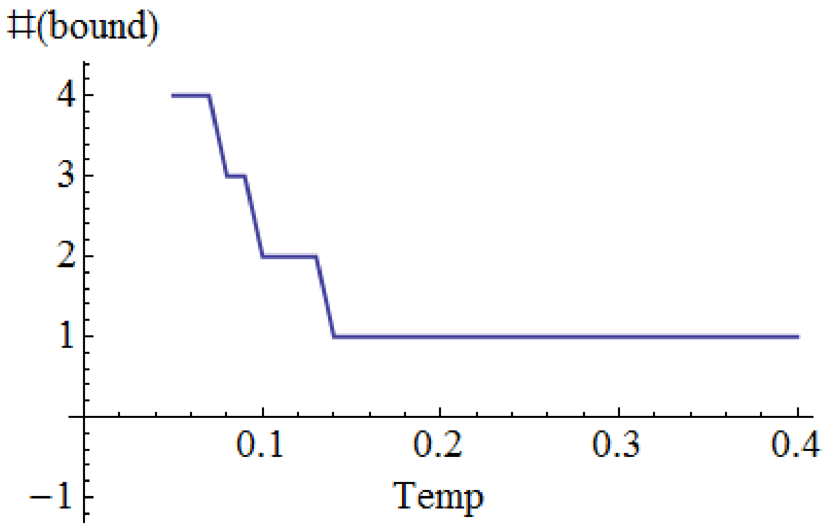
Dynamics of QQbar with SL equation

Now considering the effect of the fluctuations-dissipation **combined** with the mean field contribution:

- Potential:

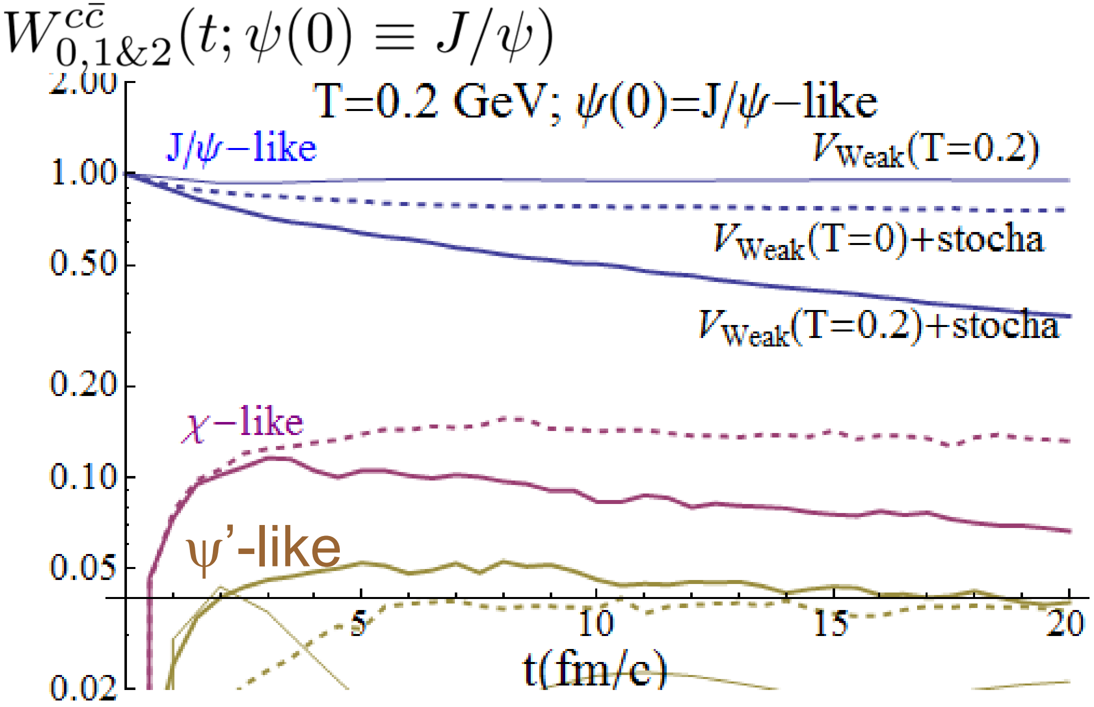


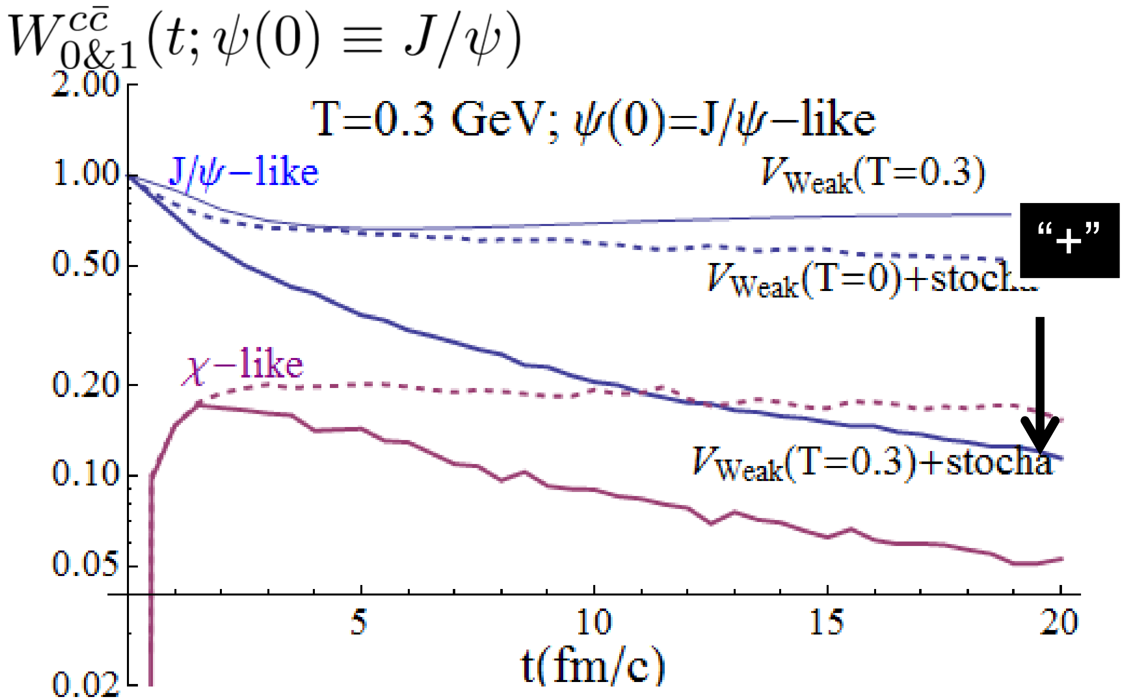




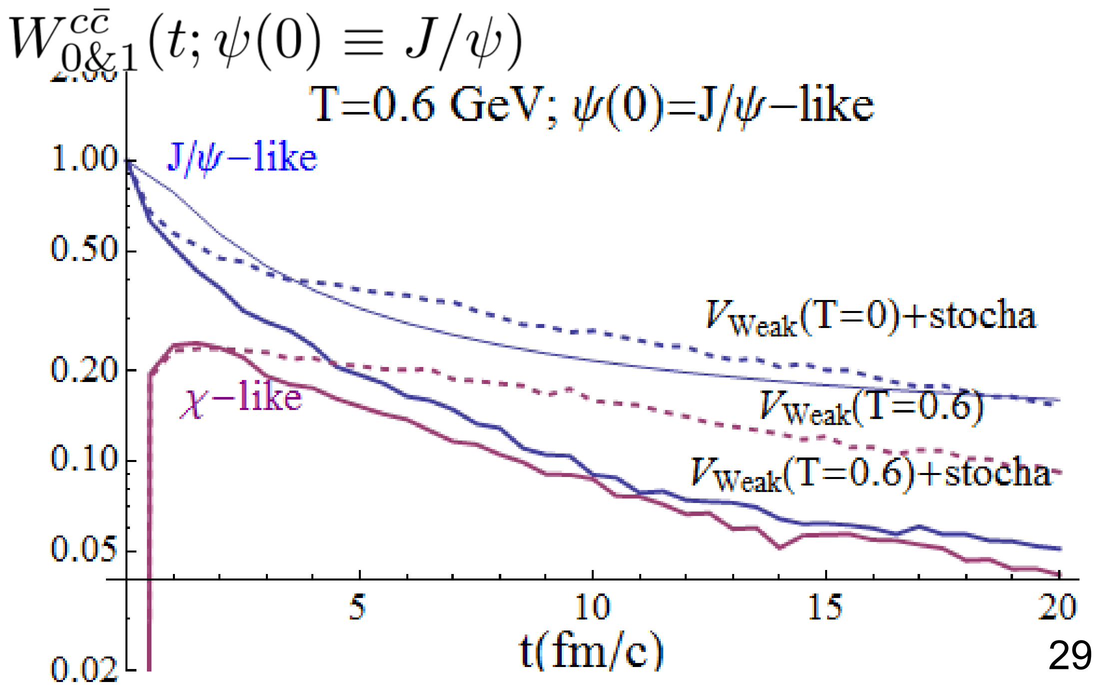
No (2S) state in the medium, but projection on T=0-(2S) does not necessarily vanishes

Evolution of the weights with V(T) and initial eigenstate

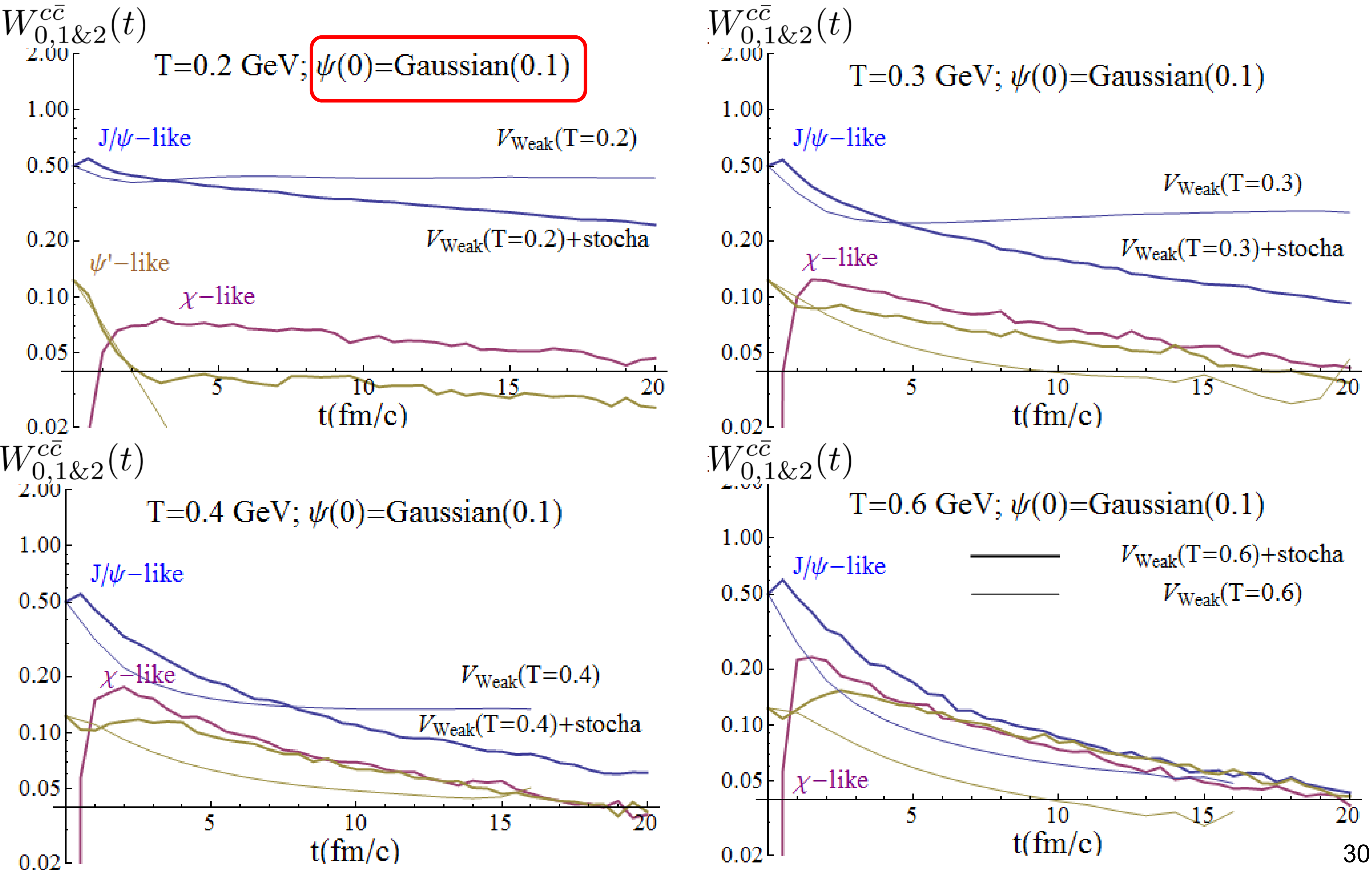




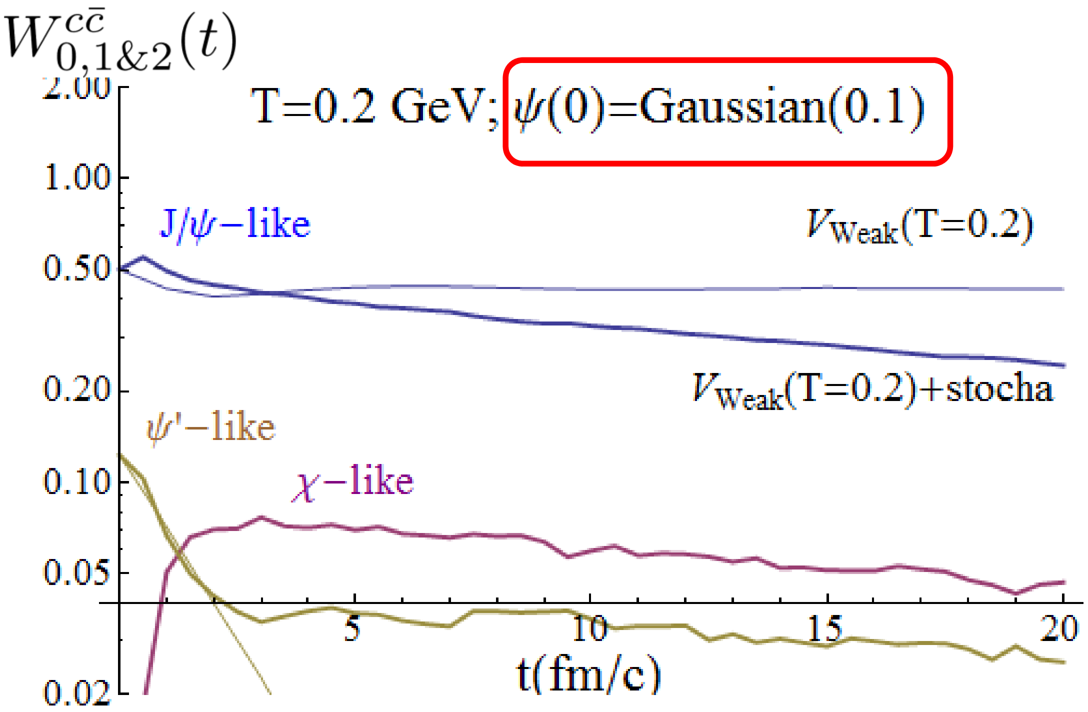
- Same features as with V(0), but…
- …both features combine to lead to higher suppression
- ⇔ Asymptotic decay proceed with larger "width" Γ
- Saturation of Γ for large T (D_s decrease at large T)

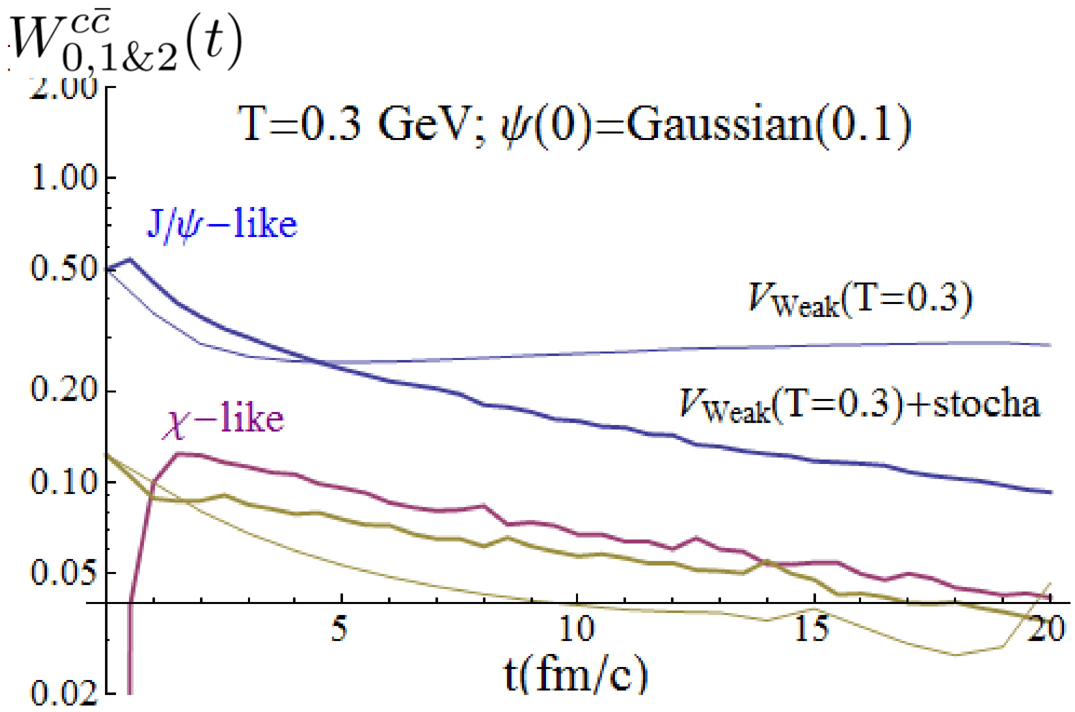


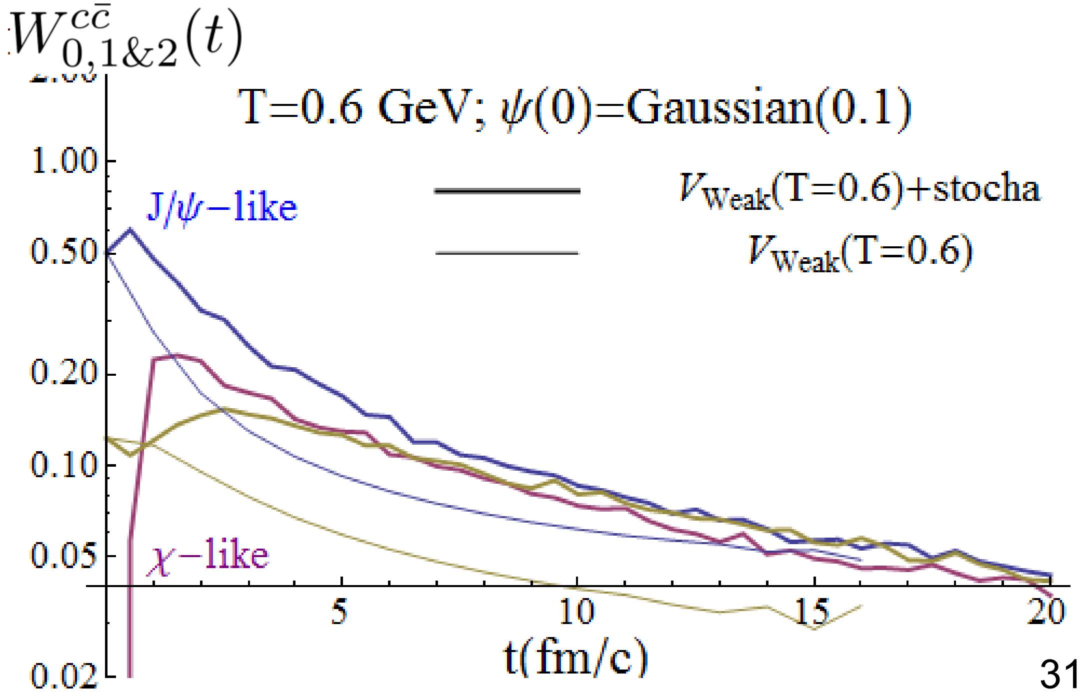
weights for V(T) and **more realistic initial state**



weights for V(T) and **more realistic initial state**



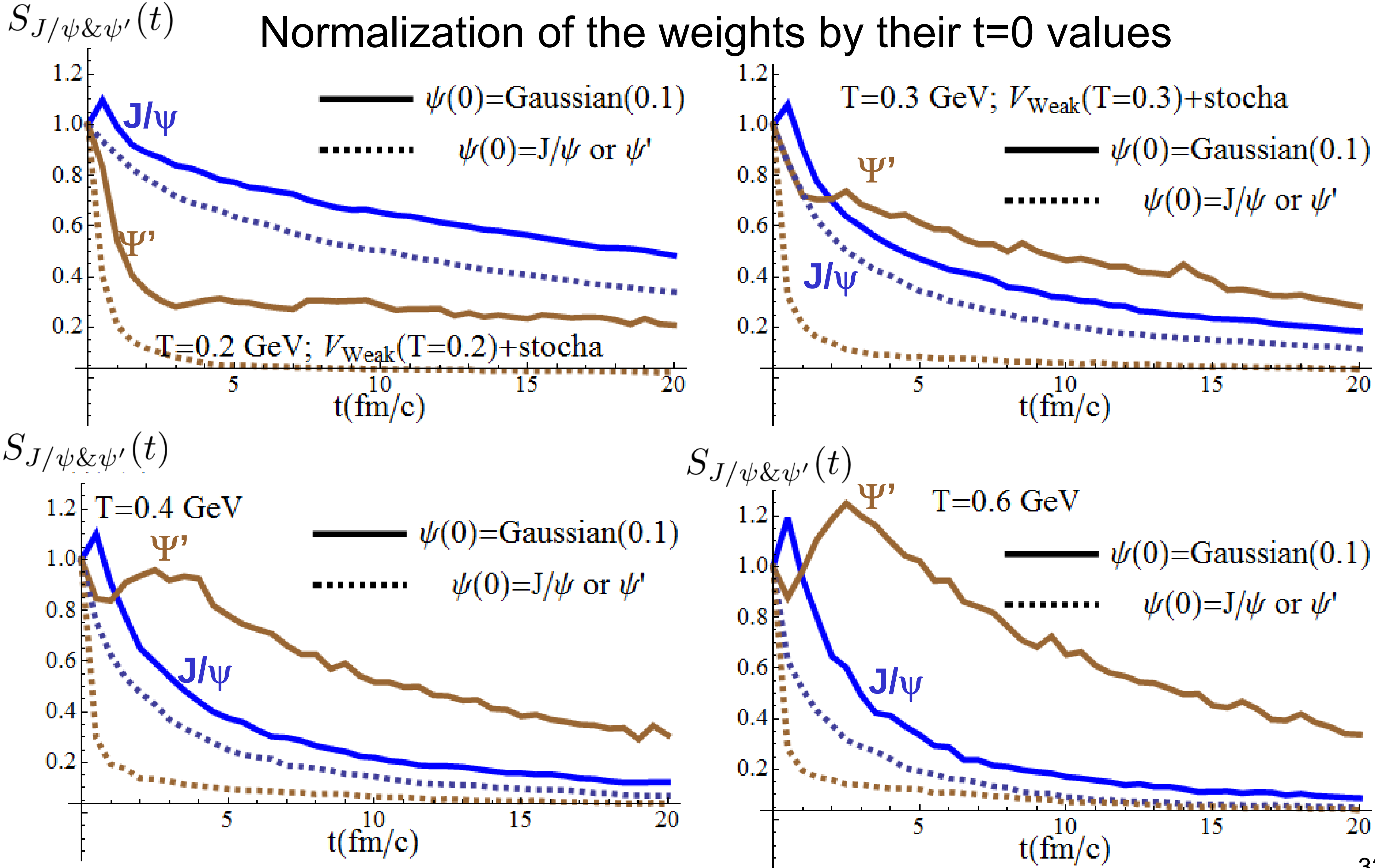




- As compared to the pure mean-field, the thermal forces can lead to an overpopulation of the initial J/ψ component at intermediate times (also true for other components)
- Universal long-time decay

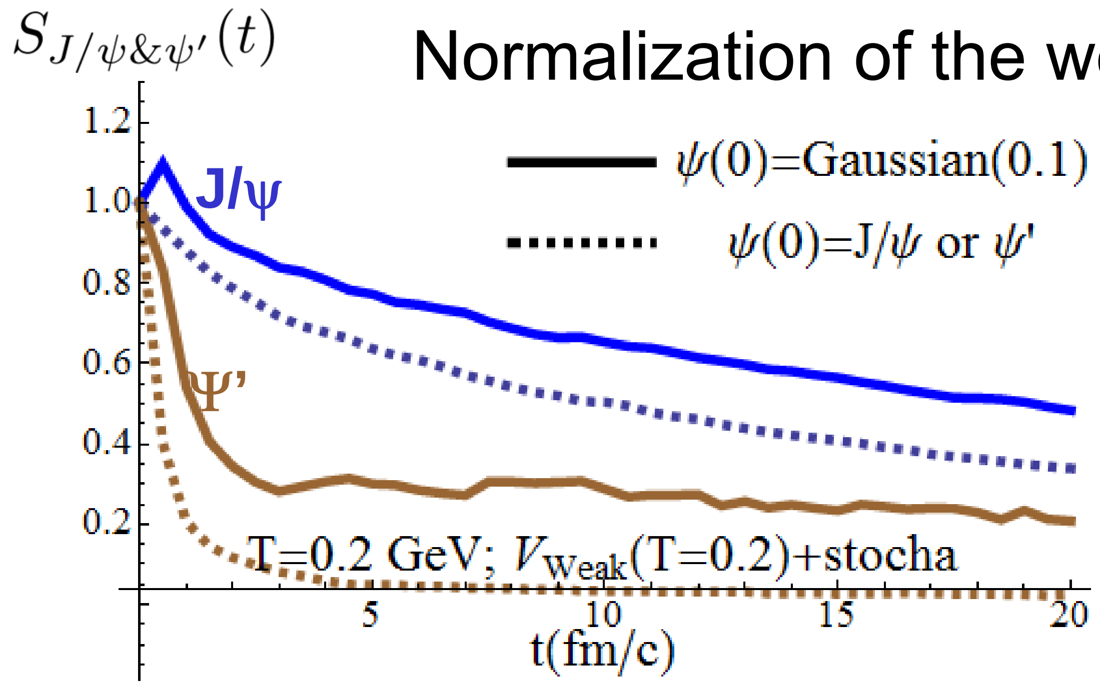
Suppression of states as a function of time ($1c\bar{c}$ in the HB)

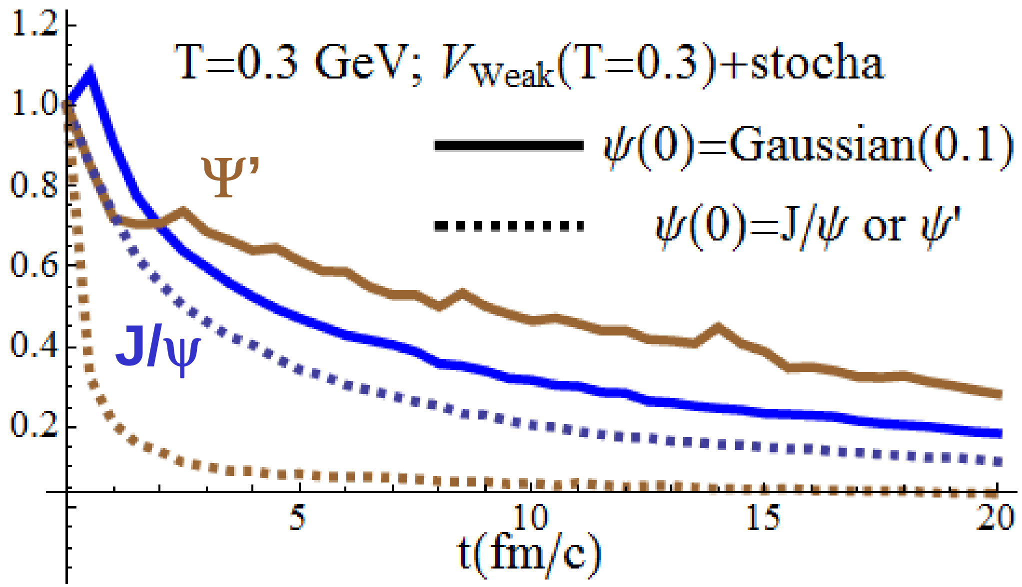
Normalization of the weights by their t=0 values

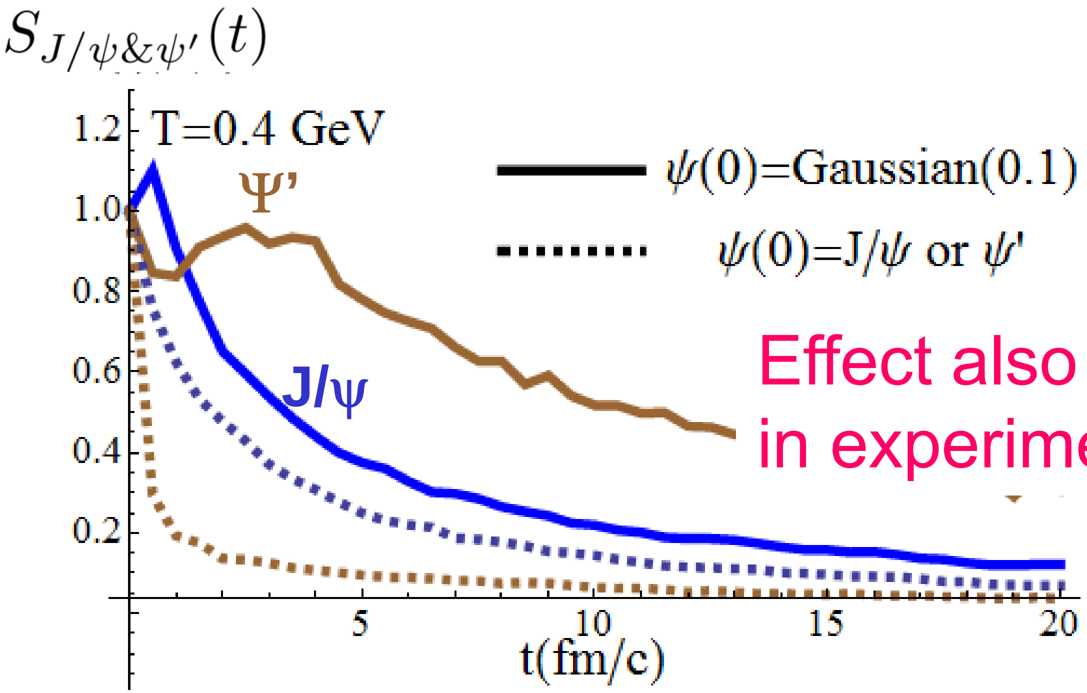


Suppression of states as a function of time (1$c\bar{c}$ in the HB)

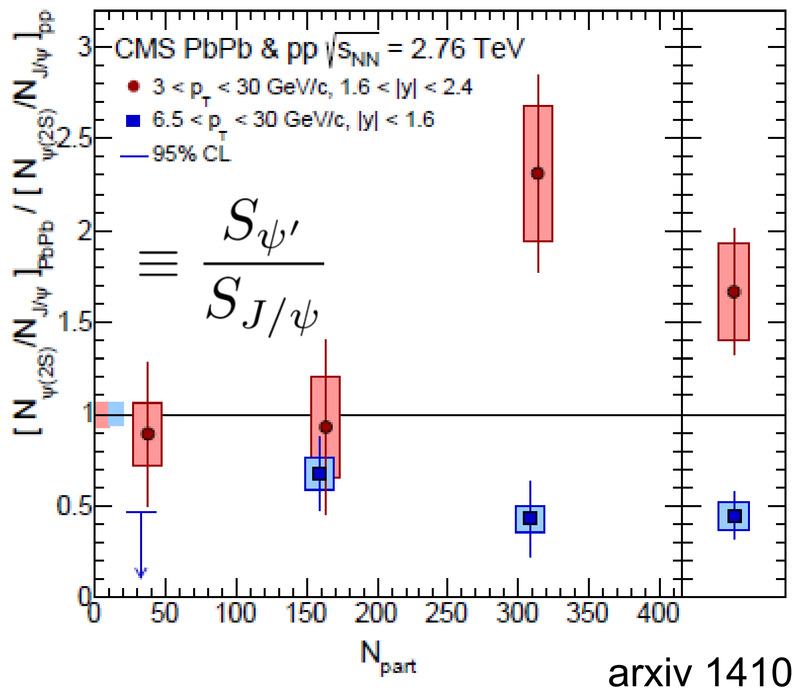
Normalization of the weights by their t=0 values







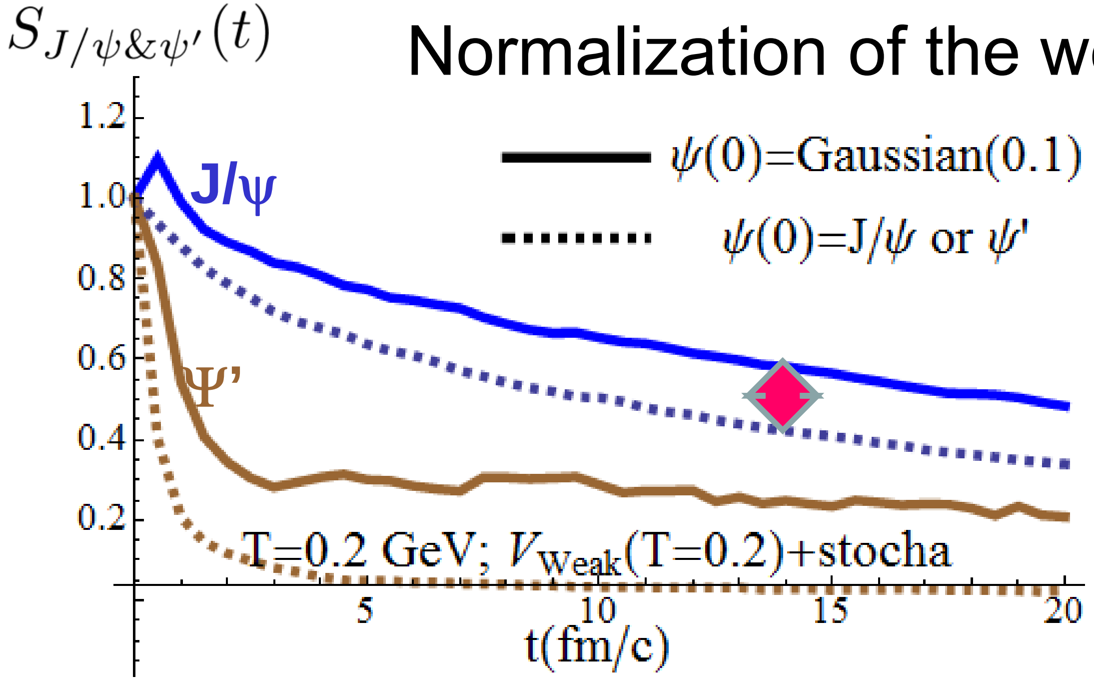
Effect also seen in experiment ?

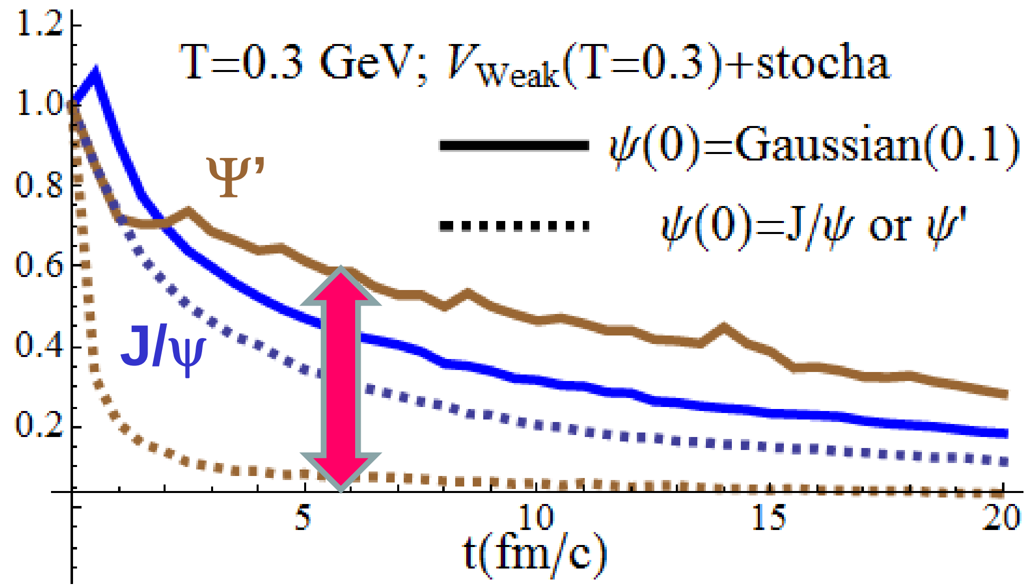


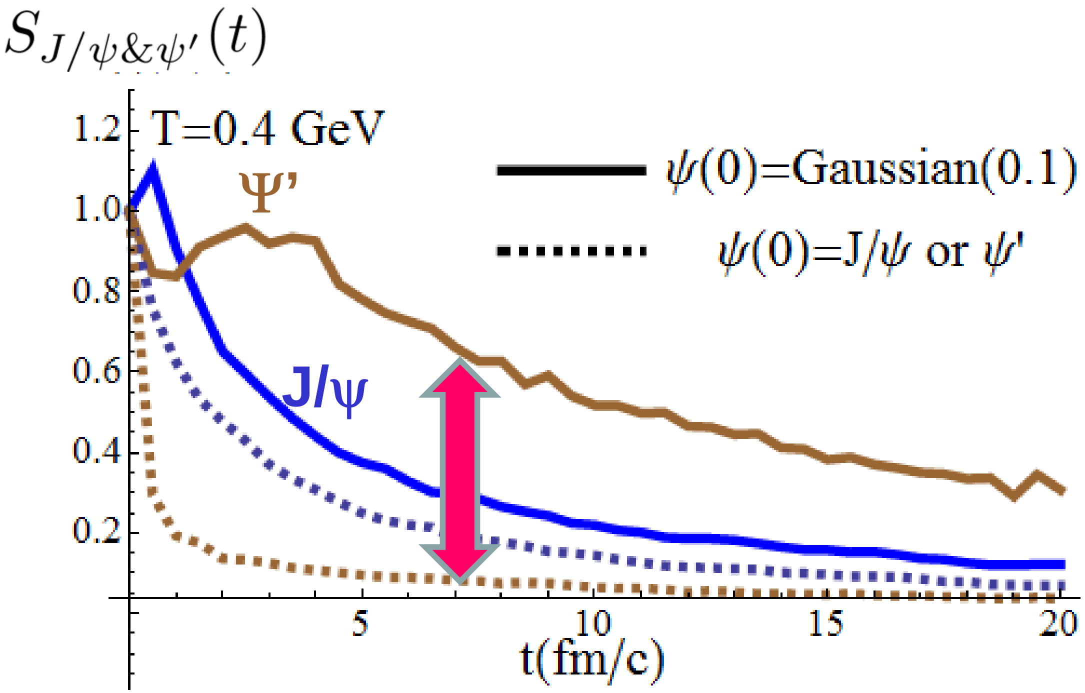
arxiv 1410.1804

Suppression of states as a function of time (1$c\bar{c}$ in the HB)

Normalization of the weights by their t=0 values

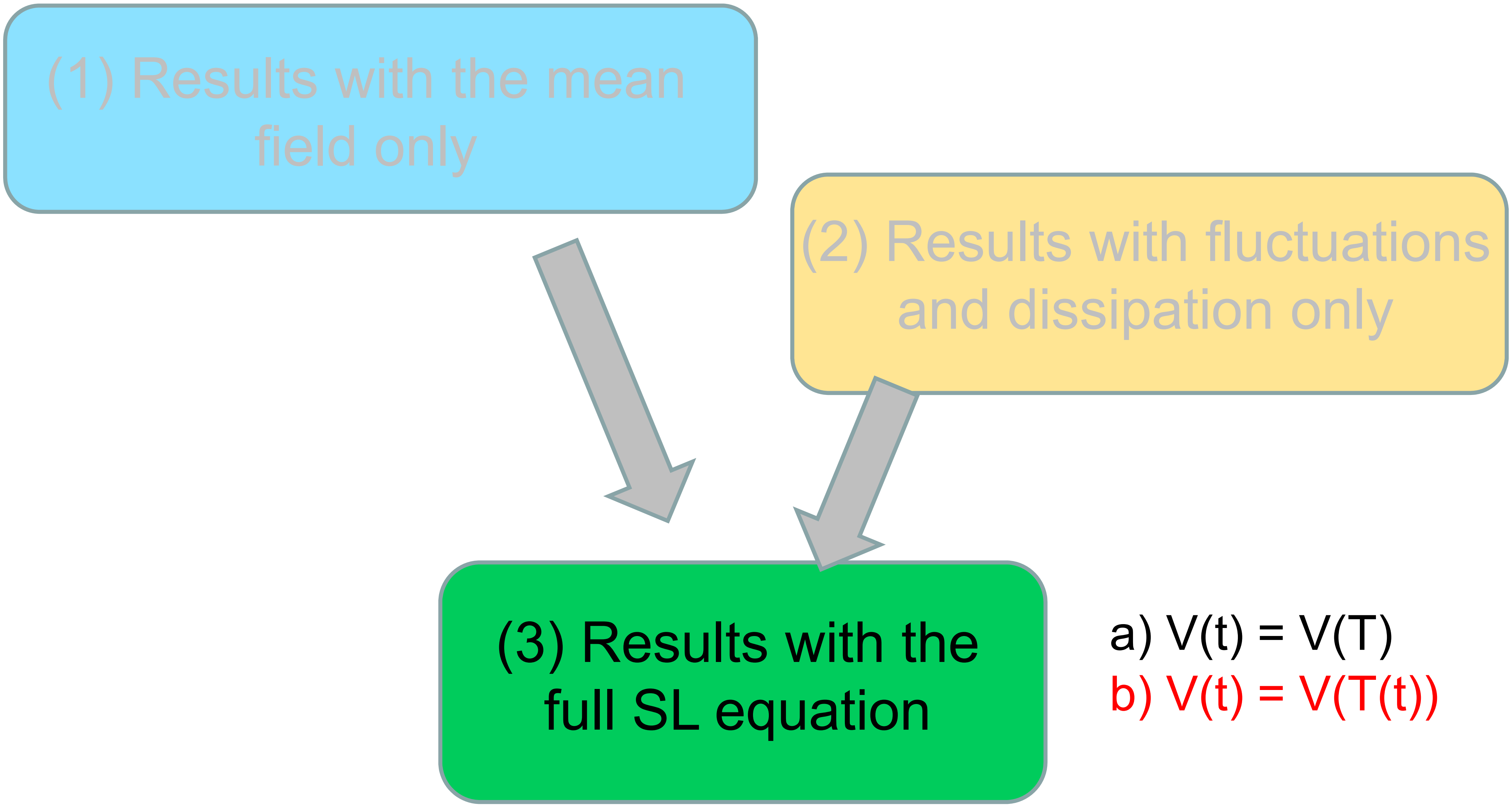






- Our understanding: can only be due to the quantum nature of the ccbar system
- S vastly depends on the initial quantum state !!! Kills the (unjustified assumption) of quantum decoherence at t=0

Road map

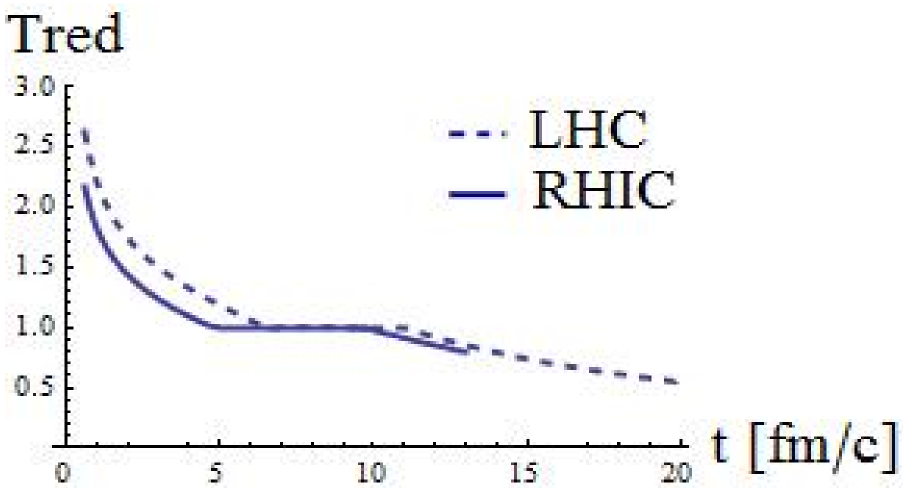


a) $V(t) = V(T)$

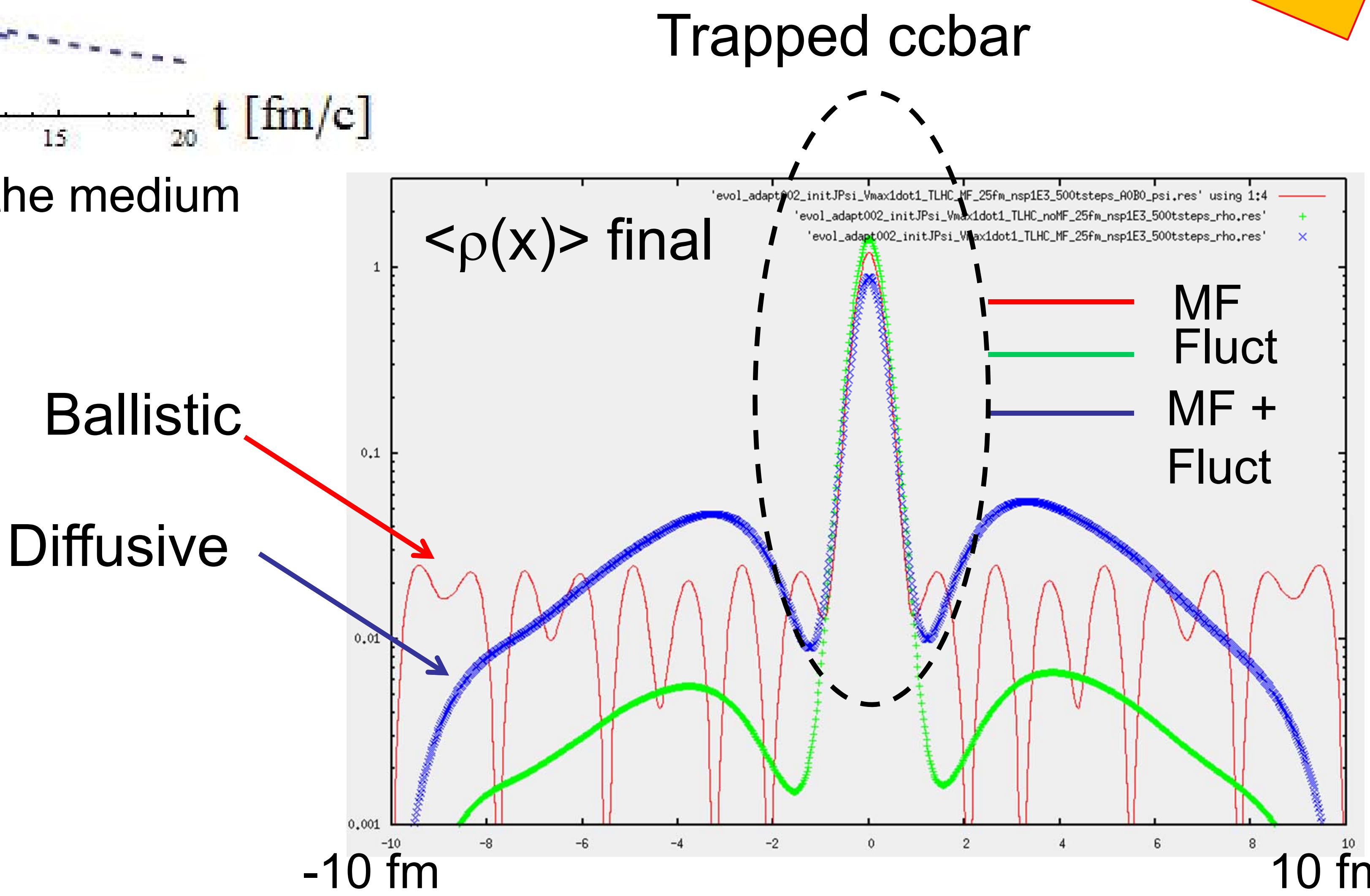
b) $V(t) = V(T(t))$

density for $V(T_{LHC}(t))$ and initial J/ψ (like)

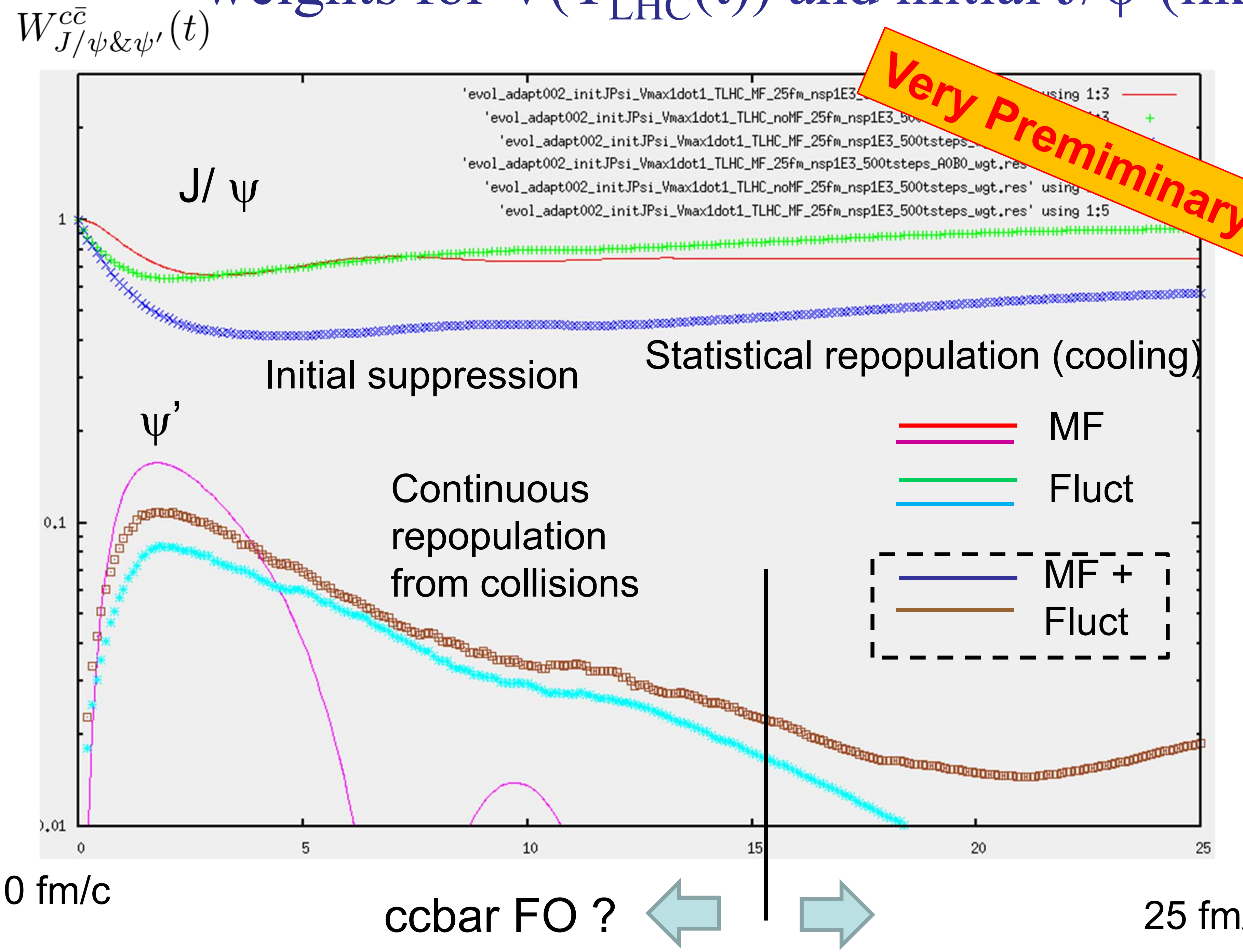
Very Preminimary



At the hottest of the medium

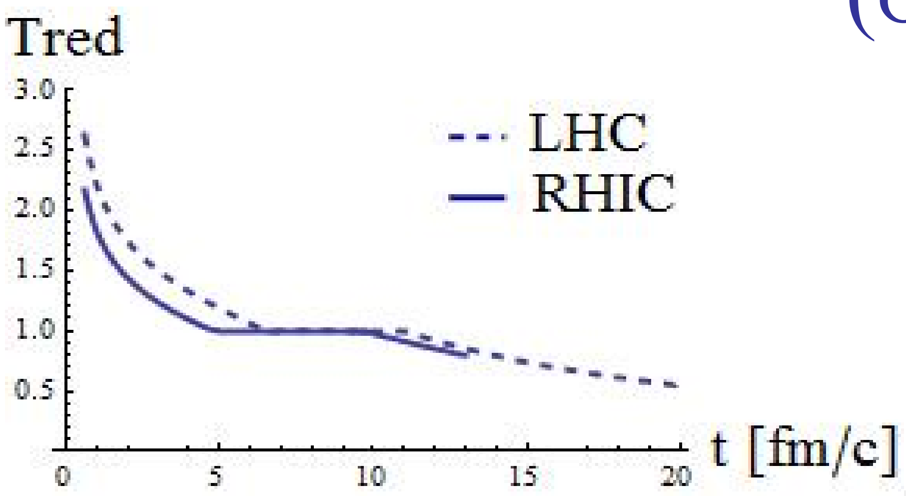


weights for V(T_{LHC}(t)) and initial J/ψ (like)



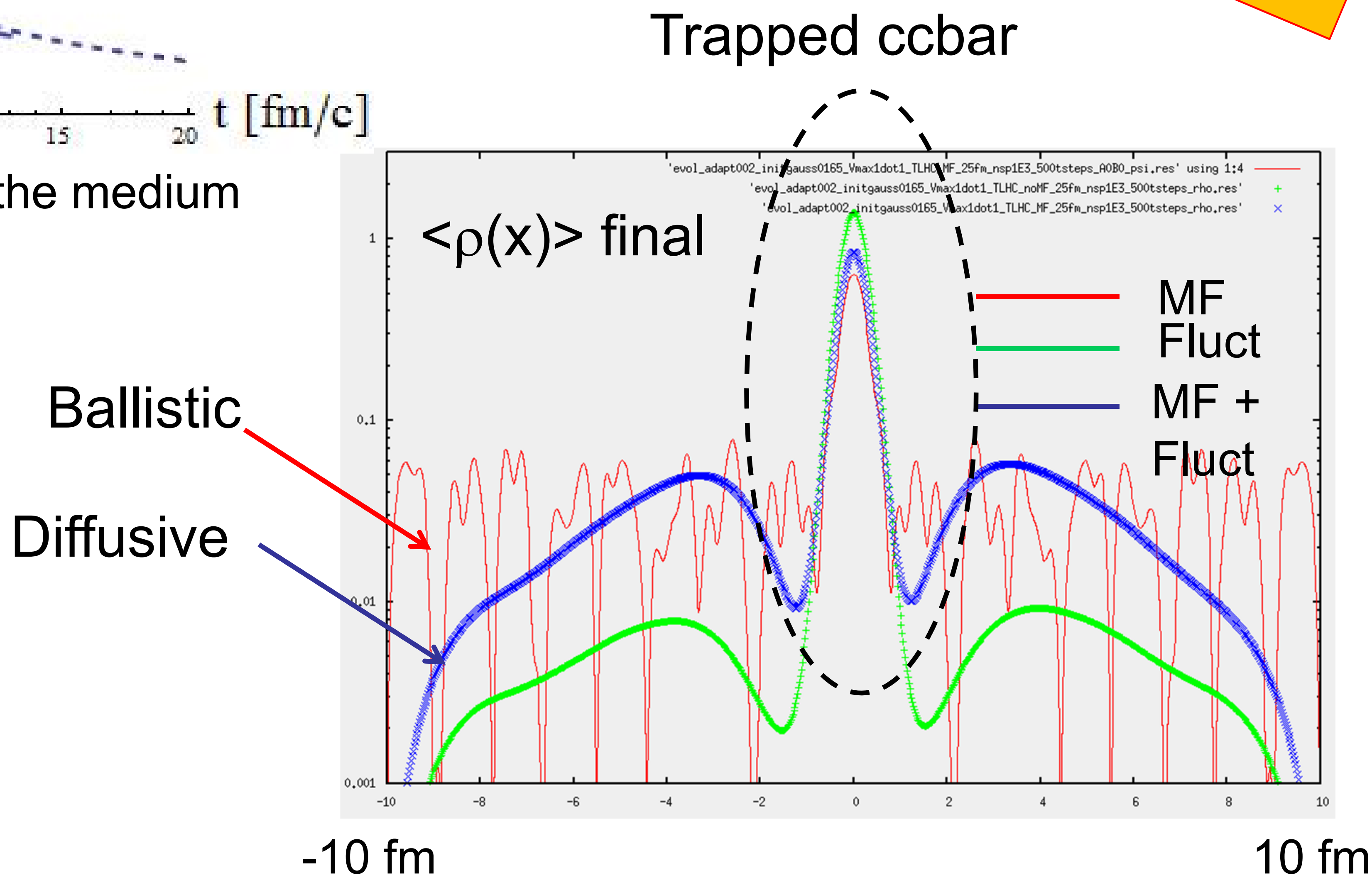
Contrarily to V(T): opposite trends for J/ψ and ψ'

density for $V(T_{LHC}(t))$ and initial Gaussian state (σ=0.165fm)



At the hottest of the medium

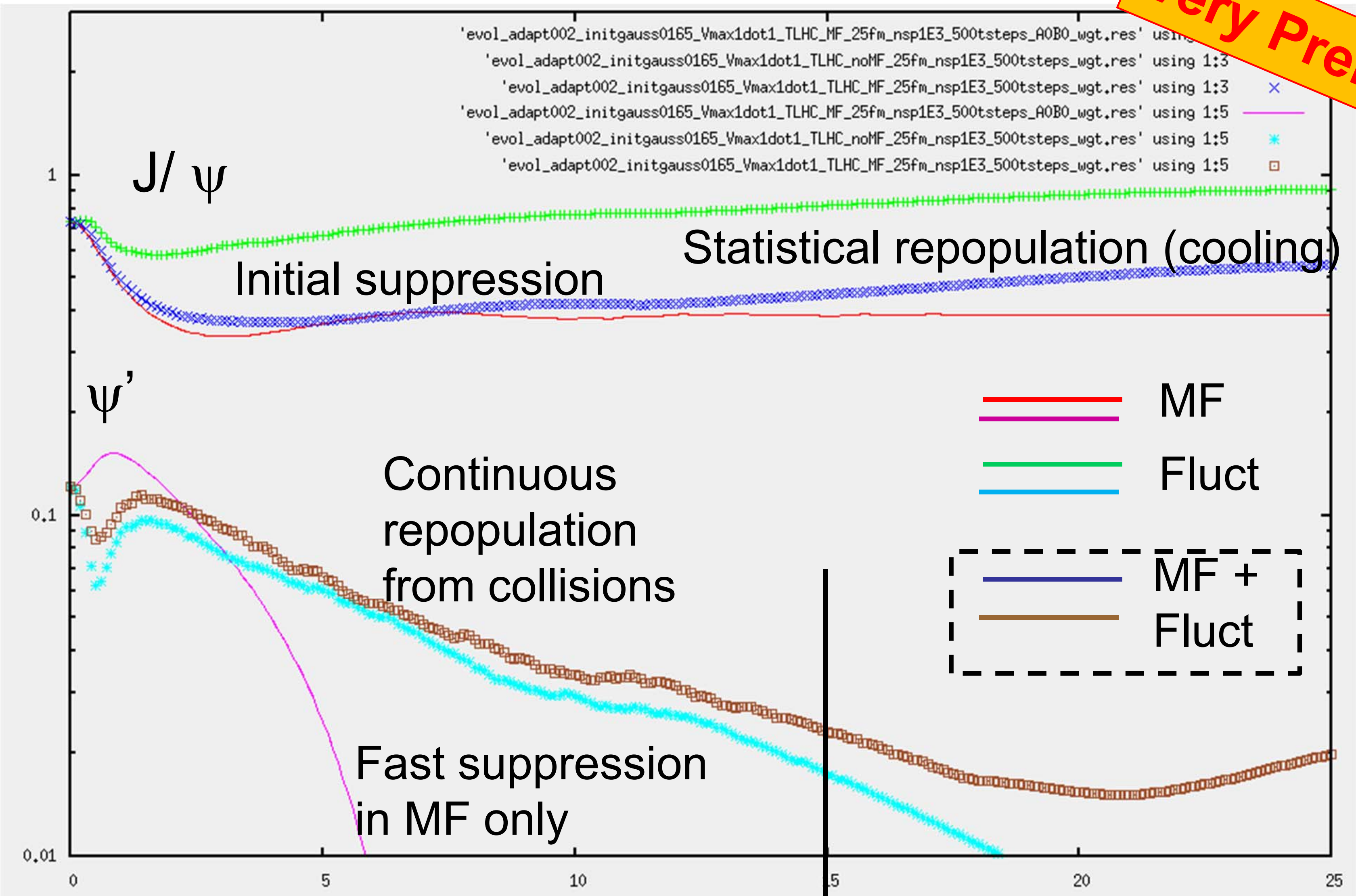
Very Preminiary



weights for V(T_{LHC}(t)) and initial Gaussian state (σ=0.165fm)

$W^{c\bar{c}}_{J/\psi\&\psi'}(t)$

Very Preminiminary



At final stage

$$\frac{S_{\psi'}}{S_{J/\psi}} < 1$$

for this particular T(t)

0 fm/c

ccbar FO ?

25 fm/c

Conclusions and Future

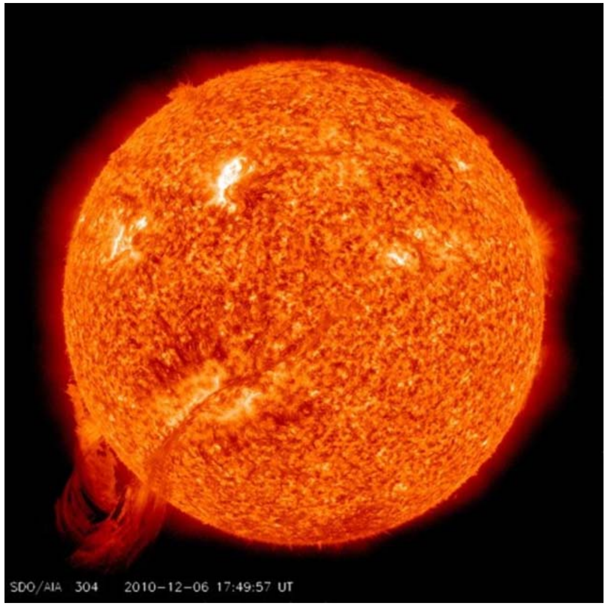
- Framework satisfying all the fundamental properties of quantum evolution in contact with a heat bath, “easy” to implement numerically
- First tests passed with success
- Rich suppression pattern found both in all types of environments, go much beyond standard simplifying assumptions (f.i. in-medium cross sections)
- Assumption of early decoherence: ruled out.

- Future:
  - Identify the limiting cases and make contact with the other models (a possible link between statistical hadronization and dynamical models)
  - Implementation in evolution scenario of a 4D QGP
  - Make contact with NRQCD

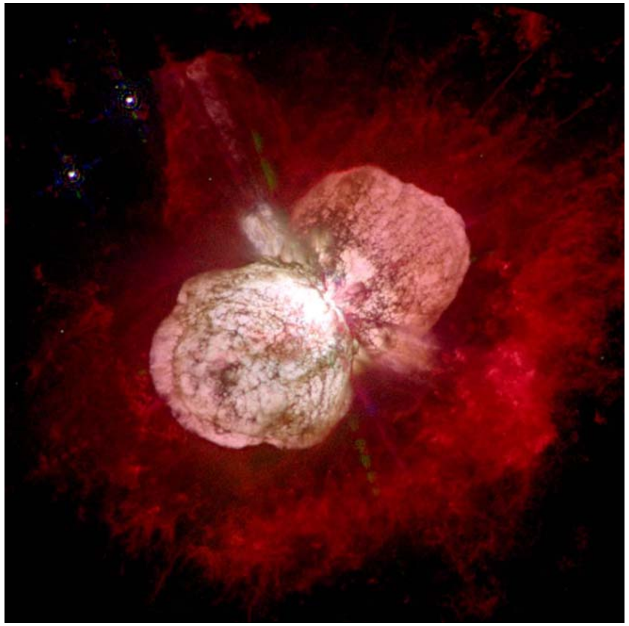
Back up

Caviats & Uncertainties

What does the sequential suppression in a stationary QGP has to do with reality anyhow ?



Picture

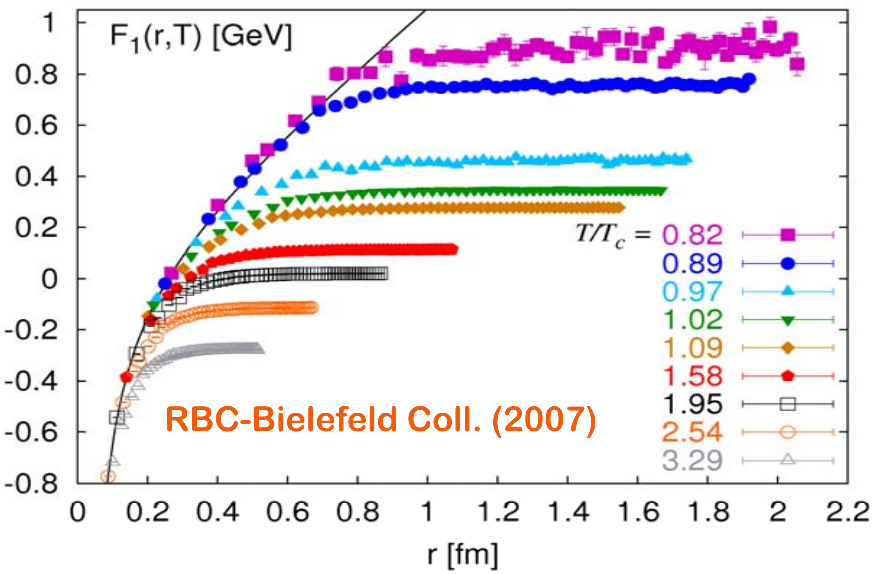


Reality

Need for a genuine time-dependent scenario

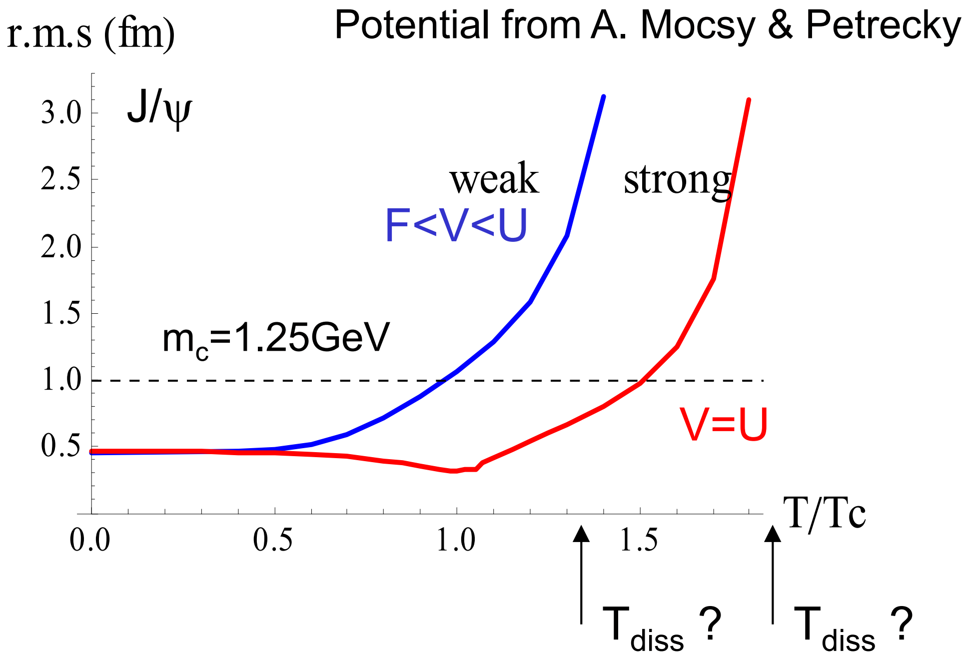
Caviats & Uncertainties

I. Quarkonia in *stationnary* medium are not well understood from the fundamental finite-T LQCD



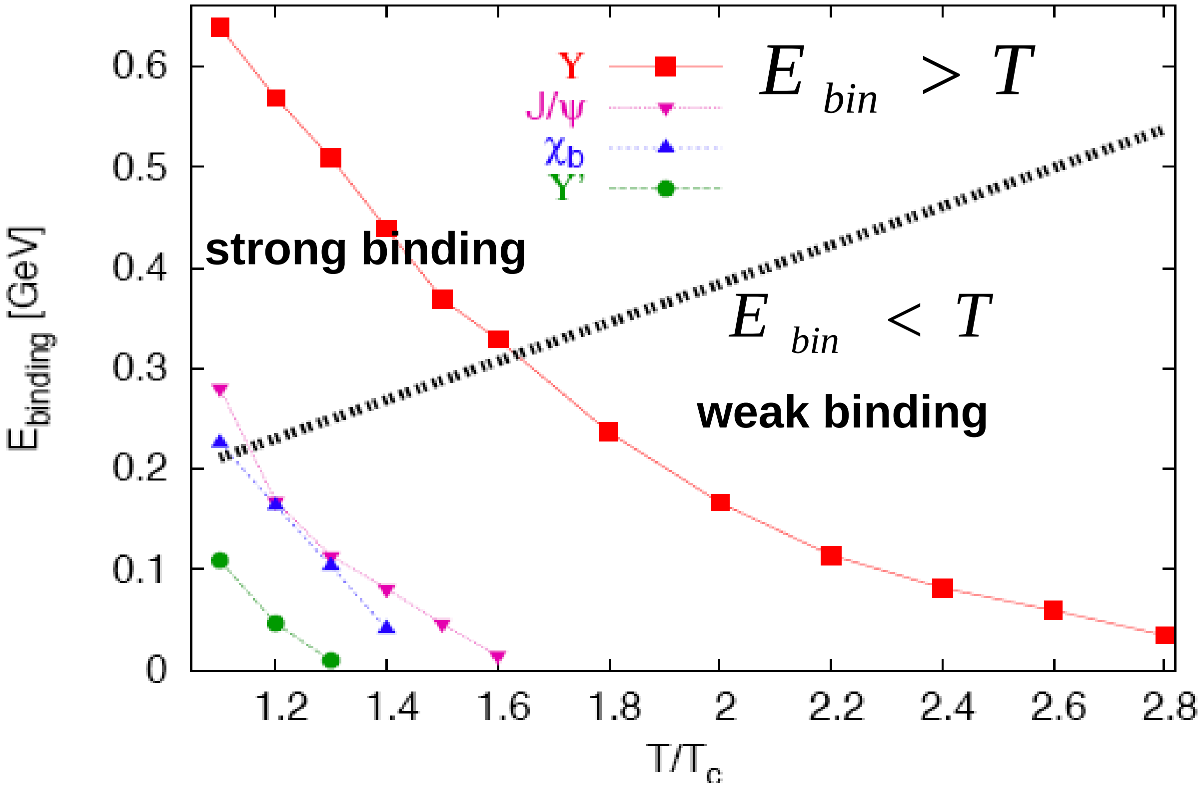
From free energy $\Rightarrow$ V(r,T) ?

Several prescriptions in litterature



Caviats & Uncertainties

II. Criteria for quarkonia "existence" (as an effective degree of freedom) in *stationnary* medium is even less understood

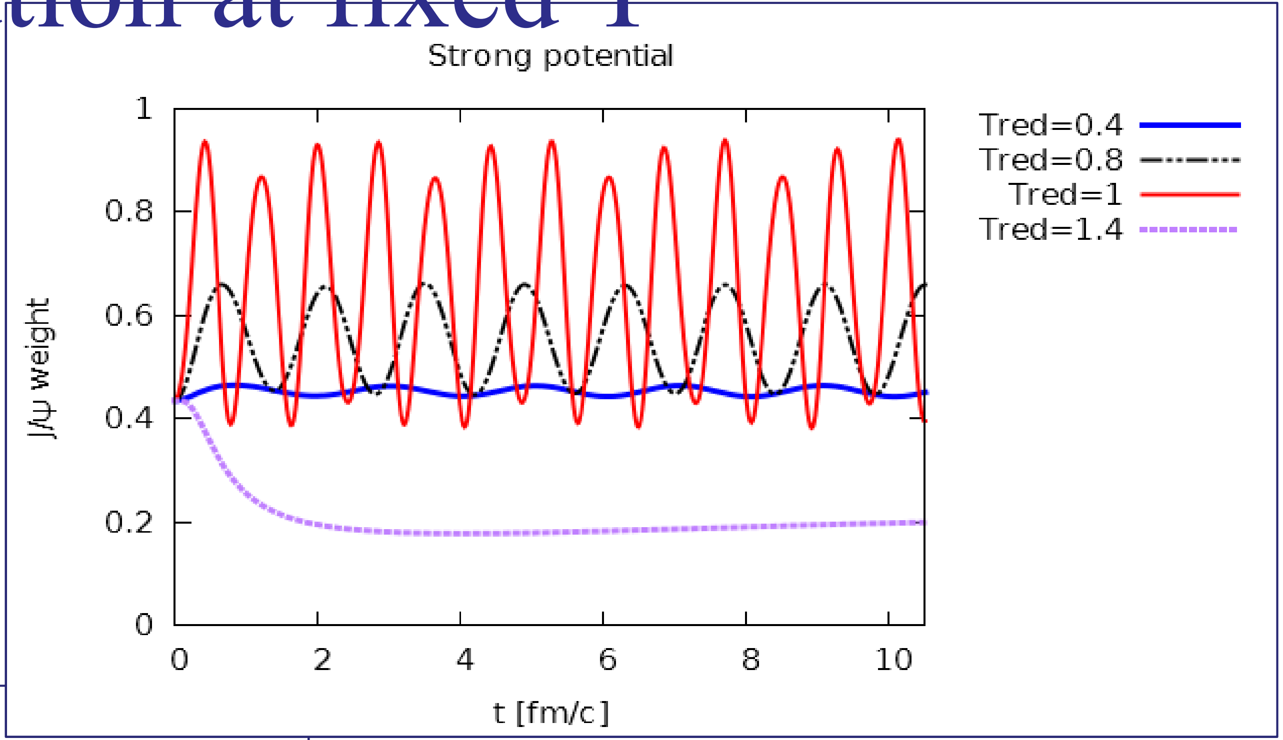


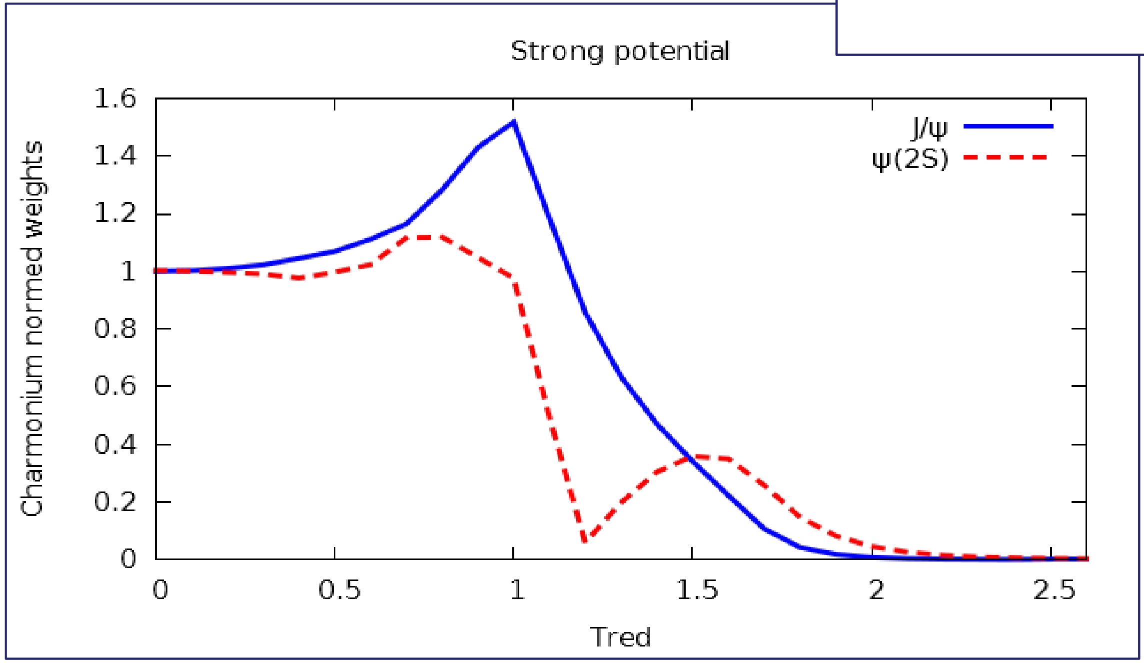
From A. Mocsy (Bad Honnef 2008)

Evolution at fixed T

Charmonia and strong color potential (V=U)

At fixed temperatures



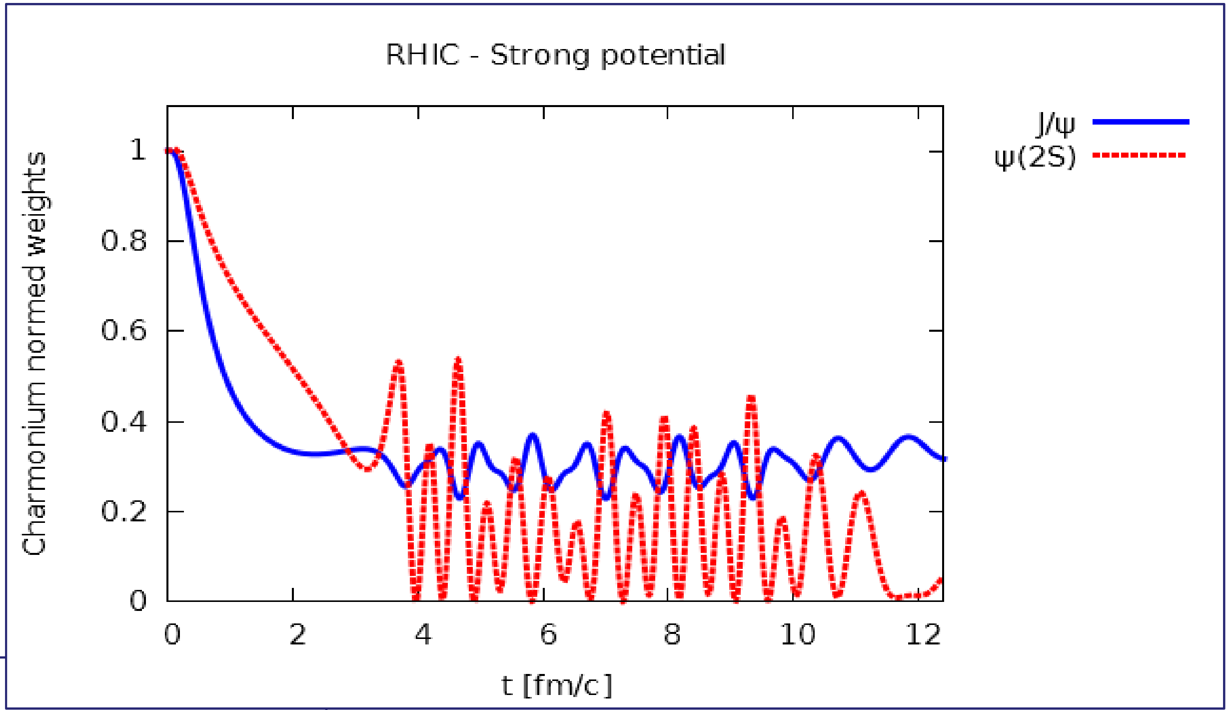


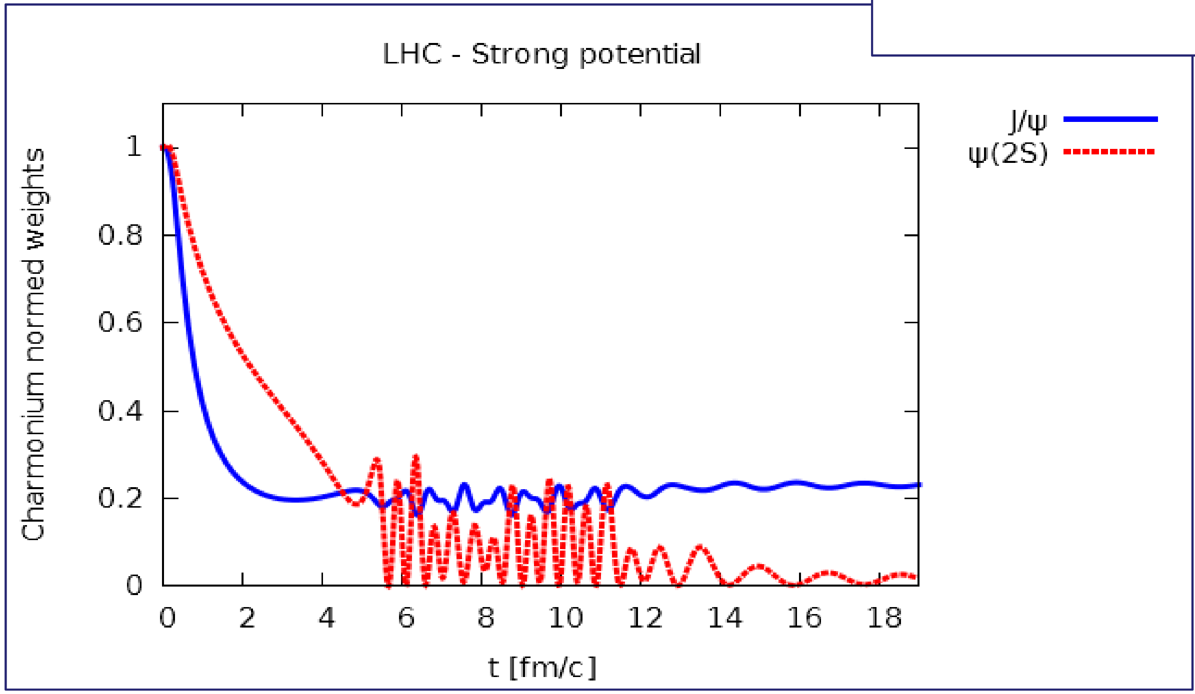
The normed weights at t->∞ function of the temperature

Evolution in realistic T scenarios

Charmonia and strong color potential (V=U)

RHIC temperature scenario



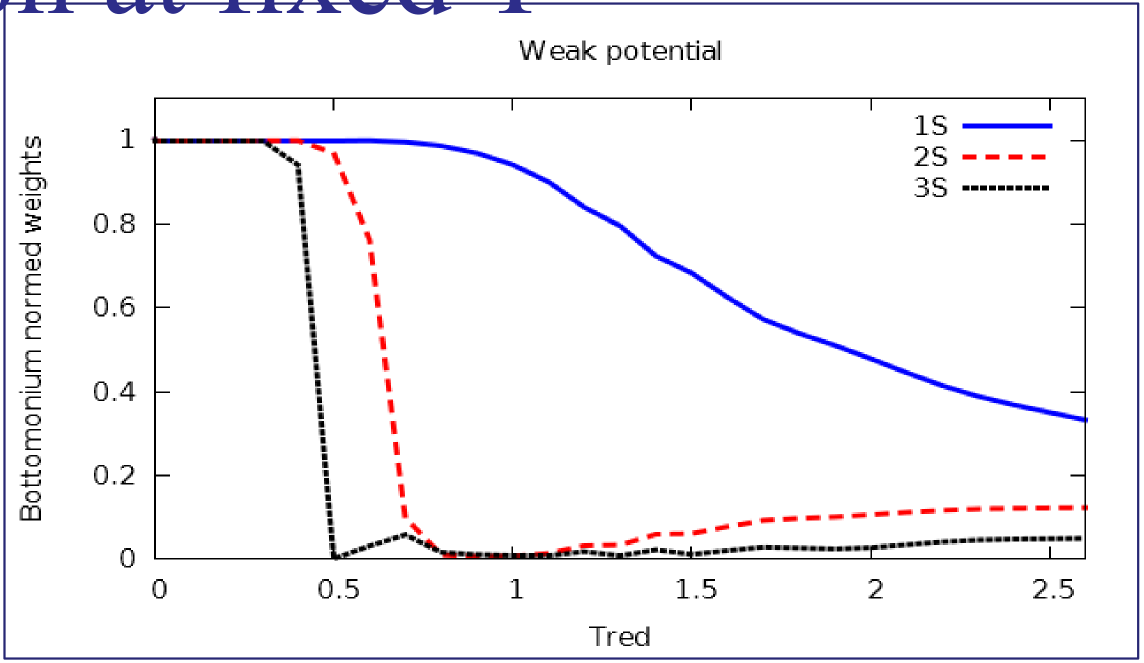


LHC temperature scenario

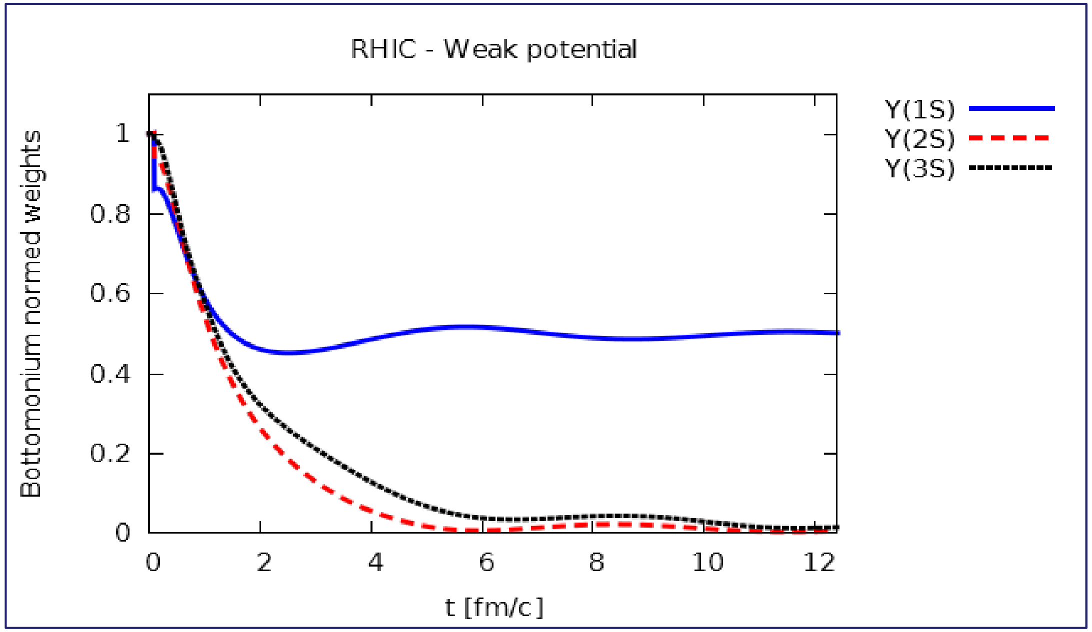
Evolution at fixed T

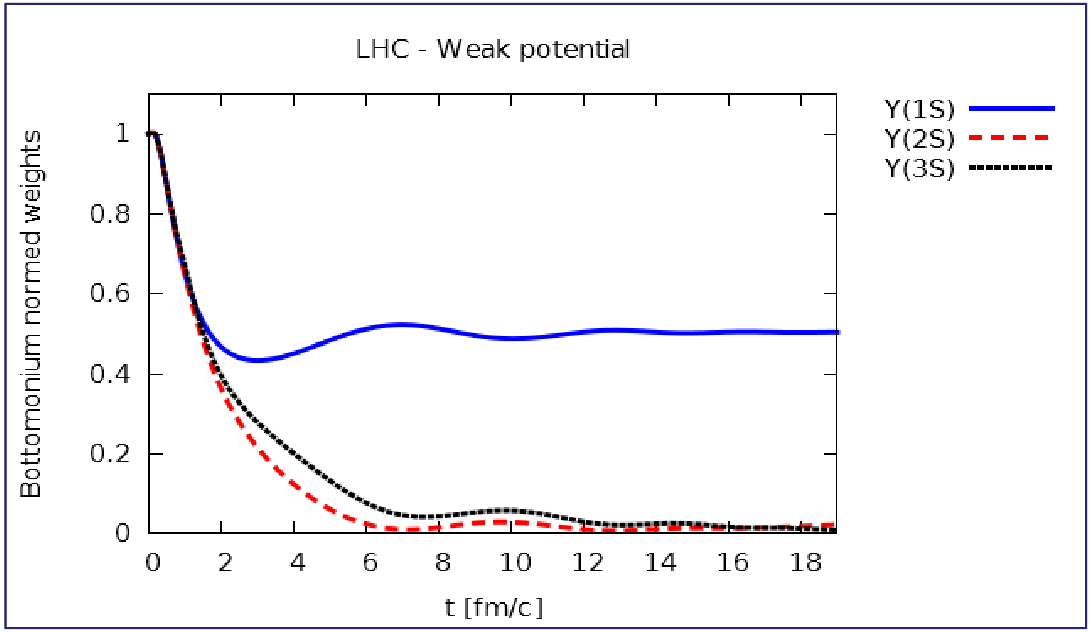
Bottomonia and weak color potential (F<V<U)

The normed weights at t->∞ function of the temperature



Temperature scenarios

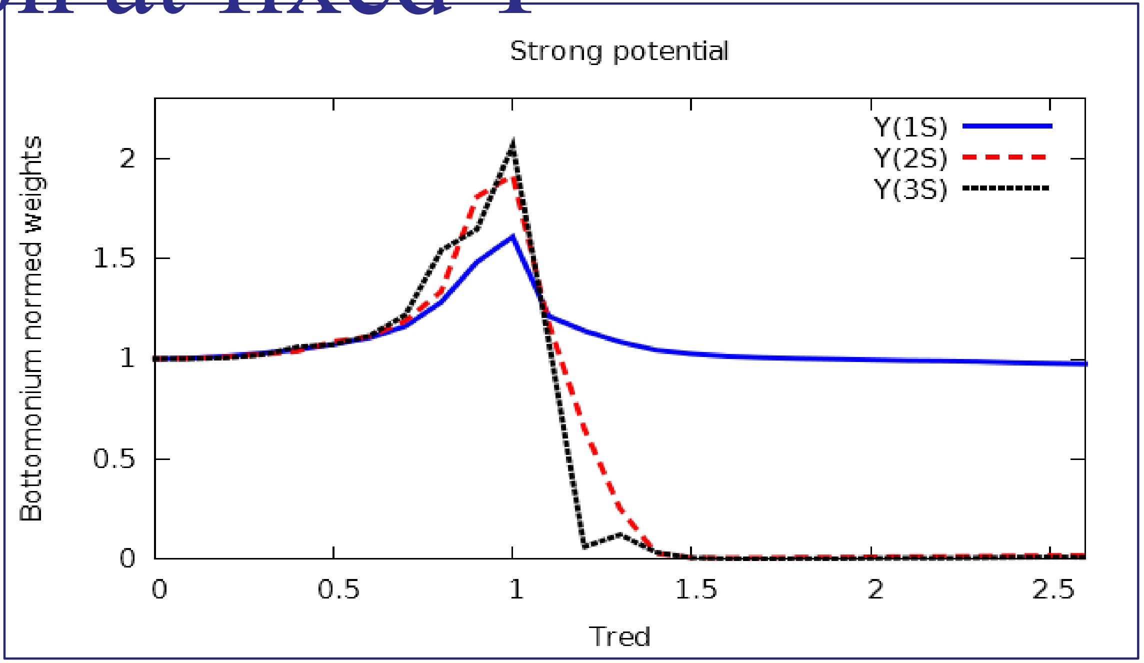




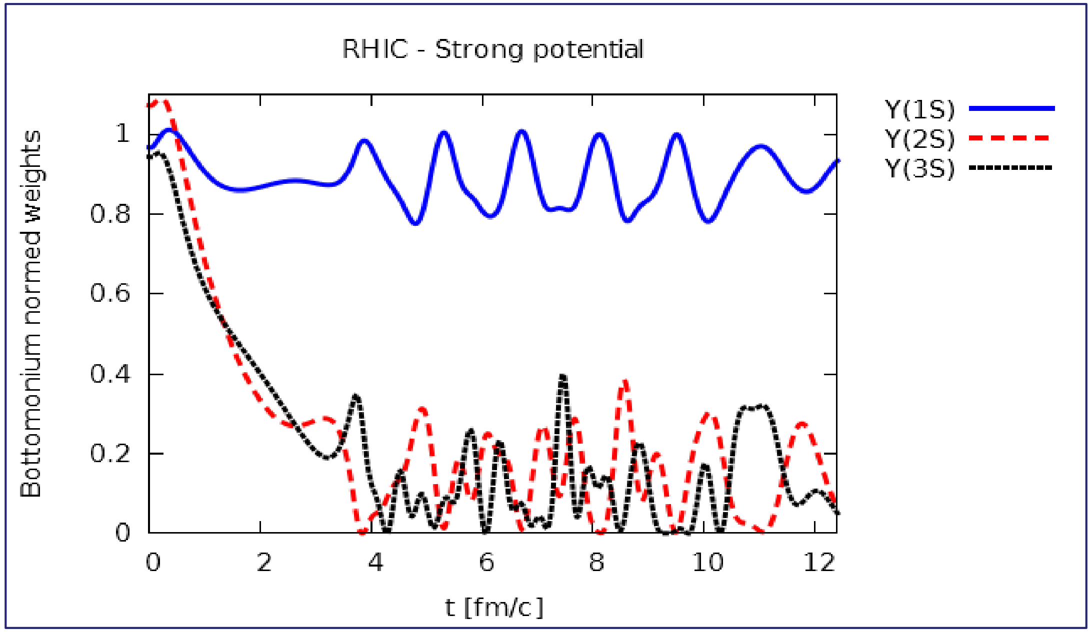
Evolution at fixed T

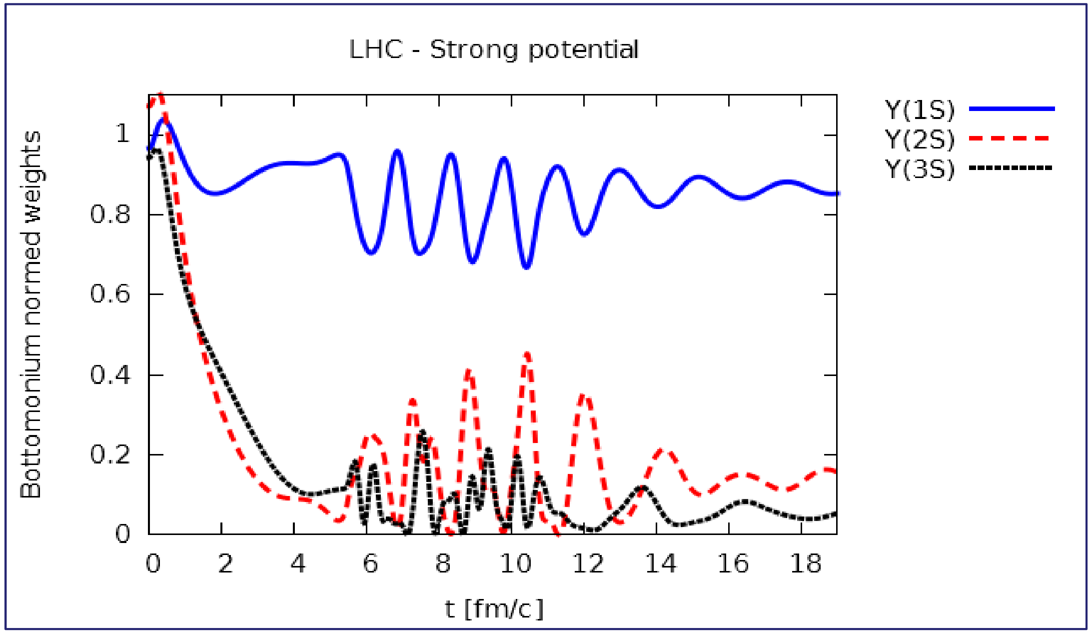
Bottomonia and strong color potential (V=U)

The normed weights at t->∞ function of the temperature



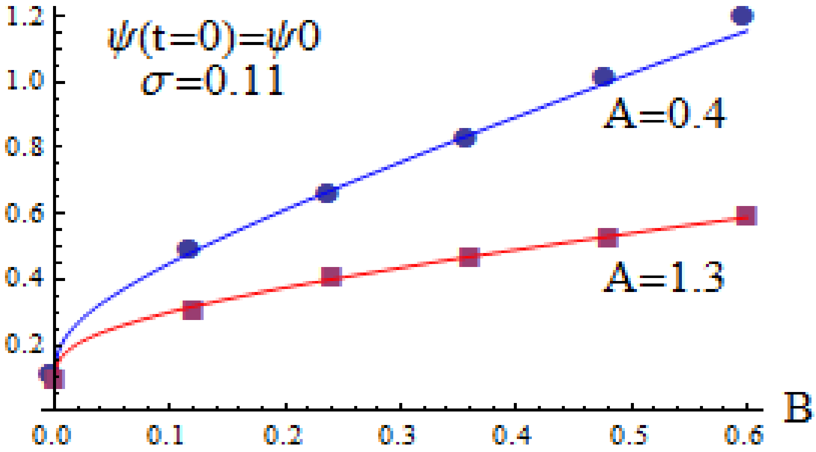
Temperature scenarios



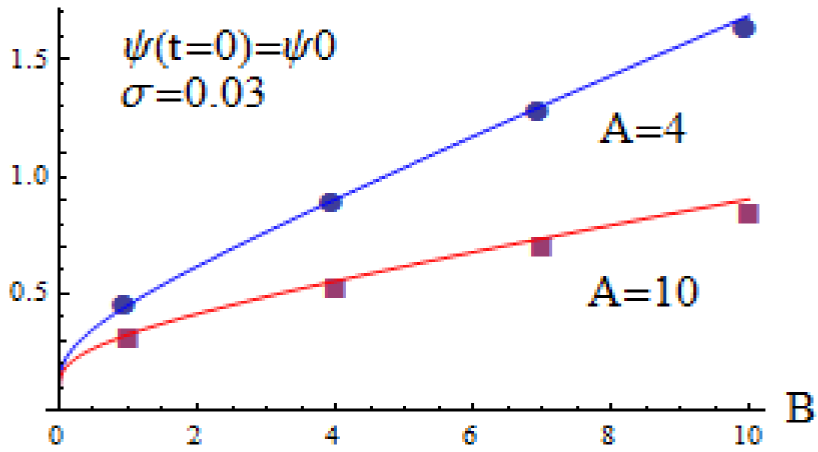


Effects of the autocorrelation

Measured temperature [ħω] at t->∞



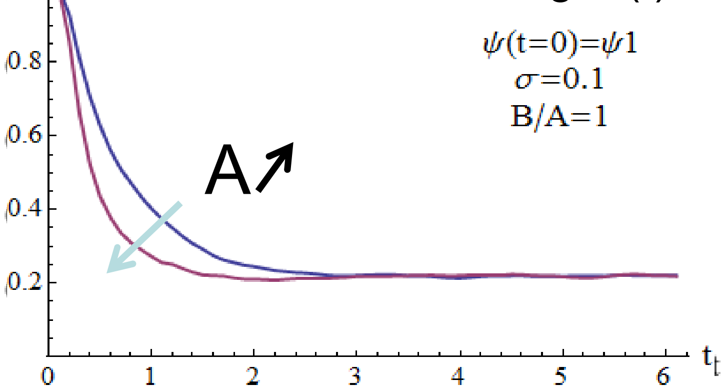
Measured temperature [ħω] at t->∞



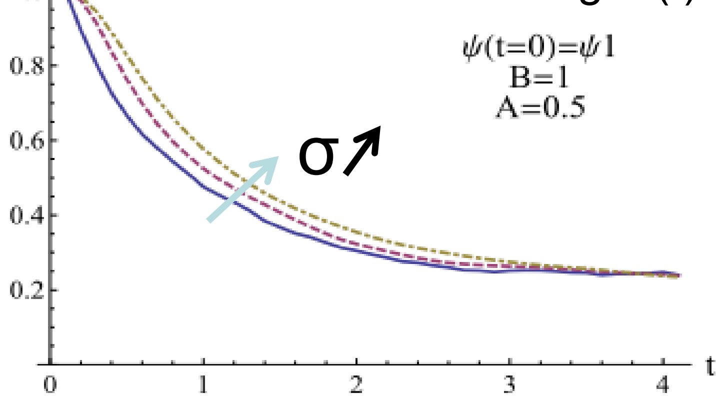
Ok for $\sigma \ll \tau_{\text{relax}}$

Tune B/A or σ to adjust the relaxation time

First excited state weight (t)



First excited state weight (t)



At a finite time:

high pt => high velocity => smaller σ => more excited states => more suppression

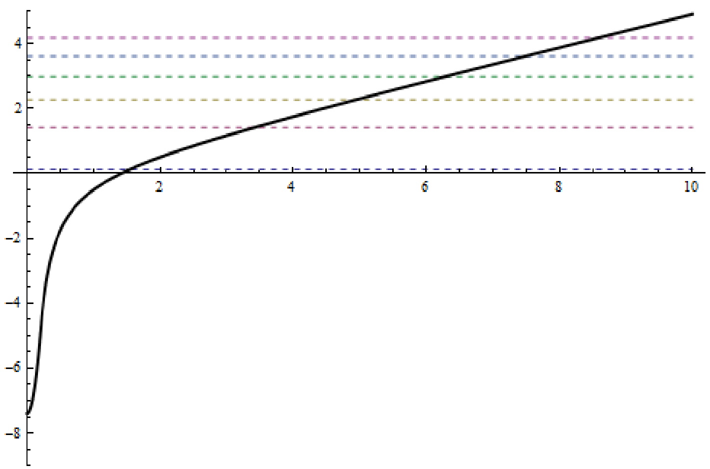
low pt => small velocity => higher σ => less excited states => less suppression <u>(=> need for regeneration ?)</u>

numerical tests of thermalization

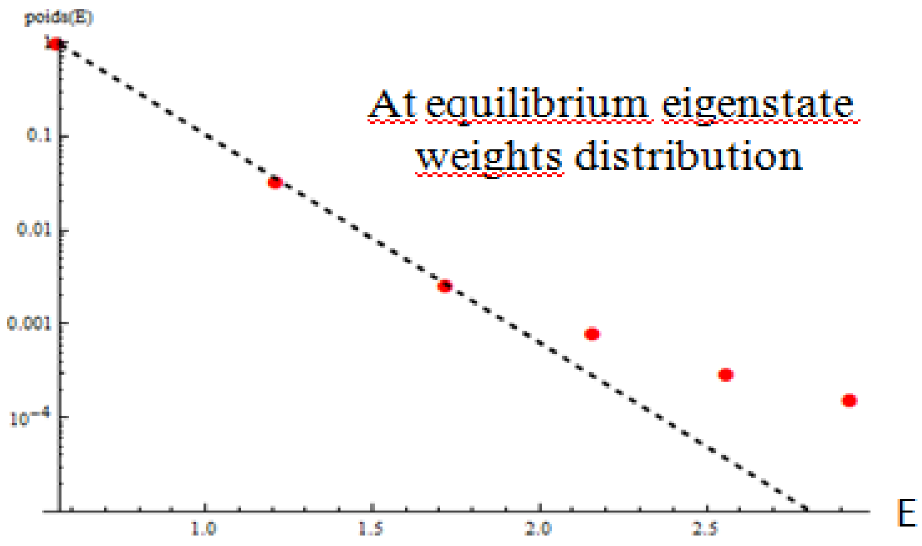
Other potentials

V(x)

Quarkonia approx



Asymptotic Boltzmann distributions ?

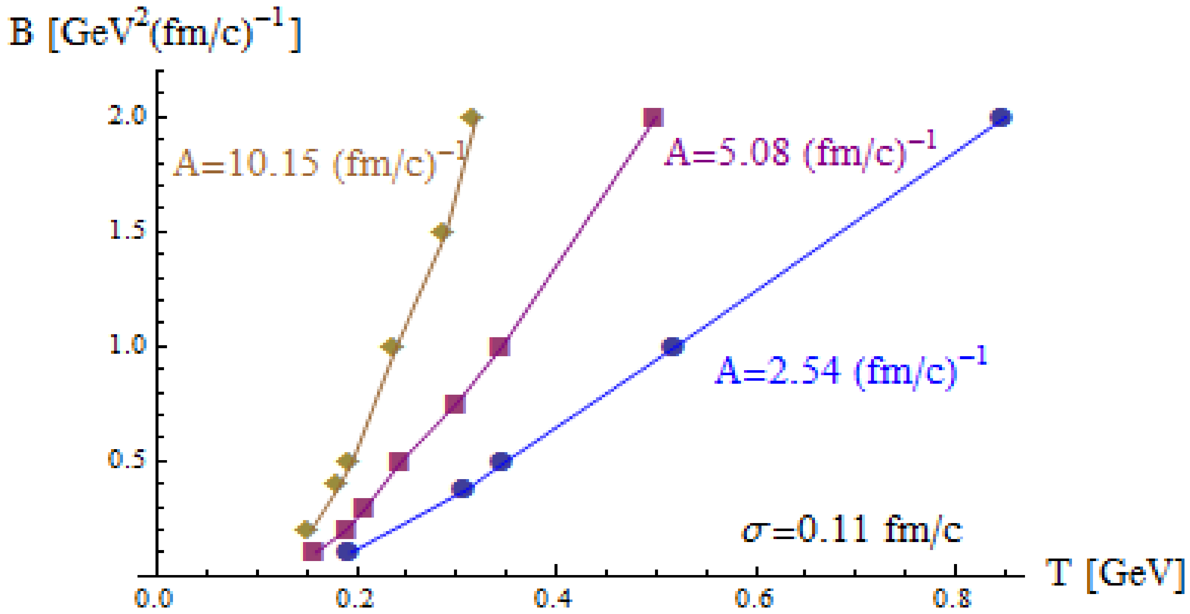


Yes; Light discrepancies from 3rd excited states for states at small T of the order of Tc

Tsallis distribution ?

Properties of the SL equation

Mastering numerically the fluctuation-dissipation relation for the Quarkonia approximated potential:



B univoquely extracted from (A,T) (as in usual quantum noise)

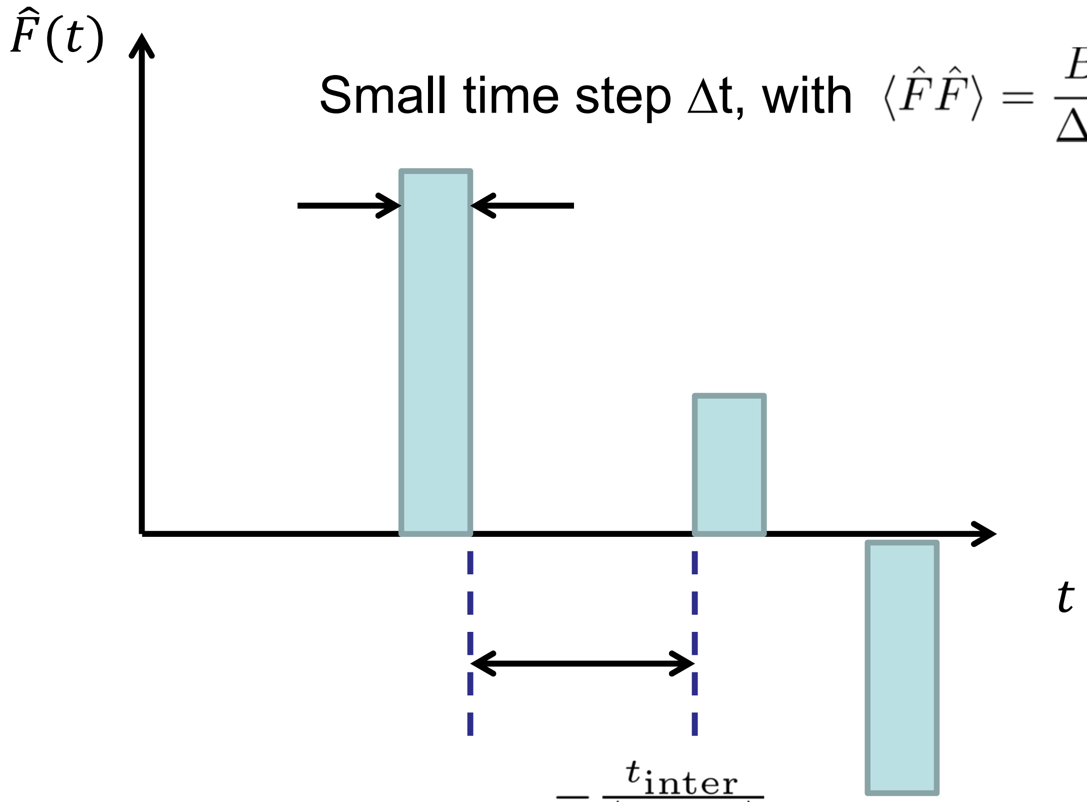
➢ Reducable to a small number of properties encoding the interactions with the heat bath:

- ❑ Temperature T
- ❑ Drag coefficient A
- ❑ Autocorrelation time σ

How legitimate (and legitimated) is it to use cross sections in *dense* medium ?

Simple toy model: Harmonic oscillator + external random forces $\hat{F}(t)$

Small time step Δt, with $\langle \hat{F}\hat{F} \rangle = \frac{B}{\Delta t}$



T_{inter}, distributed like $e^{-\frac{t_{inter}}{\langle t_{inter} \rangle}}$

Starting from ground state

After 1rst interaction $\quad \mathrm{Prob}(0 \to X) \propto \frac{B\Delta t}{m\omega}$

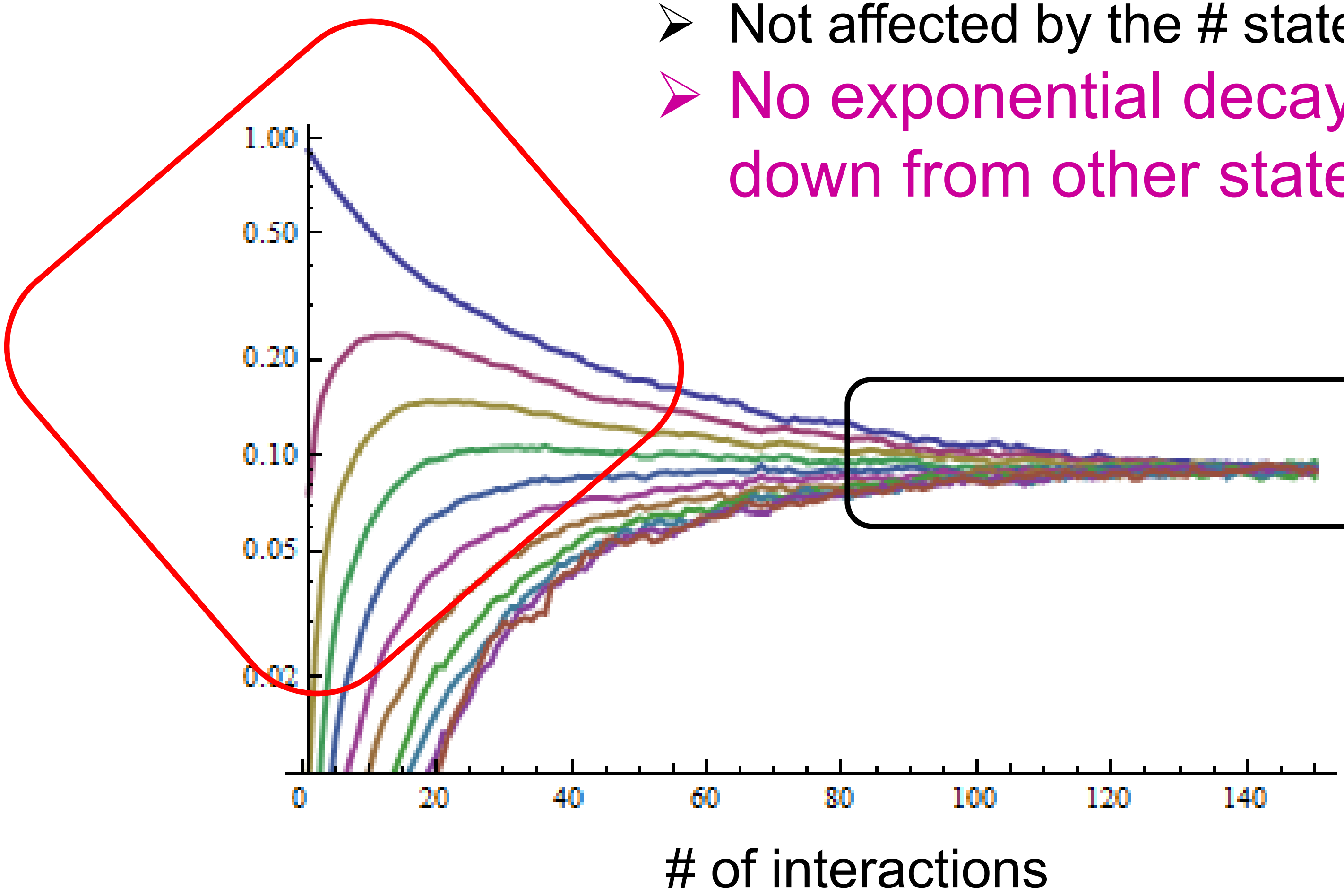
Markoffian hypothesis

After 1rst interaction $\quad \mathrm{Prob}(0) \sim e^{-\frac{B\Delta t}{m\omega}n}$

How legitimate (and legitimated) is it to use cross sections in *dense* medium ?

Results with $<t_{inter}>$ not << $1/\omega$: (basis of 11 lowest states)

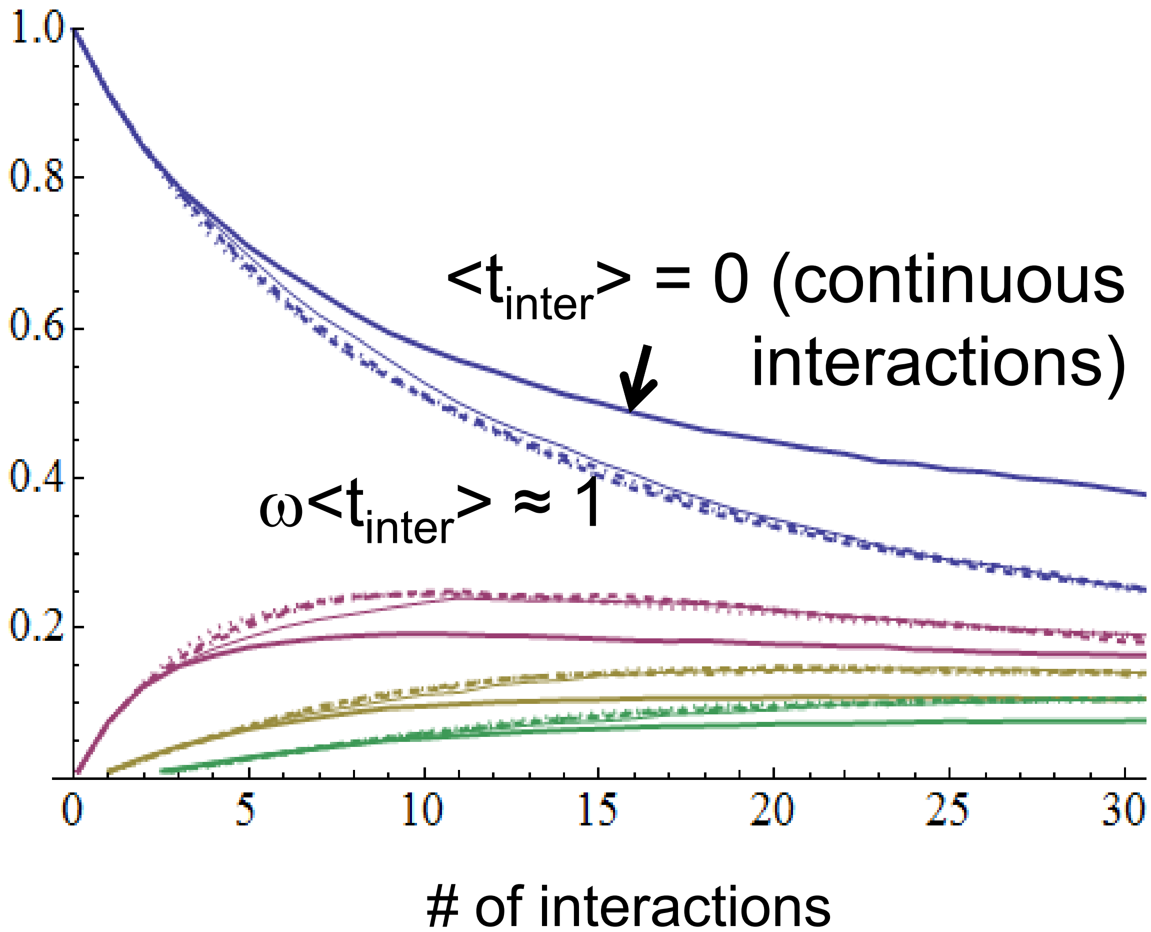
- Not affected by the # states
- No exponential decay (continuous feed down from other states)



Converges towards equilibrated distribution (no dissipation => T=∞ => all states populated with equal probability)

How legitimate (and legitimated) is it to use cross sections in *dense* medium ?

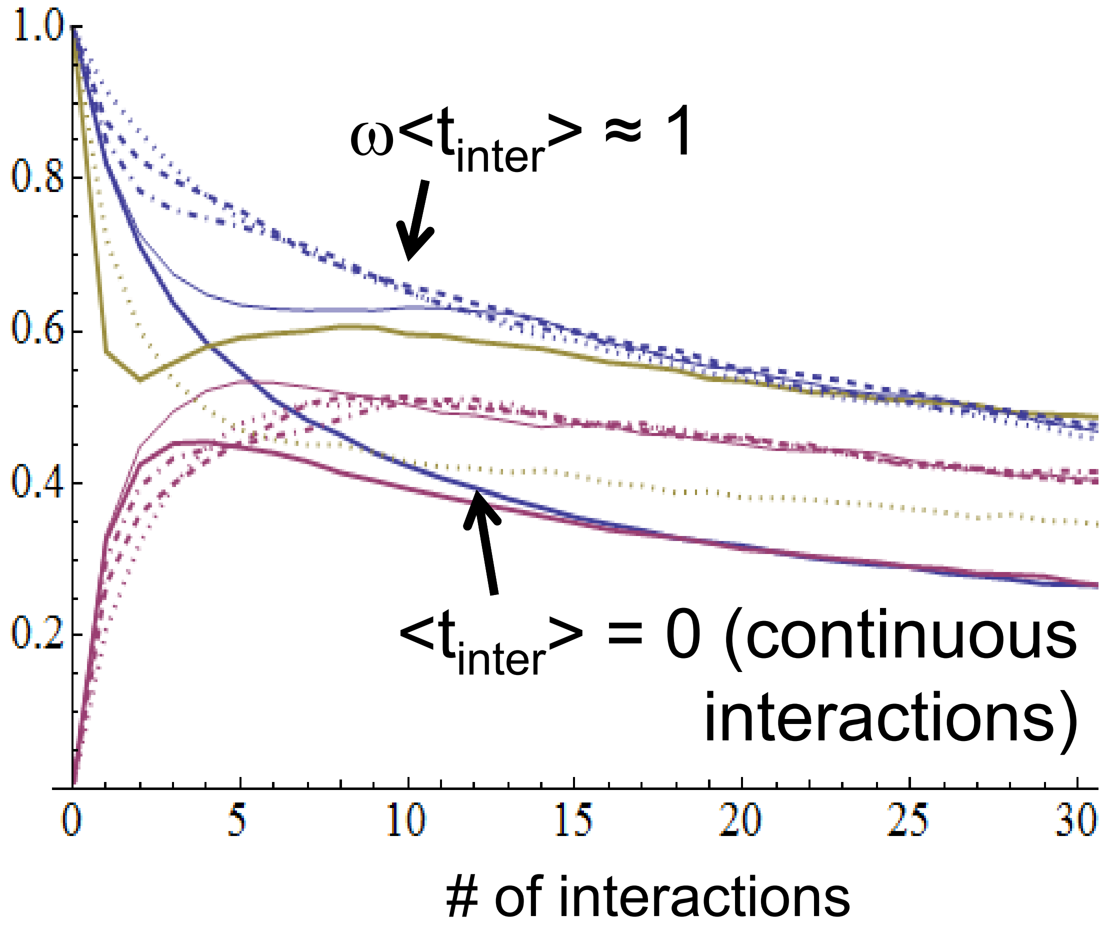
Results with for various $<t_{inter}>$, with $\psi=\psi_0$



- The case $\omega<t_{inter}> \approx 1$ can be understood in terms of transition probabilities (master equations)
- The case $\omega<t_{inter}> << 1$ requires genuine quantum treatment

How legitimate (and legitimated) is it to use cross sections in *dense* medium ?

Results with for various $<t_{inter}>$, with even $\psi=\Sigma\, a_i\, \psi_i$



Same conclusions, larger effects