Field Theory and EW Standard Model-III

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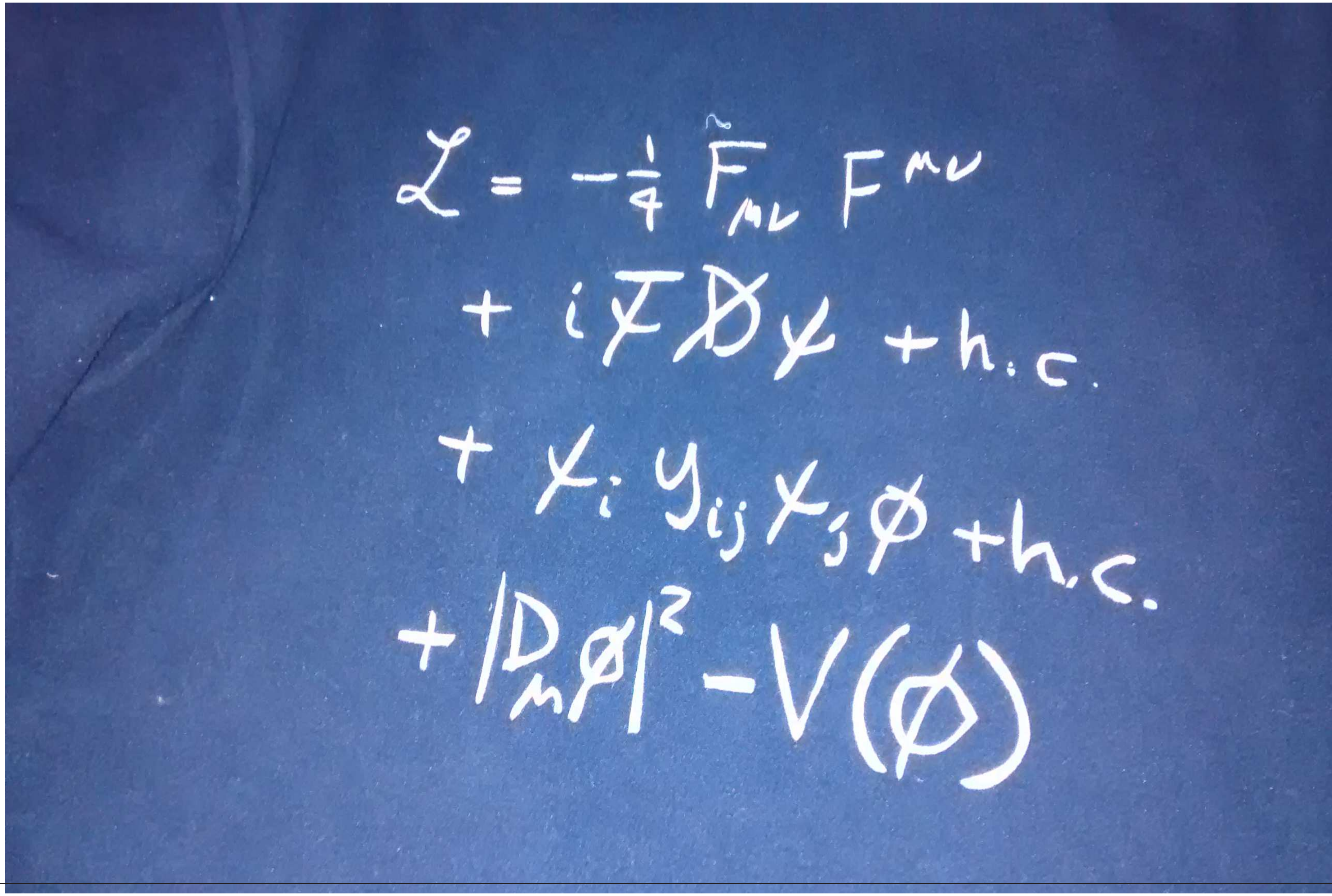

$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$
$+ i \bar{\psi} \not{D} \psi + h.c.$
$+ \psi_i y_{ij} \psi_j \phi + h.c.$
$+ |D_\mu \phi|^2 - V(\phi)$

What we have covered so far:

a) Setting up the notation of the SM Lagrangian, including Higgs mechanism

b)The miracles of the particle spectrum of the SM: Anomaly cancellation and the Custodial symmetry!

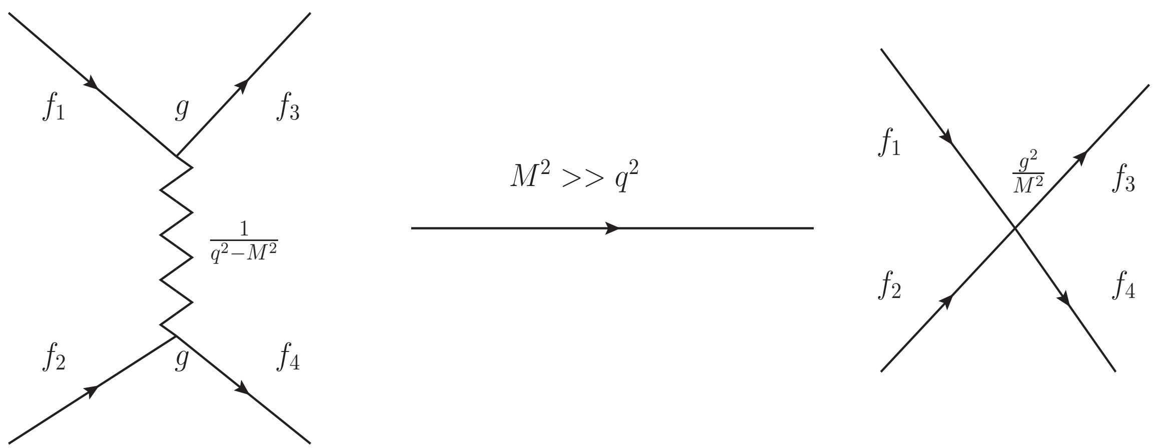
e) Test of EW unification with the determination of $\sin\theta_w$ **and resultant test of a unified gauge field theoretic description of Electro Weak interactions.**

Prediction of new particles and their masses in the SM:

c) How one can understand the development of the SM also in terms of taming the bad high energy behavior of the scattering amplitudes!

d) GIM and prediction of M_c from the observed mass difference K_L–K_S. The 'first' use of an **indirect** effect to predict a mass!

Hypothesis: The current-current interaction is just an approximation to a real ampiltude with a very heavy boson. Good agreement of Current Current interaction idea with data meant that **IVB** was necessarily heavy. IVB: Klein/Schwinger



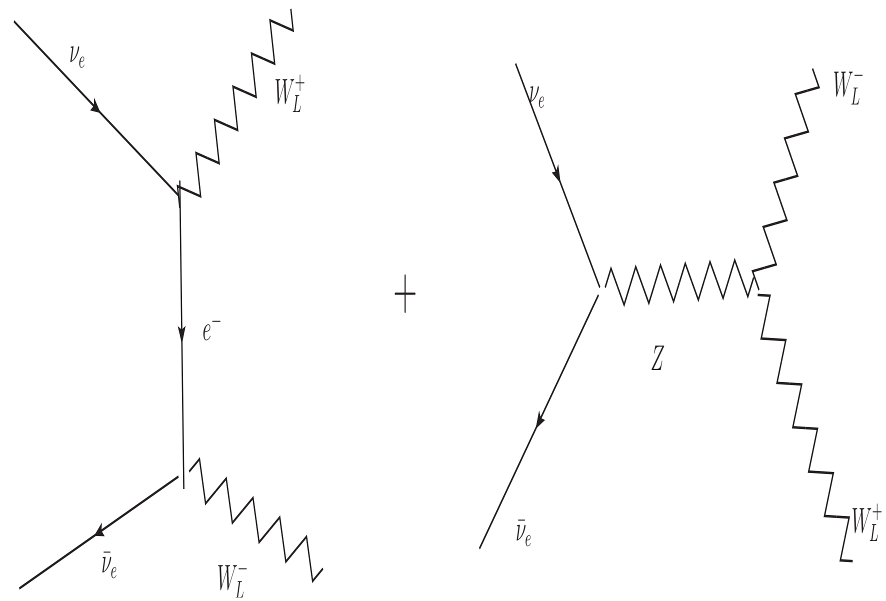
$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}}(J^W_\mu J^{W\mu\dagger}) = \frac{G_F}{\sqrt{2}}(\bar{\psi}_3\gamma_\mu\psi_1)(\bar{\psi}_4\gamma^\mu\psi_2); \quad \frac{G_F}{\sqrt{2}} \propto \frac{g^2}{M^2}$$

Fermi theory (Current-Current Interactions): $\mathcal{M}(\nu e \to \nu e)$ violates tree level unitarity for $\sqrt{s} \simeq 250 \sim G_F^{-1/2}$ GeV $\Rightarrow$ massive gauge bosons. Mass? Somewhere below this!

1)Even with a mass for the IVB, $\mathcal{M}(\nu_e\bar{\nu}_e \to W^+W^-)$ grows too fast with energy and violates unitarity.

2)s channel exchange of a Z boson in gauge theory with precisely the non abelian gauge couplings of $SU(2)_L \times U(1)$ gauge group restores the unitary behaviour. Divergence for $WW \to WW$ MUCH worse, also cured!.

The ZW^+W^- vertex has a ten structure of the $W^3W^1W^2$ ver with a coupling $g_2 \cos\theta_w$.
-i$g_2 \cos\theta_w (k-k')_\alpha g_{\mu\nu} - (2k'+k)_\nu g_{\alpha}$ $(2k+k')_\mu g_{\alpha\nu}$



J.S. Bell: Nuclear Physics, **B60**, 427, 1973:

Showed that in a renomalisable theory tree level amplitudes satisfy unitarity.

But three sets of authors asked the opposite question: What can we deduce by demanding that tree level amplitudes satisfy unitarity.

J. M. Cornwall, D. N. Levin and G. Tiktopoulos, PRL **30**, 1268 (1973), Phys.Rev. **D10**, 1145 (1974),

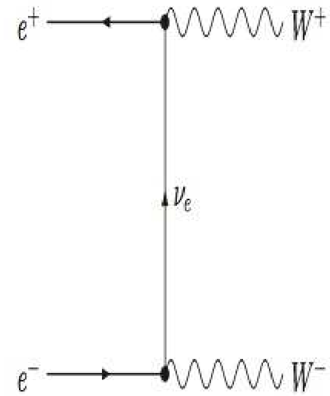
C. Llewllyn Smith : PLB **46**, 233 (1973)

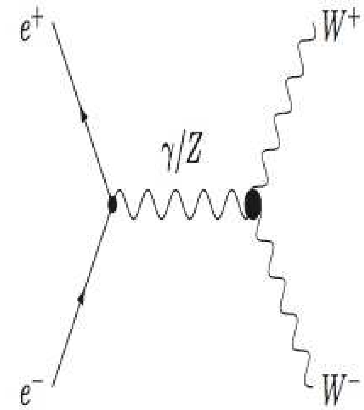
S.D. Joglekar,: Ann. Phys. **83**, 427 (1974)

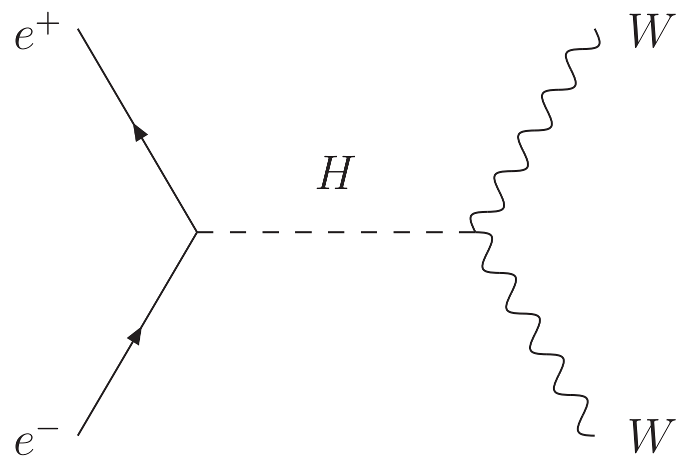
They showed that such demands uniquely indicate spontaneously broken gauge theories.

For example:

Unitarity of $\mathcal{A}(e^+e^- \to W_L W_L)$, for example, $\Rightarrow$ Divergences cancel only if there is a $J = 0$ amplitude (s channel exchange of a Spin 0 particle) whose coupling to matter/gauge particles is proportional to their masses $\Rightarrow$ **Existence of a Higgs boson.**







But no knowledge on the scale! ie. the mass of the Higgs!
Only the couplings.

SSB Gauge theory makes scattering amplitudes well behaved at high energy, even with massive gauge bosons!.

In fact Higgs couplings to matter and gauge bosons required to be proportional to masses to have unitarity.

This proportionality is indeed one of the prediction of the renormalisable $SU(2)_L \times U(1)_Y$ where the EW symmetry broken spontaneously by the Higgs mechanism!

There seems to be a lot of information in 'unitarity' requirement!

With massive W indeed the cross-section rise with s is tempered.

One gets a logarithmic violation of unitarity as opposed to the the power violation seen earlier.

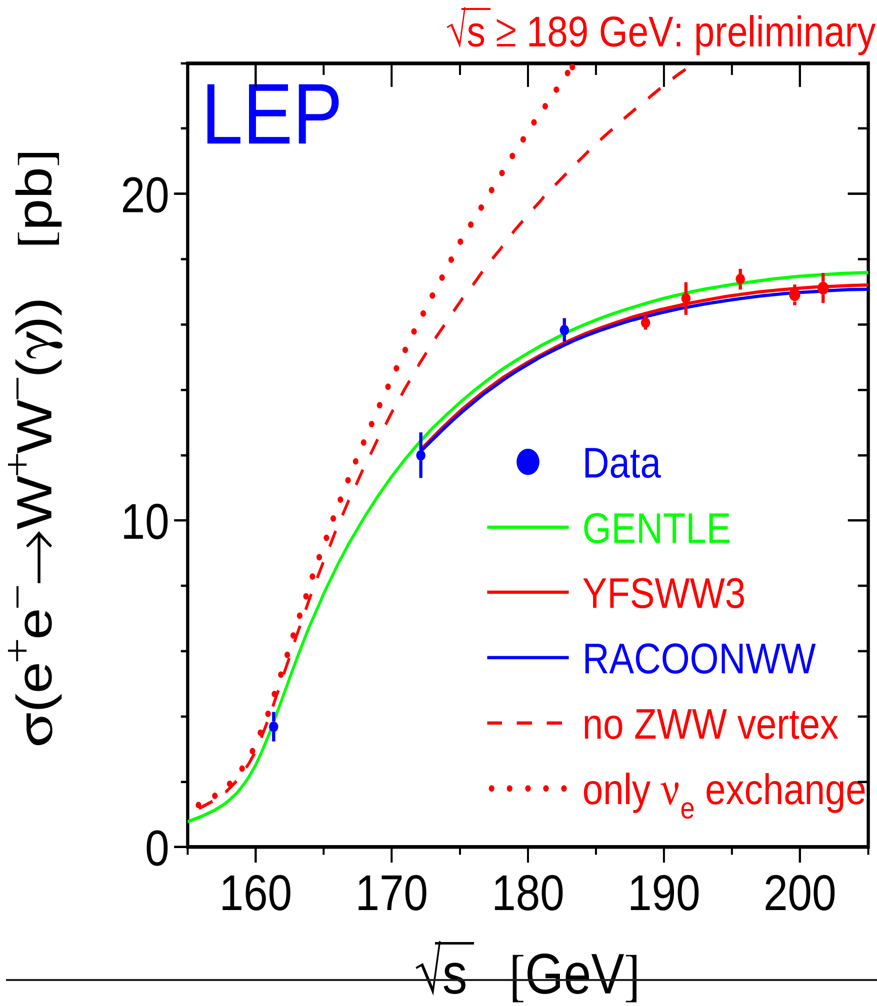
This in principle unitarity unitarity violation gets cured by the running of the coupling g_2. scale dependence of the coupling due to quantum corrections.

This whole thing points at deep connection between unitarity and renormalisability.

(book by Greiner)

Currently Arkani Hamed et al have been able to somehow show that only field theories with particles with spin $0, 1/2, 3/2, 1$ and 2 are consistent with unitarity demands on scattering amplitudes!

Direct 'Proof' of Symmetry and Symmetry breaking!!



Proof that electroweak symmetry exists and that it is broken.

The triple gauge boson ZWW coupling tames the bad high energy behaviour of the cross-section caused by the t-channel diagram. Direct proof for the ZWW coupling.

This and precision testing, confirm basics of the SM

The next point we wish to discuss is how the SM 'naturally' accommodates the observed suppression of the flab our changing neutral current processes: FCNC

Current eigenstates that couple to the W are two doublets:

$$\begin{pmatrix} u \\ d\cos\theta_c + s\sin\theta_c \end{pmatrix} \qquad \begin{pmatrix} c \\ -d\sin\theta_c + s\cos\theta_c \end{pmatrix}$$

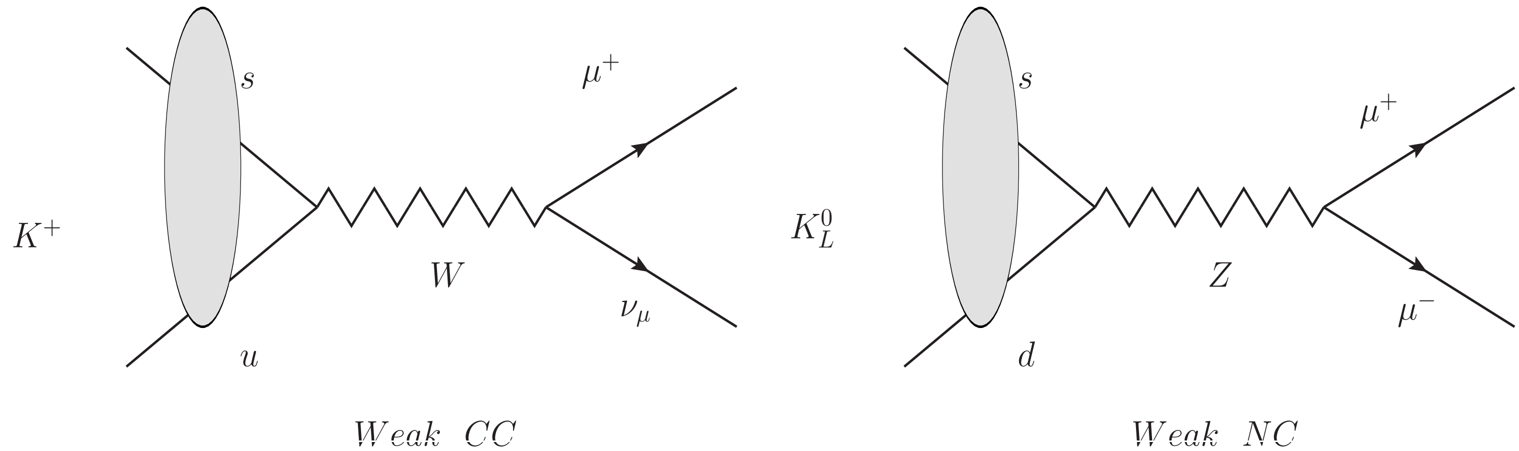
Quark mixing in two generations can then be represented by

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

$$J_\mu^{CC} = \bar{d}'\gamma_\mu(1-\gamma_5)u + \bar{s}'\gamma_\mu(1-\gamma_5)c$$

(recall the $f\bar{f}'W$ vertices we saw yesterday)

Charm is postulated. This does the trick of making Flavor Changing Neutral Current vanish at least by making sure that a vertex $d\bar{s}Z$ does not exist. Can more complicated diagrams produce FCNC?

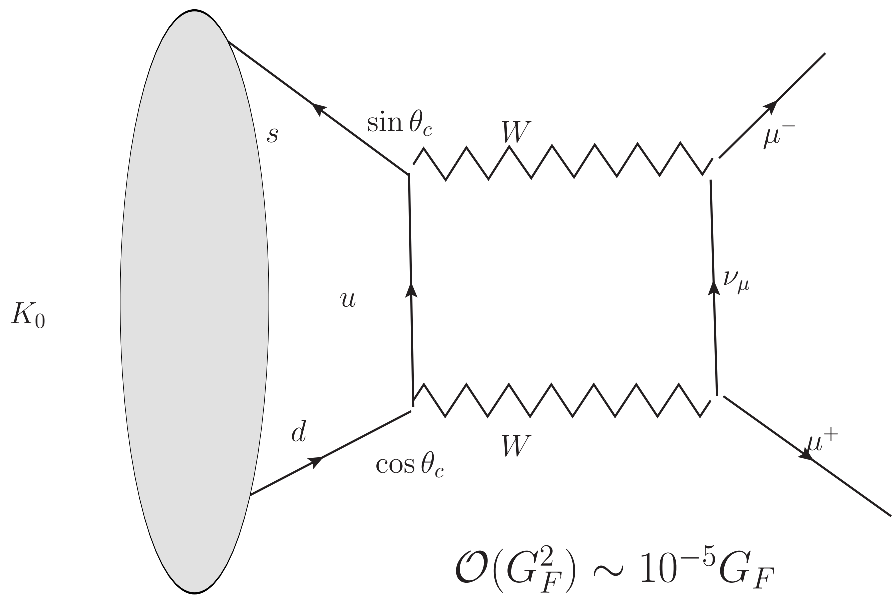


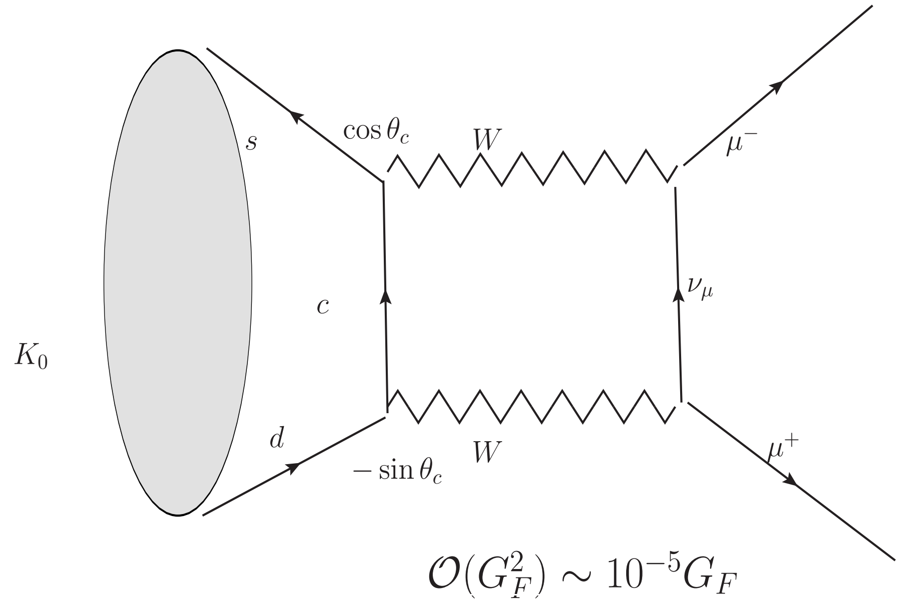
K^+, $\bar{s}u$ bound state. $K^+ \to \mu^+\nu_\mu$: weak charged current decay, $\Delta S = 1$ K^0 is a $\bar{s}d$ bound state.

If a weak neutral current with $\Delta S \neq 0$ (Flavor Changing Neutral Current: FCNC) were to exist with the same strength as the weak charged current it would cause problems. **WHY?**

$K_L^0 \to \mu^+\mu^-$ happens very rarely (one part in 10^9 among all K_L decays)

Once we have two quark doublets, tree level FCNC vanishes automaticall





What happens with loops? (I have drawn only representative ones!)

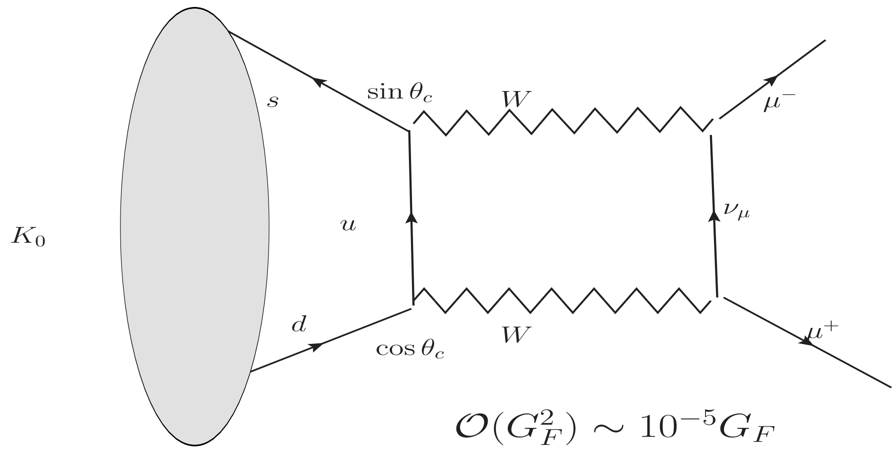
If charm contribution is absent the prediction for this flavour changing decay will be much too big compared to data.

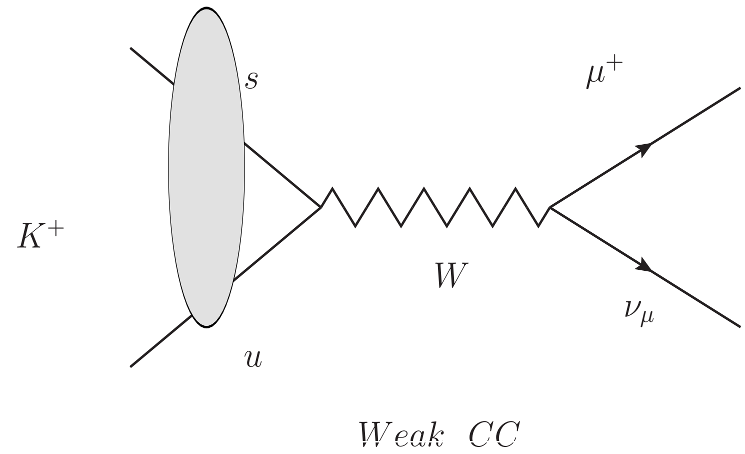
Absence of Flavour Changing Neutral Currents is granted in the EW theory ONLY IF CHARM exists. Will be exactly zero if $m_c = m_u$.

For any physics beyond SM this is always a constraint that HAS to be satisfied.

This cancellation is an example of the Glashow-Iliopoulos-Maiani (GIM) cancellation mechanism.

A very simplistic presentation given here.





Recall we already know that for a unified theory $M_W > 37.4$ GeV.

The loop amplitude, in the approximation $m_u^2 << M_W^2$ is

$$\propto \frac{g_2^2}{M_W^4} \cos\theta_c \sin\theta_c g_2^2 \times M_W^2 (1 + \mathcal{O}(m_u^2/M_W^2))$$

Recall $G_F = \frac{g_2^2}{M_W^2}$

The amplitude $\mathcal{M}_{\mu\mu} \sim G_F^2 \cos\theta_c \sin\theta_c M_W^2$

To be compared with the amplitude for $K_S \to \pi^+\pi^-$ occurring bias charged current interaction. Will happen at tree level and is given by a diagram similar to that for the K^+ decay shown in the right panel.

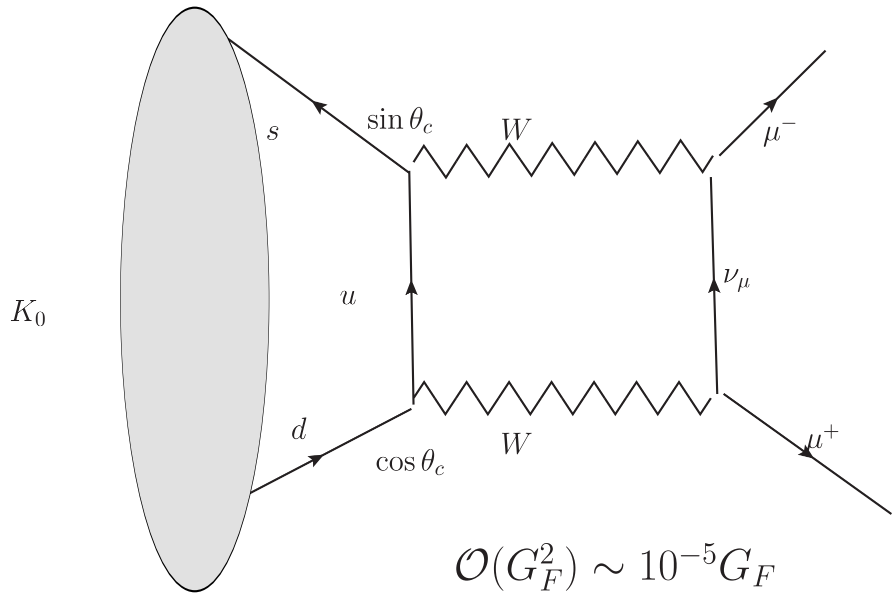
$\mathcal{M}_{CC} \sim G_F$

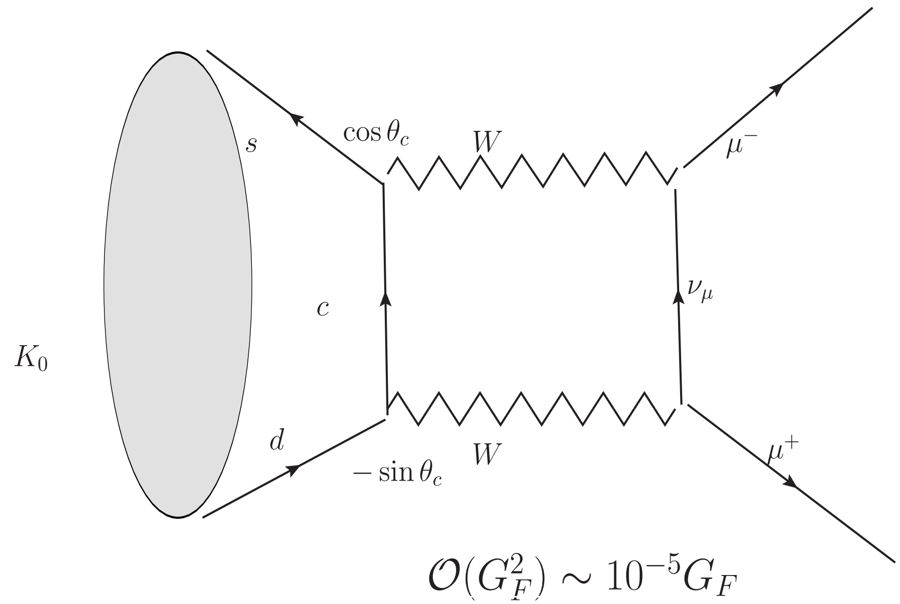
Then the branching ratio of $K^0 \to \mu^+\mu^-$ is

$\sim | \sim G_F \sin\theta_c M_W^2|^2 \sim |g_2^2 \sin\theta_c|^2 \sim 10^{-5}$

Observed suppression is 10^{-9}. $G_F \sim 10^{-5}$.

It is the large factor of M_W^2 that is removing this suppression from the coupling!

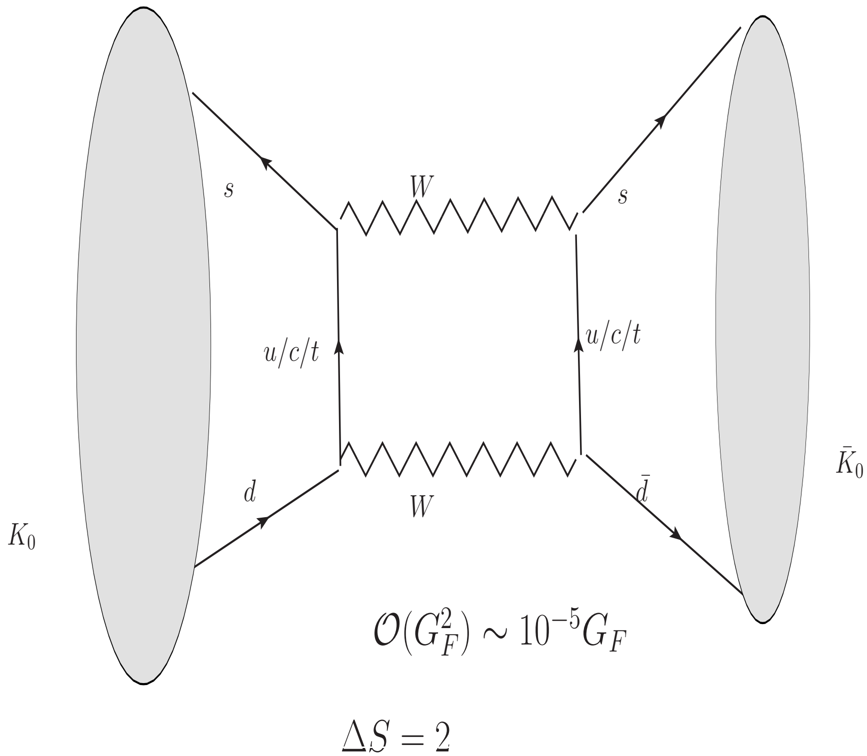




When one adds the loop contribution in the amplitude coming from u as well as c, now something interesting happens.

The large term independent of the mass of the quark in the loop cancels between these two diagrams!

Will be zero when $m_u = m_c$ and proportional to $m_c^2 - m_u^2$. So the mass dimension two factor in the amplitude $\mathcal{M}_{\mu\mu}$ is no longer the large M_W^2



Again the the quark mass independent term will cancel between the two diagrams and the resulting contribution $\propto (m_c^2 - m_u^2)$.

$$\frac{\Delta M_K}{M_K} = \frac{G_F^2}{4\pi} m_c^2 \cos^2\theta_c \sin^2\theta_c f_K^2$$

$= 7 \times 10^{-15}$ (experimental value)

Predicts $m_c \sim 1.6$ GeV.

The November revolution: Discovery of Charmonium at 3.1 GeV was the first step in validation of the SM as a renormalisable Gauge theory!

NEEDS GIM!

In the six quark picture the mass difference ΔM_K again gets its dominant part for the mass difference from the c diagrams, due to the fact that mixing between the first two and third generation is very small.

The term in loop amplitude which is independent of the quark mass goes to zero due to the unitarity of the CKM matrix.

As we learnt yesterday the unitarity of the matrix is guaranteed as it is simply the rotation between two bases: mass basis and interaction basis.

Now you will see why $B_s \to \mu^+\mu^-$ receives so much attention. The SM prediction is known and small!

For $B-physics$ the dominant contribution comes from t, the heaviest quark.

Loop contributions to rare processes, FCNC among that a probe of BSM for this reason!

Is Rabi's question answered? We somehow show that we need a new quark. Can we say FCNC ordered a new quark?

Still no answer to Rabi's question 'who ordered the μ

But what this tells is that the left handed fermions have to appear as doublets.

Second point : Quantum properties of the $SU(2)_L \times U(1)$ gauge theory imply that the number of generations should be equal for quarks and leptons. (Anomaly cancellation!)

This is peculiar feature of gauge field theories which treat the left and the right handed quarks differently.

I.e. the current coupling to a vector boson V is $\bar{f}\gamma_\mu(g_V^V - g_A^V\gamma_5)f$ with $g_A^V \neq 0$

Very simplistic description.

Consider a triangle loop diagrams with external legs being gauge bosons and the internal loop contains fermions

(draw it on the board)

This triangle diagram amplitude diverges and this can spoil the renomalisabiltiy of gauge theories.

The divergent piece exists even in the limit of zero quark mass.

So every possible triangle diagrams will contain this divergent piece.

The Miracle

When one adds contributions of all the triangle diagrams for a given triplet of external gauge boson legs, over one complete generation of quarks and leptons this piece independent of m_f cancels completely.

This is anomaly cancellation.

You need $N_c = 3$.