Field Theory and EW Standard Model-IV

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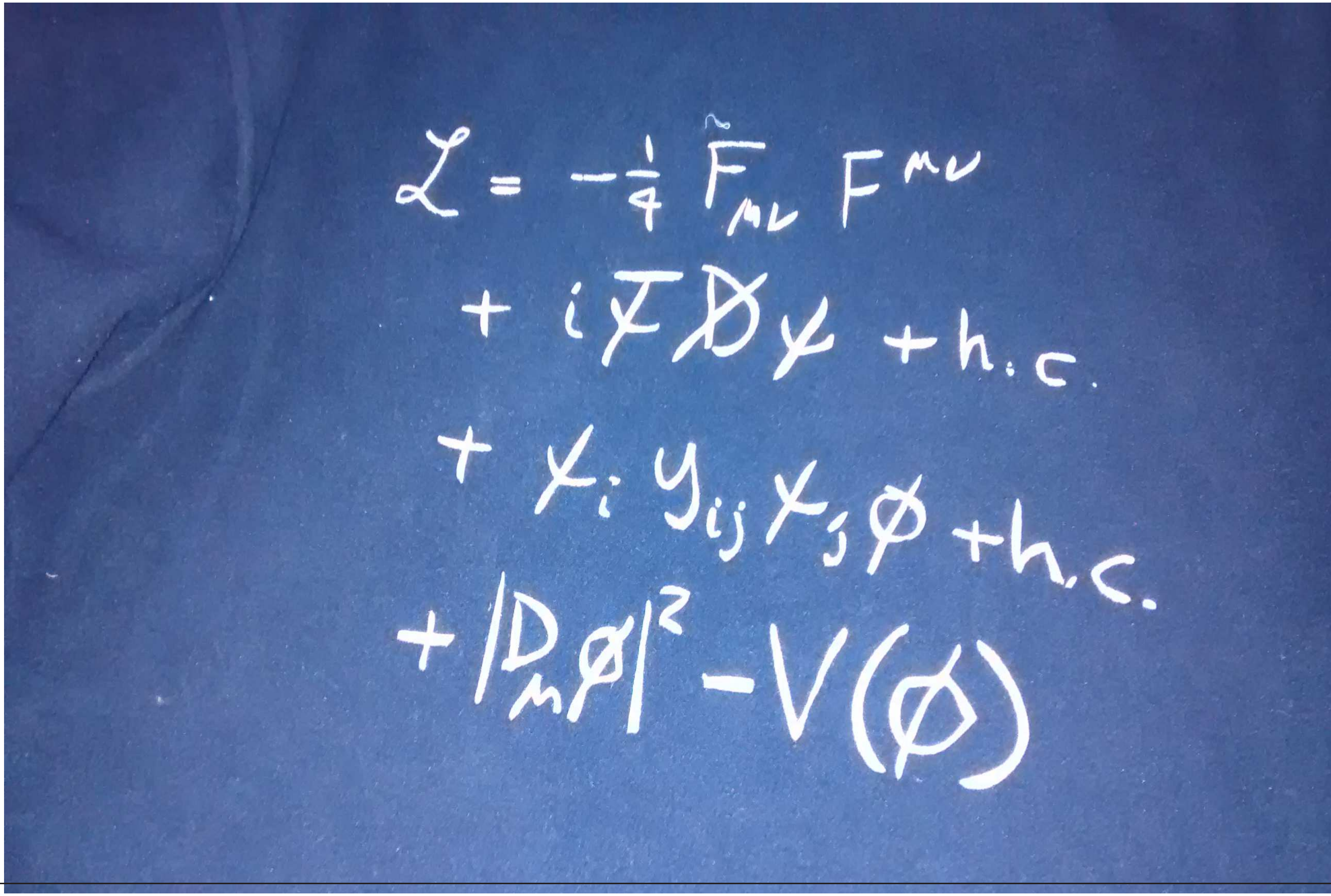

$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$
$+ i\bar{\psi} \not{D} \psi + h.c.$
$+ \psi_i y_{ij} \psi_j \phi + h.c.$
$+ |D_\mu \phi|^2 - V(\phi)$

What we have covered so far:

a) Setting up the notation of the SM Lagrangian, including Higgs mechanism

b)The miracles of the particle spectrum of the SM: Anomaly cancellation and the Custodial symmetry!

c) Test of EW unification with the determination of $\sin\theta_w$ **and resultant test of a unified gauge field theoretic description of Electro Weak interactions.**

Prediction of new particles and their masses in the SM:

d) How one can understand the development of the SM also in terms of taming the bad high energy behavior of the scattering amplitudes!

e) Fermion mass generation via Higgs mechanism and Yukawa Interactions. generation mixing in the quark sector!

f) GIM and prediction of M_c from the observed mass difference K_L–K_S. The 'first' use of an indirect effect to predict a mass!

Lecture 3:

a) Radiative corrections in a spontaneously broken gauge theory, oblique corrections and precision testing of the SM.

b)'Indirect' determination of the mass of the top and Higgs!

Lecture 4:

a)Theoretical bounds on the Higgs mass

b) Implications of the measured mass of the Higgs for the SM! i.e the scale unto which SM can be consistent without any additional physics!

In the interaction basis the Lagrangian is

$$\mathcal{L}_f = i\bar{\mathcal{Q}}'_{iL} D_\mu \gamma^\mu \mathcal{Q}'_{iL} + i\bar{u}'_{iR} D_\mu \gamma^\mu u'_{iR} + \ldots$$

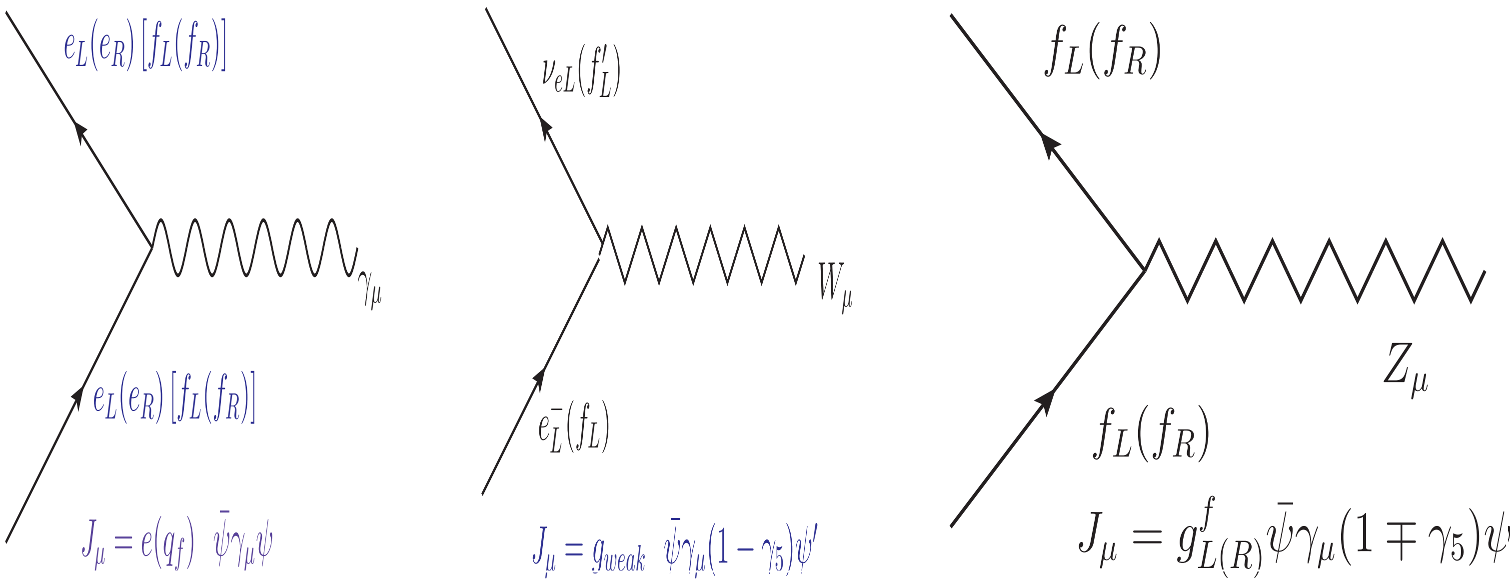
For example $D_\mu \mathcal{Q}'_{iL} = \left[\partial_\mu - i\frac{g_2}{2}\vec{\sigma}\cdot\vec{W}_\mu - i\frac{g_1}{6}B_\mu\right]\mathcal{Q}'_{iL}$

One can rewrite this in terms of the mass basis u_i, d_i and $W^\pm_\mu, Z^\mu, A_\mu$ and derive the $Zf\bar{f}$ and $Wf\bar{f}'$ vertices.

$$\begin{pmatrix} d'_1 \\ d'_2 \\ d'_3 \end{pmatrix} = \mathcal{V}^{CKM} \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix} = \begin{pmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \end{pmatrix} \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$$

and

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_w & -\sin\theta_w \\ \sin\theta_w & \cos\theta_w \end{pmatrix} \begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix}$$



Like the photon the Z couples f to f and NOT f to f'. No flavour change.

Unlike the photon Z couples differently to the left and right handed fermions.

Like the W, Z coupling too violates parity, but unlike the W not necessarily maximally.

Charged current Interaction: $J_\mu^{CC}\ W^{+\mu} + \text{h.c.}$ where

$$J_\mu^{cc} = \frac{g_2}{2\sqrt{2}} \mathcal{V}_{ij}^{CKM} \bar{u}_i' \gamma_\mu d_j .$$

Neutral Current Interaction: $J_\mu^{NC}\ Z^\mu + J_\mu^{em}\ A^\mu$

$$J_\mu^{NC} = \frac{g_2}{2\cos\theta_w} \left[g_V^f \bar{f} \gamma_\mu f - g_A^f \bar{f} \gamma_\mu \gamma_5 f \right]$$

	ν	e^-	u_i	d_i
g_A	1/2	-1/2	$+1/2$	-1/2
g_V	1/2	$-1/2 + 2\sin^2\theta_w$	$1/2 - 4/3\sin^2\theta_w$	$-1/2 + 2/3\sin^2\theta_w$

Recall

$$M_W = g_2 v/2 = \left(g_2^2\sqrt{2}/8G_F\right)^{1/2} = \frac{37.4}{\sin\theta_W} GeV$$

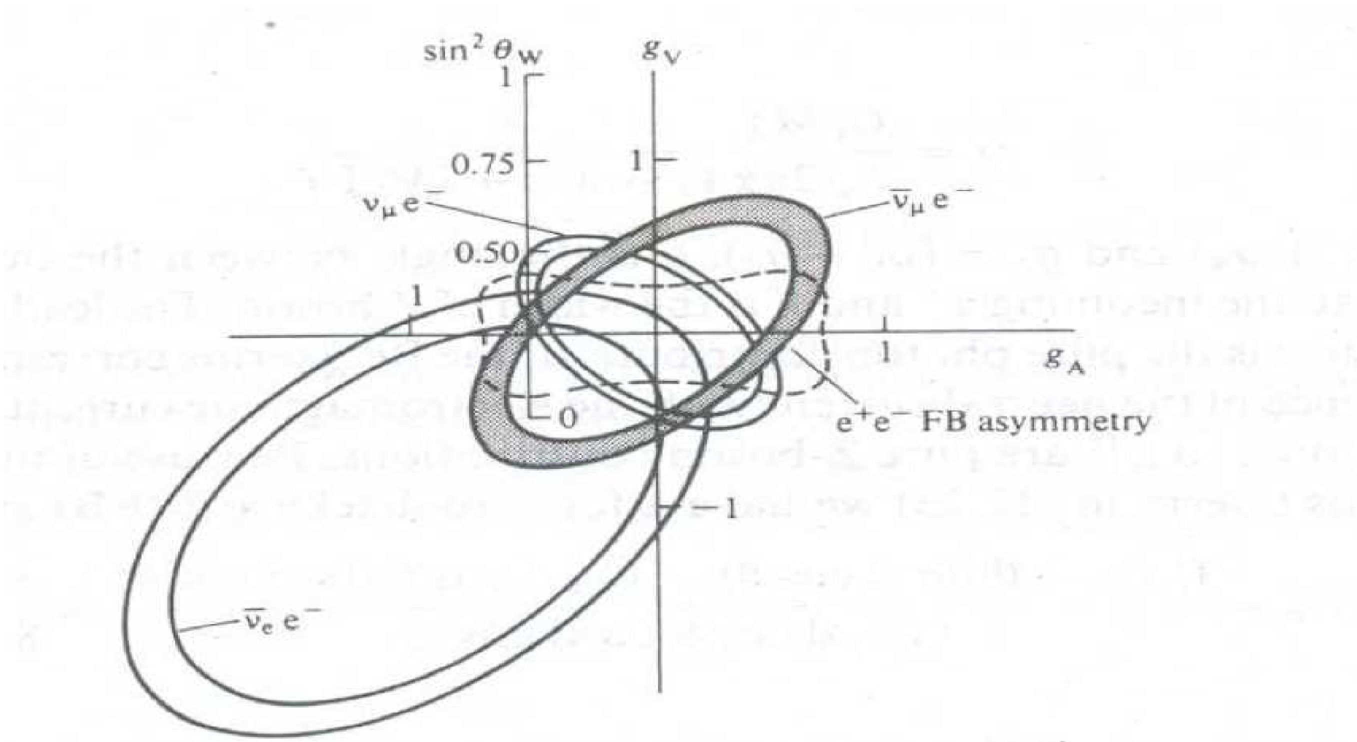
$$M_Z = \frac{M_W}{\cos\theta_W}$$

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_w} = 1$$

Thus given one free parameter $\sin\theta_w$ the neutral current couplings as well as the masses of the W and Z are predicted in the SM at the tree level.

G_F Fermi coupling constant in the β decay. Value extracted using muon life time τ_μ

Determination of Neutral Current couplings and hence $\sin^2\theta_W$ (circa 1981). Also gave $\rho \sim 1$.



Determine $\sin^2\theta_W$ using data from 1) $\bar{\nu}_\mu e^- \to \bar{\nu}_\mu e^-$; 2)$\nu_\mu e^- \to \nu_\mu e^-$; 3)$\bar{\nu}_e e^- \to \bar{\nu}_e e^-$; 4) $e^+ e^- \to \mu^+ \mu^-$.

g_V, g_A functions of $\sin\theta_W^2$ and q_f.

Using this $\sin^2\theta_W$ predict M_W, M_Z.

$M_W = 82 \pm 2$ GeV/c^2

$M_Z = 92 \pm 2$GeV/c^2

UA-1 and UA-2 experiments found W/Z with these masses, thus consistent with $\rho \sim 1$.

Test of unified Gauge theory but not a Quantum Gauge Theory!

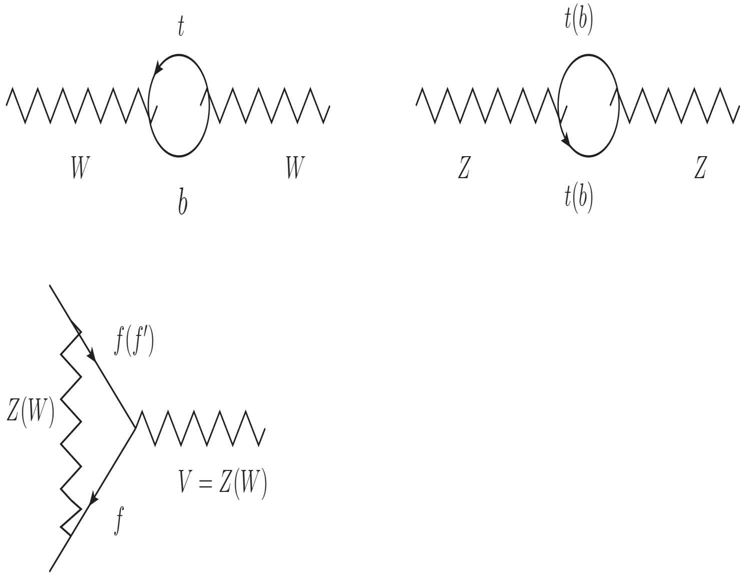
In making these comparisons one has used the tree level expressions for all the quantities.

eg. $M_W = \frac{g_2 v}{2}$

All these tree level relations change due to quantum corrections.

Renormalisability guarantees that these corrections are finite!

The gauge invariance in turn guarantees the renormalisability and as per our discussions needs the Higgs boson.



$$\rho_{corr} = 1 + \Delta\rho$$

$$\Delta\rho \simeq \frac{3 G_F M_t^2}{8\pi^2\sqrt{2}} = 0.01$$

There is also a diagram with h in the loop.

The corrections for the Z and W are different. The dominant corrections come from loop containing the heaviest quarks t, b (and sub dominant ones from h) ρ changes from value 1. (Veltman: screening theorem about the h contribution being small) Before top quark was found, its value was indirectly obtained from measuring ρ.

The corrections can be calculated only if theory is renormalisable. Renormalisability (Proved by 't Hooft and Veltman)

Precision measurements at the LEP-I of Z properties and all the neutral current couplings as well as precision measurements of the properties of the W at LEP-200, tested these corrections!

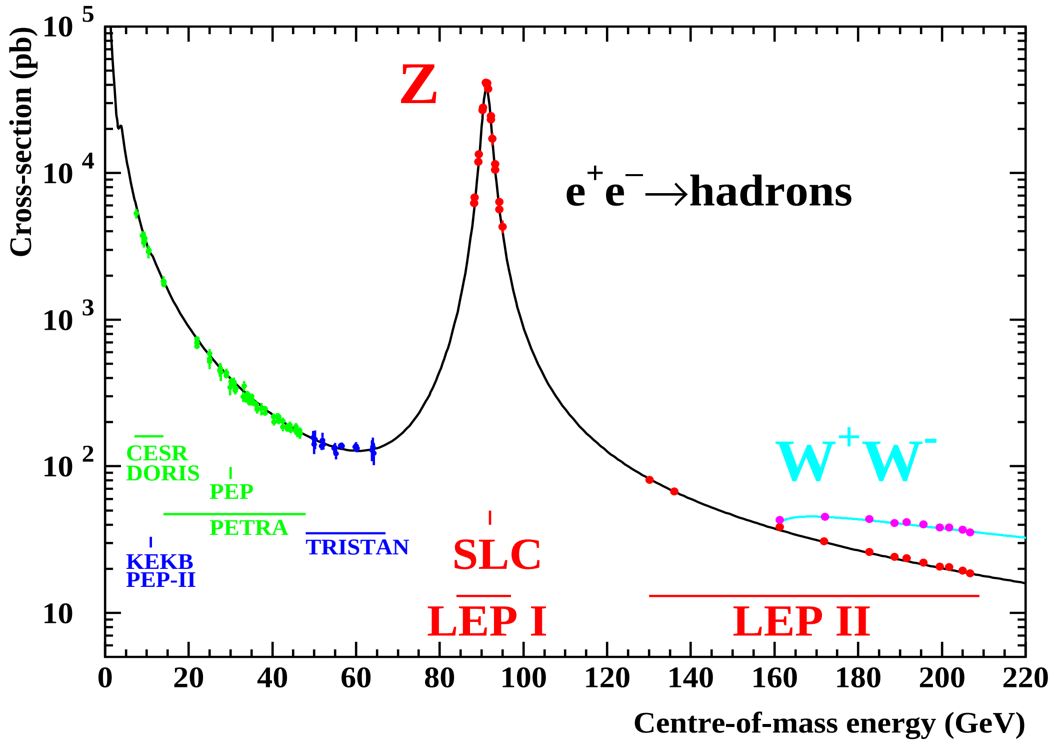
A test at the loop level of the relations should indicate a a finite mass for the Higgs if the theory is indeed renormalisable and would be an an indirect proof for the Higgs!

High precision measurements require high precision calculations.

Higher order QED and QCD corrections highly important and non-trivial.

Good understanding of QCD to calculate correctly what the detectors observe: jets.

Extensive collaborative studies between experimentalists and theorists LEP Yellow Reports.

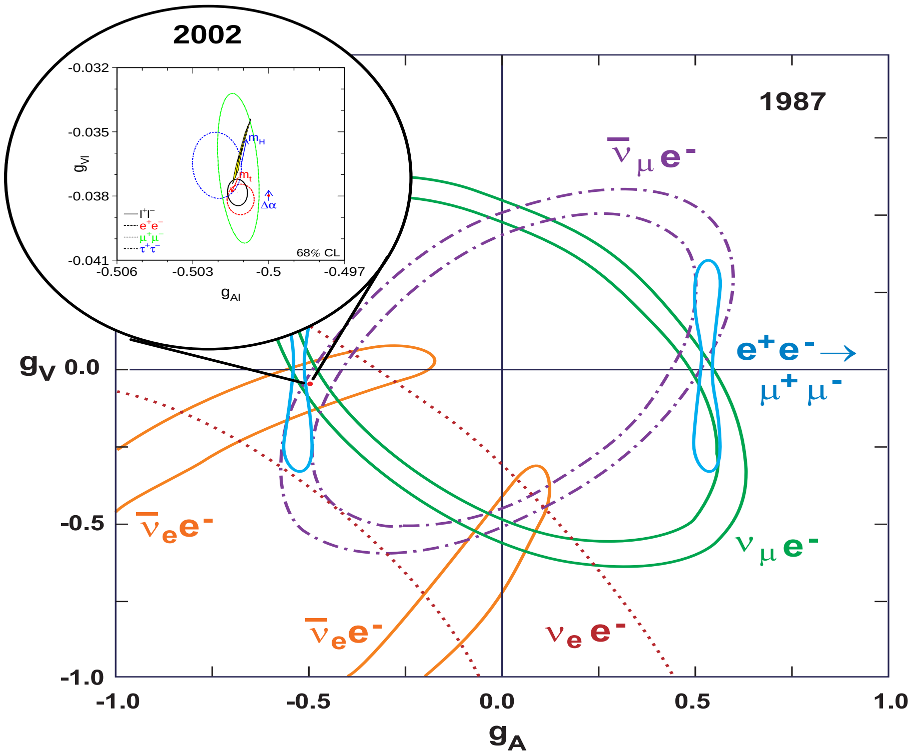


Solid line is the SM fit. Phys. Rept. 427, 257 (2006).

Large electromagnetic and QCD radiative corrections,

Initial state radiation makes the curve asymmetric near the resonance.

These measurements tested the tree level couplings and more!



Enormously more precise measurements.

Agreement with SM prediction would have been impossible unless the predicted values included higher order corrections, calculated in perturbation theory.

Recall correction to $\Delta\rho$ is 1% . The measurement is accurate to 1 part in 100 or better to see confirm this. Large mass of the t made this effect measurable!

Analog of $(g-2)_\mu$ for QED!

Logical steps in Precision testing of the SM and the indirect limits:

- SM has three parameters g_2, g_1 and v. All the SM couplings, gauge boson masses functions of these.

- A large number of EW observables measured quite accurately.

- M_Z, α_{em} and G_F are most accurately measured. Trade g_2, g_1 and v for these.

- All observables depend on these three apart from M_f (mainly M_t) *and* M_h, and of course α_s.

- Calculate all observables using **1 loop EW** radiative corrections which can be computed in a renormalisable quantum field theory.

- Compare with data, make a SM fit. Tests the SM at loop level.

Given α_{em}, M_Z, G_μ one can calculate M_W using tree level relations.

$\alpha_{em} = 1/137.0359895(61)$, $G_\mu = 1.16637(1) \times 10^{-5} GeV^{-2}; MZ = 91.1875 \pm 0.0021$ GeV

Calculate M_W using the tree level relation

$$\frac{G_\mu}{\sqrt{2}} = \frac{g_2^2}{8M_W^2} = \frac{\pi\alpha}{2M_W^2(1 - M_W^2/M_Z^2)}$$

$M_W^{tree} = 80.939$ GeV and $M_W^{expt} = 80.385 \pm 0.015$ GeV.

Loop corrections needed. Renormalisability guarantees that all the corrections are finite!

Loop corrections can be calculated consistently only in a renormalizable theory.

Depend on m_h logarithmically and on m_t quadratically.

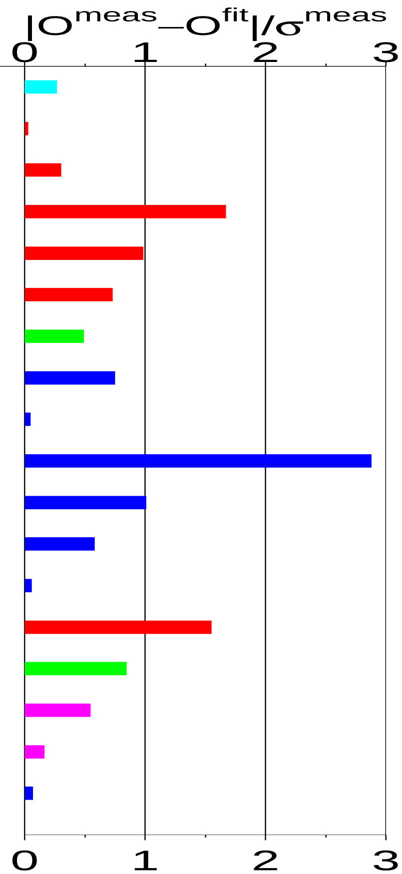
Compare measured values of M_W, m_t against calculated from EWPT for different values of m_h.

Precision measurements and precision calculations!

LEP legacy, augmented by Tevatron precision measurements!

Now by the LHC. In future by ILC,FCC,CLIC..?

	Measurement	Fit
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02750 ± 0.00033	0.02759
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4959
σ_{had}^{0} [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.742
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01645
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1481
R_b	0.21629 ± 0.00066	0.21579
R_c	0.1721 ± 0.0030	0.1723
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1038
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1481
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.385 ± 0.015	80.377
Γ_W [GeV]	2.085 ± 0.042	2.092
m_t [GeV]	173.20 ± 0.90	173.26



March 2011

see **http://lepewwg.web.cern.ch**

March 2011:

$M_W = 80.385 \pm 0.015$ GeV (measured), $80.377 GeV (theory)$

$m_t = 173.20 \pm 0.90$ GeV (measured) 172.26 GeV (theory)

In fact before top mass was measured at the Tevatron the fits made a prediction for it. The agreement between measurement and prediction was a triumph. Veltman and 't Hooft got the Nobel prize only after that!

The current values now are a little different. Tevatron, LHC have added to the precision. But my main point here was to just show the level at which the precision measurements test the SM

The value of χ^2 at the minimum is not great! so some people were bothered by it pre July 2012!

Absence of FCNC $\Rightarrow$ quarks must come in isospin doublets, charm was predicted amid top was expected to be present once b was found

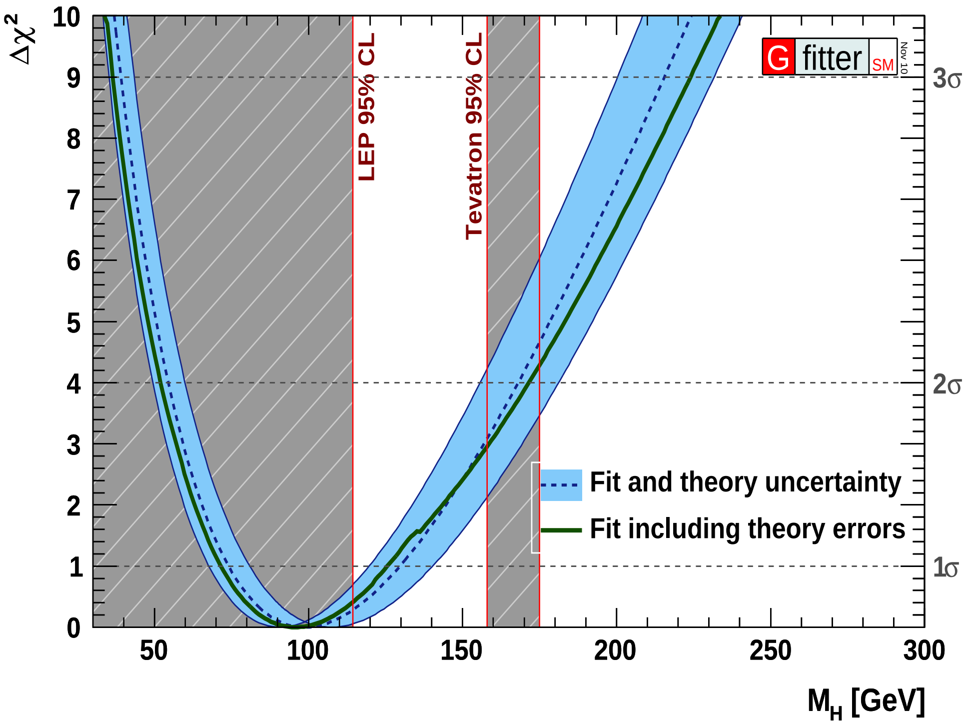
Indirect information on M_c, M_t from flavour changing neutral processes. Agreement with experimentally measured values 'proves' gauge theory.

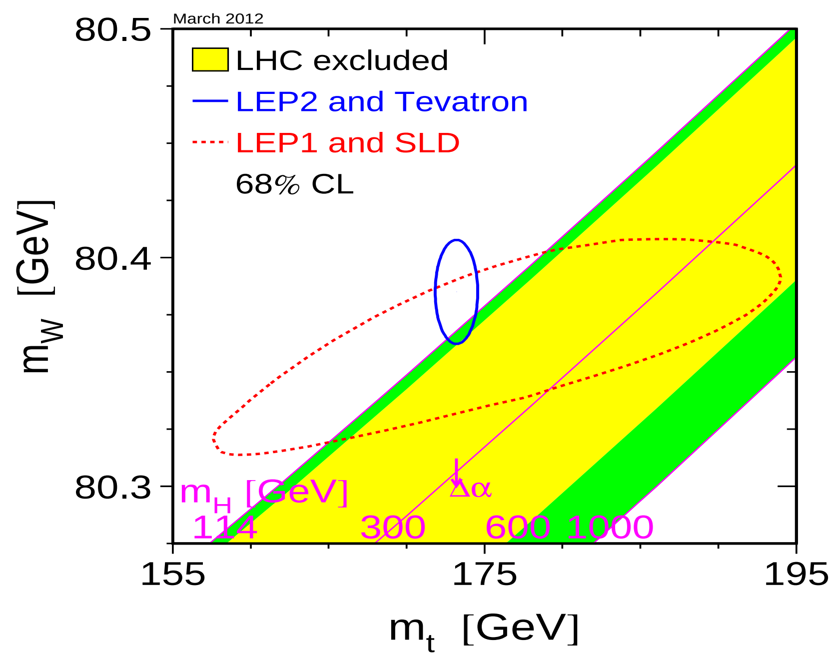
CP violation in meson systems can be explained in terms of the SM parameters and measured CKM mixing in quark sector.

M_W, M_Z predicted in terms of $\sin\theta_W$

M_t predicted from precision measurement of M_W, M_Z.

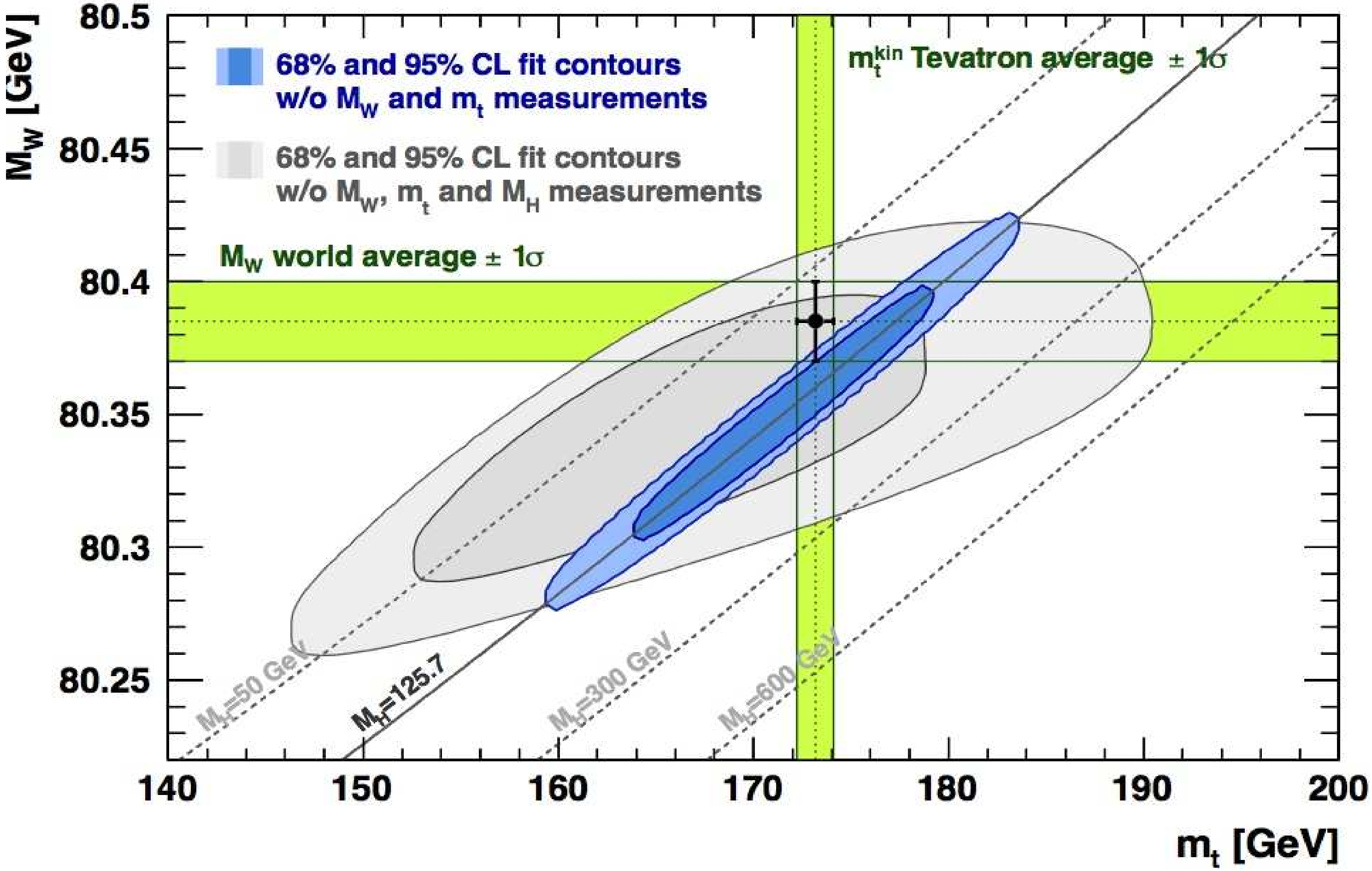
Prediction for where Higgs mass should be?





Exptal. Limits: from nonobservation of Higgs in direct searches and indirect limits from LEP/Tevatron precision measurements.

Before the observation of signal at the LHC: **Precision EW measurements like LIGHT Higgs**. For the SM to be correct Higgs HAD to be light!



Implication number 1 of the observed m_h value : SM rocks!

Remember: the Higgs mass range allowed by precision measurements can change when one goes away from the SM.

In fact a lot of effort had gone on , in constructing models how one can remove these constraints. Not only that many of these will not be required, but some are now even ruled out, by the observation of the light state.

Implication of the observed light state for BSM:

Model with fourth sequential generation with a single Higgs doublet got ruled out with 126 GeV (low mass) scalar.

This still does not tell us whether the SM is all that there is?

I.e is the SM a self-consistent theory all the way to Planck scale

OR

Does it need something more?

The observed mass of the Higgs MAY be able to tell us something about it!

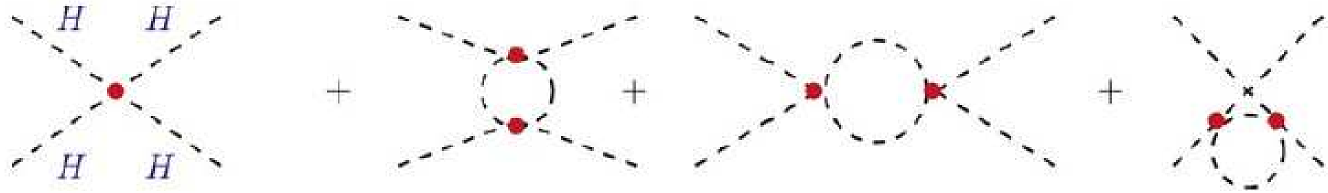
Higgs mass bounded from theoretical considerations:

Pertubative Uniatrity: Demanding that W^+W^- scattering amplitude satisfy perturbative unitarity in fact one can derive the particle content of one generation of the SM Tiktopoulos, Cornwall as well as S.D. Joglekar $\sim$ 1974

But the unitarity is guaranteed ONLY for $m_h < 780$ GeV B.Lee and Thacker

The small value of the mass of the observed state means that the SM satisfies tree level unitarity without any trouble!

Triviality and Stability Bounds: demanding that the quartic coupling in the Higgs potential $V_h = \lambda v h^3 + \lambda/4 h^4$ remains perturbative and positive, under loop corrections. The corrections come from:



At large m_h and large λ considerations of triviality give an upper bound. That used to be of great concern !

With the small observed mass it is the stability bound!

Remember: $M_h^2 = \lambda v^2$. For large λ the loop corrections dominated by the h–loops.

At one loop running of λ given by:

$$\frac{d\lambda(Q^2)}{d\log Q^2} = \frac{3}{4\pi}\lambda^2(Q^2)$$

Solving this, one gets

$$\lambda(Q^2) = \frac{\lambda(v^2)}{[1 - \frac{3}{4\pi^2}\lambda(v^2)\log(\frac{Q^2}{v^2})]}$$

For large $Q^2 \gg v^2$ then $\lambda(Q^2)$ develops a pole (the Landau pole).

If we demand that λ remain always in perturbative regime, we can ONLY have $\lambda = 0$. Theory will be trivial.

One can take an alternate view:

Demand that the scale at which λ blows up is above a given scale Λ.

For a given M_h the scale at which the pole lies

$$\Lambda_C = v \,\exp\left(\frac{2\pi^2}{3\lambda}\right) = v \exp\left(\frac{4\pi^2 v^2}{3M_h^2}\right)$$

Using $\Lambda_C = \Lambda = 10^{16}$ GeV, we will find $M_h \lesssim 200$ GeV. Upper Bound: called triviality bound

Thus just the mass of M_h can give indication of the scale of new physics beyond the SM

When M_h is small and λ not large, the fermion/gauge boson loops are important. Fermions loops come with a negative sign!

Now the RGE for λ is given by

$$\frac{d\lambda(Q^2)}{d\log(Q^2)} \simeq \frac{1}{16\pi^2}[12\lambda^2 + 6\lambda\lambda_t^2 - 3\lambda_t^4 - \frac{3}{2}\lambda(3g_2^2 + g_1^2) + \frac{3}{16}(2g_2^4 + (g_2^2 + g_1^2)^2)]$$

λ_t is the Yukawa coupling for the top. At small M_h and hence small $\lambda(v)$, at some value of Q, λ can turn negative. Potential will be unbounded. Vacuum will be unstable

The condition is

$$M_h^2 > \frac{v^2}{8\pi^2} \log(Q^2/v^2) \left[12 m_t^2/v^4 - \frac{3}{16}(2g_2^4 + (g_2^2 + g_1^2)^2)\right].$$

If we demand that the $\lambda(Q)$ is positive upto Λ_C we then get a lower bound.

For example:

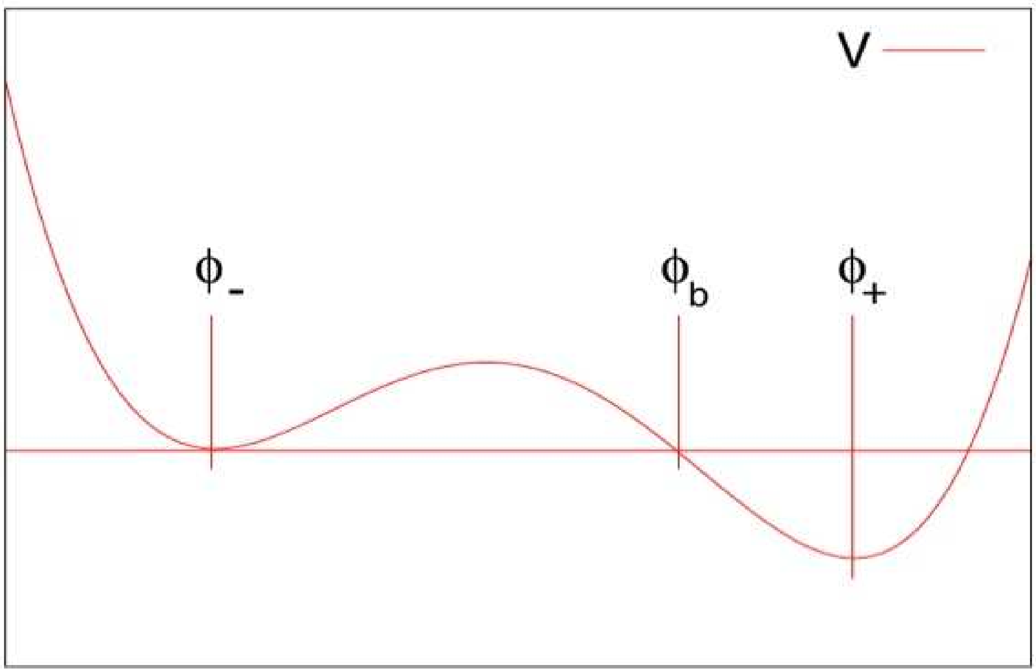
$\Lambda_C = 10^3 GeV$, $M_h \gtrsim$ 70 GeV

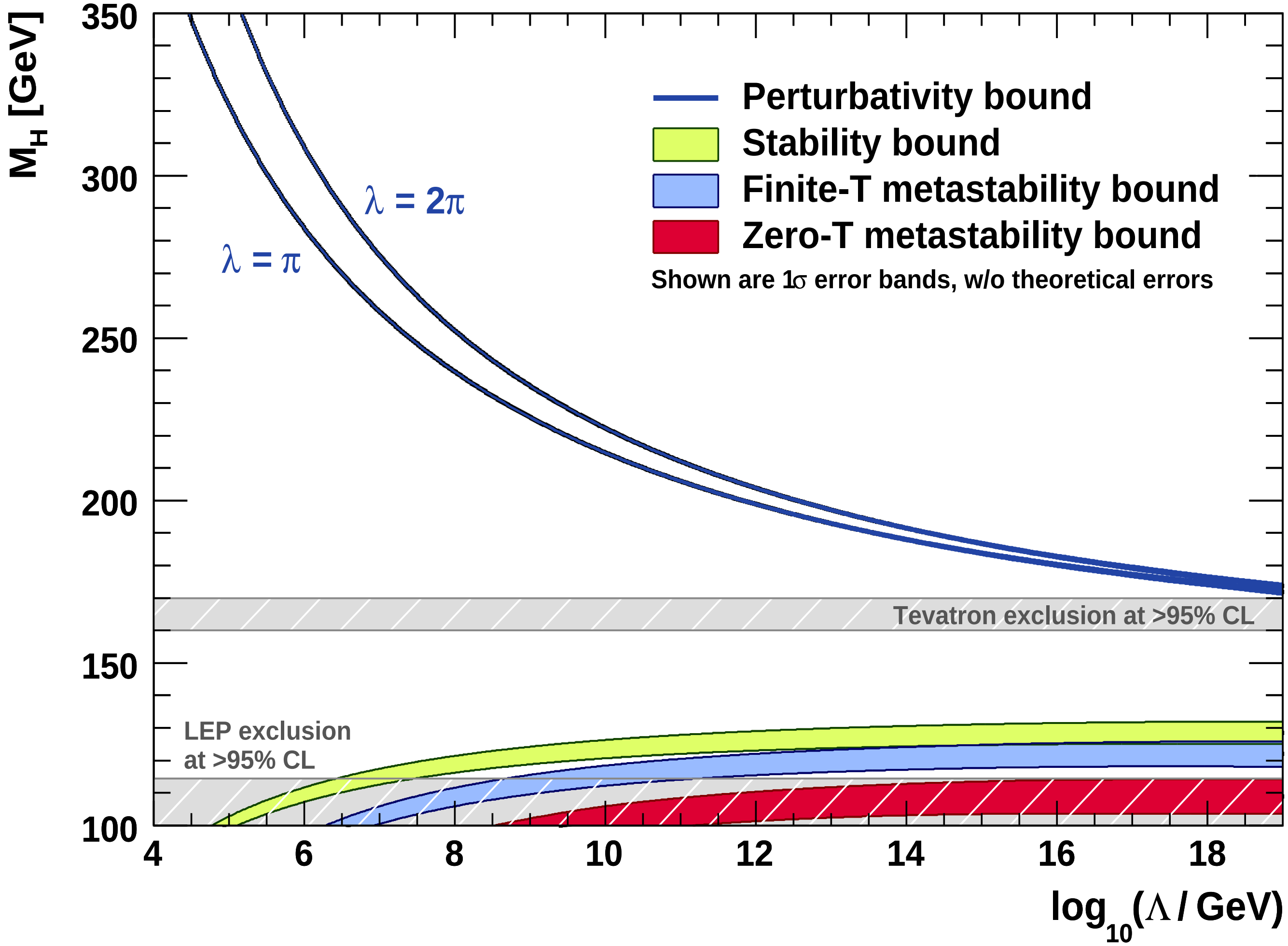
Earliest calculations of stability bounds by Linde, Weinberg.

Maini et al, Altarelli-Isidori, M. Sher, Quiros...: analysis of stability and triviality bound using RGE, metastability...

Planck scale dynamics might stabilise the vacuum for $|\Phi| >> v$ and we might be living in a metastable vacuum which has a life time bigger than that of the Universe.

How to calculate transition rates: Coleman showed us in 1977!



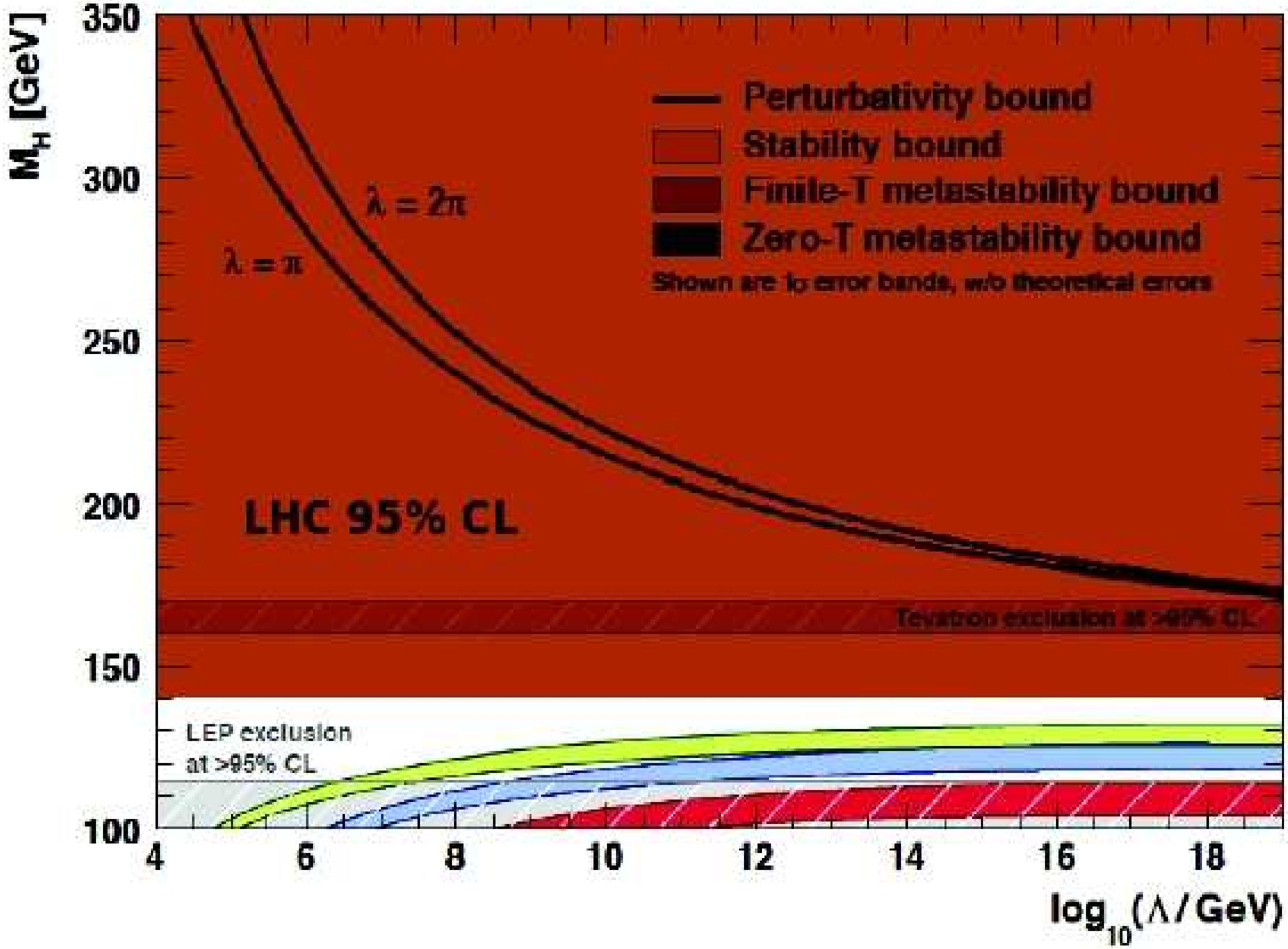


From a paper by Ellis, Giudice et al, PLB 679, 369-375 (2009). Includes higher order effects compared to the formulae here.

In view of the rather small values of m_h indicated by EWPT, need for more accurate calculation of these limits was required.

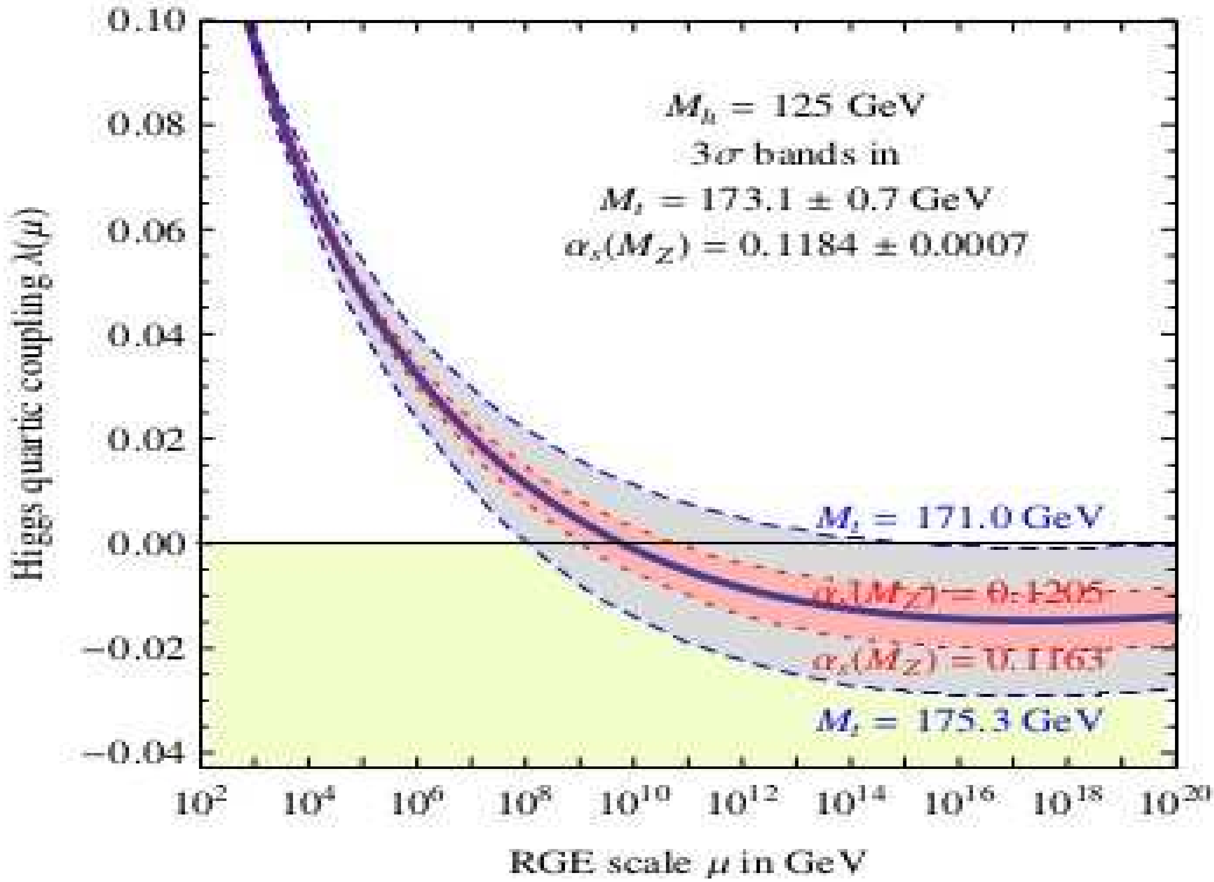
These limits critically depend also on $m_t^{\bar{M}S}$

State of the art in 2009: (Ellis, Giudice et al:0906.0954)



So the reported value around 125/126 GeV is very very special.

De Grassie et al (1205.6497) Complete NNLO analysis. Major progress. Theoretical error on the obtained bounds due to missing higher order corrections reduced to 1 GeV



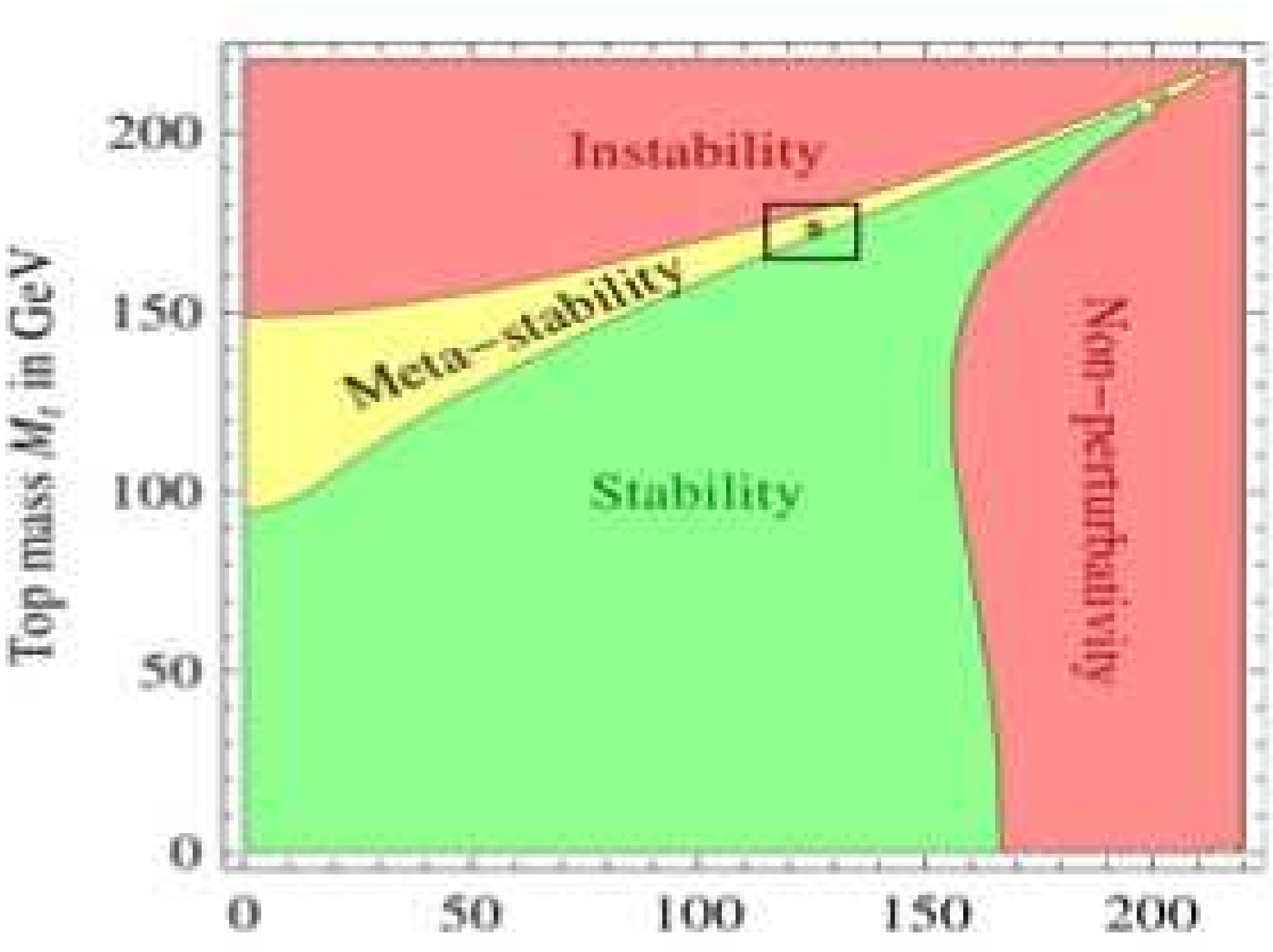
$$M_h\ [\text{GeV}] > 129.4 + 1.4\left(\frac{M_t\ [\text{GeV}] - 173.1}{0.7}\right) - 0.5\left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007}\right) \pm 1.0_{\text{th}}$$

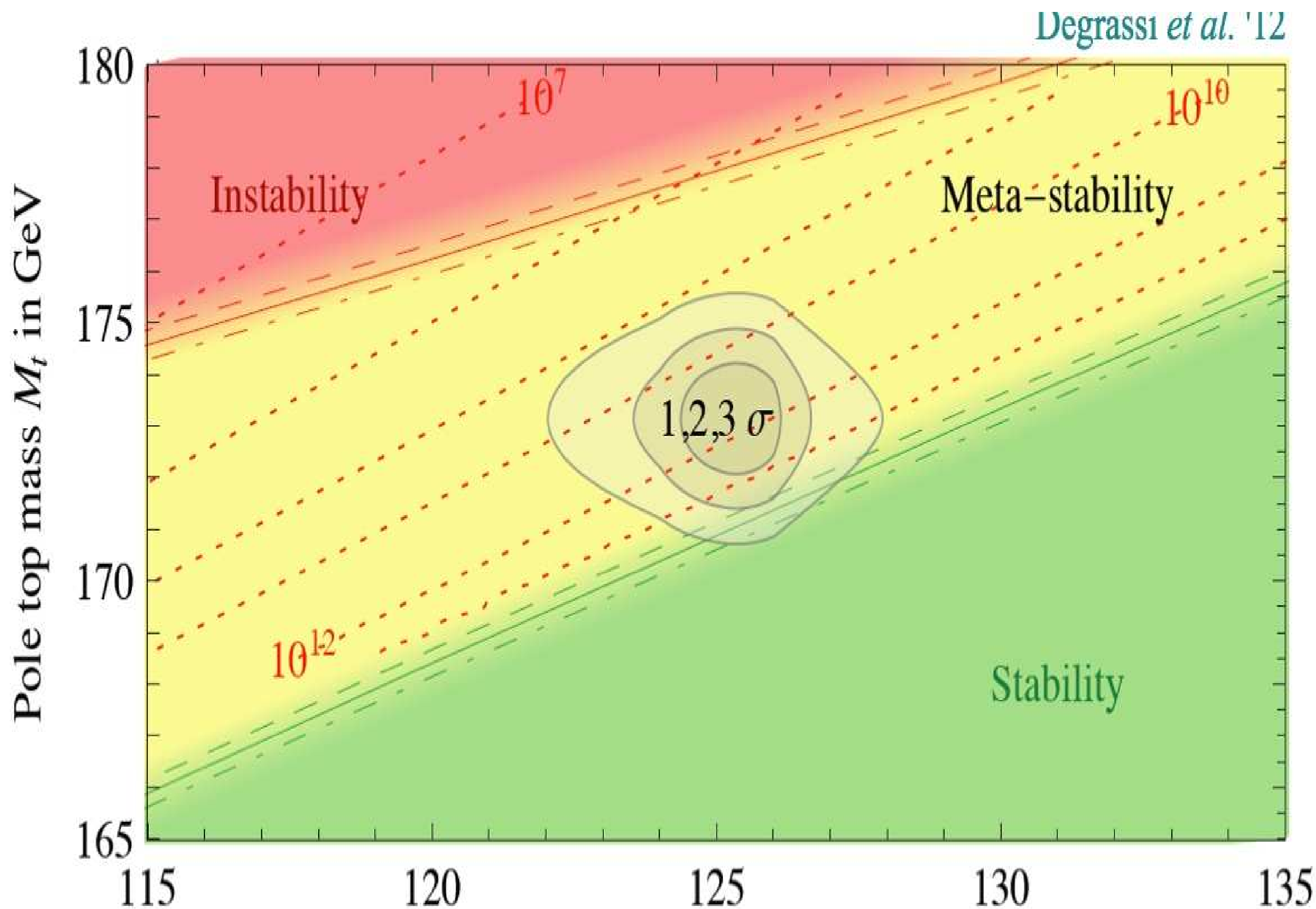
Use errors on pole mass $\Delta m_t = \pm 0.7$ GeV

So for $m_h < 126$ GeV vacuum stability of the SM all the way to Planck Scale is excluded at 98% c.l.

The exact scale where λ crosses zero, though not M_{pl} seems close to it in the SM depending on exact value of m_h.

This may be relevant for consideration of BSM or models of inflation etc.


Instability
Meta–stability
Stability
Non–perturbativity
Top mass M_t in GeV
200
150
100
50
0
0
50
100
150
200


Degrassi et al. '12
Pole top mass M_t in GeV
Instability
Meta−stability
Stability
1,2,3 σ
10^7
10^{10}
10^{12}
180
175
170
165
115
120
125
130
135

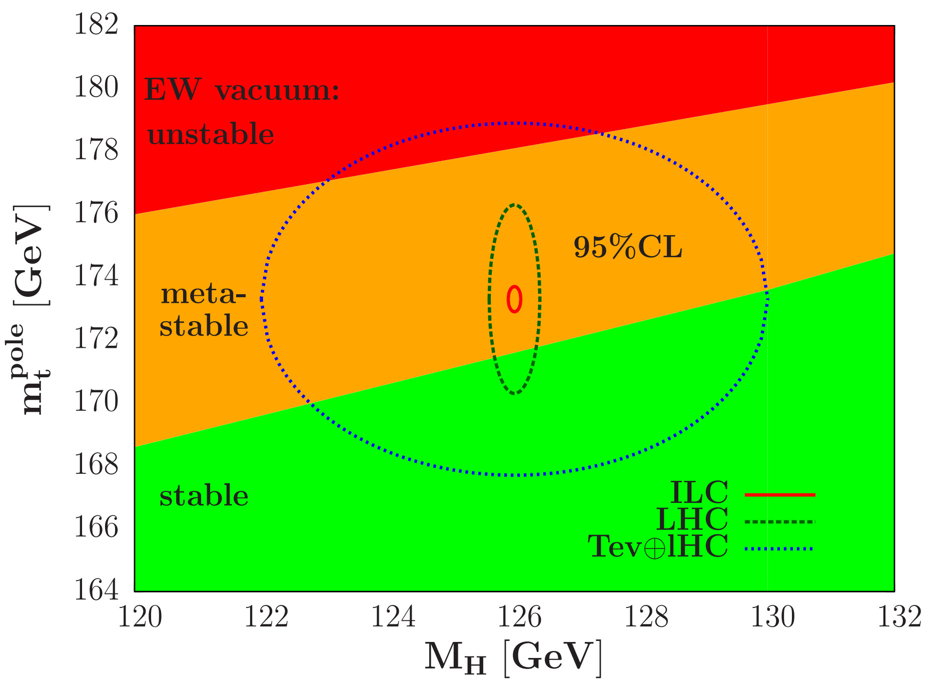
Reconsider the stability bounds given by Giudice et al. They used errors on m_t as measured at the Tevatron/LHC : the so called kinematic mass.

Moch et al : extract the $\bar{M}S$ mass of the top quark from the measurement of the top quark cross-sections at the Tevatron and the NNLO calculation. Led to larger errors!

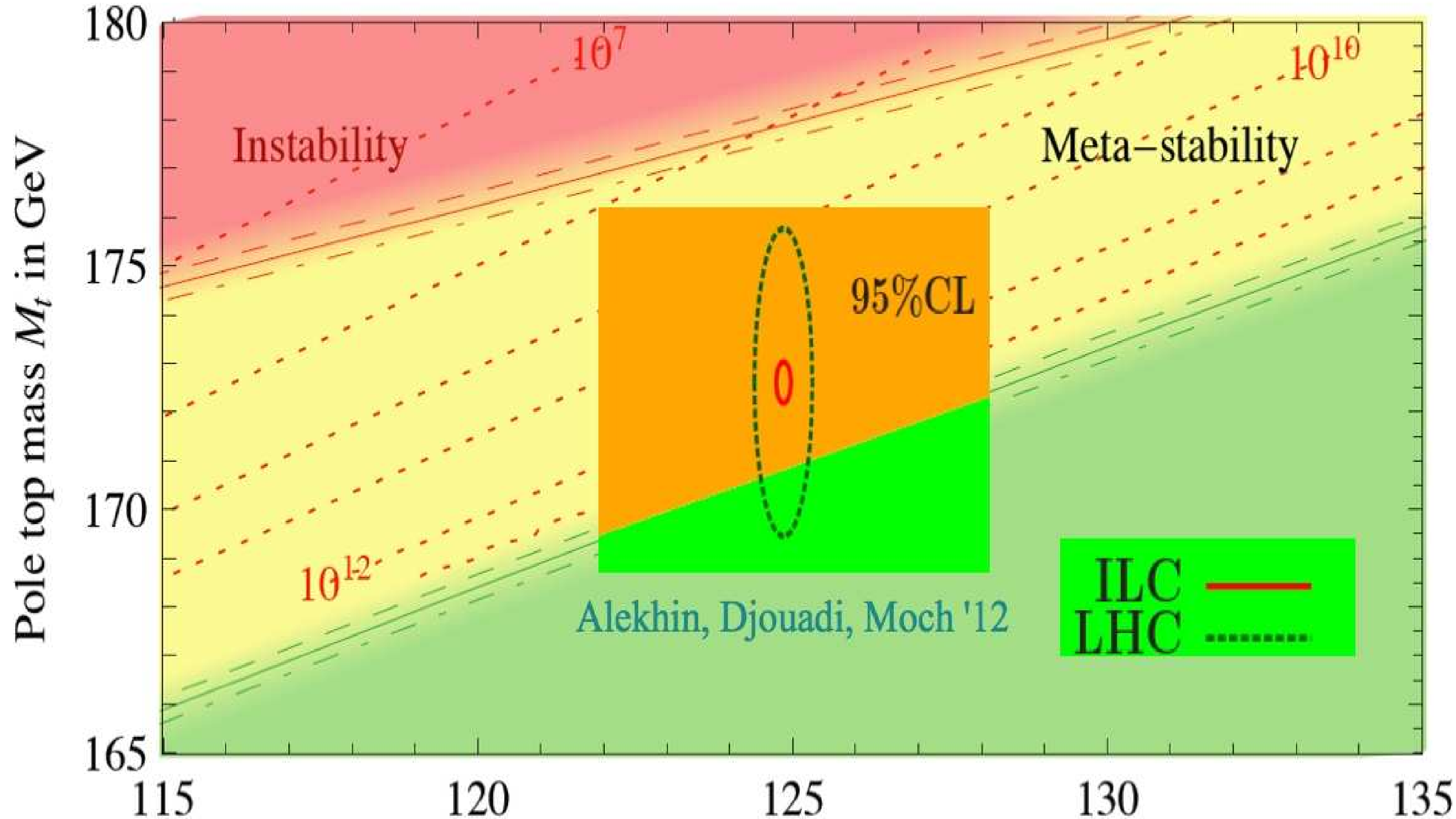
Estimate: $m_t^{pole} = 173.3 \pm 2.8$ GeV.

Vacuum stability constraint now becomes $m_h > 129.4 \pm 5.6$ GeV.

So the conclusion about the scale up to which SM is valid without getting into conflict with vacuum stability is weakened.



So the precision measurement of the mass at the ILC can really shed light whether higgs mass point to the **NEED** of BSM physics at a **particular scale**.


Pole top mass M_t in GeV
180
175
170
165
115
120
125
130
135
Instability
Meta−stability
10^7
10^{10}
10^{12}
95%CL
Alekhin, Djouadi, Moch '12
ILC
LHC

In any case the days of Standard Model are coming to an end in some sense!

Hopefully the case will be 'The King is Dead', 'Long live the King' !

Already the mass of the observed state can be used to answer the question about the scale unto which the SM is valid.

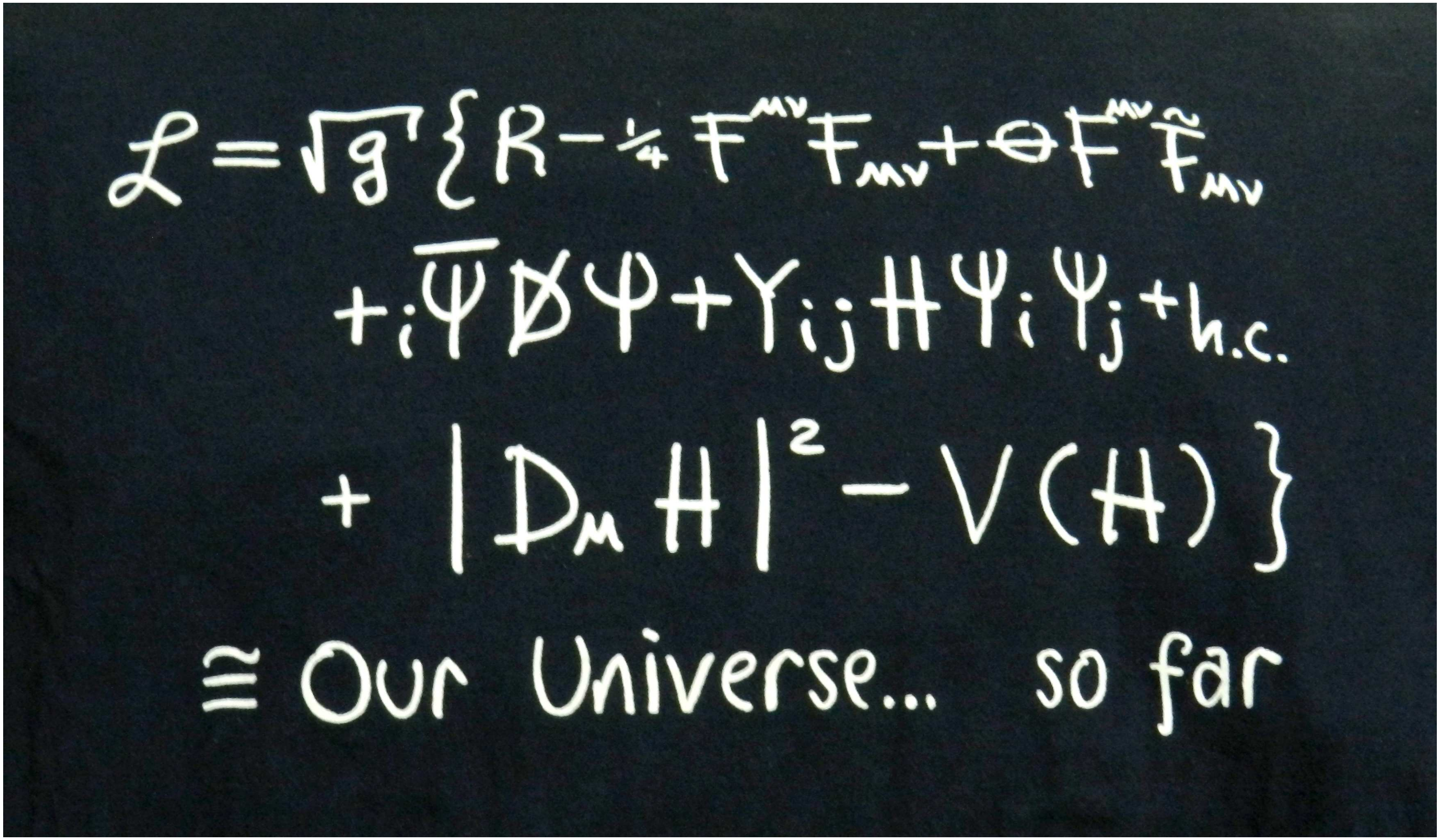
Just like the **gauge principle** and the **unitarity** were the guiding principle so far now the **'light' scalar** might be the guiding principle for future developments!

We should get a peek at the BSM land through the 'window' of measurement of the properties of the Higgs!

Exciting days ahead for sure!

If 14 TeV LHC should also fail to find 'direct' evidence for the BSM physics we would really have to understand what is so special about the Standard Model and hopefully that answer won't be Anthropic Principle !


BSM
HIGGS


$\mathcal{L} = \sqrt{g}\{R - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \Theta F^{\mu\nu}\tilde{F}_{\mu\nu}$
$+ i\bar{\Psi}\not{D}\Psi + Y_{ij}H\Psi_i\Psi_j + h.c.$
$+ |D_\mu H|^2 - V(H)\}$
$\cong$ Our Universe... so far

1)Gauge Theory of Elementary Particle Physics, T.P. Cheng and Ling-Fong Li

2)M. Peskin and S. Schröder, An Intorduction to Quantum Field Theory

3)The Standard Model, C. Burgess and G. Moore

4)Gauge Theories of Weak Inteactions: W. Greiner and B. Mueller

5)Quarks and Leptons, F. Halzen and A. Martin: phenomenological aspects of electro-weak interaction physics.

6)Introduction to High Energy Physics, D.H. Perkins

7)'Standard Model of Particle Physics' : R.M. Godbole and S. Mukhi (To be published by CUP)