Cosmology

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Outline

- Lecture 1
  - Expanding Universe
  - Dark matter: evidence
  - WIMPs
- Lecture 2
  - Axions
    - Theory
    - Cosmology
    - Search
  - Warm dark matter
    - Sterile neutrino
    - Gravitino
  - Dark matter summary
  - Baryon asymmetry of the Universe
    - Generalities.
    - Electroweak baryon number non-conservation

Outline, cont'd

- Lecture 3
  - Electroweak baryogenesis. What can make it work?
  - Before the hot epoch
    - Cosmological perturbations
      - Regimes of evolution
      - Acoustic oscillations: evidence for pre-hot epoch
    - Inflation and alternatives
    - BICEP-2 saga
  - Conclusions

Appendices

- Calculation of WIMP mass density
- Estimating axion mass density
- Wave equation in expanding Universe
- Dark energy
- CMB anisotropies, BAO and recent Universe
- Anthropic principle/Environmentalism

Lecture 1

Expanding Universe

- The Universe at large is homogeneous, isotropic and expanding.

  3d space is Euclidean (observational fact!)

  Sum of angles of a triangle = 180^o, even for triangles as large as the size of the visible Universe.

  All this is encoded in space-time metric (Friedmann–Lemâitre–Robertson–Walker)

$$ds^2 = dt^2 - a^2(t)\mathbf{dx}^2$$

  $\mathbf{x}$: comoving coordinates, label distant galaxies.

  $a(t)dx$: physical distances.

  $a(t)$: scale factor, grows in time; a_0: present value (matter of convention)

- The Universe at large is homogeneous, isotropic and expanding. 3d space is Euclidean (observational fact!) Space-time metric

$$ds^2 = dt^2 - a^2(t)\mathbf{dx}^2$$

$a(t)dx$: physical distances.

$a(t)$: scale factor, grows in time; a_0: present value

$$z(t) = \frac{a_0}{a(t)} - 1 : \quad \text{redshift}$$

Light of wavelength λ emitted at time t has now wavelength $\lambda_0 = \frac{a_0}{a(t)}\lambda = (1+z)\lambda$.

$$H(t) = \frac{\dot{a}}{a} : \quad \text{Hubble parameter, expansion rate}$$

- Present value

$$H_0 = (67.3 \pm 1.2)\ \frac{\text{km/s}}{\text{Mpc}} = (14 \cdot 10^9\ \text{yrs})^{-1}$$

$1\ \text{Mpc} = 3 \cdot 10^6\ \text{light yrs} = 3 \cdot 10^{24}\ \text{cm}$

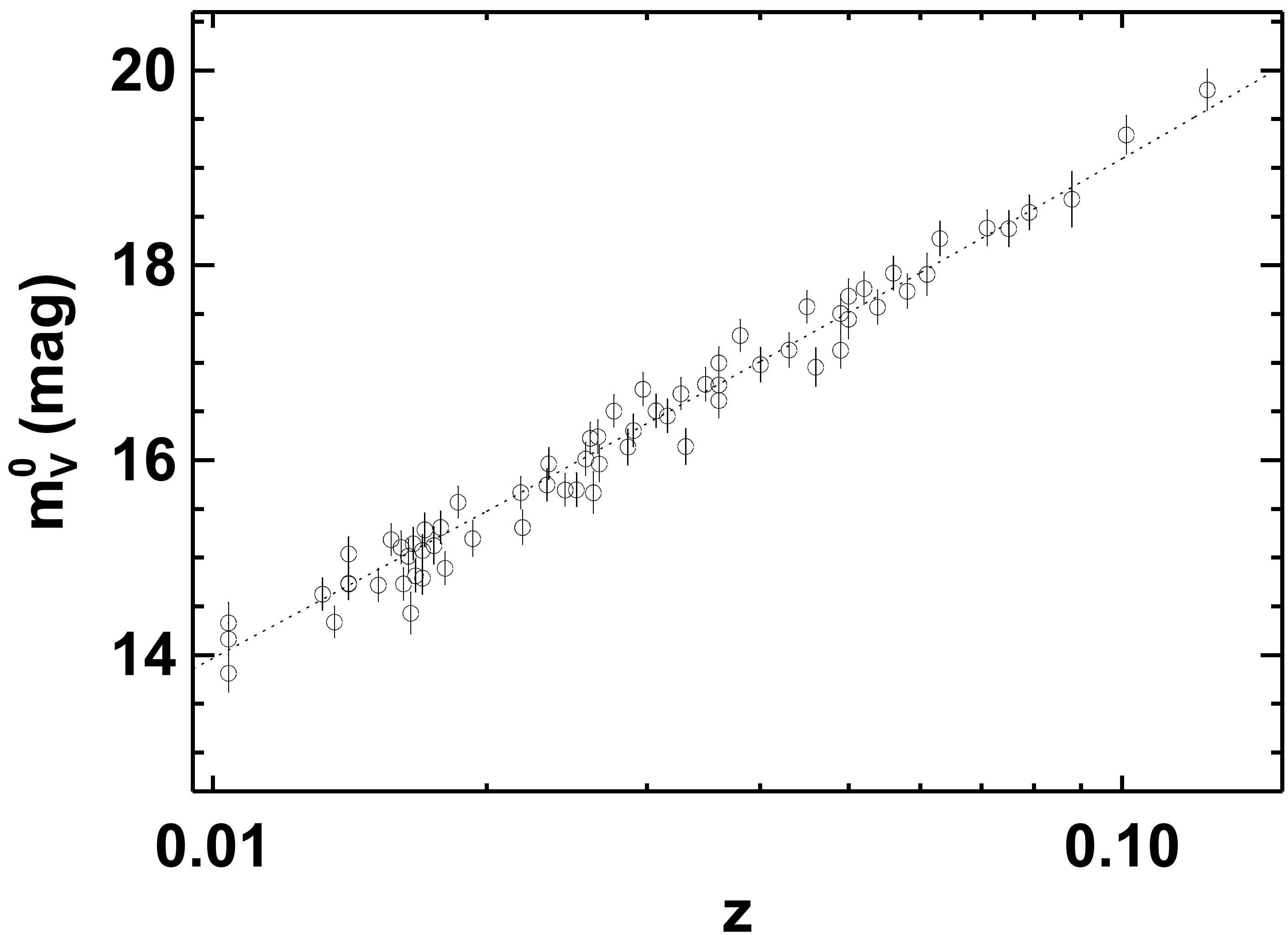
- Hubble law (valid at $z \ll 1$)

$$z = H_0 r$$

Figs. a,b,c

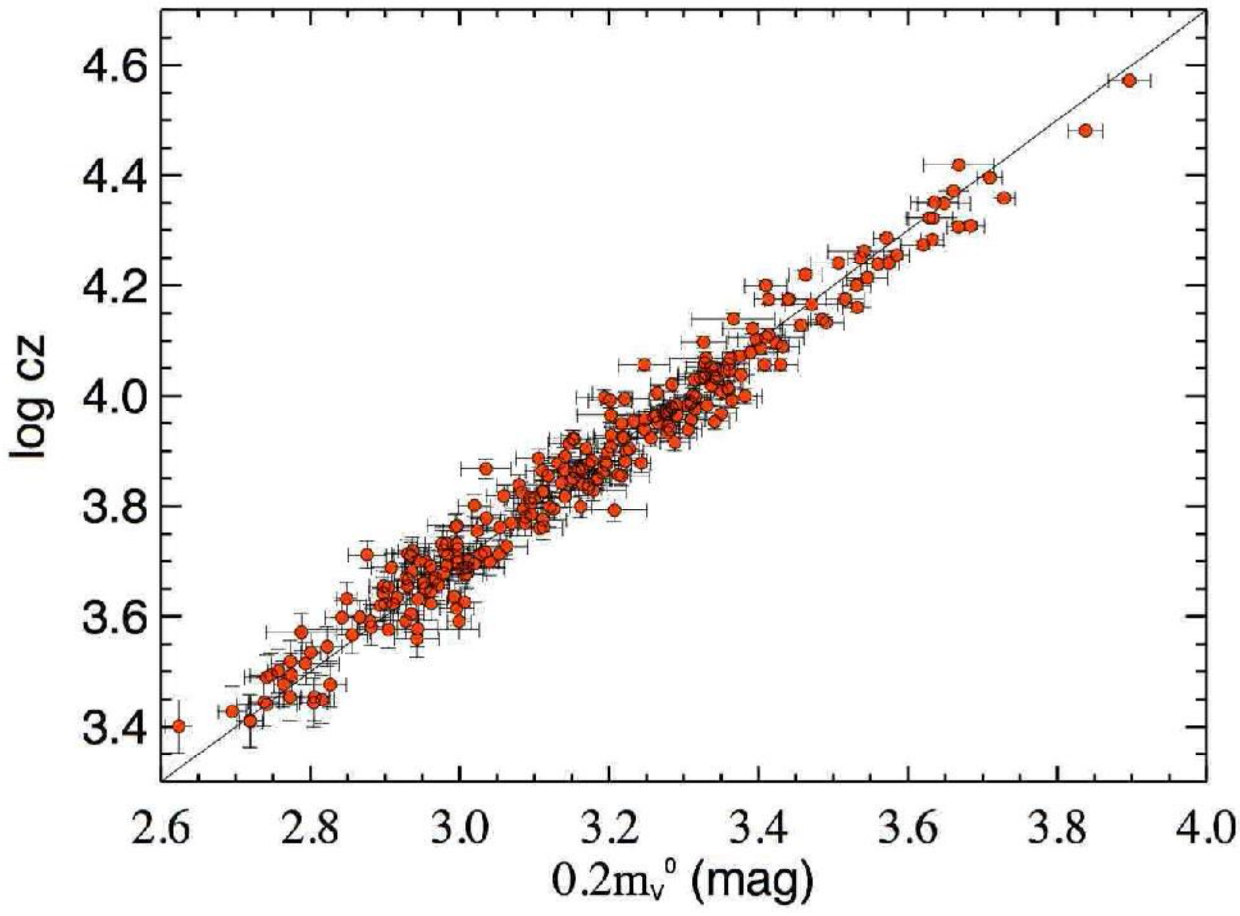
Problem: prove the Hubble law

Hubble diagram for SNe1a

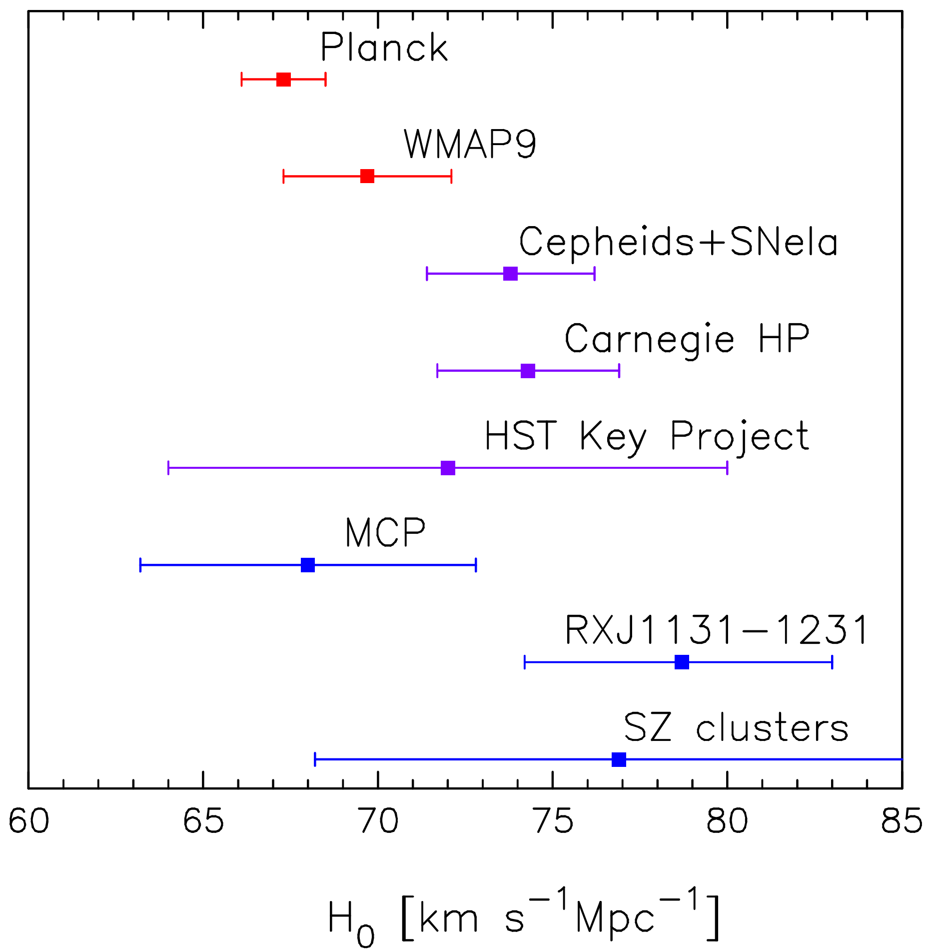


$\text{mag} = 5\log_{10} r + \text{const}$

Hubble diagram, recent



Systematics still large



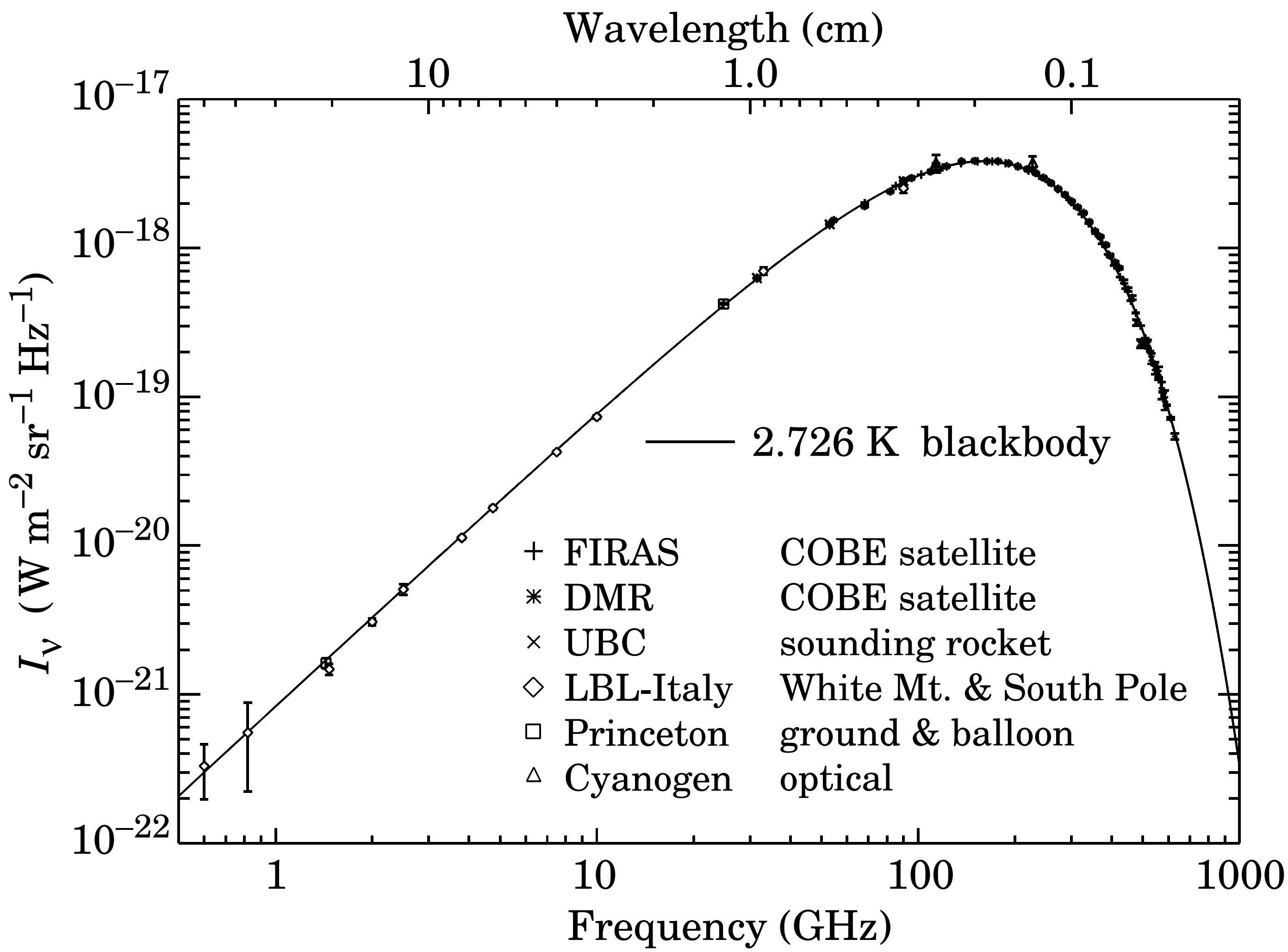
- The Universe is warm: CMB temperature today

$$T_0 = 2.7255 \pm 0.0006 \text{ K}$$

Fig.

It was denser and warmer at early times.

CMB spectrum



$T = 2.726$ K

- Present number density of photons

$$n_\gamma = \# T^3 = 410 \frac{1}{\text{cm}^3}$$

- Present entropy density

$$s = 2 \cdot \frac{2\pi^2}{45} T_0^3 + \text{neutrino contribution} = 3000 \frac{1}{\text{cm}^3}$$

In early Universe (Bose–Einstein, Fermi–Dirac)

$$s = \frac{2\pi^2}{45} g_* T^3$$

g_*: number of relativistic degrees of freedom with $m \lesssim T$;
fermions contribute with factor $7/8$.

Slow expansion $\Longrightarrow$ entropy conservation $\Longrightarrow$
Entropy density scales exactly as a^{-3}

Temperature scales approximately as a^{-1}.

Dynamics of expansion

- Friedmann equation: expansion rate of the Universe vs total energy density ρ ($M_{Pl} = G^{-1/2} = 10^{19}$ GeV):

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{8\pi}{3M_{Pl}^2}\rho$$

Einstein equations of General Relativity specified to homogeneous isotropic space-time with zero spatial curvature

- Present energy density

$$\rho_0 = \rho_c = \frac{3M_{Pl}^2}{8\pi}H_0^2 = 5 \cdot 10^{-6}\ \frac{\text{GeV}}{\text{cm}^3}$$

$$= 5\frac{m_p}{\text{m}^3}$$

$\hbar = c = k_B = 1$ in what follows

Present composition of the Universe

$$\Omega_i = \frac{\rho_{i,0}}{\rho_c}$$

present fractional energy density of i-th type of matter.

$$\sum_i \Omega_i = 1$$

- Dark energy: $\Omega_\Lambda = 0.685$
  ρ_Λ stays (almost?) constant in time
- Non-relativistic matter: $\Omega_M = 0.315$
  $\rho_M = mn(t)$ scales as $\left(\frac{a_0}{a(t)}\right)^3$
  - Dark matter: $\Omega_{DM} = 0.265$
  - Usual matter (baryons): $\Omega_B = 0.050$
- Relativistic matter (radiation): $\Omega_{rad} = 8.6 \cdot 10^{-5}$ (for massless neutrinos)
  $\rho_{rad} = \omega(t)n(t)$ scales as $\left(\frac{a_0}{a(t)}\right)^4$

Friedmann equation

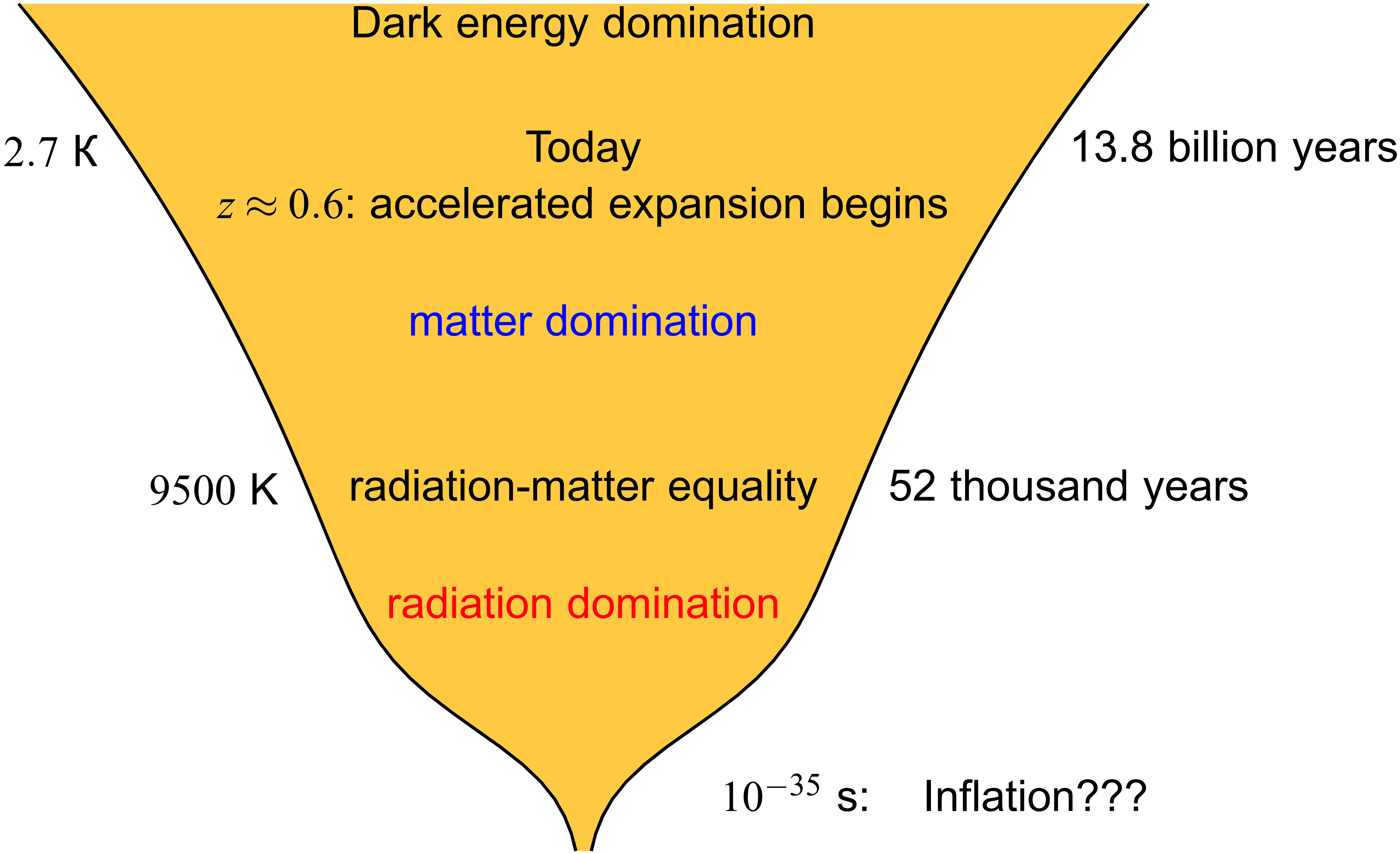
$$H^2(t) = \frac{8\pi}{3M_{Pl}^2}\left[\rho_\Lambda + \rho_M(t) + \rho_{rad}(t)\right] = H_0^2\left[\Omega_\Lambda + \Omega_M\left(\frac{a_0}{a(t)}\right)^3 + \Omega_{rad}\left(\frac{a_0}{a(t)}\right)^4\right]$$

$\ldots \Longrightarrow$ Radiation domination $\Longrightarrow$ Matter domination $\Longrightarrow$ Λ–domination

$z_{eq} = 3500$ now

$T_{eq} = 9500$ K $= 0.8$ eV

$t_{eq} = 52 \cdot 10^3$ yrs



Dark energy domination
2.7 K
Today
13.8 billion years
$z \approx 0.6$: accelerated expansion begins
matter domination
9500 K
radiation-matter equality
52 thousand years
radiation domination
10^{-35} s: Inflation???

Expansion at radiation domination

- Expansion law:

$$H^2 = \frac{8\pi}{3M_{Pl}^2}\rho \quad \Longrightarrow \quad \frac{\dot{a}^2}{a^2} = \frac{\text{const}}{a^4}$$

  Solution:

$$a(t) = \text{const} \cdot \sqrt{t}$$

- $t = 0$: Big Bang singularity

$$H = \frac{\dot{a}}{a} = \frac{1}{2t}\,, \qquad \rho \propto \frac{1}{t^2}$$

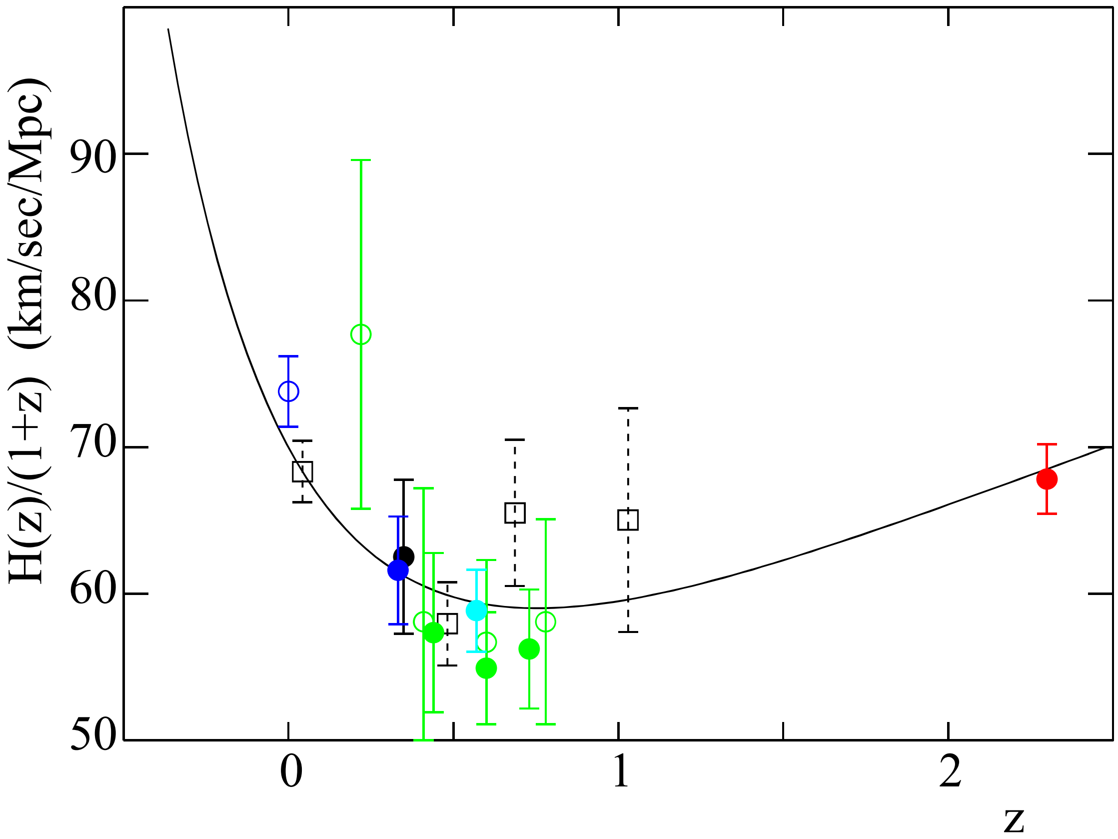
- Decelerated expansion: $\ddot{a} < 0$.

- NB: Λ-domination

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi}{3M_{Pl}^2}\rho_\Lambda = \text{const} \Longrightarrow a(t) = \mathrm{e}^{H_\Lambda t}$$

accelerated expansion Fig.

Deceleration to acceleration



$$\frac{H}{1+z} = \frac{\dot{a}}{a} \cdot \frac{a}{a_0} = \frac{\dot{a}}{a_0} = \dot{a}(z)\ ;$$ large $z \leftrightarrow$ early time

Cosmological (particle) horizon

Light travels along $ds^2 = dt^2 - a^2(t)d\mathbf{x}^2 = 0 \Longrightarrow dx = dt/a(t)$.
If emitted at $t = 0$, travels finite coordinate distance

$$\eta = \int_0^t \frac{dt'}{a(t')} \propto \sqrt{t} \quad \text{at radiation domination}$$

$\eta \propto \sqrt{t} \Longrightarrow$ visible Universe increases in time

Fig.

Physical size of causally connected region at time t (horizon size)

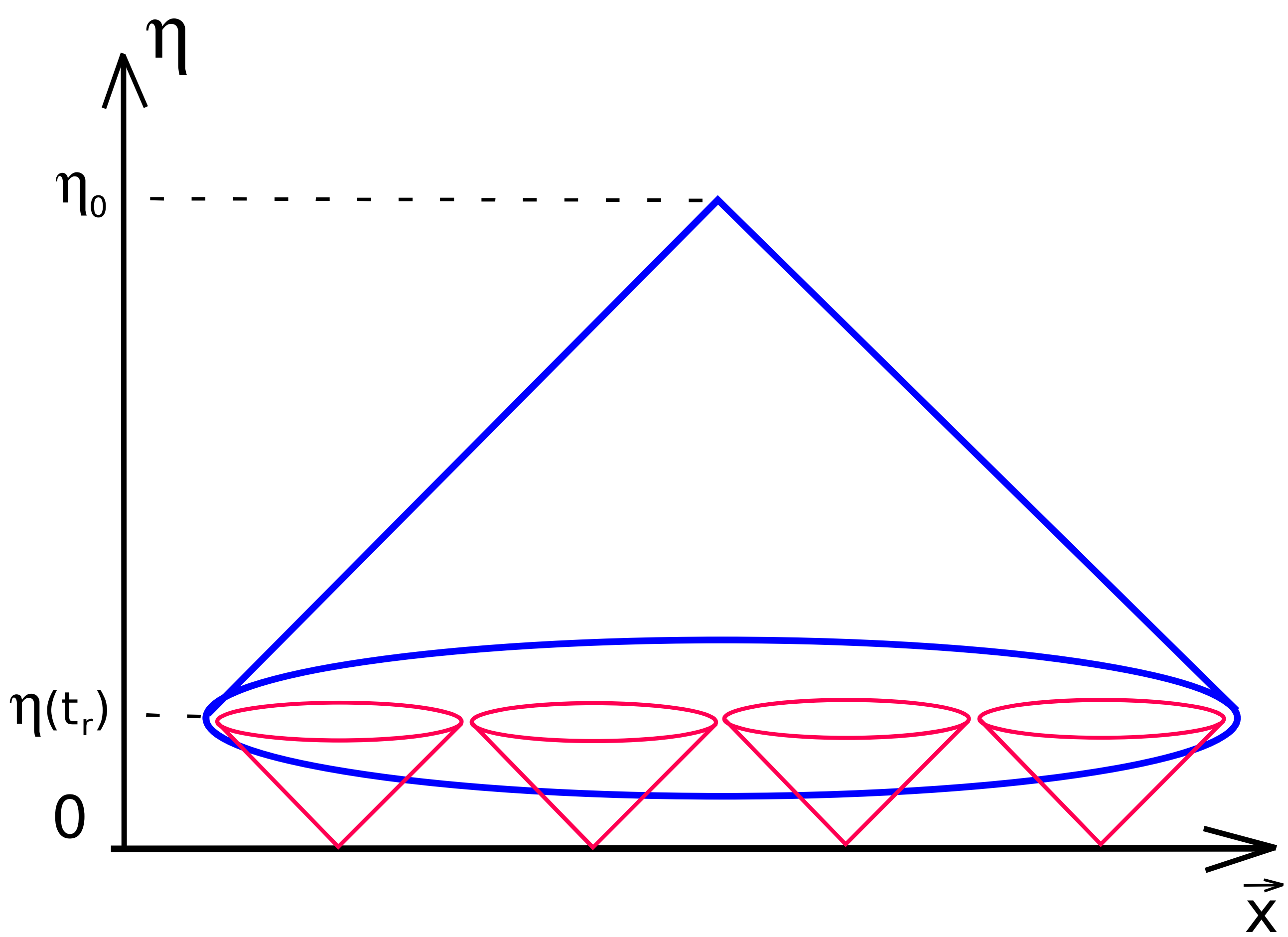
$$l_{H,t} = a(t)\int_0^t \frac{dt'}{a(t')} = 2t \quad \text{at radiation domination}$$

In hot Big Bang theory at both radiation and matter domination

$$l_{H,t} \sim t \sim H^{-1}(t)$$

Today $l_{H,t_0} \approx 15$ Gpc $= 4.5 \cdot 10^{28}$ cm

Causal structure of space-time in hot Big Bang theory



We see many regions that were causally disconnected by time t_r.
Why are they all the same?

Cornerstones of thermal history

- Big Bang Nucleosynthesis, epoch of thermonuclear reactions

$$\begin{aligned} p+n &\to {}^2H \\ {}^2H+p &\to {}^3He \\ {}^3He+n &\to {}^4He \\ \text{up to} & \quad {}^7Li \end{aligned}$$

  Abundances of light elements: measurements vs theory

  $T = 10^{10} \to 10^9$ K, $\quad t = 1 \to 300$ s

  Earliest time in thermal history probed so far Fig.

- Recombination, transition from plasma to gas.

  $z = 1090$, $T = 3000$ K, $\quad t = 380\,000$ years

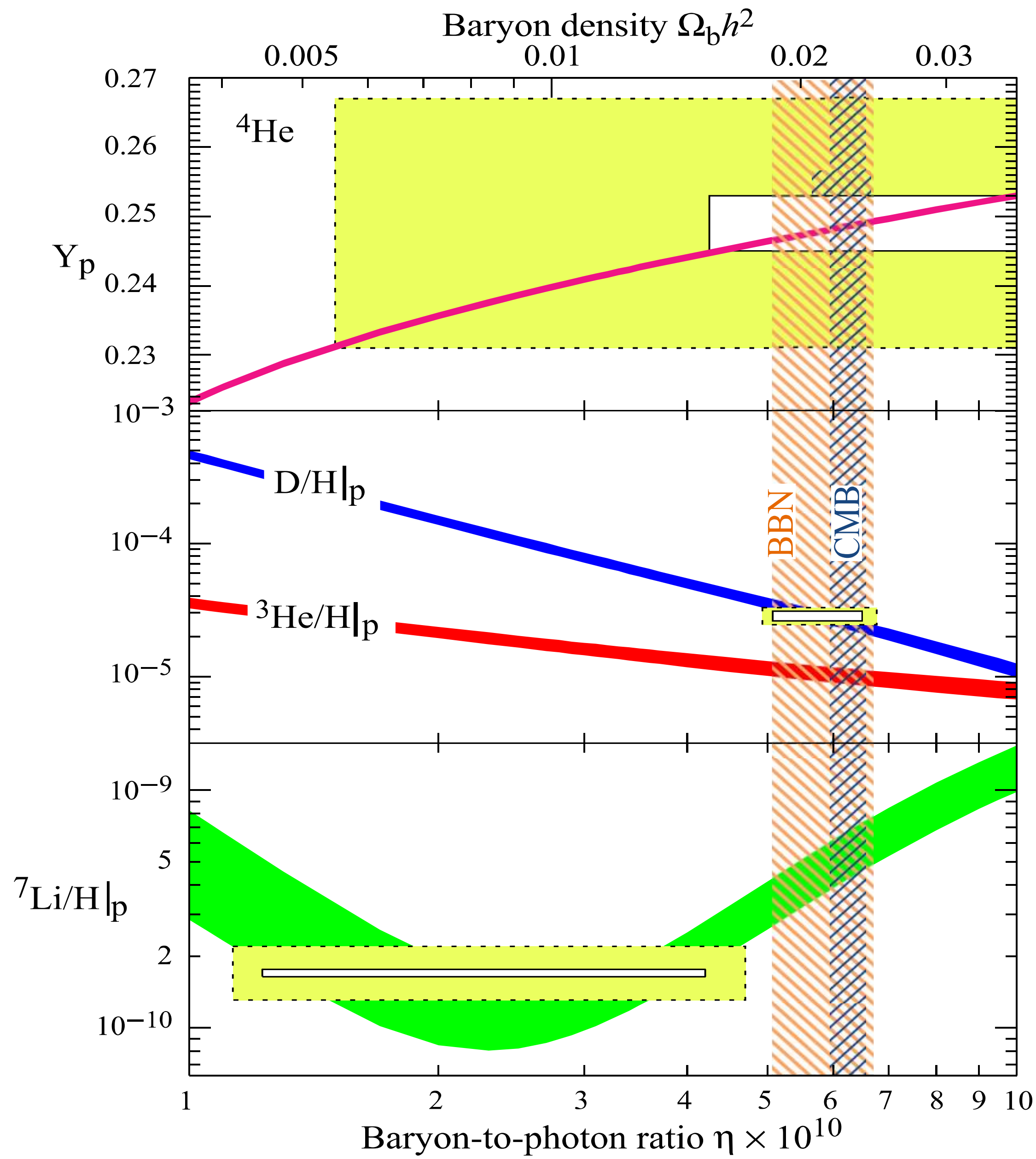
  Last scattering of CMB photons Fig.

- Neutrino decoupling: $T = 2-3$ MeV $\sim 3 \cdot 10^{10}$K, $\quad t \sim 0.1 - 1$s

- Generation of dark matter*

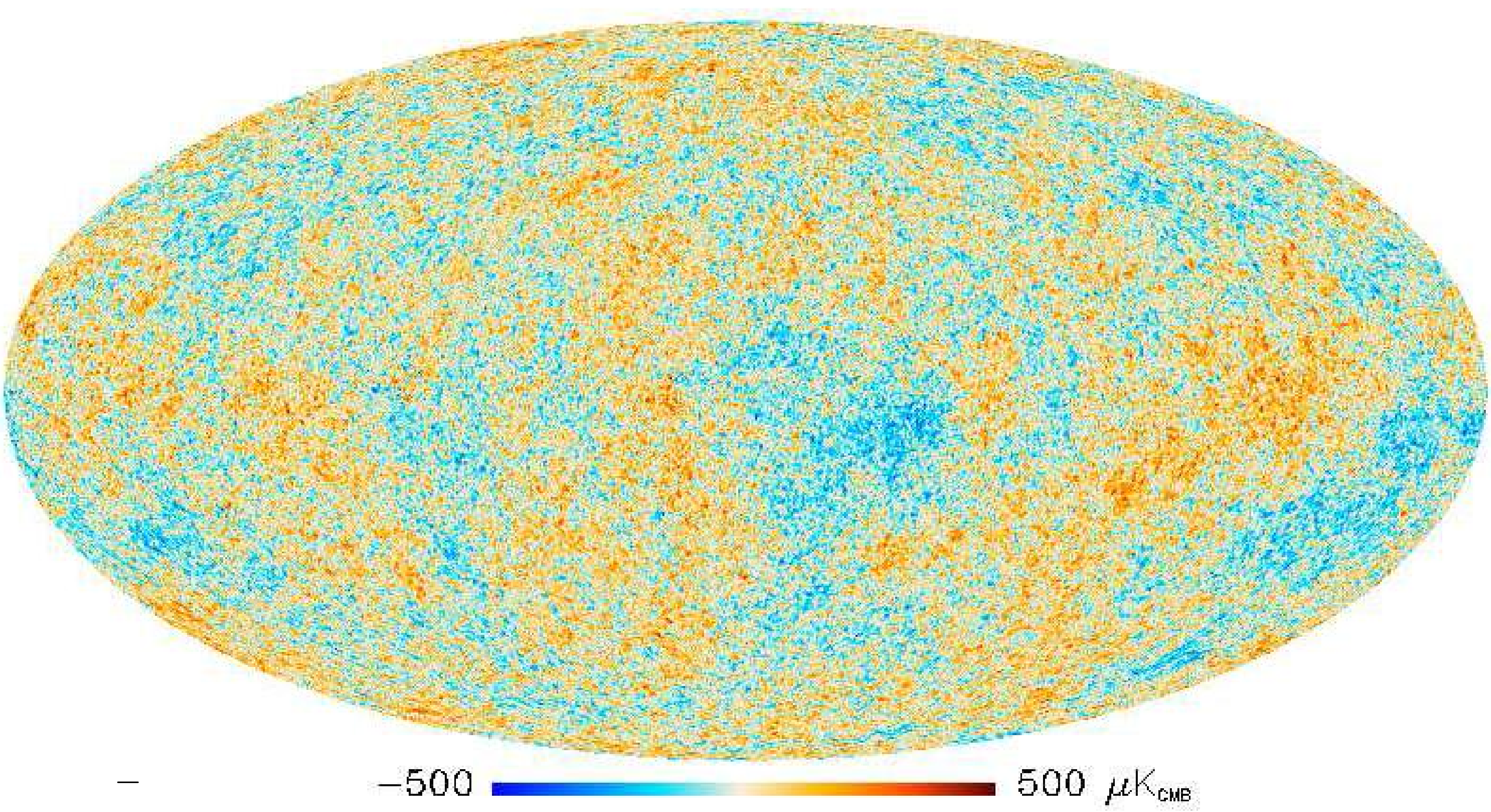
- Generation of matter-antimatter asymmetry*

*may have happend before the hot Big Bang epoch

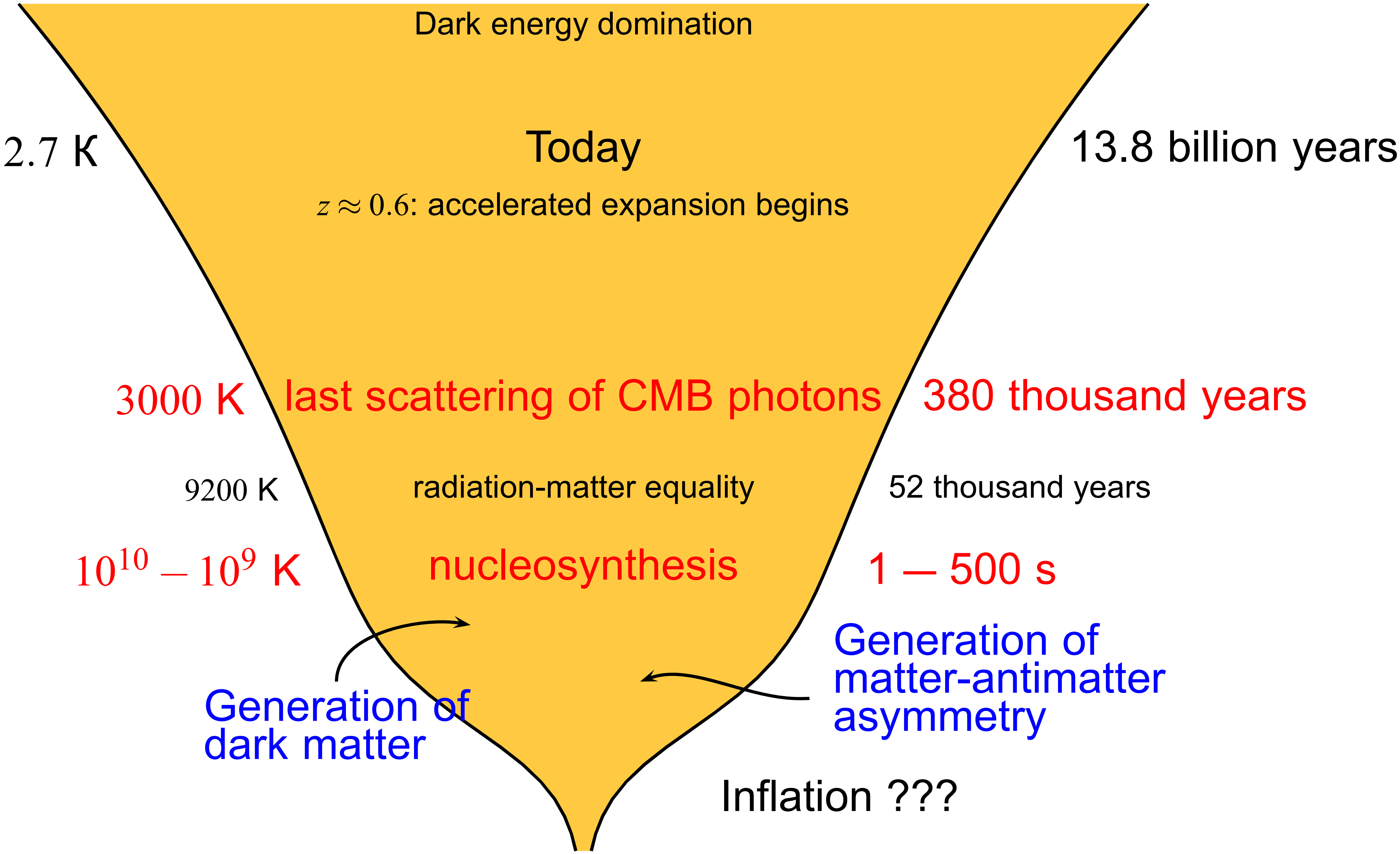


$\eta_{10} = \eta \cdot 10^{-10} =$ baryon-to-photon ratio. Consistent with CMB determination of η

$$T = 2.726^\circ K, \quad \frac{\delta T}{T} \sim 10^{-4} - 10^{-5}$$



Planck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last scattering of CMB photons
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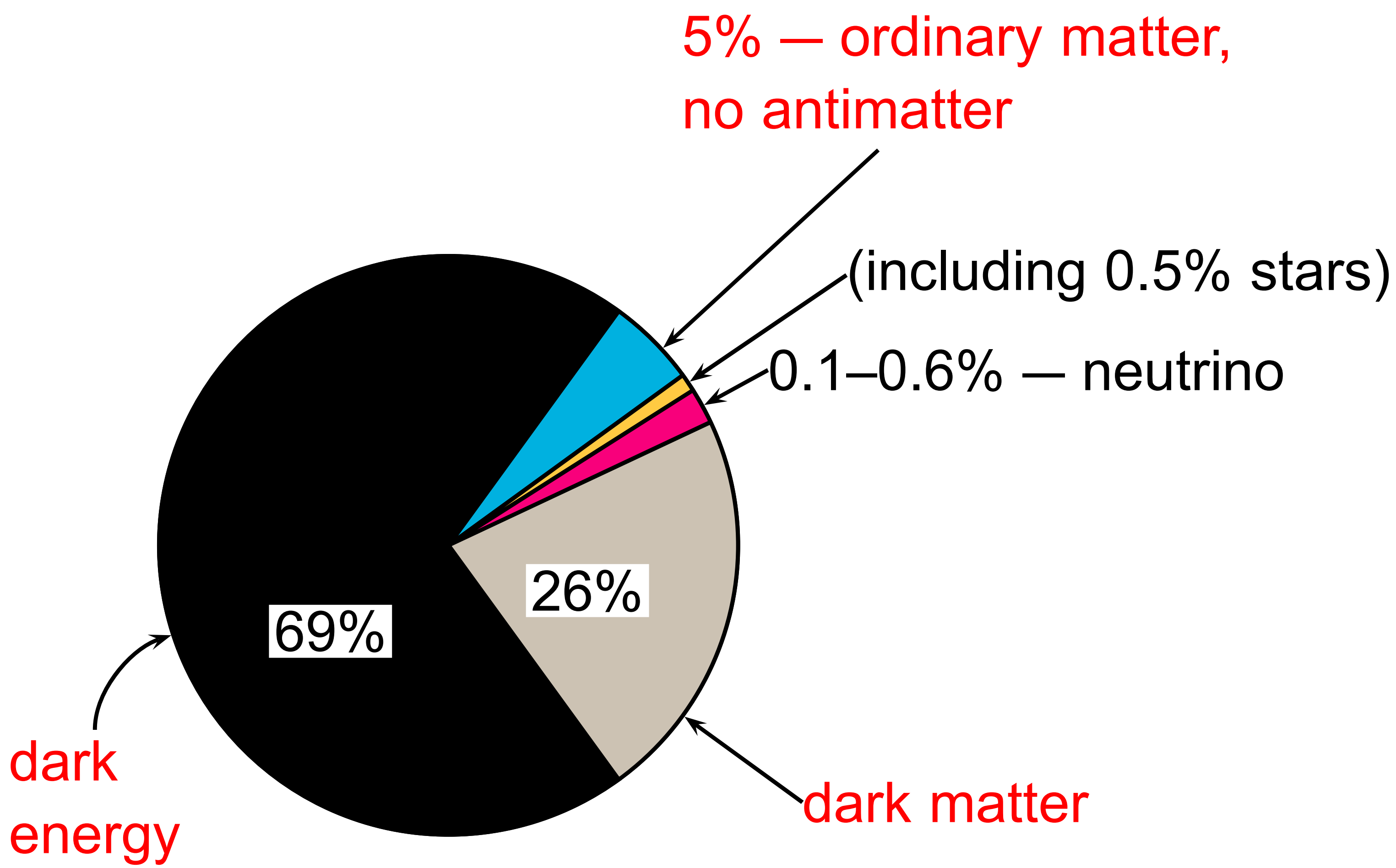
radiation-matter equality
52 thousand years
$10^{10} - 10^{9}$ K
nucleosynthesis
1 — 500 s
Generation of dark matter
Generation of matter-antimatter asymmetry
Inflation ???

Unknowns


5% — ordinary matter,
no antimatter
(including 0.5% stars)
0.1–0.6% — neutrino
26%
69%
dark matter
dark
energy

Dark matter

- Astrophysical evidence: measurements of gravitational potentials in galaxies and clusters of galaxies
  - Velocity curves of galaxies

    Fig.
  - Velocities of galaxies in clusters

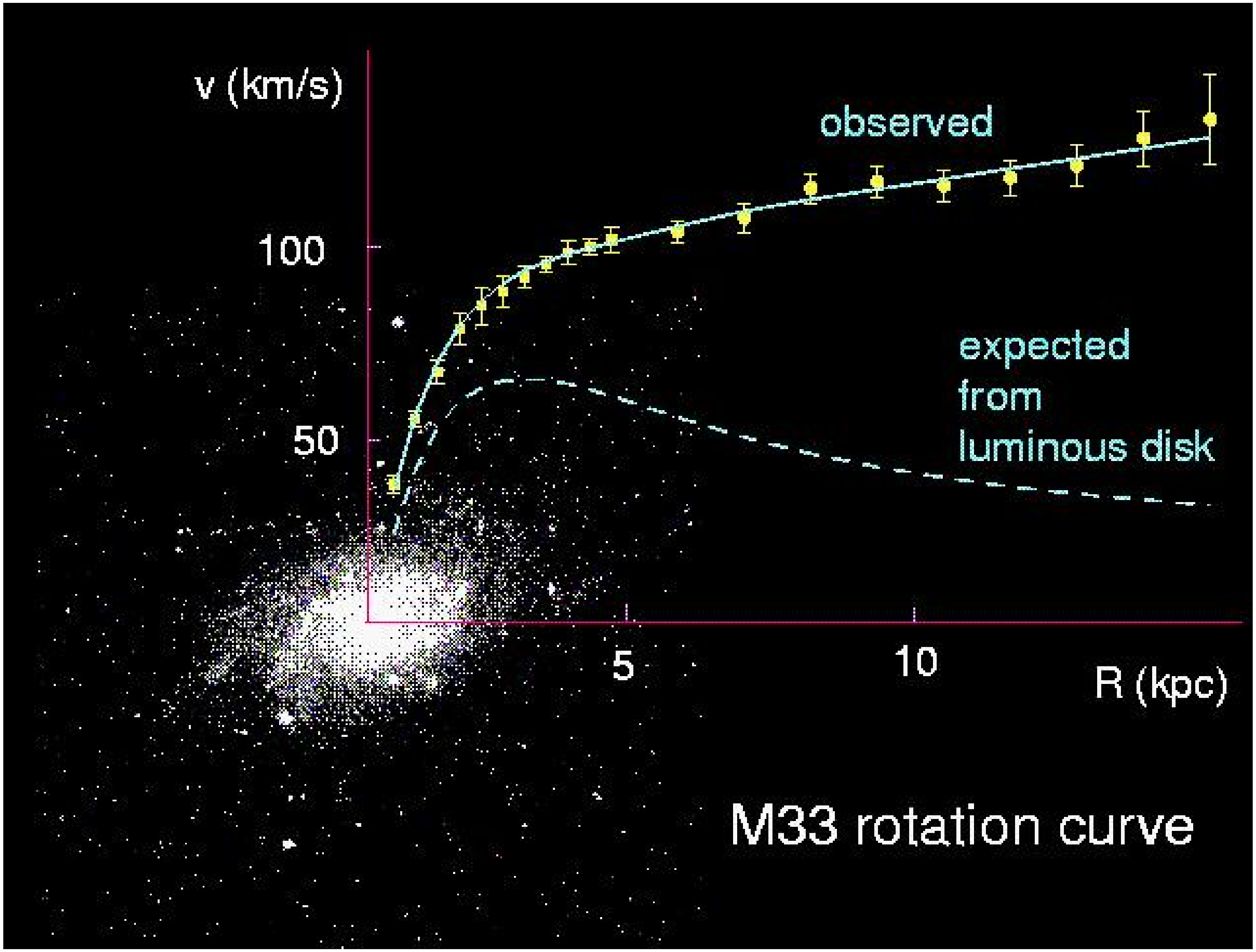
    Original Zwicky's argument, 1930's

$$v^2 = G\frac{M(r)}{r}$$

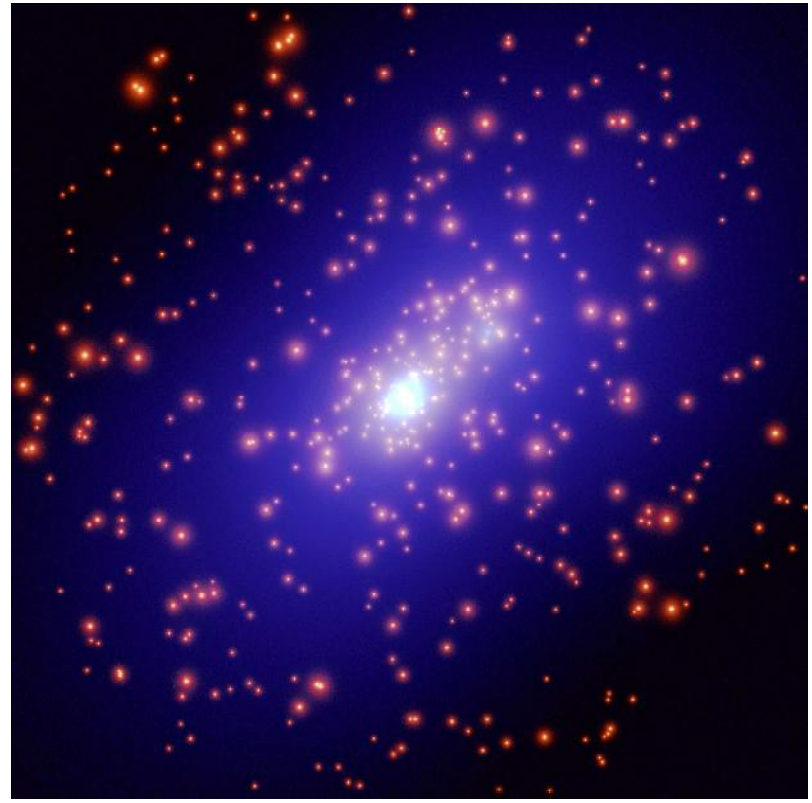
  - Temperature of gas in X-ray clusters of galaxies
  - Gravitational lensing of clusters

    Fig.
  - Etc.

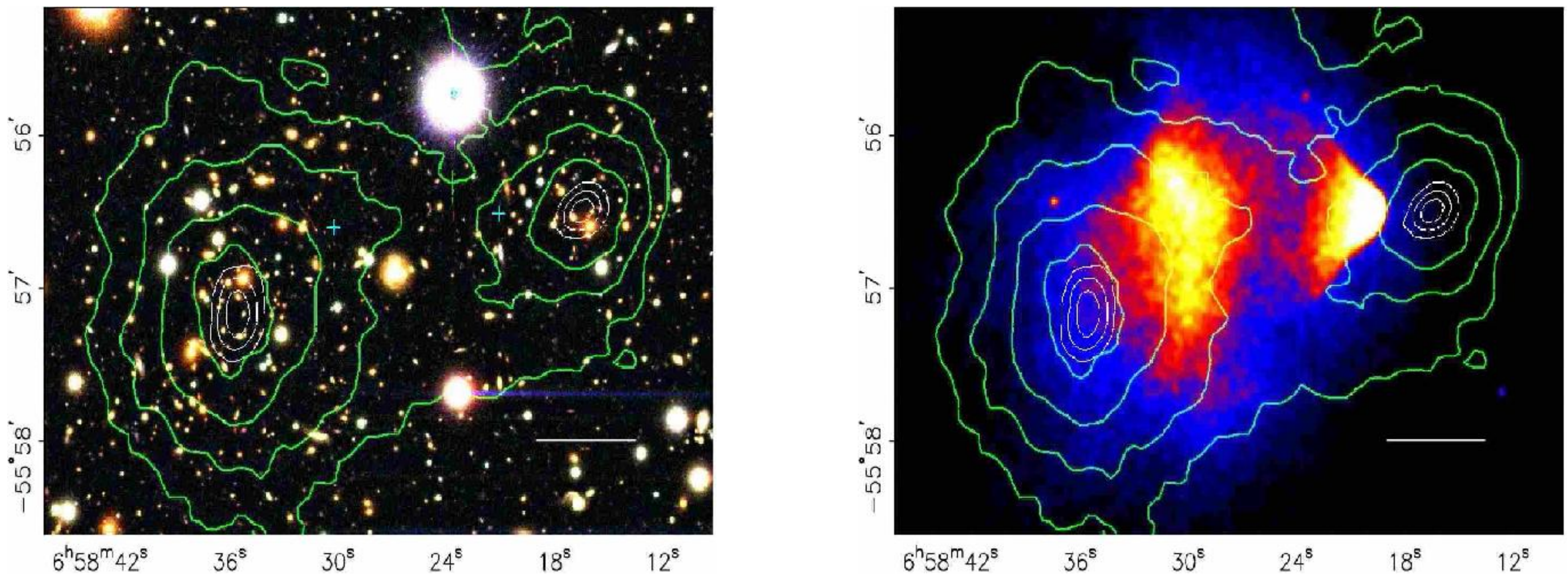
Rotation curves



Gravitational lensing



Bullet cluster



Outcome

$$\Omega_M \equiv \frac{\rho_M}{\rho_c} = 0.2 - 0.3$$

Assuming mass-to-light ratio everywhere the same as in clusters
NB: only 10 % of galaxies sit in clusters

Nucleosynthesis, CMB:

$$\Omega_B = 0.05$$

The rest is non-baryonic, $\Omega_{DM} \approx 0.26$.

Physical parameter: mass-to-entropy ratio. Stays constant in time.
Its value

$$\left(\frac{\rho_{DM}}{s}\right)_0 = \frac{\Omega_{DM}\rho_c}{s_0} = \frac{0.26 \cdot 0.5 \cdot 10^{-6}\ \text{GeV cm}^{-3}}{3000\ \text{cm}^{-3}} \simeq 4 \cdot 10^{-10}\ \text{GeV}$$

Cosmological evidence: growth of structure

CMB anisotropies: baryon density perturbations at recombination
$\approx$ photon last scattering, $T = 3000$ K, $z = 1100$:

$$\delta_B \equiv \left(\frac{\delta\rho_B}{\rho_B}\right)_{z=1100} \simeq \left(\frac{\delta T}{T}\right)_{CMB} \sim 10^{-4}$$

In matter dominated Universe, matter perturbations grow as

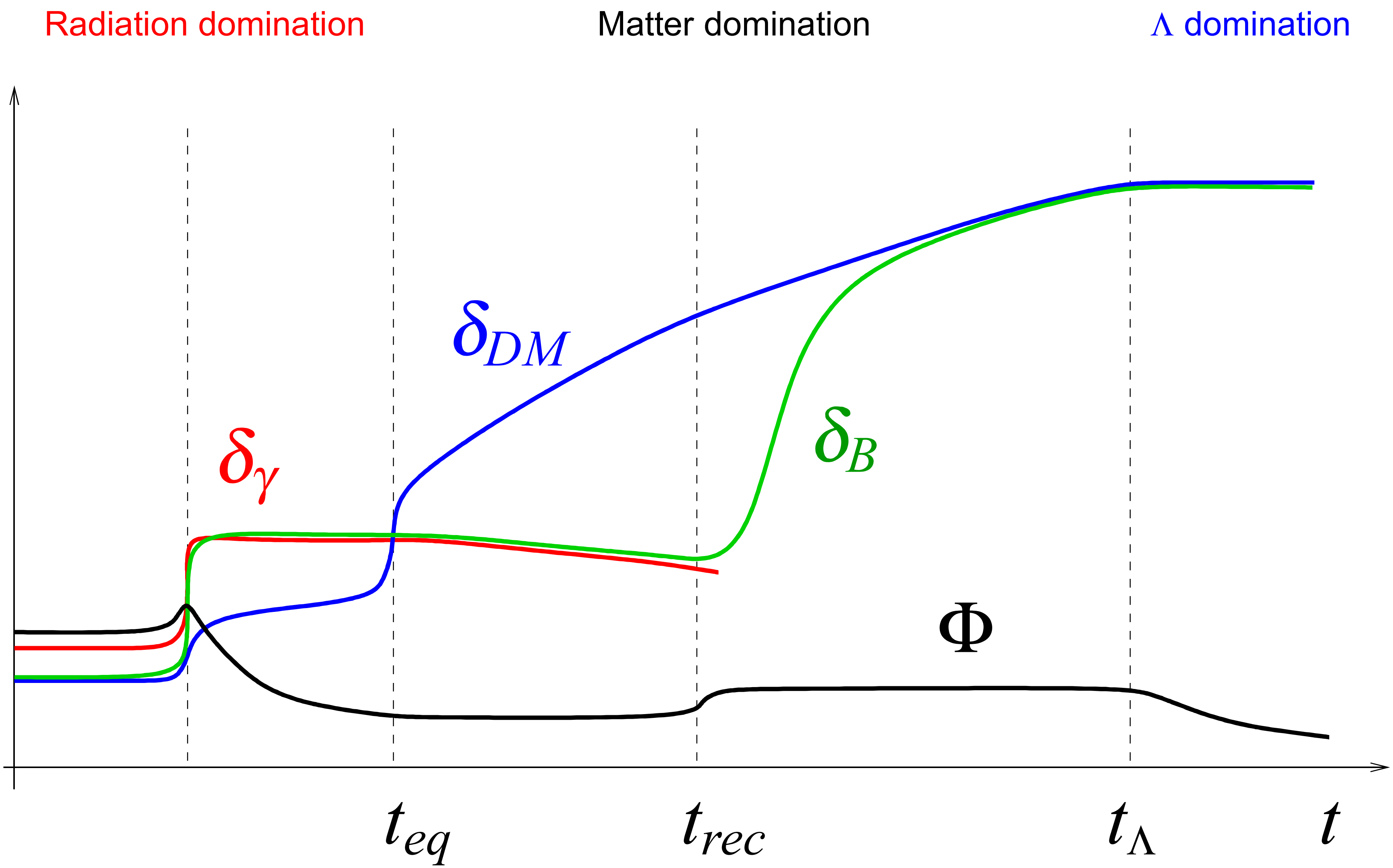
$$\frac{\delta\rho}{\rho}(t) \propto a(t)$$

Perturbations in baryonic matter grow after recombination only
If not for dark matter,

$$\left(\frac{\delta\rho}{\rho}\right)_{today} = 1100 \times 10^{-4} \sim 0.1$$

No galaxies, no stars...
Perturbations in dark matter start to grow much earlier
(already at radiation-dominated stage)

Growth of perturbations (linear regime)



NB: Need dark matter particles non-relativistic early on.

Neutrinos are not considerable part of dark matter
(way to set cosmological bound on neutrino mass,
$m_\nu \lesssim 0.1$ eV for every type of neutrino)

UNKNOWN DARK MATTER PARTICLES ARE
CRUCIAL FOR OUR EXISTENCE

If thermal relic:

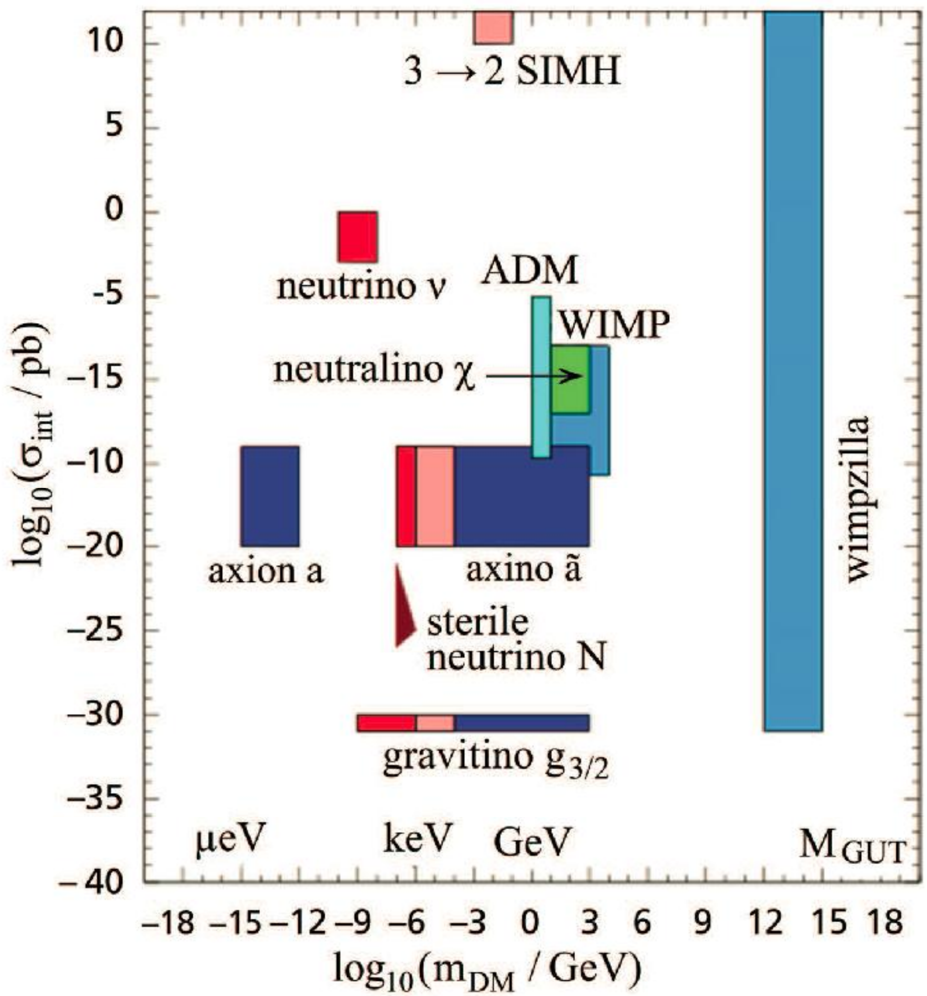
Cold dark matter, CDM

$$m_{DM} \gtrsim 100 \text{ keV}$$

Warm dark matter

$$m_{DM} \simeq 1 - 30 \text{ keV}$$

Canidates for Dark Matter particles are numerous



WIMPs

Simple but very suggestive scenario

- Assume there is a new heavy stable particle X
  - Interacts with SM particles via pair annihilation (and crossing processes)

$$X + X \leftrightarrow q\bar{q}\text{, etc}$$

  - Parameters: mass M_X; annihilation cross section at non-relativistic velocity σ
- Assume that maximum temperature in the Universe was high, $T \gtrsim M_X$
- Calculate present mass density

Outcome: mass to entropy ratio

$$\frac{M_X n_X}{s} = \# \frac{\ln(M_X M_{Pl}^* \langle\sigma v\rangle)}{\langle\sigma v\rangle \sqrt{g_*(T_f)} M_{Pl}} \; ; \qquad \# = \frac{3\sqrt{5}}{\sqrt{\pi}}$$

- Correct value, mass-to-entropy$= 4 \cdot 10^{-10}$ GeV, for

$$\sigma_0 \equiv \langle\sigma v\rangle = (1 \div 2) \cdot 10^{-36}\ \text{cm}^2 = (1 \div 2)\ \text{pb}$$

  - Weak scale cross section.

    Gravitational physics and EW scale physics combine into

$$\text{mass-to-entropy} \simeq \frac{1}{M_{Pl}} \left(\frac{\text{TeV}}{\alpha_W} \right)^2 \simeq 10^{-10}\ \text{GeV}$$

  - Mass M_X should not be much higher than 100 GeV

Weakly interacting massive particles, WIMPs.

Cold dark matter candidates

SUSY: LSP neutralinos, $X = \chi$

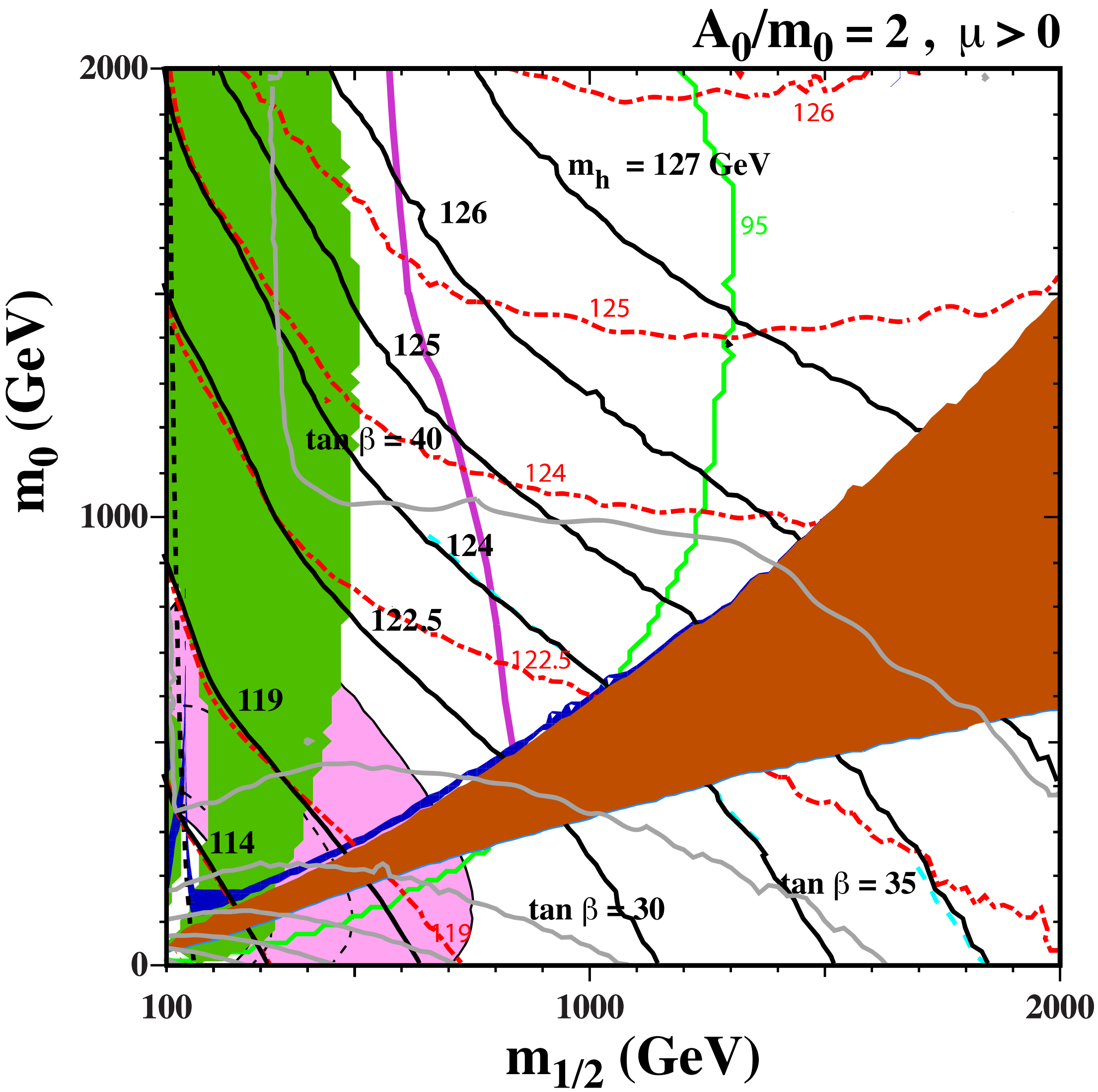
But situation is rather tense already: annihilation cross section is often too low

Important supperssion factor: $\langle \sigma v \rangle \propto v^2 \propto T/M_\chi$ because of p-wave annihilation in case $\chi\chi \to Z^* \to f\bar{f}$:

Relativistic $f\bar{f}$ $\Longrightarrow$ total angular momentum $J = 1$

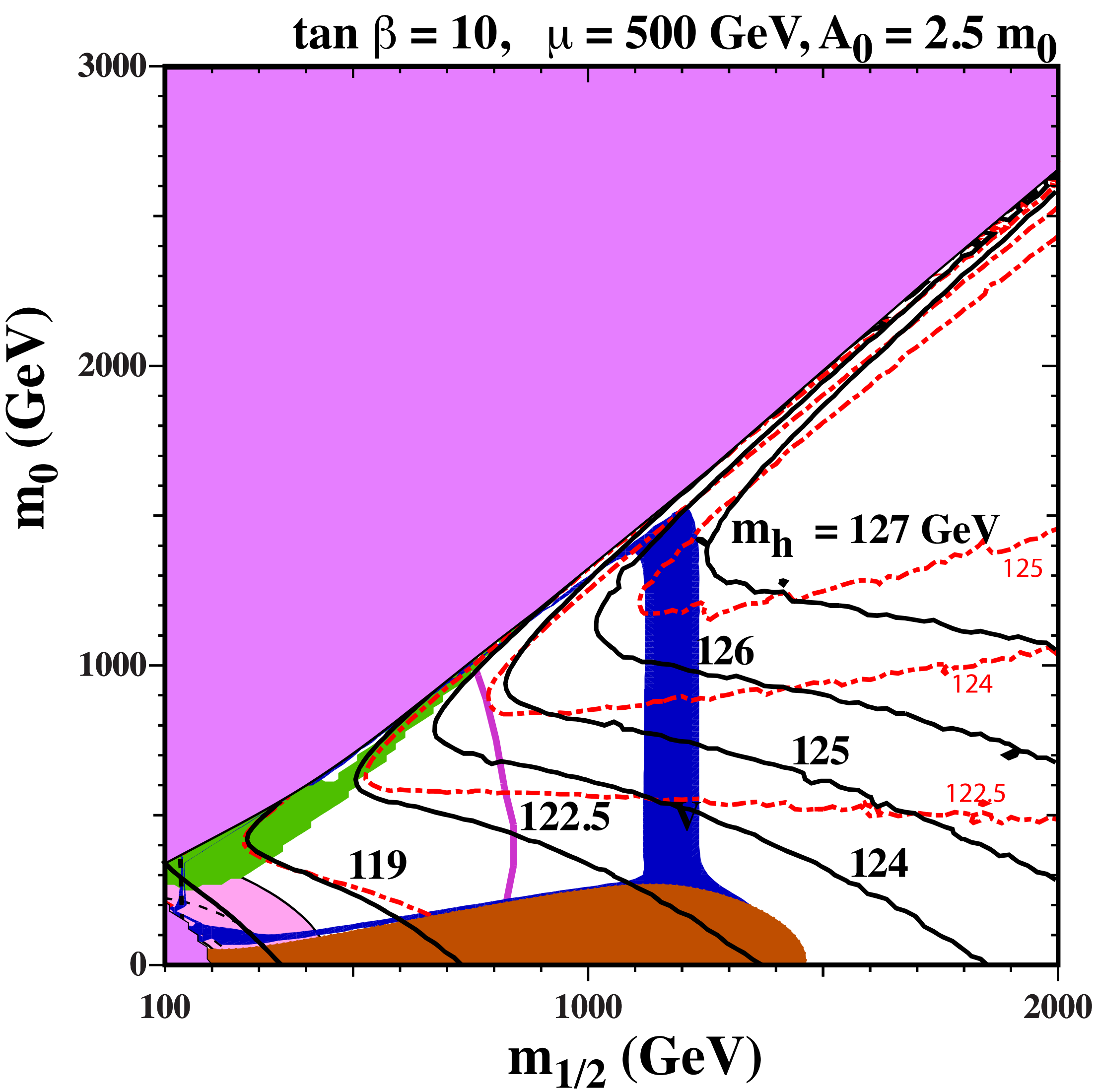
$\chi\chi$: identical fermions $\Longrightarrow$ $L = 0$, parallel spins impossible $\Longrightarrow$ p-wave

Constrained MSSM



Buchmuller et.al.' 2013

Less constrained MSSM



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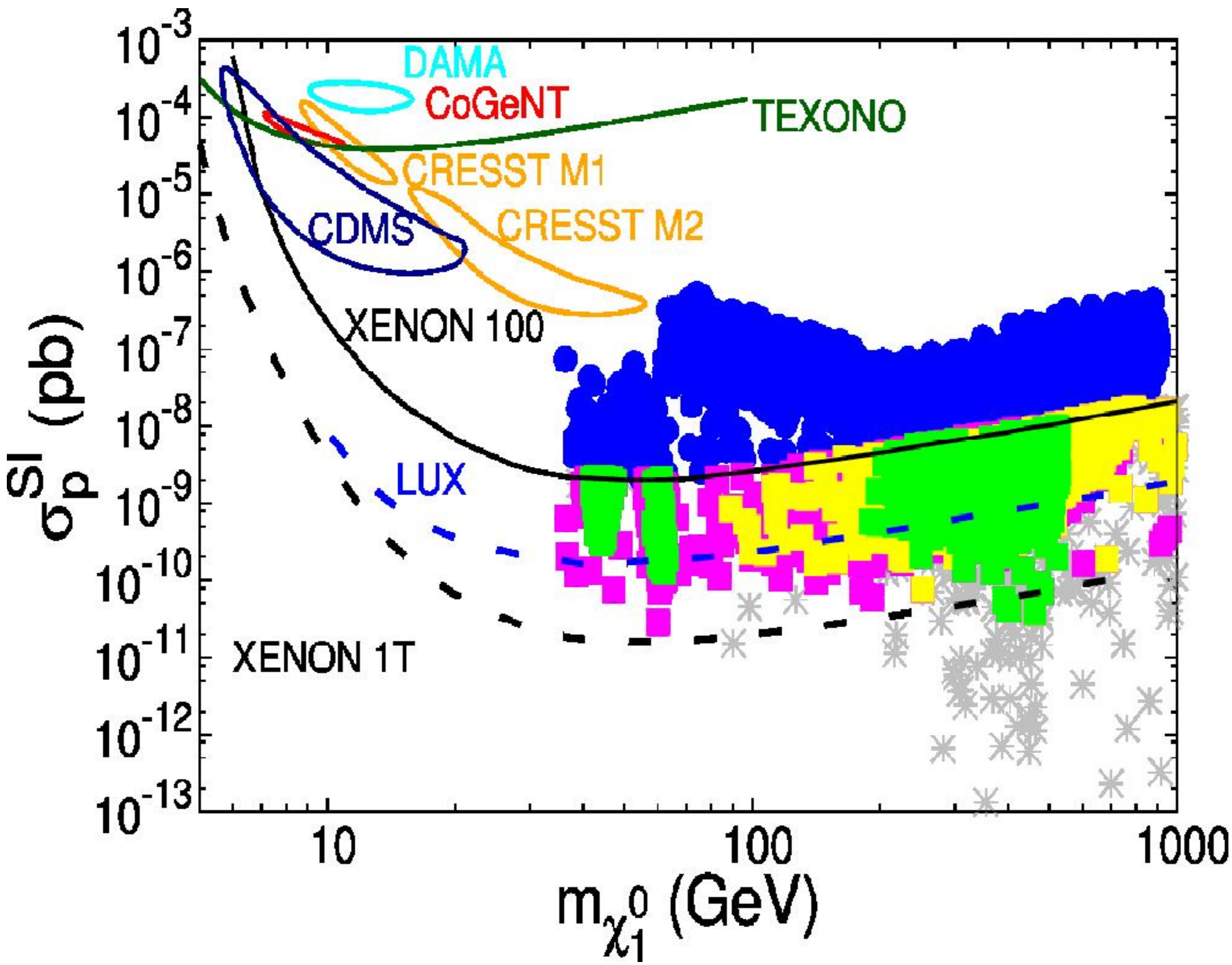
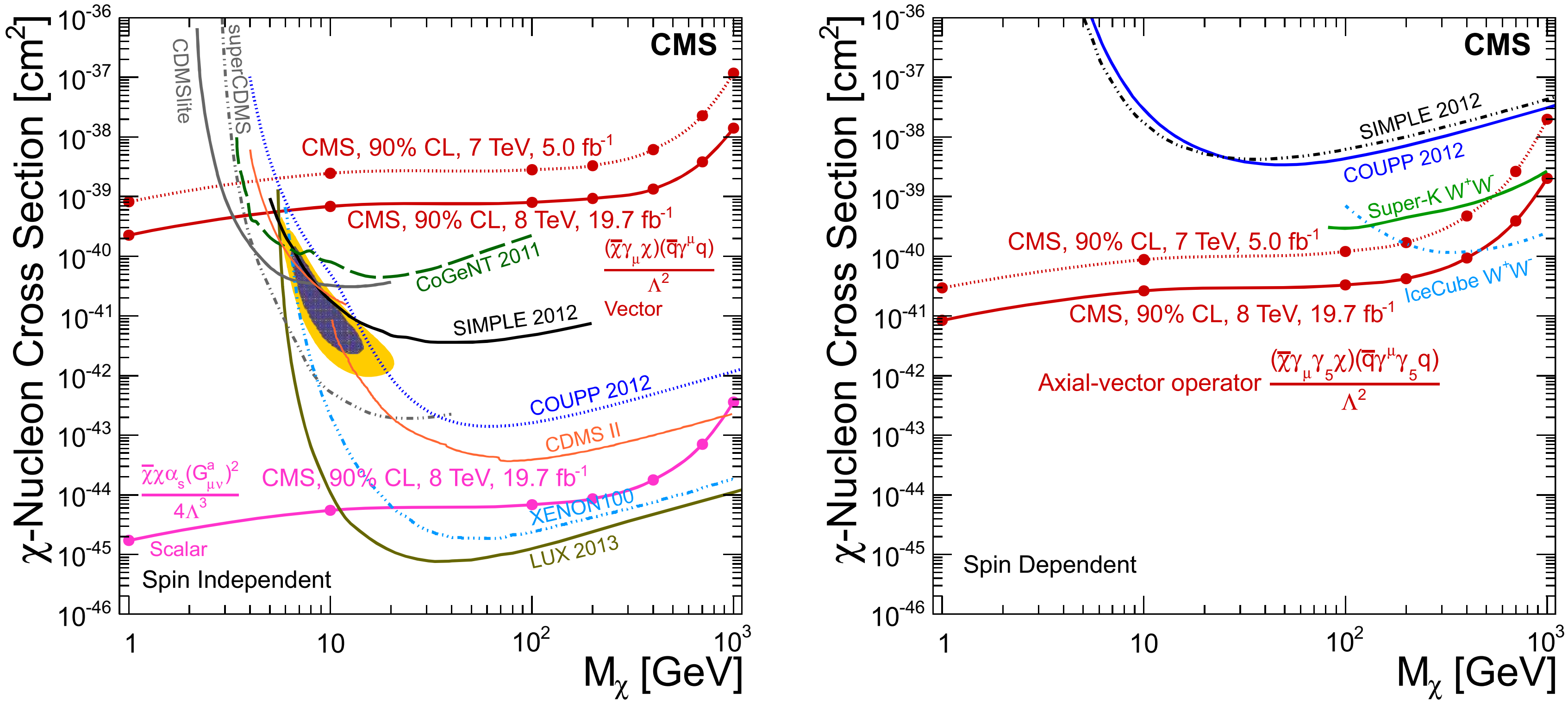
Direct searches are sensitive to SUSY



Han et.al.' 2013

The LHC becomes sensitive too



Khachatryan et.al.’ 2014

TeV SCALE PHYSICS MAY WELL BE RESPONSIBLE FOR GENERATION OF DARK MATTER

Is this guaranteed?

By no means. Other good DM candidates:

axion, sterile neutrino, gravitino.

Plus a lot of exotica...

Crucial impact of the LHC to cosmology,
direct and indirect dark matter searches