New Results from QCD Sum Rules

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The 3rd Korea-Japan Workshop on Nuclear and Hadron Physics at J-PARC

Contents

1. Introduction – Renewal of hadron spectroscopy –
2. Spectral function via QCD sum rule
3. New Approaches of sum rule analyses
4. Conclusion

QCD

- colored quarks + gluons
- scale invariance
- chiral symmetry for light quarks

Complex vacuum structure

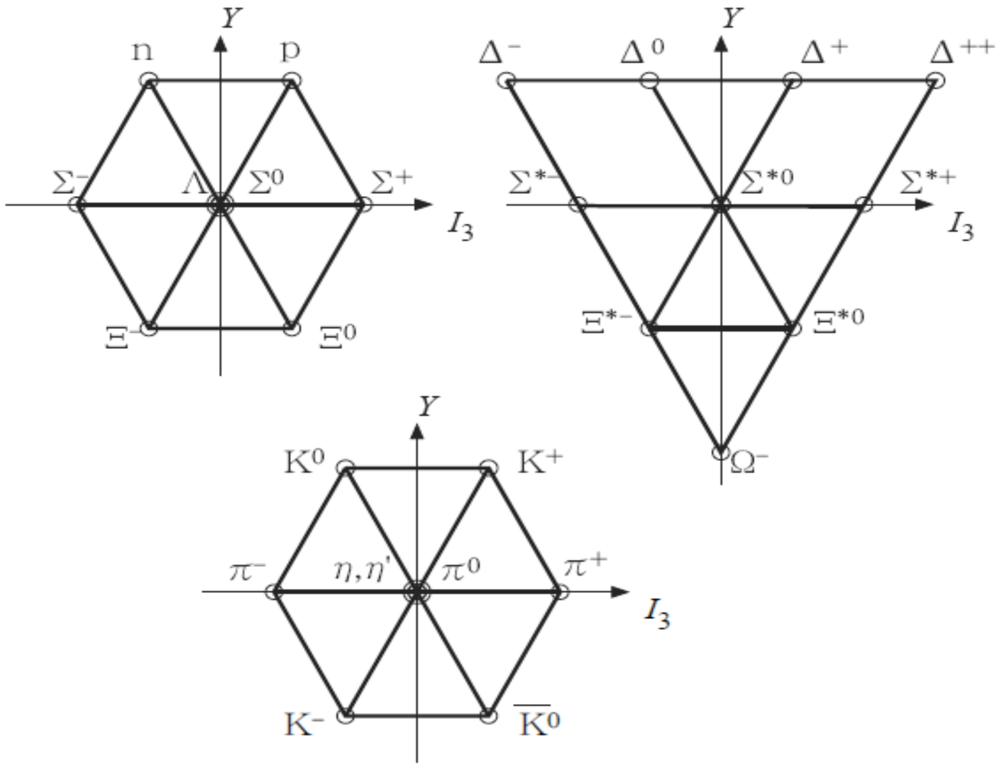
- gluon condensate - scale anomaly
- quark condensate - chiral symmetry breaking

Hadrons = elementary excitations on the vacuum

- hadron spectroscopy → symmetry of QCD
- new possibility at J-PARC, SuKEKB, GSI(PANDA)

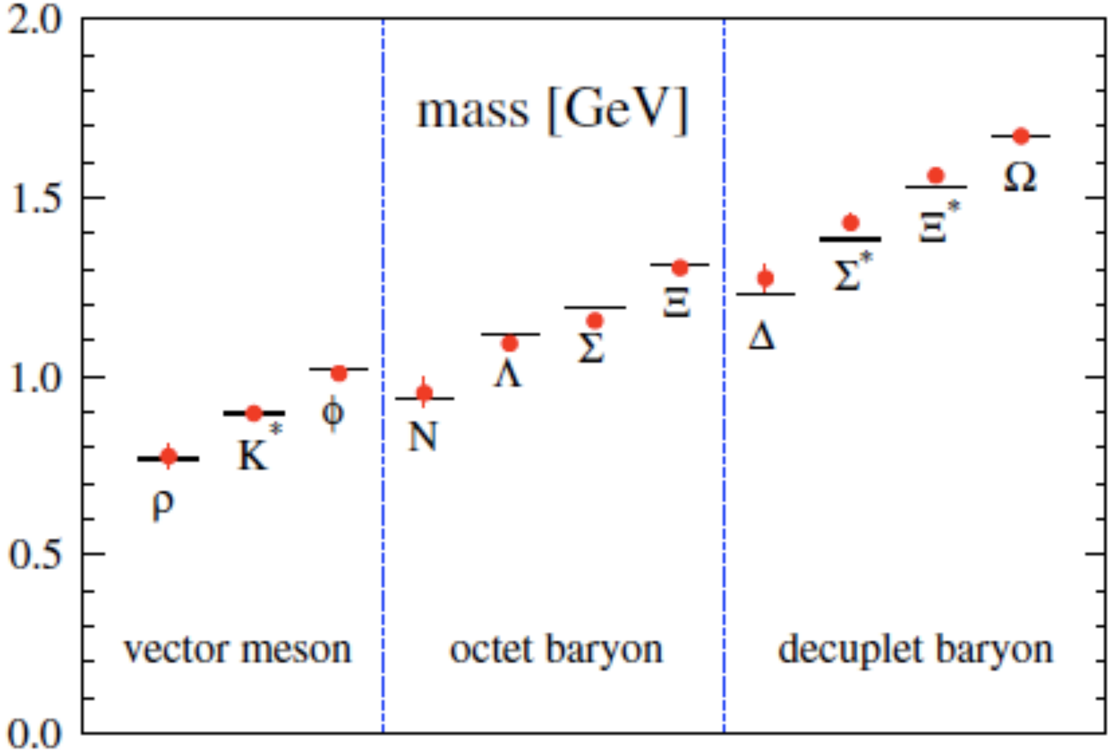
Hadron Spectroscopy

- **Ground state mesons and baryons are well understood.**
  symmetry + dynamics



Hadron Spectroscopy

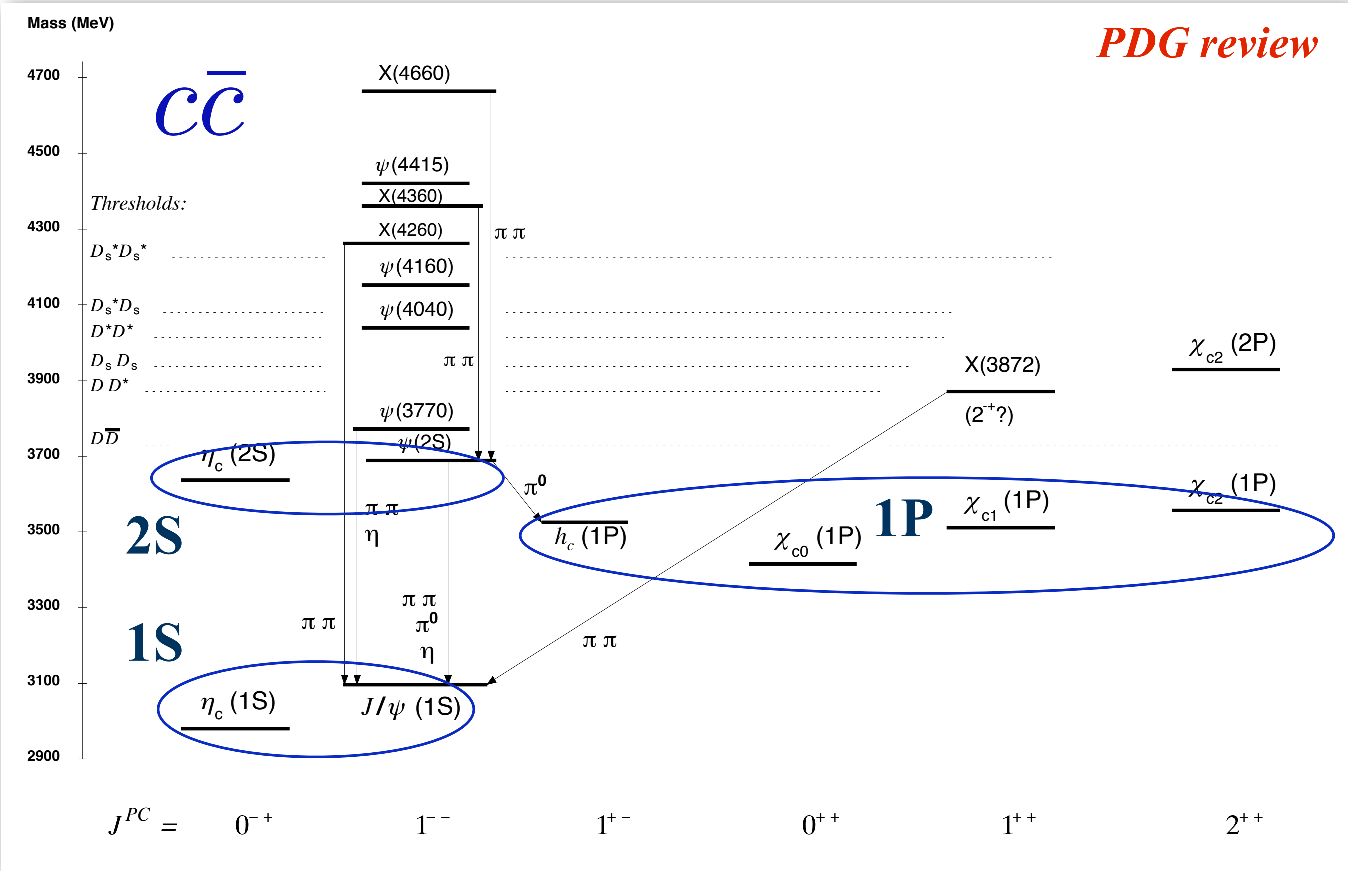
- Ground state mesons and baryons are well understood.
  symmetry + dynamics
- Lattice QCD calculation reproduces them



PACS- CS collaboration, Phys. Rev. D79 (2009) 034503

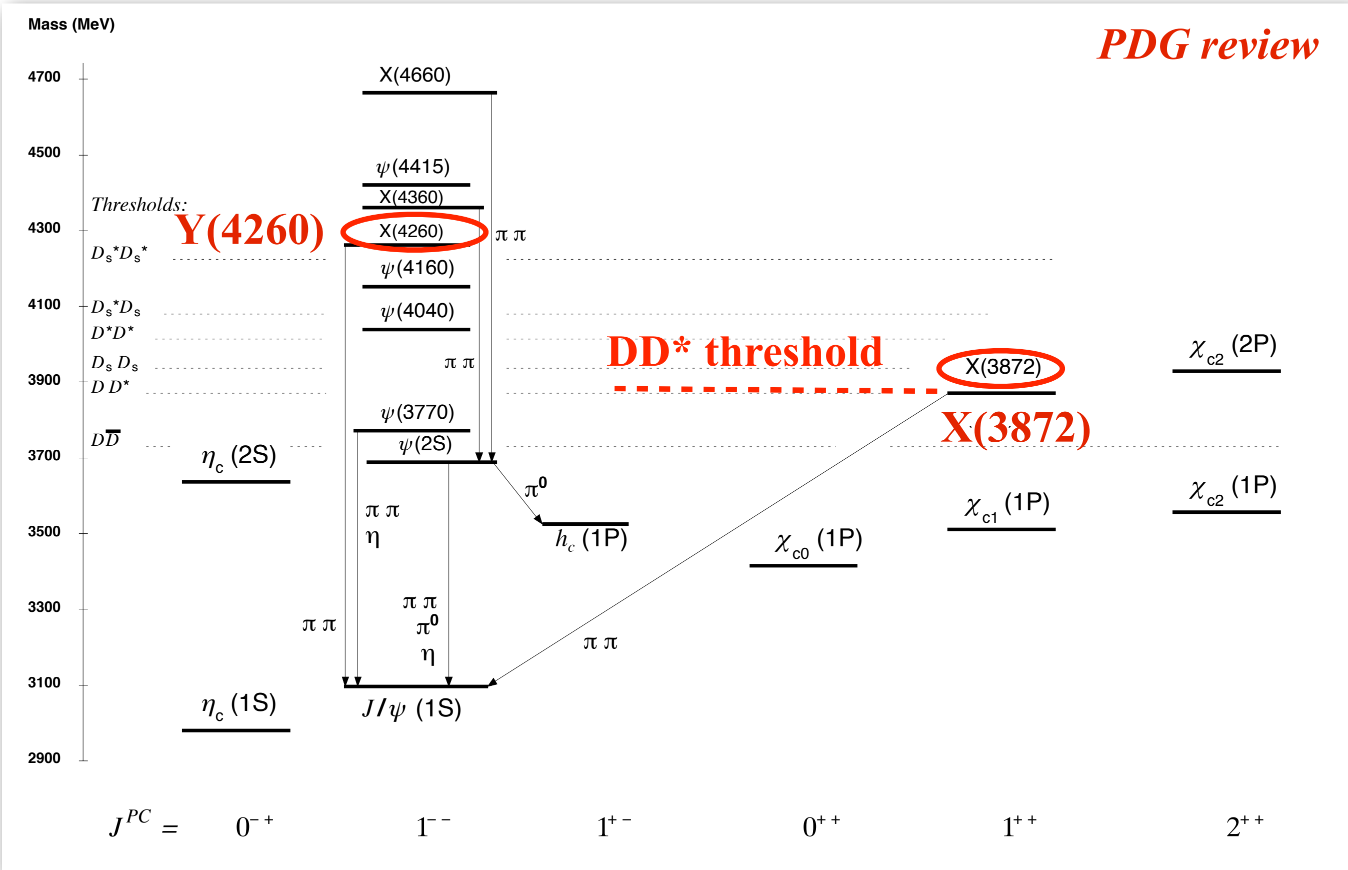
Hadron Spectroscopy

- **Charmonium (c-c^{bar}) below the D-D^{bar} threshold**
  a textbook example of the quark model



New Quarkonium-like mesons

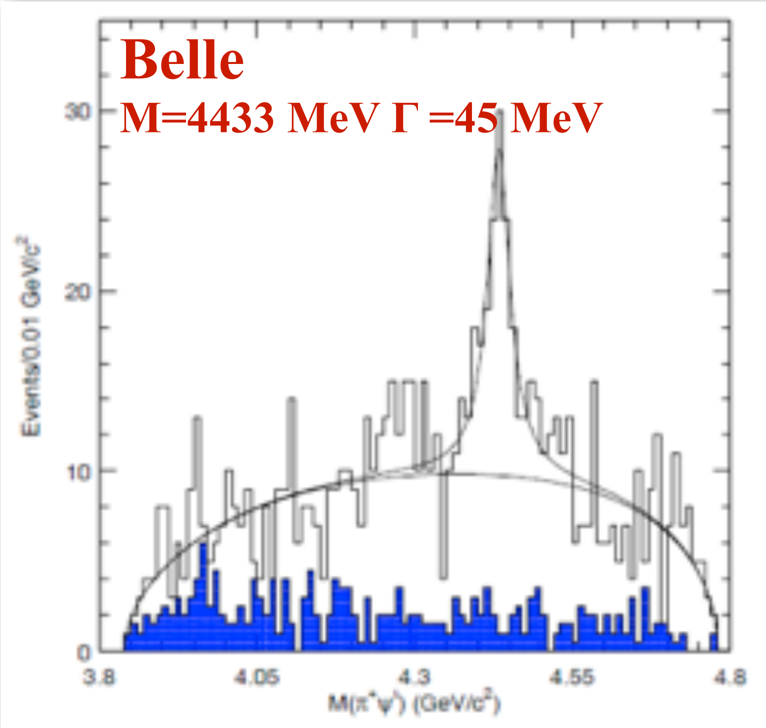
- **Some of the newly observed resonances above DD^{bar} threshold do not fit to the charmonium prediction, *ex*, X(3872), Y(4260), . . .**



New Quarkonium-like mesons

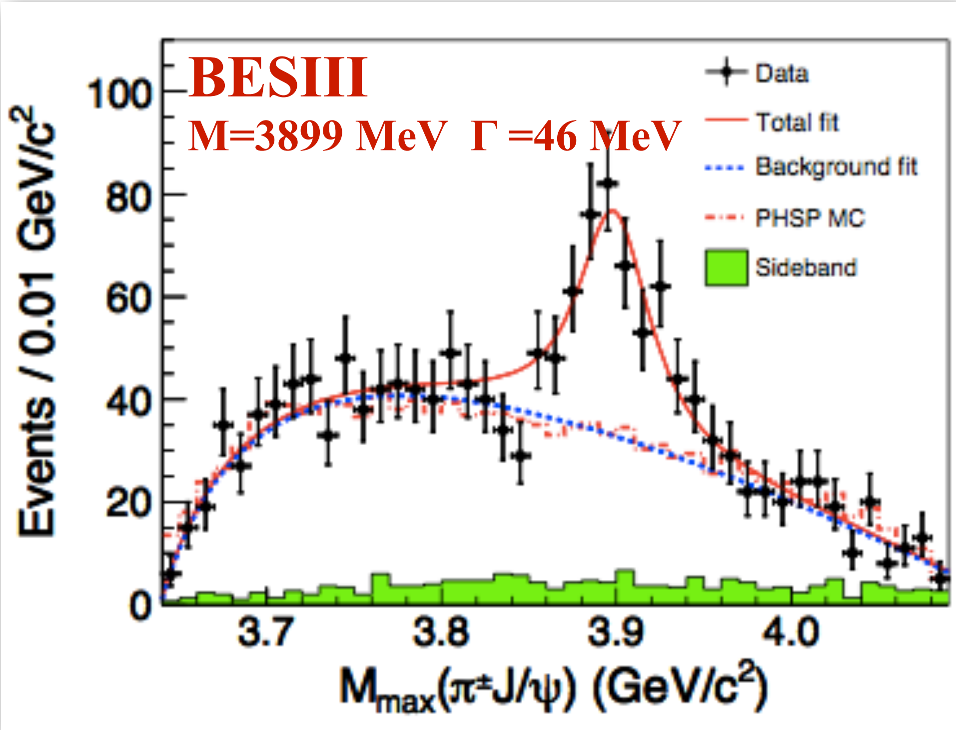
- Charged "Charmonium" ($\rightarrow c c^{bar} + \pi$)
  requires at least 4 quarks

$Z_c^+(4430)$, $Z_{c1}(4050)$, $Z_{c2}(4250)$



PRL 100 (2008) 142001

$Z_c(3900)$, $Z_c(4020)$



PRL 110 (2013) 252001

New Quarkonium-like mesons

- **Charged "Bottomonium"**

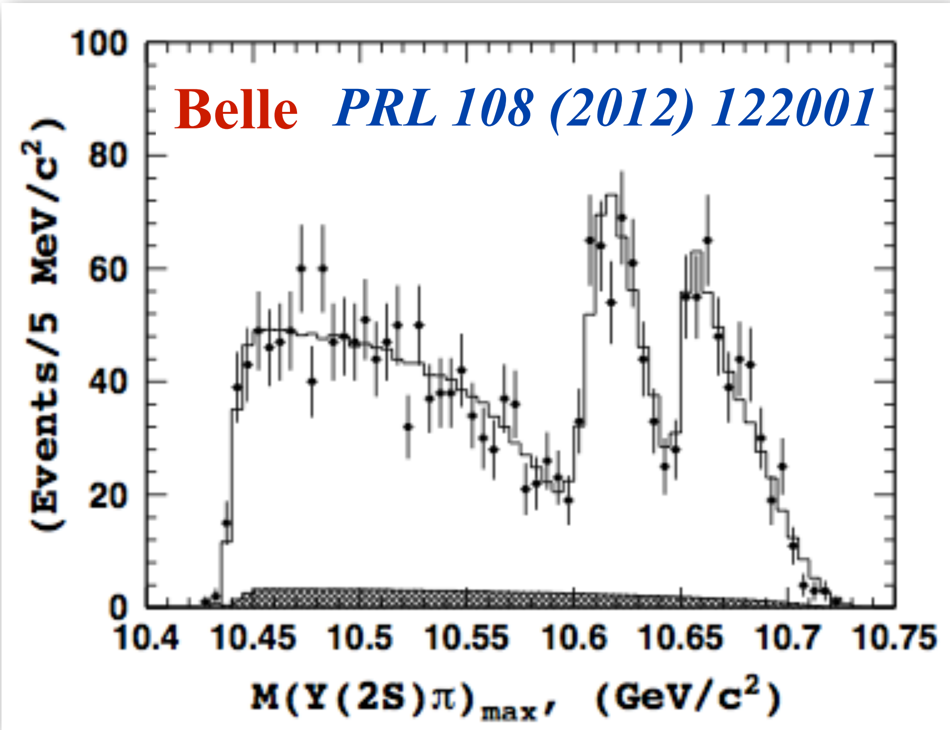
  $Z_b(10610)$, $Z_b(10650)$

  $\rightarrow \Upsilon(2s) + \pi^{\pm}$

Are they

tetra-quarks (bb^{bar}+ ud^{bar})?

or BB^*, B^*B^* molecules?



Hadron Spectroscopy

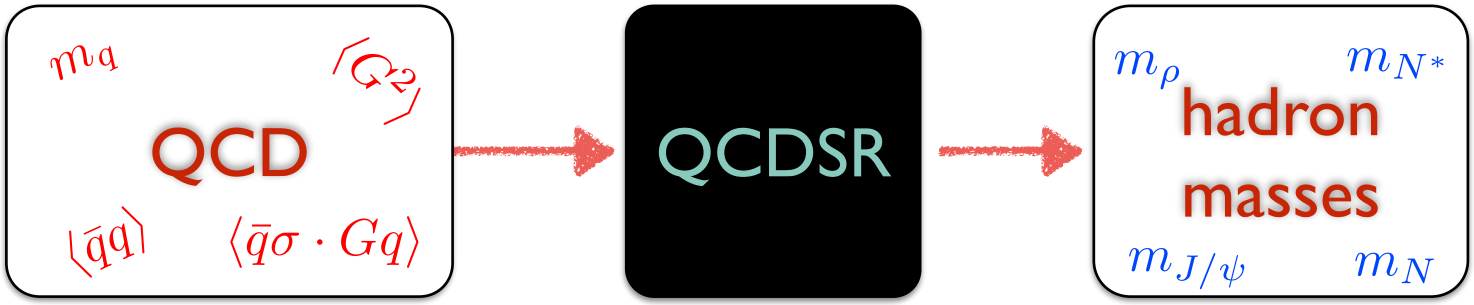
- New stage
  heavy flavors, strangeness & charm/ bottom
  chiral symmetry ⇔ heavy quark symmetry

- New facilities
  J-PARC, Super KEKB,
  J-Lab upgrade, BESIII, LHC, GSI-FAIR
  → J-PARC high momentum beam line/ new hadron hall

- Theoretical developments
  Lattice QCD at physics point, continuum limit, large volume
  Effective theories, chiral, heavy-quark symmetries
  QCD sum rules

QCD Sum Rules

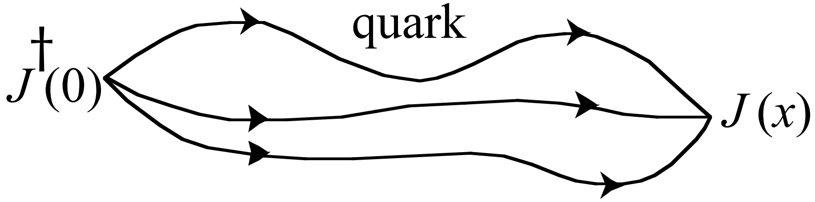
M.A. Shifman, A. Vainshtein, V. I. Zakharov,
Nucl. Phys. B147, 385 (1979)



Spectral Function

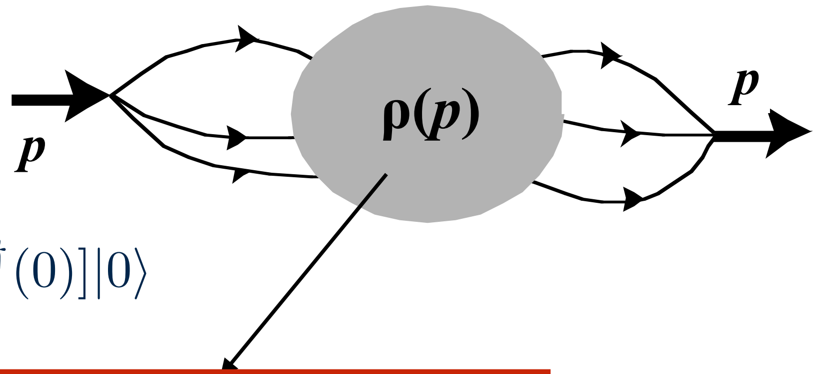
- Correlation function $\Pi(x)$ of QCD contains information of the hadron spectrum:

$$\Pi(x) = \langle 0|T[J(x)J^\dagger(0)]|0\rangle$$



- $J(x)$: interpolating field operator that determines quantum numbers

- Spectral function



$$\Pi(p) \equiv i\int d^4x\, e^{ip\cdot x}\langle 0|T[J(x)J^\dagger(0)]|0\rangle$$

$$\rho(p) = \frac{1}{\pi}\,\mathrm{Im}\,\Pi(p^2) = \sum_m \boxed{|\langle 0|J(0)|m(p)\rangle|^2\delta(p^2 - m^2)}$$

QCD Sum Rules

(1) Operator Product Expansion (OPE) (by Wilson)

Correlation function of composite operator *J*(x) can be expanded in terms of local operators $\mathcal{O}_n$, perturbatively at deep Euclid momentum, $p_E^2 = -p^2 \to \infty$.

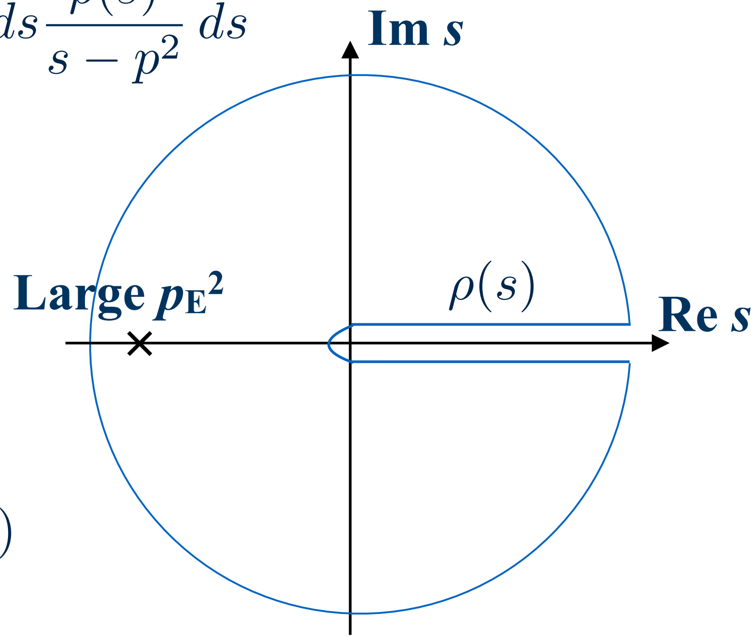
$$\Pi(p^2) = i \int d^4x \, e^{ip\cdot x} \langle 0 | T[J(x) J^\dagger(0)] | 0 \rangle = \sum_n C_n(-p^2) \langle 0 | \mathcal{O}_n | 0 \rangle$$

Non-perturbative effects are taken into account as vacuum condensates, $\langle 0 | \mathcal{O}_n | 0 \rangle$ such as $\langle 0 | \bar{q} q | 0 \rangle$, $\langle 0 | \mathrm{Tr}[G^{\mu\nu} G_{\mu\nu}] | 0 \rangle$, and $C_n(p_E^2)$ is a c-number (Wilson) coefficient.

QCD Sum Rules

(2) Dispersion Relation

$$\Pi(p^2) = \frac{1}{\pi}\int ds \frac{\mathrm{Im}\,\Pi(s)}{s-p^2}\,ds = \int ds \frac{\rho(s)}{s-p^2}\,ds$$



pole + continuum ansatz

$$\rho(s) = \lambda\delta(s-m^2) + \theta(s-s_0)\rho(s)$$

pole (bound state) at $s = m^2$

continuum (scattering states) above s_0

QCD Sum Rules

(3) Borel Transform

to improve the sum rules

eliminate the polynomial terms from the subtraction

multiply extra 1/(n-1)! factor to higher dimensional terms

suppress contribution of higher excitations

$$\mathcal{B}_{M^2}\Pi(p^2) = \lim_{\substack{-p^2,n\to\infty \\ M^2=-p^2/n}} \frac{(-p^2)^{n+1}}{n!}\left(-\frac{d}{dp^2}\right)^n \Pi(p^2)$$

$$\mathcal{B}_{M^2}\frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im}\Pi(s)}{s+p_E^2}ds = \int_0^\infty e^{-s/M^2}\rho(s)ds$$

$$\frac{1}{(-p^2+m^2)^n} \longrightarrow \frac{1}{(n-1)!(M^2)^{n-1}}e^{-m^2/M^2}$$

QCD Sum Rules

The Borel transform is applied to the OPE, we derive an integral equation of the form:

$$G(M) = \int_0^\infty d\omega\, K(M,\omega)\rho(\omega) \qquad K(M,\omega) = \frac{2\omega}{M^2}\, e^{-\omega^2/M^2}$$

- **Sum Rule for pole+cont. ansatz** $\rho(s) = \lambda\delta(s-m^2) + \theta(s-s_0)\rho(s)$

$$\boxed{G(M) = \lambda e^{-m^2/M^2} + (\text{terms with } s_0)}$$

- **The Borel Mass parameter *M* is arbitrary and the physical parameters should not depend on *M*.**

QCD Sum Rules

- **Sum Rule for ρ meson**

$$G_{OPE}(M) = \frac{1}{4\pi^2}\Big(1+\eta(\alpha_s)\Big) + \Big(2m\langle\bar{q}q\rangle + \frac{1}{12}\langle\frac{\alpha_s}{\pi}G^2\rangle\Big)\frac{1}{M^4} - \frac{112\pi}{81}\alpha_s\kappa\langle\bar{q}q\rangle^2\frac{1}{M^6} + \ldots,$$

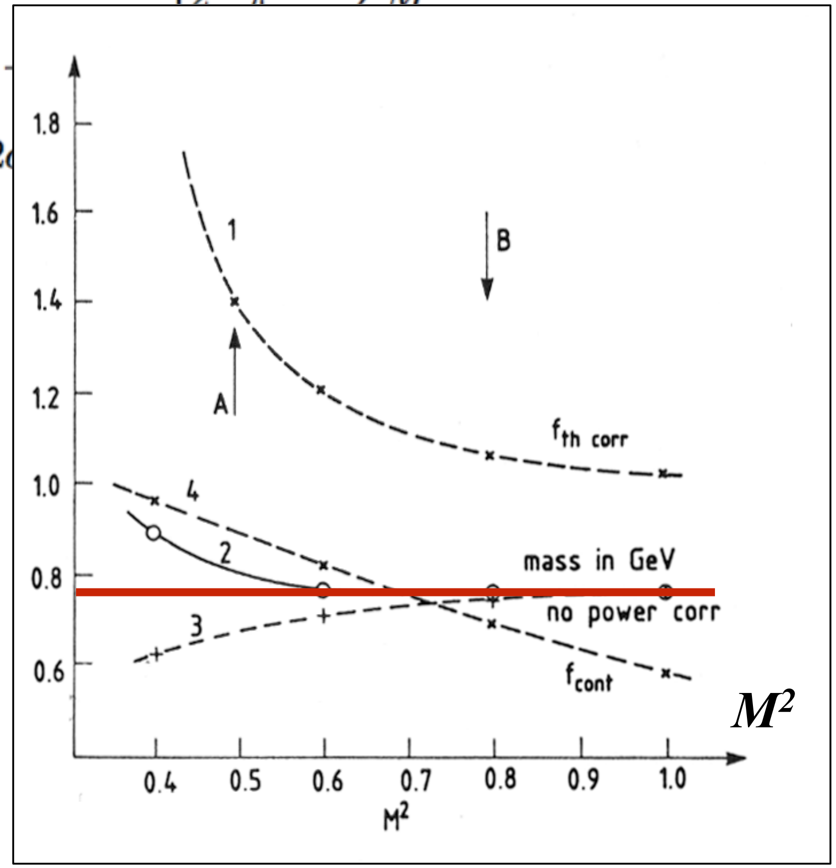
$$\eta(\alpha_s) = \frac{\alpha_s}{\pi} + 0.154\alpha_s^2 - 0.372\alpha_s^3 + \ldots.$$

QCD Sum Rules

- **Sum Rule for ρ meson**

$$G_{OPE}(M) = \frac{1}{4\pi^2}\Big(1+\eta(\alpha_s)\Big) + \Big(2m\langle\bar{q}q\rangle + \frac{1}{12}\langle\frac{\alpha_s}{\pi}G^2\rangle\Big)\frac{1}{M^4}$$
$$- \frac{112\pi}{81}\alpha_s\kappa\langle\bar{q}q\rangle^2\frac{1}{M^6}$$
$$\eta(\alpha_s) = \frac{\alpha_s}{\pi} + 0.154\alpha_s^2 - 0.372$$

The ρ meson mass is determined from the plateau in the *M*² dependent curve of *m*.



by Reinders, Rubinstein, Yazaki

MEM analyses

- The conventional analysis of QCDSR is successful in reproducing the ground states, but is not applicable to excited states.
- It cannot tell whether there exists hadron states, or not, nor whether the hadronic spectrum is modified under changes of external conditions, such as temperature, density, magnetic fields, . . .
- We need to determine the spectral function itself without assuming its functional form.

MEM analyses

P. Gubler, M.O., Prog. Theor. Phys., 124 (2010) 995
arXiv: 1005.2459v1
P. Gubler, K. Morita and M.O., Phys. Rev. Lett., 107 (2011) 092003
arXiv:1104.4436
K. Ohtani, P. Gubler, M.O., Eur. Phys. J. A47 (2011) 114
arXiv:1104.5577
K. Suzuki, P. Gubler, K. Morita,M.O., Nucl. Phys. A897 (2013) 28
arXiv:1204.1173
K. Ohtani, P. Gubler, M.O., Phys. Rev. D87 (2013) 034027
arXiv:1209.1463
K. Araki, K. Ohtani, P. Gubler, M.O., in preparation

MEM analyses

- **The Borel sum rule is reduced to a mathematical problem to invert the integral relation:**

$$G(M) = \int_0^\infty d\omega\, K(M,\omega)\rho(\omega)$$

$\rho(\omega) \geq 0$

$$K(M,\omega) = \frac{2\omega}{M^2}\, e^{-\omega^2/M^2}$$

insensitive to ω=0

***G*(*M*) is given by the QCD OPE, and *ρ*(ω) is estimated.**

A similar problem in the lattice QCD:

by Asakawa, Hatsuda, Nakahara

$$G(\tau) = \sum_{\vec{x}} \langle 0|O(\vec{x},\tau)O^\dagger(0,0)|0\rangle = \int_0^\infty d\omega K(\tau,\omega)\rho(\omega)$$

$$K(\tau,\omega) = e^{-\omega\tau}$$

MEM analyses

- If the OPE is given completely and precisely, then it would be straightforward to obtain the spectral function by inverting the sum rule.

- It is, however, not possible because the OPE is incomplete; truncated with sizable ambiguities on the condensates values.

- We therefore use the Bayesian inference method to estimate the most probable spectral function which satisfies the sum rule.

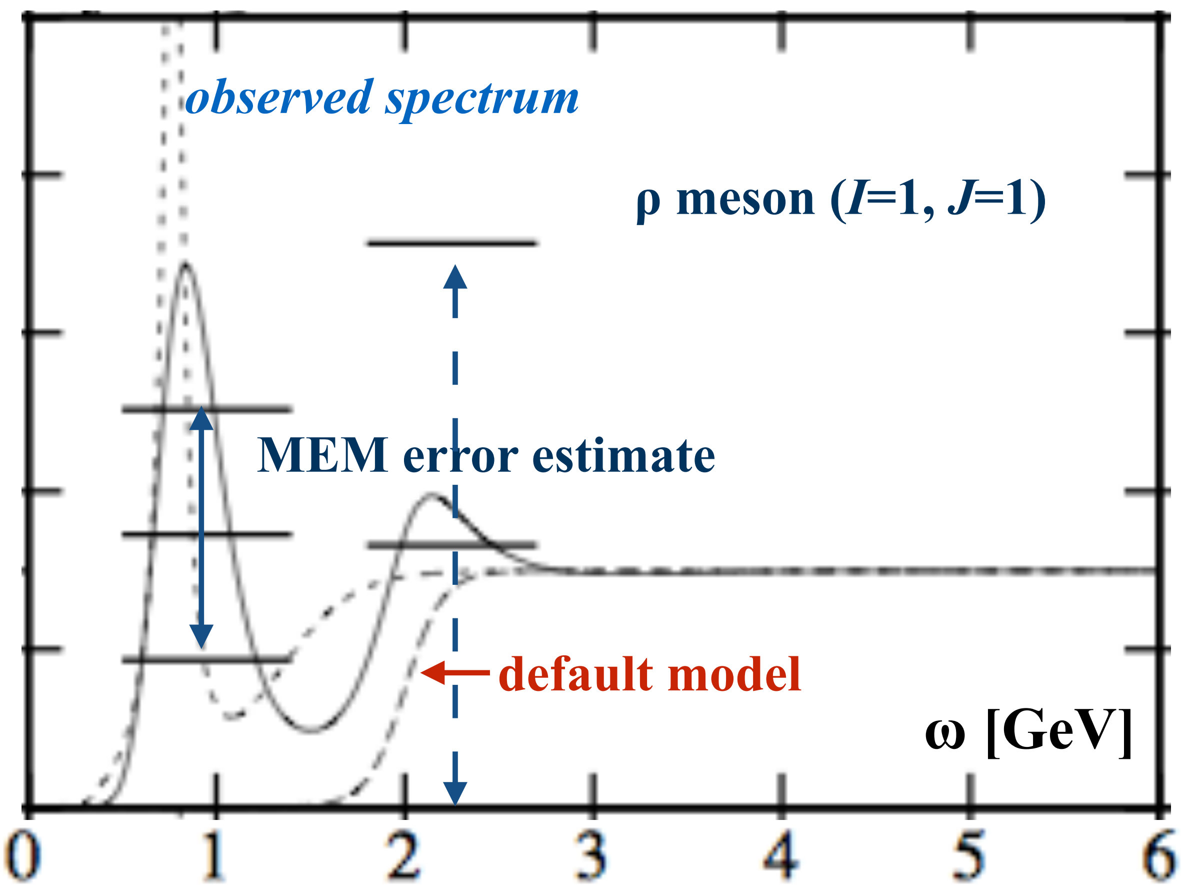
- The Bayesian inference relies on maximizing "Free Energy" with the Shannon-Jaynes information entropy

$$S[\rho] = \int_0^\infty d\omega[\rho(\omega) - m(\omega) - \rho(\omega)\log(\rho(\omega)/m(\omega))]$$

m*(ω): *default model

MEM analyses

- ρ meson sum rule

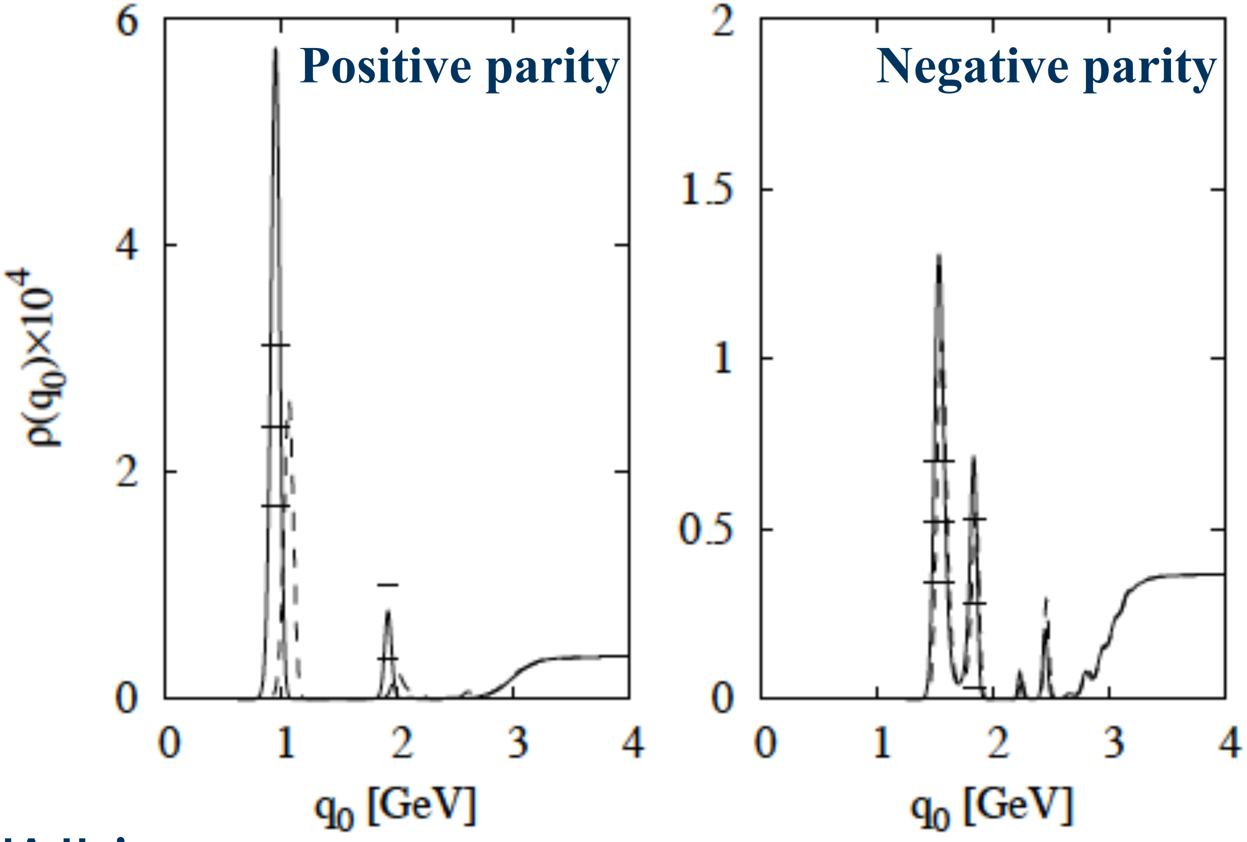


P. Gubler, M.O., Prog. Theor. Phys., 124 (2010) 995

Positive and Negative Parity Nucleon

K. Ohtani, P. Gubler, M.O., Phys. Rev. D87 (2013) 034027

- Positive and negative parity spectra of the nucleon
- Phase rotated Gaussian (2-parameter) sum rules are employed.



Charmonium at Finite Temperature

- **J/ψ Suppression in Quark-Gluon Plasma**

T. Matsui and H. Satz, Phys. Lett. B 178, 416 (1986)

Suppression of charmonium production is a plausible signature of QGP creation, as the charmonium bound states will dissociate in QGP due to the Debye screening by light quarks. The charmonium spectrum is expected to be largely modified at high *T*.

However, a few recent studies using lattice QCD and MEM suggest that J/ψ peak remains above T_c.

M. Asakawa, T. Hatsuda, PRL 92 012001 (2004)

S. Datta et al, Phys. Rev. D69, 094507 (2004)

T. Umeda et al, Eur. Phys. J. C39S1, 9 (2005)

A. Jakovác et al, Phys. Rev. D75, 014506 (2007)

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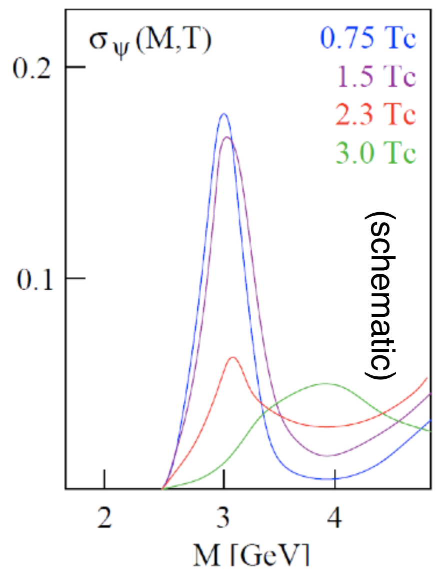

M. Asakawa, T. Hatsuda, PRL 92 012001 (2004)

S. Datta et al, Phys. Rev. D69, 094507 (2004)

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A. Jakovác et al, Phys. Rev. D75, 014506 (2007)

H. Satz, Nucl.Part.Phys. 32, 25 (2006)

Charmonium at Finite Temperature

- **Sum Rules for charmonium**

P. Gubler, K. Morita and M.O., Phys. Rev. Lett., 107 (2011) 092003

$$M(\nu) = \int_0^\infty e^{-\nu t} \rho(4m_c^2 t) dt \qquad (\nu \equiv \frac{M^2}{4m_c^2})$$

$$M(\nu) = A(\nu)\left[1 + a(\nu)\alpha_s(\nu) + b(\nu)\frac{\langle \frac{\alpha_s}{\pi} G^2 \rangle}{m_c^4} + d(\nu)\frac{\langle g^3 G^3 \rangle}{m_c^6}\right]$$

perturbative term including αs correction

Non-perturbative corrections including condensates up to dim 6

Developed and analyzed in:

M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. B147, 385 (1979); B147, 448 (1979).

L.J. Reinders, H.R. Rubinstein, S. Yazaki, Nucl. Phys. B 186, 109 (1981).

R.A. Bertlmann, Nucl. Phys. B 204, 387 (1982).

J. Marrow, J. Parker and G. Shaw, Z. Phys. C 37, 103 (1987).

Charmonium at Finite Temperature

- **Finite T ≠ 0**
  The application of QCD sum rules has been developed in:
  T. Hatsuda, Y. Koike, S.H. Lee, NPB 394, 221 (1993).

$$M(\nu) = A(\nu)\Big[1 + a(\nu)\alpha_s(\nu) + b(\nu)\underline{\frac{\langle\frac{\alpha_s}{\pi}G^2\rangle_T}{m_c^4}} + c_n(\nu)\underline{\frac{\langle\frac{\alpha_s}{\pi}G^2\rangle_{T,2}}{m_c^4}} + d(\nu)\cancel{\frac{\langle g^3G^3\rangle_T}{m_c^6}}\Big]$$

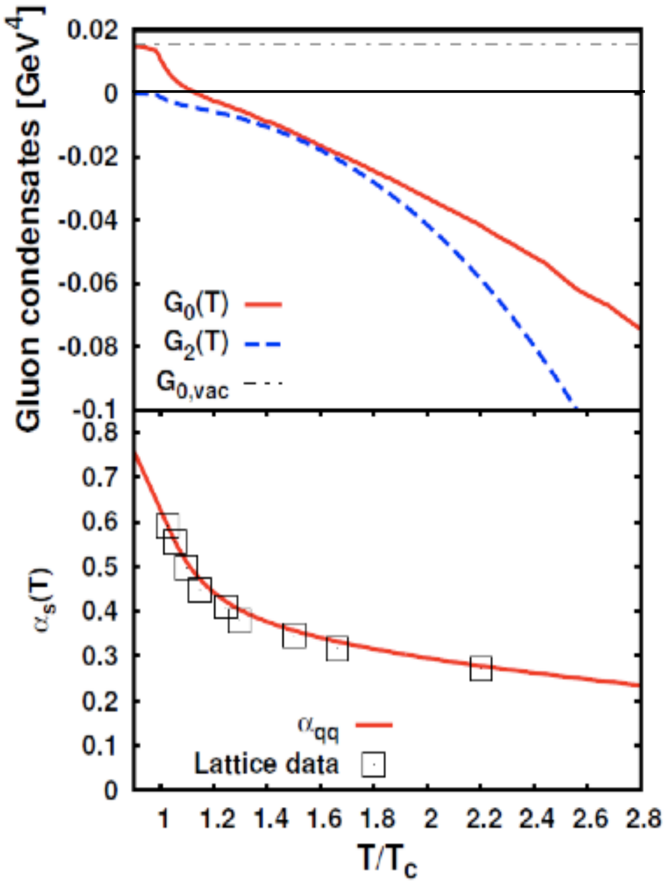
suppressed by **1/*m*$_c^6$**

$$\boxed{\begin{aligned}\langle\tfrac{\alpha_s}{\pi}G^2\rangle_T &= \langle\tfrac{\alpha_s}{\pi}G^2\rangle_{\text{vac.}} - \tfrac{8}{11}(\epsilon - 3p)\\ \langle\tfrac{\alpha_s}{\pi}G^2\rangle_{T,2} &= -\tfrac{\alpha_s(T)}{\pi}(\epsilon + p)\end{aligned}}$$

K. Morita, S.H. Lee, Phys. Rev. Lett. 100, 022301 (2008); Phys. Rev. C 77, 064904 (2008).

Charmonium at Finite Temperature

- The values of ε(T) and p(T) are obtained from quenched lattice calculations:



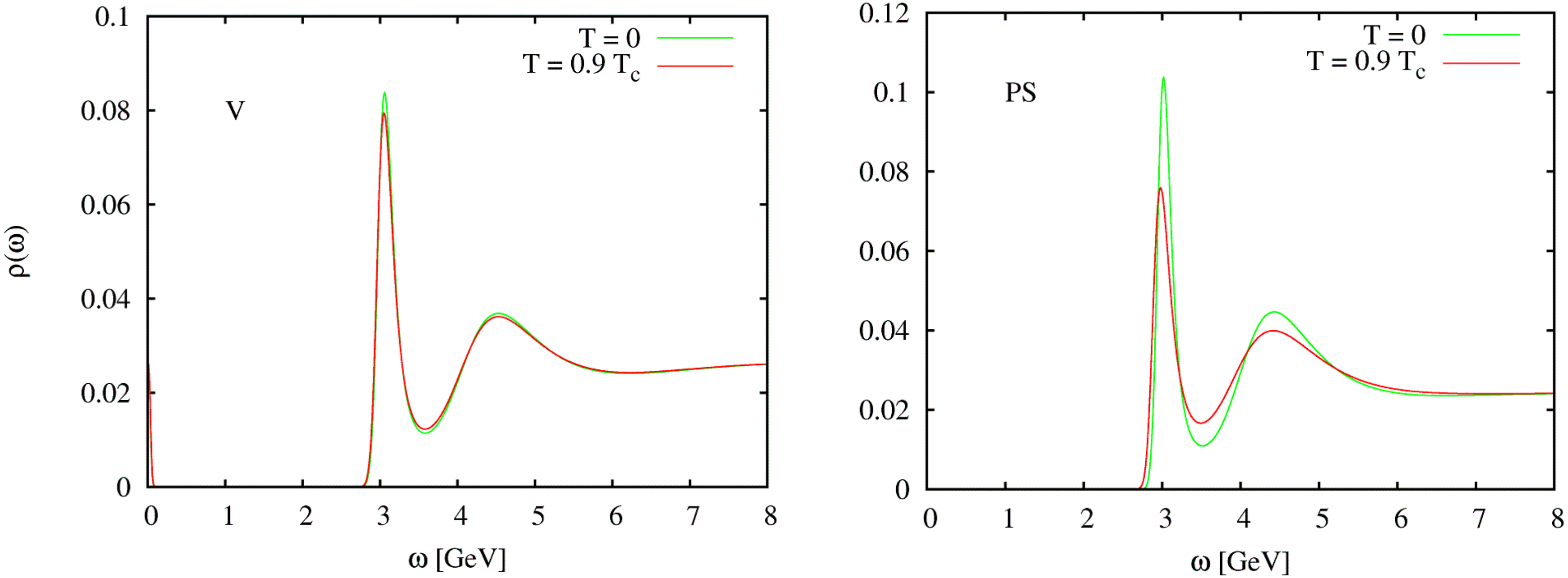
G. Boyd et al, Nucl. Phys. B 469, 419 (1996).

O. Kaczmarek et al, Phys. Rev. D 70, 074505 (2004).

A sudden decrease of the gluon condensates above T_c is observed.

Taken from K. Morita, S.H. Lee, Phys. Rev. D82, 054008 (2010).

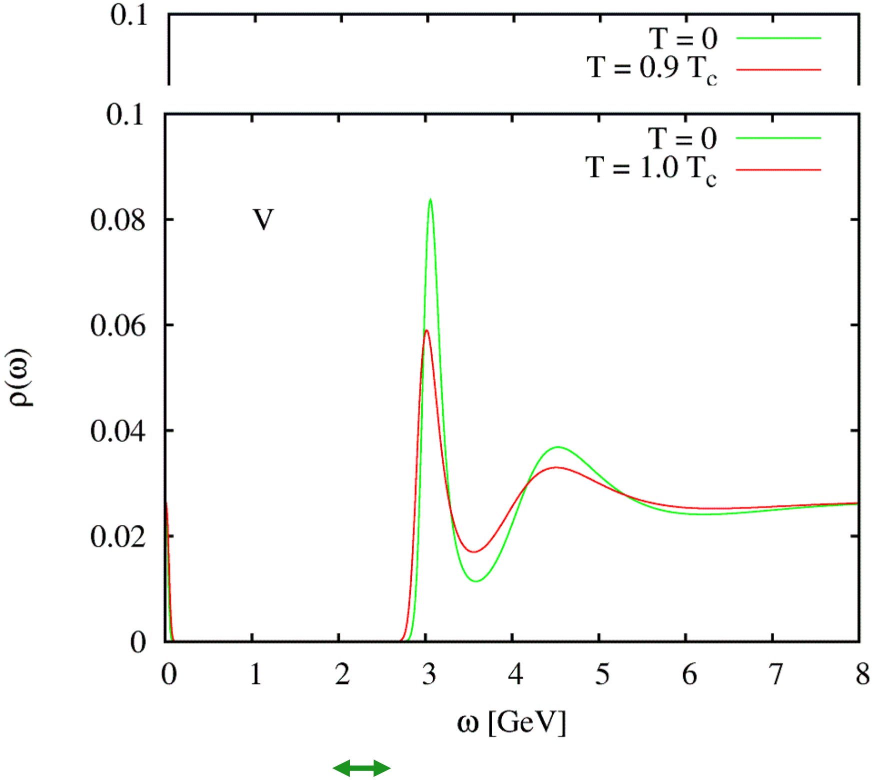
Charmonium at Finite Temperature

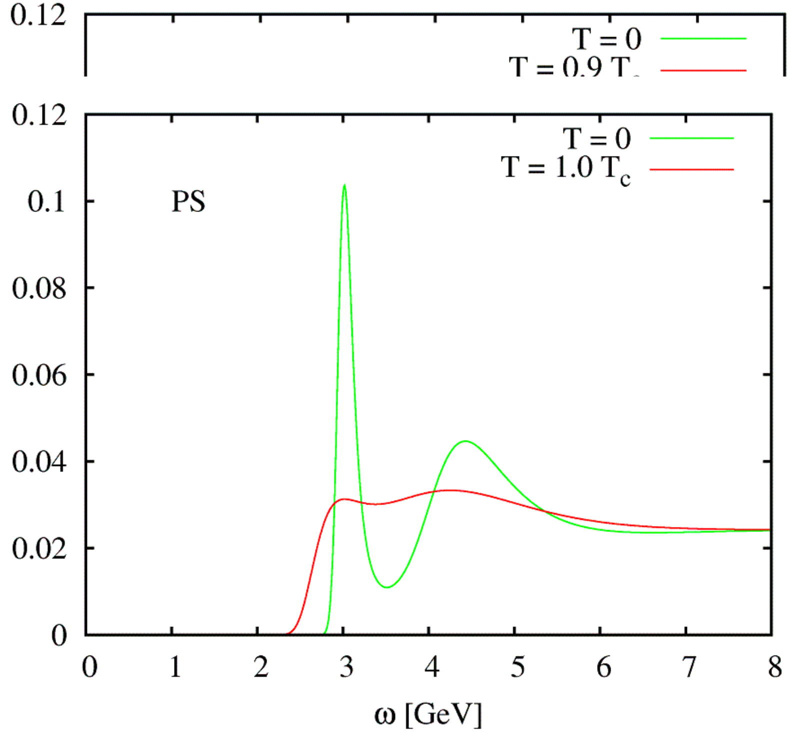


negative shift of ~40 MeV,

consistent with the analysis of Morita and Lee.

Charmonium at Finite Temperature

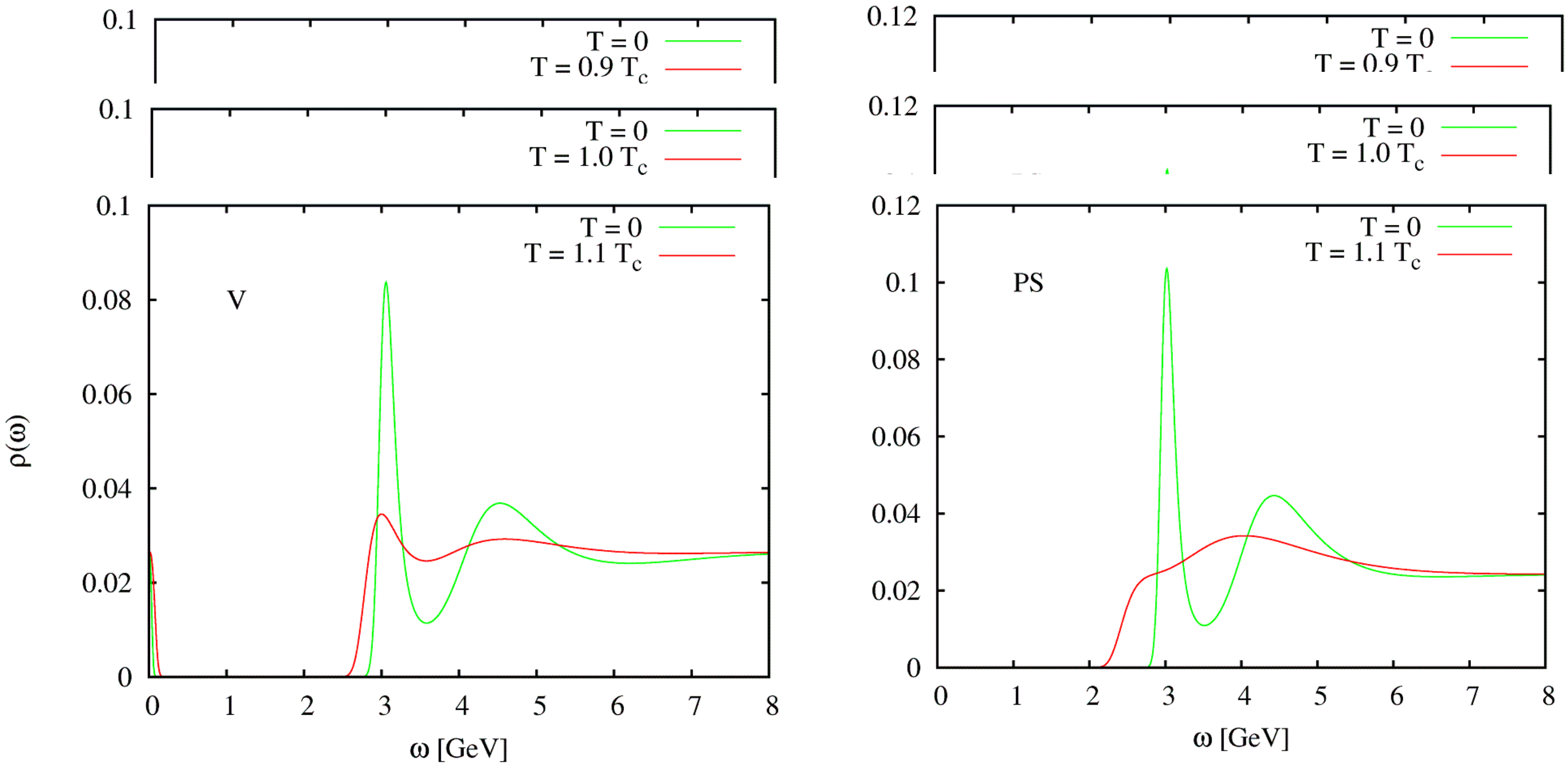




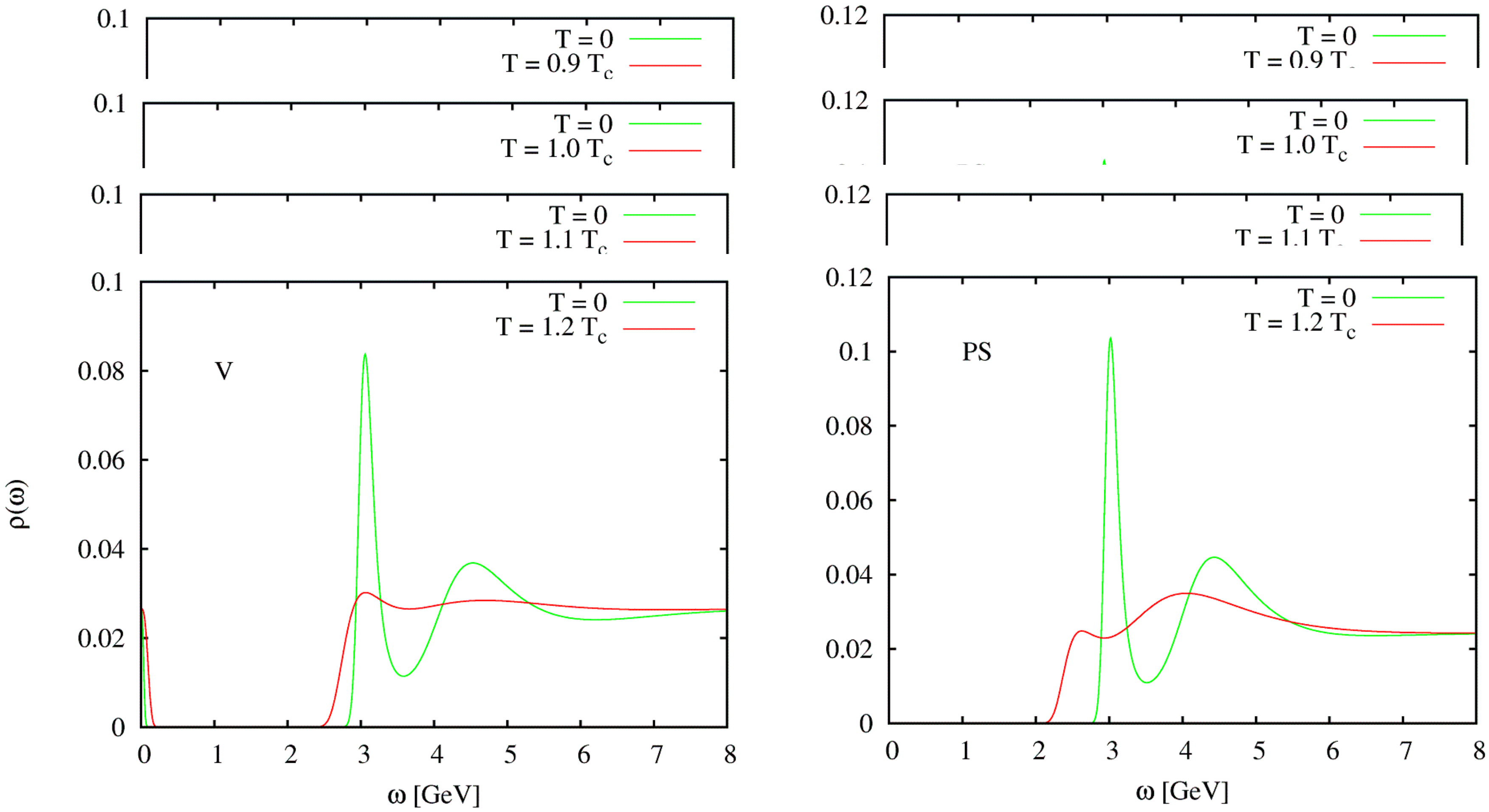
Melting

negative shift of ~50 MeV

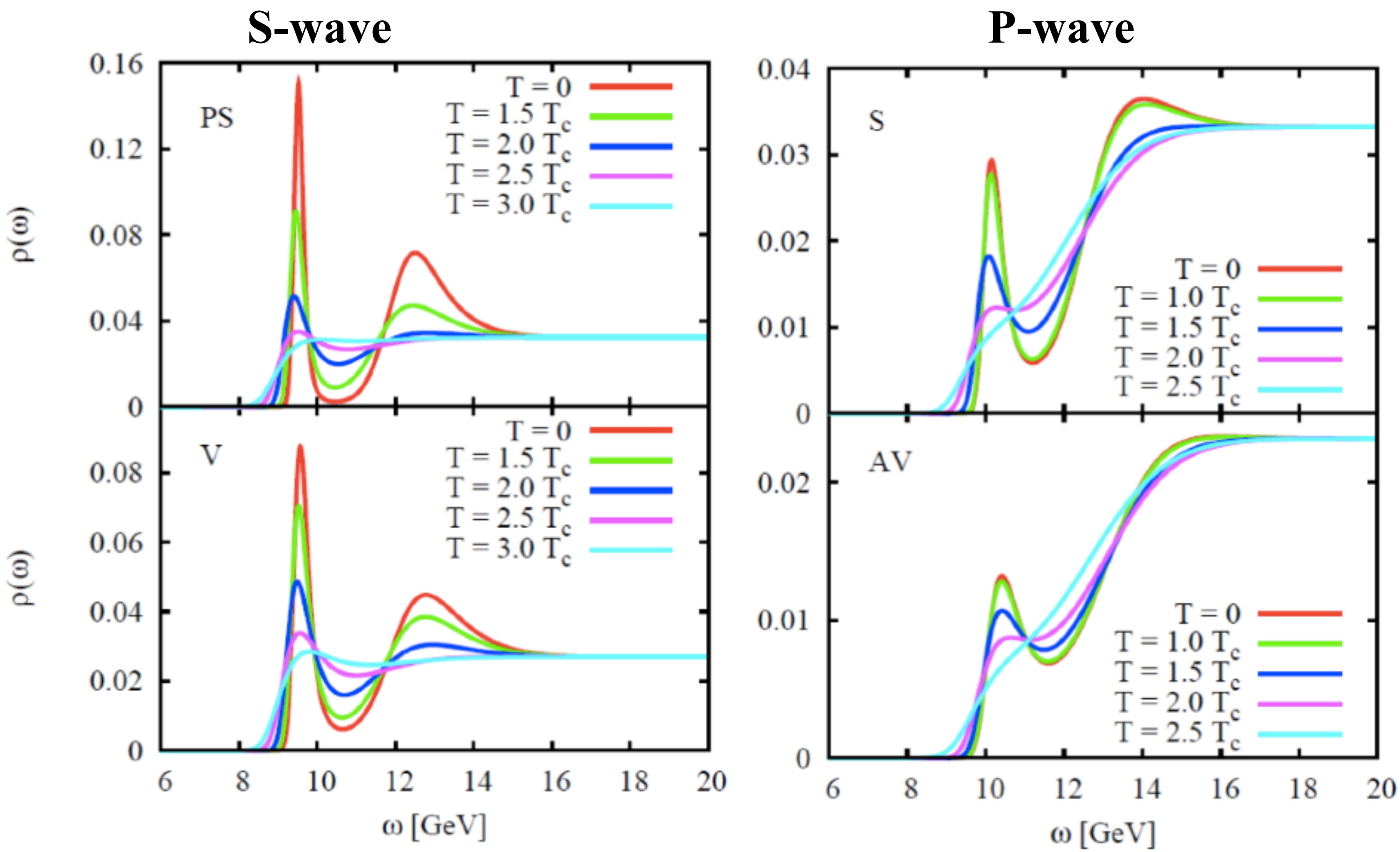
Charmonium at Finite Temperature



Charmonium at Finite Temperature

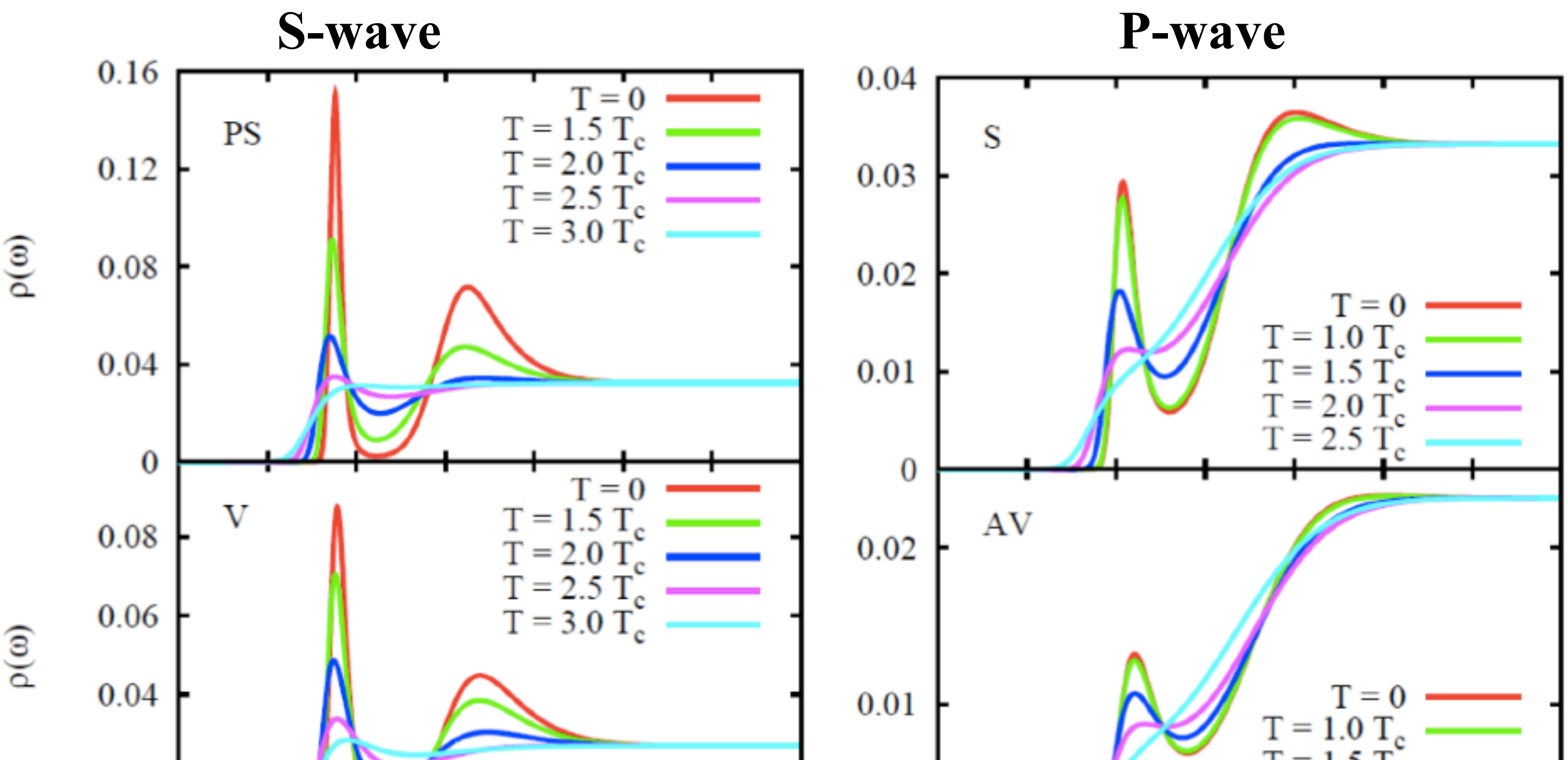


Results for bottomonium



K. Suzuki, P. Gubler, K. Morita and M.O., Nucl. Phys. A897, 28 (2013).

Results for bottomonium

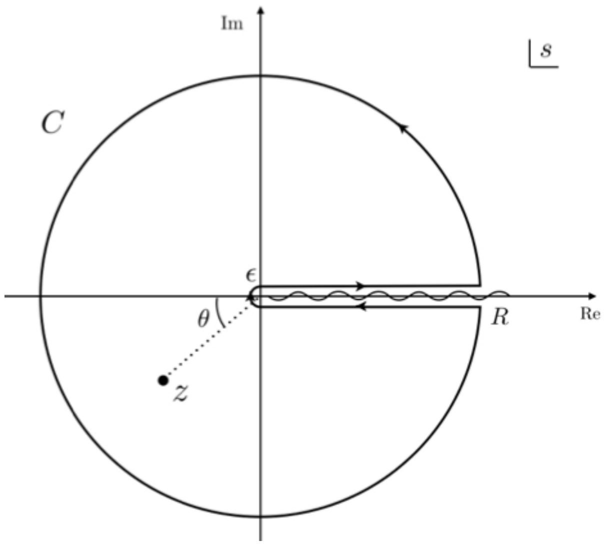


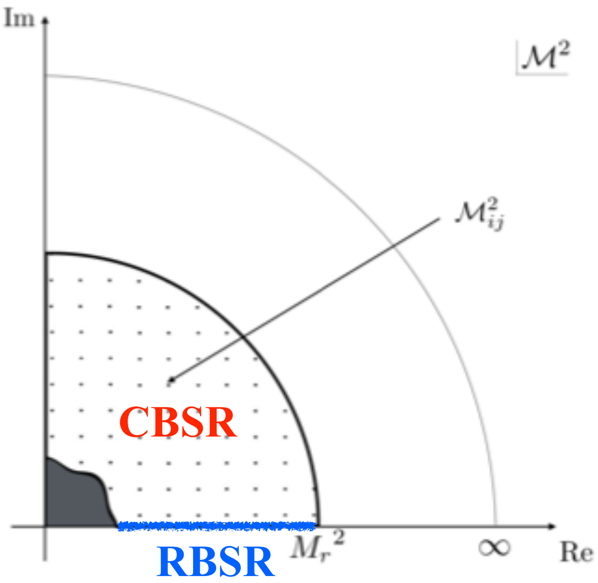
The $b\bar{b}$ (bb^{bar}) mesons survive at higher temperatures than the charmonia, because the gluon condensate terms of the sum rule are suppressed by $1/m_b^2$ factor.

K. Suzuki, P. Gubler, K. Morita and M.O., Nucl. Phys. A897, 28 (2013).

Complex Borel Sum Rules

- Quality of the MEM spectrum depends on the quantity of information extracted from the OPE.
- So far the full power of MEM is not exploited.
- We have formulated a new analysis method using the complex values of p^2 in OPE, which are mapped to complex Borel parameters.

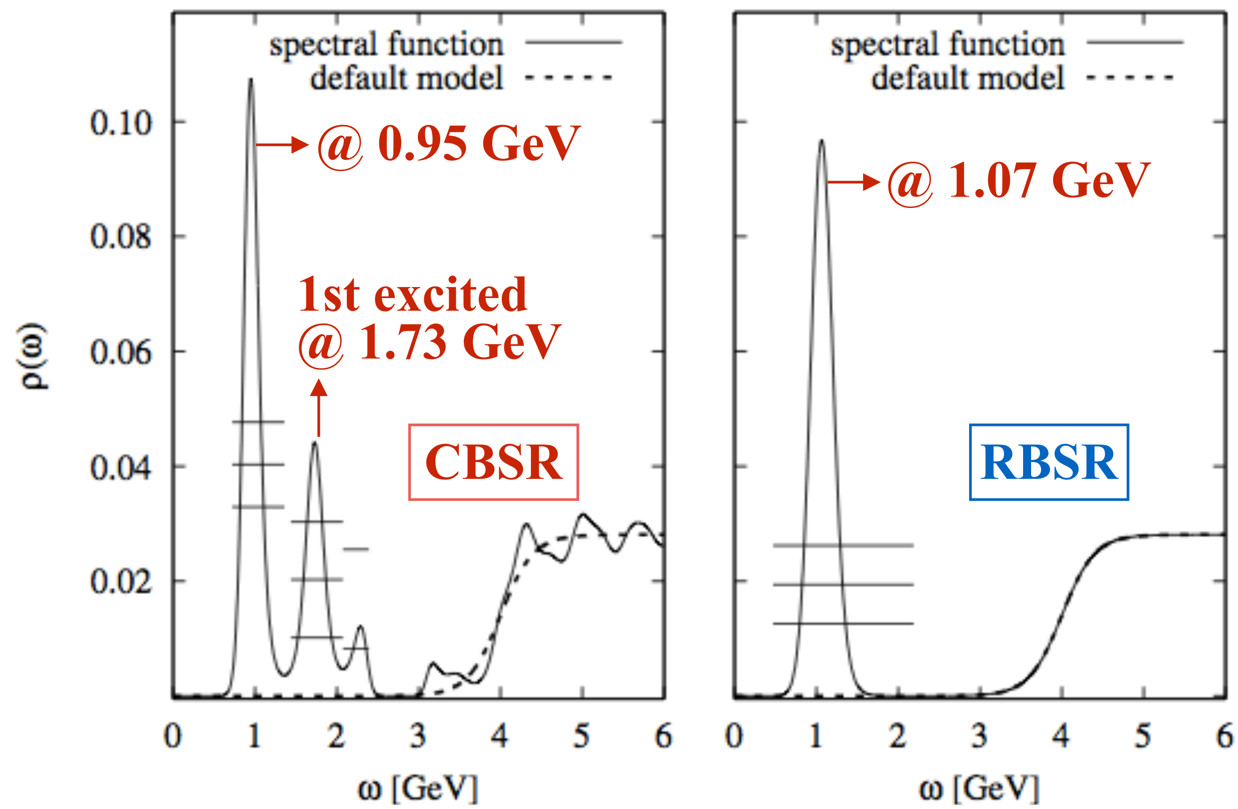




Complex Borel Sum Rules

- It is found that the new sum rule has better resolution
- Excited states and continuum spectrum may be reproduced.

Spectral function of ϕ (1^-) meson *K. Araki et al., To be published*

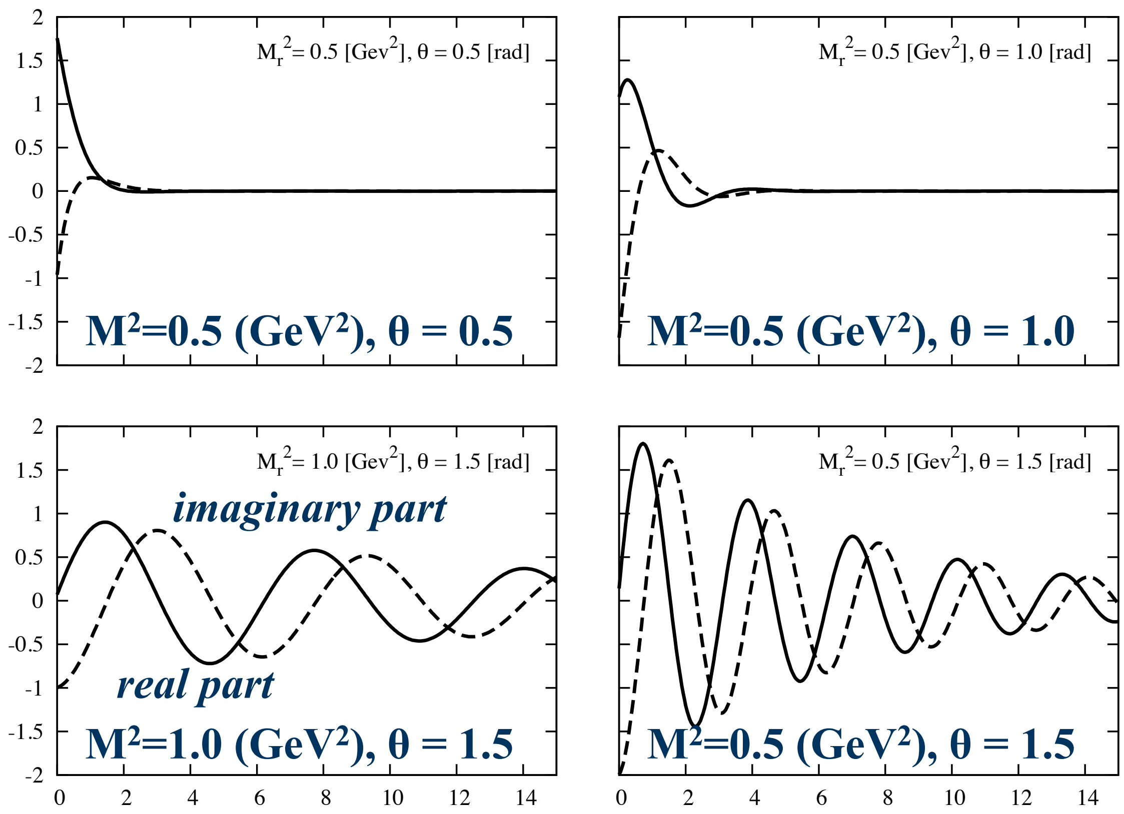


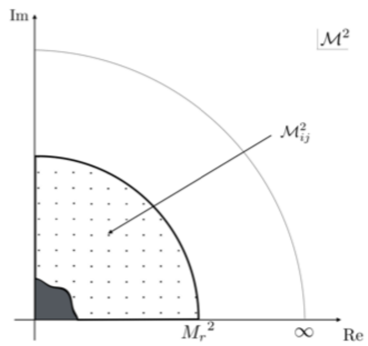
ϕ (1^-) observed
ϕ(1019)
ϕ(1680 ± 20)

Complex Borel Sum Rules

- **The complex integral kernels can extract more information on the spectral function.**

CBSR integral kernels

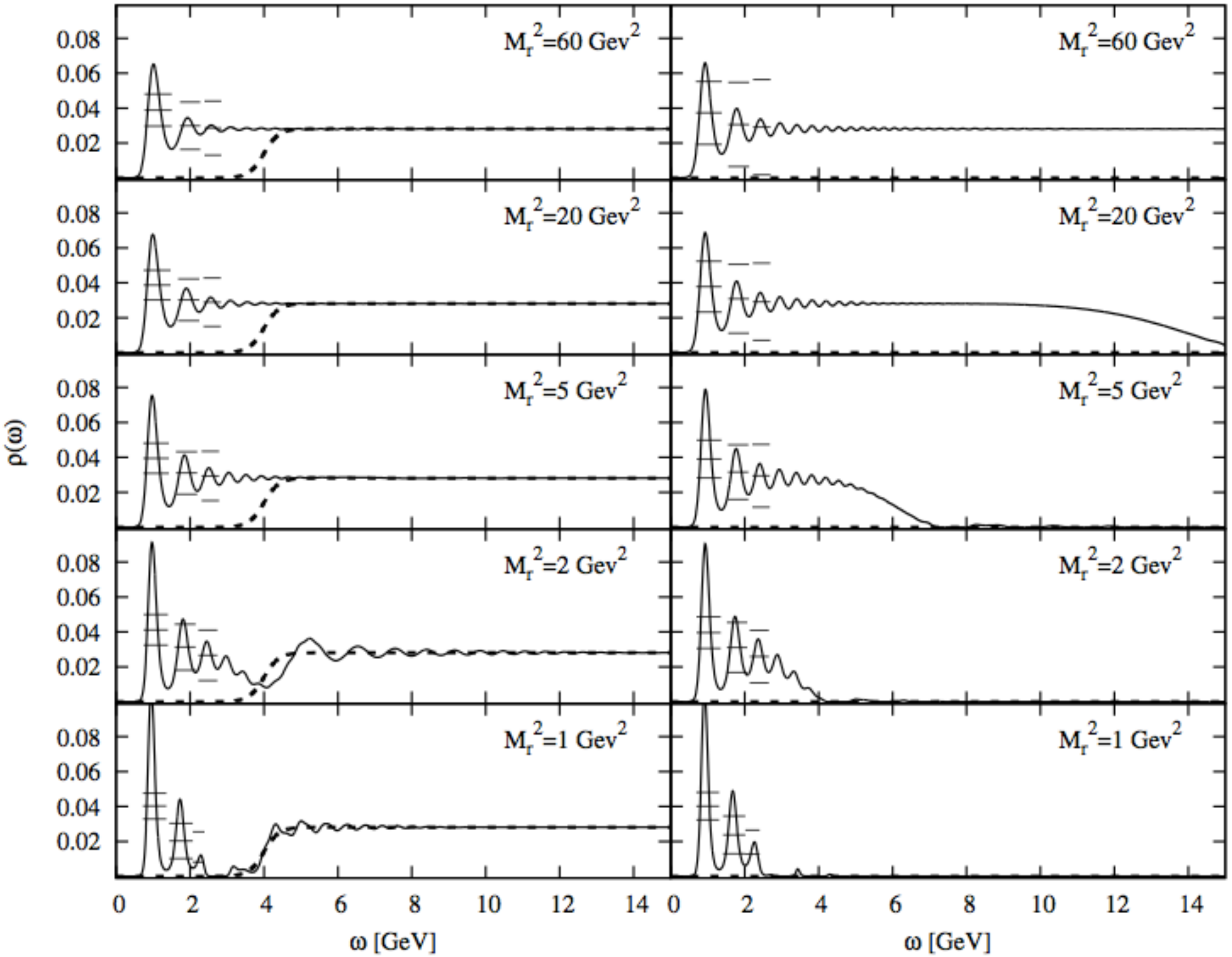




K. Araki et al., To be published

Complex Borel Sum Rules

- 1st excited state and the continuum spectrum



Conclusion

- Hadron spectroscopy is a renewed hot subject. Many new experimental facilities are ready. Theoretical development is achieving direct connection from QCD to the spectrum.
- QCD sum rules are semi-analytic approach to obtain the spectral function. The nonperturbative effects are taken into account through the quark and gluon condensates in the vacuum.
- A new MEM approach gives us opportunity to compute the spectral function without assuming its explicit shape. Extension to the complex Borel plane empowers the MEM method to obtain excited/continuum spectra.
- Recent applications of the MEM QCD-SR include the nucleon and its excited states, charmonium/bottomonium at finite temperature and continuum part of the spectrum.