Field Theory and EW Standard Model

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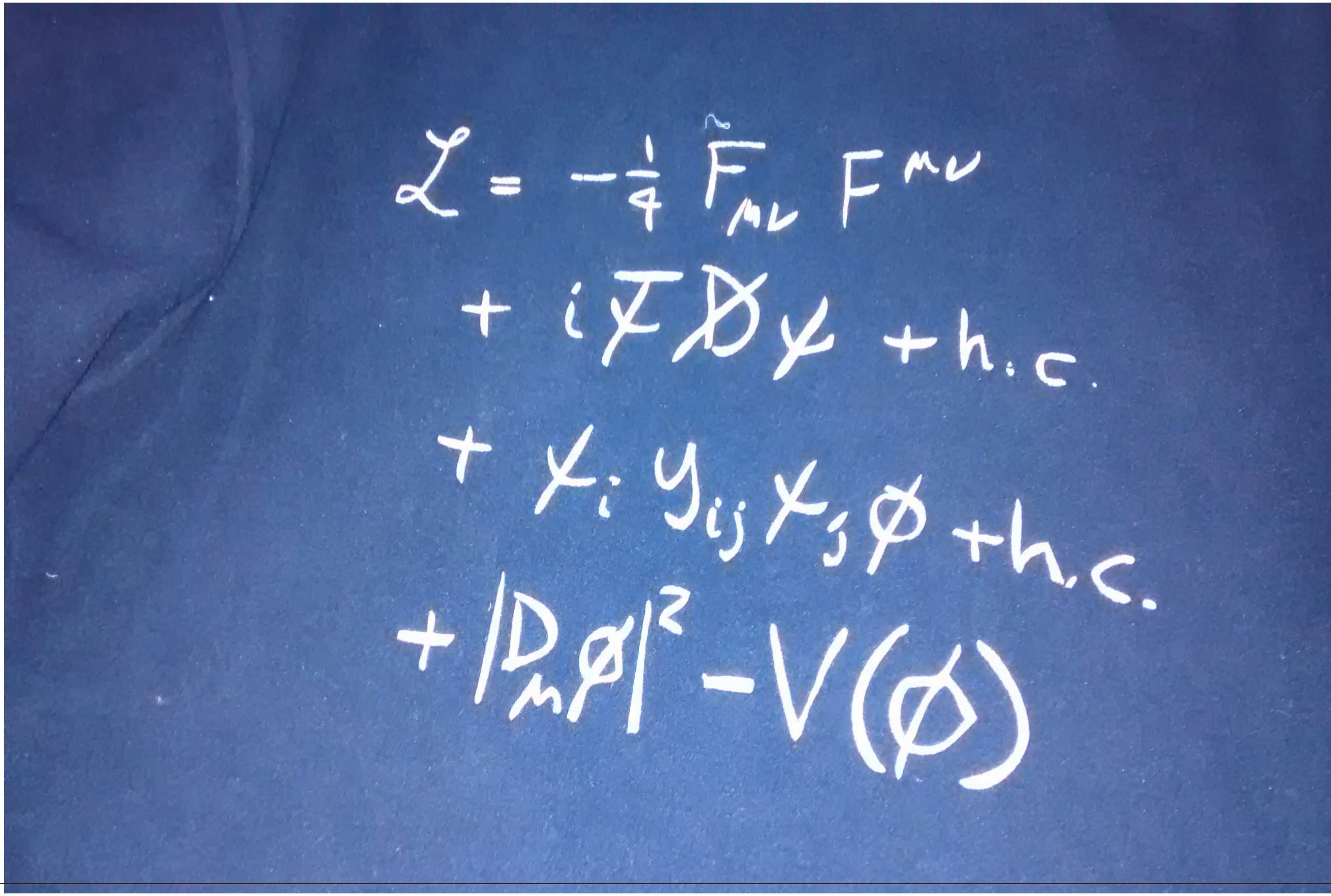

$\mathcal{L} = -\frac{1}{4} \tilde{F}_{\mu\nu} F^{\mu\nu}$
$+ i\bar{\psi} \not{D} \psi + h.c.$
$+ \psi_i y_{ij} \psi_j \phi + h.c.$
$+ |D_\mu \phi|^2 - V(\phi)$

Lectures 1,2:

a) Setting up the notation of the SM Lagrangian, including Higgs mechanism

b)The miracles of the particle spectrum of the SM: Anomaly cancellation and the Custodial symmetry!

c) How one can understand the development of the SM also in terms of taming the bad high energy behavior of the scattering amplitudes!

Prediction of new particles and their masses in the SM:

d) GIM and prediction of M_c from the observed mass difference $K_L - K_S$. The 'first' use of an **indirect** effect to predict a mass!

e) Test of EW unification with the determination of $\sin\theta_w$ and resultant test of a unified gauge field theoretic description of Electro Weak interactions.

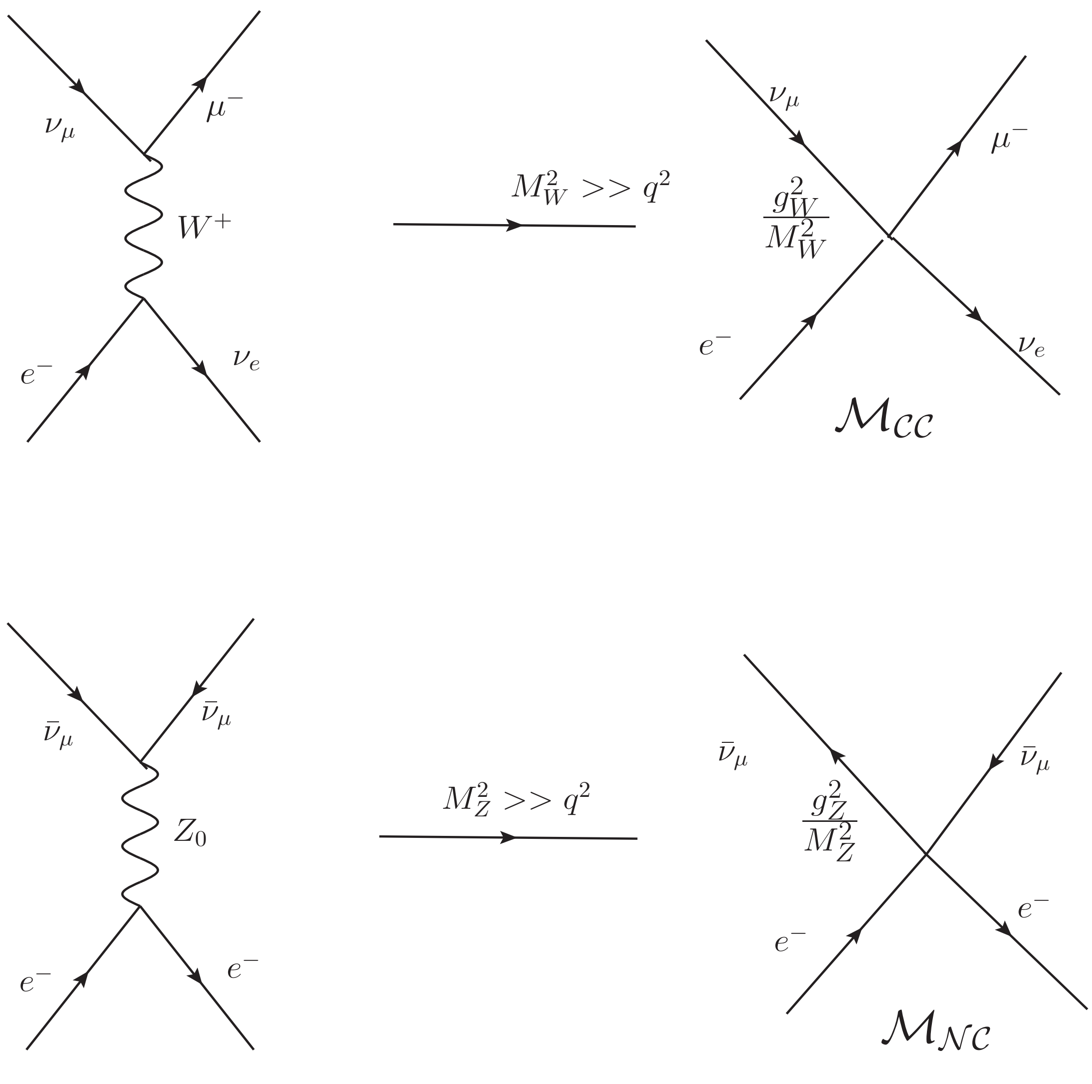
The particle spectrum after the Spontaneous symmetry breaking is easiest seen in the unitary gauge.

$$\mathcal{L}_{gauge}^{massless} + \mathcal{L}_{\Phi} \xrightarrow{\text{SSB}} \mathcal{L}_{gauge}^{massive} + \mathcal{L}_{h}$$

4 massless gauge bosons	4 scalar fields	3 massive, 1 massless gauge bosons	1 physical scalar

Count the degrees of freedom and you will see it is 12 both before and after the SSB.

The W and Z exchange generate effective interaction Hamiltonian which can describe various processes like $e^- + \nu_\mu \to \mu^- \nu_e$ a charged current process and $e^- + \nu_\mu \to e^- + \nu_\mu$ a neutral current process.



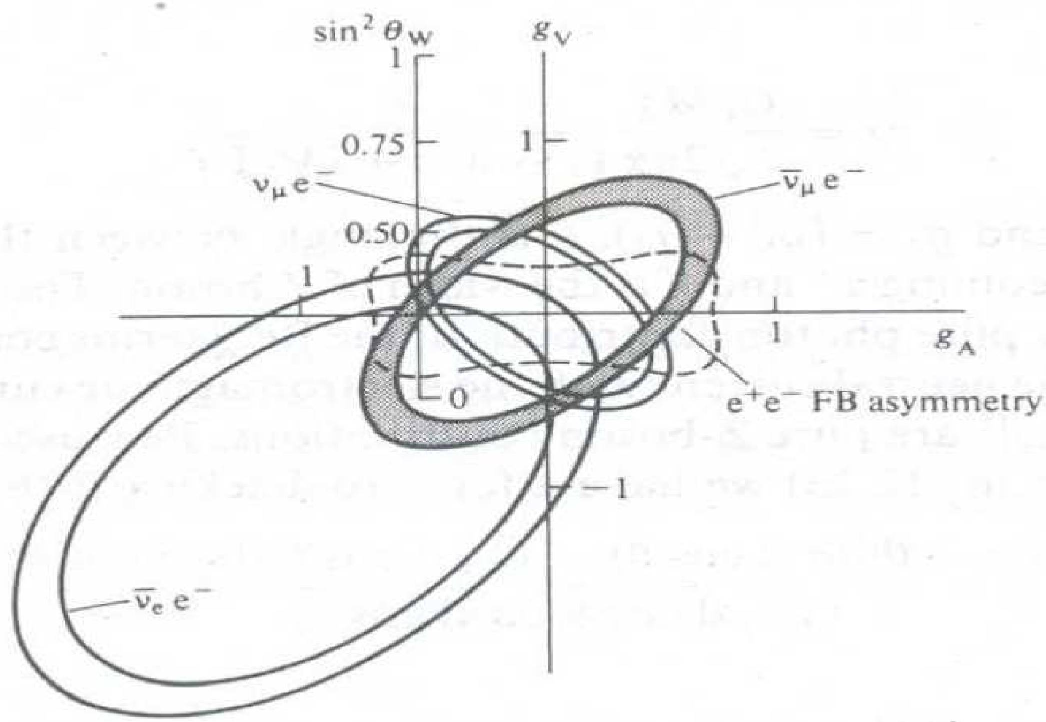
$$\rho = \frac{g_Z^2}{2 g_W^2} \frac{M_W^2}{M_Z^2}$$
$$= \frac{M_W^2}{M_Z^2 \cos^2 \theta_W}$$

ρ measures the ratio of strengths of the coupling in $\mathcal{M}_{\mathcal{CC}}$ and $\mathcal{M}_{\mathcal{NC}}$.

For the WS model $\rho = 1$ and Only if we choose doublet for the Higgs representation.

Measurements of neutral current processes gave $\rho \simeq 1$ and indicated correctness of this picture!

Determination of Neutral Current couplings and hence $\sin^2\theta_W$ (circa 1981). Also gave $\rho \sim 1$.



Predicted $M_W = 82 \pm 2, M_Z = 92 \pm 2$ GeV. Test of unified Gauge theory but not a Quantum Gauge Theory!

We can trace the $\rho = 1$ to an accidental symmetry of the Higgs potential for a doublet.

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

$$\Phi^\dagger \Phi = \Re e(\phi_1)^2 + \Im m(\phi_1)^2 + \Re e(\phi_2)^2 + \Im m(\phi_2)^2$$

Has an $SO(4)$ symmetry for the rotation of a vector

$$X = \begin{pmatrix} \Re e(\phi_2) \\ \Im m(\phi_2) \\ \Re e(\phi_1) \\ \Im m(\phi_1) \end{pmatrix}$$

Of course after the Spontaneous symmetry breaking this symmetry is lost as ONLY one field gets nonzero vev.

However an $SO(3)$ symmetry is still retained as the remaining all three do not get a nonzero vev.

This is reflected in the equality of mass term for all W_μ^a. This guarantees that $\rho = 1$.

Thus whatever may be the mass generation mechanism as long as one respects this $\rho = 1$.

This symmetry is broken, e.g., from mass splitting between the two members of $TL = 1/2$ doublet of $SU(2)_L$

In the limit of these being small, ρ will remain close to one in the SM even after radiative corrections!

$$\mathcal{L}_\Phi|_{unitary gauge} = \mathcal{L}_h + \left[M_W^2 W_\mu^+ W^{-\mu} + 1/2 M_Z^2 Z^\mu Z_\mu\right]\left(1 + \frac{h}{v}\right)^2$$

Not just the mass but the coupling of a Higgs to a gauge boson pair also comes from the Covariant Derivative term.

$g_{hVV} = \frac{M_V^2}{v}(-2i).$

Thus hVV coupling is proportional to the gauge boson masses!

It was really Weinberg's genius that he saw that exactly the same mechanism can be used to give masses to the fermions by postulating a gauge invariant term for interaction between the fermionic matter fields and the Higgs field!

$$\mathcal{L}^e_{yukawa} = -f^{*e}\mathcal{L}'_{1L}\Phi e_{1R} + h.c.$$

with $e_{1R} = e_R$ and $\mathcal{L}_{1L} = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$

In the unitary gauge then with the choice of

$$\Phi' = \begin{pmatrix} 0 \\ 1/\sqrt{2}(v + h(x)) \end{pmatrix}$$

we get

$$\mathcal{L}^{e}_{yukawa} = -\frac{f^{*e} v}{\sqrt{2}} (\bar{e}_L e_R)(1 + h/v) + h.c.$$

Thus we have $m_e = +f^{*e} v/\sqrt{2}$ and the hee coupling is just m_e.

Thus the leptons now have a mass. it is generated from a term which is gauge invariant.

Just like the vector boson masses this mass issue too is sorted out in a gauge invariant way

Some extra work is needed for the case of quarks:

$$\mathcal{L}_{yukawa} = -f^{*d}_{ij}\mathcal{Q}'_{iL}\Phi d'_{jR} - f^{*u}_{ij}\mathcal{Q}'_{iL}\tilde{\Phi}u'_{jR} + h.c.$$

where $\tilde{\Phi} = i\sigma_2\Phi^*$.

Note : Both the terms are $SU(2)_L$ invariant (by construction).

One term involves Φ and the other $\tilde{\Phi}$ because we want the $\mathcal{L}$ to be invariant under $SU(2)_L \times U(1)_Y$ transformation.

$Y_{\mathcal{Q}_L} = +1/3$, $Y_{d_R} = -2/3$, $Y_{uR} = +2/3$:

The first term has $Y = 0$ with Φ The second term has $Y = 0$ only with $\tilde{\Phi}$ which has $Y_{\tilde{\Phi}} = -Y_{\Phi} = -1$.

In the unitary gauge then with the choice of

$$\Phi' = \begin{pmatrix} 0 \\ 1/\sqrt{2}(v + h(x)) \end{pmatrix}$$

we then get

$$\mathcal{L}_{yukawa} = -\frac{f_{ij}^{*d}}{\sqrt{2}} v \; \bar{d}'_{iL}(1 + h/v) d'_{jR} + -\frac{f_{ij}^{*u}}{\sqrt{2}} v \; \bar{u}'_{iL}(1 + h/v) u'_{jR} + h.c.$$

The mass matrices are:

$m_{ij}^{d} = +\frac{f_{ij}^{*d}}{\sqrt{2}} v$ and $m_{ij}^{d} = +\frac{f_{ij}^{*d}}{\sqrt{2}} v$

In general f_{ij}^{*d}, f_{ij}^{*u} are completely arbitrary matrices in the generation space.

The mass matrices are not diagonal in the basis d_i', u_i', in the most general case.

The states $d_i', u_i', i = 1 - 3$ are clearly not mass eigenstates.

So this means that mass eigenstates $d_i, u_i, i = 1, 3$ are linear combinations of d_i', u_i'.

Recall that interaction is always given by the covariant derivative term.

$$\mathcal{L}_f = i\bar{\mathcal{Q}}'_{iL} D_\mu \gamma^\mu \mathcal{Q}'_{iL} + i\bar{u}'_{iR} D_\mu \gamma^\mu u'_{iR} + ...$$

For example $D_\mu \mathcal{Q}'_{iL} = \left[\partial_\mu - i\frac{g_2}{2}\vec{\sigma}\cdot\vec{W}_\mu - i\frac{g_1}{6}B_\mu\right]\mathcal{Q}'_{iL}$

The charged current interaction term written using the covariant derivative is for example:

$$\mathcal{L}^{cc} = \sqrt{2} g_2 \bar{u}'_{iL} \gamma^{mu} d'_{iL} W^+_\mu + h.c$$

One can prove that in the most general case, after diagonalisation of both the m^d, m^u matrices this becomes

$$\mathcal{L}^{CC} = \sqrt{2} g_2 \bar{u}_{iL} \gamma^\mu d_{jL} \times \mathcal{V}_{ij}^{CKM} + h.c.$$

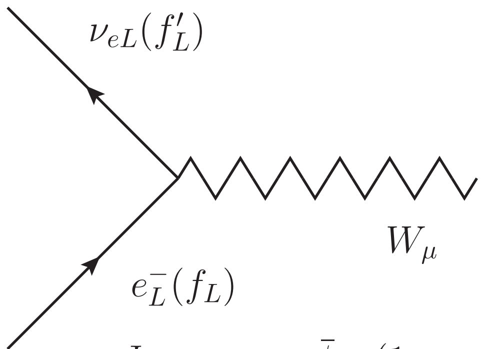
$\mathcal{V}^{CKM}$ is the Cabibbo-Kobayashi-Masakawa mixing matrix.

In your flavor physics course you will hear much more about it!

In fact for two generation case this is just the Cabibbo mixing matrix.

Step back to pre gauge theory days:

V-A theory had told that the basic unit of charged current weak interactions is a doublet of left handed fermions.



$$J_\mu = g_{weak} \;\; \bar{\psi}\gamma_\mu(1-\gamma_5)\psi'$$

$$\Delta S = \Delta Q \text{ if } f = s, f' = u$$

f and f' differ in charge by one unit.

For strange quark case $\Delta S = 1$

Called Charged Current J^{CC}_μ.

Cabbibo's important observation Phys. rev. Lett. 10, 531 (1963)

The strength of all the three types of weak transitions $\Delta S = 0, \Delta S = 1$ and pure leptonic was equal *iff* appropriate doublet was

$$\begin{pmatrix} u \\ d\cos\theta_c + s\sin\theta_c \end{pmatrix}$$

θ_c called Cabbibo angle, experimentally determined value : $\sim 12^\circ$.

Interaction eignestate: $d\cos\theta_c + s\sin\theta_c$

Bjorken and Glashow (1964) Postulated a new quark c, with same quantum numbers as the u quark , which forms a doublet with orthogonal combination $s\cos\theta_c - d\sin\theta_c$.

Thus phenomenologically generation mixing was observed even before the Weinberg model was postulated.

The mass generation mechanism for quarks seems to naturally accommodate the quark mixing!

If $\mathcal{V}^{CKM}$ has to accommodate CP violation then it needs to be complex matrix. This needs three generations.

So this was the prediction of of Kobayashi and Maskawa that if the SM has to 'describe' the observed CP violation in terms of quark mixing one needs three generations!

So far we discussed how $V - A$ theory implied possibility of a Gauge Theory of weak interactions.

This led to the postulation of a unified gauge theory of electro weak interactions $SU(2)_L \times U(1)_Y$.

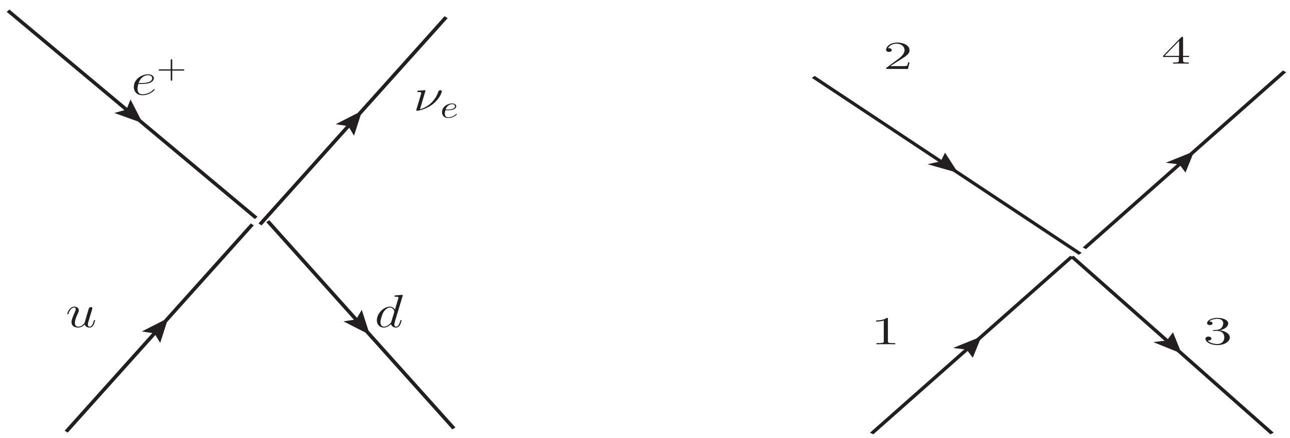
The non zero masses of gauge bosons and fermions posted a problem in writing a gauge invariant theory.

The Higgs mechanism provided a way to generate these masses without breaking gauge invariance and hence made possible that these theories may be renomalisable.

Now I want to approach the discussion from a different point of view. For that I want to discuss high energy behaviour of weak amplitudes.

Started from Fermi's (almost correct) proposal that all the β decay data can be explained by

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}}(J^W_\mu J^{W\mu\dagger}) = \frac{G_F}{\sqrt{2}}(\bar{\psi}_n \gamma_\mu \psi_p)(\bar{\psi}_{\nu_e}\gamma^\mu \psi_e) \equiv \frac{G_F}{\sqrt{2}}(\bar{\psi}_3 \gamma_\mu \psi_1)(\bar{\psi}_4 \gamma^\mu \psi_2)$$



$n(udd) \to p(uud) + e^- + \bar{\nu}_e$ decay could be understood (once quark model was known) as a transition of d quark to u with an emission of an $e^-\bar{\nu}_e$ pair!

Four fermion point interaction describes the β decay

$n \to p + e^- + \bar{\nu}_e$

The same Lagrangian can be correctly used to calculate

$\nu_e + n \to p + e^-$

Similarly lagrangian for

$\mu \to \nu_\mu + \bar{\nu}_e + e^-$

can be used to calculate

$e^- + \nu_\mu \to \nu_e + \mu^-$

In case of current-current interaction there is no other mass scale other than s the square of cm energy

$$\sigma_{tot} = \frac{G_F^2}{2\pi} s.$$

This cross-section grows with energy. This must come into conflict with unitarity.

Unitarity demands that all partial wave amplitudes should be bounded by 1.

For s-wave scattering this translates into

$$\frac{d\sigma}{dt}\Big|_{t=0} > \frac{\sigma_{tot}^2}{16\pi}$$

Calculate that this means

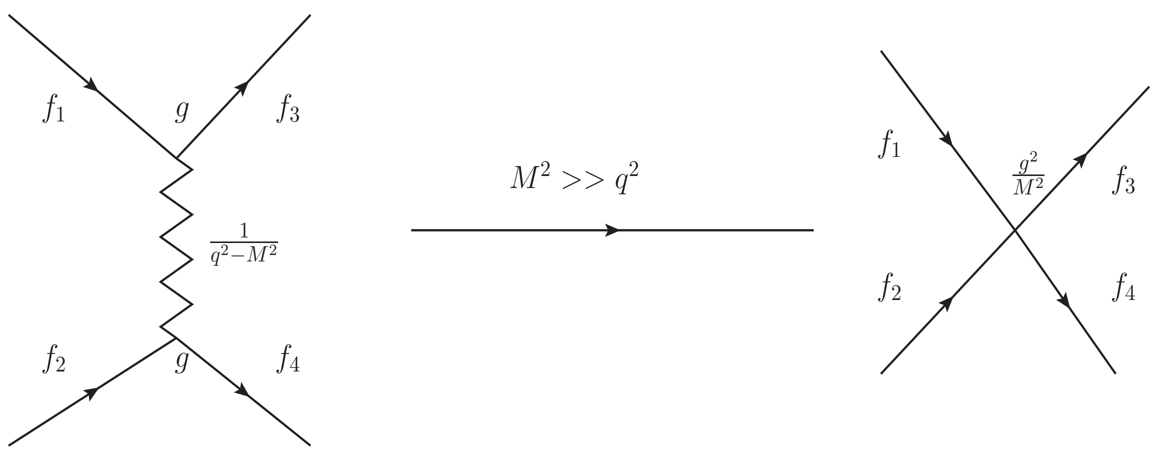
$s < \frac{4\pi\sqrt{2}}{G_F} \to \sqrt{s} \simeq 300GeV$.

What does this mean? Surely $s = 2m_e E_{\nu_\mu}$ can be anything.

So what this means that unitarity is violated beyond this energy. So the theory must be modified below it.

This is what led Schwinger to propose that vector bosons are massive!

Hypothesis: The current-current interaction is just an approximation to a real ampiltude with a very heavy boson. Good agreement of Current Current interaction idea with data meant that **IVB** was necessarily heavy. IVB: Klein/Schwinger

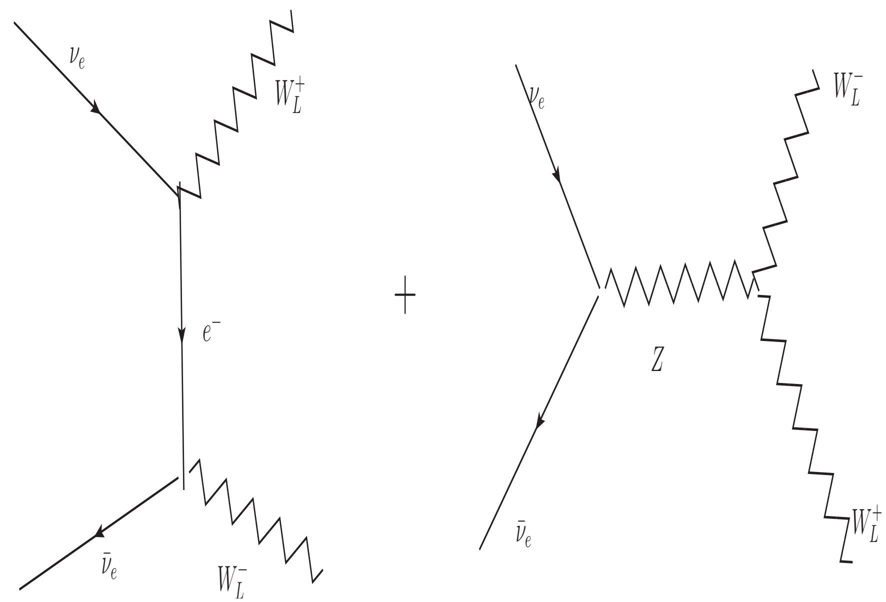


$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}}(J^W_\mu J^{W\mu\dagger}) = \frac{G_F}{\sqrt{2}}(\bar{\psi}_3\gamma_\mu\psi_1)(\bar{\psi}_4\gamma^\mu\psi_2); \quad \frac{G_F}{\sqrt{2}} \propto \frac{g^2}{M^2}$$

Fermi theory (Current-Current Interactions): $\mathcal{M}(\nu e \to \nu e)$ violates tree level unitarity for $\sqrt{s} \simeq 250 \sim G_F^{-1/2}$ GeV $\Rightarrow$ massive gauge bosons. Mass? Somewhere below this!

1)Even with a mass for the IVB, $\mathcal{M}(\nu_e \bar{\nu}_e \to W^+ W^-)$ grows too fast with energy and violates unitarity.

2)s channel exchange of a Z boson in gauge theory with precisely the non abelian gauge couplings of $SU(2)_L \times U(1)$ gauge group restores the unitary behaviour. Divergence for $WW \to WW$ MUCH worse, also cured!.



J.S. Bell: Nuclear Physics, **B60**, 427, 1973:

Showed that in a renomalisable theory tree level amplitudes satisfy unitarity.

But three sets of authors asked the opposite question: What can we deduce by demanding that tree level amplitudes satisfy unitarity.

J. M. Cornwall, D. N. Levin and G. Tiktopoulos, PRL **30**, 1268 (1973), Phys.Rev. **D10**, 1145 (1974),

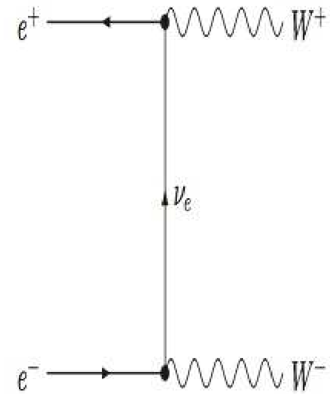
C. Llewllyn Smith : PLB **46**, 233 (1973)

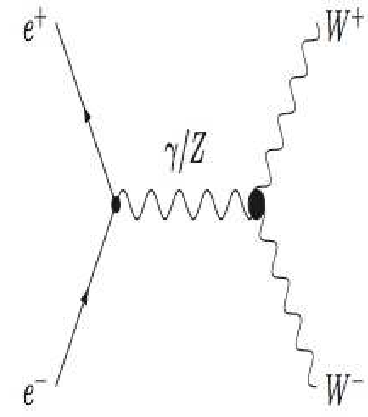
S.D. Joglekar,: Ann. Phys. **83**, 427 (1974)

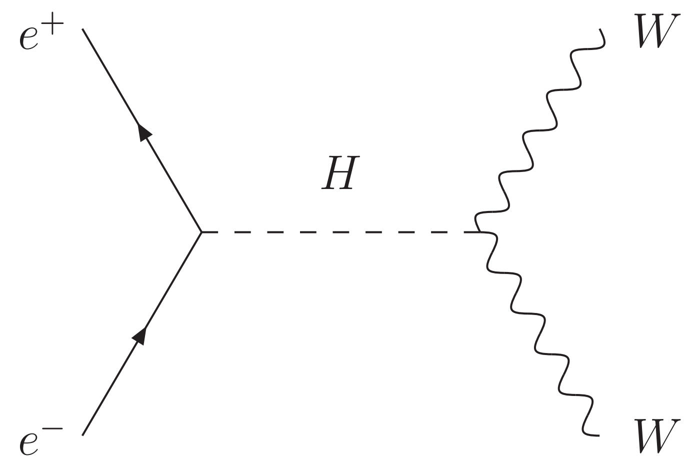
They showed that such demands uniquely indicate spontaneously broken gauge theories.

For example:

Unitarity of $\mathcal{A}(e^+e^- \to W_L W_L)$, for example, $\Rightarrow$ Divergences cancel only if there is a $J = 0$ amplitude (s channel exchange of a Spin 0 particle) whose coupling to matter/gauge particles is proportional to their masses $\Rightarrow$ **Existence of a Higgs boson.**







But no knowledge on the scale! ie. the mass of the Higgs!
Only the couplings.

SSB Gauge theory makes scattering amplitudes well behaved at high energy, even with massive gauge bosons!.

In fact Higgs couplings to matter and gauge bosons required to be proportional to masses to have unitarity.

This proportionality is indeed one of the prediction of the renormalisable $SU(2)_L \times U(1)_Y$ where the EW symmetry broken spontaneously by the Higgs mechanism!

There seems to be a lot of information in 'unitarity' requirement!

With massive W indeed the cross-section rise with s is tempered.

One gets a logarithmic violation of unitarity as opposed to the the power violation seen earlier.

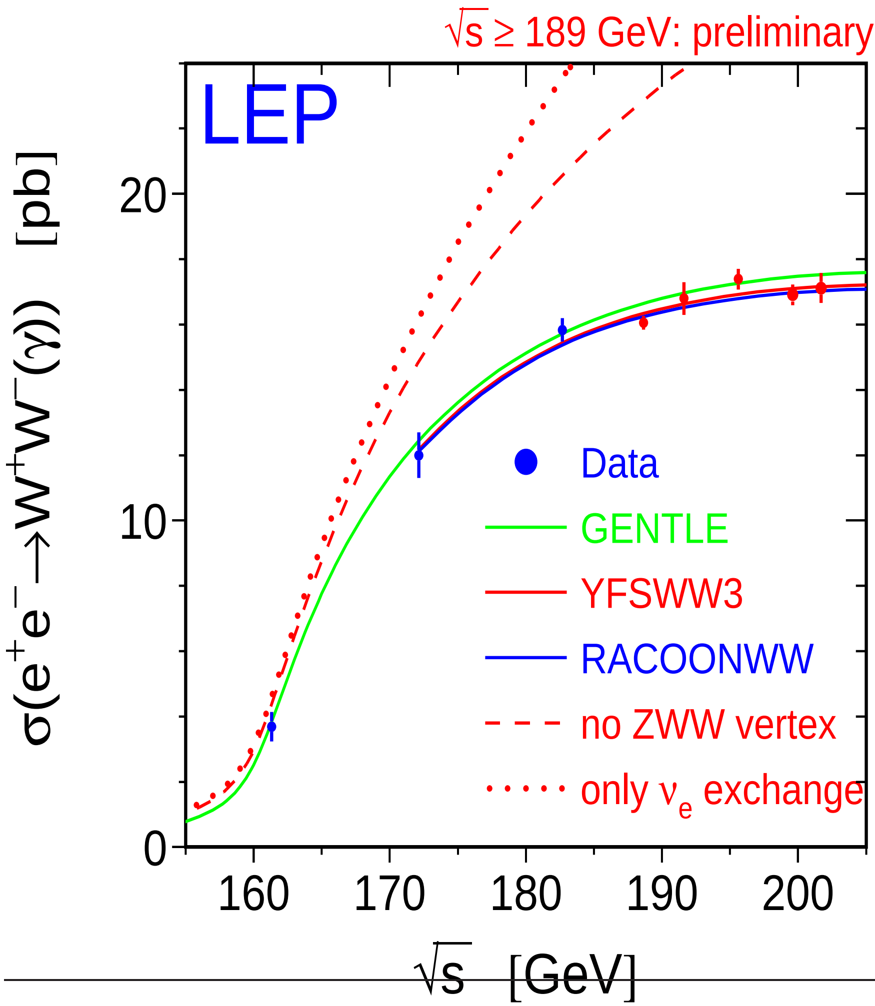
This in principle unitarity unitarity violation gets cured by the running of the coupling g_2. scale dependence of the coupling due to quantum corrections.

This whole thing points at deep connection between unitarity and renormalisability.

(book by Greiner)

Currently Arkani Hamed et al have been able to somehow show that only field theories with particles with spin $0, 1/2, 3/2, 1$ and 2 are consistent with unitarity demands on scattering amplitudes!

Direct 'Proof' of Symmetry and Symmetry breaking!!



Proof that electroweak symmetry exists and that it is broken.

The triple gauge boson ZWW coupling tames the bad high energy behaviour of the cross-section caused by the t-channel diagram. Direct proof for the ZWW coupling.

This and precision testing, confirm basics of the SM

The next point we wish to discuss is how the SM 'naturally' accommodates the observed suppression of the flab our changing neutral current processes: FCNC

Current eigenstates that couple to the W are two doublets:

$$\begin{pmatrix} u \\ d\cos\theta_c + s\sin\theta_c \end{pmatrix} \qquad \begin{pmatrix} c \\ -d\sin\theta_c + s\cos\theta_c \end{pmatrix}$$

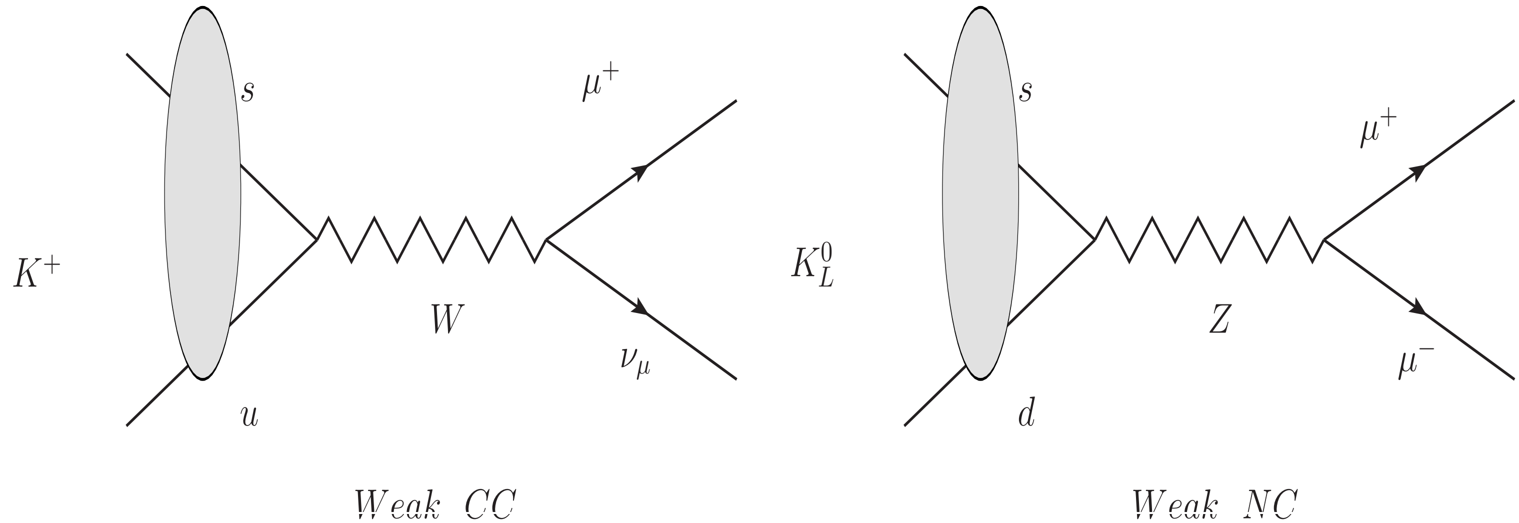
Quark mixing in two generations can then be represented by

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

$$J_\mu^{CC} = \bar{d}'\gamma_\mu(1-\gamma_5)u + \bar{s}'\gamma_\mu(1-\gamma_5)c$$

(recall the $f\bar{f}'W$ vertices we saw yesterday)

Charm is postulated. This does the trick of making Flavor Changing Neutral Current vanish at least by making sure that a vertex $d\bar{s}Z$ does not exist. Can more complicated diagrams produce FCNC?



K^+, $\bar{s}u$ bound state. $K^+ \to \mu^+\nu_\mu$: weak charged current decay, $\Delta S = 1$

K^0 is a $\bar{s}d$ bound state. If a weak neutral current with $\Delta S \neq 0$ (Flavor Changing Neutral Current: FCNC) were to exist with the same strength as the weak charged current it would cause problems.

$K_L^0 \to \mu^+\mu^-$ happens very rarely (one part in 10^8 among all K_L decays)

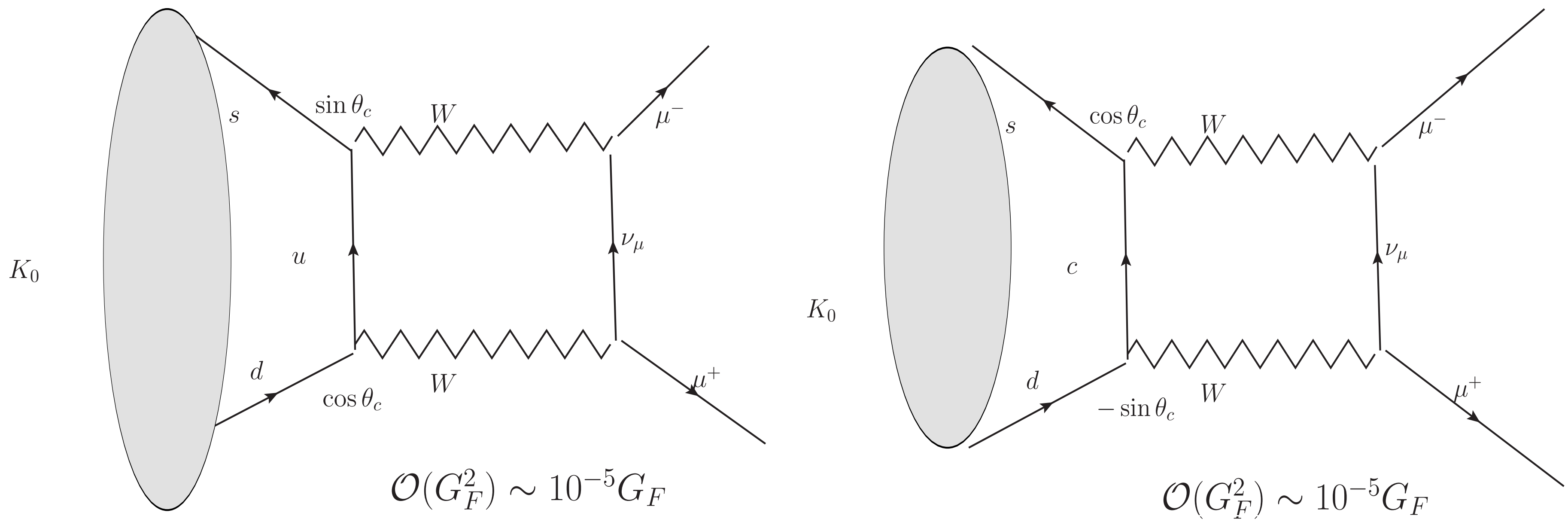
Once we have two quark doublets, tree level FCNC vanishes automatically.☺

Is Rabi's question answered? We somehow show that we need a new quark. Can we say FCNC ordered a new quark?

Still no answer to Rabi's question 'who ordered the μ

But what this tells is that the left handed fermions have to appear as doublets.

Second point : Quantum properties of the $SU(2)_L \times U(1)$ gauge theory imply that the number of generations should be equal for quarks and leptons. (Anomaly cancellation!)



What happens with loops?

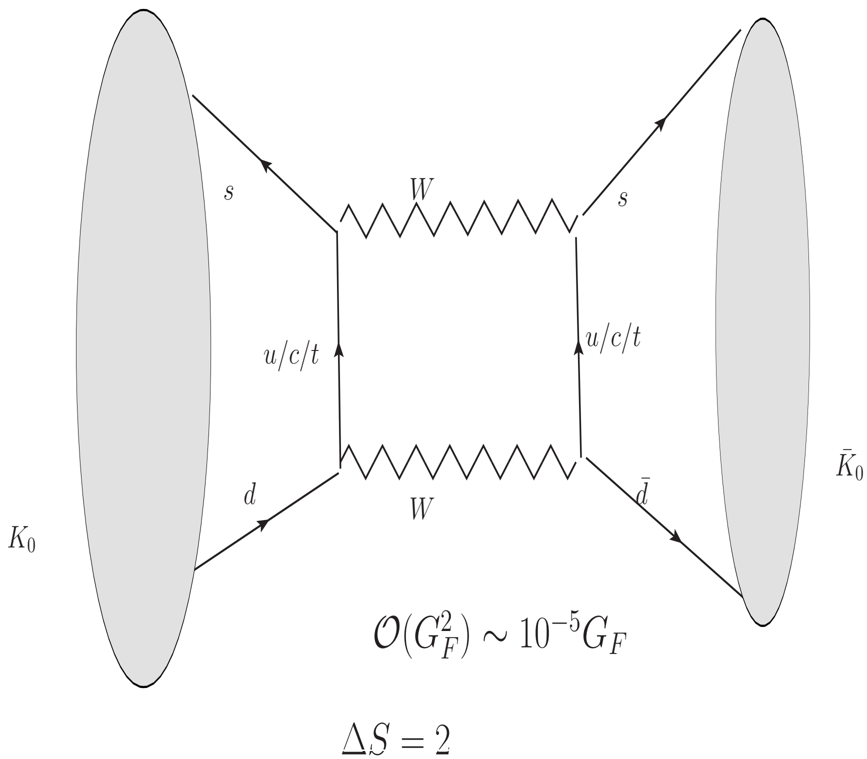
If charm contribution is absent the prediction for this flavour changing decay will be much too big compared to data.

Absence of Flavour Changing Neutral Currents is granted in the EW theory ONLY IF CHARM exists. Will be exactly zero if $m_c = m_u$.

For any physics beyond SM this is always a constraint that HAS to be satisfied.

This cancellation is an example of the Glashow-Iliopoulos-Maiani (GIM) cancellation mechanism.

A very simplistic presentation given here.



Loop yields a finite result only in the four quark picture the result of the calculation and is

$$\frac{\Delta M_K}{M_K} = \frac{G_F^2}{4\pi} m_c^2 \cos^2\theta_c \sin^2\theta_c f_K^2$$

$= 7 \times 10^{-15}$

Predicts $m_c \sim 1.6$ GeV.

The November revolution: Discovery of Charmonium at 3.1 GeV was the first step in validation of the SM as a renormalisable Gauge theory!

NEEDS GIM!