Deformations of
sd and *pf* shell Λ hypernuclei with AMD

Masahiro Isaka (RIKEN)

Grand challenges of hypernuclear physics

Interaction: To understand baryon-baryon interaction

- **2 body interaction between baryons (nucleon, hyperon)**
  - hyperon-nucleon (YN)
  - hyperon-hyperon (YY)

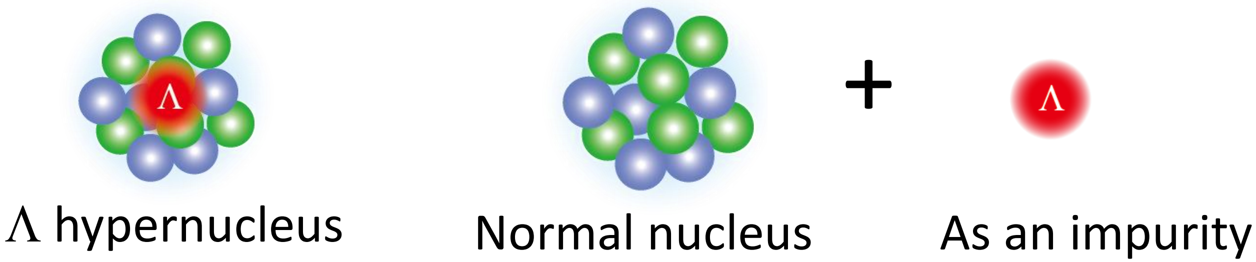
  } A major issue in hypernuclear physics

Structure: To understand many-body system of nucleons and hyperon

- **Addition of hyperon(s) shows us new features of nuclear structure**

  Ex.) Structure change by hyperon(s)
  - No Pauli exclusion between N and Y
  - YN interaction is different from NN

  } "Hyperon as an impurity in nuclei"

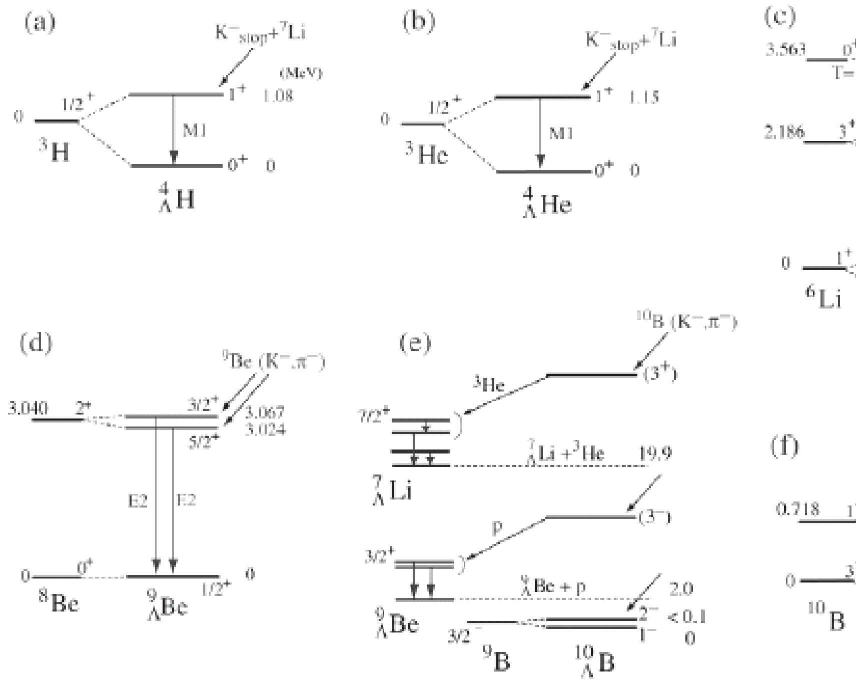


Today's talk: deformation of Λ hypernuclei

Recent achievements in (hyper)nuclear physics

Knowledge of ΛN effective interaction

- **Study of light (*s*, *p*-shell) Λ hypernuclei**
  - Accurate solution of few-body problems [1]
  - ΛN G-matrix effective interactions [2]
  - Increases of experimental information [3]



Development of theoretical models

- **Through the study of unstable nuclei**

  Ex.: Antisymmetrized Molecular Dynamics (AMD)[4]
  - AMD can describe dynamical changes of various structure
  - No assumption on clustering and deformation

Recent developments enable us to study structure of Λ hypernuclei

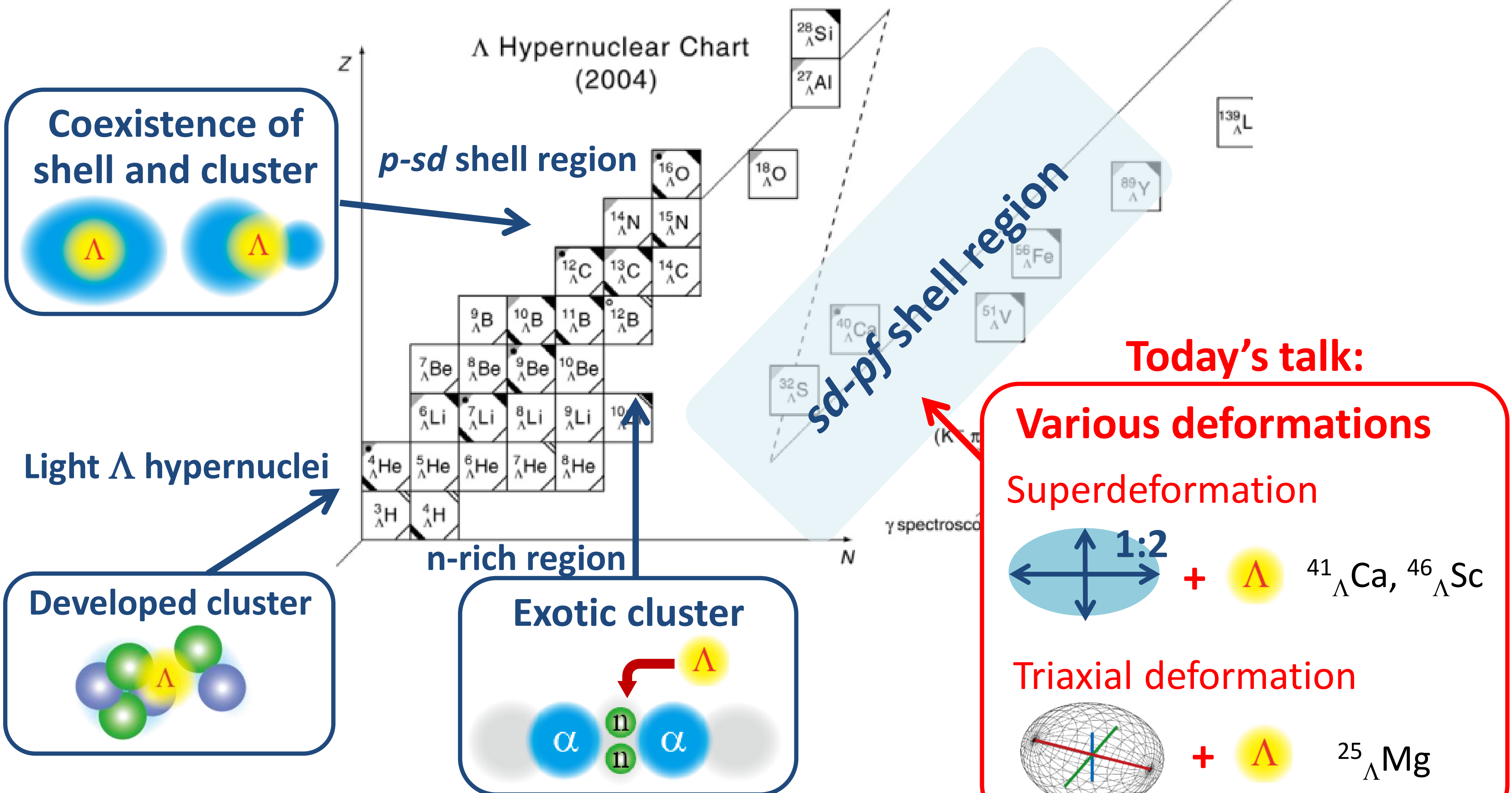
[1] E. Hiyama, NPA **805** (2008), 190c, [2] Y. Yamamoto, *et al.*, PTP Suppl. **117** (1994), 361.,
[3] O. Hashimoto and H. Tamura, PPNP **57** (2006), 564., [4] Y. Kanada-En'yo *et al.*, PTP **93** (1995), 115.

Toward heavier and exotic Λ hypernuclei

<u>Experiments at J-PARC, JLab and Mainz etc.</u>

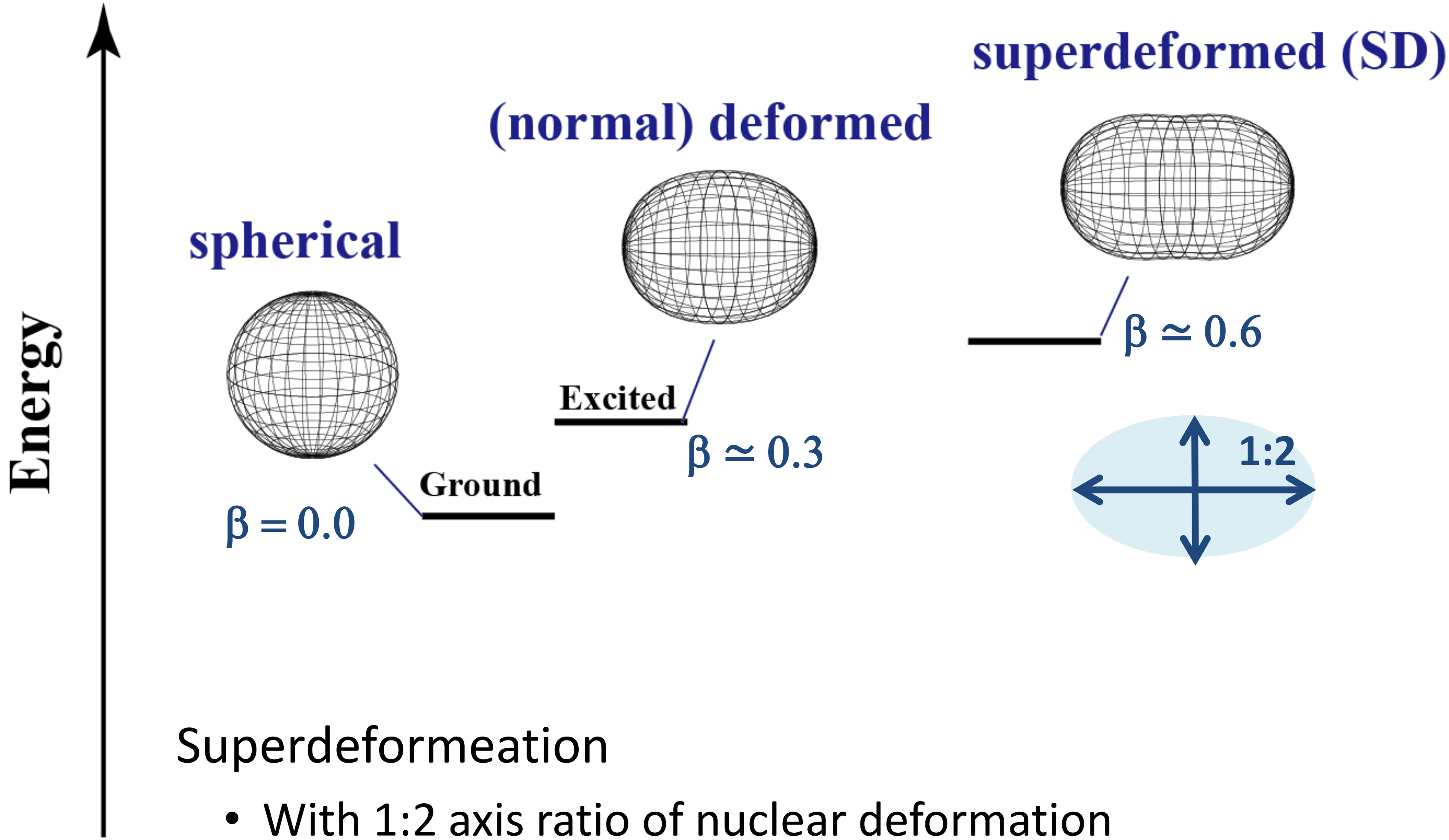
- **Hypernuclear chart will be extended to heavier regions**

"Structure of hypernuclei"



Taken from O. Hashimoto and H. Tamura, PPNP **57**(2006),564.

Superdeformation



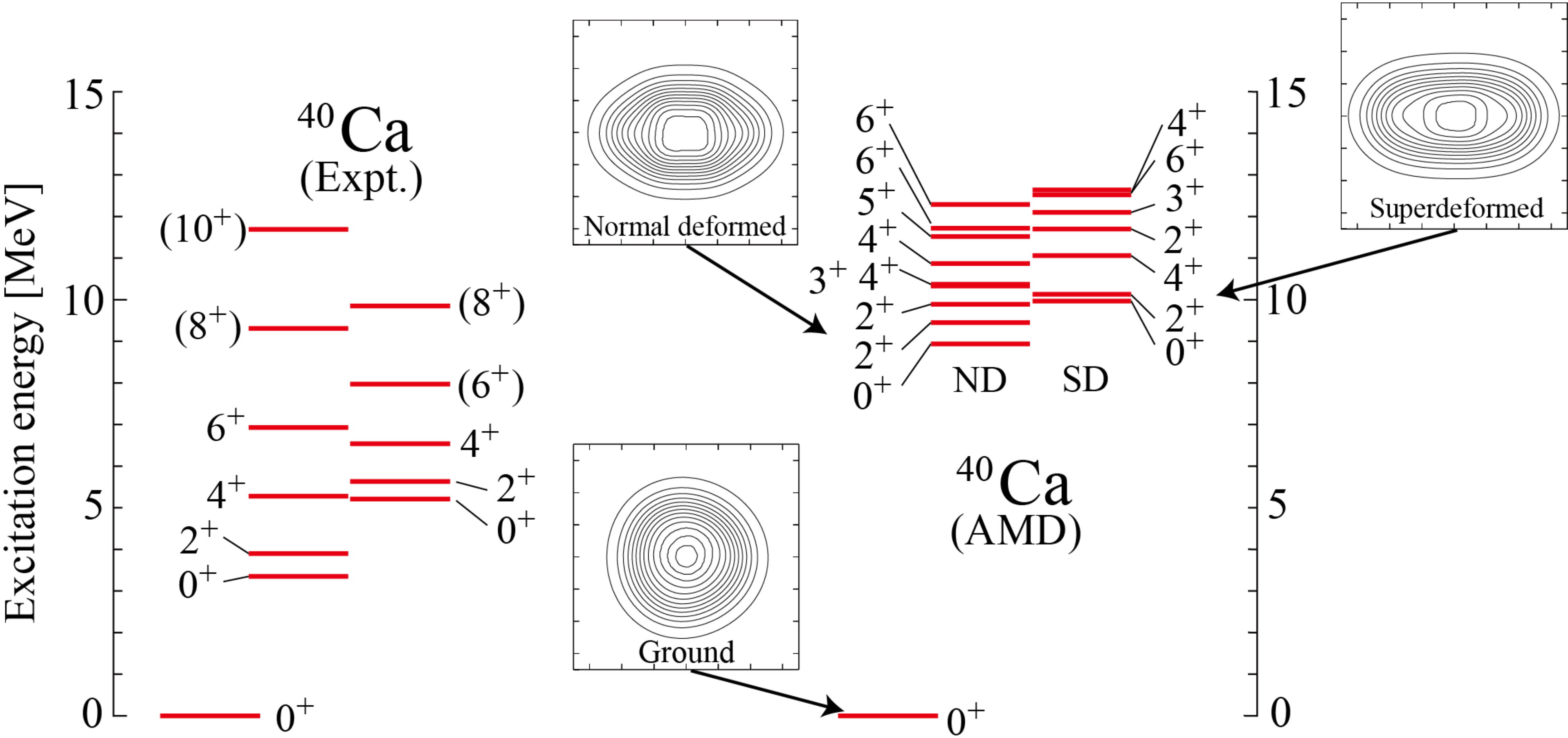
Superdeformeation

- With 1:2 axis ratio of nuclear deformation
- Observed in several sd-pf shell nuclei

Example: Superdeformed (SD) states

Ex.) ^{40}Ca — J. R. MacDonald, *et al*., PRC**3**, 219(1971), E. Ideguchi, *et al*., PRL**87**, 222501(2011)
W. Gerace and A. Green, NPA**93**, 110(1967); NPA**123**, 241 (1969)

- **Observed in ^{40}Ca, while the ground state is spherical**



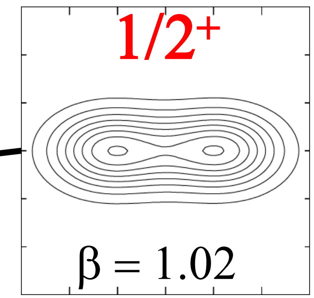
If a Λ particle is added, what's happen?

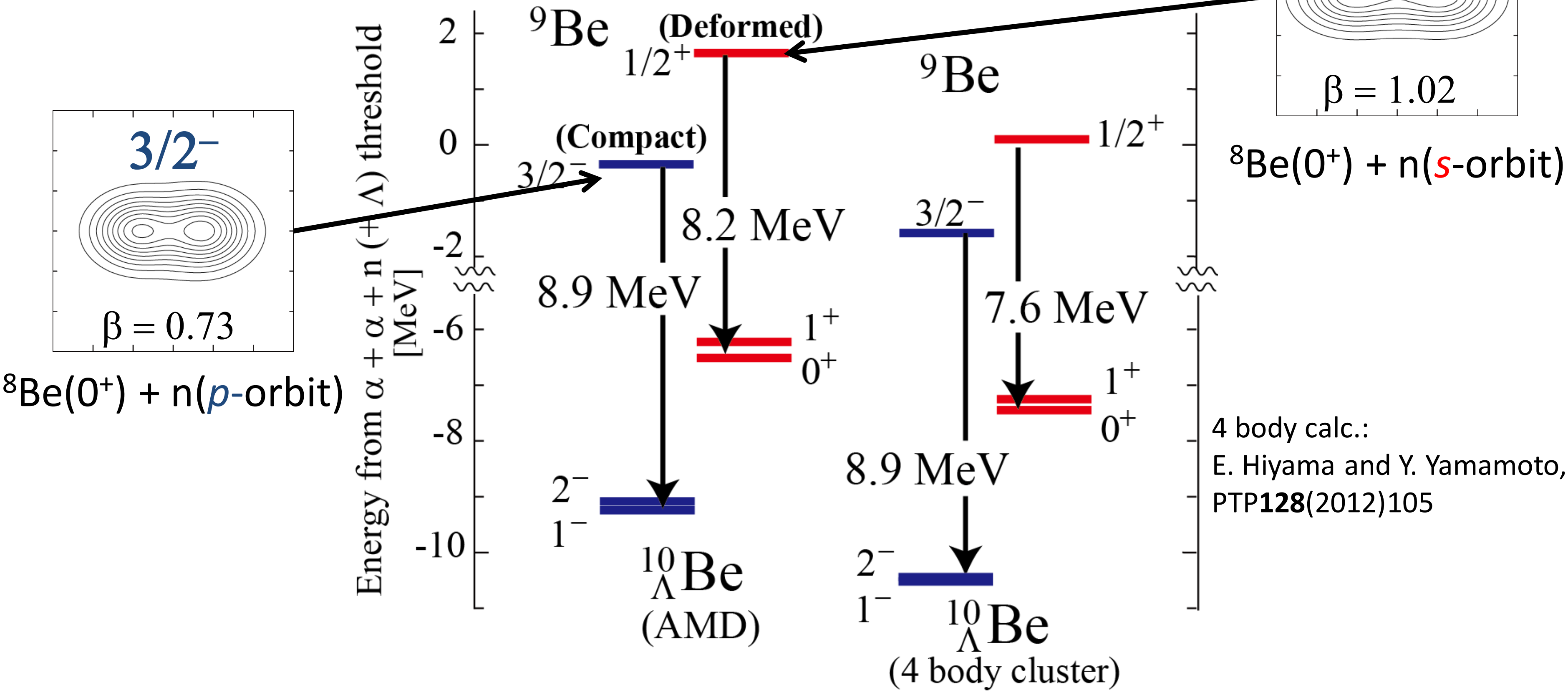
AMD calculation for ^{40}Ca: basically follows Y. Taniguchi, *et al*., PRC **76**, 044317 (2007)

B_Λ with different deformations(structures)

- **Λ binging energy (B_Λ) in light Λ hypernuclei with cluster structure**
  - **Λ coupled to the compact state is more deeply bound**
    ⟶ **Changes of excitation spectra**

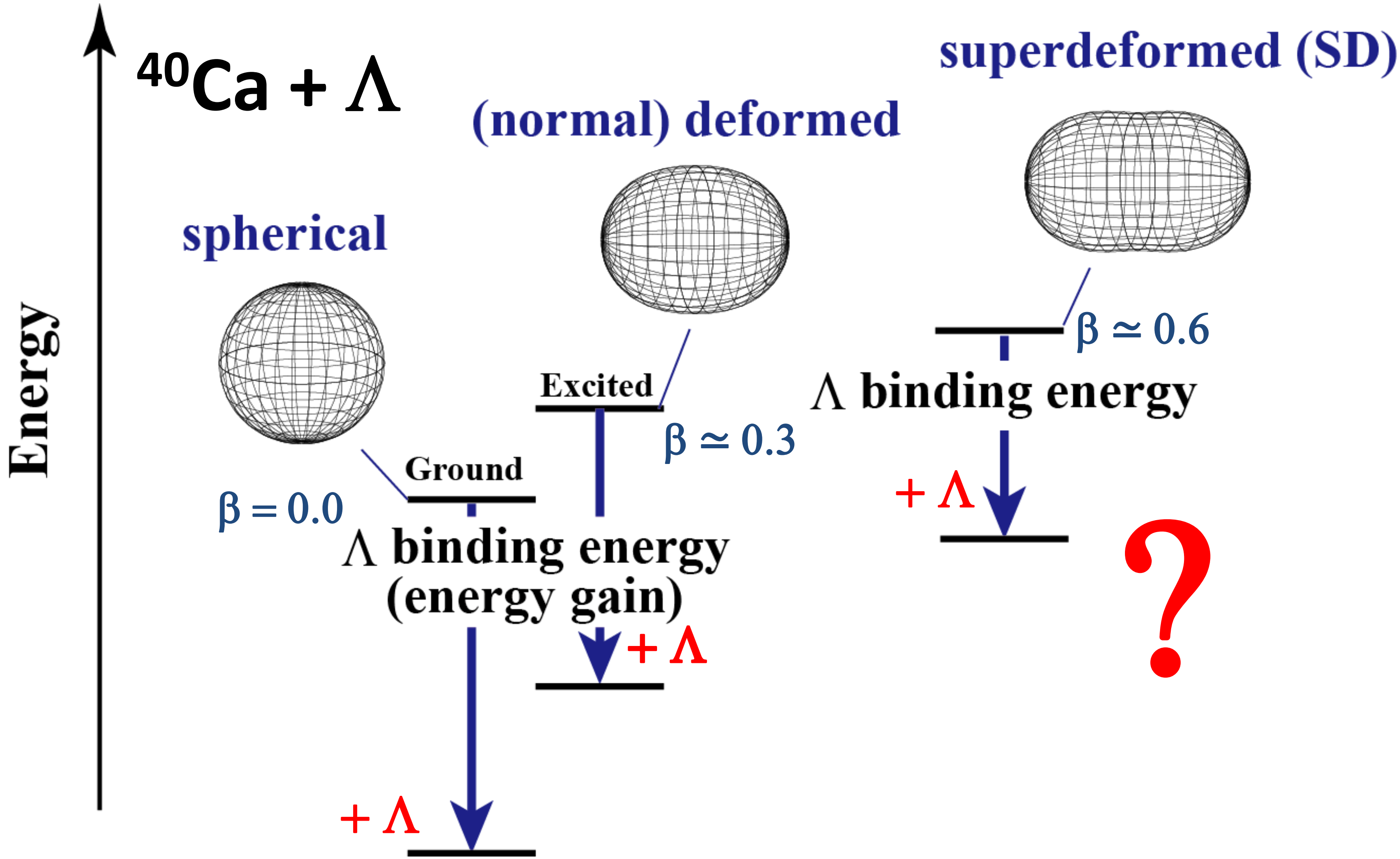
Ex.) $^{10}_{\Lambda}$Be (α + α + n + Λ)





How about difference of B_Λ in superdeformed states?

Superdeformation



Purposes: to reveal difference of B_Λ and deformation change by Λ

Our method: antiysymmetrized molecular dynamics (AMD)

Theoretical Framework: HyperAMD

M.Isaka, *et al.*, PRC**83**(2011) 044323
M. Isaka, *et al.*, PRC**83**(2011) 054304

We extended the AMD to hypernuclei

HyperAMD (Antisymmetrized Molecular Dynamics for hypernuclei)

◆Hamiltonian

$$\hat{H} = \hat{T}_N + \hat{V}_{NN} + \hat{T}_\Lambda + \hat{V}_{\Lambda N} - \hat{T}_g$$

NN : Gogny D1S

ΛN : YNG-ESC08c

◆Wave function

- **Nucleon part : Slater determinant**

  Spatial part of single particle w.f. is described as Gaussian packet

$$\varphi_N(\vec{r}) = \frac{1}{\sqrt{A!}} \det\left[\varphi_i(\vec{r}_j)\right]$$

$$\varphi_i(r) \propto \exp\left[-\sum_{\sigma=x,y,z} \nu_\sigma (r - Z_i)_\sigma^2\right] \chi_i \eta_i \qquad \chi_i = \alpha_i \chi_\uparrow + \beta_i \chi_\downarrow$$

- **Single-particle w.f. of Λ hyperon:**

  Superposition of Gaussian packets

$$\varphi_\Lambda(r) = \sum_m c_m \varphi_m(r)$$

$$\varphi_m(r) \propto \exp\left[-\sum_{\sigma=x,y,z} \mu \nu_\sigma (r - z_m)_\sigma^2\right] \chi_m \qquad \chi_m = a_m \chi_\uparrow + b_m \chi_\downarrow$$

- **Total w.f. :**

$$\psi(\vec{r}) = \sum_m c_m \varphi_m(r_\Lambda) \otimes \frac{1}{\sqrt{A!}} \det\left[\varphi_i(\vec{r}_j)\right]$$

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- **Single-particle w.f. of Λ hyperon:**
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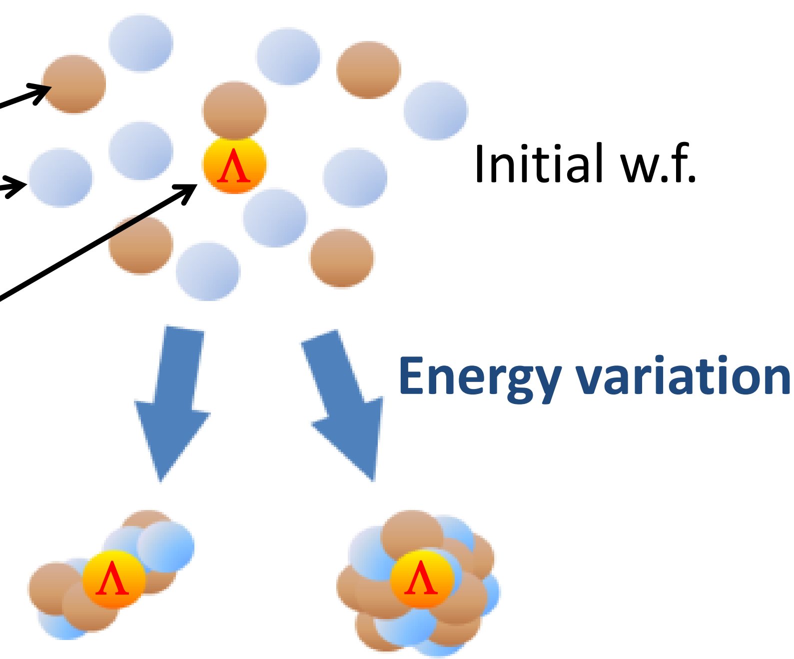

Various deformations and/or cluster structure

Theoretical Framework: HyperAMD

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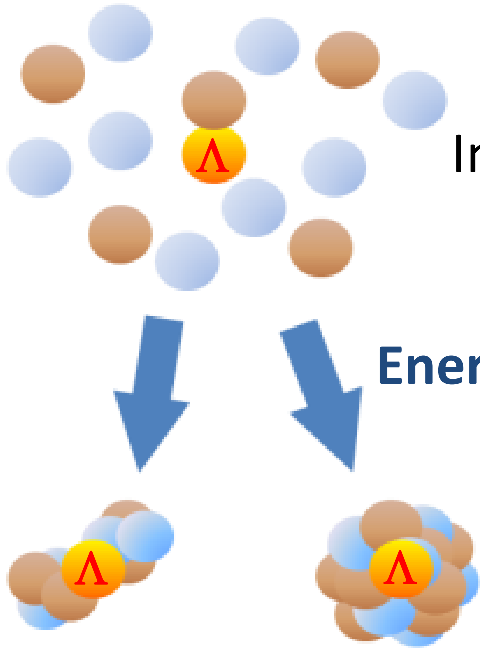
◆ Procedure of the calculation

Variational Calculation

- Imaginary time development method $\frac{dX_i}{dt} = \frac{\kappa}{\hbar}\frac{\partial H^{\pm}}{\partial X_i^*}$ $\kappa < 0$
- Variational parameters: $X_i = Z_i, z_i, \alpha_i, \beta_i, a_i, b_i, \nu_i, c_i$



Initial w.f.: randomly generated

Energy variation

Various deformations
and/or cluster structure

Theoretical Framework: HyperAMD

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↓

Angular Momentum Projection

$$\left|\Phi_K^s; JM\right\rangle = \int d\Omega D_{MK}^{J*}(\Omega) R(\Omega) \left|\Phi^{s+}\right\rangle$$

↓

Generator Coordinate Method(GCM)

- Superposition of the w.f. with different configuration
- Diagonalization of $H_{sK,s'K'}^{J\pm}$ and $N_{sK,s'K'}^{J\pm}$

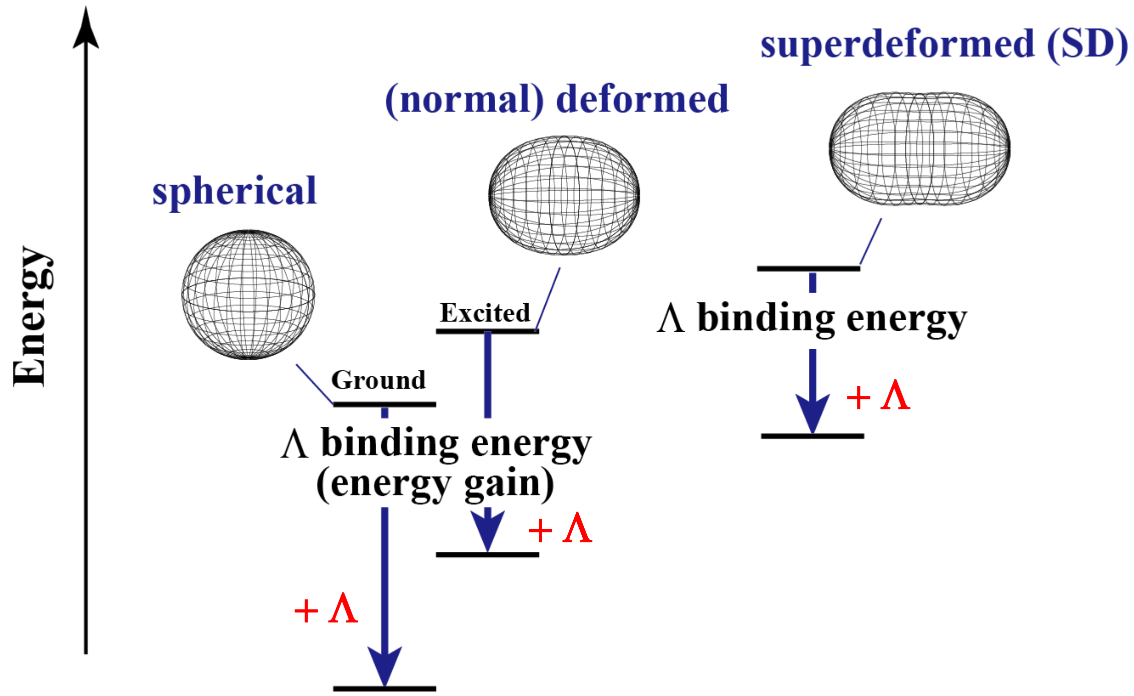
$$H_{sK,s'K'}^{J\pm} = \left\langle \Phi_K^s; J^{\pm}M \middle| \hat{H} \middle| \Phi_{K'}^{s'}; J^{\pm}M \right\rangle$$

$$N_{sK,s'K'}^{J\pm} = \left\langle \Phi_K^s; J^{\pm}M \middle| \Phi_{K'}^{s'}; J^{\pm}M \right\rangle$$

$$\left|\Psi^{J\pm M}\right\rangle = \sum_{sK} g_{sK} \left|\Phi_K^s; J^{\pm}M\right\rangle$$

Results and Discussions

B_Λ is different depending on deformations ?



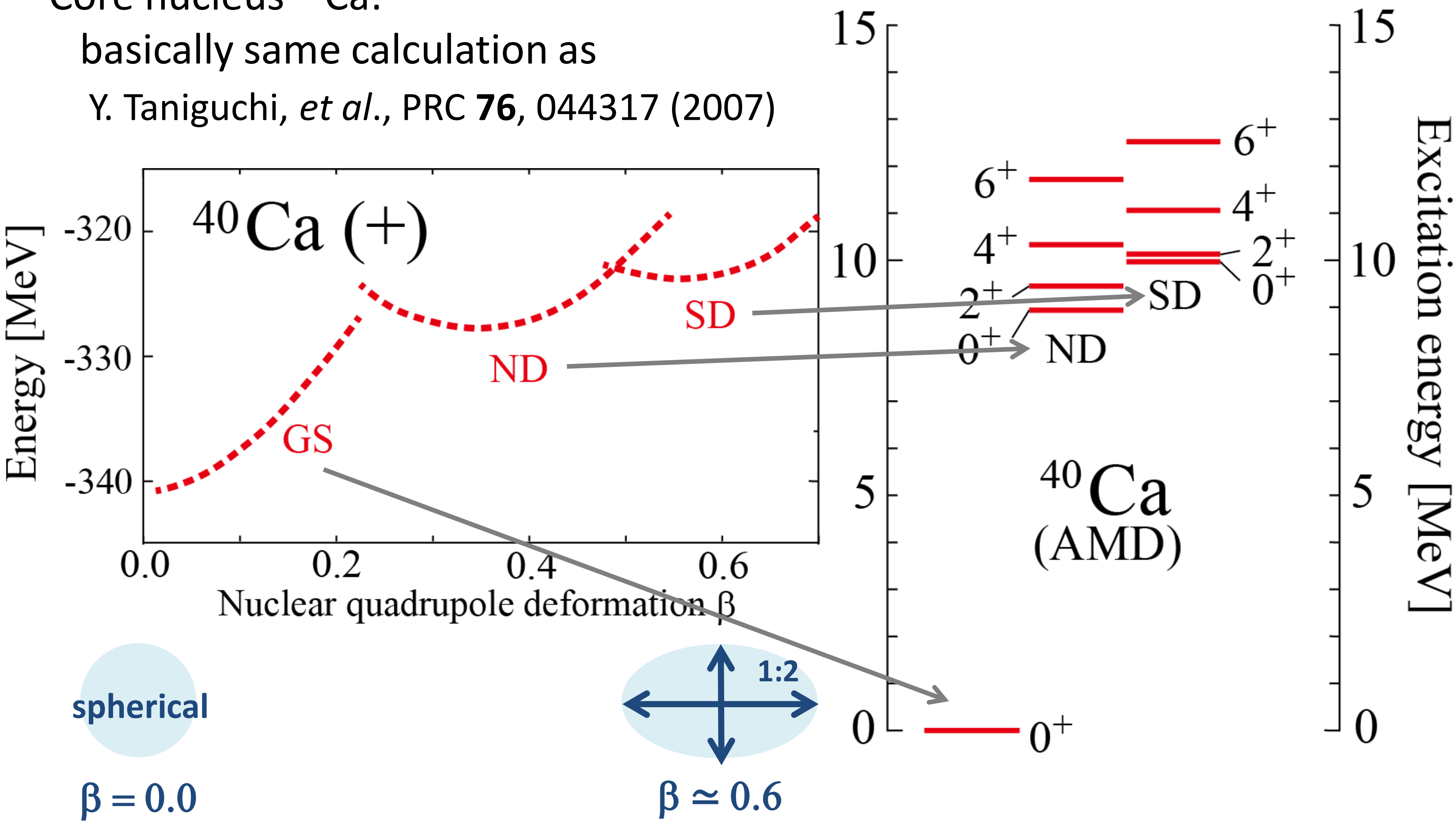
1. Energy curves and Λ single particle energy
2. B_Λ and excitation spectra

M.I., K. Fukukawa, M. Kimura, E. Hiyama, H. Sagawa, and Y. Yamamoto, PRC**89**, 024310 (2014)

ND and SD states of ^{40}Ca

- **Ground, normal deformed and superdeformed states are obtained**

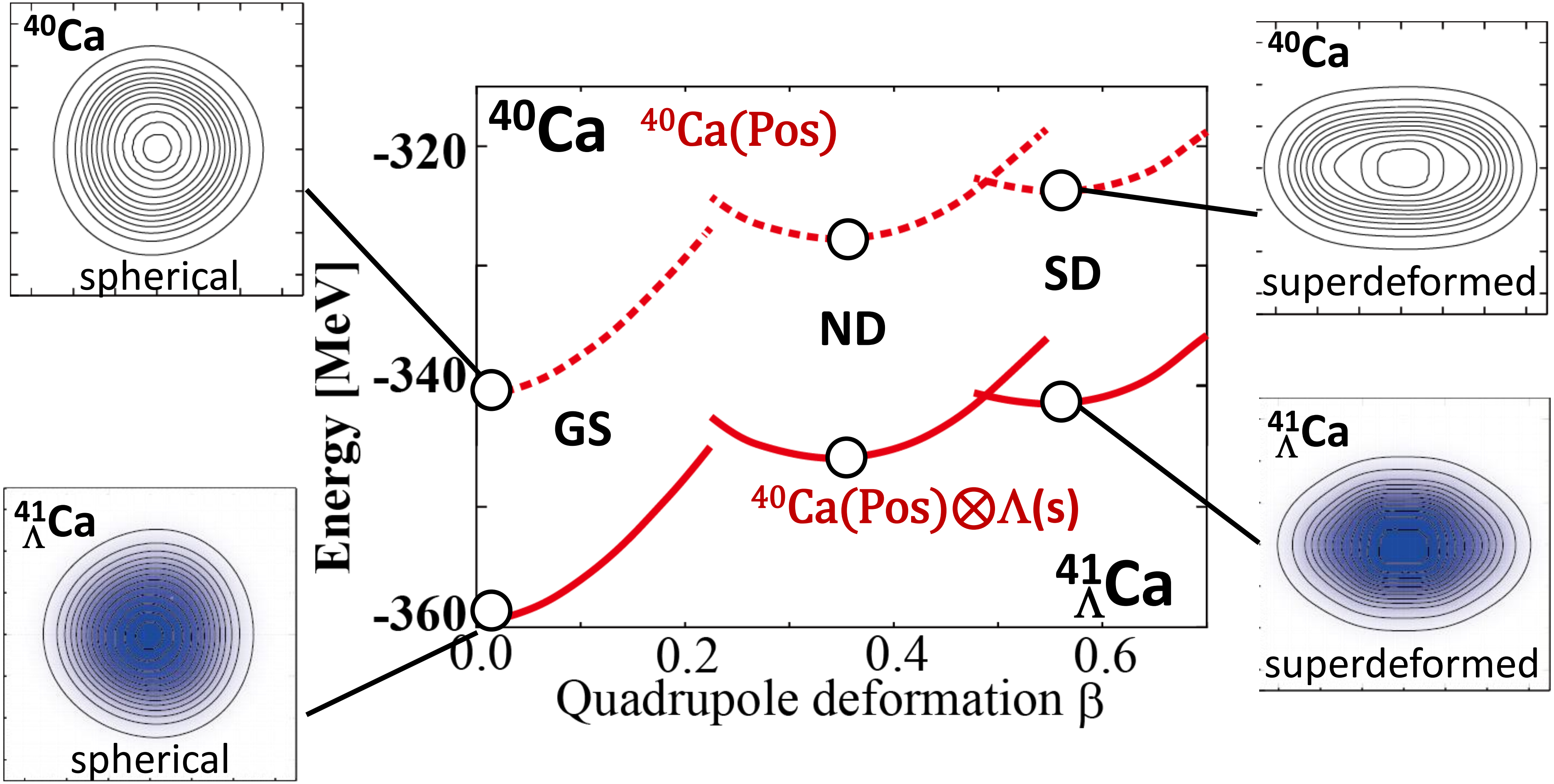
Core nucleus ^{40}Ca:
basically same calculation as
Y. Taniguchi, *et al.*, PRC **76**, 044317 (2007)



Energy curves of $^{41}{}_{\Lambda}$Ca as a function of β

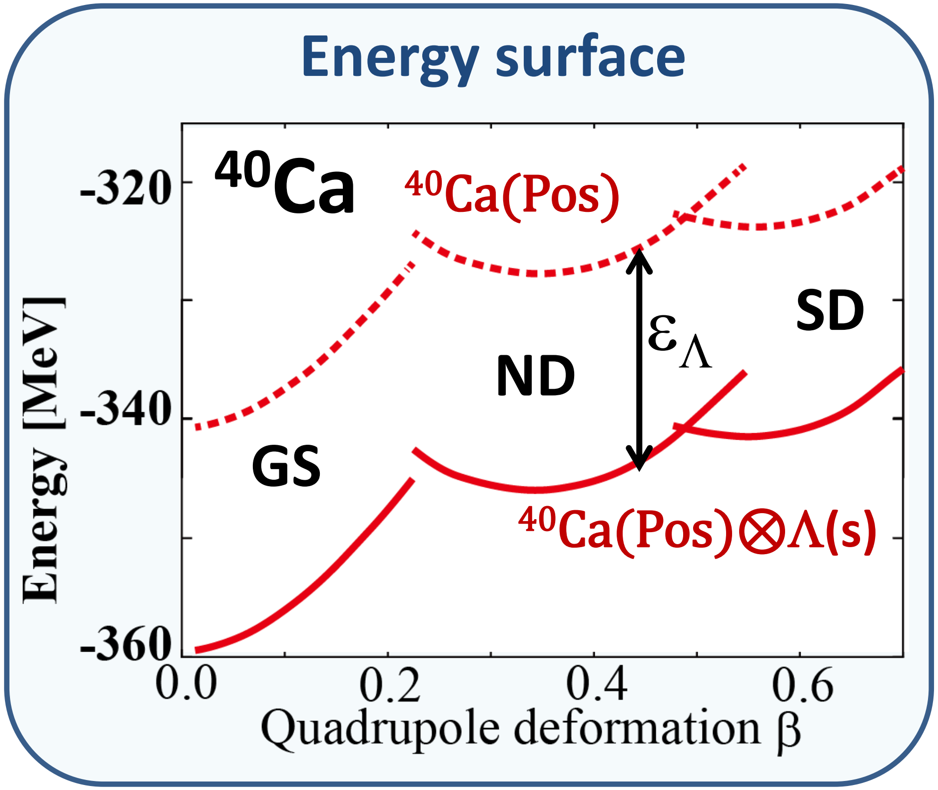
$^{41}_{\Lambda}$Ca

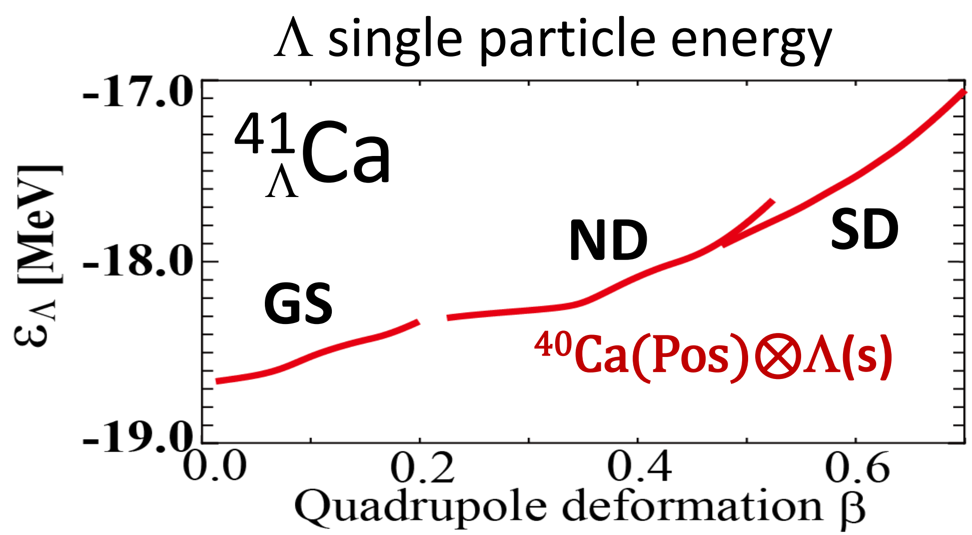
- **"GS⊗Λ", "ND ⊗Λ " and "SD ⊗Λ " curves are obtained**
  → SD states will appear in $^{41}{}_{\Lambda}$Ca
- **Energy (local) minima are almost unchanged**



Λ single particle energy

- **Definition:** $\epsilon_\Lambda(\beta) = E({}^{46}_{\Lambda}\mathrm{Sc})(\beta) - E({}^{45}\mathrm{Sc})(\beta)$
- **General trend: ε_Λ changes within 1 - 2 MeV as β increases**

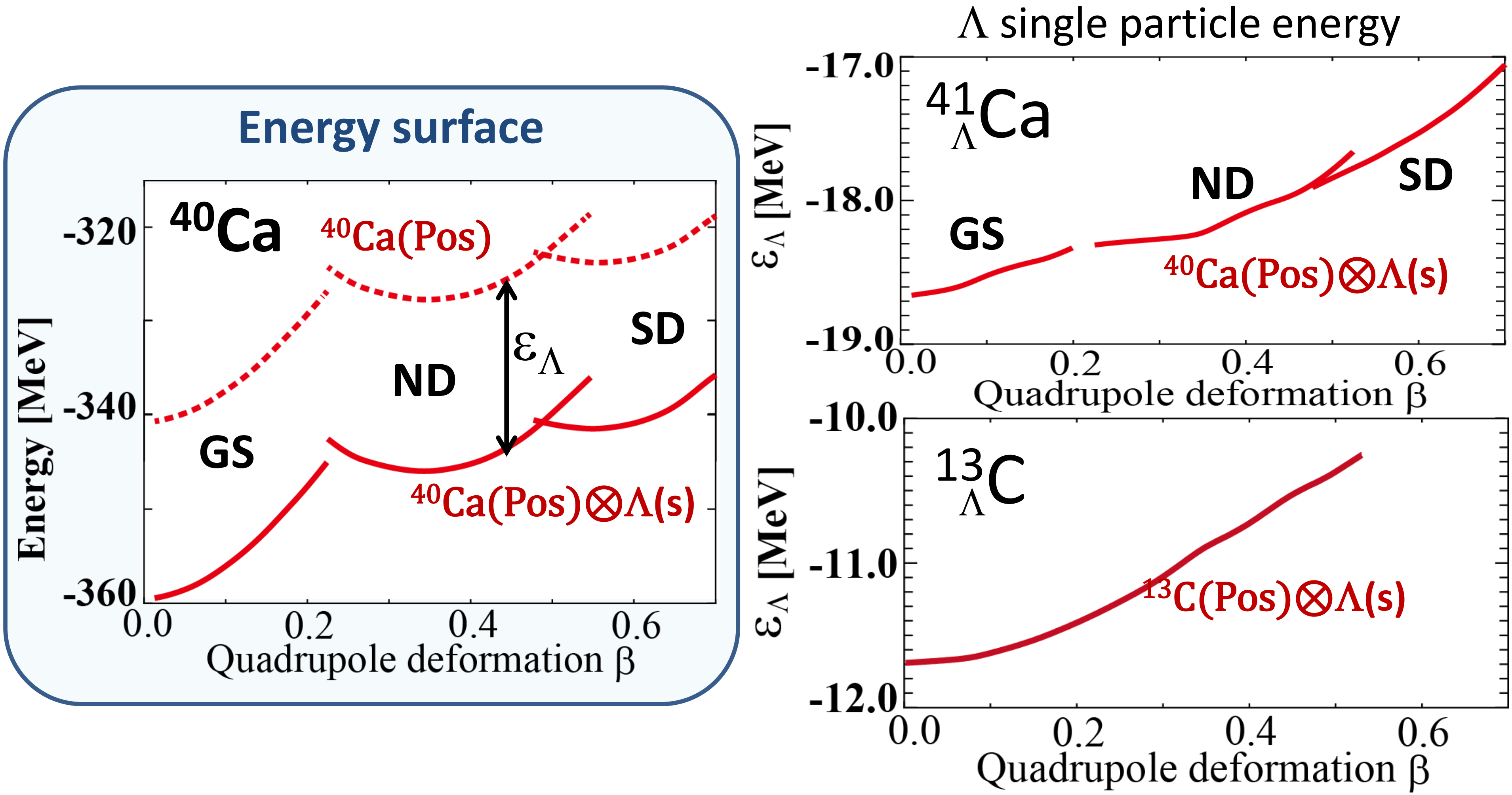




Λ single particle energy

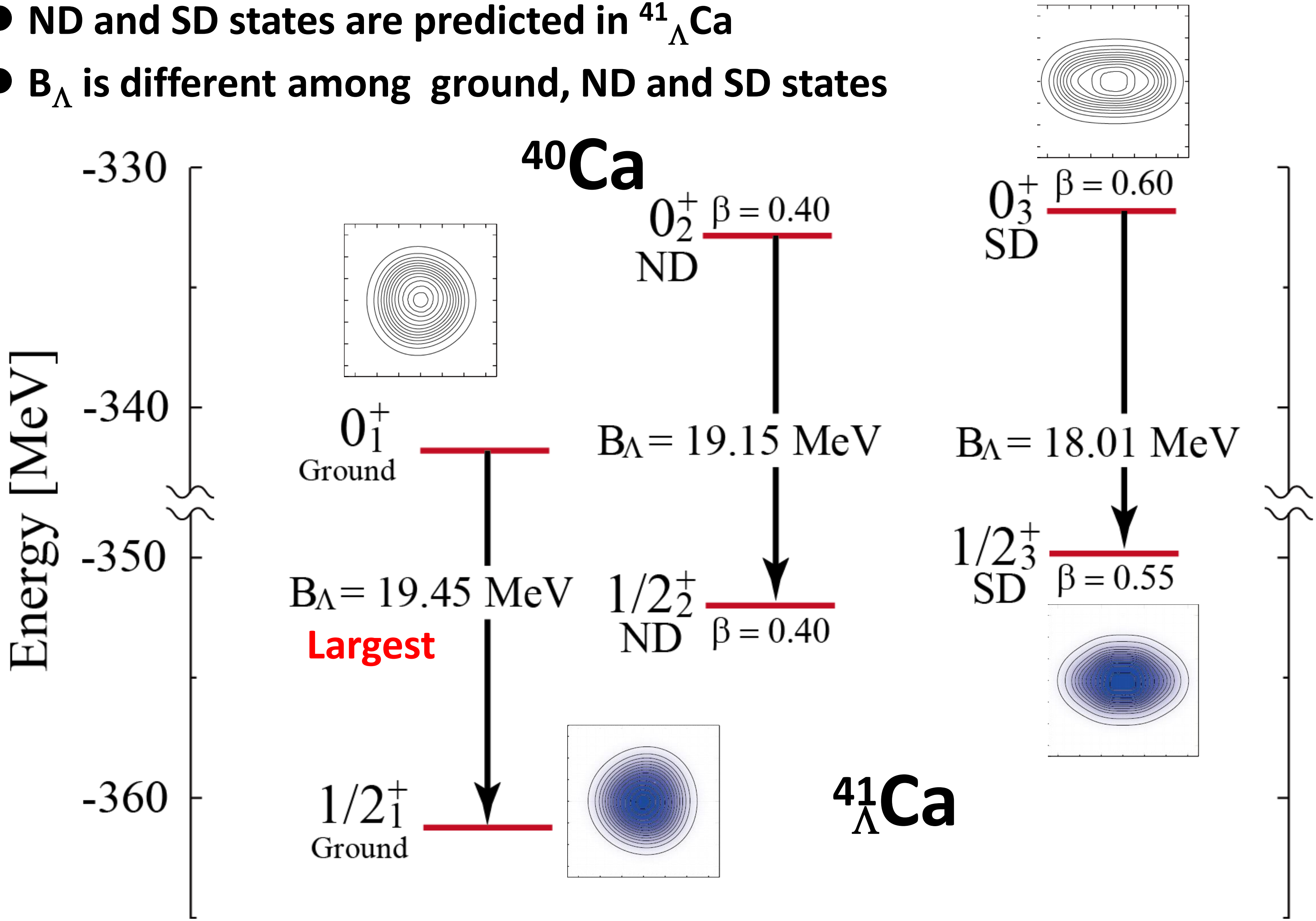
- **Definition:** $\epsilon_\Lambda(\beta) = E(^{46}_{\Lambda}\mathrm{Sc})(\beta) - E(^{45}\mathrm{Sc})(\beta)$
- **General trend: ε_Λ changes within 1 - 2 MeV as β increases**

→ **Similar to the *p* shell Λ hypernuclei**



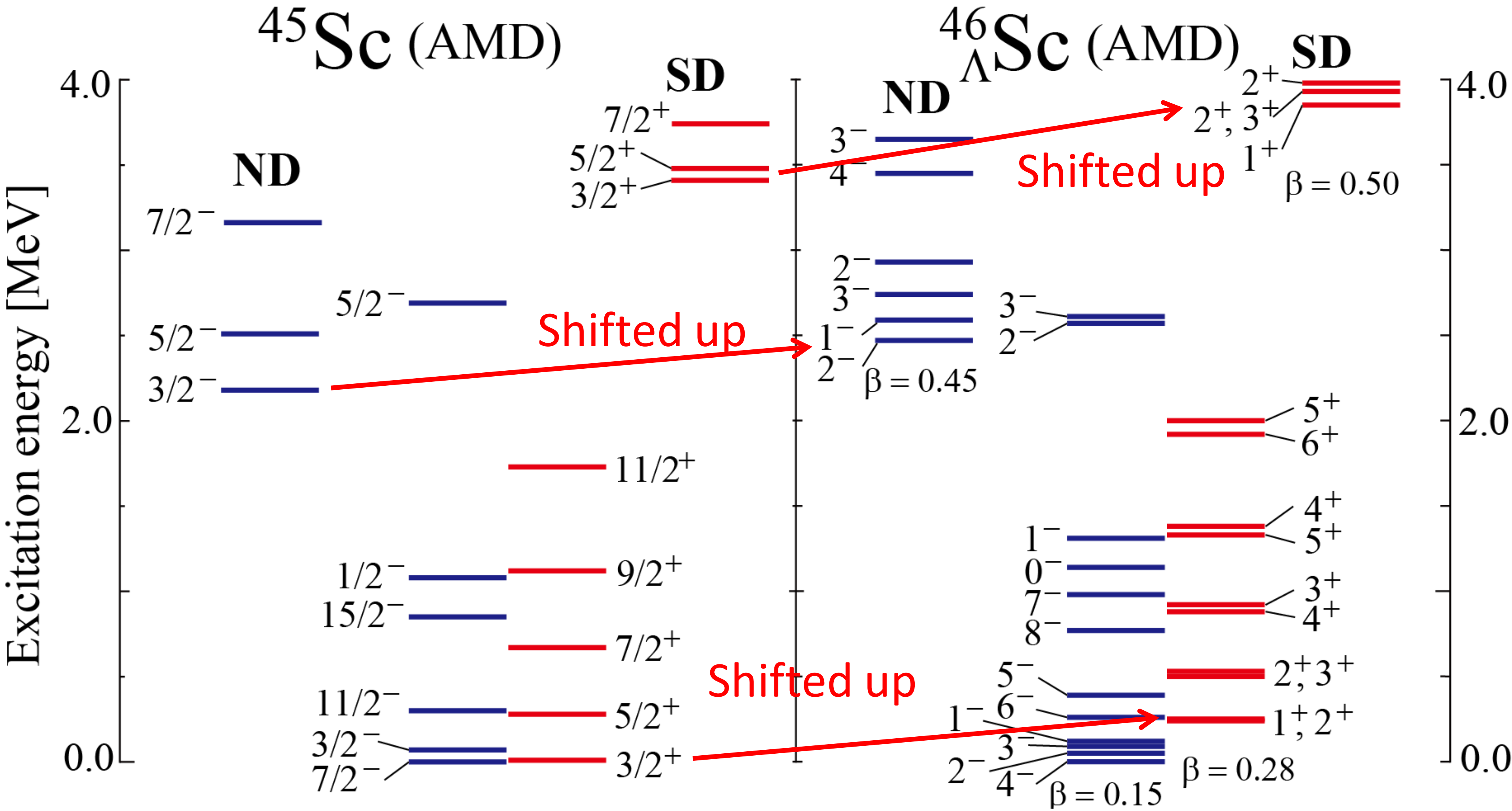
Difference of Λ binding energy B_Λ

- ND and SD states are predicted in $^{41}_{\Lambda}$Ca
- B_Λ is different among ground, ND and SD states



Excitation spectra of $^{46}{}_{\Lambda}$Sc and $^{48}{}_{\Lambda}$Sc

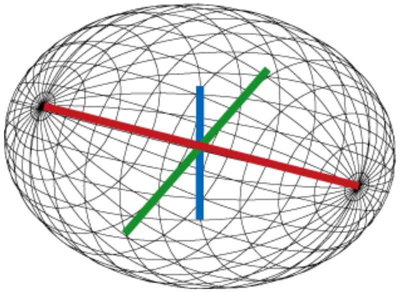
- **Difference of B_{Λ} leads to the energy shift up of the deformed states**
- **Similar phenomena in $^{48}{}_{\Lambda}$Sc**



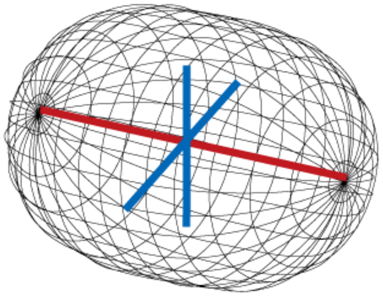
We hope these states in Sc Λ hypernuclei are observed at JLab

Results and Discussions

Changes of triaxial deformed hypernucleus $^{25}{}_{\Lambda}$Mg



Triaxial deformation



cf. Prolate deformation

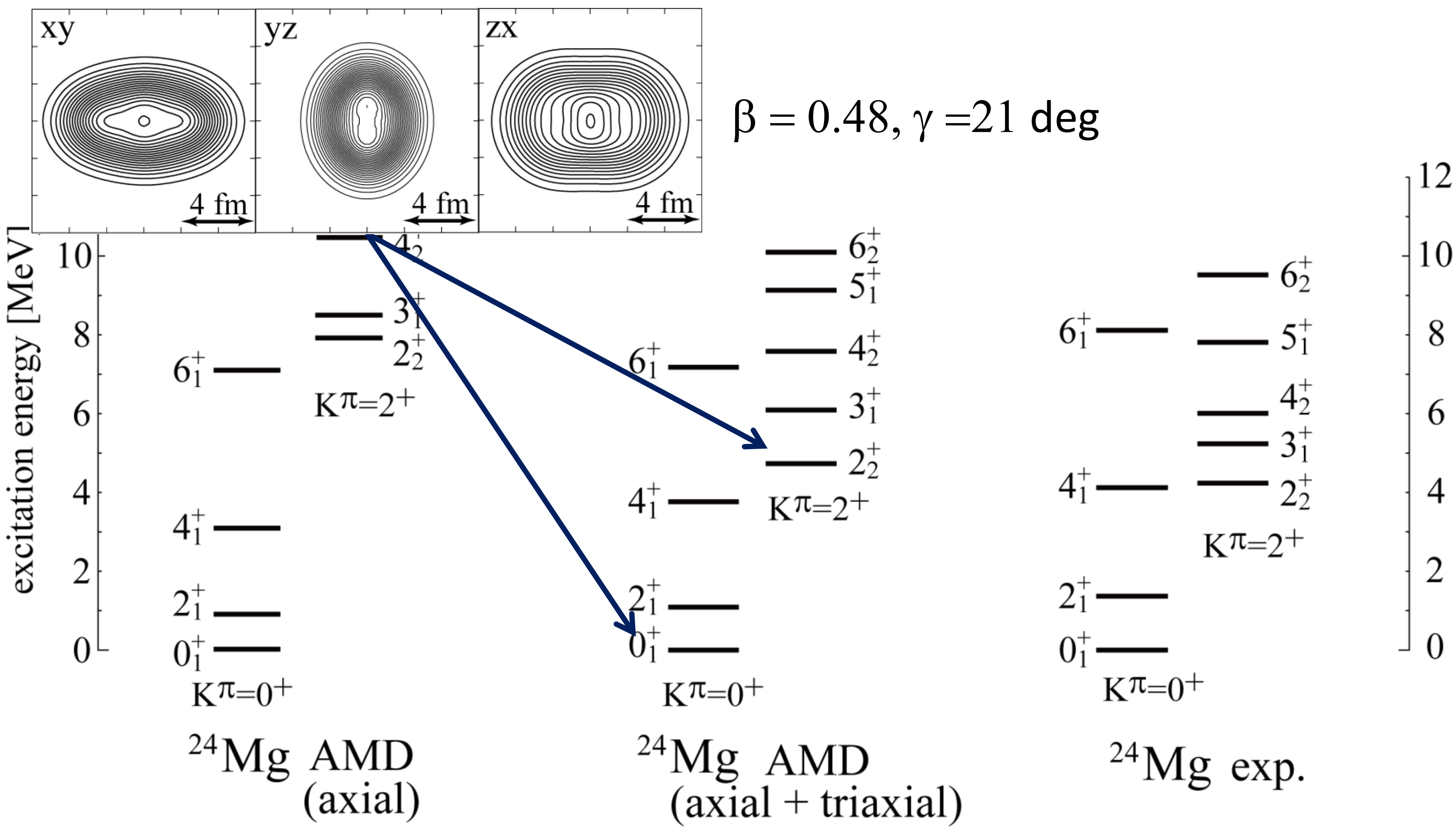
How does a Λ particle modify triaxial deformation and affect excitation spectra?

M. I., M. Kimura, A. Dote and A. Ohnishi, PRC **85** (2012), 034303.

Triaxial deformation

Ex.) ^{24}Mg   Candidate of triaxial deformed nuclei

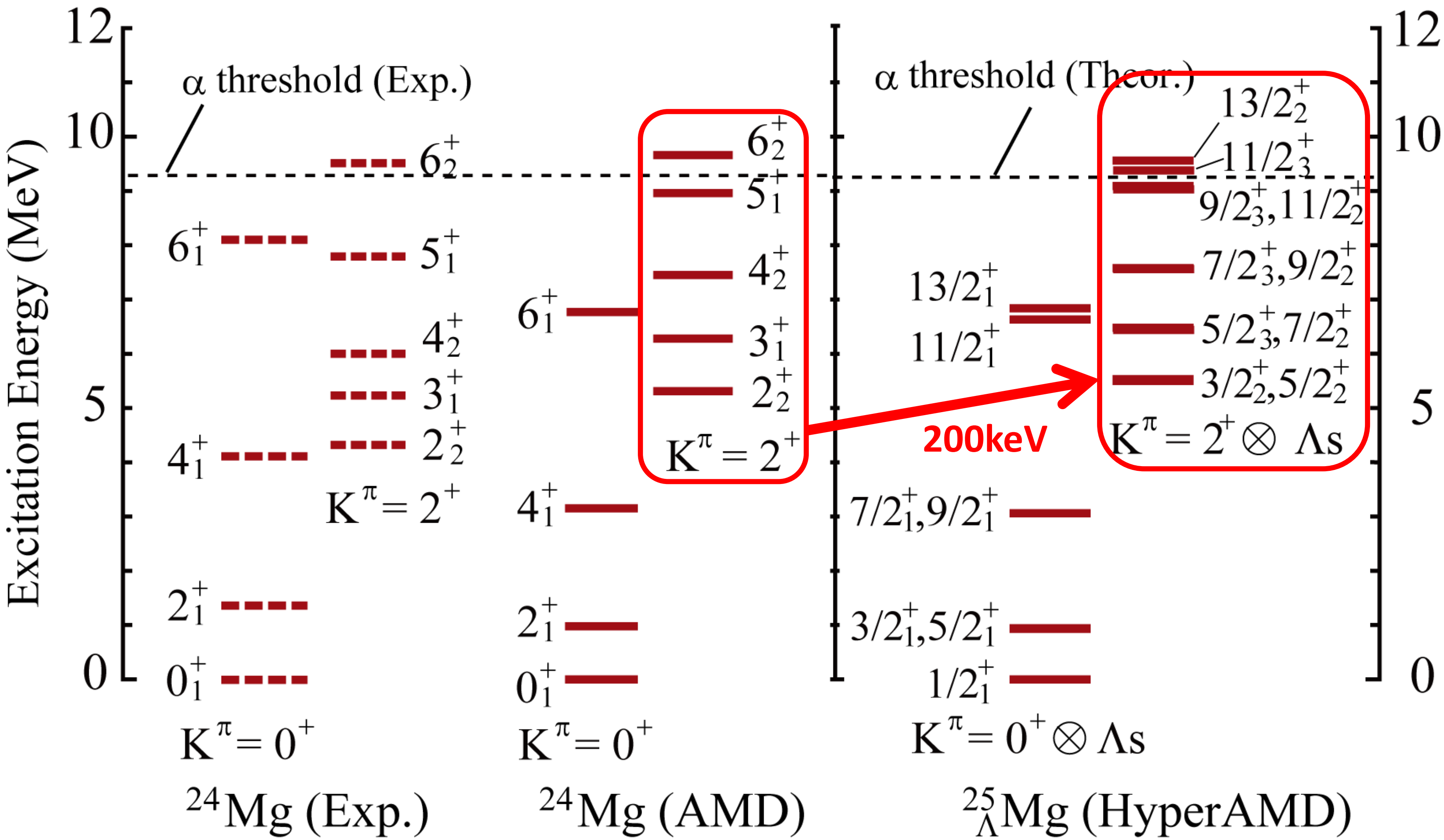
- **Low lying $K^{\pi}=2^{+}$ band: a sign of triaxial deformation**
- **$K^{\pi}=0^{+}$ and $K^{\pi}=2^{+}$ bands have almost the same (triaxial) deformation**



Does a Λ particle change the excitation spectra?

Results: Excitation spectra of $^{25}{}_{\Lambda}$Mg

$K^{\pi}=2^{+}\otimes\Lambda$ band is shifted up by about 200 keV due to the difference of B_{Λ} between the $K^{\pi}=0^{+}$ and $K^{\pi}=2^{+}$ bands

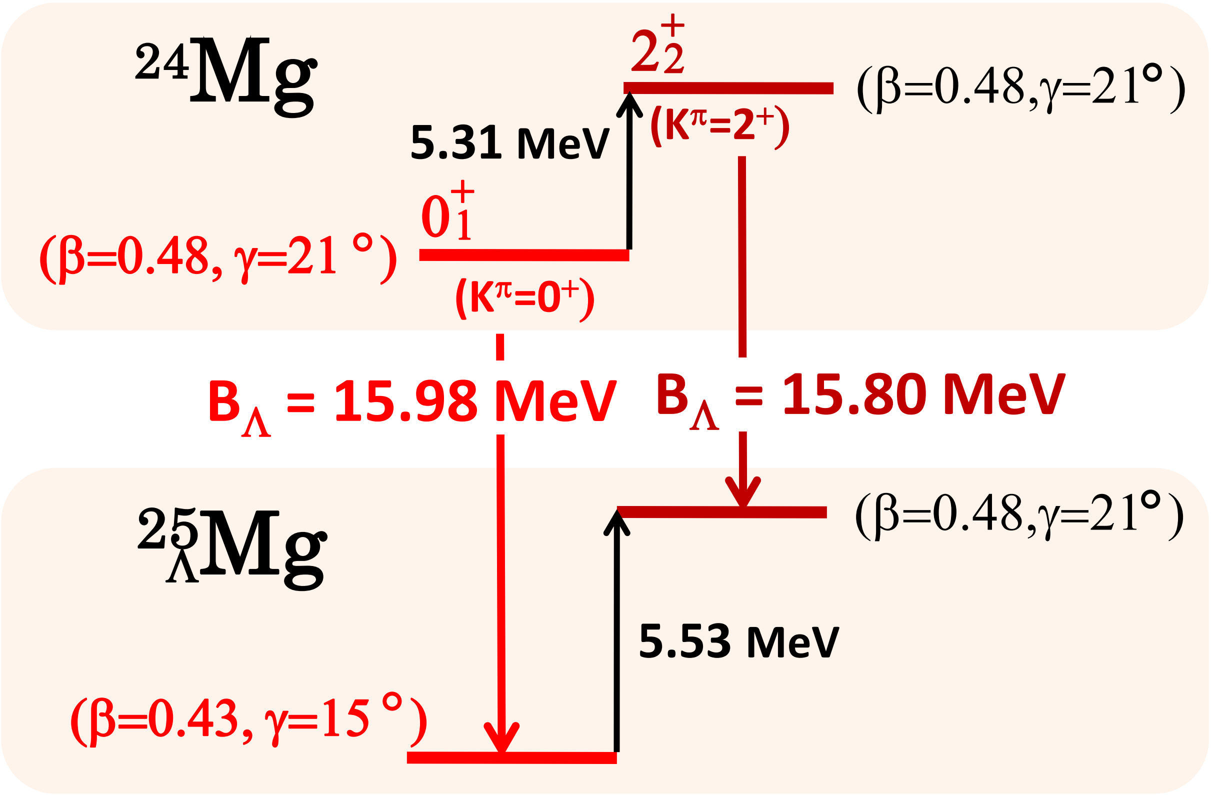


Two bands: Almost the same (triaxial) deformation ⟶ Why shifted up?

Results: Reasons for the shift up

◆ Difference of the Λ binding energy B_Λ

- B_Λ in the $K^\pi=0^+\otimes\Lambda$ band is larger than that in the $K^\pi=2^+\otimes\Lambda$ band
- Difference of B_Λ comes from the difference of deformation change between the two bands



We hope the shift up of the $K^\pi=2^+$ band is observed at J-PARC

Summary

◆ Summary

- AMD + GCM was used to study deformations of *sd-pf* shell Λ hypernuclei

Λ added to SD states: $^{41}{}_{\Lambda}$Ca, $_{\Lambda}$Sc

- Prediction of SD states in Λ hypernuclei: $^{41}{}_{\Lambda}$Ca and $^{46}{}_{\Lambda}$Sc
  - Deformation is almost unchanged by Λ in Ca and Sc hypernuclei
  - B_{Λ} is different depending on deformations: smaller in SD states

Λ added to triaxial deformation: $^{25}{}_{\Lambda}$Mg

- B_{Λ} is different between the $K^{\pi}=0^{+}$ and $K^{\pi}=2^{+}$ bands, while they have almost the same (triaxial) deformation
  - This is due to the difference of deformation change

◆ Future plan

- To predict the production cross sections
- Comparison of B_{Λ} with cluster states: $^{13}{}_{\Lambda}$C (Hoyle + Λ)