Higgs Physics
and
Beyond the Standard Model

(mainly SUSY)

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Part 3

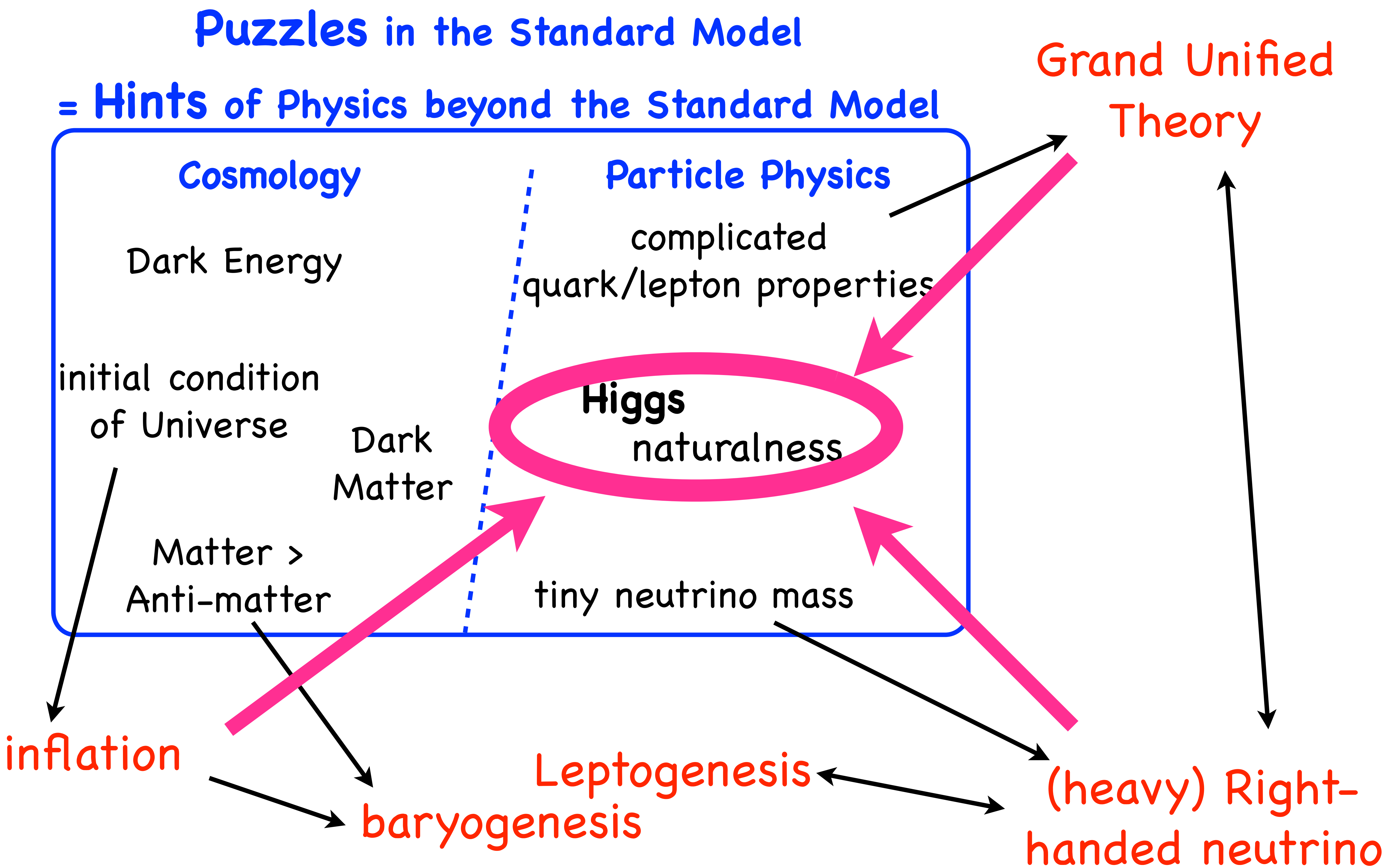
CAUTION

In today's lecture there are many equations/formulas (and not many figures/pictures...).

PLEASE interrupt and ask questions before everybody falls asleep......

Plan

0. **Introduction**
1. **Higgs**
2. **Beyond the Standard Model**
   2.1. puzzles in SM = hints of BSM.
   done
   2.2. renormalization and naturalness


Puzzles in the Standard Model
= Hints of Physics beyond the Standard Model
Cosmology
Particle Physics
Dark Energy
complicated quark/lepton properties
initial condition of Universe
Dark Matter
Higgs
naturalness
Matter > Anti-matter
tiny neutrino mass
Grand Unified Theory
inflation
Leptogenesis
baryogenesis
(heavy) Right-handed neutrino

Plan

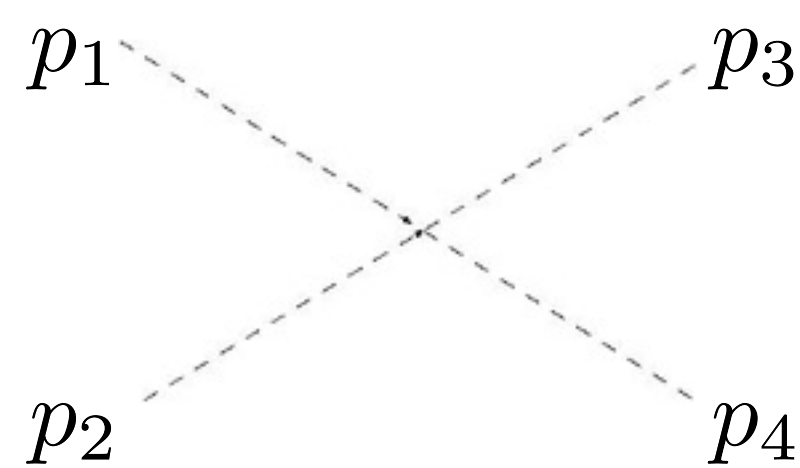
0. **Introduction**
1. **Higgs**
2. **Beyond the Standard Model**
   - 2.1. puzzles in SM = hints of BSM.
   - 2.2. renormalization and naturalness
     - (i) renormalization
     - (ii) naturalness

(i) renormalization

This part is based on A.Zee's textbook, Chap. III
& a lecture note by R.Kitano (for HEP spring school, May 2013, Biwako, Japan)

Consider a 2-body scattering in a scalar ϕ^4 theory.

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$$



The **tree level** amplitude is

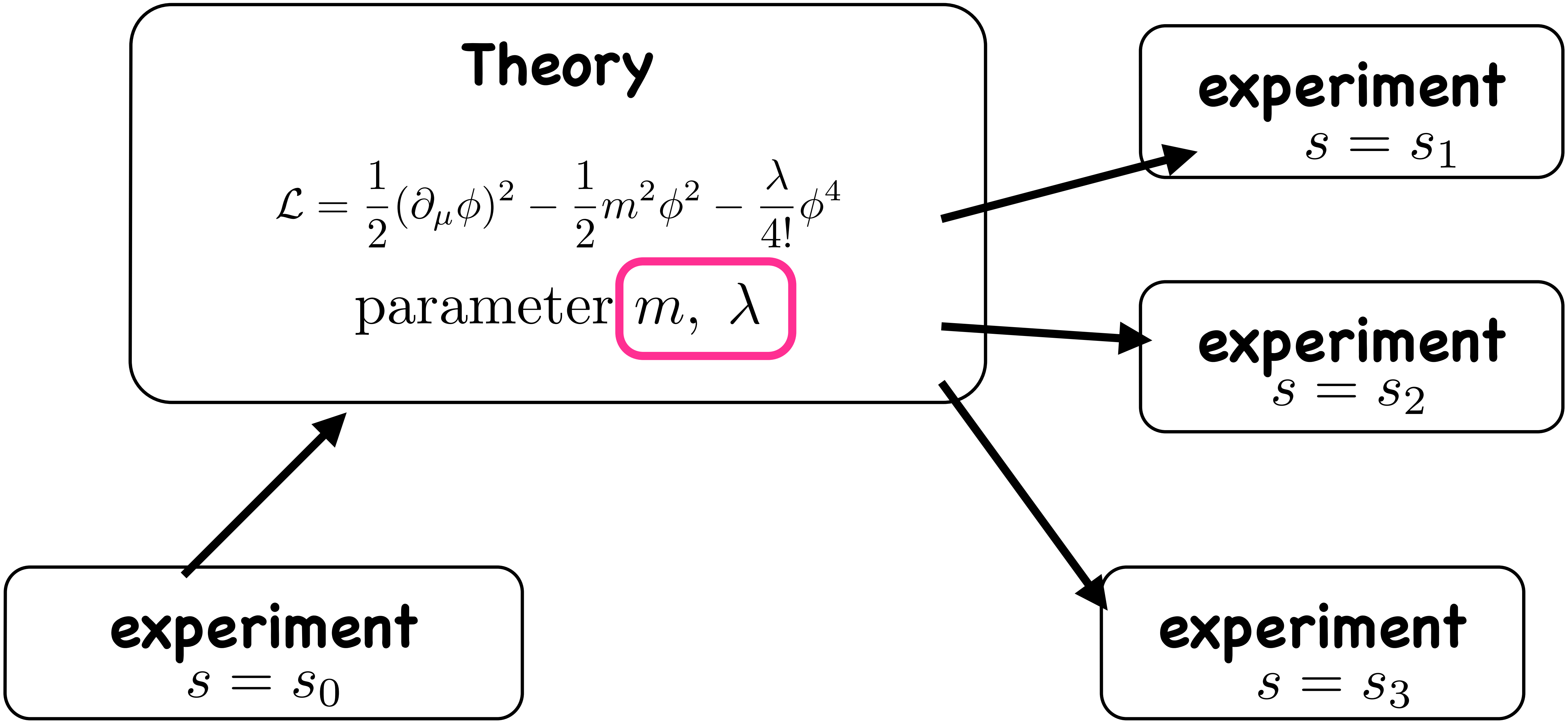
$$\mathcal{M} = -\lambda + \mathcal{O}(\lambda^2)$$

The differential cross section is

$$d\sigma = \frac{1}{16\pi} \cdot \frac{d\Omega}{4\pi} \cdot \frac{1}{s} \underbrace{\cdot|\mathcal{M}|^2,}_{\lambda^2+\mathcal{O}(\lambda^3)} \qquad s=(p_1+p_2)^2$$

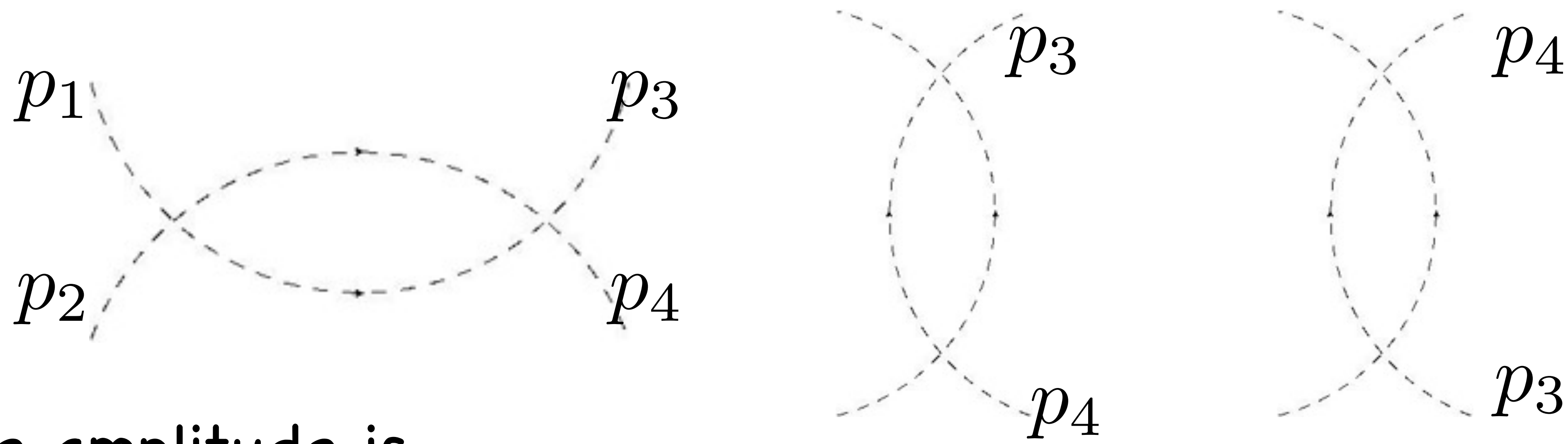
By measuring scattering cross section at e.g., $s = s_0$, we can fix λ.

(i) renormalization



(i) renormalization

Now let's see the next order in perturbation theory.



The amplitude is

$$\mathcal{M} = -\lambda + \lambda^2 \cdot \frac{-i}{2} \int \frac{d^4k}{(2\pi)^4} \cdot \frac{1}{k^2 - m^2 + i\epsilon} \cdot \frac{1}{(k+p_1+p_2)^2 - m^2 + i\epsilon} + \mathcal{O}(\lambda^3) + (s \to t, u)$$

Now the integral **diverges**!

$$\mathcal{M} \sim \int^{\infty} \frac{d^4k}{k^4} \sim \int^{\infty} \frac{dk}{k} \sim \log(\infty)$$

But that's OK.

Suppose that the theory is valid only up to a scale Λ,

and cut off the momentum integration. $$\int^{\infty} dk \longrightarrow \int^{\Lambda} dk$$

(regularization)

(i) renormalization

Then the amplitude becomes (neglecting the mass, for simplicity),

$$\mathcal{M} = -\lambda + C\lambda^2 \left[\log\left(\frac{\Lambda^2}{s}\right) + \log\left(\frac{\Lambda^2}{t}\right) + \log\left(\frac{\Lambda^2}{u}\right)\right] + \mathcal{O}(\lambda^3) \qquad C = 1/32\pi^2$$

LHS can be measured (by scattering at $s = s_0$, for instance).

RHS depends on the artificial cut-off, Λ.

Is that OK? Can this theory still make a prediction?

No problem. We can still compare between experiments.

Theory

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$$

experiment
$s = s_1$

experiment
$s = s_2$

(i) renormalization

Then the amplitude becomes (neglecting the mass, for simplicity),

$$\mathcal{M} = -\lambda + C\lambda^2 \left[\log\left(\frac{\Lambda^2}{s}\right) + \log\left(\frac{\Lambda^2}{t}\right) + \log\left(\frac{\Lambda^2}{u}\right)\right] + \mathcal{O}(\lambda^3) \qquad C = 1/32\pi^2$$

LHS can be measured (by scattering at $s = s_0$, for instance).

RHS depends on the artificial cut-off, Λ.

Is that OK? Can this theory still make a prediction?

No problem. We can still compare between experiments.

$$\text{exp.1:}\ \mathcal{M}(s_1,t_1,u_1) = -\lambda + C\lambda^2 \left[\log\left(\frac{\Lambda^2}{s_1}\right) + \log\left(\frac{\Lambda^2}{t_1}\right) + \log\left(\frac{\Lambda^2}{u_1}\right)\right] + \mathcal{O}(\lambda^3)$$

$$\text{exp.2:}\ \mathcal{M}(s_2,t_2,u_2) = -\lambda + C\lambda^2 \left[\log\left(\frac{\Lambda^2}{s_2}\right) + \log\left(\frac{\Lambda^2}{t_2}\right) + \log\left(\frac{\Lambda^2}{u_2}\right)\right] + \mathcal{O}(\lambda^3)$$

In each eqs, RHS depends on the artificial cut-off Λ.

But if we subtract,...

(i) renormalization

$$\text{exp.1:}\ \mathcal{M}(s_1,t_1,u_1) = -\lambda + C\lambda^2\left[\log\left(\frac{\Lambda^2}{s_1}\right) + \log\left(\frac{\Lambda^2}{t_1}\right) + \log\left(\frac{\Lambda^2}{u_1}\right)\right] + \mathcal{O}(\lambda^3)$$

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In each eqs, RHS depends on the artificial cut-off Λ.

But if we subtract,...

$$\begin{aligned}\mathcal{M}(s_2,t_2,u_2) &= \mathcal{M}(s_1,t_1,u_1) + C\lambda^2\left[\log\left(\frac{s_1}{s_2}\right) + \log\left(\frac{t_1}{t_2}\right) + \log\left(\frac{u_1}{u_2}\right)\right] + \mathcal{O}(\lambda^3)\\ &= \mathcal{M}(s_1,t_1,u_1) + C\mathcal{M}(s_1,t_1,u_1)^2\left[\log\left(\frac{s_1}{s_2}\right) + \log\left(\frac{t_1}{t_2}\right) + \log\left(\frac{u_1}{u_2}\right)\right] + \mathcal{O}(\mathcal{M}(s_1,t_1,u_1)^3)\end{aligned}$$

The exp.2 observable is completely determined by the exp.1 observable.
Dependences on the cut-off Λ and λ disappear!

Though the intermediate calculation involves an artificial cut-off Λ,

the final relation between exp.1 and exp.2 is independent of Λ.

This is the **"renormalization"**.

(i) renormalization

$$\mathcal{M}(s_2,t_2,u_2) = \mathcal{M}(s_1,t_1,u_1) + C\lambda^2\left[\log\left(\frac{s_1}{s_2}\right)+\log\left(\frac{t_1}{t_2}\right)+\log\left(\frac{u_1}{u_2}\right)\right]+\mathcal{O}(\lambda^3)$$

$$= \mathcal{M}(s_1,t_1,u_1) + C\mathcal{M}(s_1,t_1,u_1)^2\left[\log\left(\frac{s_1}{s_2}\right)+\log\left(\frac{t_1}{t_2}\right)+\log\left(\frac{u_1}{u_2}\right)\right]+\mathcal{O}(\mathcal{M}(s_1,t_1,u_1)^3)$$

OK, we can now compare experimentally measurable quantities.
But what is the coupling λ then?

Recall

$$\mathcal{M}(s_0,t_0,u_0) = -\lambda + C\lambda^2\left[\log\left(\frac{\Lambda^2}{s_0}\right)+\log\left(\frac{\Lambda^2}{t_0}\right)+\log\left(\frac{\Lambda^2}{u_0}\right)\right]+\mathcal{O}(\lambda^3)$$

Thus

$$\lambda = -\mathcal{M}(s_0,t_0,u_0) + C\lambda^2\left[\log\left(\frac{\Lambda^2}{s_0}\right)+\log\left(\frac{\Lambda^2}{t_0}\right)+\log\left(\frac{\Lambda^2}{u_0}\right)\right]+\mathcal{O}(\lambda^3)$$

$$= -\mathcal{M}(s_0,t_0,u_0) + C\mathcal{M}(s_0,t_0,u_0)^2\left[\log\left(\frac{\Lambda^2}{s_0}\right)+\log\left(\frac{\Lambda^2}{t_0}\right)+\log\left(\frac{\Lambda^2}{u_0}\right)\right]+\mathcal{O}(\mathcal{M}^3)$$

Fine. λ is now expressed in terms of observable $\mathcal{M}(s_0,t_0,u_0)$.

But it depends on Λ!

$$\boxed{\lambda = \lambda(\Lambda)}$$

(i) renormalization

But it depends on Λ! $\boxed{\lambda = \lambda(\Lambda)}$

No problem. Physics does not depend on Λ.

The combination $\big(\Lambda, \lambda(\Lambda)\big)$ determines the observable.
and the observables are independent of Λ.

By this requirement we can also obtain a differential equation for $\lambda(\Lambda)$.

$$\mathcal{M}(s_0,t_0,u_0) = -\lambda(\Lambda) + C\lambda(\Lambda)^2\left[\log\left(\frac{\Lambda^2}{s_0}\right) + \log\left(\frac{\Lambda^2}{t_0}\right) + \log\left(\frac{\Lambda^2}{u_0}\right)\right] + \mathcal{O}(\lambda^3)$$

$$\to 0 = \frac{d\mathcal{M}(s_0,t_0,u_0)}{d\log\Lambda} = -\frac{d\lambda}{d\log\Lambda} + 6C\lambda^2 + 2C\lambda\frac{d\lambda}{d\log\Lambda}\left[\log\left(\frac{\Lambda^2}{s_0}\right) + \cdots\right] + \mathcal{O}(\lambda^3)$$

$$\to \boxed{\frac{d\lambda}{d\log\Lambda} = 6C\lambda^2 + \mathcal{O}(\lambda^3)}$$

Renormalization Group Equation.

As far as this is satisfied, observables are independent of Λ.

(i) renormalization

We regularized the divergent integral by a momentum cut-off Λ.

There are other regularizations.

Dimensional regularization:

We don't discuss what it is, but the basic idea is the same.

- There is an artificial parameter μ with a mass dimension one.
- The combination $\left(\mu, \lambda(\mu)\right)$ determines the observable, and the observables are independent of μ.

- The μ-dependence is given by the Renormalization Group Equation.

$$\frac{d\lambda(\mu)}{d\log\mu} = 6C\lambda^2 + \mathcal{O}(\lambda^3)$$

(i) renormalization

We regularized the divergent integral by a momentum cut-off Λ.

There are other regularizations.

Dimensional regularization:

REMARK: Although the observable is independent of μ, it is better to use μ close to the energy scale of your interest when you calculate by perturbation theory.

$$\mathcal{M}(s,\cdots) = -\lambda(\mu) + C\lambda(\mu)^2\left[\log\left(\frac{\mu^2}{s}\right) + \cdots\right] + \mathcal{O}(\lambda^3)$$

When $\mu \sim$ s, this log factor is small
—> better convergence.

e.g., For hard processes at LHC, $\alpha_s(\mu)$ with μ >> 1 GeV is used.

👉 Cf. Lectures by Prof. Skands and Prof. Lee.

Plan

0. Introduction
1. Higgs
2. Beyond the Standard Model
    2.1. puzzles in SM = hints of BSM.
    2.2. renormalization and naturalness
        (i) renormalization
        (ii) naturalness

done

(ii) naturalness

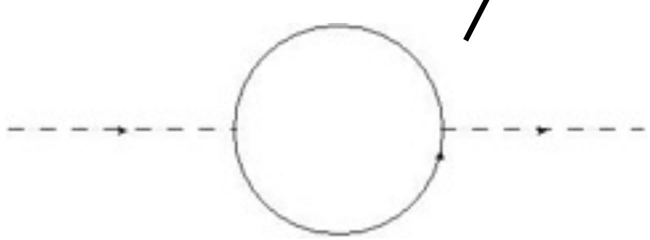
Now let's discuss the **naturalness** for the Higgs mass parameter.

$$V(H) = -m^2|H|^2 + \lambda_H |H|^4.$$

Consider with the **cut-off regularization.**
The correction to the mass parameter is

$$\delta m^2 = -\frac{3}{8\pi^2}\left(y_t^2 - \lambda_H - \frac{3}{8}g^2 - \frac{3}{8}g'^2 + \cdots \right)\Lambda^2$$

NOTE:
quadratic dependence
(not logarithmic)



top loop

Higgs loop

gauge boson loops

(ii) naturalness

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NOTE:
quadratic dependence
(not logarithmic)

Let's consider the largest top contribution.
The corrected mass parameter is then...

$$m^2 = m^2(\Lambda) - \frac{3}{8\pi^2} y_t^2 \Lambda^2 + \cdots$$

(ii) naturalness

$$m^2 = m^2(\Lambda) - \frac{3}{8\pi^2} y_t^2 \Lambda^2 + \cdots$$

m^2: $(\sim 100\ \mathrm{GeV})^2$

$\frac{3}{8\pi^2} y_t^2 \Lambda^2$: $(\sim 100000\ldots\ldots\ \mathrm{GeV})^2$

depending on the cut-off

$m^2(\Lambda)$: $(\sim 100000\ldots\ldots\ \mathrm{GeV})^2 + (\sim 100\ \mathrm{GeV})^2$

If $m^2(\Lambda)$ is a fundamental parameter,

(for instance, if our space-time becomes somehow latticed at very small scale Λ^{-1})

this is unnatural.

$\boxed{\text{naturalness problem}}$

(ii) naturalness

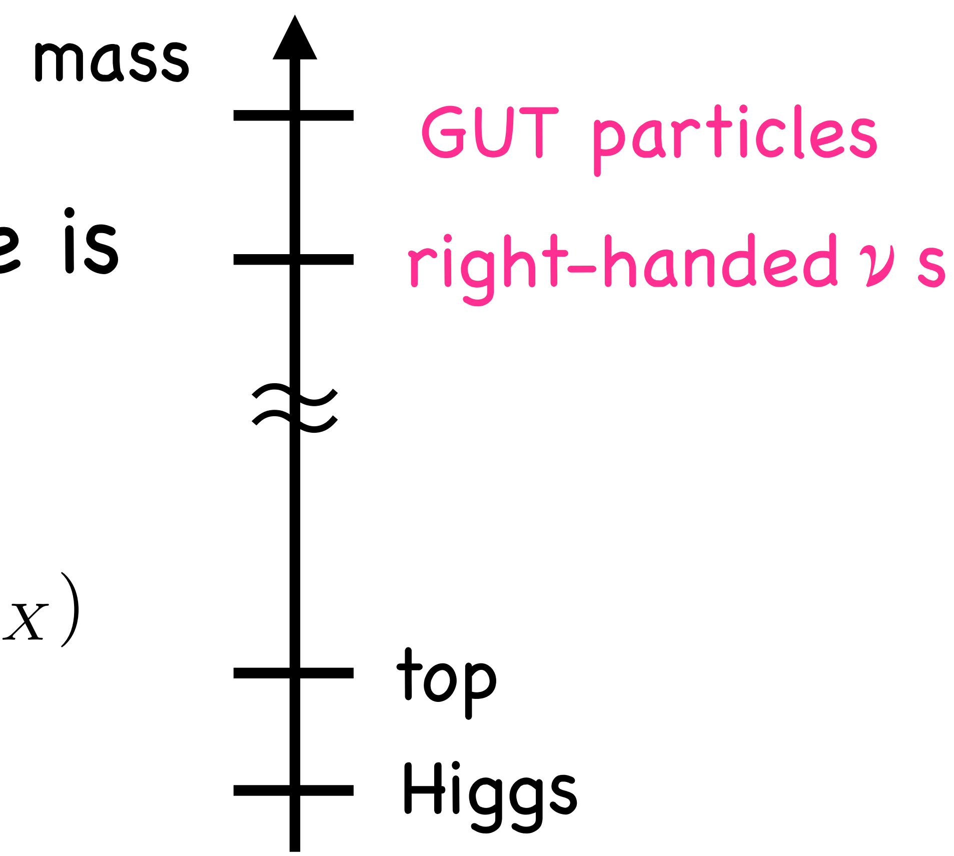
$$m^2 = m^2(\Lambda) - \frac{3}{8\pi^2} y_t^2 \Lambda^2 + \cdots$$

naturalness problem

Remark

Λ^2 term may be an artifact of cut-off regularization.
For instance, it doesn't exist in dimensional regularization.

But even if the Λ^2 term is absent,
a large correction exists as far as there is
a **heavy particles** coupled to Higgs.



$$m^2 = m^2(\mu) + C_X m_X^2 \log\left(\frac{\mu^2}{m_X^2}\right) + \cdots \quad (\text{for } \mu > m_X)$$

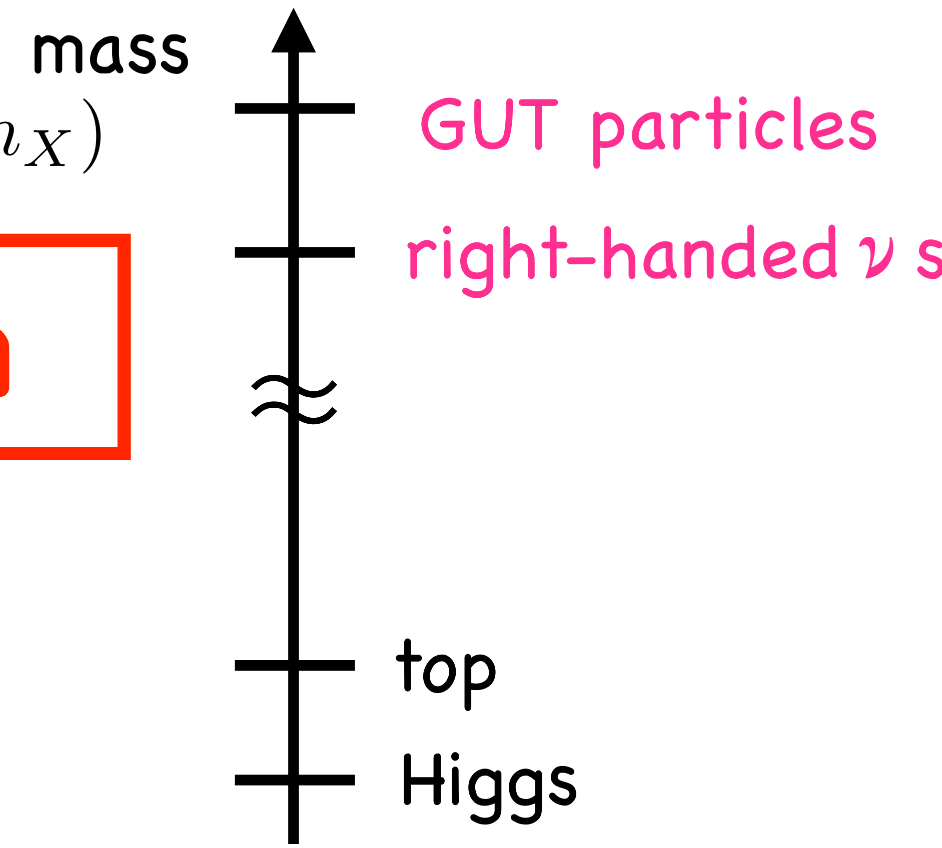
X's coupling to Higgs

X's mass

[See e.g., 1303.7244, 1402.2658 for recent discussions]

(ii) naturalness

$$m^2 = m^2(\mu) + C_X m_X^2 \log\left(\frac{\mu^2}{m_X^2}\right) + \cdots \quad (\text{for } \mu > m_X)$$



naturalness problem

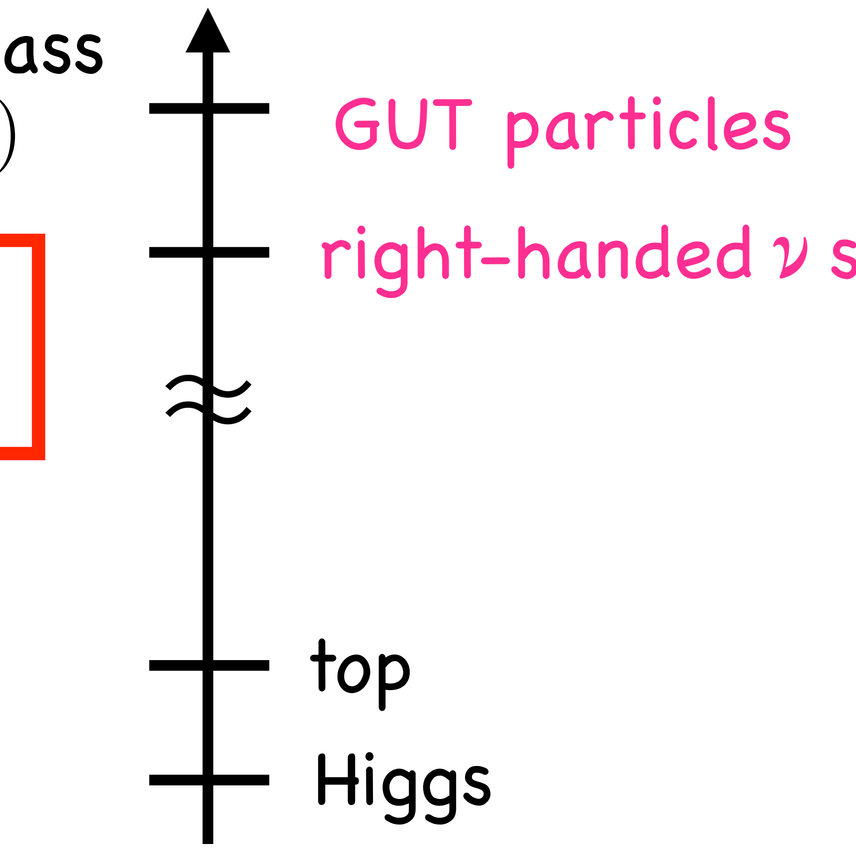
If we think m(μ) at μ > m_X is more fundamental than m(μ) at weak scale μ,

(For instance, in QCD, $m_{u,d}$ at $\mu > \Lambda_{QCD}$ seems more fundamental than the pion masses.....)

this is unnatural.

(ii) naturalness

$$m^2 = m^2(\mu) + C_X m_X^2 \log\left(\frac{\mu^2}{m_X^2}\right) + \cdots \qquad (\text{for } \mu > m_X)$$



naturalness problem

solutions

- Don't mind.

  There is no problem in experimental observables.
  (Don't listen too much to theorists...... 😜.)

- Landscape + anthropic principle

  We live in a fine-tuned vacuum because otherwise we cannot live.

- No such heavy particles

  or 4d perturbative QFT breaks down anyway before that.

- cancellation among loop corrections

  §3
  - SUSY
  - little Higgs (top correction canceled.)
  - . . .

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done

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0. Introduction
1. Higgs
2. Beyond the Standard Model

done

3. SUSY
4. SUSY after Higgs discovery

3. Supersymmetry

Fermion <-> Boson

Standard Model

		spin		
quark	q	1/2 <-> 0	squark	$\widetilde{q}$
lepton	ℓ	1/2 <-> 0	slepton	$\widetilde{\ell}$
Higgs	H	0 <-> 1/2	higgsino	$\widetilde{h}$
gauge bosons γ, Z, W, g		1 <-> 1/2	gaugino $\widetilde{B}, \widetilde{W}, \widetilde{g}$	

MSSM (minimal SUSY Standard Model)

Plan

0. Introduction
1. Higgs
2. Beyond the Standard Model
3. SUSY
   - 3.1. motivations
   - 3.2. supersymmetry
   - 3.3. MSSM (minimal SUSY Standard Model)
   - 3.4 MSSM Lagrangian

3. Supersymmetry

§ 3.1. motivations

(i) naturalness of Higgs boson mass

(ii) coupling unification

(iii) · · ·

3. Supersymmetry

§ 3.1. motivations

(i) naturalness of Higgs boson mass

- In terms of cut-off regularization,

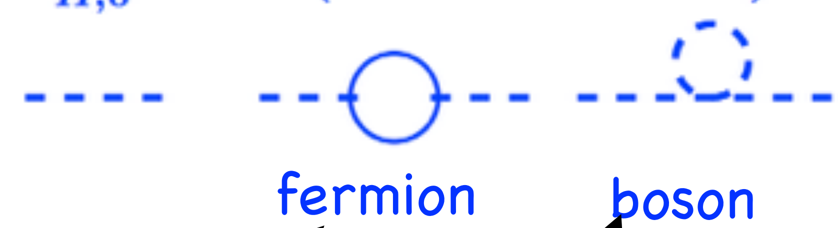
fine-tuning problem

$$m_H^2 = m_{H,0}^2 + \Lambda^2 \qquad (\Lambda \gg m_H)$$

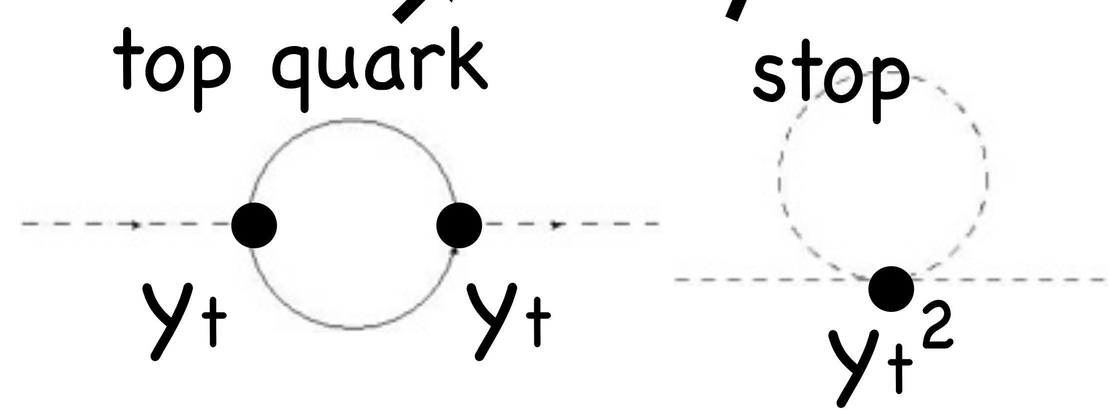
(fine tuning like 1.0000000000000001 - 1)

→ solved by the supersymmetry !

$$m_H^2 = m_{H,0}^2 + (\Lambda^2 - \Lambda^2)$$



For instance,...



$y_t^2 \Lambda^2 \quad - y_t^2 \Lambda^2$

- In terms of dim. reg. + heavy particle X,

$\delta m^2(\mu) \sim (m_X\,[\text{scalar}]^2 - m_X\,[\text{fermions}]^2) \log (\mu / m_X)$

3. Supersymmetry

§ 3.1. motivation

(ii) coupling unification

- gauge field kinetic term of GUT (SU(5))

$$\left(F_{\mu\nu}F^{\mu\nu} \to \frac{1}{g^2} F_{\mu\nu}F^{\mu\nu} \text{ by field redefinition } A_\mu \to \frac{1}{g} A_\mu \right)$$

$$\frac{1}{g_{\mathrm{GUT}}^2} \sum_{a=1}^{24} F^a_{\mu\nu} F^{a\mu\nu}$$

$$= \frac{1}{g_3^2} G^a_{\mu\nu} G^{a\mu\nu} + \frac{1}{g_2^2} W^a_{\mu\nu} W^{a\mu\nu} + \frac{1}{g_1^2} B_{\mu\nu} B^{\mu\nu} + (X, Y \text{ gauge bosons})$$

$$\boxed{\Longrightarrow \quad g_1 = g_2 = g_3 = g_{\mathrm{GUT}} \quad @ \quad \mu = \text{unification scale} \quad (= \text{the scale where GUT is broken})}$$

- running gauge coupling

R.G.eq $$\frac{d}{d \log \mu} \alpha_i^{-1} = -\frac{1}{2\pi} b_i \qquad \text{(1-loop)}$$

3. Supersymmetry

§ 3.1. motivation

(ii) coupling unification

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R.G.eq $$\frac{d}{d\log\mu}\alpha_i^{-1} = -\frac{1}{2\pi}b_i \qquad \text{(1-loop)}$$

	INPUT $\alpha_i(m_Z)$	b_i^{SM}	b_i^{MSSM}
SU(3)	$\simeq 0.118$	-7	-3
SU(2)	$\dfrac{\alpha}{\sin^2\theta_W} \simeq \dfrac{1/128}{0.23}$	$-19/6$	$+1$
U(1)	$\alpha_1 = \left(\dfrac{5}{3}\right)\alpha_Y = \left(\dfrac{5}{3}\right)\dfrac{\alpha}{\cos^2\theta_W}$	$+41/10$	$+33/5$

$g_1 = \sqrt{\frac{5}{3}}g_Y, \quad b_1 = \frac{3}{5}b_Y$

see Appendix yesterday (mini review of GUT)

3. Supersymmetry

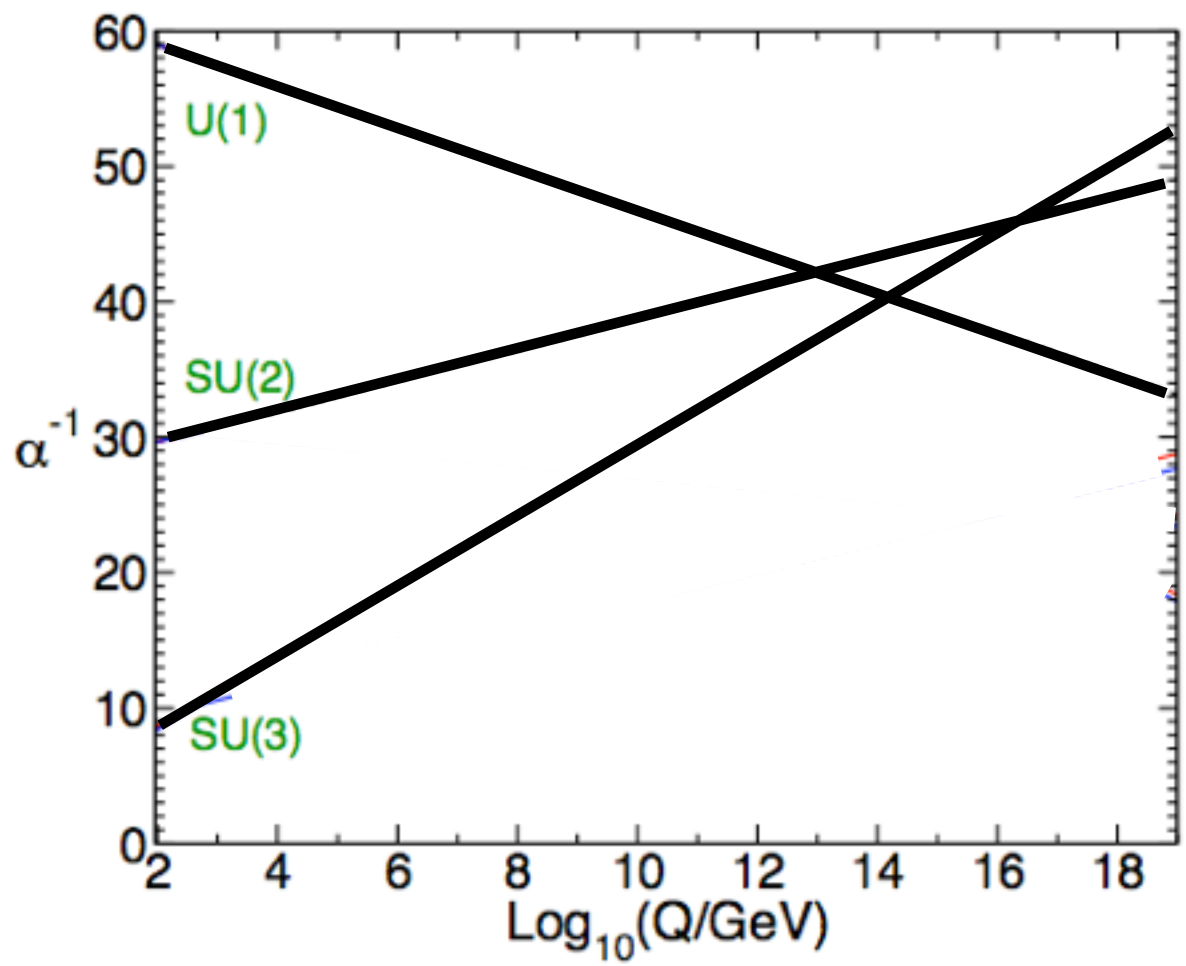
§ 3.1. motivation

(ii) coupling unification

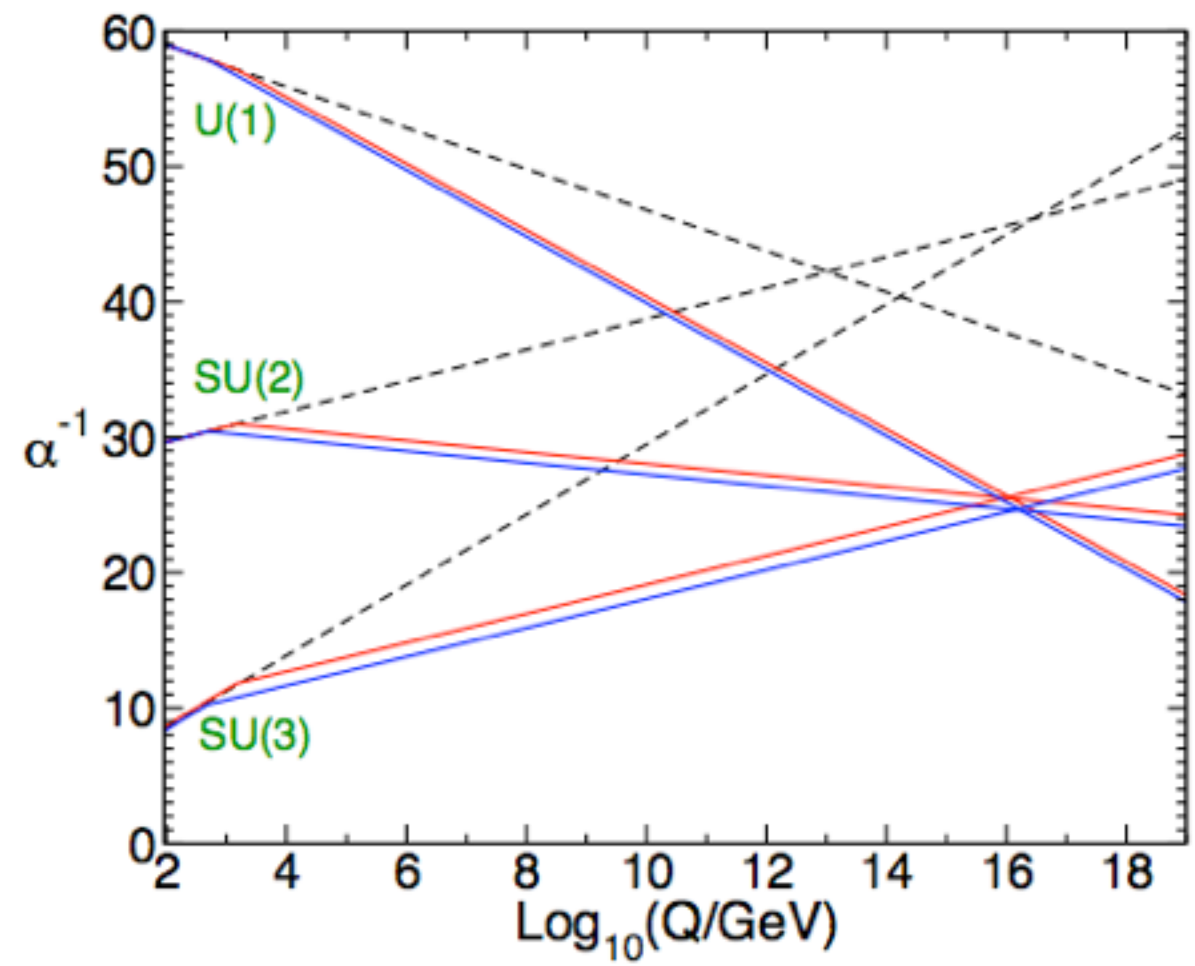
- running gauge coupling

R.G.eq $$\frac{d}{d\log\mu}\alpha_i^{-1} = -\frac{1}{2\pi}b_i \qquad \text{(1-loop)}$$

Standard Model



Standard Model + SUSY



Plan

0. **Introduction**
1. **Higgs**
2. **Beyond the Standard Model**
3. **SUSY**

   3.1. motivations

   done

   3.2. supersymmetry

   3.3. MSSM (minimal SUSY Standard Model)

   3.4 MSSM Lagrangian

3. Supersymmetry

§ 3.2. SUSY

(i) simplest model (in 4-dim)

$$\phi \longleftrightarrow \psi$$

complex scalar $\longleftrightarrow$ 2-component Weyl fermion

$$\underline{\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi + i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi}$$

free massless

(notataion

$\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi = \psi^\dagger_{\dot{\alpha}} (\bar{\sigma}^\mu)^{\dot{\alpha}\alpha} \partial_\mu \psi_\alpha$ (sum is taken as ${}_{\dot{\alpha}}{}^{\dot{\alpha}}$ and ${}^{\alpha}{}_{\alpha}$)

$$\sigma^\mu = (\mathbf{1}, \sigma^i), \quad \bar{\sigma}^\mu = (\mathbf{1}, -\sigma^i), \quad \sigma^i = \left(\begin{pmatrix} & 1 \\ 1 & \end{pmatrix}, \begin{pmatrix} & -i \\ i & \end{pmatrix}, \begin{pmatrix} 1 & \\ & -1 \end{pmatrix} \right)$$

$$g_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$$

$$\psi^\alpha = \epsilon^{\alpha\beta}\psi_\beta, \quad \psi^\dagger_{\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}}\psi^{\dagger\dot{\beta}}, \quad \epsilon^{12} = -\epsilon^{21} = \epsilon_{21} = -\epsilon_{12} = 1, \quad \epsilon^{ii} = \epsilon_{ii} = 0$$

)

(i) simplest model (in 4-dim)

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi + i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi$$

What kind of symmetry does it have ?

example: U(1) symmetry (scalar sector)

$$\phi \to e^{i\alpha} \phi$$

or

$$\delta\phi = i\alpha\phi$$

Lagrangian is invariant under this U(1) transformation.

$$\begin{aligned}\delta\mathcal{L} &= \partial_\mu(\delta\phi^*)\partial_\mu\phi + \partial_\mu\phi^*\partial_\mu(\delta\phi) \\ &= \partial_\mu(-i\alpha\phi^*)\partial_\mu\phi + \partial_\mu\phi^*\partial_\mu(i\alpha\phi) \quad = 0\end{aligned}$$

(i) simplest model (in 4-dim)

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi + i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi$$

What kind of symmetry does it have ?

SUSY transformation $\boxed{\text{Fermion <-> Boson}}$

(i) simplest model (in 4-dim)

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi + i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi$$

What kind of symmetry does it have ?

SUSY transformation $\boxed{\text{Fermion <-> Boson}}$

$$\begin{cases} \delta\phi & = \chi^\alpha \psi_\alpha \\ \delta\psi_\alpha & = -i(\sigma^\mu \chi^\dagger)_\alpha \partial_\mu \phi \end{cases}$$

χ_α SUSY transformation parameter.

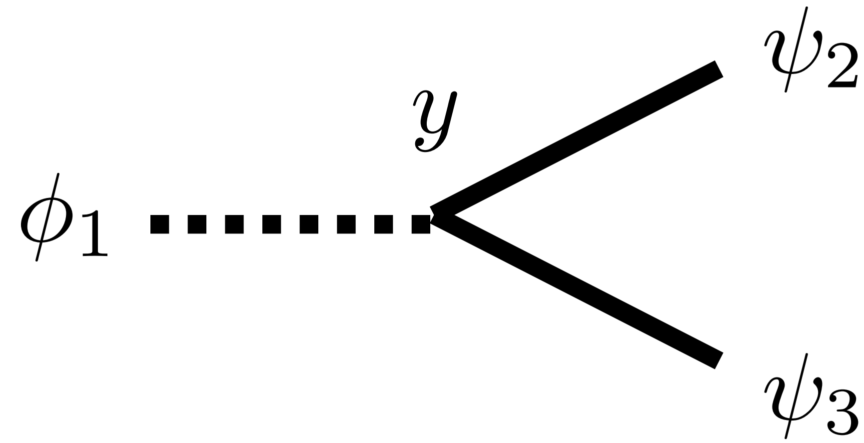
- 2-component
- anti-commuting (fermionic)

Then,...

$$\begin{aligned} \delta\mathcal{L} &= \delta\left(\partial_\mu \phi^* \partial^\mu \phi\right) + \delta\left(i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi\right) \\ &= \partial_\mu(\delta\phi^*)\partial^\mu \phi + \partial_\mu \phi^* \partial^\mu(\delta\phi) + i(\delta\psi^\dagger)\bar{\sigma}^\mu \partial_\mu \psi + i\psi^\dagger \bar{\sigma}^\mu \partial_\mu(\delta\psi) \\ &= \partial_\mu(\chi^\dagger \psi^\dagger)\partial^\mu \phi + \partial_\mu \phi^* \partial^\mu(\chi\psi) + \ldots \\ &= 0 \end{aligned}$$

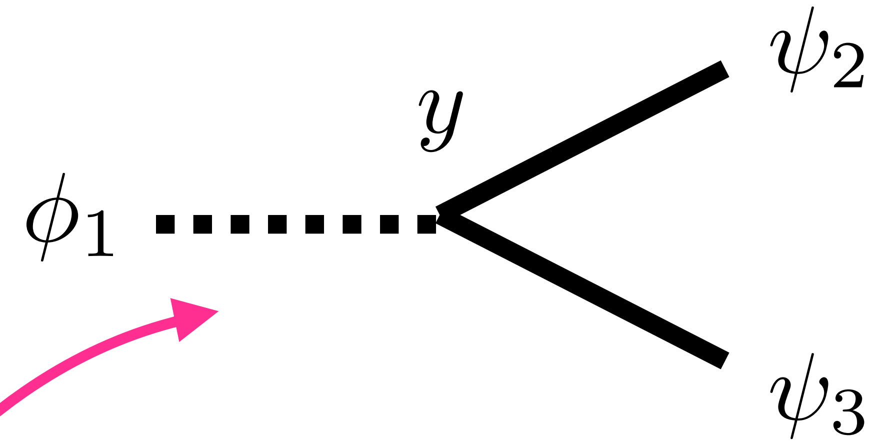
(ii) interaction

$$\mathcal{L}_{\rm int} = -y \cdot \phi_1 \psi_2 \psi_3$$



(ii) interaction

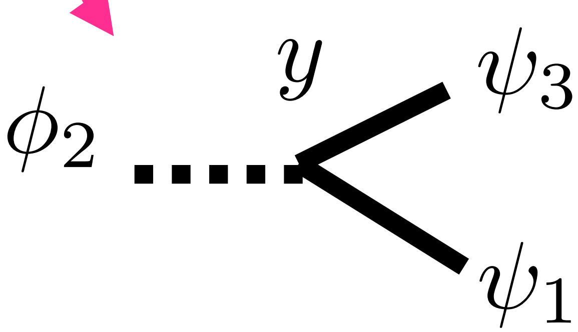
$$\mathcal{L}_{\rm int} = -y \cdot \phi_1 \psi_2 \psi_3$$

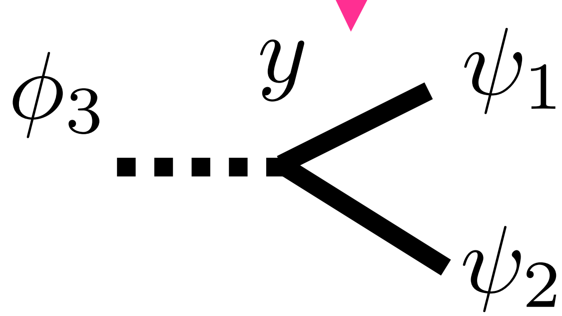


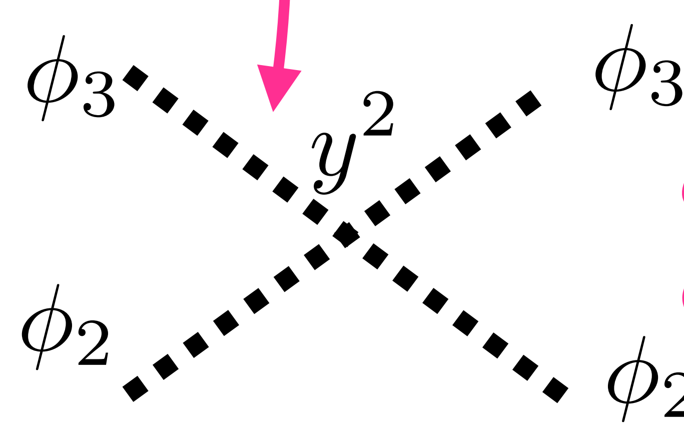
"supersymmetrize"

$$\begin{aligned}\mathcal{L}_{int} = & - y \cdot \phi_1 \psi_2 \psi_3 \quad && - |y|^2 |\phi_2|^2 |\phi_3|^2 \\ & - y \cdot \phi_2 \psi_3 \psi_1 && - |y|^2 |\phi_3|^2 |\phi_1|^2 \\ & - y \cdot \phi_3 \psi_1 \psi_2 && - |y|^2 |\phi_1|^2 |\phi_2|^2\end{aligned}$$

With all these terms, it is invariant under SUSY transformation.



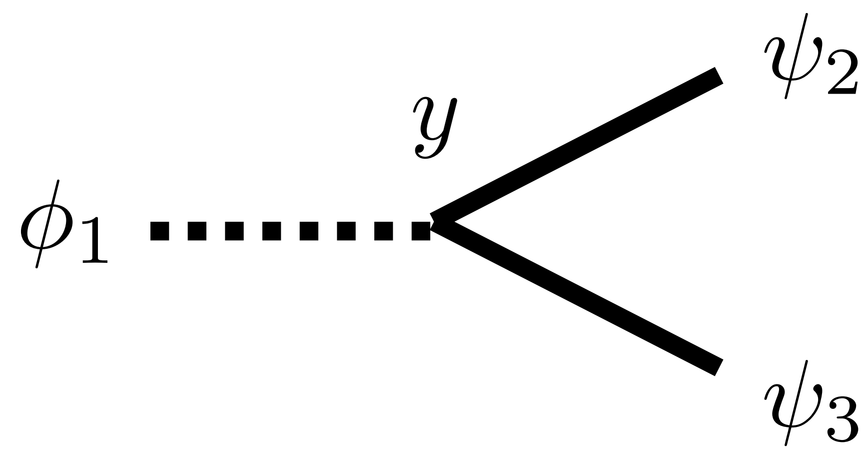




all the same coupling

(ii) interaction

$$\mathcal{L}_{\rm int} = -y \cdot \phi_1 \psi_2 \psi_3$$



"supersymmetrize"

$$\begin{aligned}\mathcal{L}_{int} = & - y \cdot \phi_1 \psi_2 \psi_3 \qquad - |y|^2 |\phi_2|^2 |\phi_3|^2 \\ & - y \cdot \phi_2 \psi_3 \psi_1 \qquad - |y|^2 |\phi_3|^2 |\phi_1|^2 \\ & - y \cdot \phi_3 \psi_1 \psi_2 \qquad - |y|^2 |\phi_1|^2 |\phi_2|^2 \end{aligned}$$

With all these terms, it is invariant under SUSY transformation.

$$\boxed{W = y \cdot \Phi_1 \Phi_2 \Phi_3}$$

superpotential

$$\mathcal{L}_{\rm int} = -\frac{\partial^2 W(\phi)}{\partial \phi_i \partial \phi_j} \psi_i \psi_j - \left| \frac{\partial W}{\partial \phi_i} \right|^2$$

$\Phi_i \sim (\phi_i, \psi_i)$

superfield: contains boson-fermion pair

(iii) mass term

$$W = M\Phi_1\Phi_2$$

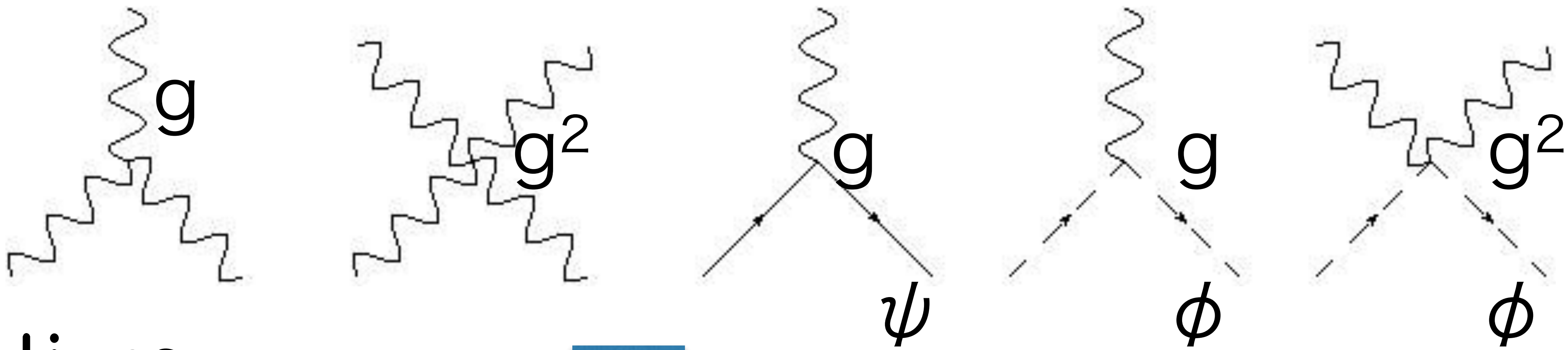
$$\to \mathcal{L} = -\frac{\partial^2 W}{\partial\phi_i\partial\phi_j}\psi_i\psi_j - \left|\frac{\partial W}{\partial\phi_i}\right|^2$$

$$= -M\psi_1\psi_2 - |M|^2|\phi_1|^2 - |M|^2|\phi_2|^2$$

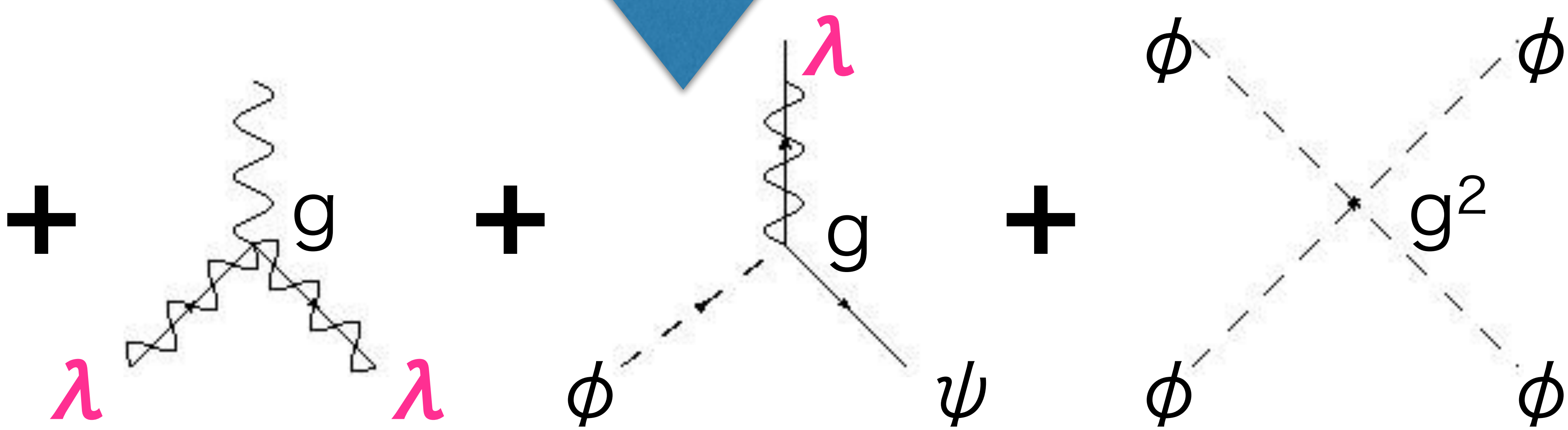
fermion mass = boson mass

(iv) gauge sector

gauge interactions



"supersymmetrize"



gaugino
(= fermionic partner of gauge boson)

With all these terms, the Lagrangian is invariant under SUSY transformation.

(v) summary

ϕ (scalar) $\longleftrightarrow$ ψ (fermion)

A_μ (gauge) $\longleftrightarrow$ λ (fermion)

They have the same masses,
and the same couplings.

Plan

0. Introduction
1. Higgs
2. Beyond the Standard Model
3. SUSY
    - 3.1. motivations
    - 3.2. supersymmetry
    - 3.3. MSSM (minimal SUSY Standard Model)
    - 3.4 MSSM Lagrangian

done

§ 3.3. MSSM

Standard Model

		spin		
quark	q	1/2 <-> 0	squark	$\widetilde{q}$
lepton	ℓ	1/2 <-> 0	slepton	$\widetilde{\ell}$
Higgs	H	0 <-> 1/2	higgsino	$\widetilde{h}$
gauge bosons γ, Z, W, g		1 <-> 1/2	gaugino $\widetilde{B}, \widetilde{W}, \widetilde{g}$	

MSSM (minimal SUSY Standard Model)

§ 3.3. MSSM

More precisely..........

Standard Model

		spin		
quark	q	1/2 <-> 0	squark	$\widetilde{q}$
lepton	ℓ	1/2 <-> 0	slepton	$\widetilde{\ell}$
Higgs	$\cancel{H}$	0 <-> 1/2	higgsino	$\widetilde{h}$
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MSSM (minimal SUSY Standard Model)

§ 3.3. MSSM

More precisely.........

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MSSM (minimal SUSY Standard Model)

§ 3.3. MSSM

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MSSM (minimal SUSY Standard Model)

comment 1: Why 2 Higgs ?

reason 1. Yukawa coupling.

$$\mathcal{L}^{\rm SM} = -y_u H^* \bar{u} Q - y_d H \bar{d} Q$$

conjugate

but in SUSY,

$$W = y_u \cancel{H^* Q u^c} + y_d H Q d^c$$

$W(\phi)$ must be a function of ϕ, not , ϕ^*

$$W = y_u H_u Q u^c + y_d H_d Q d^c$$

2 Higgs

§ 3.3. MSSM

		spin		
quark	q	1/2 <-> 0	squark	$\widetilde{q}$
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MSSM (minimal SUSY Standard Model)

comment 1: Why 2 Higgs ?

reason 2. Anomaly

$$1\ \text{Higgs}: \quad H \Longleftrightarrow \widetilde{h} = (1,2)_{1/2} \quad \text{gauge anomaly}$$

$$2\ \text{Higgs}: \quad H_u \Longleftrightarrow \widetilde{h_u} = (1,2)_{1/2} \quad \text{anomaly}$$

$$H_d \Longleftrightarrow \widetilde{h_d} = (1,2)_{-1/2} \quad \text{cancellation}$$

§ 3.3. MSSM

		spin	
quark	q	1/2 <-> 0	squark $\widetilde{q}$
lepton	ℓ	1/2 <-> 0	slepton $\widetilde{\ell}$
Higgs	H_u, H_d	0 <-> 1/2	higgsino $\widetilde{h}$
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MSSM (minimal SUSY Standard Model)

comment 2:

mass eigenstates

$$\text{neutral fermions} \quad \widetilde{h_u^0}, \widetilde{h_d^0}, \widetilde{B}, \widetilde{W}^0 \Longrightarrow \quad \widetilde{\chi}^0_{1,2,3,4}$$ neutralinos

$$\text{charged fermions} \quad \widetilde{h_u^+}, \widetilde{h_d^-}, \widetilde{W}^{\pm} \Longrightarrow \quad \widetilde{\chi}^{\pm}_{1,2}$$ charginos

Plan

0. **Introduction**
1. **Higgs**
2. **Beyond the Standard Model**
3. **SUSY**

   3.1. motivations

   3.2. supersymmetry

   3.3. MSSM (minimal SUSY Standard Model)

   done

   3.4 MSSM Lagrangian