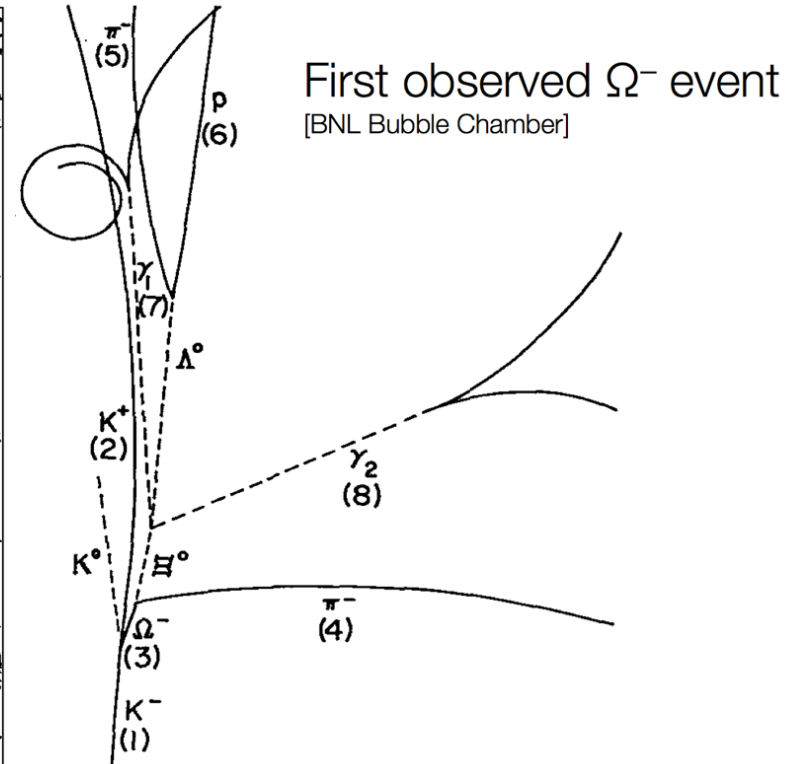
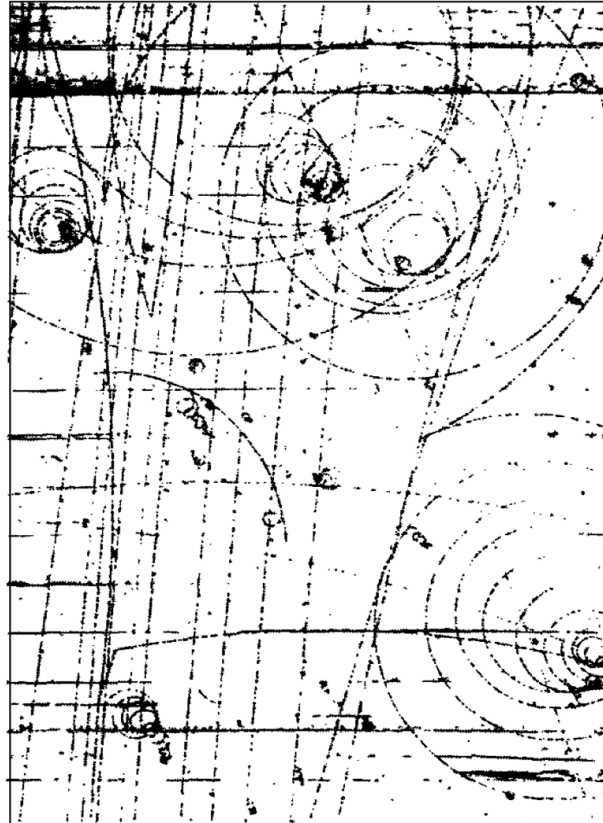


# INSTRUMENTATION & DETECTORS for HIGH ENERGY PHYSICS I



# WHAT IS A PARTICLE DETECTOR ?

An apparatus able to  
detect the passage of a particle  
and/or localise it  
and/or measure its momentum or energy  
and/or identify its nature  
and/or measure its time of arrival  
.....



# HOW ARE PARTICLES DETECTED ?

In order to detect a particle it must  
interact with the material of the detector  
transfer energy in some recognisable way and leave a *signal*.

Detection of particles happens via their energy loss in the material they traverse.

Charged particles

Photons

Hadrons

Neutrinos

Ionization, Bremsstrahlung, Cherenkov, ...

Photo/Compton effect, pair production

Nuclear interactions

Weak interactions

multiple  
interactions

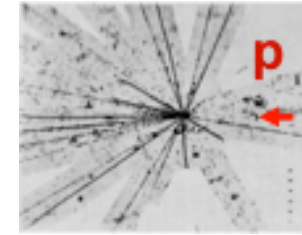
single  
interactions...

multiple  
interactions

# FOUR STEPS

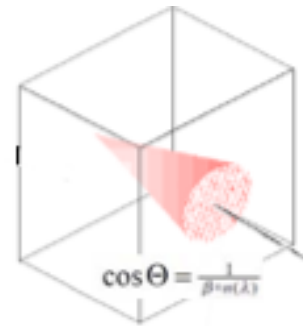
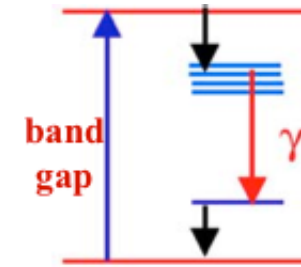
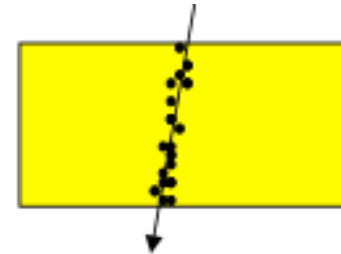
## Lesson 1

1. Particles interact with matter  
depends on particle and material



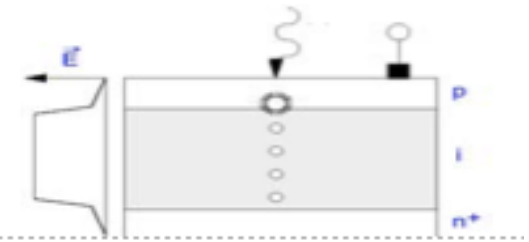
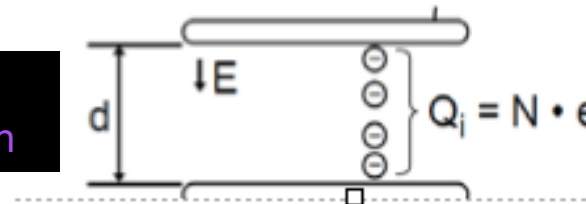
## Lesson 2

2. Energy loss transfer to detectable signal  
depends on the material



## Lesson 3

3. Signal collection  
depends on signal and type of detection

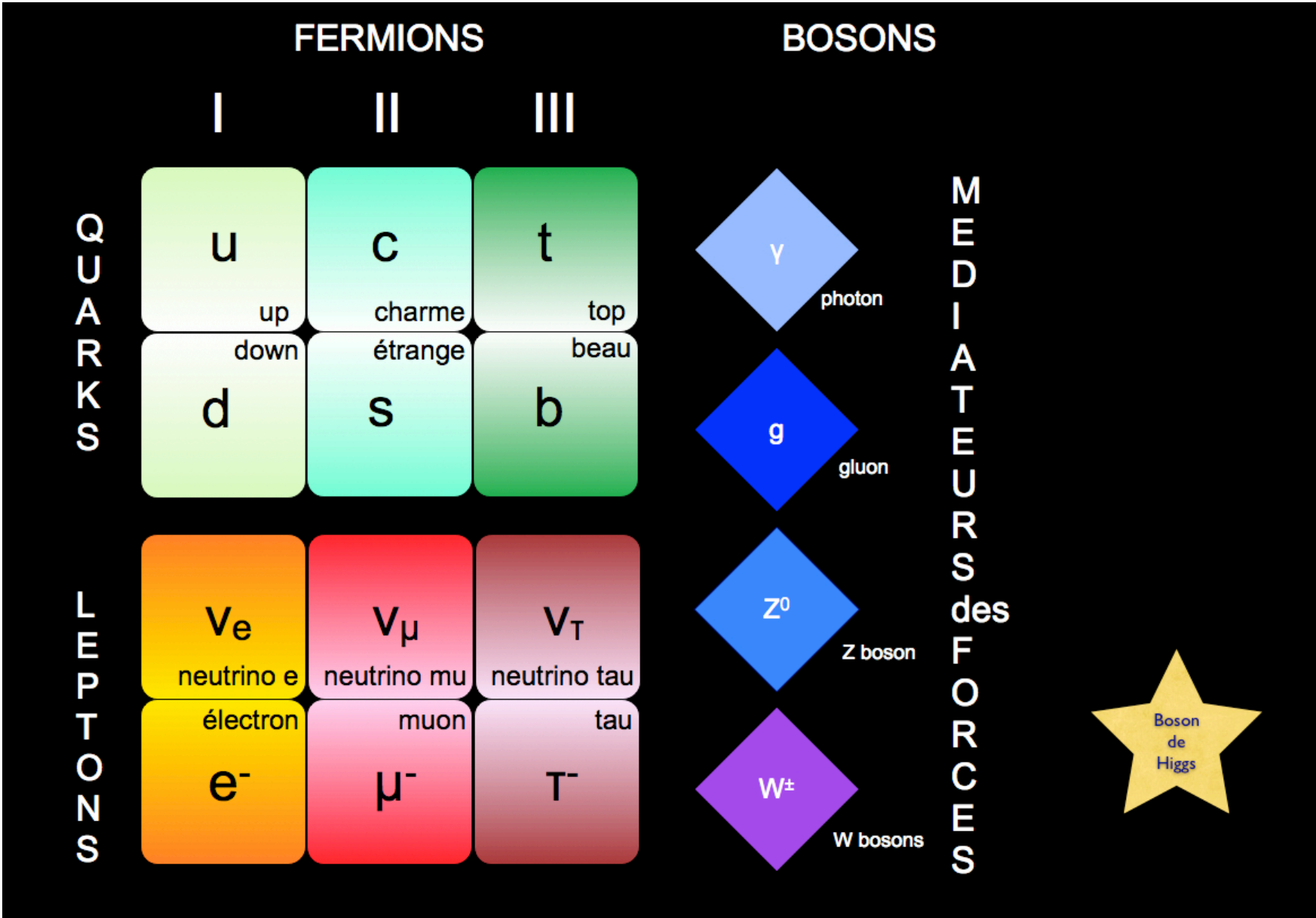


4. BUILD a SYSTEM  
depends on physics, experimental conditions,....

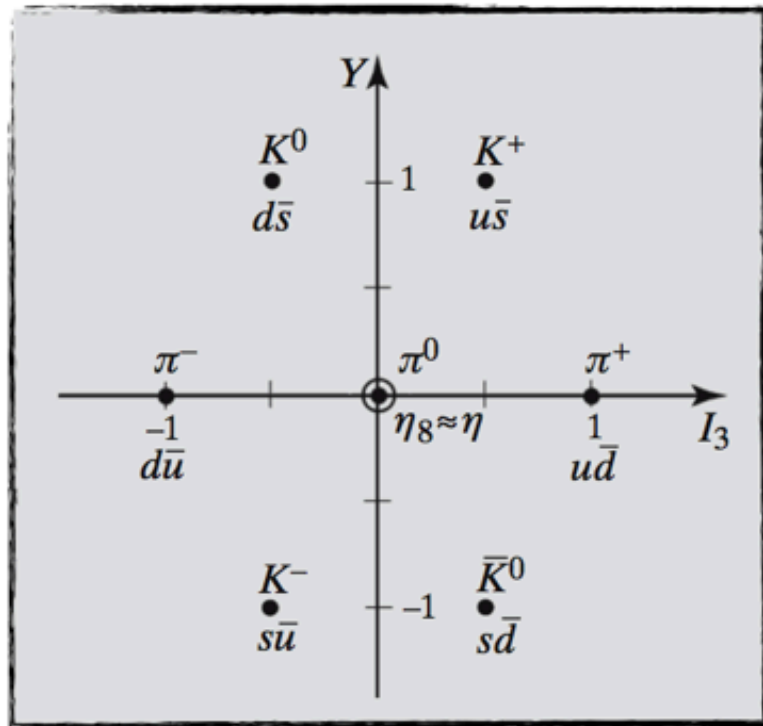




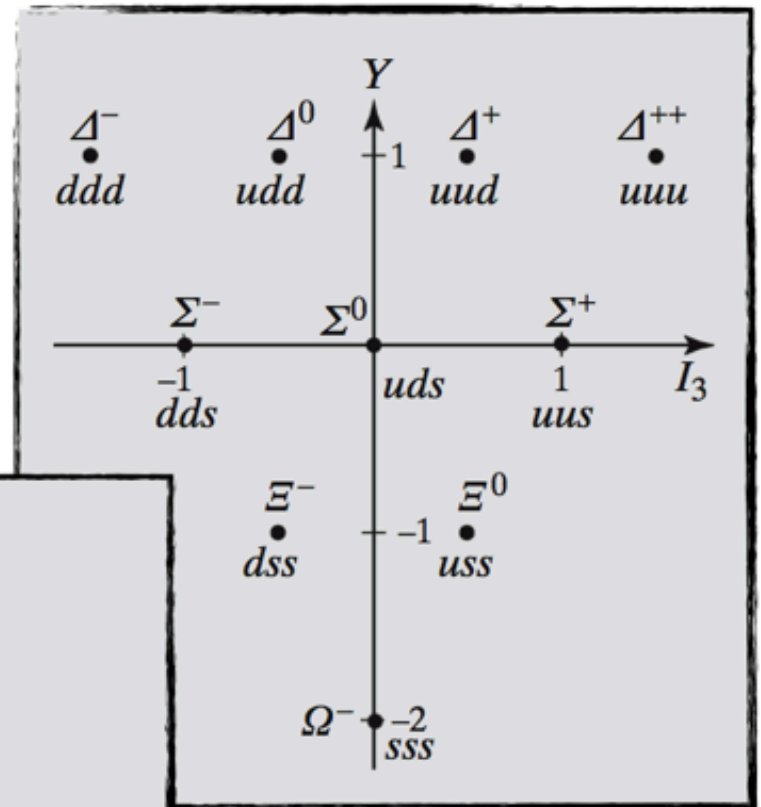
# ELEMENTARY PARTICLES and FORCES



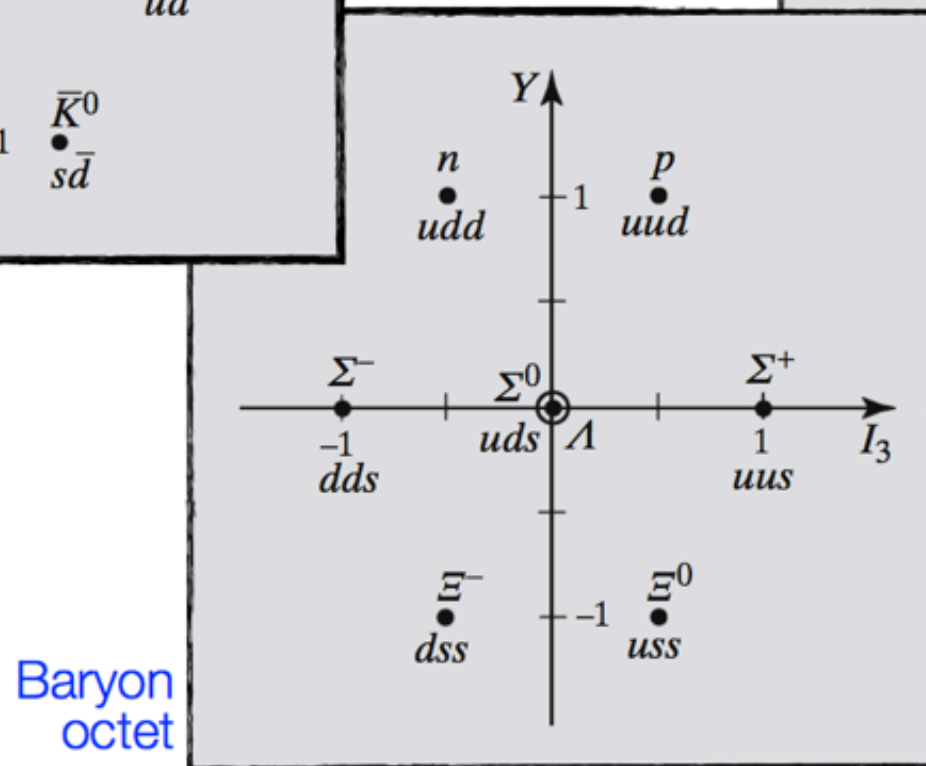
# PARTICLES



Meson  
octet



Baryon  
decuplet



Baryon  
octet

H,  $p, W^\pm, Z^0, q, e, \mu, \tau, \nu_e, \nu_\mu, \nu_\tau, \pi^\pm, \pi^0, \eta, f_0(660), g(770),$   
 $\omega(782), \eta'(958), f_0(980), a_0(980), \phi(1020), h_1(1170), b_1(1235),$   
 $a_1(1260), f_2(1270), f_1(1285), \eta(1295), \pi(1300), a_2(1320),$   
 $f_0(1370), f_1(1420), \omega(1420), \eta(1440), a_0(1450), g(1450),$   
 $f_0(1500), f_2'(1525), \omega(1650), \omega_3(1670), \pi_2(1670), \phi(1680),$   
 $g_3(1690), g(1700), f_0(1710), \pi(1800), \phi_3(1850), f_2(2010),$   
 $a_4(2040), f_4(2050), f_2(2300), f_2(2340), K^\pm, K^0, K_S^0, K_L^0, K^*(892),$   
 $K_1(1270), K_1(1400), K^*(1410), K_0^*(1430), K_2^*(1430), K^*(1680),$   
 $K_2(1770), K_3^*(1780), K_2(1820), K_4^*(2045), D^\pm, D^0, D^*(2007)^0,$   
 $D^*(2010)^\pm, D_1(2420)^0, D_2^*(2460)^0, D_2^*(2460)^\pm, D_s^\pm, D_s^{*\pm},$   
 $D_{s1}(2536)^\pm, D_{s1}(2573)^\pm, B^\pm, B^0, B^*, B_S^0, B_c^\pm, \eta_c(1s), J/\psi(1s),$   
 $\chi_{c0}(1P), \chi_{c1}(1P), \chi_{c2}(1P), \psi(2S), \psi(3770), \psi(4040), \psi(4160),$   
 $\psi(4415), \Upsilon(1S), \chi_{b0}(1P), \chi_{b1}(1P), \chi_{b2}(1P), \Upsilon(2S), \chi_{b0}(2P),$   
 $\chi_{b2}(2P), \Upsilon(3S), \Upsilon(4S), \Upsilon(10860), \Upsilon(11020), p, n, N(1440),$   
 $N(1520), N(1535), N(1650), N(1675), N(1680), N(1700), N(1710),$   
 $N(1720), N(2190), N(2220), N(2250), N(2600), \Delta(1232), \Delta(1600),$   
 $\Delta(1620), \Delta(1700), \Delta(1905), \Delta(1910), \Delta(1920), \Delta(1930), \Delta(1950),$   
 $\Delta(2420), \Lambda, \Lambda(1405), \Lambda(1520), \Lambda(1600), \Lambda(1670), \Lambda(1690),$   
 $\Lambda(1800), \Lambda(1810), \Lambda(1820), \Lambda(1830), \Lambda(1890), \Lambda(2100),$   
 $\Lambda(2110), \Lambda(2350), \Sigma^+, \Sigma^0, \Sigma^-, \Sigma(1385), \Sigma(1660), \Sigma(1670),$   
 $\Sigma(1750), \Sigma(1775), \Sigma(1915), \Sigma(1940), \Sigma(2030), \Sigma(2250), \Xi^0, \Xi^-,$   
 $\Xi(1530), \Xi(1690), \Xi(1820), \Xi(1950), \Xi(2030), \Omega^-, \Omega(2250)^-,$   
 $\Lambda_c^+, \Lambda_c^0, \Sigma_c(2455), \Sigma_c(2520), \Xi_c^+, \Xi_c^0, \Xi_c'^+, \Xi_c'^0, \Xi(2645),$   
 $\Xi_c(2790), \Xi_c(2815), \Omega_c^0, \Lambda_b^0, \Xi_b^0, \Xi_b^-, t, \bar{t}$

There are many more

+ the ones we have not yet observed



# KNOWN PARTICLES

<http://pdg.lbl.gov>

~ 180 Selected Particles

HOW CAN A PARTICLE  
DETECTOR

DISTINGUISH

THE PARTICLES WE KNOW

MEASURE PROPERTIES of  
PHYSICS PROCESSES

IDENTIFY THE EXISTENCE  
OF A NEW PARTICLE



H,  $p$ ,  $n$ ,  $W^\pm$ ,  $Z^0$ ,  $g$ ,  $e$ ,  $\mu$ ,  $\tau$ ,  $\nu_e$ ,  $\nu_\mu$ ,  $\nu_\tau$ ,  $\pi^\pm$ ,  $\pi^0$ ,  $\eta$ ,  $f_0(660)$ ,  $\rho(770)$ ,  $\omega(782)$ ,  $\eta'(958)$ ,  $f_0(980)$ ,  $a_0(980)$ ,  $\phi(1020)$ ,  $h_1(1170)$ ,  $b_1(1235)$ ,  $a_1(1260)$ ,  $f_2(1270)$ ,  $f_1(1285)$ ,  $\eta(1295)$ ,  $\pi(1300)$ ,  $a_2(1320)$ ,  $f_0(1370)$ ,  $f_1(1420)$ ,  $\omega(1420)$ ,  $\eta(1440)$ ,  $a_0(1450)$ ,  $\rho(1450)$ ,  $f_0(1500)$ ,  $f_2'(1525)$ ,  $\omega(1650)$ ,  $\omega_3(1670)$ ,  $\pi_2(1670)$ ,  $\phi(1680)$ ,  $\rho_3(1690)$ ,  $\rho(1700)$ ,  $f_0(1710)$ ,  $\pi(1800)$ ,  $\phi_3(1850)$ ,  $f_2(2010)$ ,  $a_4(2040)$ ,  $f_4(2050)$ ,  $f_2(2300)$ ,  $f_2(2340)$ ,  $K^\pm$ ,  $K^0$ ,  $K_S^0$ ,  $K_L^0$ ,  $K^{*0}(892)$ ,  $K_1(1270)$ ,  $K_1(1400)$ ,  $K^{*0}(1410)$ ,  $K_0^*(1430)$ ,  $K_2^*(1430)$ ,  $K^{*0}(1680)$ ,  $K_2(1770)$ ,  $K_3^*(1780)$ ,  $K_2(1820)$ ,  $K_4^*(2045)$ ,  $D^\pm$ ,  $D^0$ ,  $D^{*0}(2007)$ ,  $D^{*0}(2010)^{\pm}$ ,  $D_1(2420)^0$ ,  $D_2^*(2460)^0$ ,  $D_2^*(2460)^{\pm}$ ,  $D_s^{\pm}$ ,  $D_s^{*\pm}$ ,  $D_{s1}(2536)^{\pm}$ ,  $D_{s1}(2573)^{\pm}$ ,  $B^\pm$ ,  $B^0$ ,  $B^{*0}$ ,  $B_S^0$ ,  $B_c^{\pm}$ ,  $\eta_c(1S)$ ,  $J/\psi(1S)$ ,  $\chi_{c0}(1P)$ ,  $\chi_{c1}(1P)$ ,  $\chi_{c2}(1P)$ ,  $\psi(2S)$ ,  $\psi(3770)$ ,  $\psi(4040)$ ,  $\psi(4160)$ ,  $\psi(4415)$ ,  $\Upsilon(1S)$ ,  $\chi_{b0}(1P)$ ,  $\chi_{b1}(1P)$ ,  $\chi_{b2}(1P)$ ,  $\Upsilon(2S)$ ,  $\chi_{b0}(2P)$ ,  $\chi_{b1}(2P)$ ,  $\Upsilon(3S)$ ,  $\Upsilon(4S)$ ,  $\Upsilon(10860)$ ,  $\Upsilon(11020)$ ,  $p$ ,  $n$ ,  $N(1440)$ ,  $N(1520)$ ,  $N(1535)$ ,  $N(1650)$ ,  $N(1675)$ ,  $N(1680)$ ,  $N(1700)$ ,  $N(1710)$ ,  $N(1720)$ ,  $N(2190)$ ,  $N(2220)$ ,  $N(2250)$ ,  $N(2600)$ ,  $\Delta(1232)$ ,  $\Delta(1600)$ ,  $\Delta(1620)$ ,  $\Delta(1700)$ ,  $\Delta(1905)$ ,  $\Delta(1910)$ ,  $\Delta(1920)$ ,  $\Delta(1930)$ ,  $\Delta(1950)$ ,  $\Delta(2420)$ ,  $\Lambda$ ,  $\Lambda(1405)$ ,  $\Lambda(1520)$ ,  $\Lambda(1600)$ ,  $\Lambda(1670)$ ,  $\Lambda(1690)$ ,  $\Lambda(1800)$ ,  $\Lambda(1810)$ ,  $\Lambda(1820)$ ,  $\Lambda(1830)$ ,  $\Lambda(1890)$ ,  $\Lambda(2100)$ ,  $\Lambda(2110)$ ,  $\Lambda(2350)$ ,  $\Sigma^+$ ,  $\Sigma^0$ ,  $\Sigma^-$ ,  $\Sigma(1385)$ ,  $\Sigma(1660)$ ,  $\Sigma(1670)$ ,  $\Sigma(1750)$ ,  $\Sigma(1775)$ ,  $\Sigma(1915)$ ,  $\Sigma(1940)$ ,  $\Sigma(2030)$ ,  $\Sigma(2250)$ ,  $\Xi^0$ ,  $\Xi^-$ ,  $\Xi(1530)$ ,  $\Xi(1690)$ ,  $\Xi(1820)$ ,  $\Xi(1950)$ ,  $\Xi(2030)$ ,  $\Omega^-$ ,  $\Omega(2250)^-$ ,  $\Lambda_c^+$ ,  $\Lambda_c^0$ ,  $\Sigma_c(2455)$ ,  $\Sigma_c(2520)$ ,  $\Xi_c^+$ ,  $\Xi_c^0$ ,  $\Xi_c^{*+}$ ,  $\Xi_c^{*0}$ ,  $\Xi_c^{*+}$ ,  $\Xi_c^{*0}$ ,  $\Xi(2645)$ ,  $\Xi_c(2790)$ ,  $\Xi_c(2815)$ ,  $\Omega_c^0$ ,  $\Lambda_b^0$ ,  $\Xi_b^0$ ,  $\Xi_b^-$ ,  $t\bar{t}$

There are many more

+ the ones we have not yet observed

W. Riegler/CERN



# LIMITED SIZE DETECTOR

Among these 180 listed particles,

27 have a long enough lifetime 

such that, for GeV energies, they travel more than one micrometer

Among these 27,

14 have  $c\tau < 0.5$  mm and leave a very short track in the detector

All Particles with  $c\tau > 1\mu\text{m}$  @ GeV Level

| Particle                       | Mass (MeV)  | Life time $\tau$ (s)                        | $c\tau$               |
|--------------------------------|-------------|---|-----------------------|
| $\gamma$                       | 0           | $\infty$                                    | $\infty$              |
| $\pi^\pm (u\bar{d}, d\bar{u})$ | 140         | $2.6 \cdot 10^{-8}$                         | 7.8 m                 |
| $K^\pm (u\bar{s}, \bar{u}s)$   | 494         | $1.2 \cdot 10^{-8}$                         | 3.7 m                 |
| $K^0 (d\bar{s}, \bar{d}s)$     | 497         | $5.1 \cdot 10^{-8}$<br>$8.9 \cdot 10^{-11}$ | 15.5 m<br>2.7 cm      |
| $D^\pm (c\bar{d}, \bar{c}d)$   | 1869        | $1.0 \cdot 10^{-12}$                        | 315 $\mu\text{m}$     |
| $D^0 (c\bar{u}, \bar{c}u)$     | 1864        | $4.1 \cdot 10^{-13}$                        | 123 $\mu\text{m}$     |
| $D_s^\pm (c\bar{s}, \bar{c}s)$ | 1969        | $4.9 \cdot 10^{-13}$                        | 147 $\mu\text{m}$     |
| $B^\pm (u\bar{b}, \bar{u}b)$   | 5279        | $1.7 \cdot 10^{-12}$                        | 502 $\mu\text{m}$     |
| $B^0 (b\bar{d}, \bar{b}d)$     | 5279        | $1.5 \cdot 10^{-12}$                        | 462 $\mu\text{m}$     |
| $B_s^0 (s\bar{b}, \bar{s}b)$   | 5370        | $1.5 \cdot 10^{-12}$                        | 438 $\mu\text{m}$     |
| $B_c^\pm (c\bar{b}, \bar{c}b)$ | $\sim 6400$ | $\sim 5 \cdot 10^{-13}$                     | 150 $\mu\text{m}$     |
| $p (uud)$                      | 938.3       | $> 10^{33}$ y                               | $\infty$              |
| $n (udd)$                      | 939.6       | 885.7 s                                     | $2.655 \cdot 10^8$ km |
| $\Lambda^0 (uds)$              | 1115.7      | $2.6 \cdot 10^{-10}$                        | 7.89 cm               |
| $\Sigma^+ (uus)$               | 1189.4      | $8.0 \cdot 10^{-11}$                        | 2.404 cm              |
| $\Sigma^- (dds)$               | 1197.4      | $1.5 \cdot 10^{-10}$                        | 4.434 cm              |
| $\Xi^0 (uss)$                  | 1315        | $2.9 \cdot 10^{-10}$                        | 8.71 cm               |
| $\Xi^- (dss)$                  | 1321        | $1.6 \cdot 10^{-10}$                        | 4.91 cm               |
| $\Omega^- (sss)$               | 1672        | $8.2 \cdot 10^{-11}$                        | 2.461 cm              |
| $\Lambda_c^+ (udc)$            | 2285        | $\sim 2 \cdot 10^{-13}$                     | 60 $\mu\text{m}$      |
| $\Xi_c^+ (usc)$                | 2466        | $4.4 \cdot 10^{-13}$                        | 132 $\mu\text{m}$     |
| $\Xi_c^0 (dcs)$                | 2472        | $\sim 1 \cdot 10^{-13}$                     | 29 $\mu\text{m}$      |
| $\Omega_c^0 (ssc)$             | 2688        | $6.0 \cdot 10^{-14}$                        | 19 $\mu\text{m}$      |
| $\Lambda_b (uab)$              | 5620        | $1.2 \cdot 10^{-12}$                        | 368 $\mu\text{m}$     |

"Secondary Vertices"

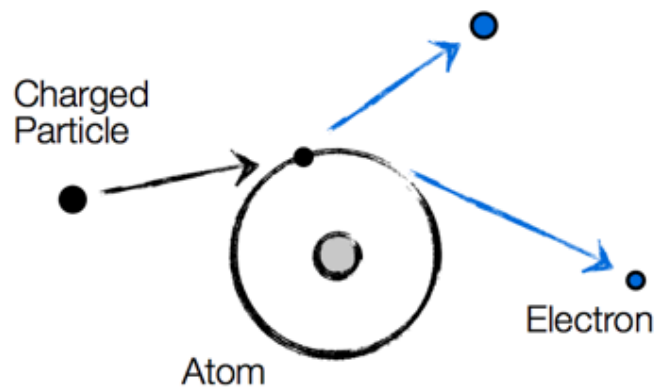
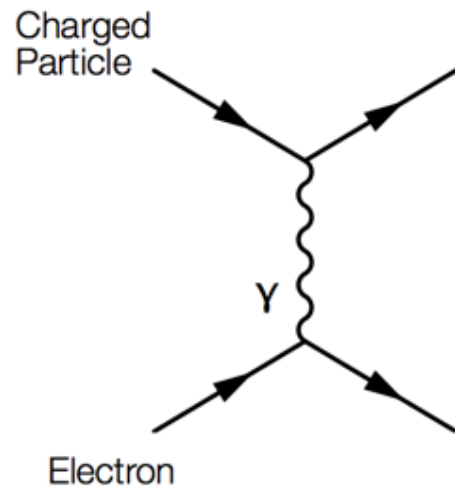
# THE 13 PARTICLES A DETECTOR MUST BE ABLE TO MEASURE AND IDENTIFY

|           |  |                                  |
|-----------|--|----------------------------------|
| $e^\pm$   | $m_e = 0.511 \text{ MeV}$                | } EM                             |
| $\mu^\pm$ | $m_\mu = 105.7 \text{ MeV} \sim 200 m_e$ |                                  |
| $\gamma$  | $m_\gamma = 0, Q = 0$                    |                                  |
| $\pi^\pm$ | $m_\pi = 139.6 \text{ MeV} \sim 270 m_e$ | } EM, Strong<br>$\sim 3.5 m_\pi$ |
| $K^\pm$   | $m_K = 493.7 \text{ MeV} \sim 1000 m_e$  |                                  |
| $p^\pm$   | $m_p = 938.3 \text{ MeV} \sim 2000 m_e$  |                                  |
| $K^0$     | $m_{K^0} = 497.7 \text{ MeV} \quad Q=0$  | } Strong                         |
| $n$       | $m_n = 939.6 \text{ MeV} \quad Q=0$      |                                  |

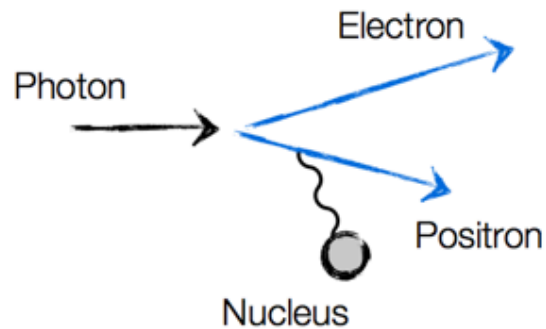
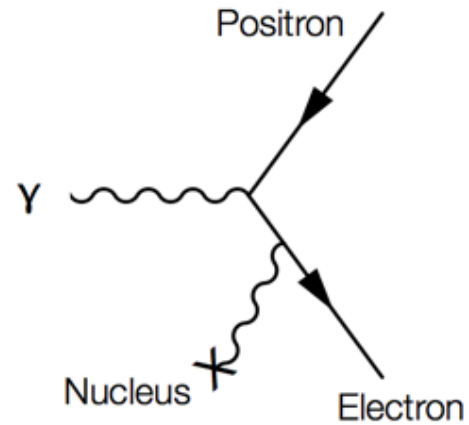
The Difference in  
Mass, Charge, Interaction  
is the key to the Identification

# EXAMPLES OF INTERACTIONS

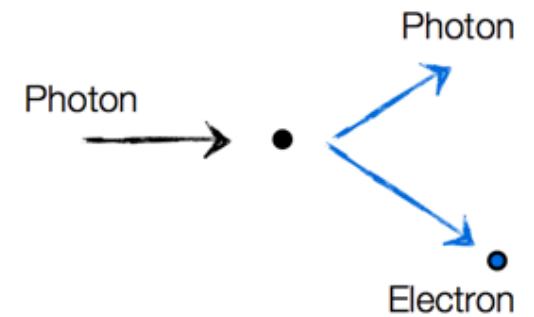
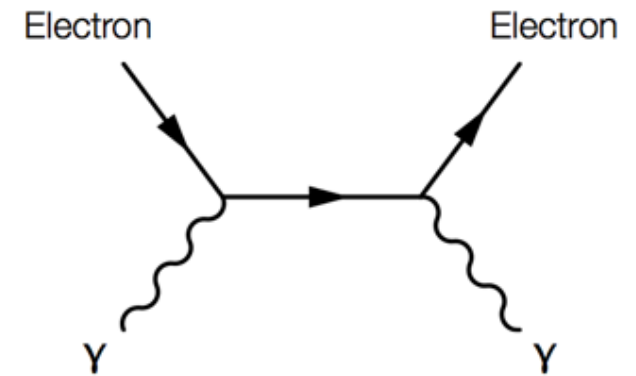
Ionization:



Pair production:



Compton scattering:



# HEP & SI UNITS

| Quantity    | HEP units                            | SI Units                       |
|-------------|--------------------------------------|--------------------------------|
| length      | 1 fm                                 | $10^{-15}$ m                   |
| energy      | 1 GeV                                | $1.602 \cdot 10^{-10}$ J       |
| mass        | $1 \text{ GeV}/c^2$                  | $1.78 \cdot 10^{-27}$ kg       |
| $\hbar=h/2$ | $6.588 \cdot 10^{-25} \text{ GeV s}$ | $1.055 \cdot 10^{-34}$ Js      |
| c           | $2.988 \cdot 10^{23} \text{ fm/s}$   | $2.988 \cdot 10^8 \text{ m/s}$ |
| $\hbar c$   | 0.1973 GeV fm                        | $3.162 \cdot 10^{-26}$ Jm      |

## Natural units ( $\hbar = c = 1$ )

|        |  |
|--------|--|
| mass   | 1 GeV  |
| length | $1 \text{ GeV}^{-1} = 0.1973 \text{ fm}$             |
| time   | $1 \text{ GeV}^{-1} = 6.59 \cdot 10^{-25} \text{ s}$ |



# MEASURING PARTICLES

Particles are characterized by

Mass

[Unit: eV/c<sup>2</sup> or eV]

Momentum

[Unit: eV/c or eV]

Energy

[Unit: eV]

Charge

[Unit: e]

[+ Spin, Lifetime ...]

$$\text{eV} = 1.6 \cdot 10^{-19} \text{ J}$$

$$c = 299\,792\,458 \text{ m/s}$$

$$e = 1.602176487(40) \cdot 10^{-19} \text{ C}$$

Relativistic kinematics:

$$E^2 = \vec{p}^2 c^2 + m^2 c^4$$

$$\beta = \frac{v}{c} \quad \gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

$$E = m\gamma c^2 = mc^2 + E_{\text{kin}}$$

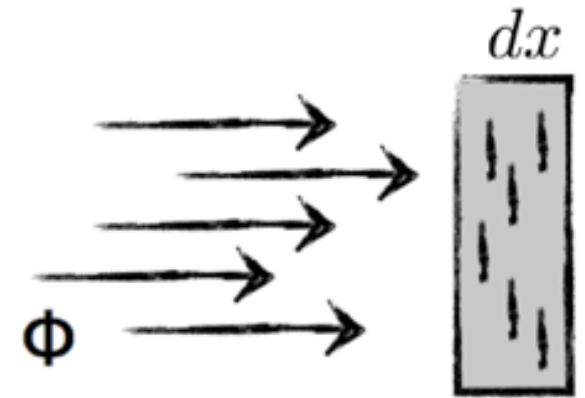
Particle Identification via  
measurement of

e.g.  $(E, \vec{p}, Q)$  or  $(\vec{p}, \beta, Q)$   
 $(\vec{p}, m, Q) \dots$

$$\vec{p} = m\gamma\vec{\beta}c \quad \vec{\beta} = \frac{\vec{p}c}{E}$$

# INTERACTION CROSS-SECTION

Flux  $\Phi = \frac{1}{S} \frac{dN_i}{dt}$   $[L^{-2} t^{-1}]$



Reactions per unit of time  $\frac{dN_{\text{reac}}}{dt} = \Phi \overbrace{\sigma N_{\text{target}}}^{\text{area obscured by target particle}} dx$   $[t^{-1}]$

$[L^{-2} t^{-1}]$   $[?]$   $[L^{-1}]$   $[L]$

Reaction rate per target particle  $W_{if} = \Phi \sigma$   $[t^{-1}]$

Cross section per target particle  $\sigma = \frac{W_{if}}{\Phi}$   $[L^2]$  = reaction rate per unit of flux

1b =  $10^{-28} \text{ m}^2$  (roughly the area of a nucleus with  $A = 100$ )

# FERMI GOLDEN RULE

From non-relativistic perturbation theory...

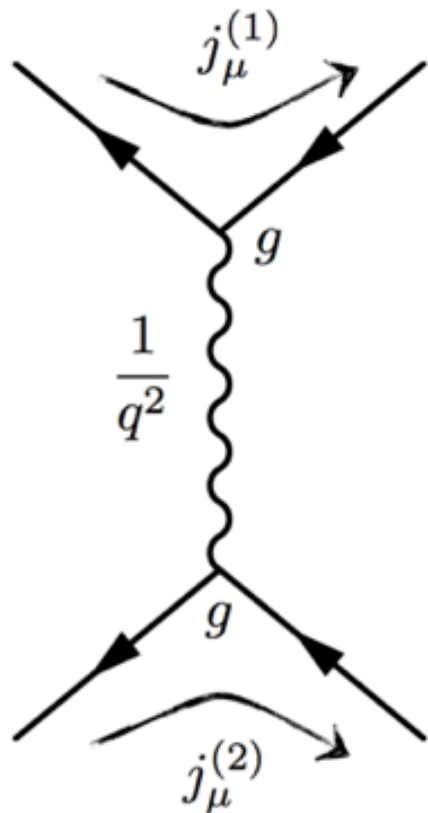
transition probability      matrix element      energy density of final states

$$W_{if} = \frac{2\pi}{\hbar} |M_{if}|^2 \frac{dN}{dE_f}$$

[t<sup>-1</sup>]

[E]

[E<sup>-1</sup>]



$$M_{if} = -i \int j_\mu^{(1)} \left( \frac{1}{q^2} \right) j_\mu^{(2)} d^4x$$

$$\sigma \sim |M_{if}|^2 \sim g^4 \left( \frac{1}{q^4} \right)$$

# CROSS-SECTION: ORDER OF MAGNITUDE

Standard

cross section unit:

$$[\sigma] = \text{mb}$$

with  $1 \text{ mb} = 10^{-27} \text{ cm}^2$

or in

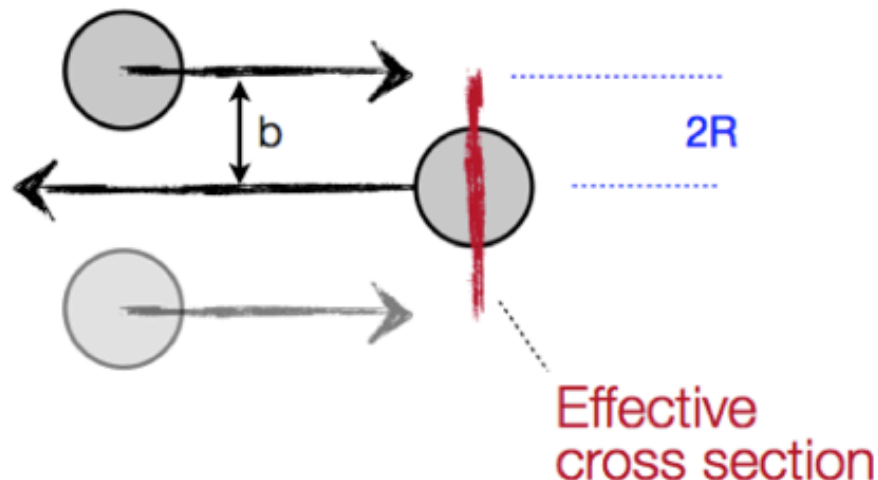
natural units:

$$[\sigma] = \text{GeV}^{-2}$$

with  $1 \text{ GeV}^{-2} = 0.389 \text{ mb}$

$$1 \text{ mb} = 2.57 \text{ GeV}^{-2}$$

Estimating the  
proton-proton cross section:



---

using:  $\hbar c = 0.1973 \text{ GeV fm}$   
 $(\hbar c)^2 = 0.389 \text{ GeV}^2 \text{ mb}$

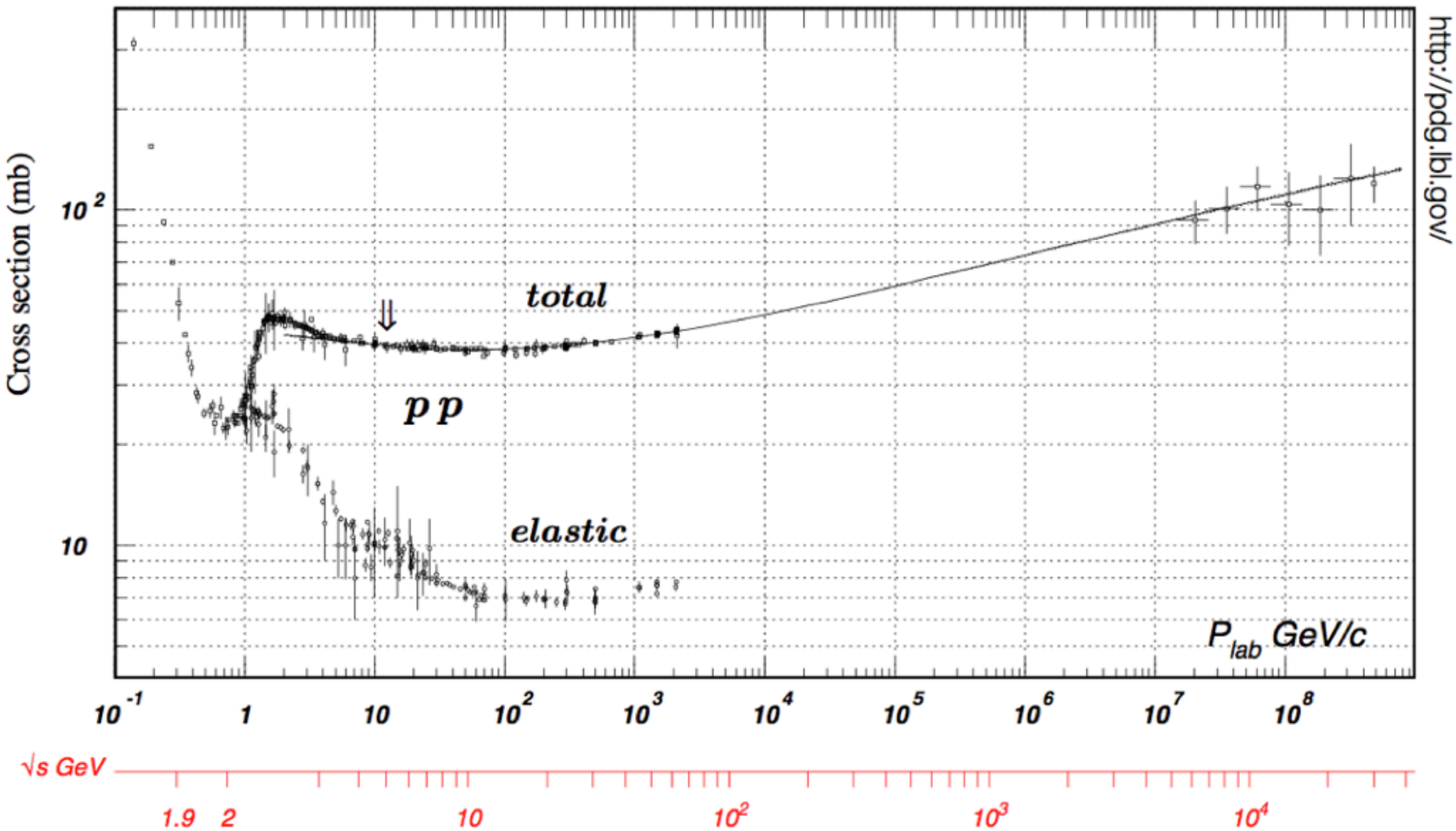
Proton radius:  $R = 0.8 \text{ fm}$

Strong interactions happens up to  $b = 2R$

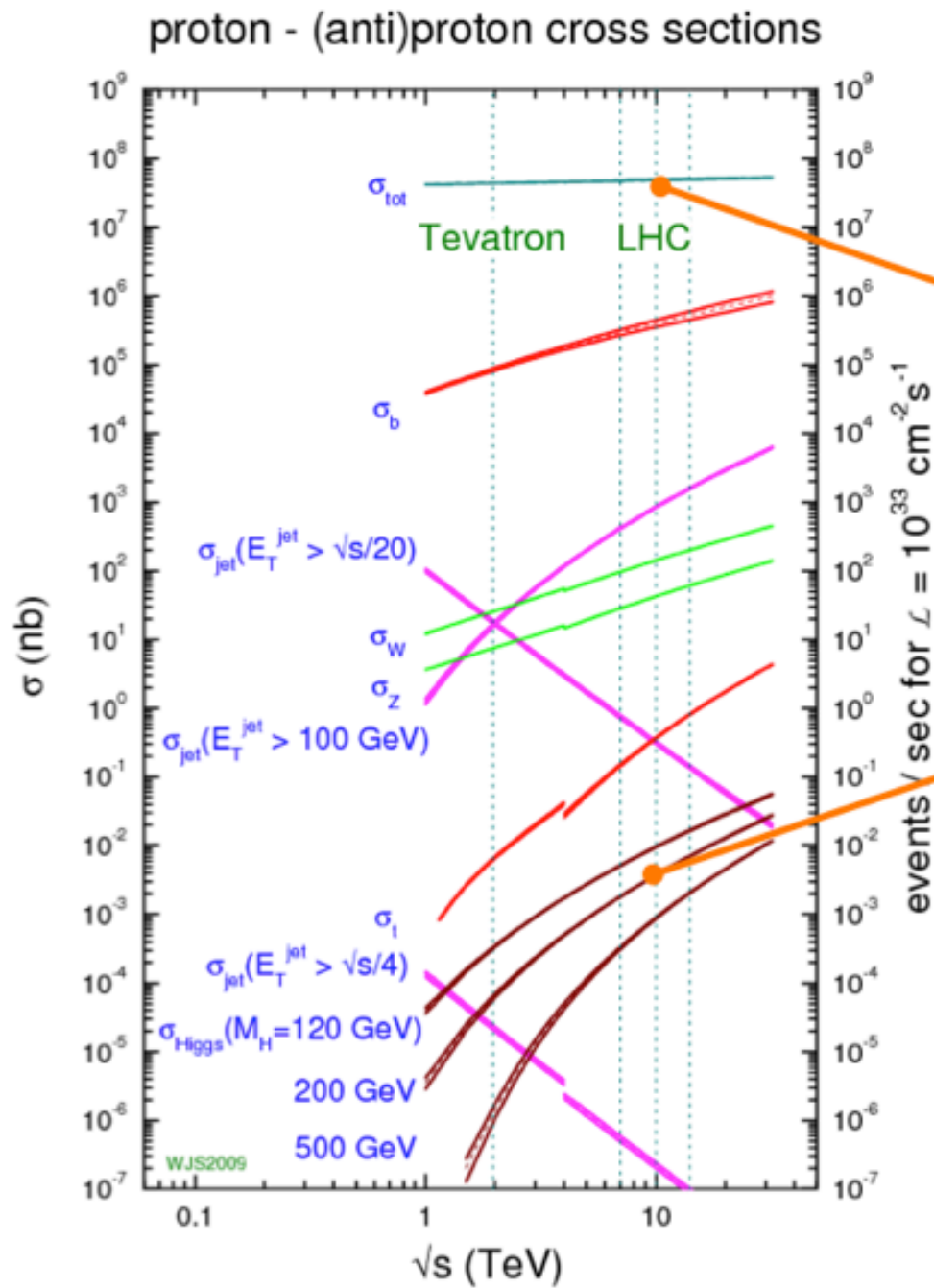
$$\begin{aligned}\sigma &= \pi (2R)^2 = \pi \cdot 1.6^2 \text{ fm}^2 \\ &= \pi \cdot 1.6^2 10^{-26} \text{ cm}^2 \\ &= \pi \cdot 1.6^2 10 \text{ mb} \\ &= 80 \text{ mb}\end{aligned}$$



# PROTON-PROTON SCATTERING CROSS-SECTION



# CROSS-SECTIONS AT THE LHC



$10^8 \text{ events/s}$

$\sim 10^{10}$

$10^{-2} \text{ events/s} \sim$

$10 \text{ events/min}$

$[m_H \sim 120 \text{ GeV}]$

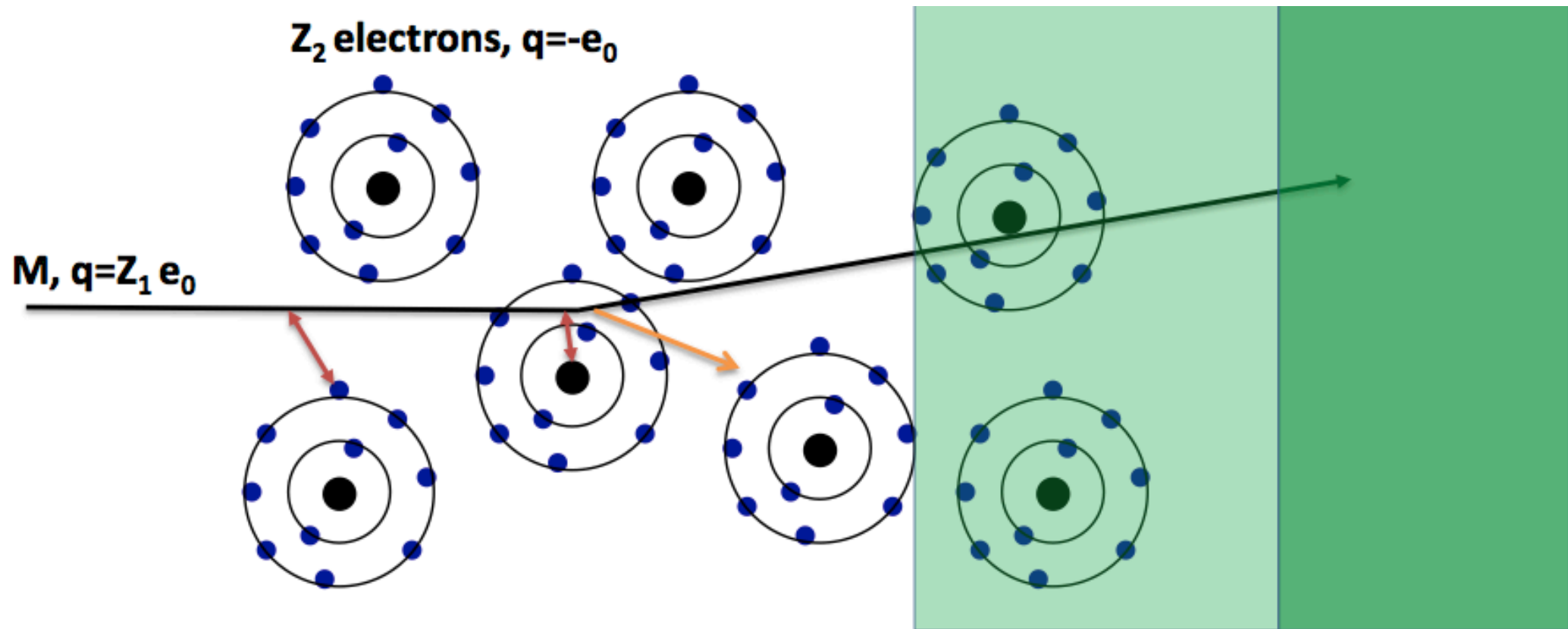
$0.2\% H \rightarrow \gamma\gamma$

$1.5\% H \rightarrow ZZ$

**TRIGGER !**

# ELECTROMAGNETIC INTERACTION

## PARTICLE - MATTER



### Interaction with the atomic electrons.

The incoming particle loses energy and the atoms are **excited** or **ionised**.

### Interaction with the atomic nucleus.

The incoming particle is deflected causing **multiple scattering** of the particle in the material.

During this scattering a **Bremsstrahlung photon** can be emitted

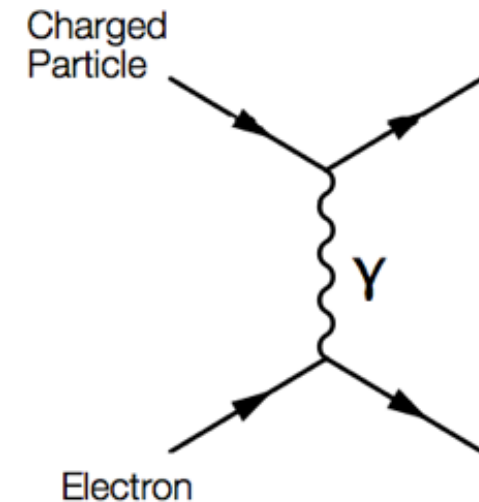
In case the particle's velocity is larger than the velocity of light in the medium, the resulting EM shockwave manifests itself as **Cherenkov radiation**. When the particle crosses the boundary between two media, there is a probability of 1% to produce an Xray photon called **Transition radiation**.

# ENERGY LOSS BY IONISATION: BETHE-BLOCH FORMULA

For now assume:  $Mc^2 \gg m_e c^2$

i.e. energy loss for heavy charged particles  
[dE/dx for electrons more difficult ...]

Interaction dominated  
by elastic collisions with electrons ...



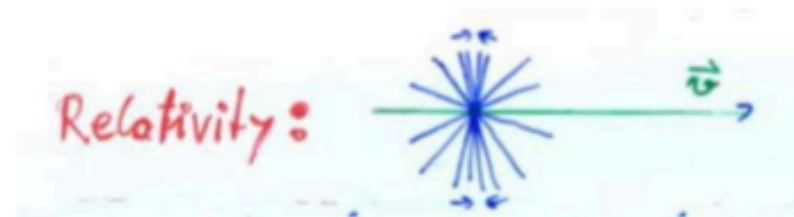
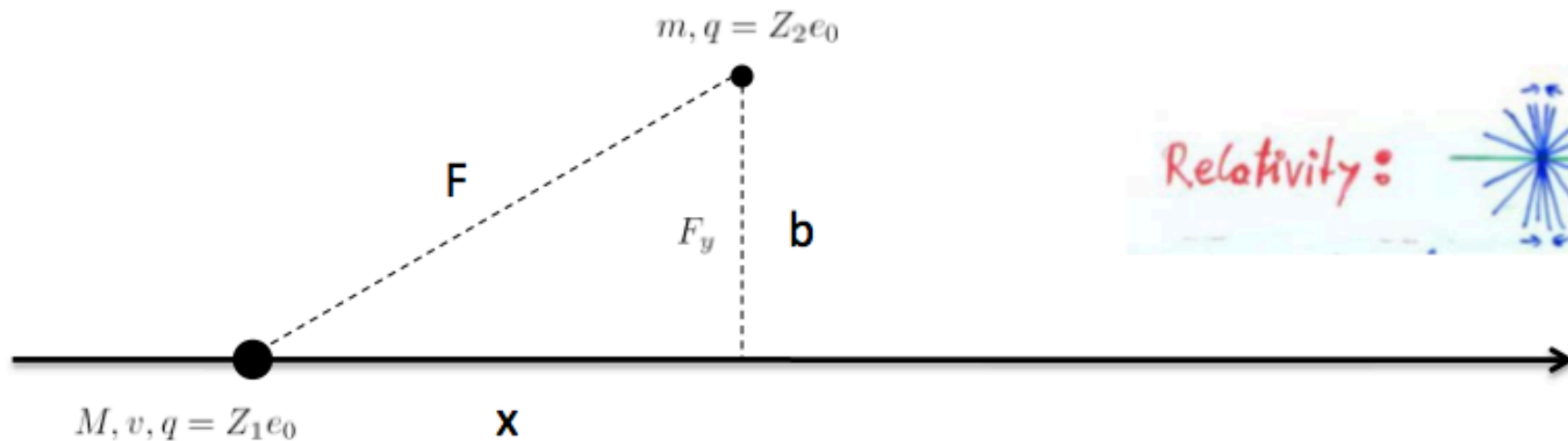
Bethe-Bloch Formula

$$-\left\langle \frac{dE}{dx} \right\rangle = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[ \frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 T_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$

$$\propto 1/\beta^2 \cdot \ln(\text{const} \cdot \beta^2 \gamma^2)$$



# IONISATION & EXCITATION



While the charged particle is passing another charged particle the Coulomb force is acting, resulting in momentum transfer.

The relativistic form of the transverse electric field does not change the momentum transfer. The transverse field is stronger, but the time of action is shorter.

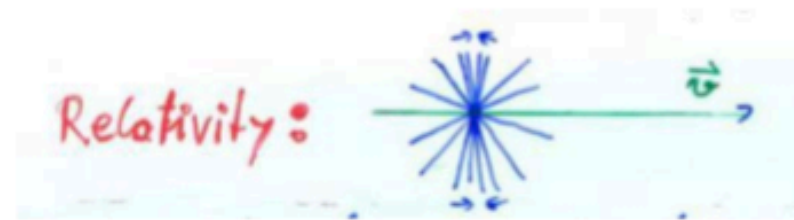
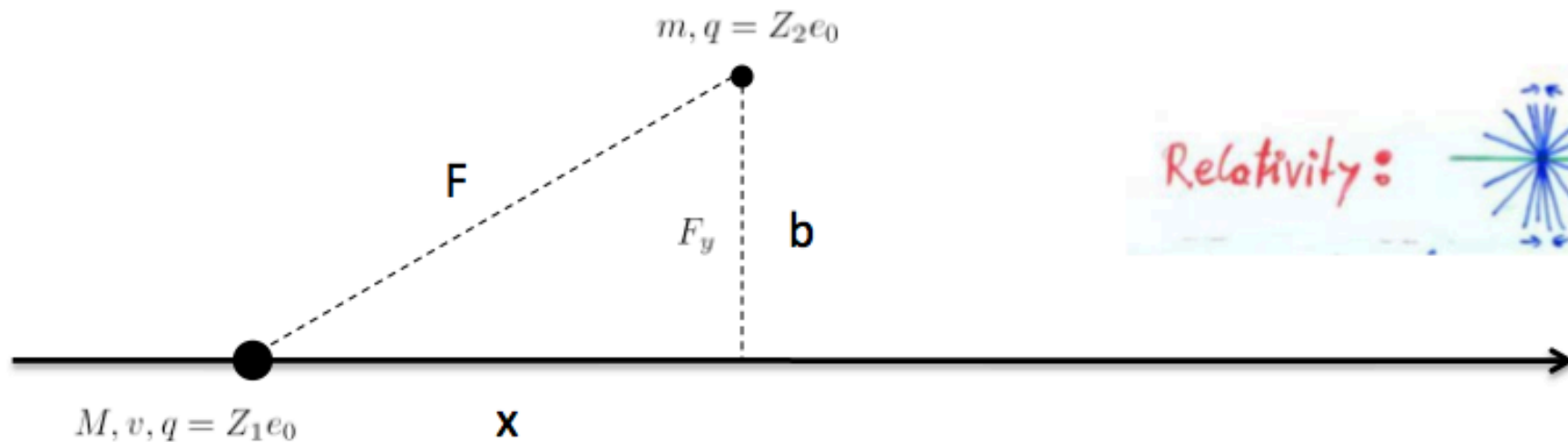
$$F_y = \frac{Z_1 Z_2 e_0^2}{4\pi\epsilon_0(b^2 + v^2 t^2)} \frac{b}{\sqrt{b^2 + v^2 t^2}}$$

$$\Delta p = \int_{-\infty}^{\infty} F_y(t) dt = \frac{2Z_1 Z_2 e_0^2}{4\pi\epsilon_0 v b}$$

$$F_y = \frac{\gamma Z_1 Z_2 e_0^2 b}{4\pi\epsilon_0(b^2 + \gamma^2 v^2 t^2)^{3/2}}$$

$$\Delta p = \int_{-\infty}^{\infty} F_y(t) dt = \frac{2Z_1 Z_2 e_0^2}{4\pi\epsilon_0 v b}$$

# IONISATION & EXCITATION



The transferred energy

The incoming particle transfers energy mainly/only to the atomic electrons.

$$\Delta E = \frac{(\Delta p)^2}{2m} = \frac{Z_2^2}{m} \frac{2Z_1^2 e_0^4}{(4\pi\epsilon_0)^2 v^2 b^2}$$

$$\Delta E(\text{electrons}) = Z_2 \frac{1}{m_e} \frac{2Z_1^2 e_0^4}{(4\pi\epsilon_0)^2 v^2 b^2}$$

$$\Delta E(\text{nucleus}) = \frac{Z_2^2}{2Z_2 m_p} \frac{2Z_1^2 e_0^4}{(4\pi\epsilon_0)^2 v^2 b^2}$$

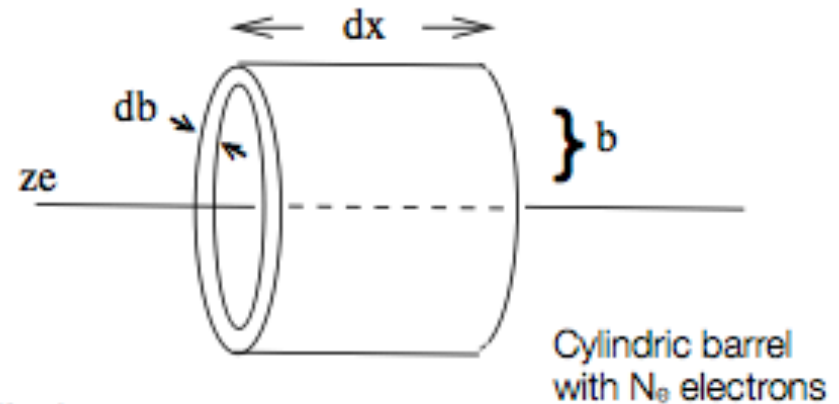
$$\frac{\Delta E(\text{electrons})}{\Delta E(\text{nucleus})} = \frac{2m_p}{m_e} \approx 4000$$

# BETHE-BLOCH FORMULA - CLASSICAL DERIVATION

Bohr 1913

Energy transfer onto **single** electron  
for **impact parameter**  $b$ :

$$\Delta E(b) = \frac{\Delta p^2}{2m_e}$$



Consider cylindric barrel  $\rightarrow N_e = n \cdot (2\pi b) \cdot db dx$

Energy loss **per path length**  $dx$  for  
**distance between**  $b$  **and**  $b+db$  in medium with **electron density**  $n$ :

Energy loss!

$$-dE(b) = \frac{\Delta p^2}{2m_e} \cdot 2\pi n b db dx = \frac{4z^2 e^4}{2b^2 v^2 m_e} \cdot 2\pi n b db dx = \frac{4\pi n z^2 e^4}{m_e v^2} \frac{db}{b} dx$$

Diverges for  $b \rightarrow 0$ ; integration only  
for relevant range  $[b_{\min}, b_{\max}]$ :

Bohr 1913

$$-\frac{dE}{dx} = \frac{4\pi n z^2 e^4}{m_e v^2} \cdot \int_{b_{\min}}^{b_{\max}} \frac{db}{b} = \frac{4\pi n z^2 e^4}{m_e v^2} \ln \frac{b_{\max}}{b_{\min}}$$

# BETHE-BLOCH FORMULA - CLASSICAL DERIVATION

Determination of relevant range  $[b_{\min}, b_{\max}]$ :

Bohr 1913

[Arguments:  $b_{\min} > \lambda_e$ , i.e. de Broglie wavelength;  $b_{\max} < \infty$  due to screening ...]

$$b_{\min} = \lambda_e = \frac{h}{p} = \frac{2\pi\hbar}{\gamma m_e v}$$

Use Heisenberg uncertainty principle or  
that electron is located within de Broglie wavelength ...

$$b_{\max} = \frac{\gamma v}{\langle \nu_e \rangle} ; \quad \left[ \gamma = \frac{1}{\sqrt{1-\beta^2}} \right]$$

Interaction time ( $b/v$ ) must be much shorter than period  
of the electron ( $\gamma/v_e$ ) to guarantee relevant energy transfer ...

[adiabatic invariance]

$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_e c^2 \beta^2} n \cdot \ln \frac{m_e c^2 \beta^2 \gamma^2}{2\pi\hbar \langle \nu_e \rangle}$$

Deviates by factor 2  
from QM derivation

Electron density:  
Effective Ionization potential:

$$n = N_A \cdot \rho \cdot Z/A !!$$

$$I \sim \hbar \langle \nu_e \rangle$$



# BETHE-BLOCH FORMULA

[see e.g. PDG 2010]

$$-\left\langle \frac{dE}{dx} \right\rangle = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[ \frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 T_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$

[· ρ]  
density

$$K = 4\pi N_A r_e^2 m_e c^2 = 0.307 \text{ MeV g}^{-1} \text{ cm}^2$$

$$T_{\max} = 2m_e c^2 \beta^2 \gamma^2 / (1 + 2\gamma m_e/M + (m_e/M)^2)$$

[Max. energy transfer in single collision]

$$N_A = 6.022 \cdot 10^{23}$$

[Avogadro's number]

$$r_e = e^2 / 4\pi\epsilon_0 m_e c^2 = 2.8 \text{ fm}$$

[Classical electron radius]

$$m_e = 511 \text{ keV}$$

[Electron mass]

$$\beta = v/c$$

[Velocity]

$$\gamma = (1 - \beta^2)^{-1/2}$$

[Lorentz factor]

z : Charge of incident particle

M : Mass of incident particle

Z : Charge number of medium

A : Atomic mass of medium

I : Mean excitation energy of medium

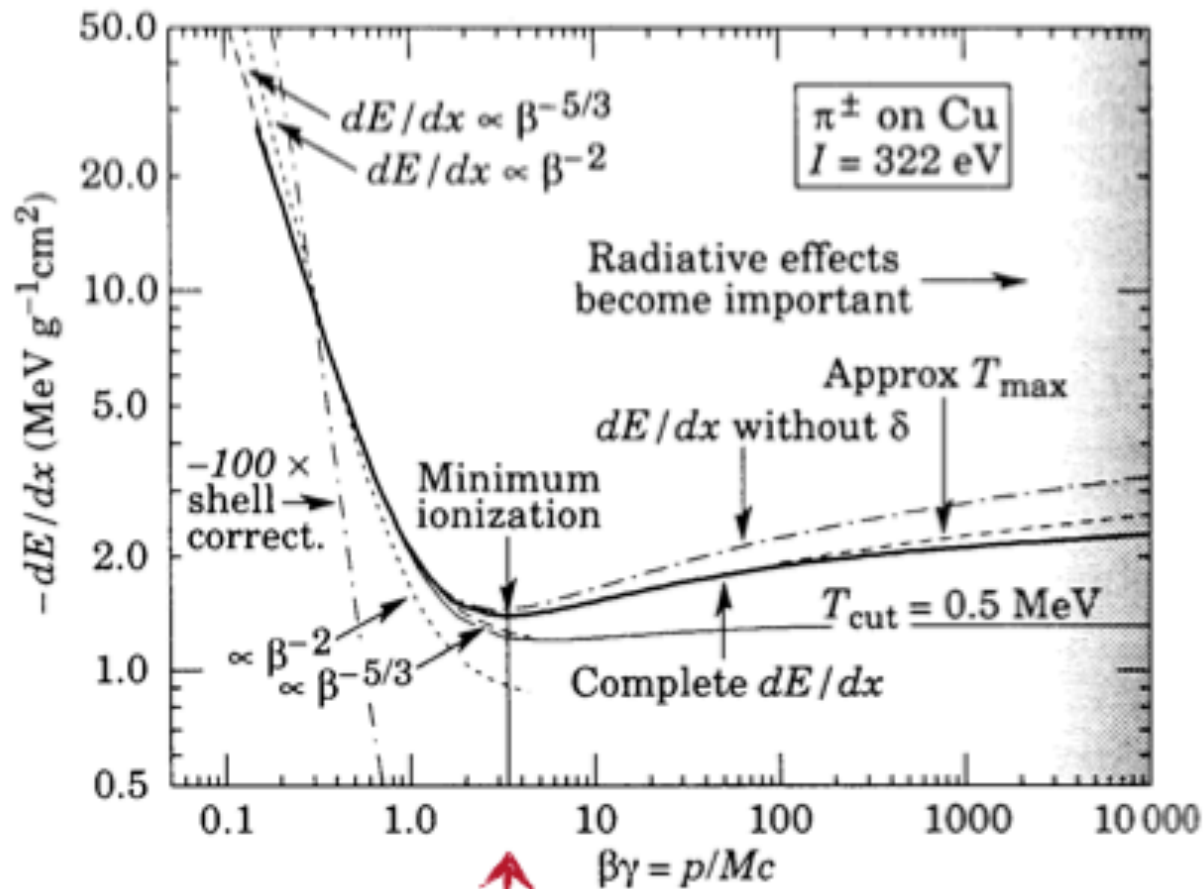
δ : Density correction [transv. extension of electric field]

Validity:

$$.05 < \beta\gamma < 500$$

$$M > m_\mu$$

# ENERGY LOSS of PIONS in Cu



$\beta\gamma = 3-4$

Minimum ionizing particles (MIP):  $\beta\gamma = 3-4$

$dE/dx$  falls  $\sim \beta^{-2}$ ; kinematic factor  
[precise dependence:  $\sim \beta^{-5/3}$ ]

$dE/dx$  rises  $\sim \ln(\beta\gamma)^2$ ; relativistic rise  
[rel. extension of transversal E-field]

Saturation at large  $(\beta\gamma)$  due to density effect (correction  $\delta$ )  
[polarization of medium]

Units:  $\text{MeV g}^{-1} \text{cm}^2$

MIP loses  $\sim 13$  MeV/cm  
[density of copper:  $8.94 \text{ g/cm}^3$ ]

# UNDERSTANDING BETHE-BLOCH

## 1/ $\beta^2$ -dependence:

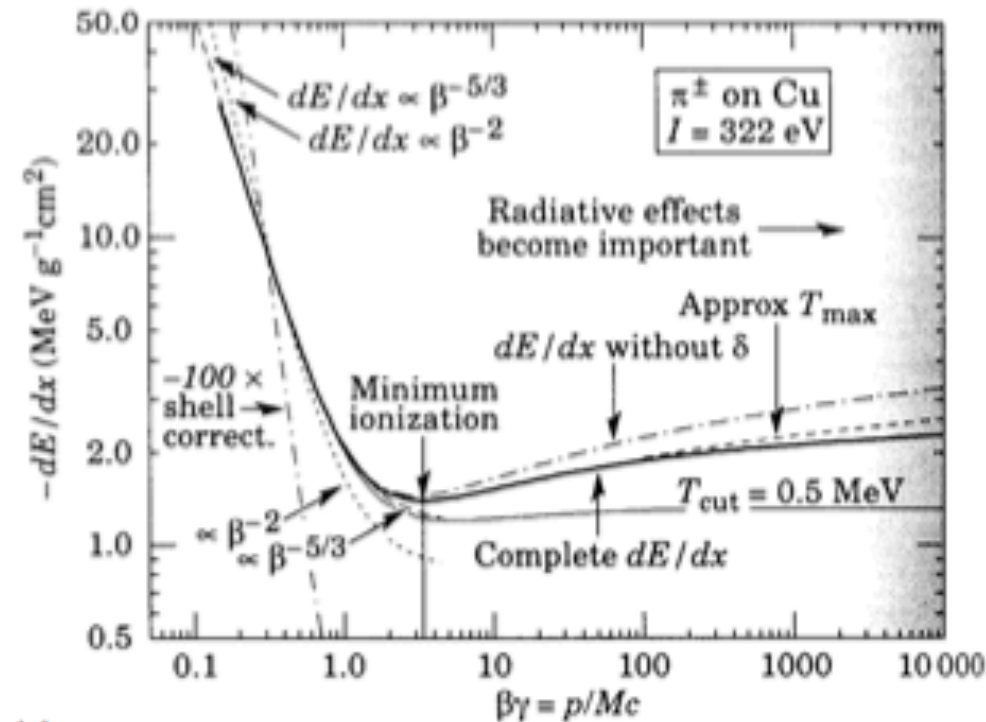
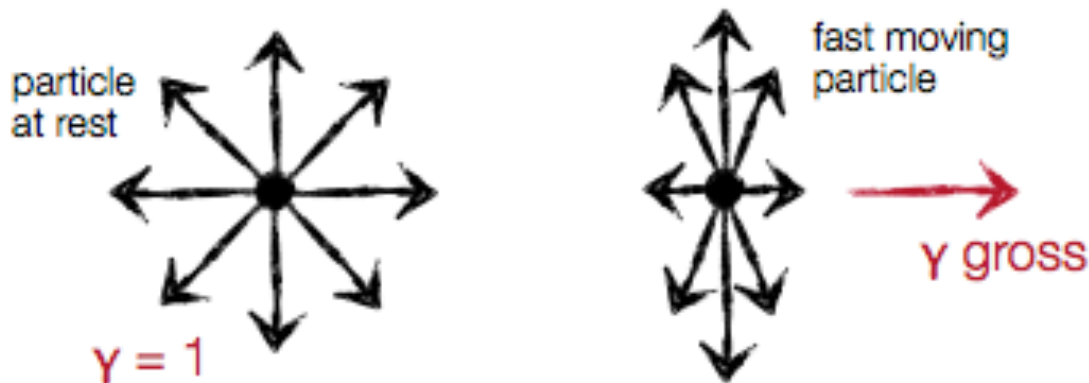
Remember:

$$\Delta p_{\perp} = \int F_{\perp} dt = \int F_{\perp} \frac{dx}{v}$$

i.e. slower particles feel electric force of atomic electron for longer time ...

## Relativistic rise for $\beta\gamma > 4$ :

High energy particle: transversal electric field increases due to Lorentz transform;  $E_y \rightarrow \gamma E_y$ . Thus interaction cross section increases ...



## Corrections:

low energy : shell corrections  
high energy : density corrections

# UNDERSTANDING BETHE-BLOCH

## Density correction:

Polarization effect ...

[density dependent]

→ Shielding of electrical field far from particle path; effectively cuts off the long range contribution ...

More relevant at high  $\gamma$  ...

[Increased range of electric field; larger  $b_{\max}$ ; ...]

For high energies:

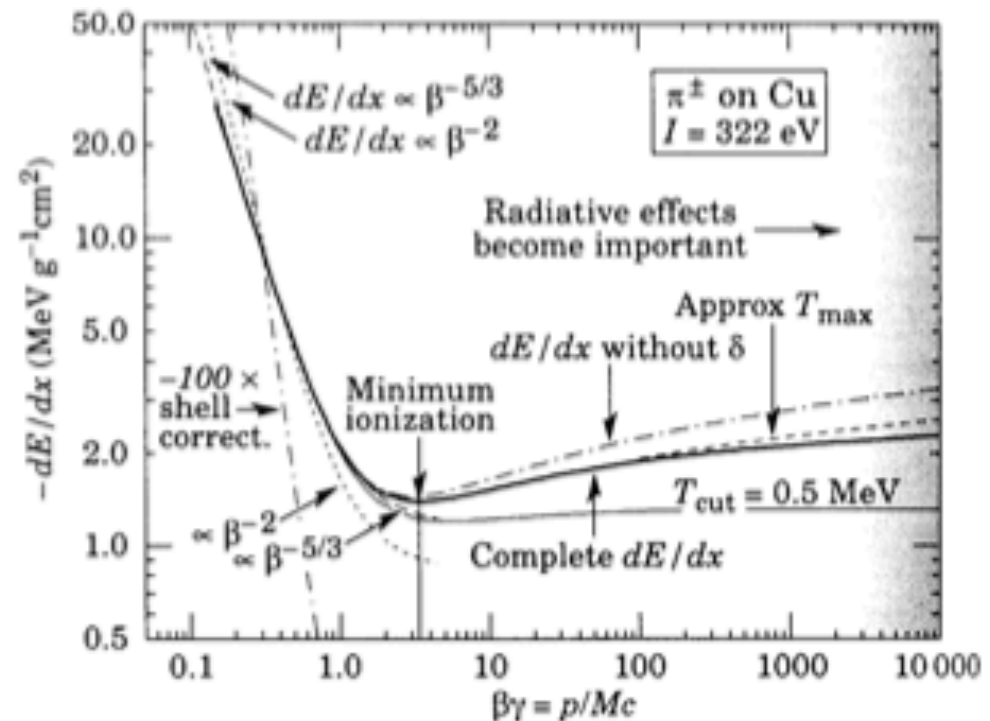
$$\delta/2 \rightarrow \ln(\hbar\omega/I) + \ln \beta\gamma - 1/2$$

## Shell correction:

Arises if particle velocity is close to orbital velocity of electrons, i.e.  $\beta c \sim v_e$ .

Assumption that electron is at rest breaks down ...

Capture process is possible ...



Density effect leads to saturation at high energy ...

Shell correction are in general small ...

# CHARGED PARTICLE ENERGY LOSS in MATERIALS

Dependence on target element

Mass  $A$

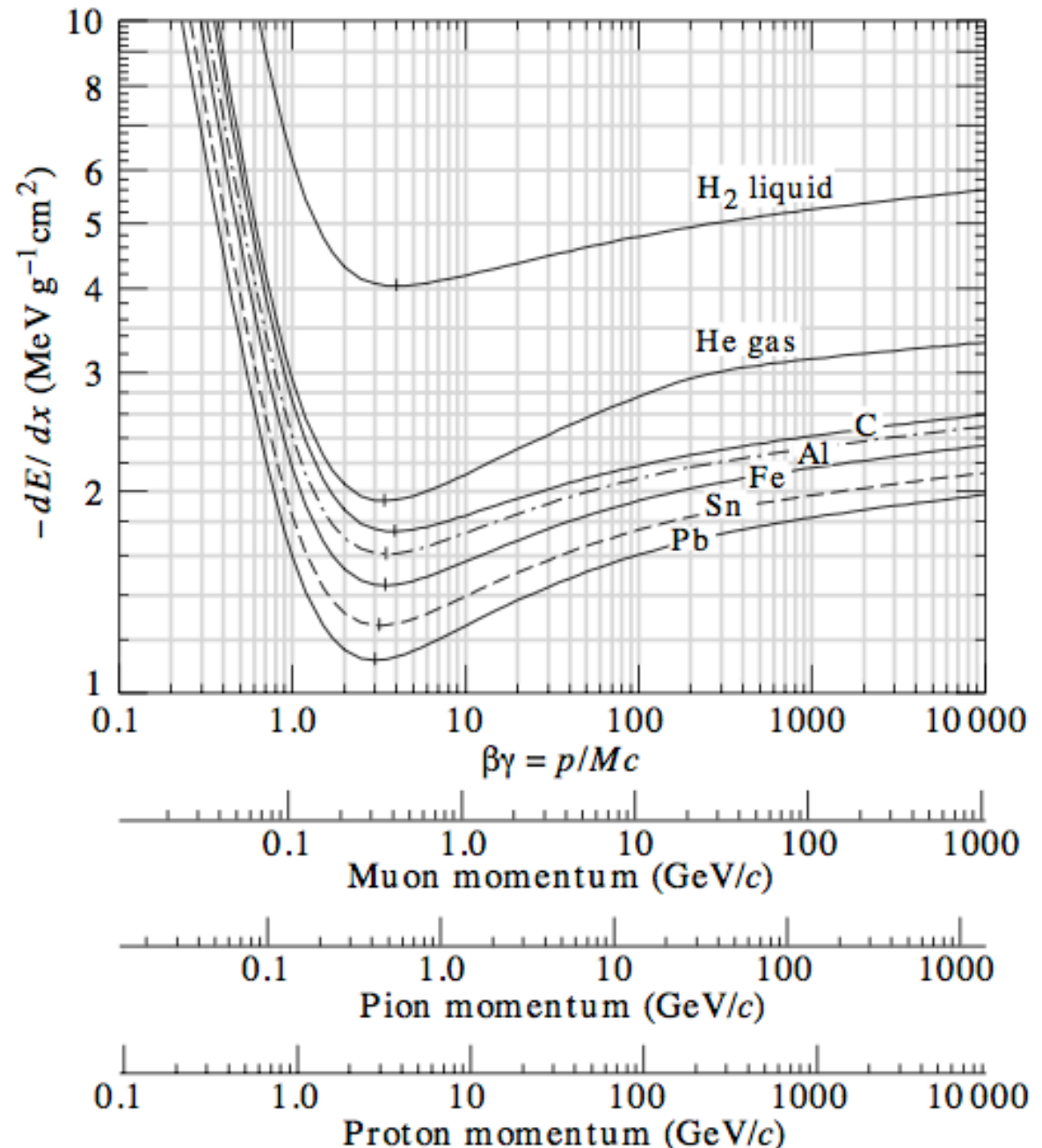
Charge  $Z$

Minimum Ionisation

$-dE/dx \sim 1\text{--}2 \text{ MeV g}^{-1}\text{cm}^2$

e.g. for Pb with  $\rho=11.35 \text{ g/cm}^3$ :

$-dE/dx \sim 13 \text{ MeV/cm}$





# MATERIAL PROPERTIES

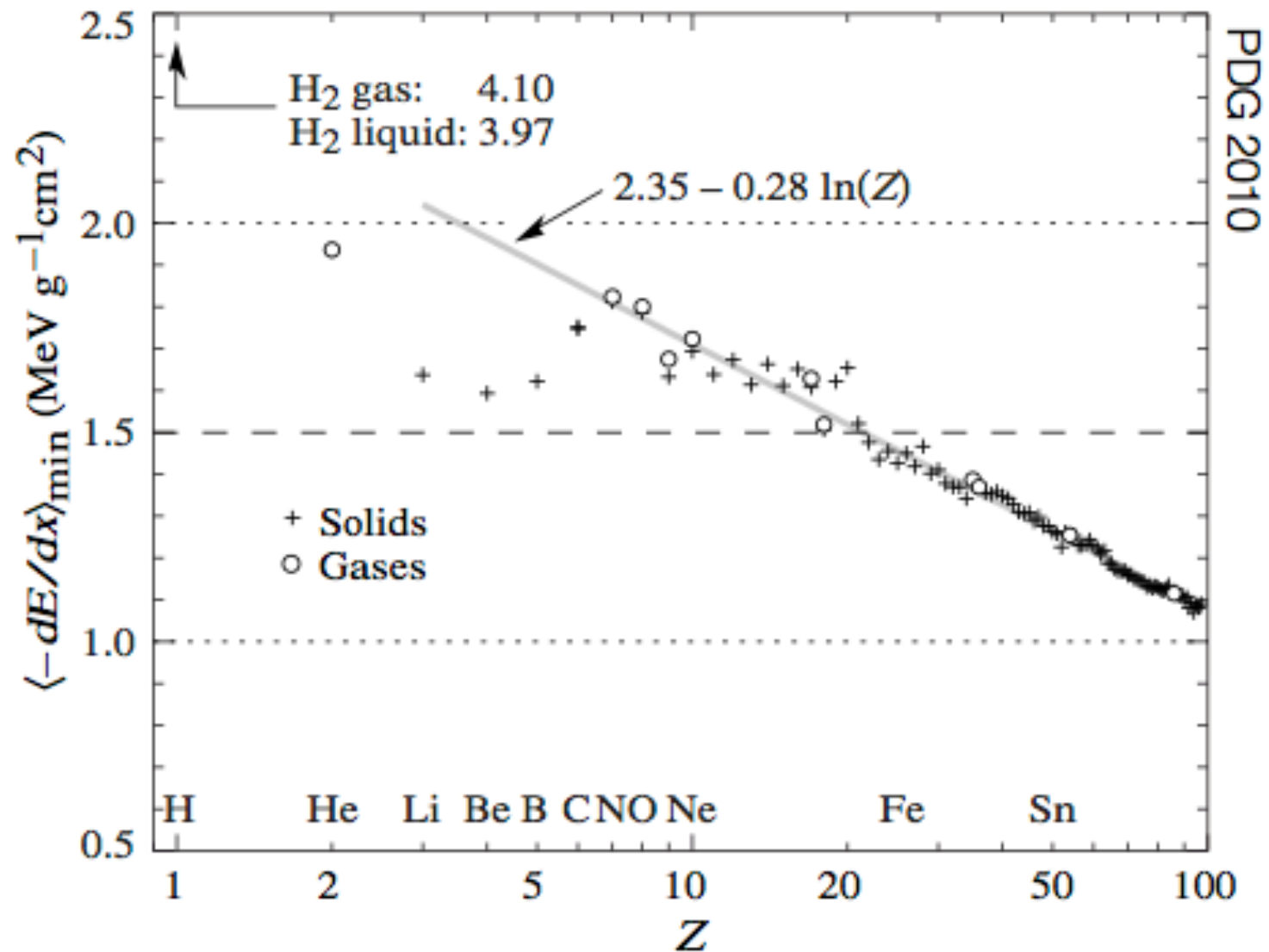
## 6. ATOMIC AND NUCLEAR PROPERTIES OF MATERIALS

**Table 6.1.** Revised May 2002 by D.E. Groom (LBNL). Gases are evaluated at 20°C and 1 atm (in parentheses) or at STP [square brackets]. Densities and refractive indices without parentheses or brackets are for solids or liquids, or are for cryogenic liquids at the indicated boiling point (BP) at 1 atm. Refractive indices are evaluated at the sodium D line. Data for compounds and mixtures are from Refs. 1 and 2. Further materials and properties are given in Ref. 3 and at <http://pdg.lbl.gov/AtomicNuclearProperties>.

| Material              | $Z$ | $A$        | $\langle Z/A \rangle$ | Nuclear <sup>a</sup><br>collision<br>length $\lambda_T$<br>{g/cm <sup>2</sup> } | Nuclear <sup>a</sup><br>interaction<br>length $\lambda_I$<br>{g/cm <sup>2</sup> } | $dE/dx _{\min}^b$<br>$\left\{ \frac{\text{MeV}}{\text{g/cm}^2} \right\}$ | Radiation length <sup>c</sup><br>$X_0$<br>{g/cm <sup>2</sup> } {cm} |          | Density<br>{g/cm <sup>3</sup> }<br>({g/ℓ}<br>for gas) | Liquid<br>boiling<br>point at<br>1 atm(K) | Refractive<br>index $n$<br>( $(n-1) \times 10^6$<br>for gas) |
|-----------------------|-----|------------|-----------------------|---|---|--|---|----------|---|---|--|
| H <sub>2</sub> gas    | 1   | 1.00794    | 0.99212               | 43.3  | 50.8  | (4.103)  | 61.28 <sup>d</sup>  | (731000) | (0.0838)[0.0899]                                      |   | [139.2]  |
| H <sub>2</sub> liquid | 1   | 1.00794    | 0.99212               | 43.3  | 50.8  | 4.034  | 61.28 <sup>d</sup>  | 866      | 0.0708  | 20.39                                     | 1.112  |
| D <sub>2</sub>        | 1   | 2.0140     | 0.49652               | 45.7  | 54.7  | (2.052)  | 122.4   | 724      | 0.169[0.179]  | 23.65                                     | 1.128 [138]  |
| He                    | 2   | 4.002602   | 0.49968               | 49.9  | 65.1  | (1.937)  | 94.32   | 756      | 0.1249[0.1786]  | 4.224                                     | 1.024 [34.9]   |
| Li                    | 3   | 6.941      | 0.43221               | 54.6  | 73.4  | 1.639  | 82.76   | 155      | 0.534   |   | —  |
| Be                    | 4   | 9.012182   | 0.44384               | 55.8  | 75.2  | 1.594  | 65.19   | 35.28    | 1.848   |   | —  |
| C                     | 6   | 12.011     | 0.49954               | 60.2  | 86.3  | 1.745  | 42.70   | 18.8     | 2.265 <sup>e</sup>                                    |   | —  |
| N <sub>2</sub>        | 7   | 14.00674   | 0.49976               | 61.4  | 87.8  | (1.825)  | 37.99   | 47.1     | 0.8073[1.250]   | 77.36                                     | 1.205 [298]  |
| O <sub>2</sub>        | 8   | 15.9994    | 0.50002               | 63.2  | 91.0  | (1.801)  | 34.24   | 30.0     | 1.141[1.428]  | 90.18                                     | 1.22 [296]   |
| F <sub>2</sub>        | 9   | 18.9984032 | 0.47372               | 65.5  | 95.3  | (1.675)  | 32.93   | 21.85    | 1.507[1.696]  | 85.24                                     | [195]  |
| Ne                    | 10  | 20.1797    | 0.49555               | 66.1  | 96.6  | (1.724)  | 28.94   | 24.0     | 1.204[0.9005]   | 27.09                                     | 1.092 [67.1]   |
| Al                    | 13  | 26.981539  | 0.48181               | 70.6  | 106.4   | 1.615  | 24.01   | 8.9      | 2.70  |   | —  |
| Si                    | 14  | 28.0855    | 0.49848               | 70.6  | 106.0   | 1.664  | 21.82   | 9.36     | 2.33  |   | 3.95   |
| Ar                    | 18  | 39.948     | 0.45059               | 76.4  | 117.2   | (1.519)  | 19.55   | 14.0     | 1.396[1.782]  | 87.28                                     | 1.233 [283]  |
| Ti                    | 22  | 47.867     | 0.45948               | 79.9  | 124.9   | 1.476  | 16.17   | 3.56     | 4.54  |   | —  |
| Fe                    | 26  | 55.845     | 0.46556               | 82.8  | 131.9   | 1.451  | 13.84   | 1.76     | 7.87  |   | —  |
| Cu                    | 29  | 63.546     | 0.45636               | 85.6  | 134.9   | 1.403  | 12.86   | 1.43     | 8.96  |   | —  |
| Ge                    | 32  | 72.61      | 0.44071               | 88.3  | 140.5   | 1.371  | 12.25   | 2.30     | 5.323   |   | —  |
| Sn                    | 50  | 118.710    | 0.42120               | 100.2   | 163   | 1.264  | 8.82  | 1.21     | 7.31  |   | —  |
| Xe                    | 54  | 131.29     | 0.41130               | 102.8   | 169   | (1.255)  | 8.48  | 2.87     | 2.953[5.858]  | 165.1                                     | [701]  |
| W                     | 74  | 183.84     | 0.40250               | 110.3   | 185   | 1.145  | 6.76  | 0.35     | 19.3  |   | —  |
| Pt                    | 78  | 195.08     | 0.39984               | 113.3   | 189.7   | 1.129  | 6.54  | 0.305    | 21.45   |   | —  |
| Pb                    | 82  | 207.2      | 0.39575               | 116.2   | 194   | 1.123  | 6.37  | 0.56     | 11.35   |   | —  |
| U                     | 92  | 238.0289   | 0.38651               | 117.0   | 199   | 1.082  | 6.00  | ≈0.32    | ≈18.95  |   | —  |

| Material  | $Z$ | $A$ | $\langle Z/A \rangle$ | Nuclear <sup>a</sup><br>collision<br>length $\lambda_T$<br>{g/cm <sup>2</sup> } | Nuclear <sup>a</sup><br>interaction<br>length $\lambda_I$<br>{g/cm <sup>2</sup> } | $dE/dx _{\min}$ <sup>b</sup><br>$\left\{ \frac{\text{MeV}}{\text{g/cm}^2} \right\}$ | Radiation length <sup>c</sup><br>$X_0$<br>{g/cm <sup>2</sup> } {cm} |                 | Density<br>{g/cm <sup>3</sup> }<br>({g/ℓ}<br>for gas) | Liquid<br>boiling<br>point at<br>1 atm(K) | Refractive<br>index $n$<br>(( $n-1$ )×10 <sup>6</sup><br>for gas) |
|---|-----|-----|-----------------------|---|---|---|---|-----------------|---|---|---|
| Air, (20°C, 1 atm.), [STP]  |     |     | 0.49919               | 62.0  | 90.0  | (1.815)   | 36.66   | [30420]         | (1.205)[1.2931]                                       | 78.8                                      | (273) [293]   |
| H <sub>2</sub> O  |     |     | 0.55509               | 60.1  | 83.6  | 1.991   | 36.08   | 36.1            | 1.00  | 373.15                                    | 1.33  |
| CO <sub>2</sub> gas   |     |     | 0.49989               | 62.4  | 89.7  | (1.819)   | 36.2  | [18310]         | [1.977]   |   | [410]   |
| CO <sub>2</sub> solid (dry ice)   |     |     | 0.49989               | 62.4  | 89.7  | 1.787   | 36.2  | 23.2            | 1.563   | sublimes                                  | —   |
| Shielding concrete <sup>f</sup>   |     |     | 0.50274               | 67.4  | 99.9  | 1.711   | 26.7  | 10.7            | 2.5   |   | —   |
| SiO <sub>2</sub> (fused quartz)   |     |     | 0.49926               | 66.5  | 97.4  | 1.699   | 27.05   | 12.3            | 2.20 <sup>g</sup>                                     |   | 1.458   |
| Dimethyl ether, (CH <sub>3</sub> ) <sub>2</sub> O   |     |     | 0.54778               | 59.4  | 82.9  | —   | 38.89   | —               | —   | 248.7                                     | —   |
| Methane, CH <sub>4</sub>  |     |     | 0.62333               | 54.8  | 73.4  | (2.417)   | 46.22   | [64850]         | 0.4224[0.717]   | 111.7                                     | [444]   |
| Ethane, C <sub>2</sub> H <sub>6</sub>   |     |     | 0.59861               | 55.8  | 75.7  | (2.304)   | 45.47   | [34035]         | 0.509(1.356) <sup>h</sup>                             | 184.5                                     | (1.038) <sup>h</sup>  |
| Propane, C <sub>3</sub> H <sub>8</sub>  |     |     | 0.58962               | 56.2  | 76.5  | (2.262)   | 45.20   | —               | (1.879)   | 231.1                                     | —   |
| Isobutane, (CH <sub>3</sub> ) <sub>2</sub> CHCH <sub>3</sub>  |     |     | 0.58496               | 56.4  | 77.0  | (2.239)   | 45.07   | [16930]         | [2.67]  | 261.42                                    | [1900]  |
| Octane, liquid, CH <sub>3</sub> (CH <sub>2</sub> ) <sub>6</sub> CH <sub>3</sub>                       |     |     | 0.57778               | 56.7  | 77.7  | 2.123   | 44.86   | 63.8            | 0.703   | 398.8                                     | 1.397   |
| Paraffin wax, CH <sub>3</sub> (CH <sub>2</sub> ) <sub><math>n \approx 23</math></sub> CH <sub>3</sub> |     |     | 0.57275               | 56.9  | 78.2  | 2.087   | 44.71   | 48.1            | 0.93  |   | —   |
| Nylon, type 6 <sup>i</sup>  |     |     | 0.54790               | 58.5  | 81.5  | 1.974   | 41.84   | 36.7            | 1.14  |   | —   |
| Polycarbonate (Lexan) <sup>j</sup>  |     |     | 0.52697               | 59.5  | 83.9  | 1.886   | 41.46   | 34.6            | 1.20  |   | —   |
| Polyethylene terephthalate (Mylar) <sup>k</sup>   |     |     | 0.52037               | 60.2  | 85.7  | 1.848   | 39.95   | 28.7            | 1.39  |   | —   |
| Polyethylene <sup>l</sup>   |     |     | 0.57034               | 57.0  | 78.4  | 2.076   | 44.64   | ≈47.9           | 0.92–0.95   |   | —   |
| Polyimide film (Kapton) <sup>m</sup>  |     |     | 0.51264               | 60.3  | 85.8  | 1.820   | 40.56   | 28.6            | 1.42  |   | —   |
| Lucite, Plexiglas <sup>n</sup>  |     |     | 0.53937               | 59.3  | 83.0  | 1.929   | 40.49   | ≈34.4           | 1.16–1.20   |   | ≈1.49   |
| Polystyrene, scintillator <sup>o</sup>  |     |     | 0.53768               | 58.5  | 81.9  | 1.936   | 43.72   | 42.4            | 1.032   |   | 1.581   |
| Polytetrafluoroethylene (Teflon) <sup>p</sup>   |     |     | 0.47992               | 64.2  | 93.0  | 1.671   | 34.84   | 15.8            | 2.20  |   | —   |
| Polyvinyltolulene, scintillator <sup>q</sup>  |     |     | 0.54155               | 58.3  | 81.5  | 1.956   | 43.83   | 42.5            | 1.032   |   | —   |
| Aluminum oxide (Al <sub>2</sub> O <sub>3</sub> )  |     |     | 0.49038               | 67.0  | 98.9  | 1.647   | 27.94   | 7.04            | 3.97  |   | 1.761   |
| Barium fluoride (BaF <sub>2</sub> )   |     |     | 0.42207               | 92.0  | 145   | 1.303   | 9.91  | 2.05            | 4.89  |   | 1.56  |
| Bismuth germanate (BGO) <sup>r</sup>  |     |     | 0.42065               | 98.2  | 157   | 1.251   | 7.97  | 1.12            | 7.1   |   | 2.15  |
| Cesium iodide (CsI)   |     |     | 0.41569               | 102   | 167   | 1.243   | 8.39  | 1.85            | 4.53  |   | 1.80  |
| Lithium fluoride (LiF)  |     |     | 0.46262               | 62.2  | 88.2  | 1.614   | 39.25   | 14.91           | 2.632   |   | 1.392   |
| Sodium fluoride (NaF)   |     |     | 0.47632               | 66.9  | 98.3  | 1.69  | 29.87   | 11.68           | 2.558   |   | 1.336   |
| Sodium iodide (NaI)   |     |     | 0.42697               | 94.6  | 151   | 1.305   | 9.49  | 2.59            | 3.67  |   | 1.775   |
| Silica Aerogel <sup>s</sup>   |     |     | 0.50093               | 66.3  | 96.9  | 1.740   | 27.25   | 136@ $\rho=0.2$ | 0.04–0.6  |   | 1.0+0.21 $\rho$   |
| NEMA G10 plate <sup>t</sup>   |     |     |                       | 62.6  | 90.2  | 1.87  | 33.0  | 19.4            | 1.7   |   | —   |

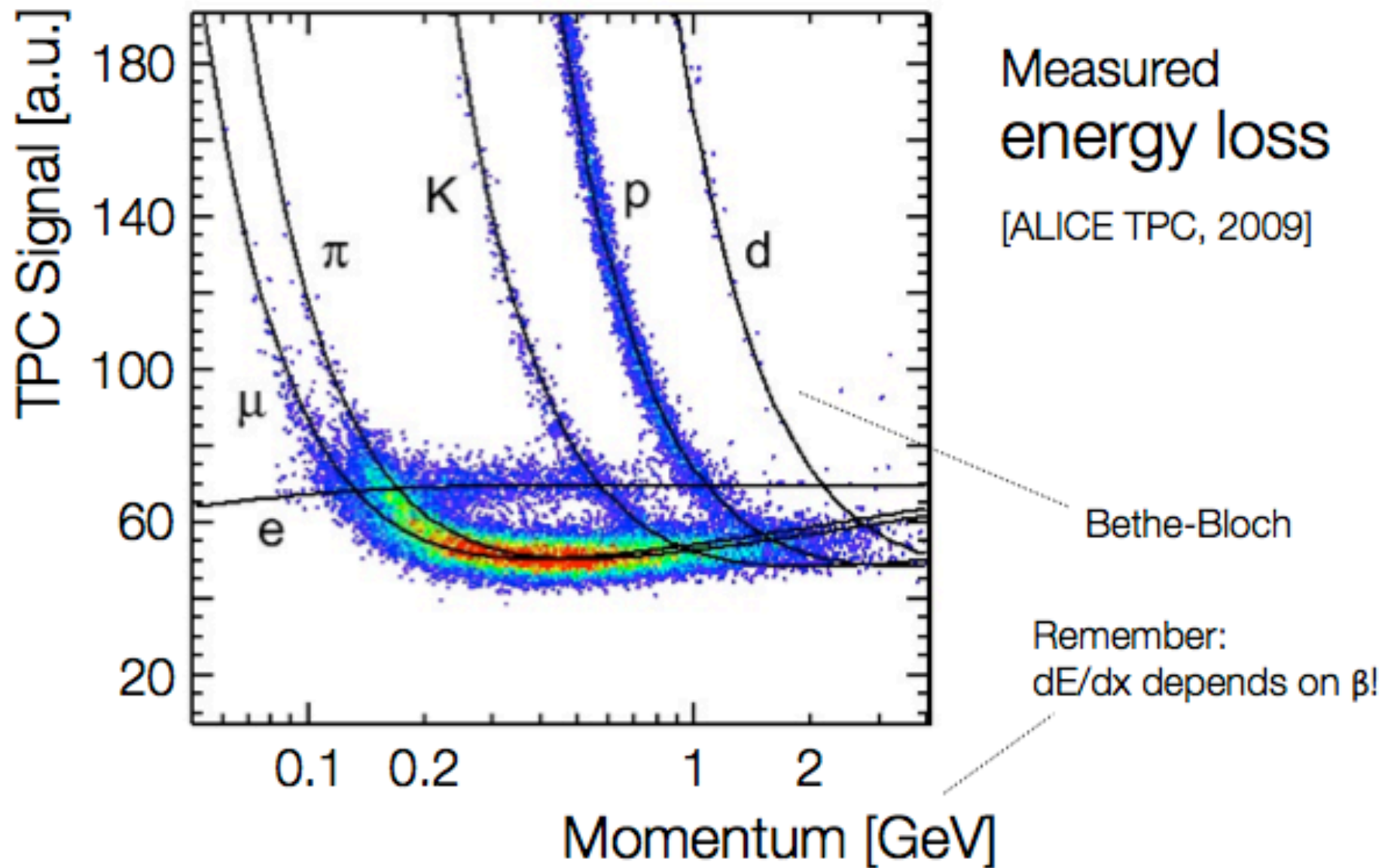
# STOPPING POWER AT MINIMUM IONISATION



Stopping power at minimum ionization for the chemical elements. The straight line is fitted for  $Z > 6$ .  
 A simple functional dependence on  $Z$  is not to be expected, since  $\langle -dE/dx \rangle$  also depends on other variables.



# $dE/dX$ and PARTICLE IDENTIFICATION

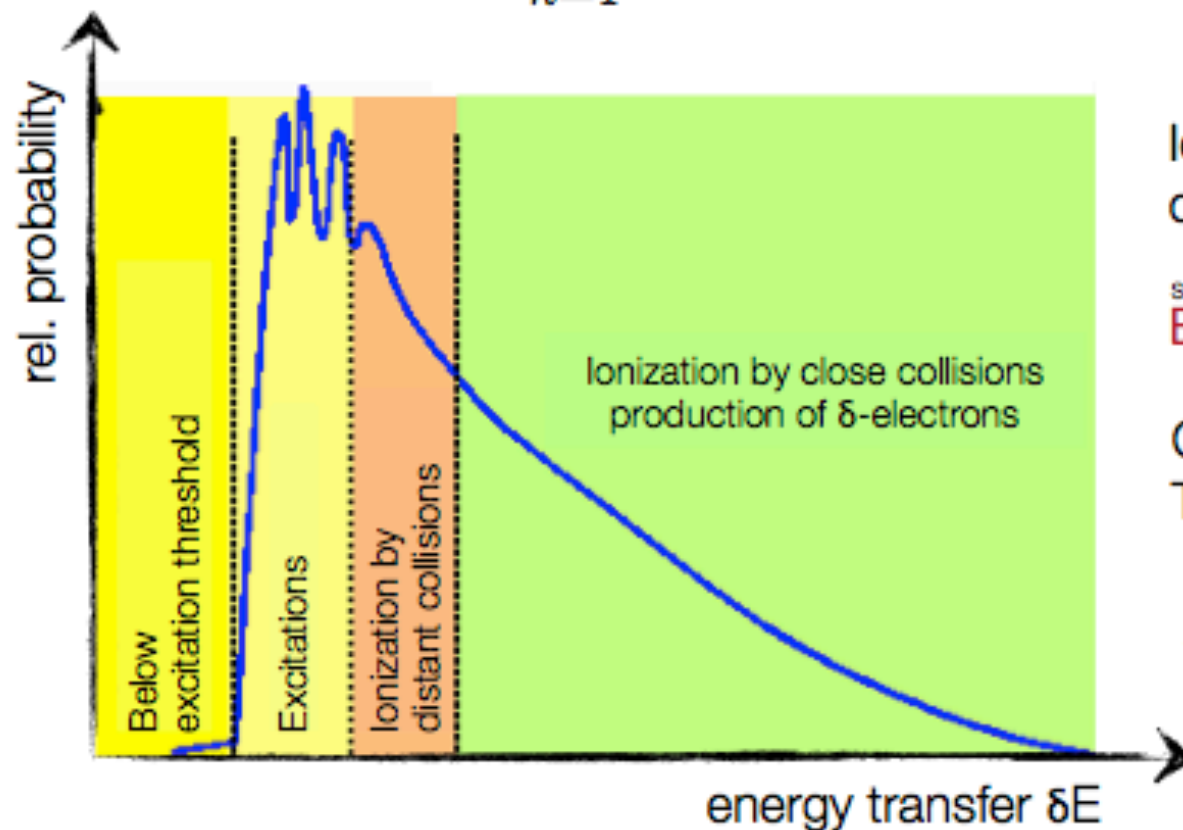


# dE/dx FLUCTUATIONS

Bethe-Bloch describes **mean** energy loss; measurement via energy loss  $\Delta E$  in a material of thickness  $\Delta x$  with

$$\Delta E = \sum_{n=1}^N \delta E_n$$

$N$  : number of collisions  
 $\delta E$  : energy loss in a single collision



Ionization loss  $\delta E$   
distributed statistically ...

so-called  
**Energy loss 'straggling'**

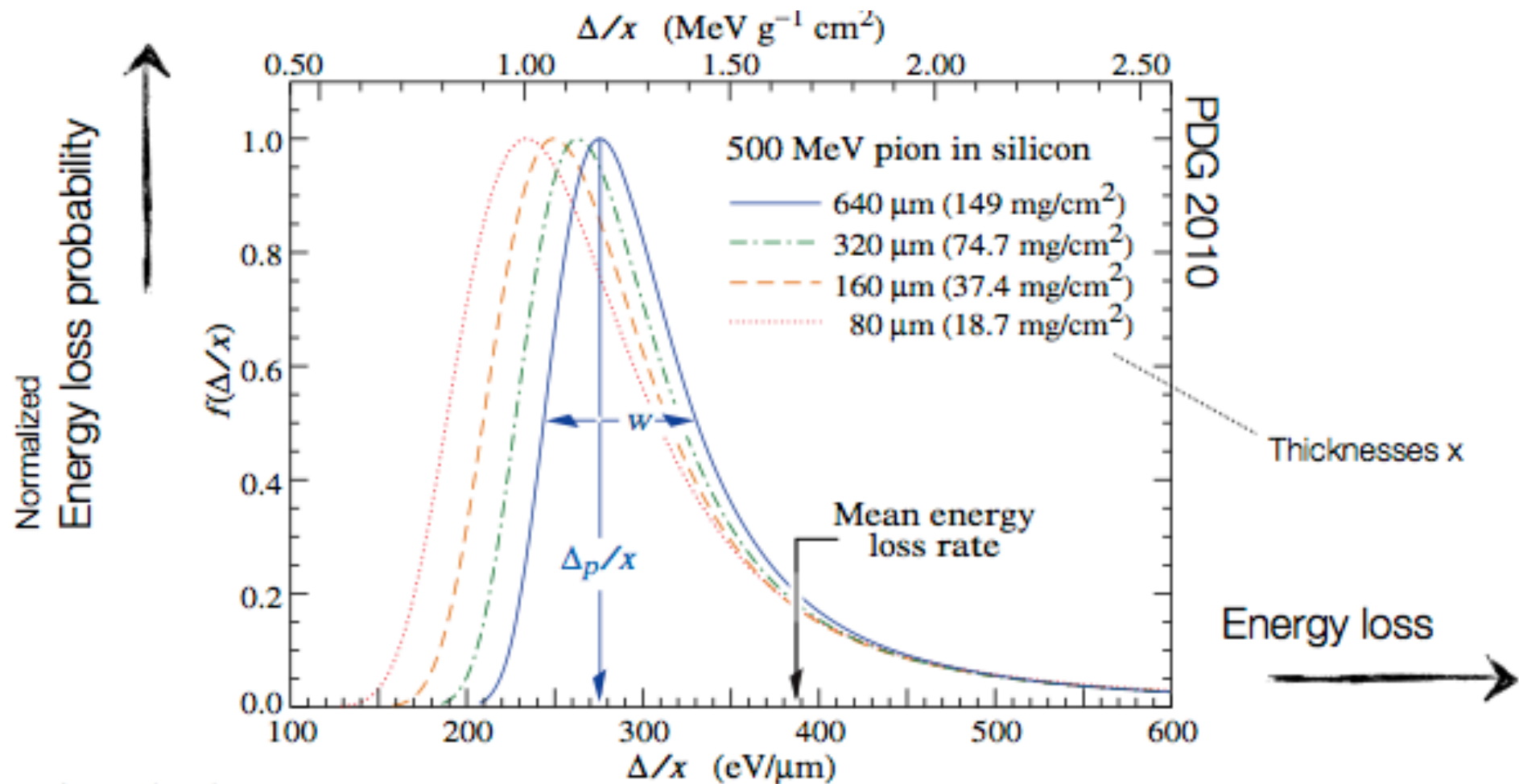
Complicated problem ...  
Thin absorbers: **Landau distribution**

Standard Gauss with mean energy loss  $E_0$   
+ tail towards high energies due to  $\delta$ -electrons

see also Allison & Cobb  
[Ann. Rev. Nucl. Part. Sci. 30 (1980) 253.]



# dE/dx FLUCTUATIONS - LANDAU DISTRIBUTION



Approximation:

$$f(\Delta/x) = \frac{1}{\sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{\Delta/x - a(\Delta/x)_{\text{mip}}}{\xi} \right)^2 \right] + e^{-\left( \frac{\Delta/x - a(\Delta/x)_{\text{mip}}}{\xi} \right)}$$

for full form  
see e.g. Leo

$\xi$ : material constant

# MEAN PARTICLE RANGE

Integrate over energy loss  
from E down to 0

$$R = \int_E^0 \frac{dE}{dE/dx}$$

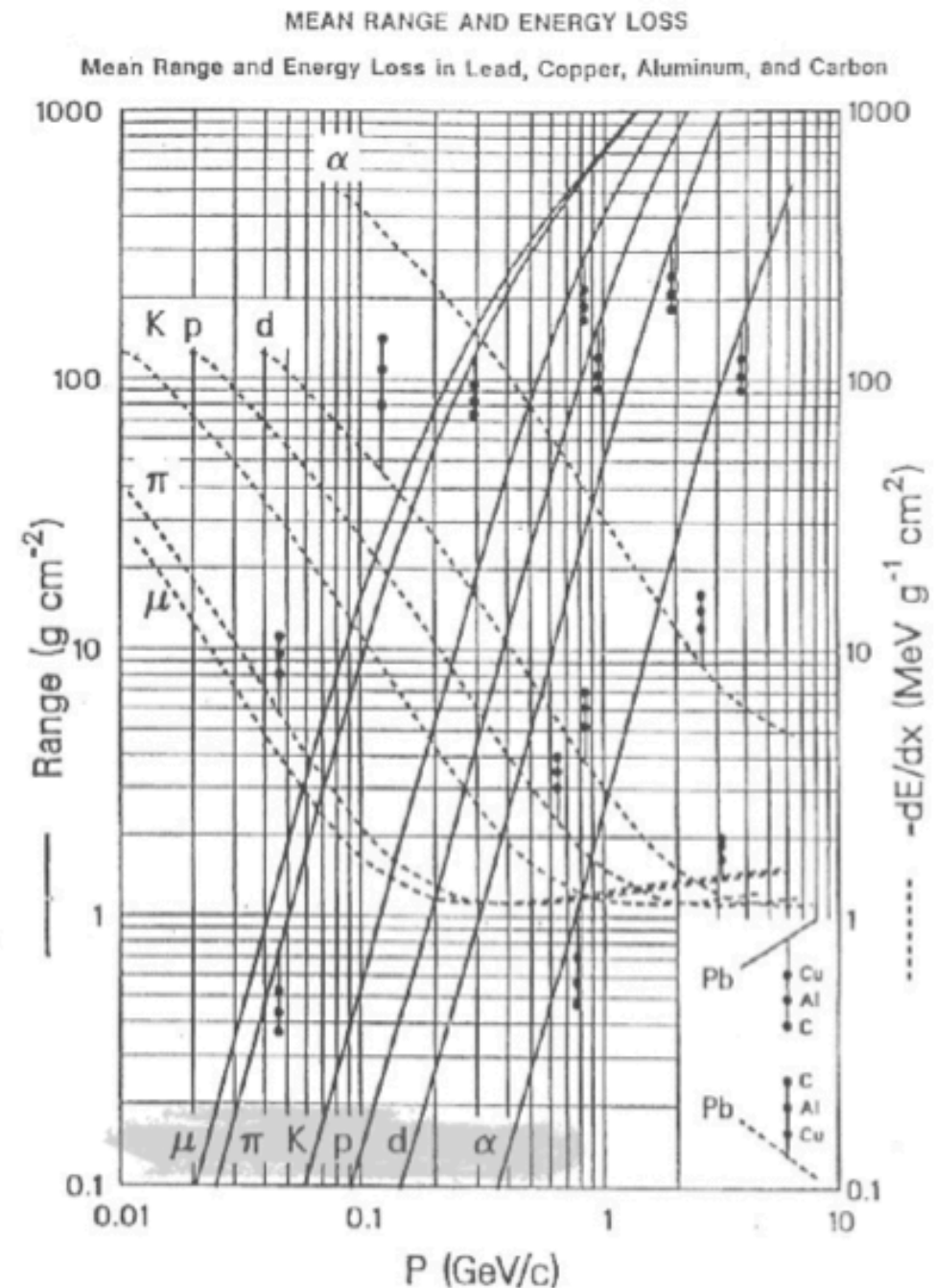
Example:

Proton with  $p = 1$  GeV

Target: lead with  $\rho = 11.34$  g/cm<sup>3</sup>

$$R/M = 200 \text{ g cm}^{-2} \text{ GeV}^{-1}$$

$$\rightarrow R = 200/11.34/1 \text{ cm} \sim 20 \text{ cm}$$



# ENERGY LOSS of ELECTRONS

Bethe-Bloch formula needs modification

Incident and target electron have same mass  $m_e$   
Scattering of identical, indistinguishable particles

$$-\left\langle \frac{dE}{dx} \right\rangle_{\text{el.}} = K \frac{Z}{A} \frac{1}{\beta^2} \left[ \ln \frac{m_e \beta^2 c^2 \gamma^2 T}{2I^2} + F(\gamma) \right]$$

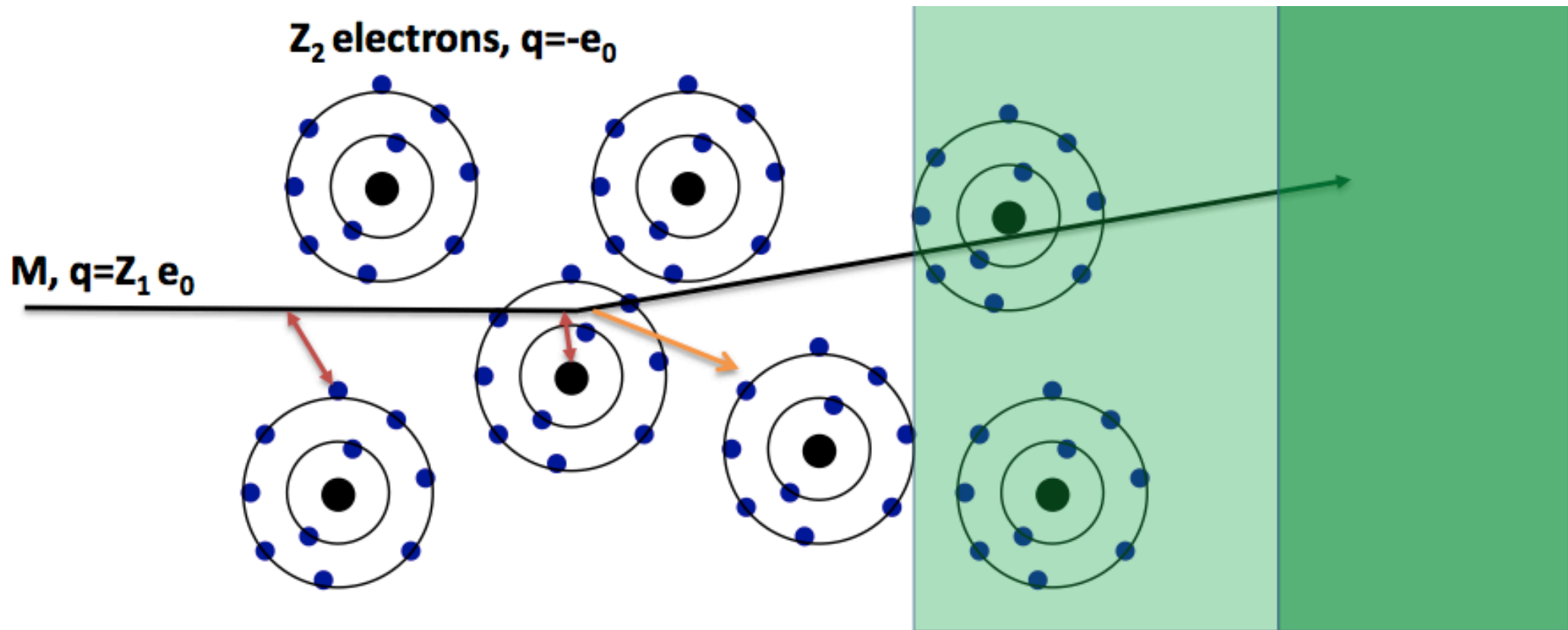
[T: kinetic energy of electron]

$$W_{\text{max}} = \frac{1}{2}T$$

Remark: different energy loss for electrons and positrons at low energy as positrons are not identical with electrons; different treatment ...

# ELECTROMAGNETIC INTERACTION

## PARTICLE - MATTER



### Interaction with the atomic electrons.

The incoming particle loses energy and the atoms are **excited** or **ionised**.

### Interaction with the atomic nucleus.

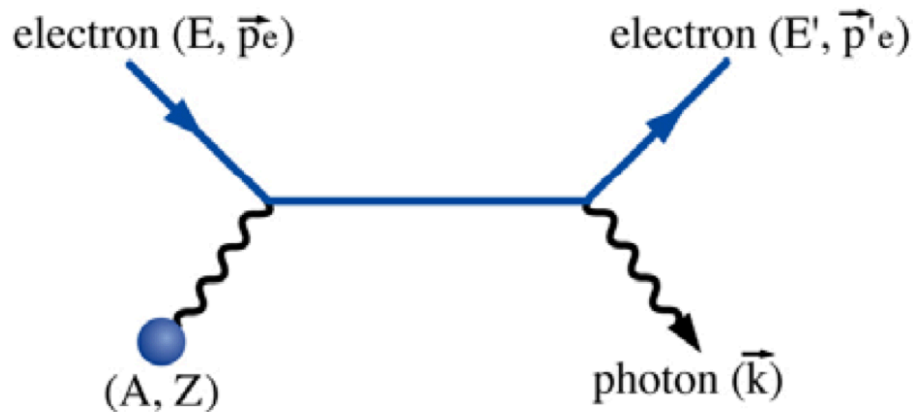
The incoming particle is deflected causing **multiple scattering** of the particle in the material.

During this scattering a **Bremsstrahlung photon** can be emitted

In case the particle's velocity is larger than the velocity of light in the medium, the resulting EM shockwave manifests itself as **Cherenkov radiation**. When the particle crosses the boundary between two media, there is a probability of 1% to produce an Xray photon called **Transition radiation**.

# BREMSSTRAHLUNG

Real photon emission in the electromagnetic field of the atomic nucleus



Electric field of the nucleus + of the electrons  $Z(Z+1)$

At large radius, electrons screen the nucleus  $\ln(183Z^{-1/3})$

$$d\sigma/dk = 4 \alpha Z(Z+1)r_e^2 \ln(183Z^{-1/3})(4/3 - 4/3y + y^2)/k$$

[D.F.]

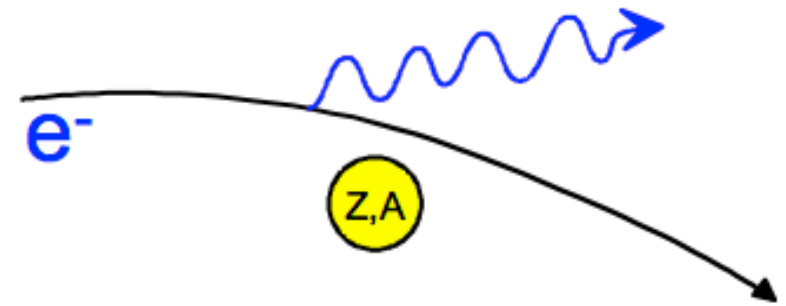
where  $y=k/E$  and  $r_e = \frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{m_e c^2} = 2.818 \cdot 10^{-15} \text{ m}$  classical radius of the electron.

For a given E, the average energy lost by radiation,  $dE$ , is obtained by integrating over  $y$ .



# BREMSSTRAHLUNG & RADIATION LENGTH

Bremsstrahlung arises if particles are accelerated in Coulomb field of nucleus



$$\frac{dE}{dx} = 4\alpha N_A \frac{z^2 Z^2}{A} \left( \frac{1}{4\pi\epsilon_0} \frac{e^2}{mc^2} \right)^2 E \ln \frac{183}{Z^{\frac{1}{3}}} \propto \frac{E}{m^2}$$

i.e. energy loss proportional to  $1/m^2 \rightarrow$  **main relevance for electrons ...**

**... or ultra-relativistic muons**

Consider electrons:

$$\frac{dE}{dx} = 4\alpha N_A \frac{Z^2}{A} r_e^2 \cdot E \ln \frac{183}{Z^{\frac{1}{3}}}$$

$$\frac{dE}{dx} = \frac{E}{X_0} \quad \text{with} \quad X_0 = \frac{A}{4\alpha N_A Z^2 r_e^2 \ln \frac{183}{Z^{\frac{1}{3}}}}$$

[Radiation length in g/cm<sup>2</sup>]

$$\Rightarrow E = E_0 e^{-x/X_0}$$

After passage of one  $X_0$  electron has lost all but  $(1/e)^{\text{th}}$  of its energy

[i.e. 63%]

# RADIATION LENGTH

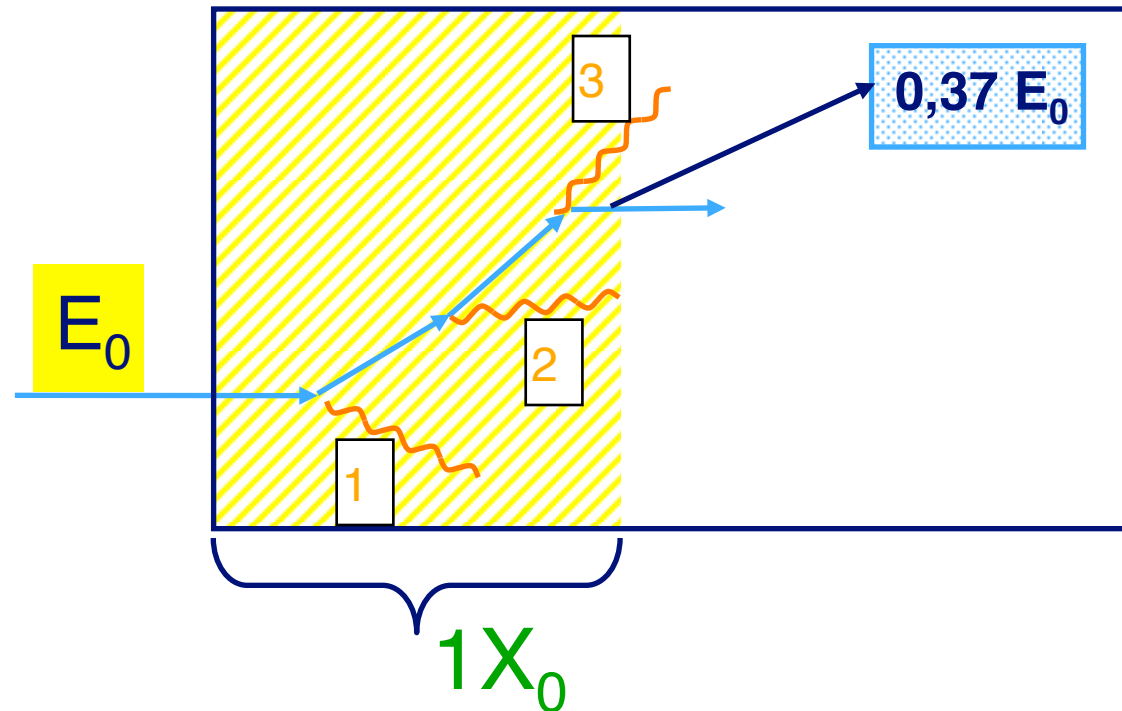
The radiation length is a “universal” distance, very useful to describe electromagnetic showers (electrons & photons)

$X_0$  is the distance after which the incident electron has radiated  $(1-1/e)$  63% of its incident energy

$$dE/dx = E/X_0$$

$$dE/E = dx/X_0$$

$$E = E_0 e^{-x/X_0}$$



|            | Air   | Eau | Al  | LAr | Fe   | Pb   | PbWO <sub>4</sub> | LAr/Pb |
|------------|-------|-----|-----|-----|------|------|-------------------|--------|
| Z          | -     | -   | 13  | 18  | 26   | 82   | -                 | -      |
| $X_0$ (cm) | 30420 | 36  | 8,9 | 14  | 1,76 | 0.56 | 0.89              | 1.9    |

# RADIATION LENGTH

## Approximation

( $X_0$  in cm: divide by  $\rho$  [g/cm<sup>3</sup>])

$$X_0 \approx \frac{180A}{Z^2} \text{ g.cm}^{-2}$$

Energy loss by radiation

$$\langle E(x) \rangle = E_0 e^{-\frac{x}{X_0}}$$

$\gamma$  Absorption ( $e^+ e^-$  pair creation)

$$\langle I(x) \rangle = I_0 e^{-\frac{7}{9} \frac{x}{X_0}}$$

For compound material

$w_j$  being the relative density

$$1/X_0 = \sum w_j / X_j$$

# CRITICAL ENERGY

Critical energy:

$$\left. \frac{dE}{dx}(E_c) \right|_{\text{Brems}} = \left. \frac{dE}{dx}(E_c) \right|_{\text{Ion}}$$

Approximation:

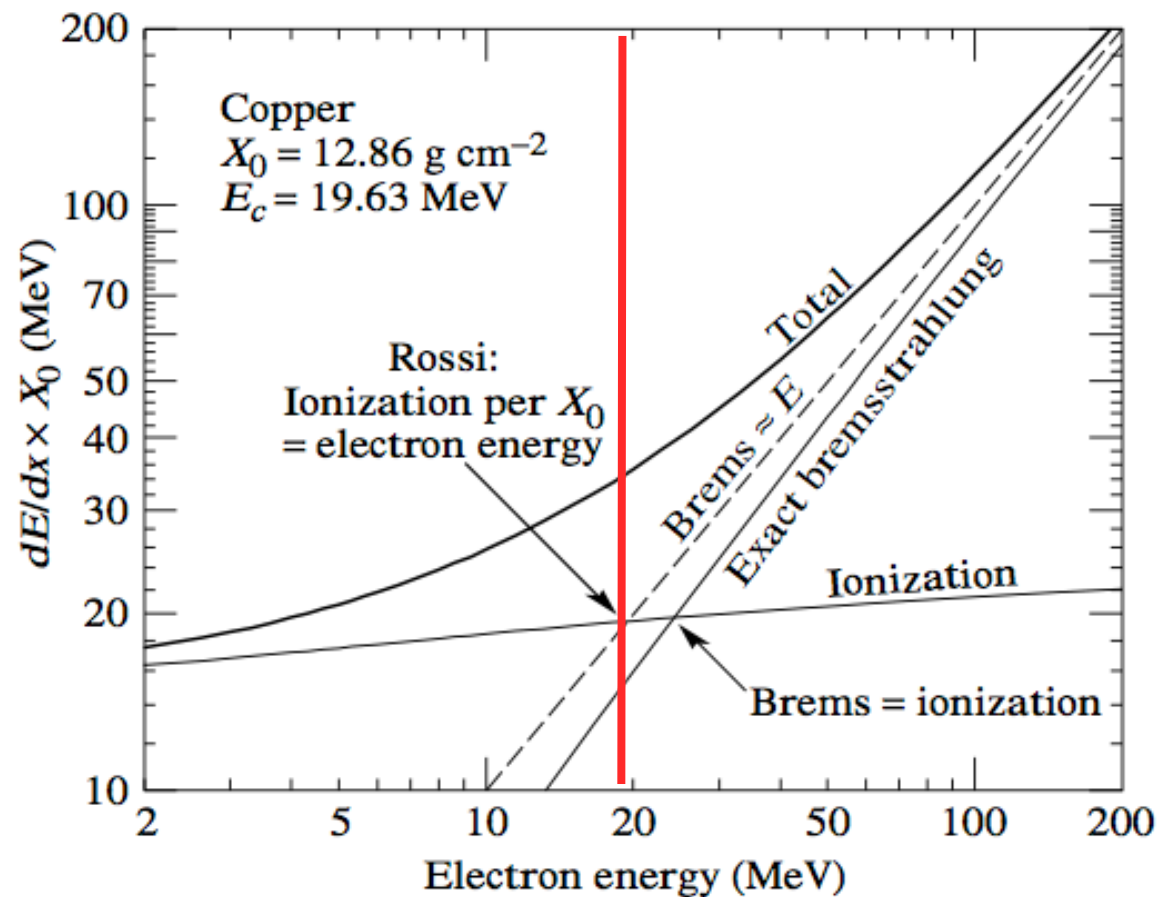
$$E_c^{\text{Gas}} = \frac{710 \text{ MeV}}{Z + 0.92}$$

$$E_c^{\text{Sol/Liq}} = \frac{610 \text{ MeV}}{Z + 1.24}$$

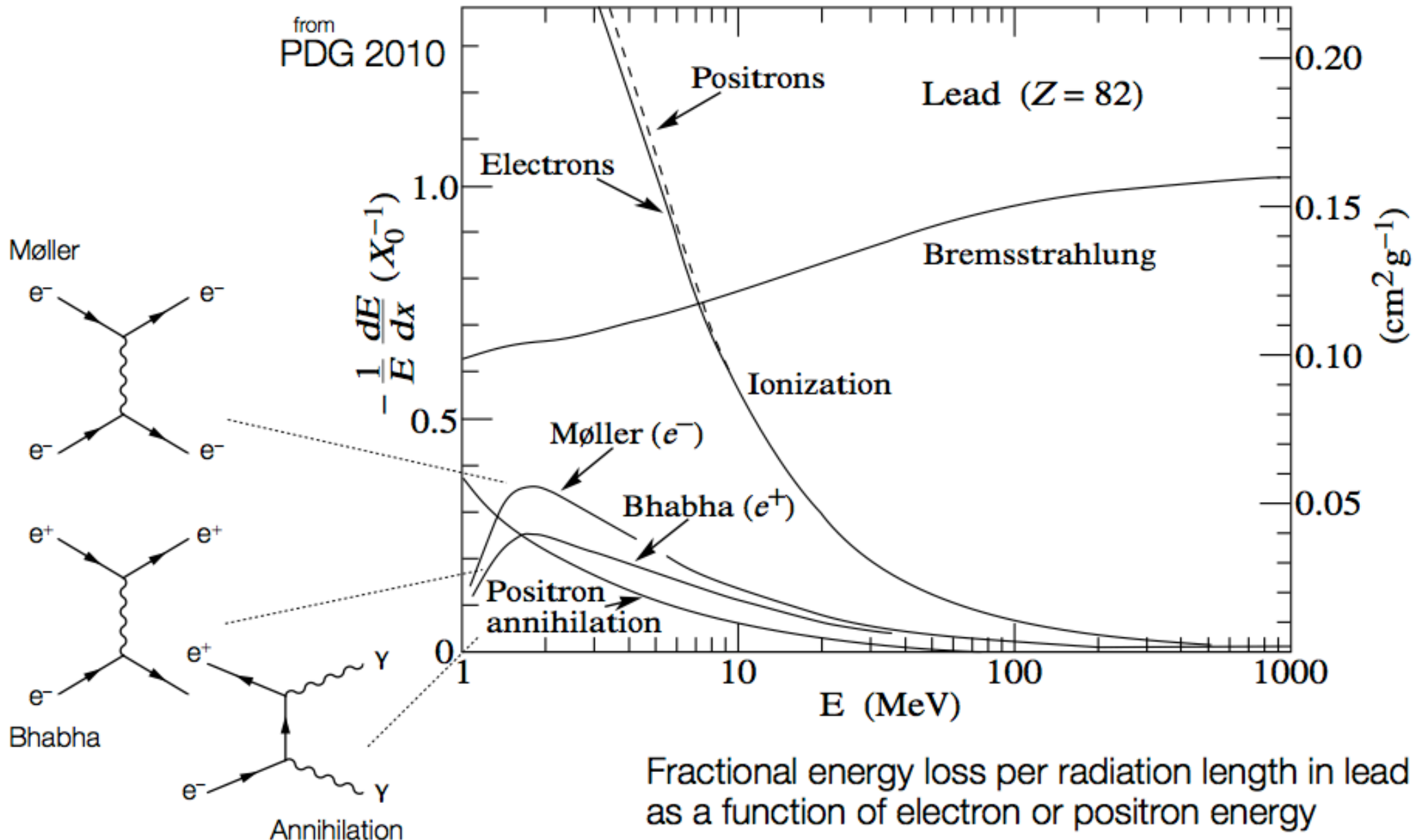
Example Copper:

$$E_c \approx 610/30 \text{ MeV} \approx 20 \text{ MeV}$$

$$\left( \frac{dE}{dx} \right)_{\text{Tot}} = \left( \frac{dE}{dx} \right)_{\text{Ion}} + \left( \frac{dE}{dx} \right)_{\text{Brems}}$$

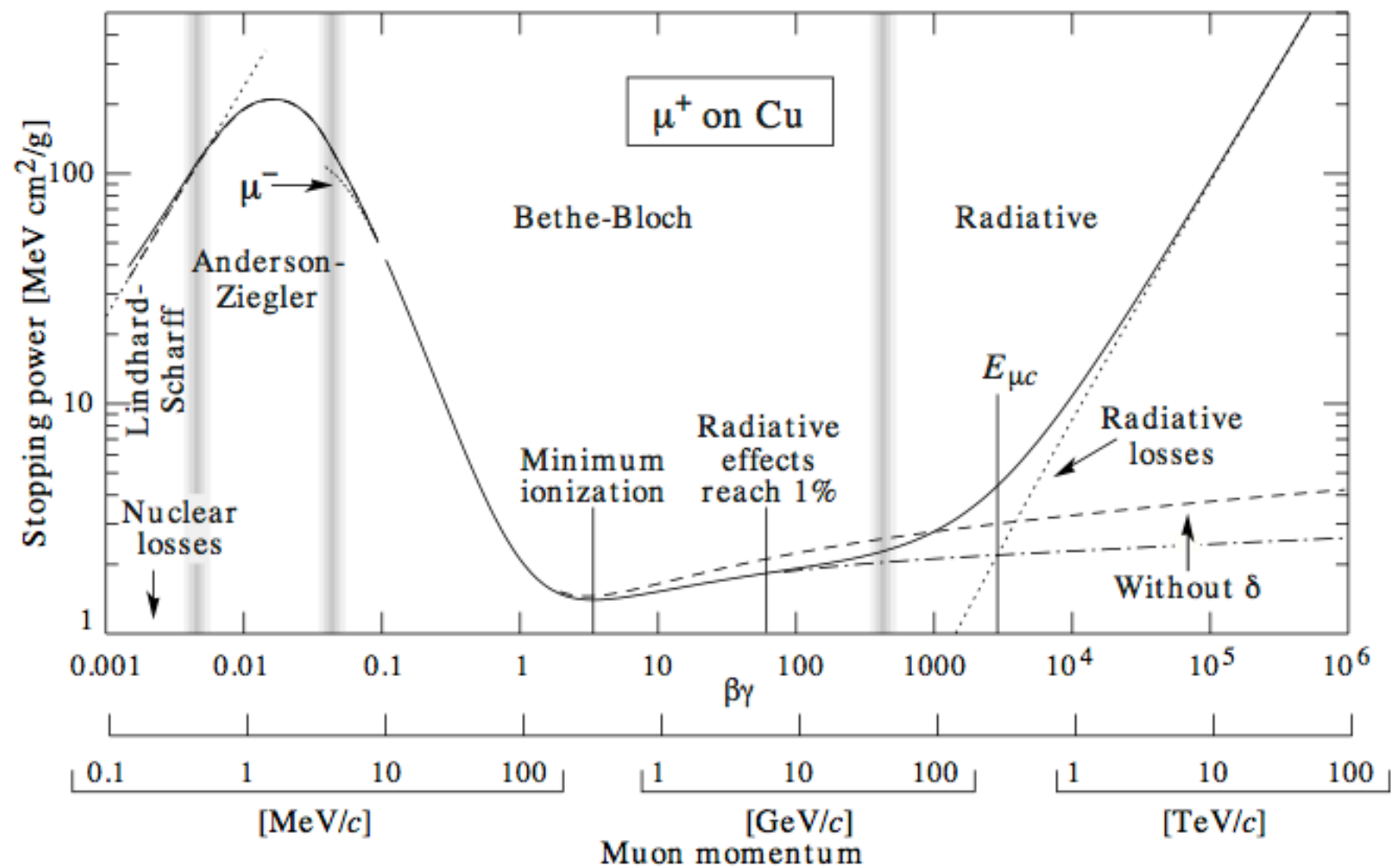


# TOTAL ENERGY LOSS FOR ELECTRONS





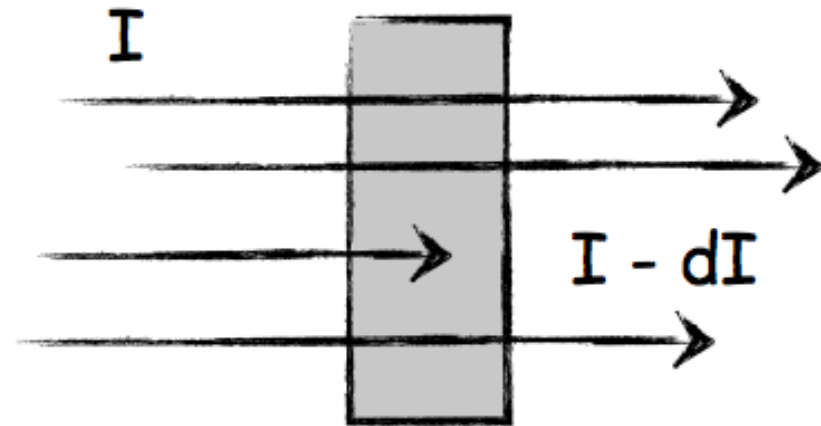
# $\mu^+$ in COPPER



# INTERACTION OF PHOTONS WITH MATTER

Characteristic for interactions of photons with matter:

A single interaction removes photon from beam !



Possible Interactions

Photoelectric Effect  
Compton Scattering  
Pair Production

Rayleigh Scattering ( $\gamma A \rightarrow \gamma A$ ;  $A$  = atom; coherent)  
Thomson Scattering ( $\gamma e \rightarrow \gamma e$ ; elastic scattering)  
Photo Nuclear Absorption ( $\gamma K \rightarrow pK/nK$ )  
Nuclear Resonance Scattering ( $\gamma K \rightarrow K^* \rightarrow \gamma K$ )  
Delbruck Scattering ( $\gamma K \rightarrow \gamma K$ )  
Hadron Pair production ( $\gamma K \rightarrow h^+h^-K$ )

$$dI = -\mu I dx$$

[  $\mu$ : absorption coefficient ]

depends on  
 $E, Z, \rho$

→ Beer-Lambert law:

$$I(x) = I_0 e^{-\mu x}$$

with  $\lambda = 1/\mu = 1/n\sigma$   
[ mean free path ]

# PHOTO-ELECTRIC EFFECT

Energy of  
outgoing electron:

$$E_e = h\nu - I_b$$

Photon energy

Binding energy  
[strongly Z dependent]

Typical energy dependence:

$$\sigma_{\text{ph}} = 2\pi r_e^2 \alpha^4 Z^5 (mc^2)/E_\gamma$$

[for  $E_\gamma \gg mc^2$ ]

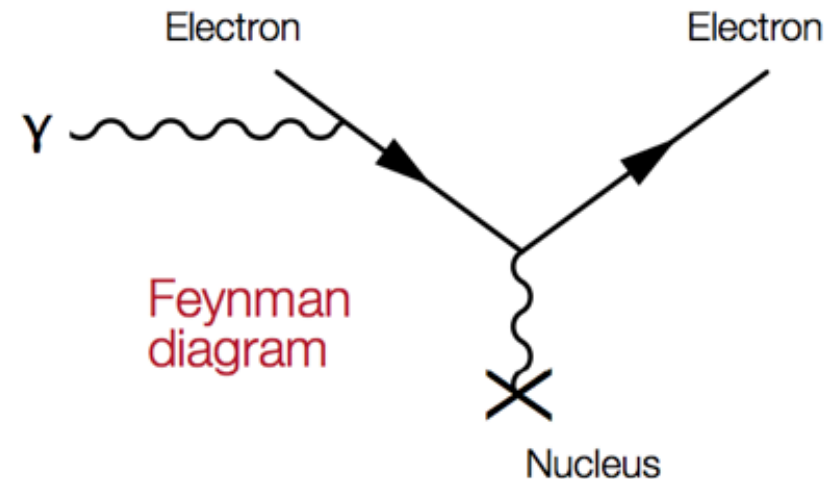
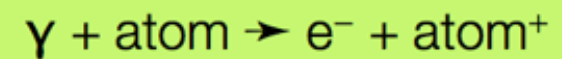
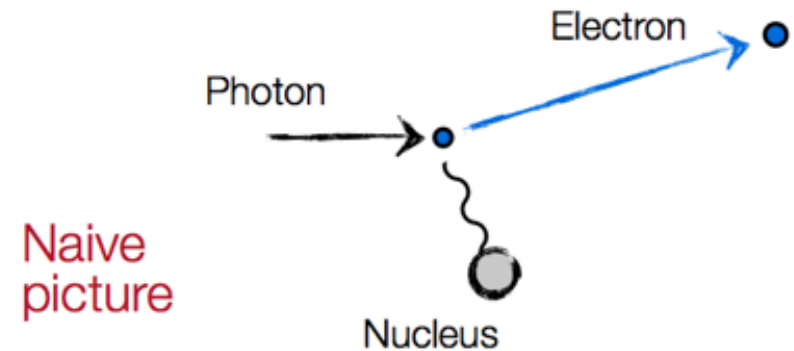
$$\sigma_{\text{ph}} = \alpha\pi a_B Z^5 (I_0/E_\gamma)^{7/2}$$

[for  $I_0 \ll E_\gamma \ll mc^2$ ]

Example values:

$a_B = 0.53 \cdot 10^{-10} \text{ m}$ ;  $I_0 = 13.6 \text{ eV}$ ;  $\alpha = 1/137$ ;  $1 \text{ b} = 10^{-28} \text{ m}^2$   
use  $E_\gamma = 100 \text{ keV}$

$$\begin{aligned} \rightarrow \sigma_{\text{ph}}(\text{Fe}) &= 29 \text{ barn} \\ \sigma_{\text{ph}}(\text{Pb}) &= 5000 \text{ barn} \end{aligned}$$



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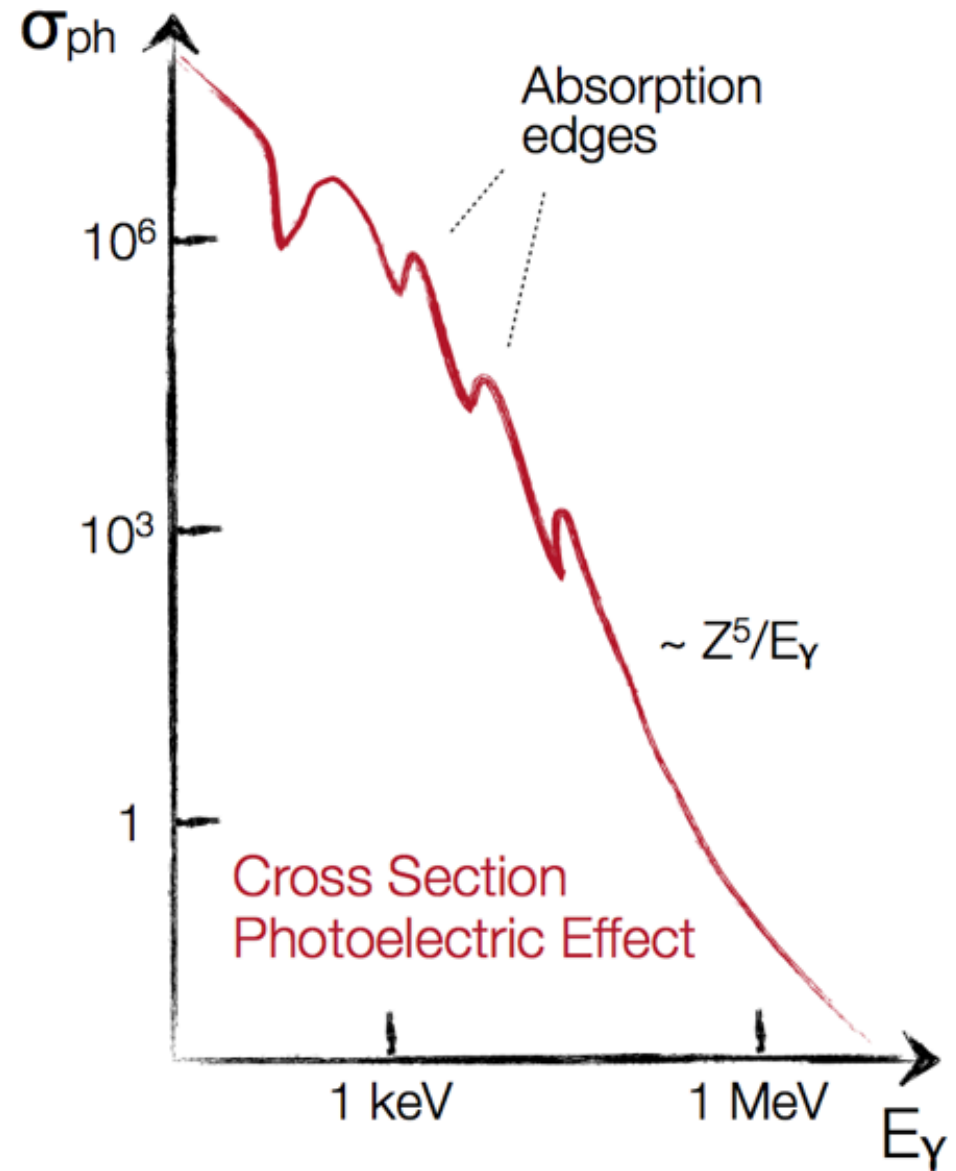
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# PAIR PRODUCTION

Cross Section:  
[for  $E_\gamma \gg m_e c^2$ ]

$$\sigma_{\text{pair}} \approx \underbrace{\frac{7}{9} \left( 4 \alpha r_e^2 Z^2 \ln \frac{183}{Z^{\frac{1}{3}}} \right)}_{A/N_A X_0}$$

[ $X_0$ : radiation length]  
[in cm or g/cm<sup>2</sup>]

Absorption coefficient:

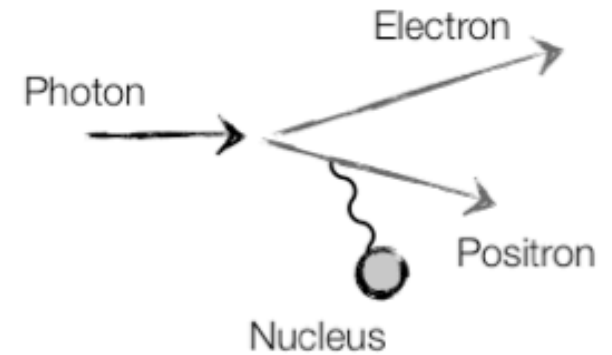
$$\mu = n\sigma \quad [\text{with } n: \text{particle density}]$$

$$\mu = \rho \cdot N_A / A \cdot \sigma_{\text{pair}}$$

$$= 7/9 \frac{1}{X_0}$$

[where now  $X_0$  is in cm]

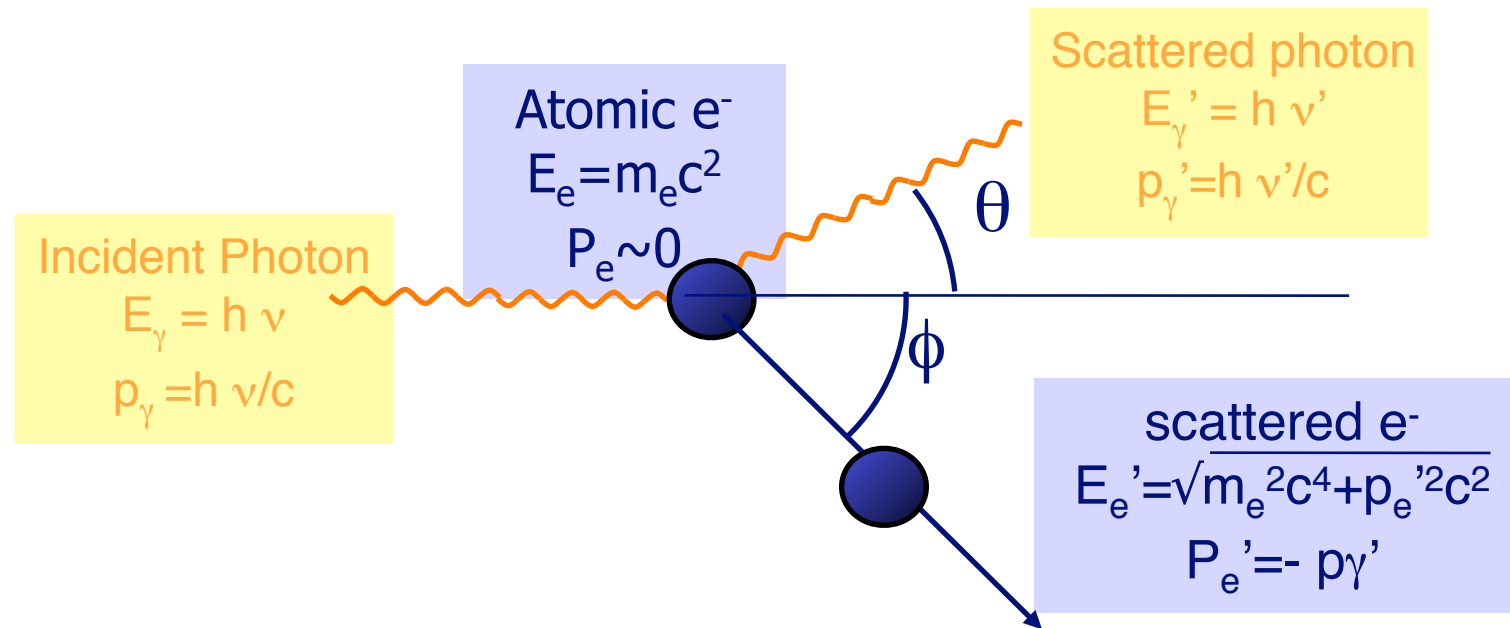
$$I(x) = I_0 e^{-\mu x}$$



|                      | $\rho$ [g/cm <sup>3</sup> ] | $X_0$ [cm]      |
|----------------------|-----------------------------|-----------------|
| H <sub>2</sub> [fl.] | 0.071                       | 865             |
| C                    | 2.27                        | 18.8            |
| Fe                   | 7.87                        | 1.76            |
| Pb                   | 11.35                       | 0.56            |
| Air                  | $1.2 \cdot 10^{-3}$         | $30 \cdot 10^3$ |



# COMPTON SCATTERING

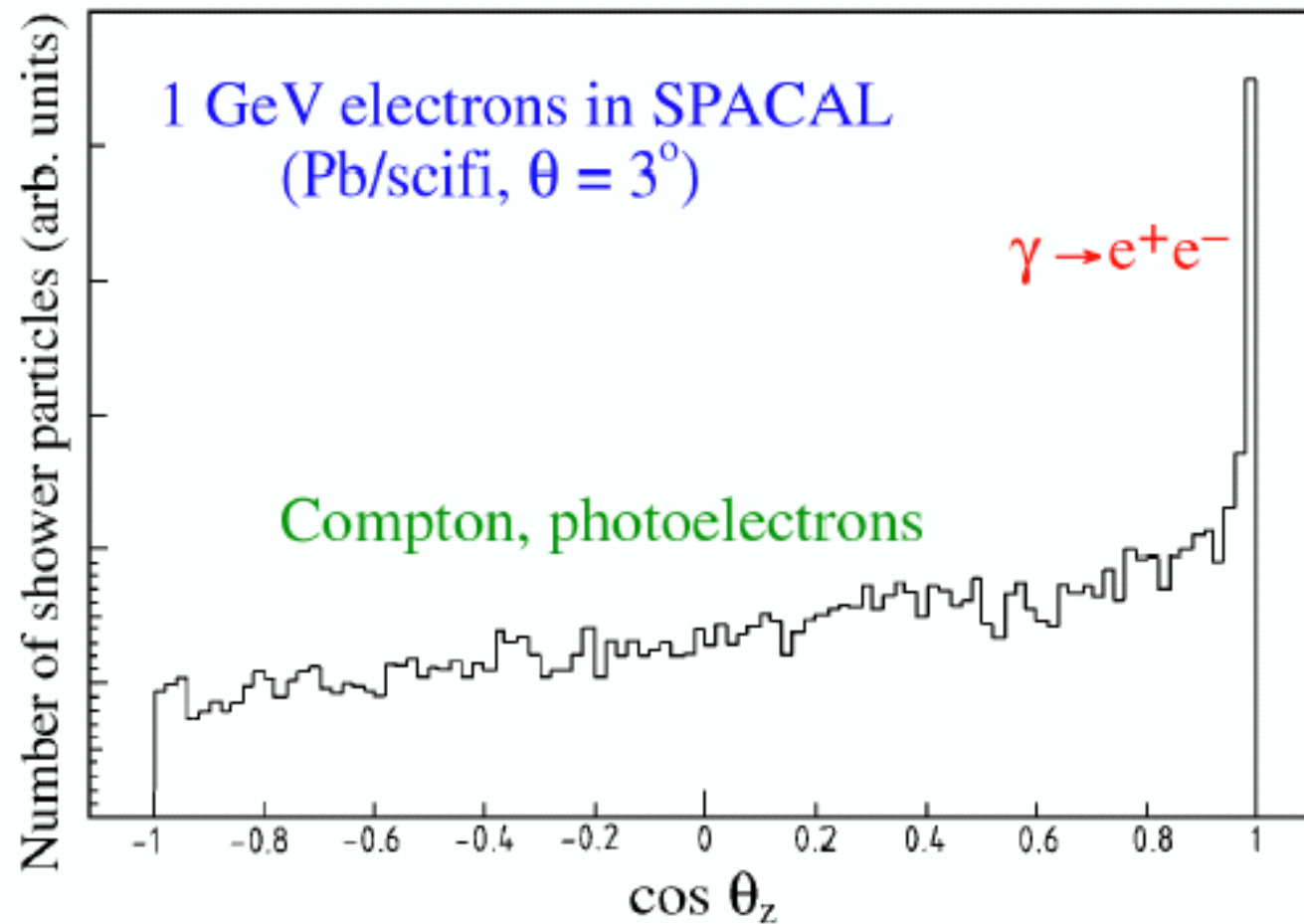


QED cross-section for  $\gamma$ -e scattering

$$\sigma_{\text{compton}} \sim Z \cdot \ln(E_\gamma) / E_\gamma$$

Process dominant at  $E_\gamma \approx 100 \text{ keV} - 5 \text{ GeV}$

# ANGULAR DISTRIBUTION



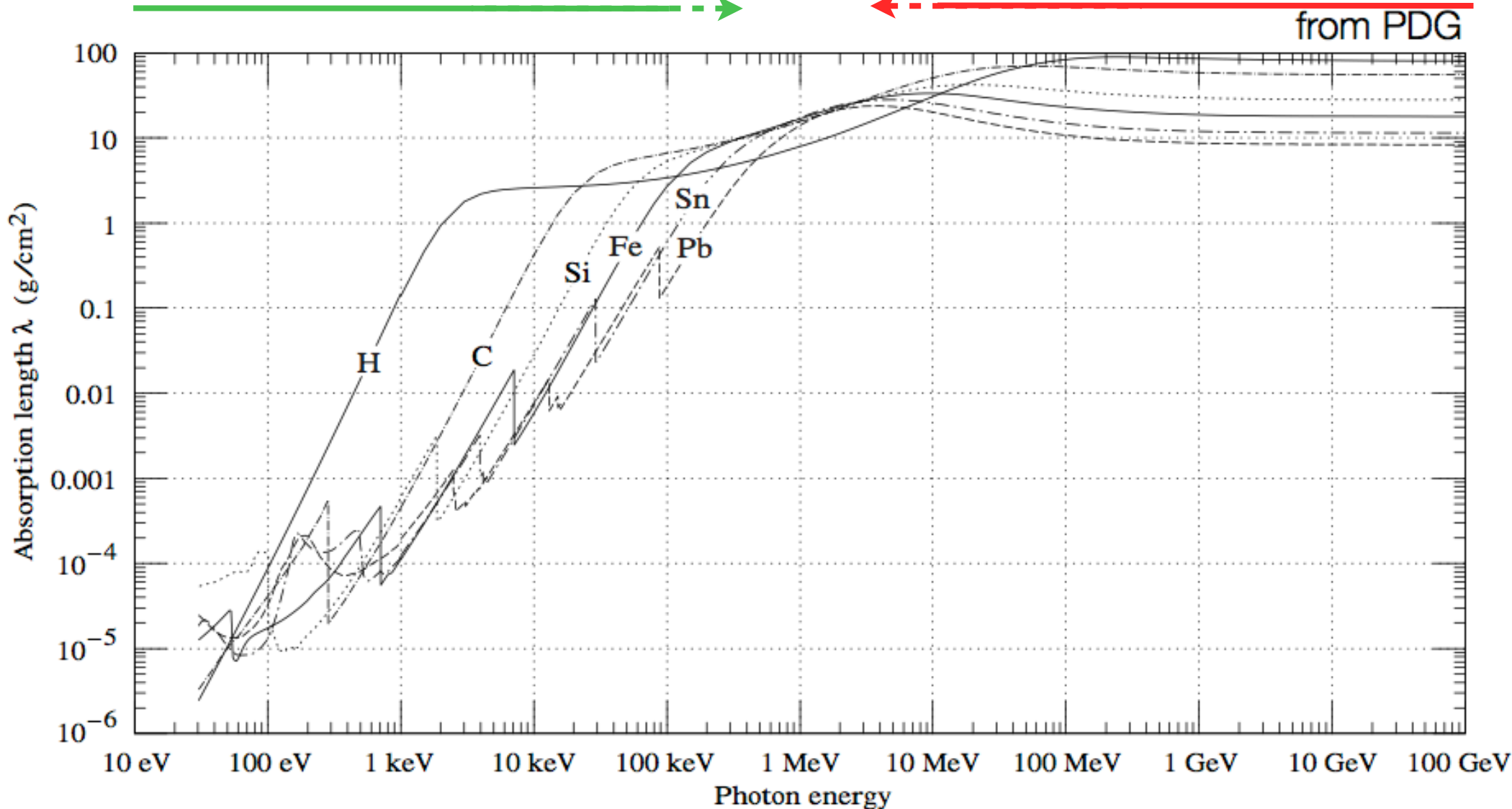
# INTERACTION OF PHOTONS WITH MATTER

Mass absorption coefficient  $\lambda = 1/(\mu/\rho)$  [g.cm<sup>2</sup>] with  $\mu = N_A \cdot \sigma / A$

$$\sigma_{Ph} \propto \frac{Z^5}{E^{3.5}}$$

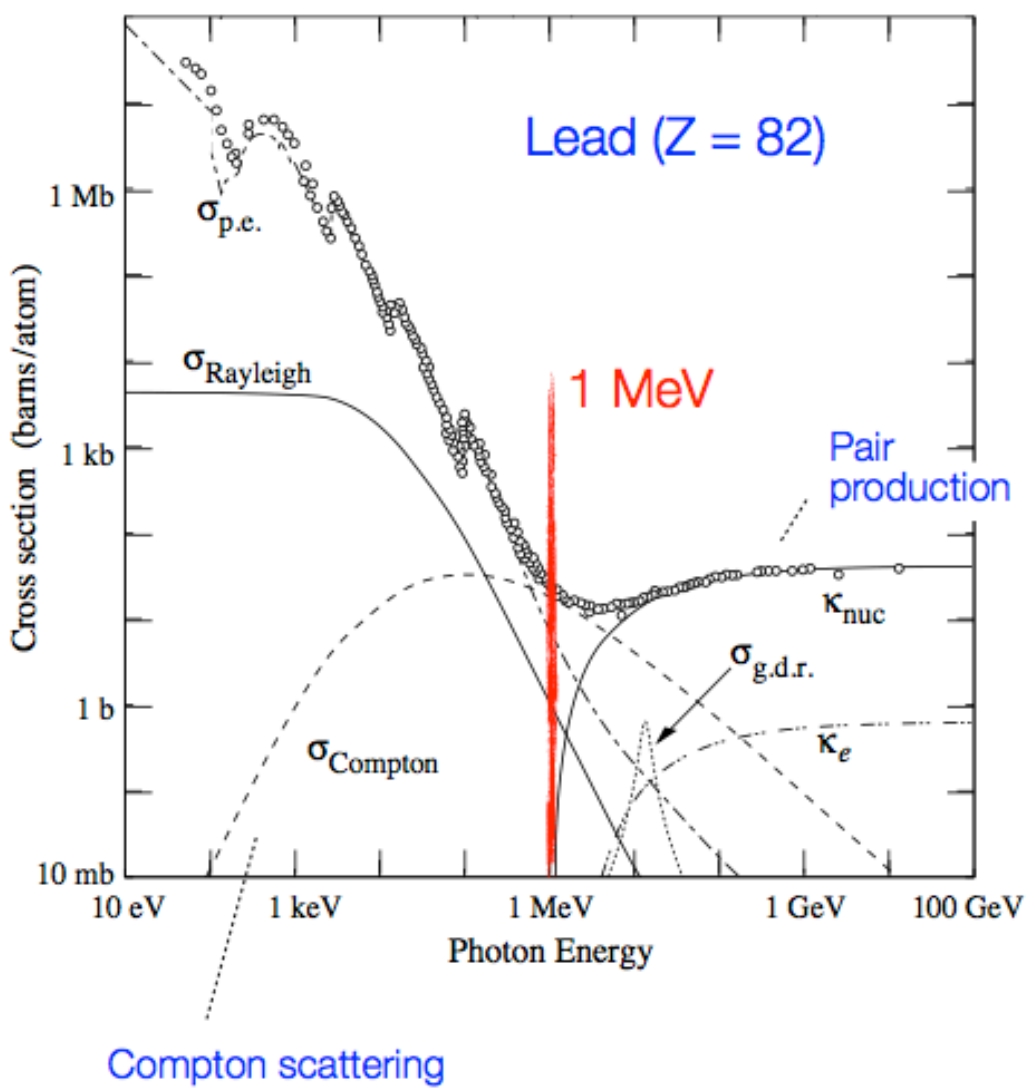
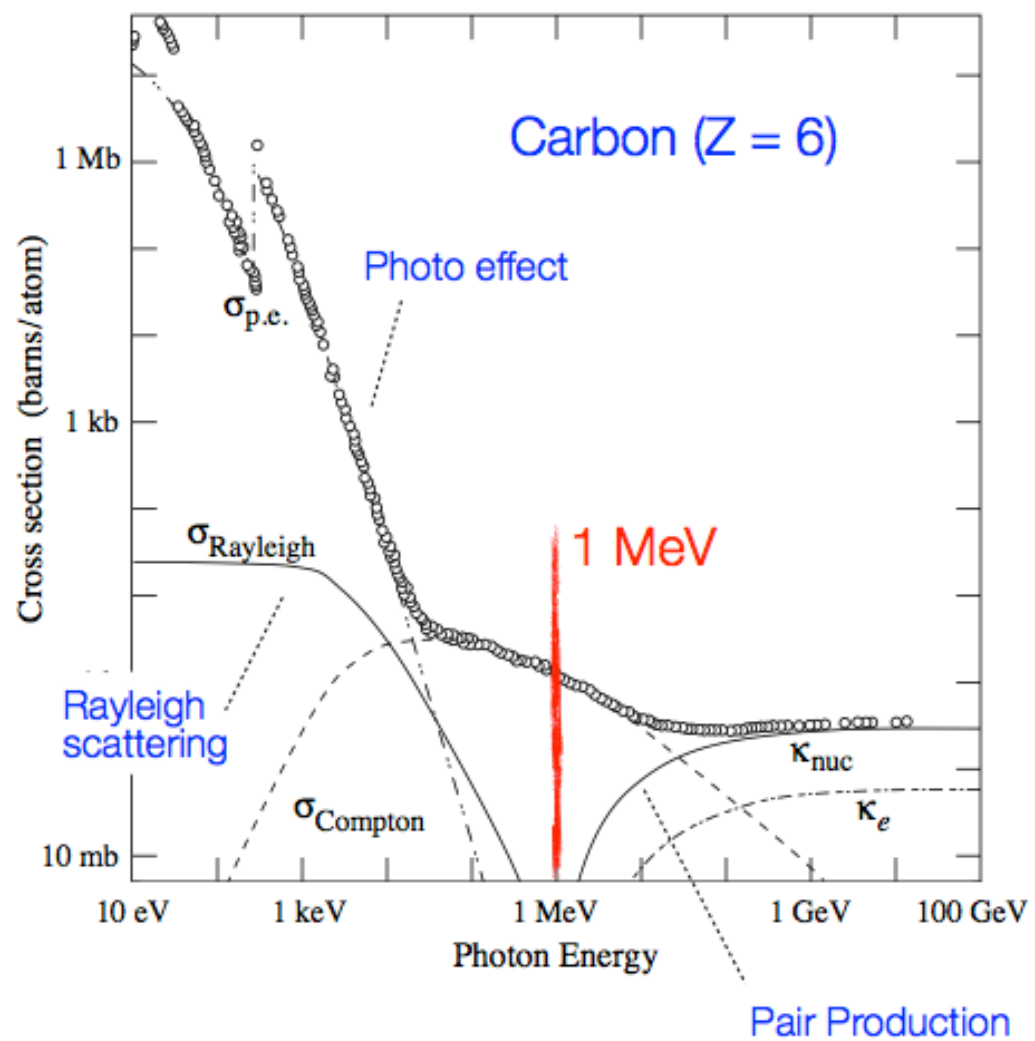
$$\sigma_{Compton} \propto \frac{\ln E}{E} \cdot Z$$

$$\sigma_{Pair} \propto Z^2$$



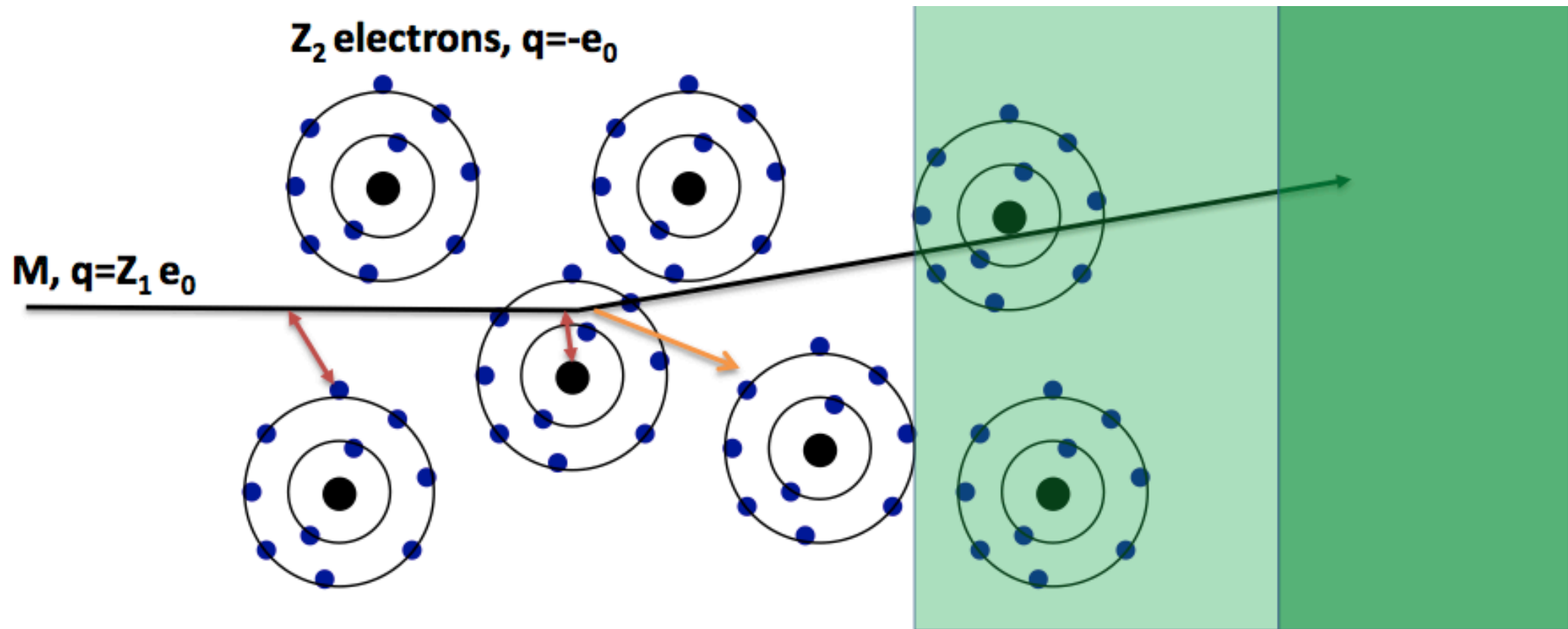
# INTERACTION OF PHOTONS WITH MATTER

Photon Total Cross Sections



# ELECTROMAGNETIC INTERACTION

## PARTICLE - MATTER



### Interaction with the atomic electrons.

The incoming particle loses energy and the atoms are **excited** or **ionised**.

### Interaction with the atomic nucleus.

The incoming particle is deflected causing **multiple scattering** of the particle in the material.

During this scattering a **Bremsstrahlung photon** can be emitted

In case the particle's velocity is larger than the velocity of light in the medium, the resulting EM shockwave manifests itself as **Cherenkov radiation**. When the particle crosses the boundary between two media, there is a probability of 1% to produce an Xray photon called **Transition radiation**.

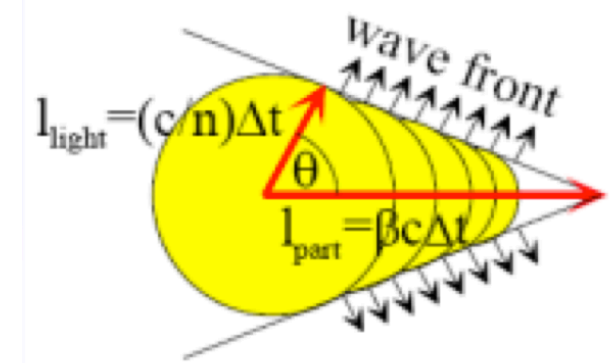
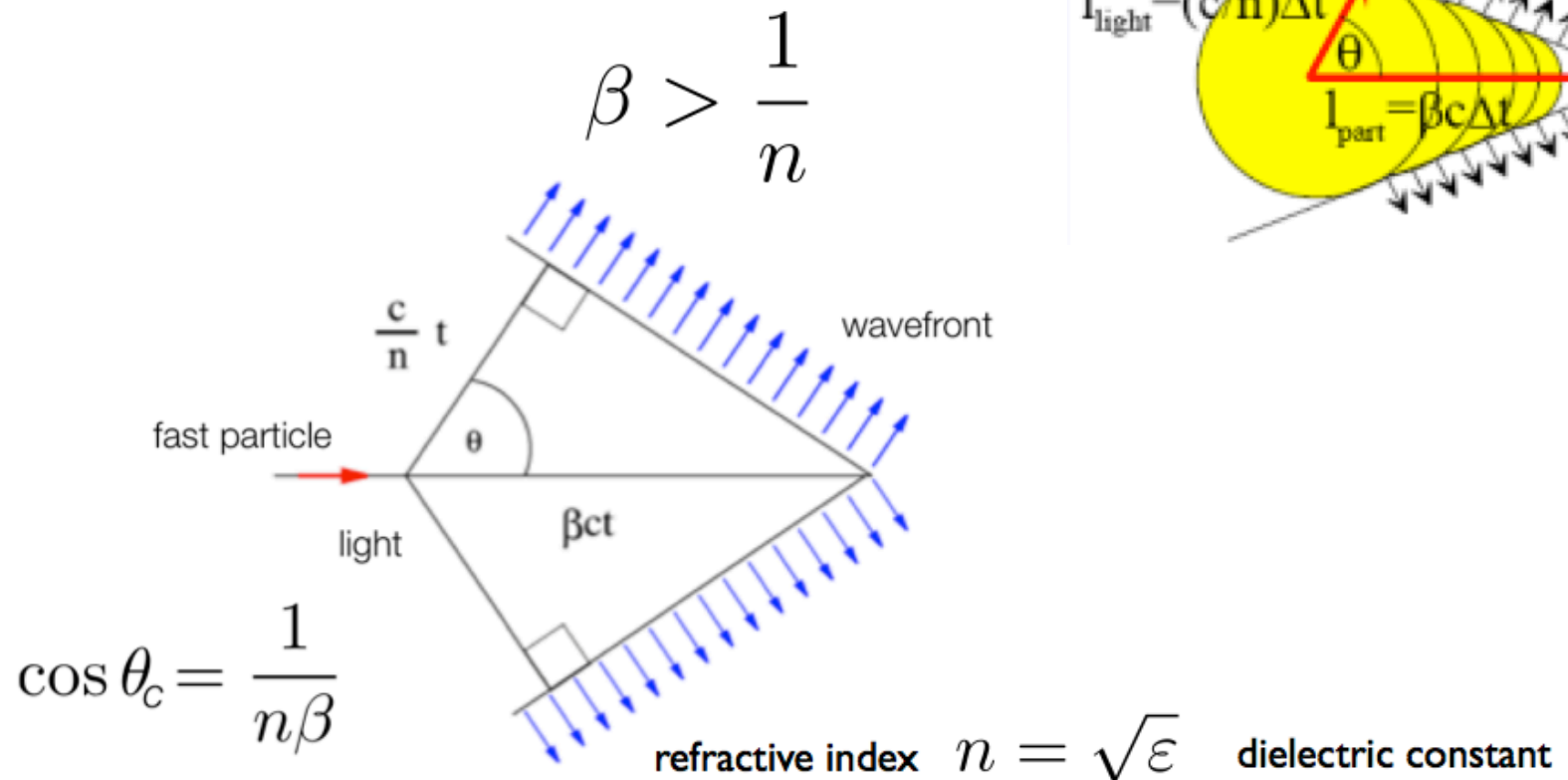


# CERENKOV RADIATION

Particles moving in a medium with **speed larger than speed of light in that medium** lose energy by emitting electromagnetic radiation

Charged particles polarise the medium generating an electrical dipole varying with time

Every point in the trajectory emits a spherical EM wave; waves constructively interfere



# CERENKOV RADIATION

Parameters of Typical Radiator

| Medium   | n        | $\beta_{thr}$ | $\theta_{max} [\beta=1]$ | $N_{ph} [eV^{-1} cm^{-1}]$ |
|----------|----------|---------------|--------------------------|----------------------------|
| Air      | 1.000283 | 0.9997        | 1.36                     | 0.208                      |
| Isobutan | 1.00127  | 0.9987        | 2.89                     | 0.941                      |
| Water    | 1.33     | 0.752         | 41.2                     | 160.8                      |
| Quartz   | 1.46     | 0.685         | 46.7                     | 196.4                      |

Note: Energy loss by Cherenkov radiation very small w.r.t. ionization (< 1%).

Example:

[Proton with  $E_{kin}=1$  GeV passing through 1 cm water]

$\beta = p/E \approx 0.875; \cos\theta_c = 1/n\beta = 0.859 \rightarrow \theta_c = 30.8^\circ$   
 $d^2N/(dE dx) = 370 \sin^2\theta_c \text{ eV}^{-1} \text{ cm}^{-1} \approx 100 \text{ eV}^{-1} \text{ cm}^{-1}$

$\rightarrow \Delta E_{loss} = \langle E \rangle d^2N/(dE dx) \Delta E \Delta x$   
 $= 2.5 \text{ eV} \cdot 100 \text{ eV}^{-1} \text{ cm}^{-1} \cdot 5 \text{ eV} \cdot 1 \text{ cm} = 1.25 \text{ keV}$

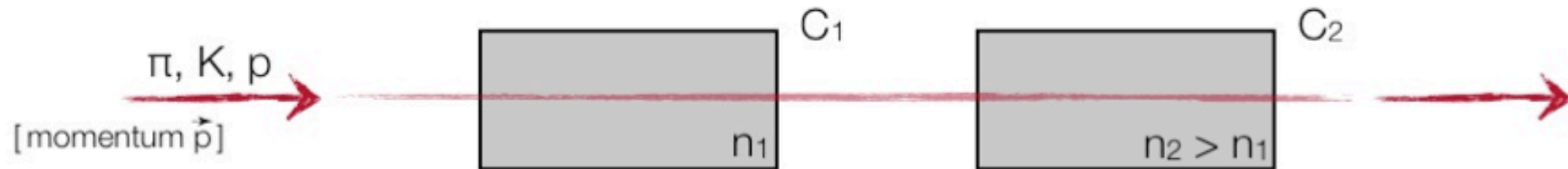
Visible light only!  
[ $E = 1 - 5 \text{ eV}; \lambda = 300 - 600 \text{ nm}$ ]

$\Delta E_{loss} < 1.25 \text{ keV}$

# IDENTIFYING PARTICLES with CERENKOV RADIATION

Threshold detection:

Observation of Cherenkov radiation  $\rightarrow \beta > \beta_{\text{thr}}$



Choose  $n_1, n_2$  in such a way that for:

$$n_2 : \quad \beta_{\pi}, \beta_K > 1/n_2 \text{ and } \beta_p < 1/n_2$$

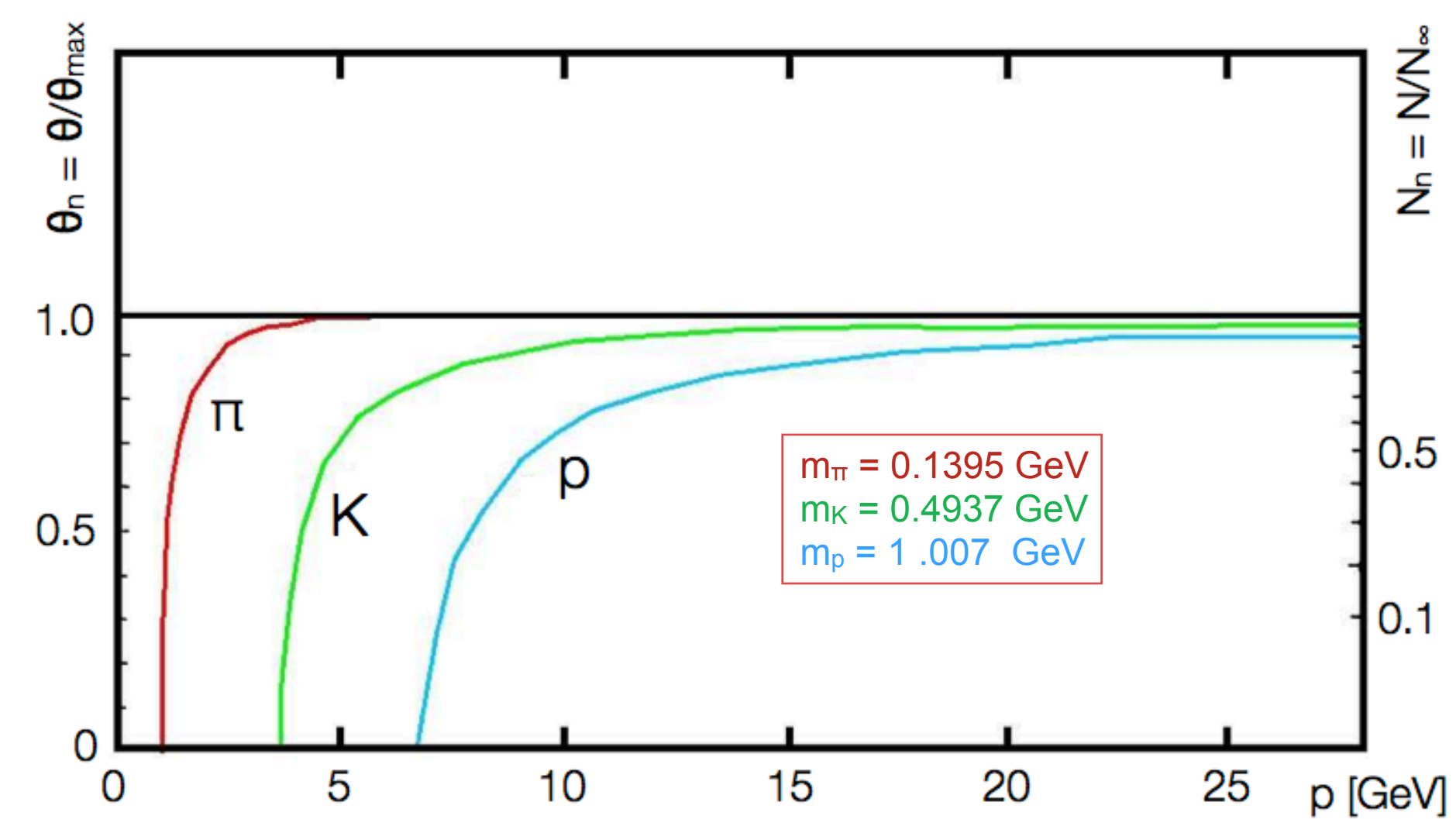
$$n_1 : \quad \beta_{\pi} > 1/n_1 \text{ and } \beta_K, \beta_p < 1/n_1$$

Light in C<sub>1</sub> and C<sub>2</sub>  $\rightarrow$  identified pion

Light in C<sub>2</sub> and not in C<sub>1</sub>  $\rightarrow$  identified kaon

Light neither in C<sub>1</sub> and C<sub>2</sub>  $\rightarrow$  identified proton

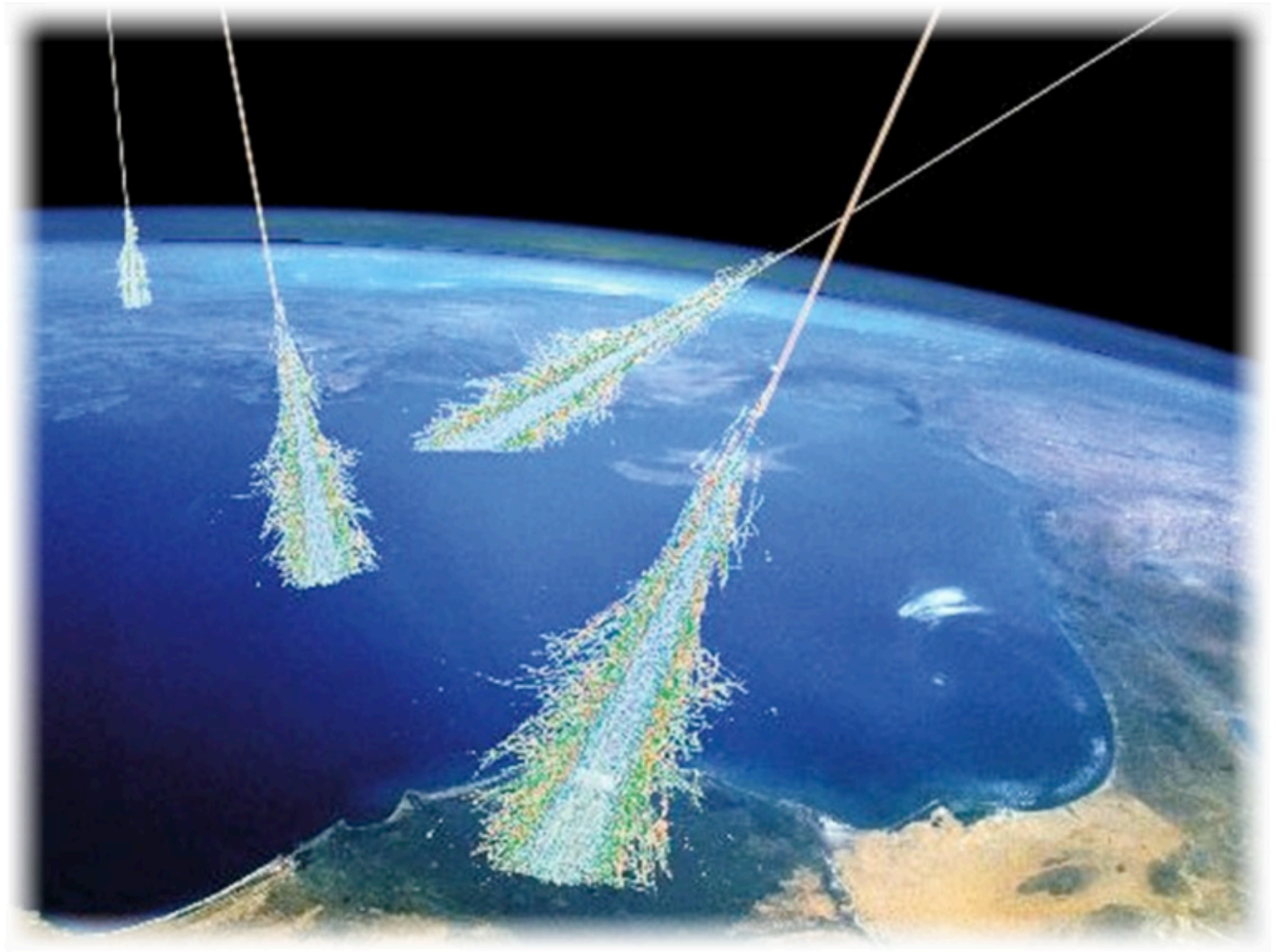
# CERENKOV RADIATION: MOMENTUM DEPENDENCE



Cherenkov angle  
Number of photons

grows with  $\beta$  and reaches asymptotic value for  $\beta = 1$   
[ $\theta_{\max} = \arccos(1/n)$ ;  $N_\infty = x \cdot 370/\text{cm} (1 - 1/n^2)$ ]

# COSMIC RAYS



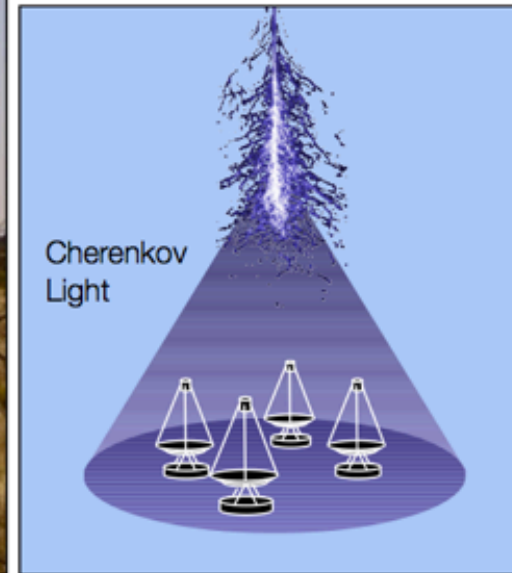


# HESS EXPERIMENT



Hess Telescopes  
Namibia

$\gamma$ -ray detection



# Transition radiation

Transition radiation occurs if a relativistic particle (**large  $\gamma$** ) passes the **boundaries between two media with different refraction indices**.

Intensity of radiation is **logarithmically proportional to  $\gamma$**

Angular distribution strongly forward peaked  
[Interference; coherence condition]

$$\theta \leq 1/\gamma$$

Coherent radiation is generated only  
over a very small formation length

$$D = \gamma c / \omega_p$$

Plasma frequency  
[from Drude model]

Volume element from which coherent  
radiation is emitted ...

$$V = \pi D \rho_{\max}^2$$

$\rho_{\max} = \gamma v / \omega$   
[transversal range ...  
... with large polarization]

Maximum energy of radiated photons  
limited by plasma frequency ...  
[results from requiring  $V \neq 0 \rightarrow \omega = \gamma \omega_p$ ]

$$E_{\max} = \gamma \hbar \omega_p$$

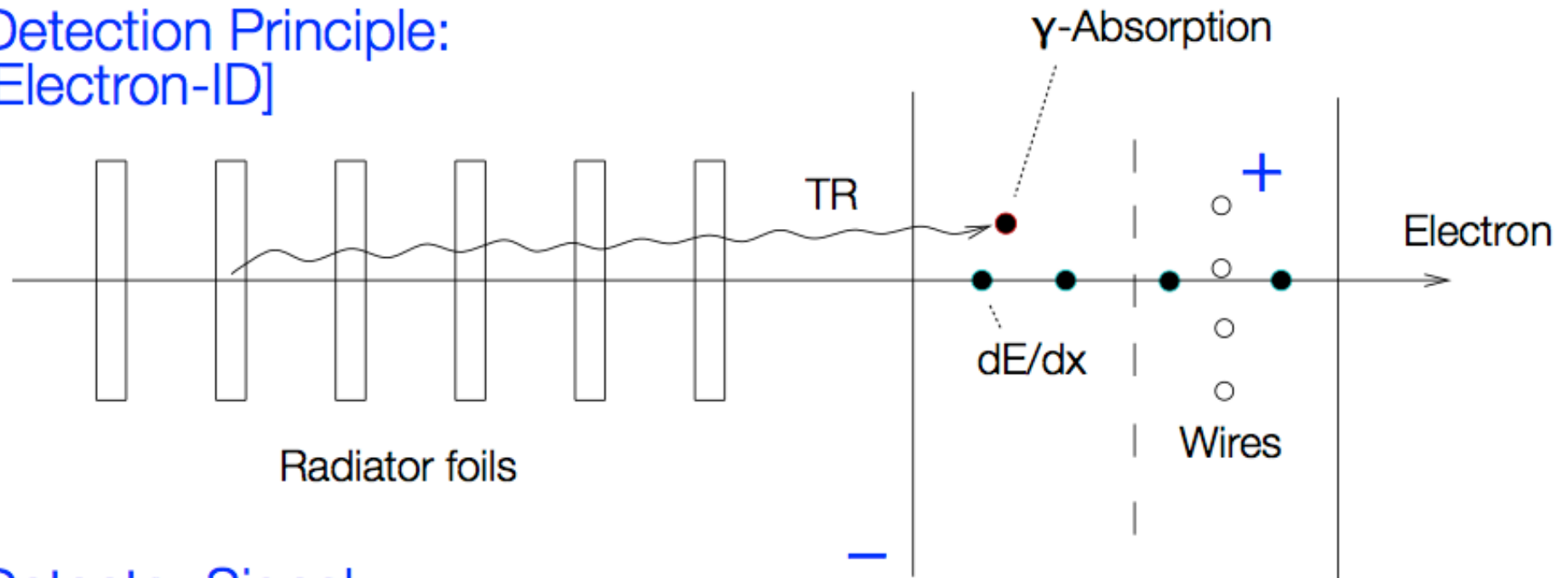
[X-Rays  $\rightarrow$  large  $\gamma$ !!]

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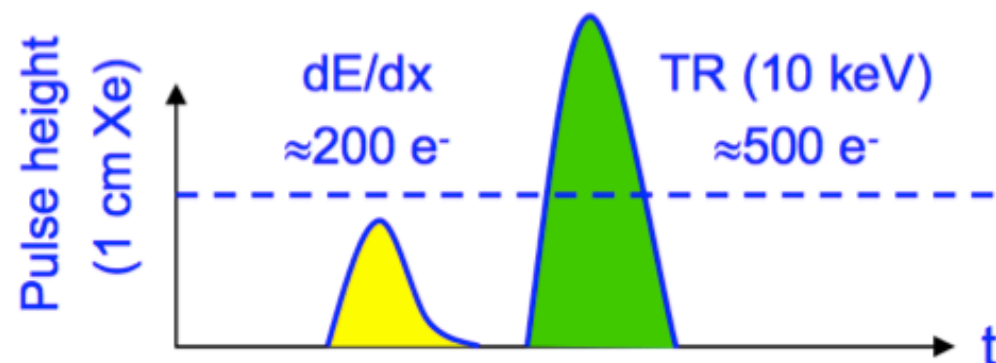
Typical values:       $\left[ \begin{array}{ll} \text{CH}_2: & \hbar \omega_p = 20 \text{ eV}; \gamma = 10^3 \\ \text{[ Air: } & \hbar \omega_p = 0.7 \text{ eV} \end{array} \right]$        $D = 10 \mu\text{m}$   
[d > D: absorption dominates]

# IDENTIFYING PARTICLES WITH TRANSITION RADIATION

## Detection Principle: [Electron-ID]



## Detector Signal:



- Detector should be sensitive to  $3 \leq E_\gamma \leq 30 \text{ keV}$ .  
✓ Gaseous detectors
- In gas  $\sigma_{\text{photo effect}} \propto Z^5$
- Gases with high  $Z$  are required  
✓ e.g. Xenon ( $Z=54$ )



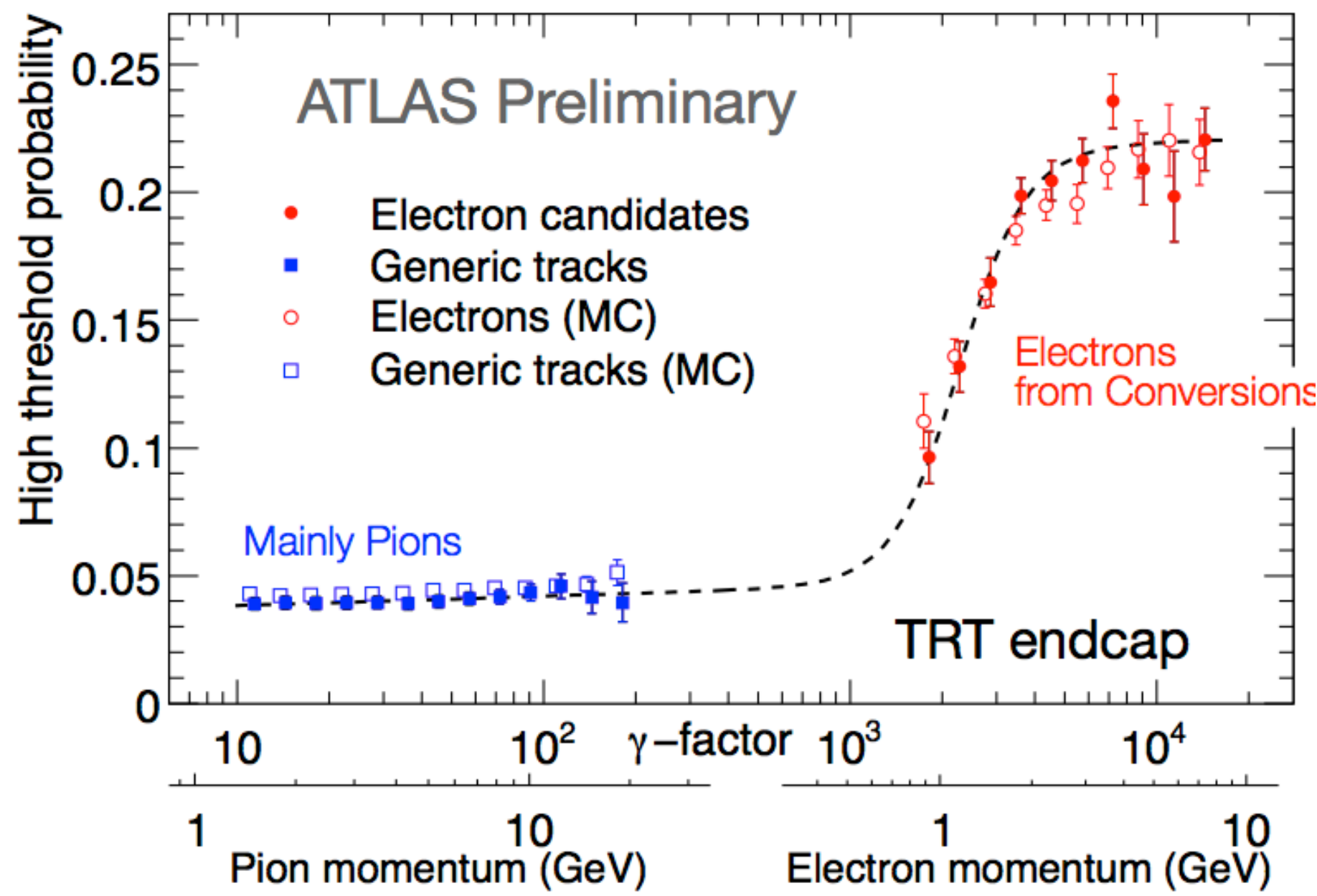
# ATLAS TRANSITION RADIATION TRACKER

Straw Tube Tracker  
with interspace filled with foam

→ Tracking & transition radiation

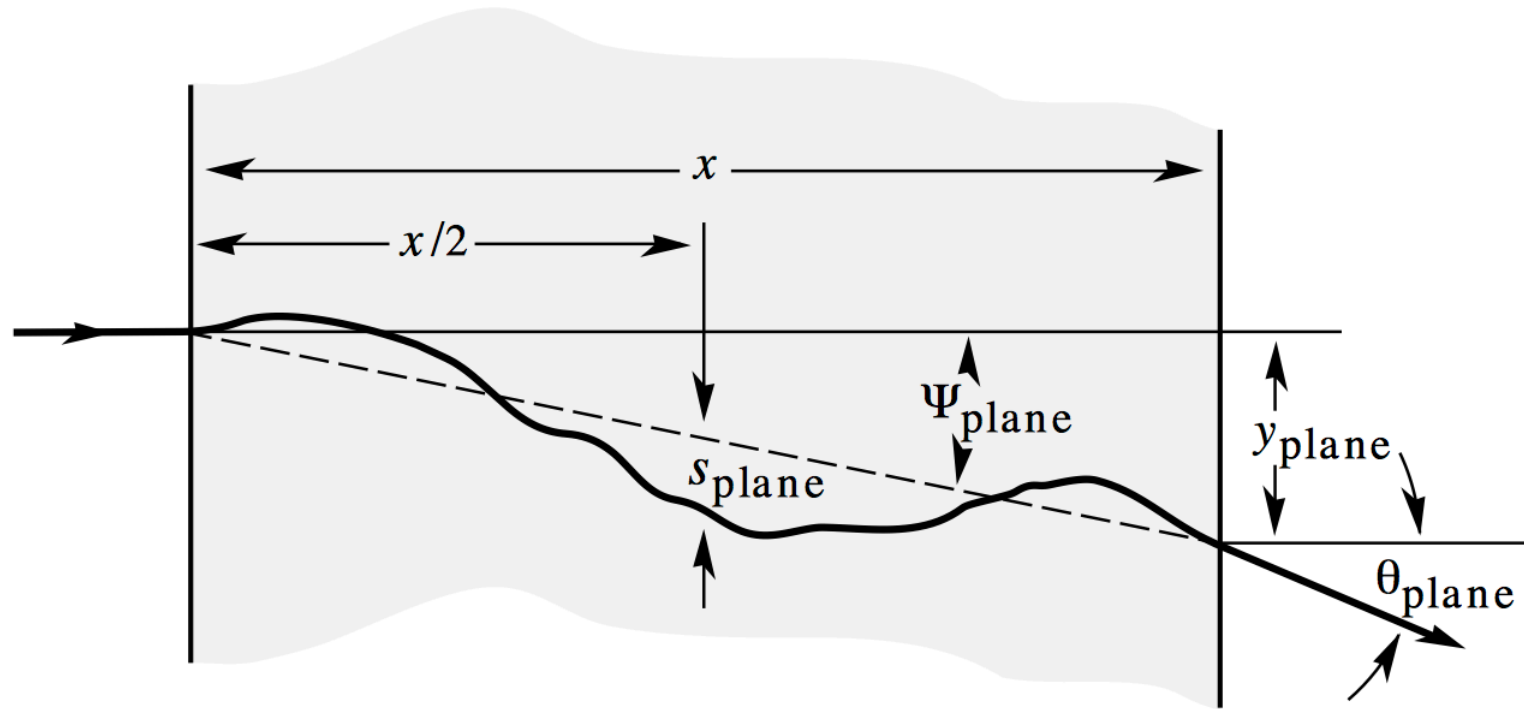


# IDENTIFYING PARTICLES WITH TRANSITION RADIATION





# MULTIPLE SCATTERING



Scattering of charged particles off the atoms in the medium causes a change of direction

The statistical sum of many such small angle scattering results in a gaussian angular distribution with a width given by

$$\theta_0 = \frac{13.6 \text{ MeV}}{\beta c p} z \sqrt{x/X_0} \left[ 1 + 0.038 \ln(x/X_0) \right]$$

## Example

$p=1 \text{ GeV}$ ,  $x=300\mu\text{m}$ ,  $\text{Si } X_0=9.4 \text{ cm} \rightarrow \theta_0=0.8 \text{ mrad}$

For a distance of 10 cm this corresponds to  $80 \mu\text{m}$ , which is significantly larger than typical resolution of Si-strip detector.

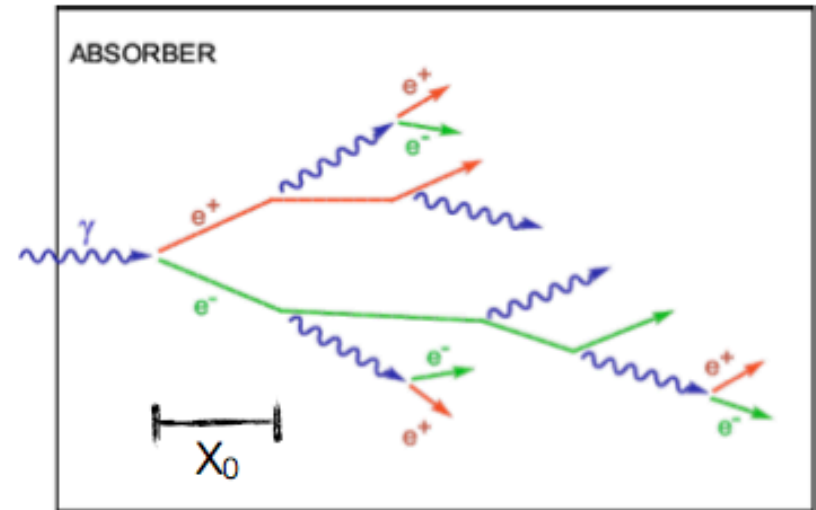
# ELECTROMAGNETIC SHOWERS

## Reminder:

Dominant processes  
at high energies ...

Photons : Pair production

Electrons : Bremsstrahlung



## Pair production:

$$\sigma_{\text{pair}} \approx \frac{7}{9} \left( 4 \alpha r_e^2 Z^2 \ln \frac{183}{Z^{\frac{1}{3}}} \right)$$

$$= \frac{7}{9} \frac{A}{N_A X_0} \quad \begin{matrix} [X_0: \text{radiation length}] \\ [\text{in cm or g/cm}^2] \end{matrix}$$

Absorption  
coefficient:

$$\mu = n\sigma = \rho \frac{N_A}{A} \cdot \sigma_{\text{pair}} = \frac{7}{9} \frac{\rho}{X_0}$$

## Bremsstrahlung:

$$\frac{dE}{dx} = 4\alpha N_A \frac{Z^2}{A} r_e^2 \cdot E \ln \frac{183}{Z^{\frac{1}{3}}} = \frac{E}{X_0}$$

$$\rightarrow E = E_0 e^{-x/X_0}$$

After passage of one  $X_0$  electron  
has only  $(1/e)^{\text{th}}$  of its primary energy ...  
[i.e. 37%]

# HADRONIC SHOWERS

Hadronic interaction:

Elastic:

$$p + \text{Nucleus} \rightarrow p + \text{Nucleus}$$

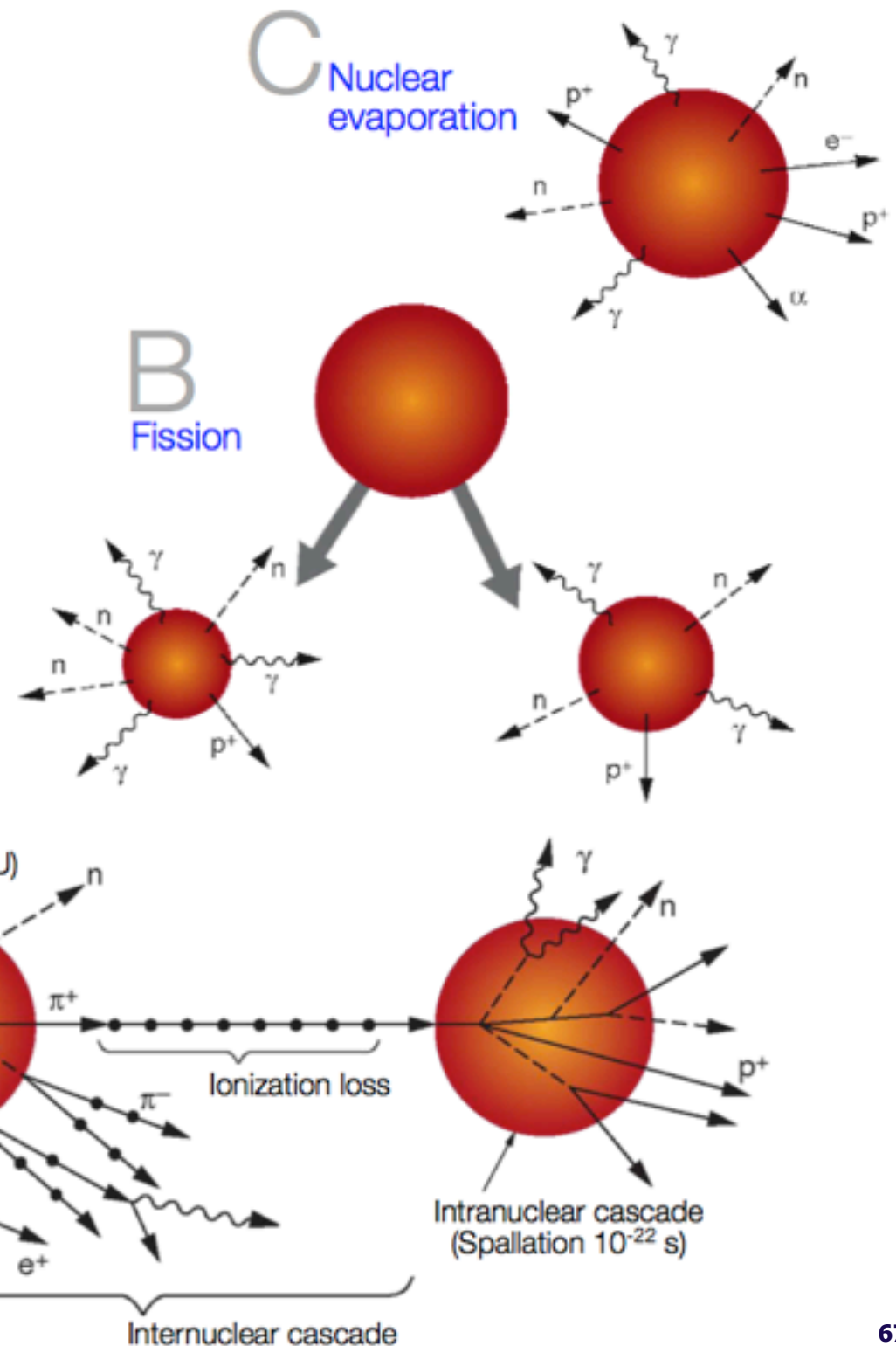
Inelastic:

$$p + \text{Nucleus} \rightarrow \pi^+ + \pi^- + \pi^0 + \dots + \text{Nucleus}^*$$

$$\left[ \begin{array}{l} \text{Nucleus}^* \rightarrow \text{Nucleus A} + n, p, \alpha, \dots \\ \quad \rightarrow \text{Nucleus B} + 5p, n, \pi, \dots \\ \quad \rightarrow \text{Nuclear fission} \end{array} \right]$$

**A** Inter- and  
intranuclear cascade

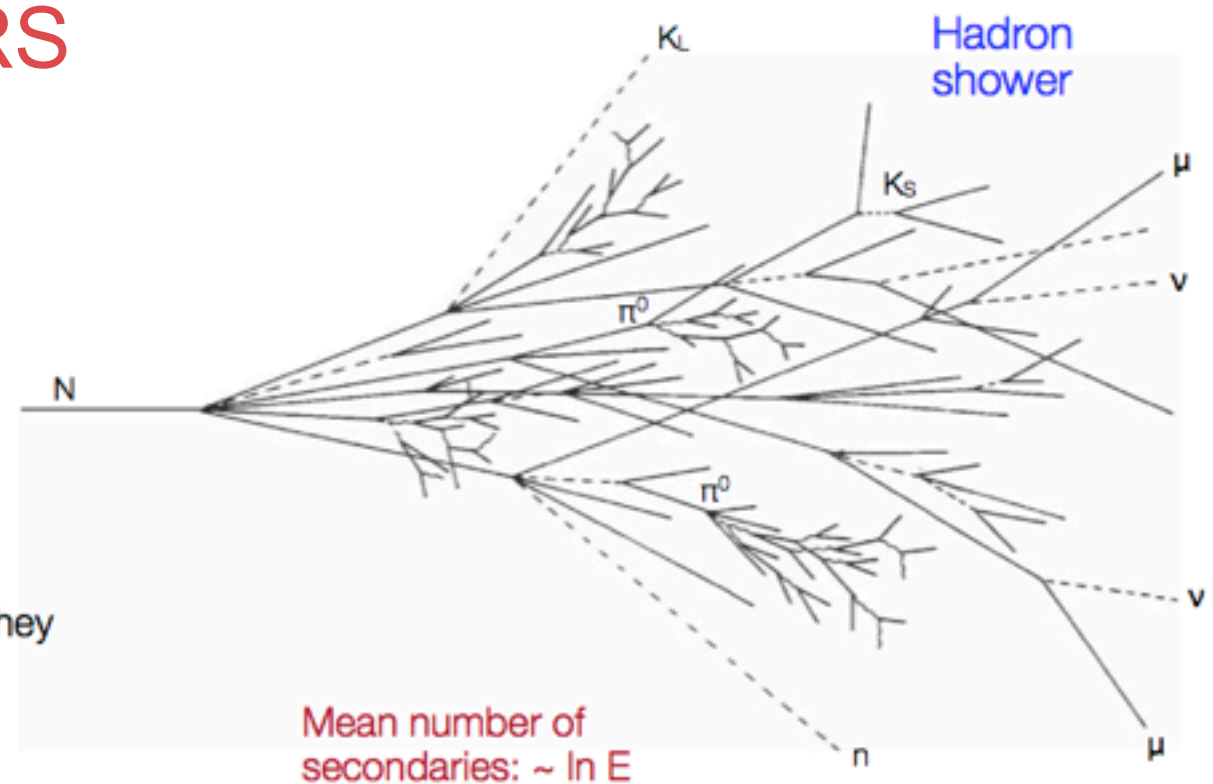
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# HADRONIC SHOWERS

## Shower development:

1.  $p + \text{Nucleus} \rightarrow \text{Pions} + N^* + \dots$
2. Secondary particles ...  
undergo further inelastic collisions until they fall below pion production threshold
3. Sequential decays ...  
 $\pi^0 \rightarrow \gamma\gamma$ : yields electromagnetic shower  
 Fission fragments  $\rightarrow \beta$ -decay,  $\gamma$ -decay  
 Neutron capture  $\rightarrow$  fission  
 Spallation ...



Substantial  
electromagnetic fraction

$$f_{\text{em}} \sim \ln E$$

[variations significant]

## Cascade energy distribution:

[Example: 5 GeV proton in lead-scintillator calorimeter]

|  |                |
|--|----------------|
| Ionization energy of charged particles ( $p, \pi, \mu$ ) | 1980 MeV [40%] |
| Electromagnetic shower ( $\pi^0, \eta^0, e$ )            | 760 MeV [15%]  |
| Neutrons   | 520 MeV [10%]  |
| Photons from nuclear de-excitation                       | 310 MeV [ 6%]  |
| Non-detectable energy (nuclear binding, neutrinos)       | 1430 MeV [29%] |
|  | <hr/>          |
|  | 5000 MeV       |

# HADRONIC INTERACTION LENGTH

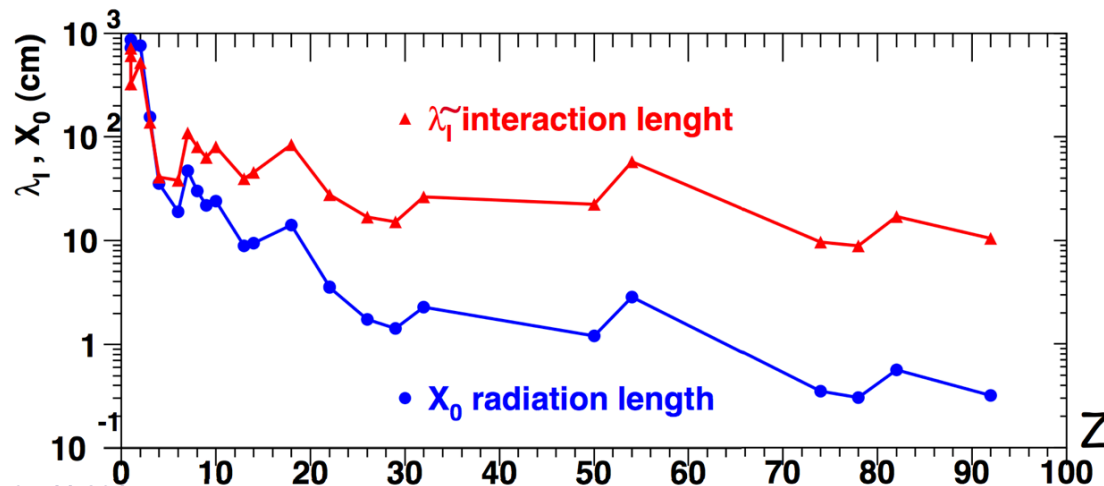
For high energies incoming hadrons, the hadronic cross-section is

- ~constant as function of energy
- ~independent of the hadron type

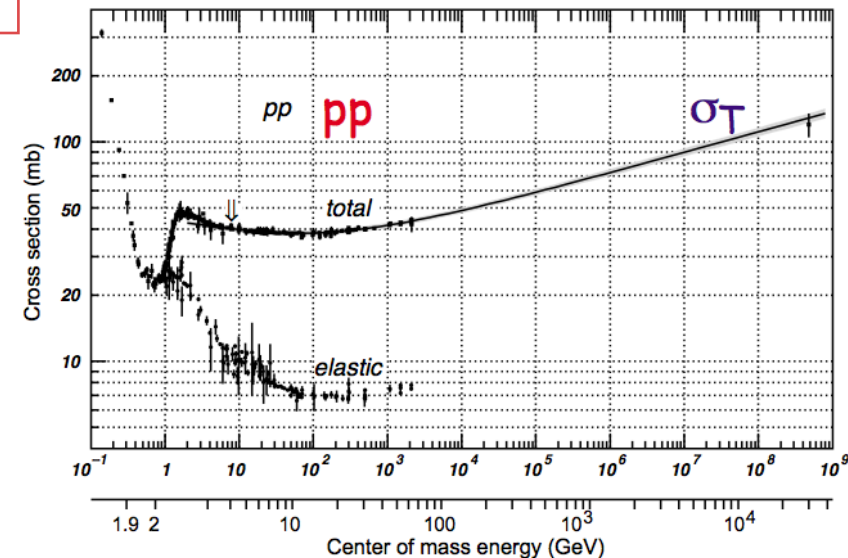
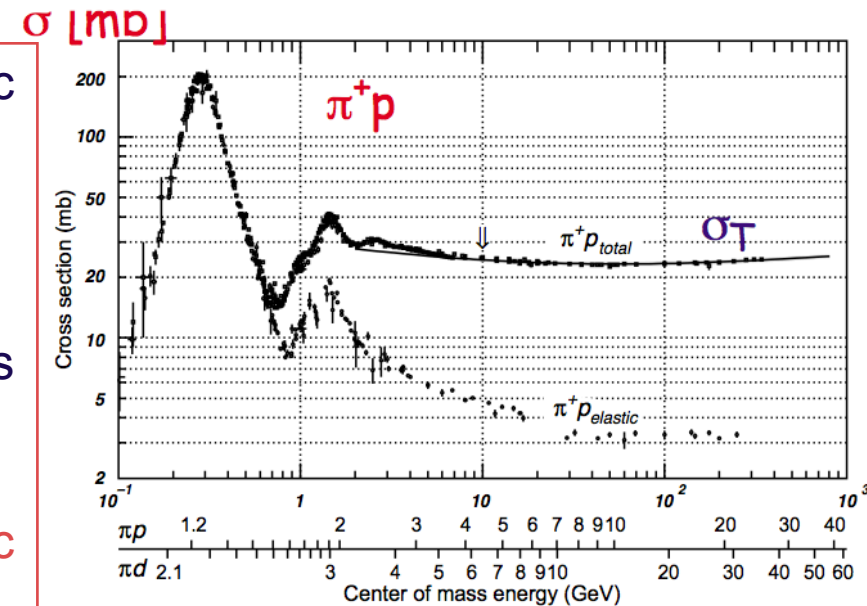
The material dependence of the total cross-section is given by  $\sigma_{\text{inel}} \approx \sigma_0 A^{0.7}$ ,  $\sigma_0 = 35 \text{ mb}$

Characterize hadronic interactions by **hadronic interaction length** in g/cm<sup>2</sup> (or in [cm] by normalising to the material density).

$$\lambda_{\text{int}} \approx \frac{A}{N_A \sigma_{\text{int}}} \approx 35 A^{1/3} \text{ g cm}^{-2}$$



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# INTERACTIONS OF PARTICLES WITH MATTER

## IONISATION AND EXCITATION

Charged particles traversing material and exciting and ionising atoms.

The average energy loss of the incoming particle by the process, is to a good approximation, described by the Bethe-Block formula.

## CHERENKOV RADIATION

If a particle propagates in a material with velocity  $>$  speed of light in this material, C radiation is emitted at a characteristic angle that depends on the particle velocity and the refractive index of the medium

## TRANSITION RADIATION

If a charged particle is crossing the boundary between two materials of different dielectric permittivity, there is a certain probability for emission of an X-ray photon.

## MULTIPLE SCATTERING AND BREMSSTRAHLUNG

Incoming particles are scattering off the atomic nuclei which are partially shielded by the atomic electrons.

This scattering imposes a lower level on the momentum resolution of the spectrometer, when measuring the particle momentum by deflection of the particle trajectory in the magnetic field.

The deflection of the particle on the nucleus results in an acceleration that causes the emission of Bremsstrahlung photons.

The photons in turn produce  $e^+e^-$  pairs in the vicinity of the nucleus, which causes the EM cascade.

This effect depends on  $m^{-2}$ : only relevant for electrons.

## HADRONIC INTERACTION

Incoming hadrons on a material will interact with the nucleus and create a shower composed of hadrons, electrons, photon.

A fraction of the energy is *lost* in the form of binding energy or neutrinos.

INTERACTIONS



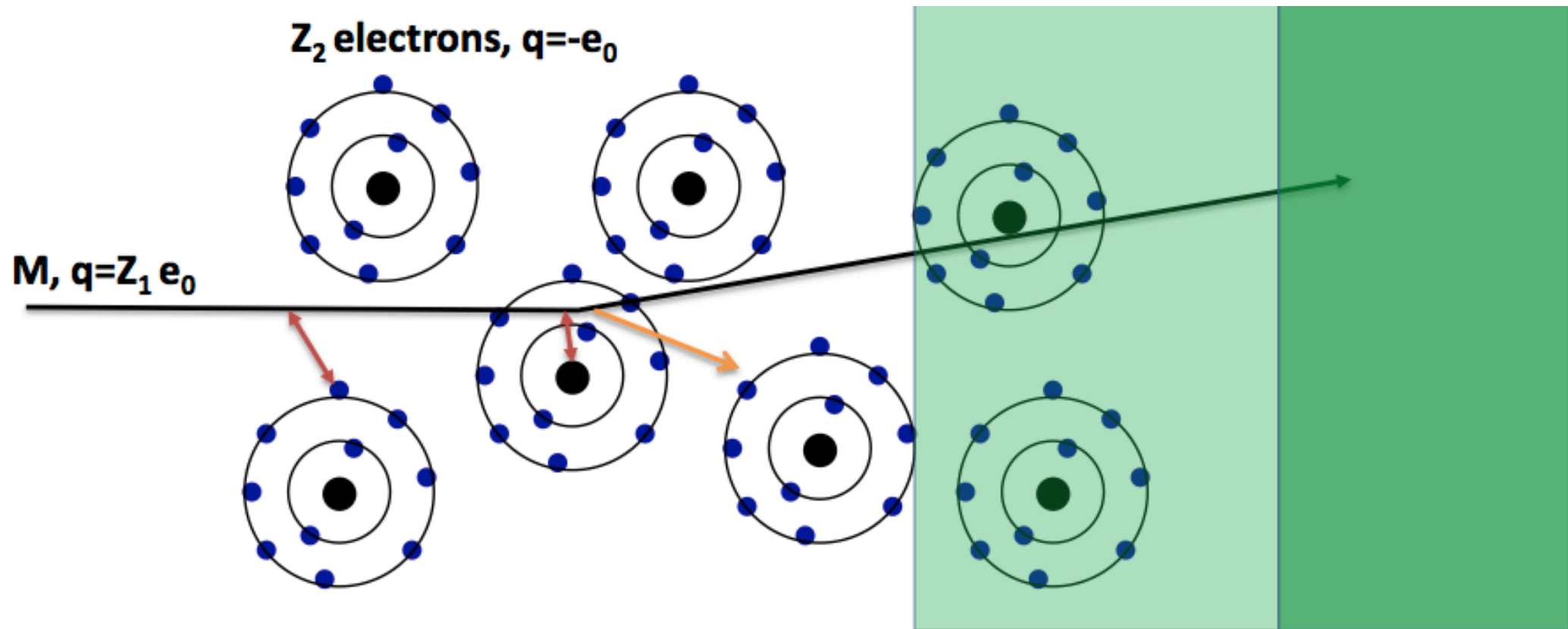
DETECTORS

# ELECTROMAGNETIC INTERACTIONS OF PARTICLES WITH MATTER

INTERACTIONS



DETECTORS



## Interaction with the atomic electrons.

The incoming particle loses energy and the atoms are **exited** or **ionised**.

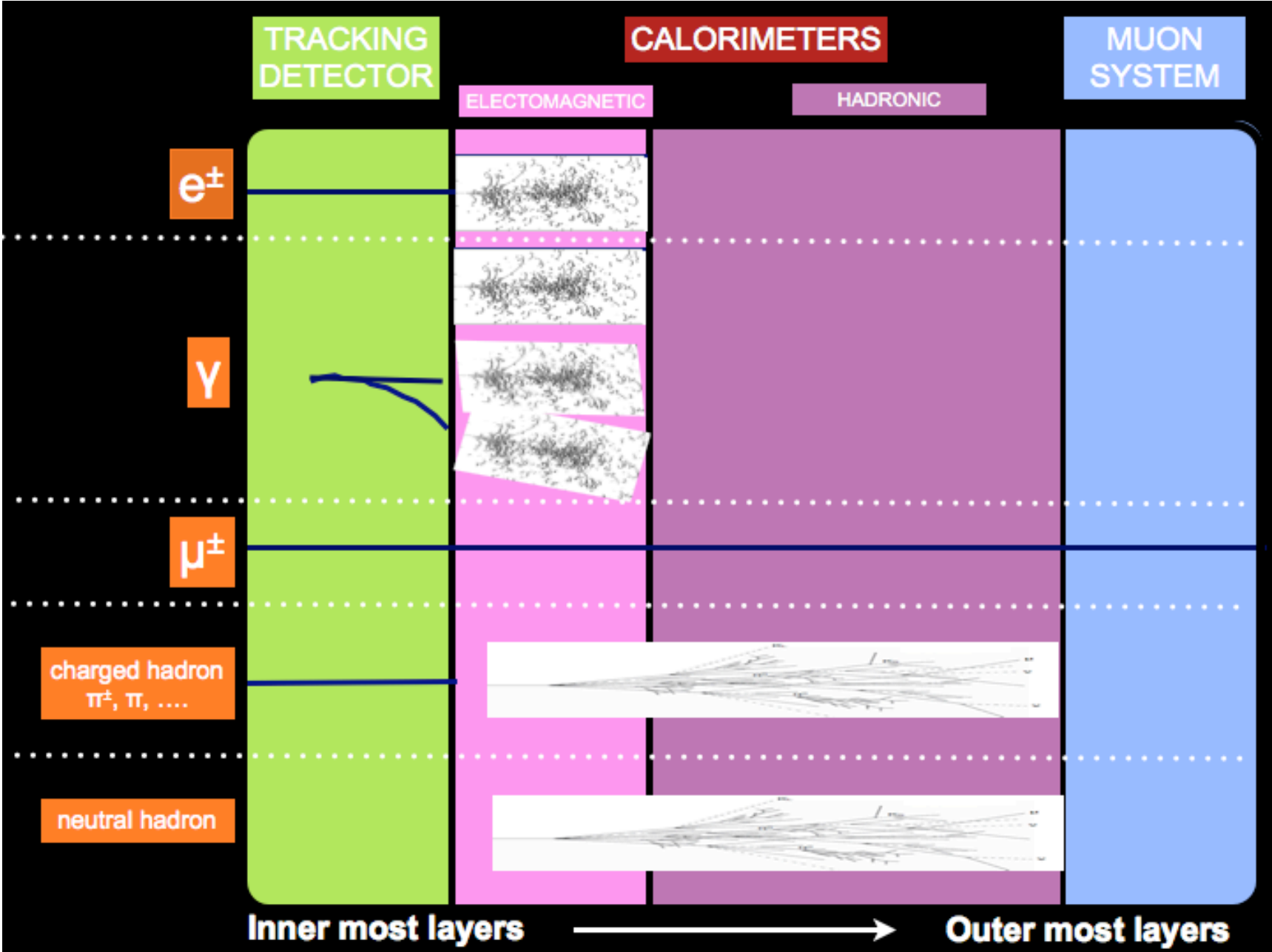
## Interaction with the atomic nucleus.

The incoming particle is deflected causing **multiple scattering** of the particle in the material.








During this scattering a **Bremsstrahlung photon** can be emitted

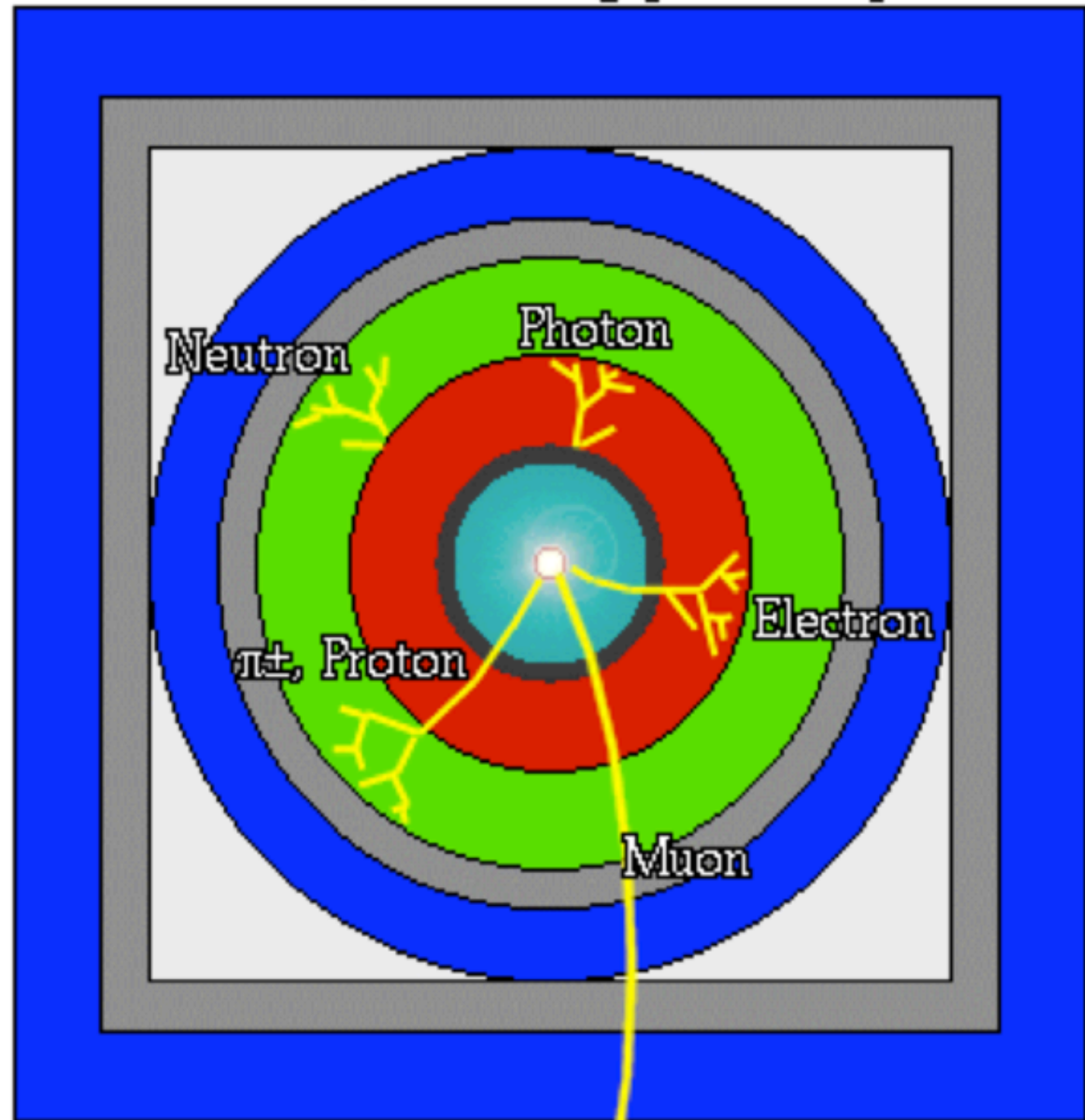
In case the particle's velocity is larger than the velocity of light in the medium, the resulting EM shockwave manifests itself as **Cherenkov radiation**. When the particle crosses the boundary between two media, there is a probability of 1% to produce an Xray photon called **Transition radiation**.

# PARTICLE DETECTION: schematic



# PARTICLE DETECTION: schematic

-  Beam Pipe (center)
-  Tracking Chamber
-  Magnet Coil
-  E-M Calorimeter
-  Hadron Calorimeter
-  Magnetized Iron
-  Muon Chambers



Muon Spectrometer

Muon

Neutrino

Hadronic Calorimeter

Proton

Neutron

Electromagnetic Calorimeter

Electron

Photon

The dashed tracks are invisible to the detector

Solenoid magnet

Tracking {  
Transition  
Radiation  
Tracker

Pixel/SCT  
detector



# CREDIT and BIBLIOGRAPHY

A lot of material in these lectures are from:

[Daniel Fournier @ EDIT2011](#)

[Marco Delmastro @ ESIPAP 2014](#)

[Weiner Raigler @ AEPSHEP2013](#)

[Hans Christian Schultz-Coulon's lectures](#)

[Carsten Niebuhr's lectures \[1\]\[2\]\[3\]](#)

[Georg Streinbrueck's lecture](#)

[Pippa Wells @ EDIT2011](#)

[Jérôme Baudot @ ESIPAP2014](#)