Study of hyperon potentials from 2+1 lattice QCD

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for HAL QCD Collaboration

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Plan of research

QCD

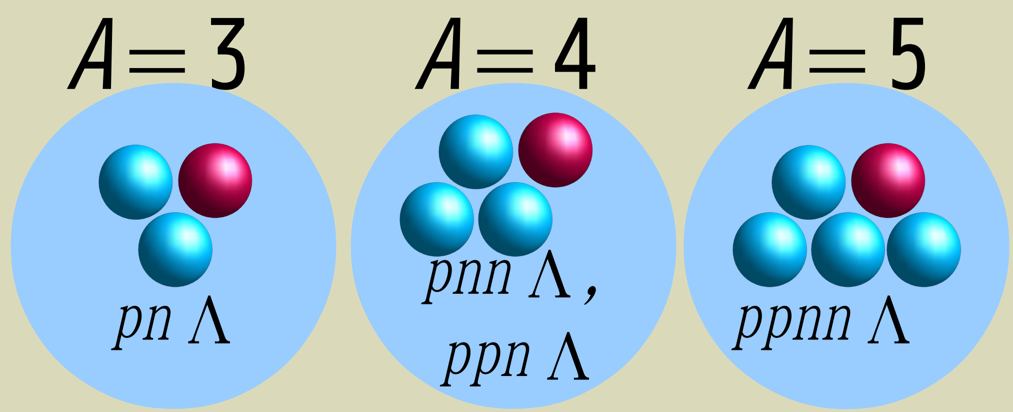
Baryon interaction

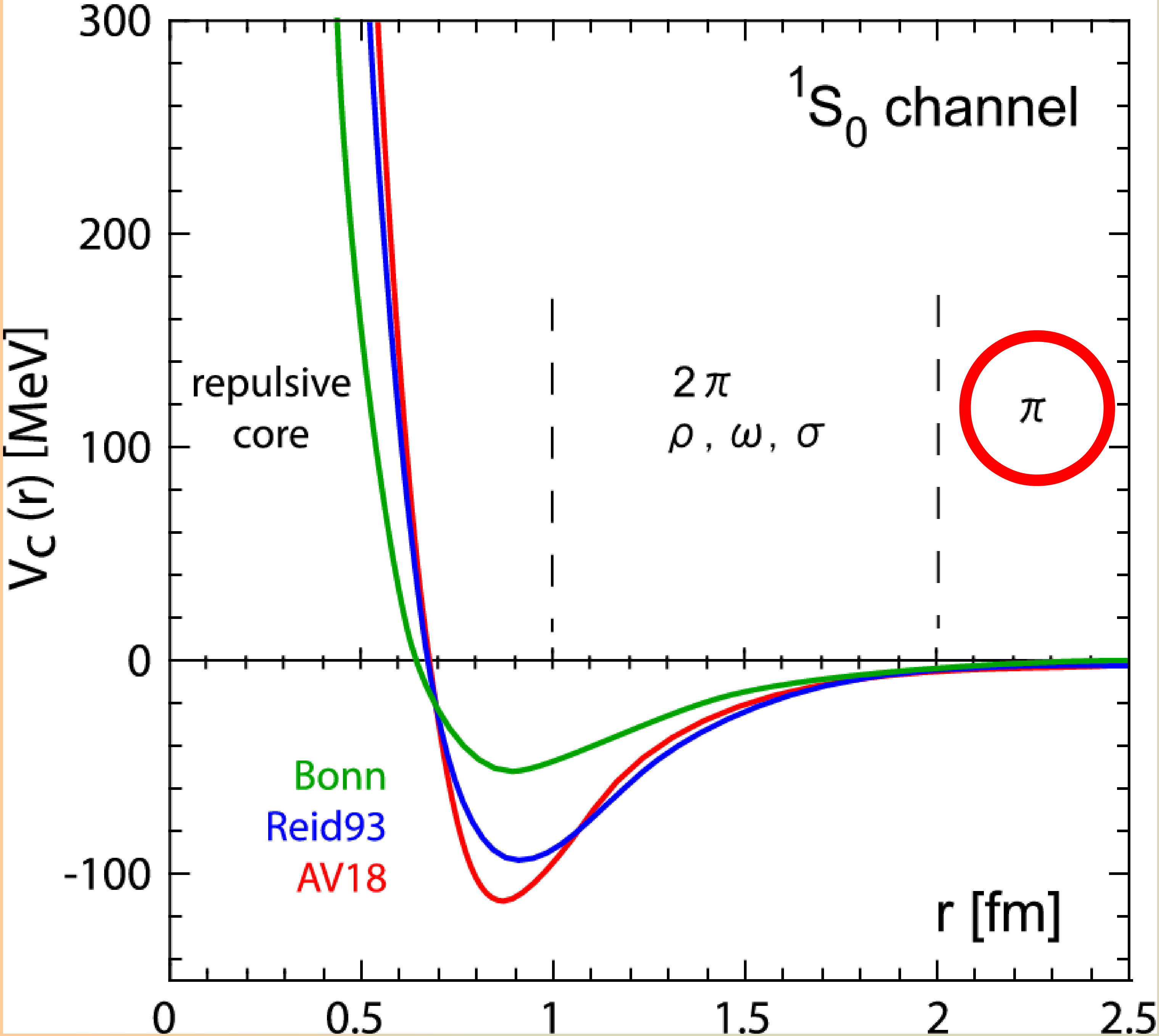
J-PARC
hyperon-nucleon (YN)
scattering

Structure and reaction of
(hyper)nuclei

Equation of State (EoS)
of nuclear matter

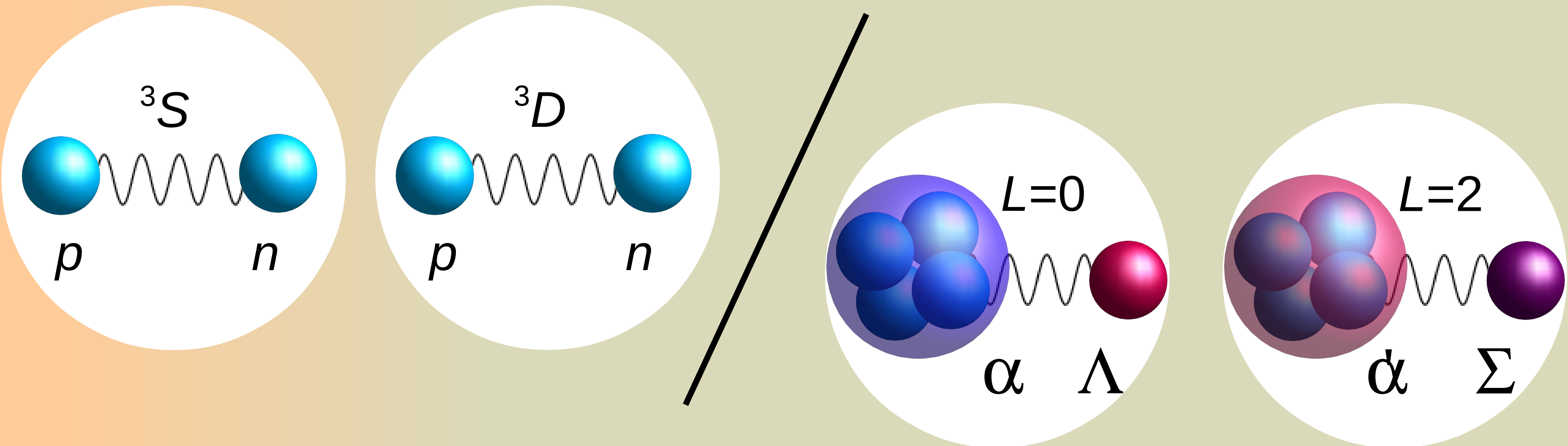
Neutron star and
supernova





300
200
100
0
-100
V_C (r) [MeV]
$^1S_0$ channel
repulsive core
2 π
ρ , ω , σ
π
Bonn
Reid93
AV18
r [fm]
0
0.5
1
1.5
2
2.5

Comparison between d=p+n and core+Y



	$\langle T_S \rangle$ (MeV)	$\langle T_D \rangle$ (MeV)	$\langle V_{NN}(\text{central}) \rangle$ (MeV)	$\langle V_{NN}(\text{tensor}) \rangle$ (MeV)	$\langle V_{NN}(\text{LS}) \rangle$ (MeV)
AV8	8.57	11.31	−4.46	−16.64	−1.02
G3RS	10.84	5.64	−7.29	−11.46	0.00

	$\langle T_{Y\text{-}c} \rangle_\Lambda$	$\langle T_{Y\text{-}c} \rangle_\Sigma + \Delta\langle H_C \rangle$	$\langle V_{YN}$(のこり)$\rangle$	$2\langle V_{\Lambda N\text{-}\Sigma N}(\text{tensor}) \rangle$
${}^5_\Lambda\text{He}$	9.11	3.88+4.68	−0.86	−19.51
${}^4_\Lambda\text{H}^*$	5.30	2.43+2.02	0.01	−10.67
${}^4_\Lambda\text{H}$	7.12	2.94+2.16	−5.05	−9.22

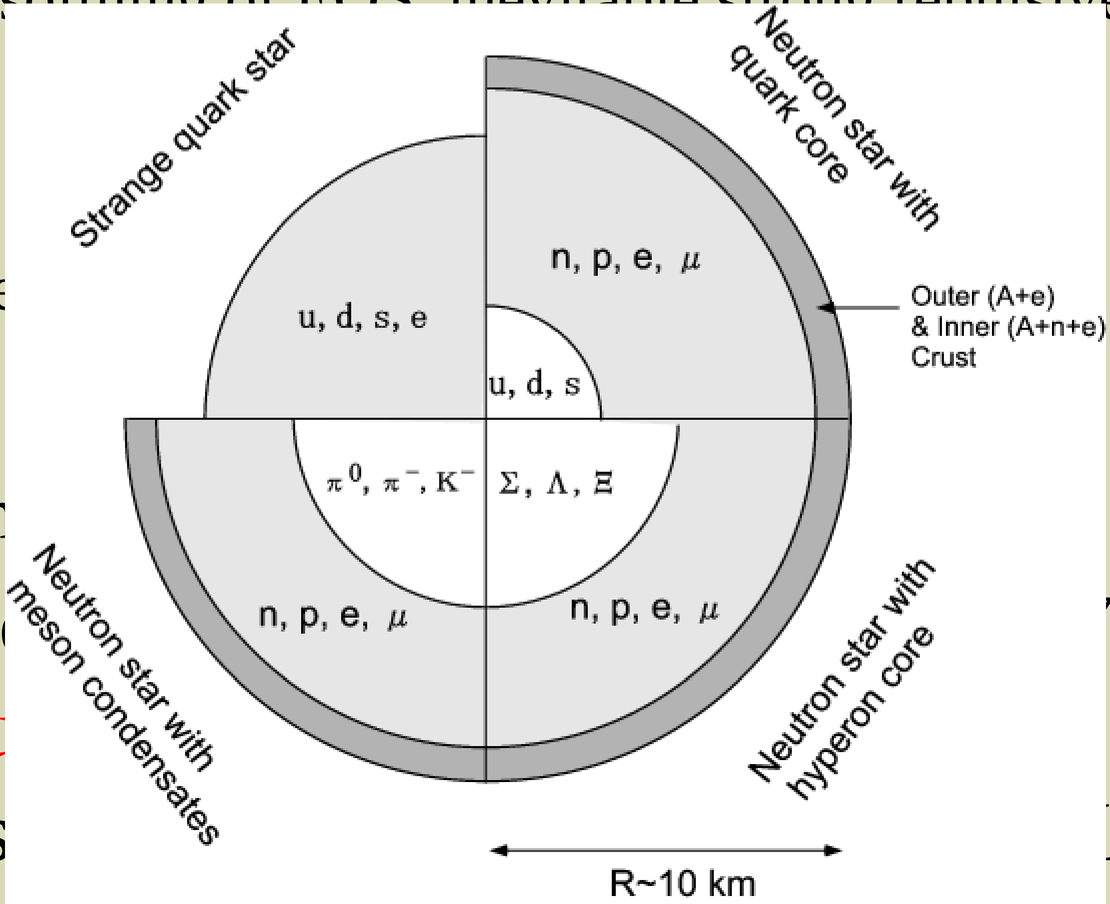
Lattice QCD calculation

Outline

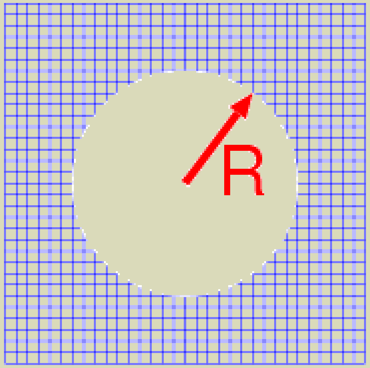
- Introduction
- Formulation --- potential (central + tensor)
- Numerical results:
  - $N\Lambda$ force (V_C + V_T)
  - $N\Sigma$ (I=3/2) force (V_C + V_T)
- Recent work on lattice QCD
  - Effective hadron block algorithm for the 4pt correlation function (NBS wave function)
  - Extention to various baryon-baryon channels
  - Hybrid parallel computation by MPI and OpenMP
- Summary

Introduction:

- Study of hyperon-nucleon (*YN*) and hyperon-hyperon (*YY*) interactions is one of the important subjects in the nuclear physics.
  - Structure of the neutron-star core,
    - Hyperon mixing, softning of EOS, inevitable strong repulsive force,
  - H-dibaryon problem,
    - To be, or not to be
- The project at J-PARC:
  - Explore the multistran
- However, the phenomen *Y* interactions has large u contrast to the nice des *N* potential.



Formulation



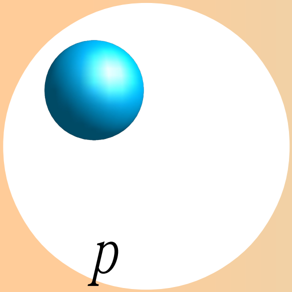
Lattice QCD simulation

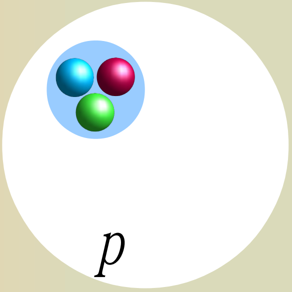
$$L = -\frac{1}{4} G^{a}_{\mu\nu} G^{a\mu\nu} + \bar{q}\gamma^{\mu}(i\partial_{\mu} - g\, t^{a} A^{a}_{\mu})q - m\bar{q}q$$

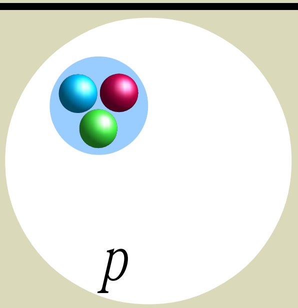
$$\langle O(\bar{q}, q, U)\rangle = \int dU\, d\bar{q}\, dq\, \mathrm{e}^{-S(\bar{q},q,U)} O(\bar{q}, q, U)$$

$$= \int dU \det D(U)\, \mathrm{e}^{-S_U(U)} O(D^{-1}(U))$$

$$= \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} O(D^{-1}(U_i))$$



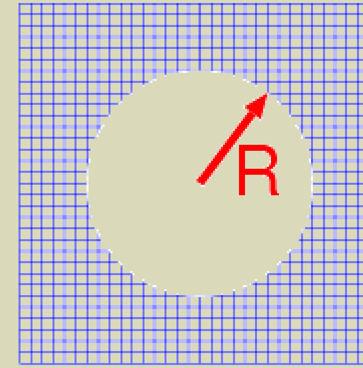
$\rightarrow \langle$ 

(t) 

$(t_0)\rangle$

Formulation

Lattice QCD simulation



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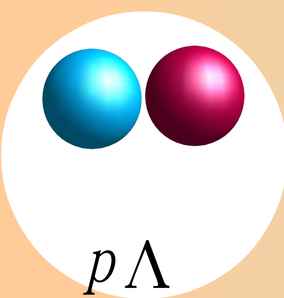
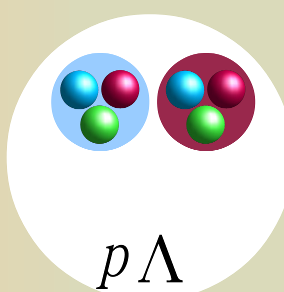
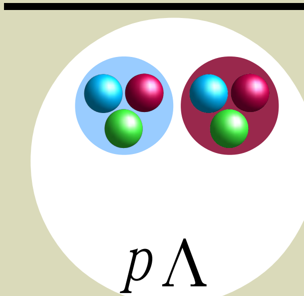


$\longrightarrow \langle$ 

(t) $\overline{\quad}$ 

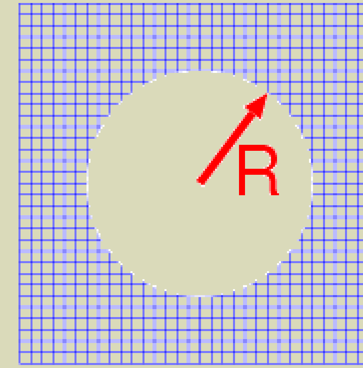
$(t_0)\rangle$

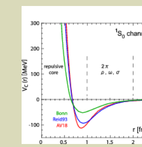
Formulation

i) basic procedure:

asymptotic region

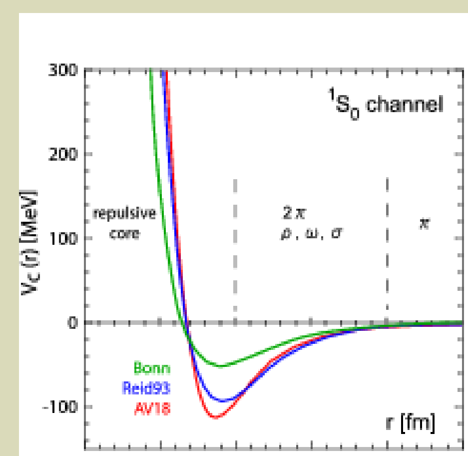
--> phase shift





ii) advanced (HAL's) procedure: interacting region

--> potential



Formulation

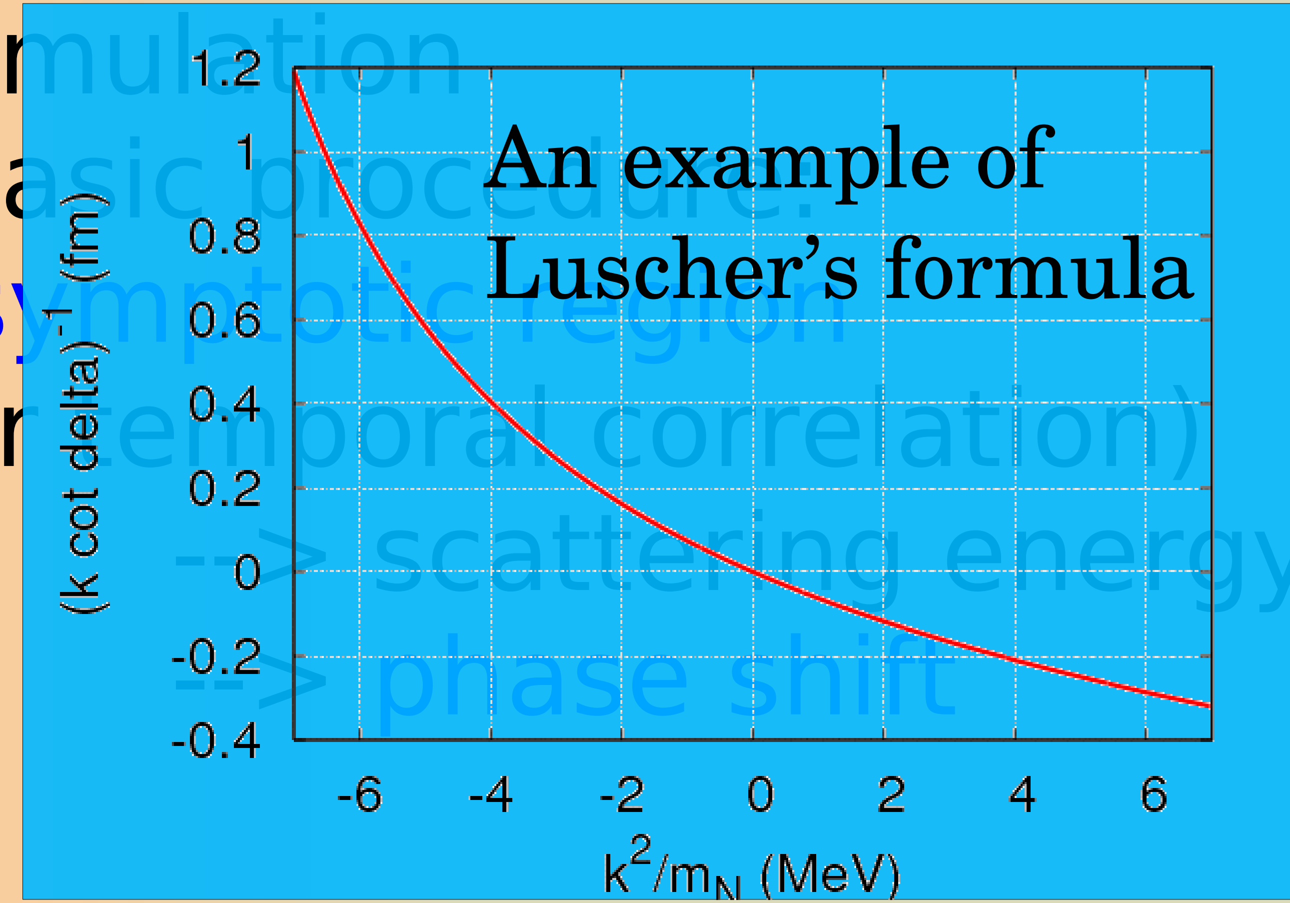
i) basic procedure:

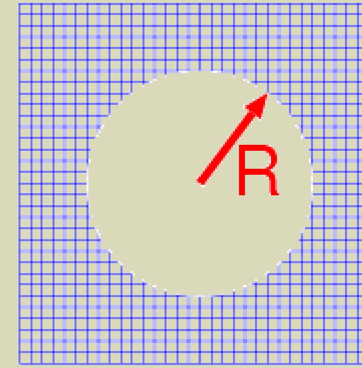
asymptotic region

(or temporal correlation)

--> scattering energy

--> phase shift





$$E = \frac{k^2}{2\mu}$$

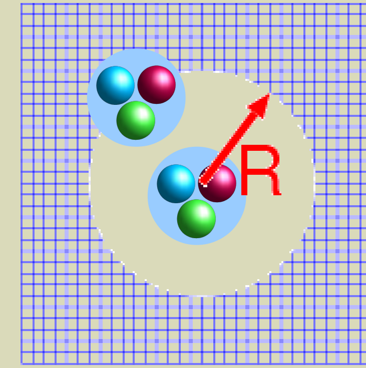
$$k \cot \delta_0(k) = \frac{2}{\sqrt{\pi} L} Z_{00}(1; (kL/(2\pi))^2) = \frac{1}{a_0} + O(k^2)$$

$$Z_{00}(1; q^2) = \frac{1}{\sqrt{4\pi}} \sum_{\vec{n} \in Z^3} \frac{1}{(n^2 - q^2)^s} \qquad \Re\, s > \frac{3}{2}$$

Luscher, NPB354, 531 (1991).

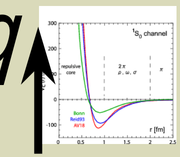
Aoki, et al., PRD71, 094504 (2005).

Formulation



Lattice QCD simulation

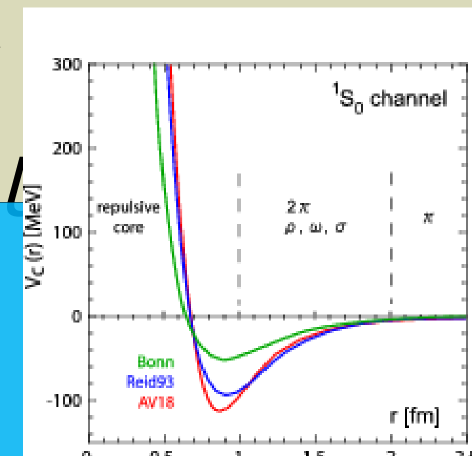
$$L=-\frac{1}{4}G^{a}_{\mu\nu}G^{a\mu\nu}+\bar{q}\gamma^{\mu}(i\partial_{\mu}-g\,t^{a}A^{a}_{\mu})q-m\bar{q}q$$



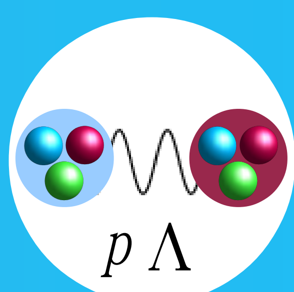
$$\langle O(\bar{q},q,U)\rangle=\int dU\, d\bar{q}\, dq\, e^{-S(\bar{q},q,U)}\, O(\bar{q},q \ldots$$

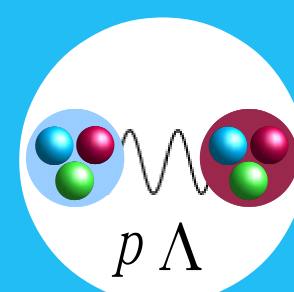
$$=\int dU\, \det D(U)\, e^{-S_U(U)}\, O(D^{-1}(U \ldots$$

$$=\lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}O(D^{-1}(U_i))$$



$$F^{(JM)}_{\alpha\beta}(\vec{r},t-t_0)$$

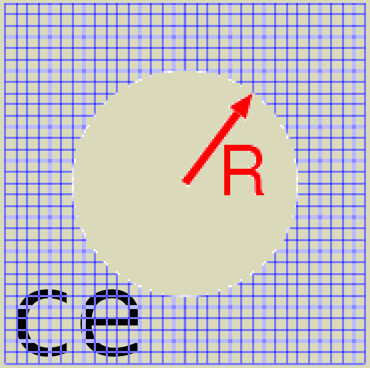
$\rightarrow \langle$ 

$(\vec{r},t)$ 

$(t_0)\rangle$

Calculate the scattering state

HAL formulation



ii) advanced procedure:

make better use of the lattice output ! (wave function)

interacting region

--> potential

Ishii, Aoki, Hatsuda, PRL99, 022001 (2007); ibid., arXiv:0805.2462[hep-ph].

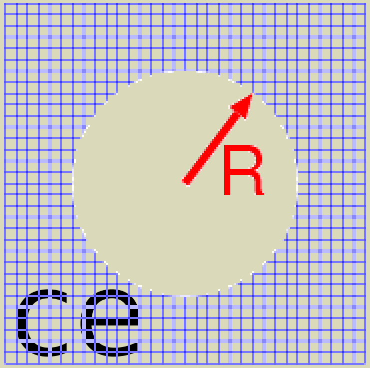
NOTE:

> Potential is not a direct experimental observable.

> Potential is a useful tool to give (and to reproduce) the physical quantities. (e.g., phase shift)

....

HAL formulation



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PRL99, 022001 (2007);
ibid., arXiv:0805.2462[hep-ph].

=>

> Phase shift
> Nuclear many-body problems

Numerical results

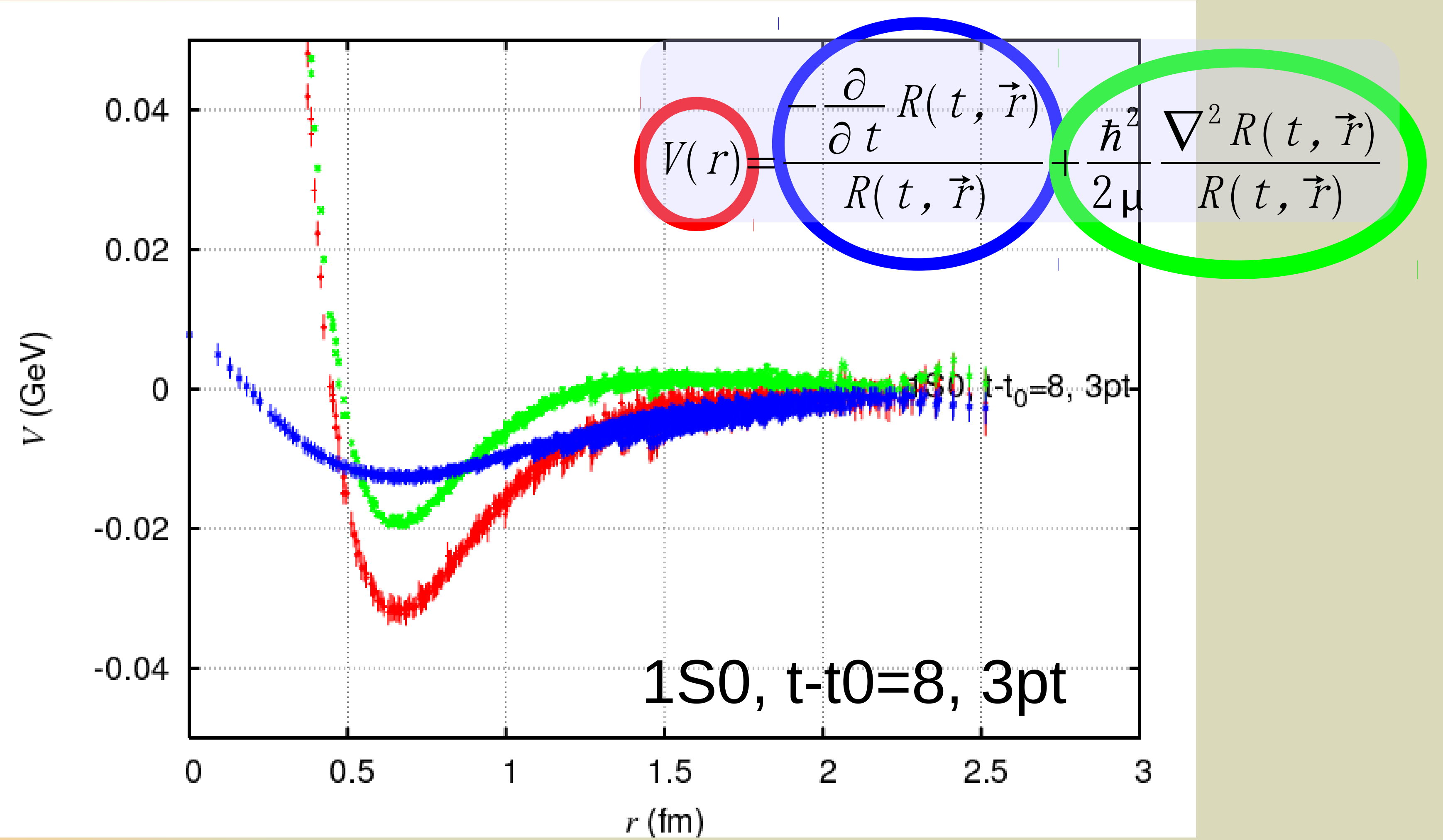
Full QCD calculations by using N_F=2+1 PACS-CS gauge configurations:

- S. Aoki, et al., (PACS-CS Collaboration),
  PRD**79**, 034503 (2009), arXiv:0807.1661 [hep-lat].
- Iwasaki gauge action at β=1.90 on $32^3 \times 64$ lattice
- O(a) improved Wilson quark action
- $1/a$ = 2.17 GeV (a = 0.0907 fm)

$(\kappa_{ud})_{N_{conf}}$	m_π	m_ρ	m_K	m_{K^*}	m_N	m_Λ	m_Σ	m_Ξ
2+1 flavor QCD by PACS-CS with $\kappa_s = 0.13640$ @present calc (Dirichlet BC along T)								
$(0.13700)_{609}$	700.0(4)	1108(3)	785.8(3)	1159(2)	1573(4)	1632(4)	1650(5)	1700(4)
$(0.13727)_{481}$	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
(0.13754)	415(1)	903(5)	639.7(8)	1024(4)	1232(10)	1354(6)	1415(7)	1512(4)
Exp.	135	770	494	892	940	1116	1190	1320

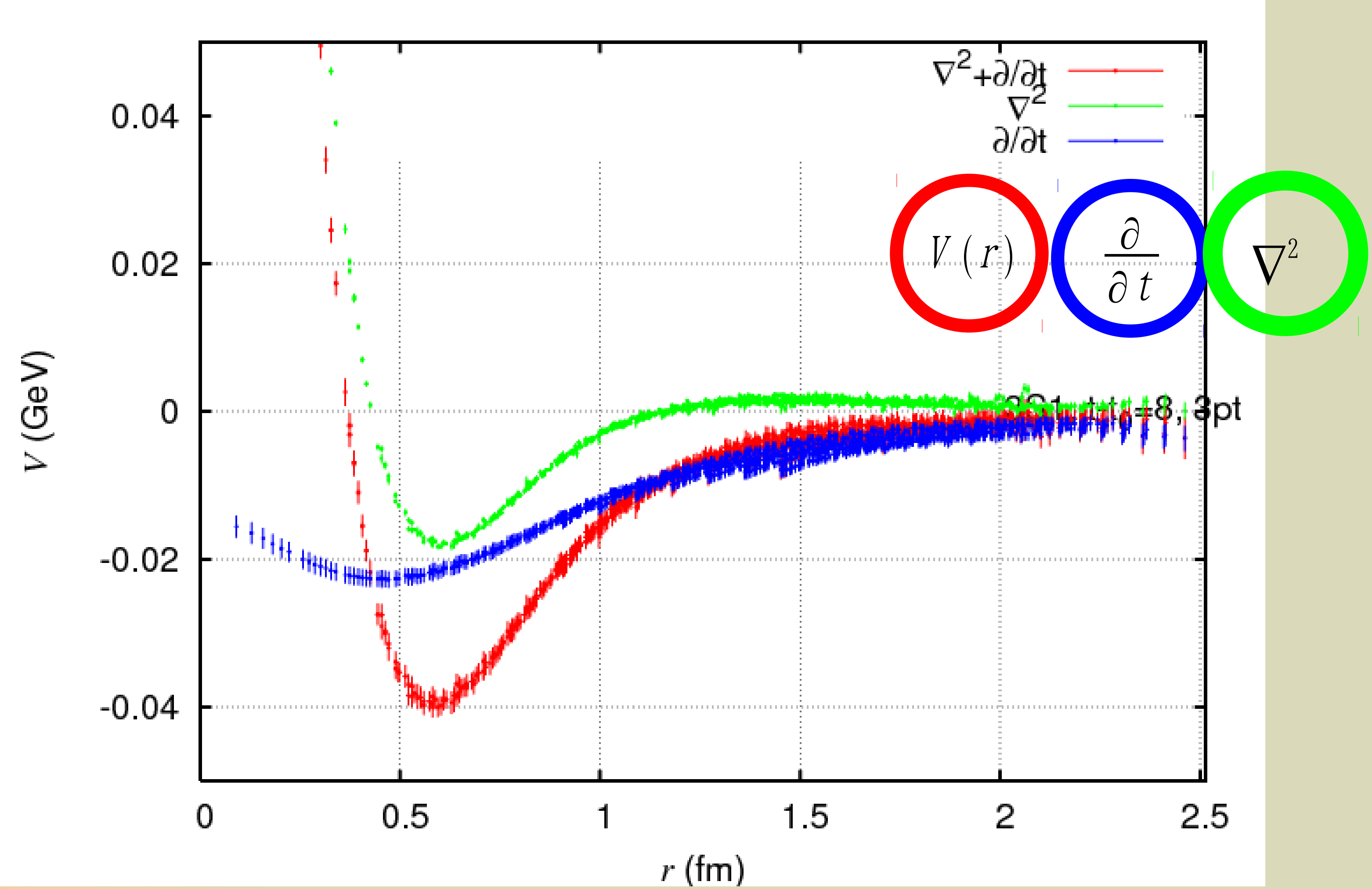
ΛN potential

$V_C(\Lambda N;\ 1S0)$



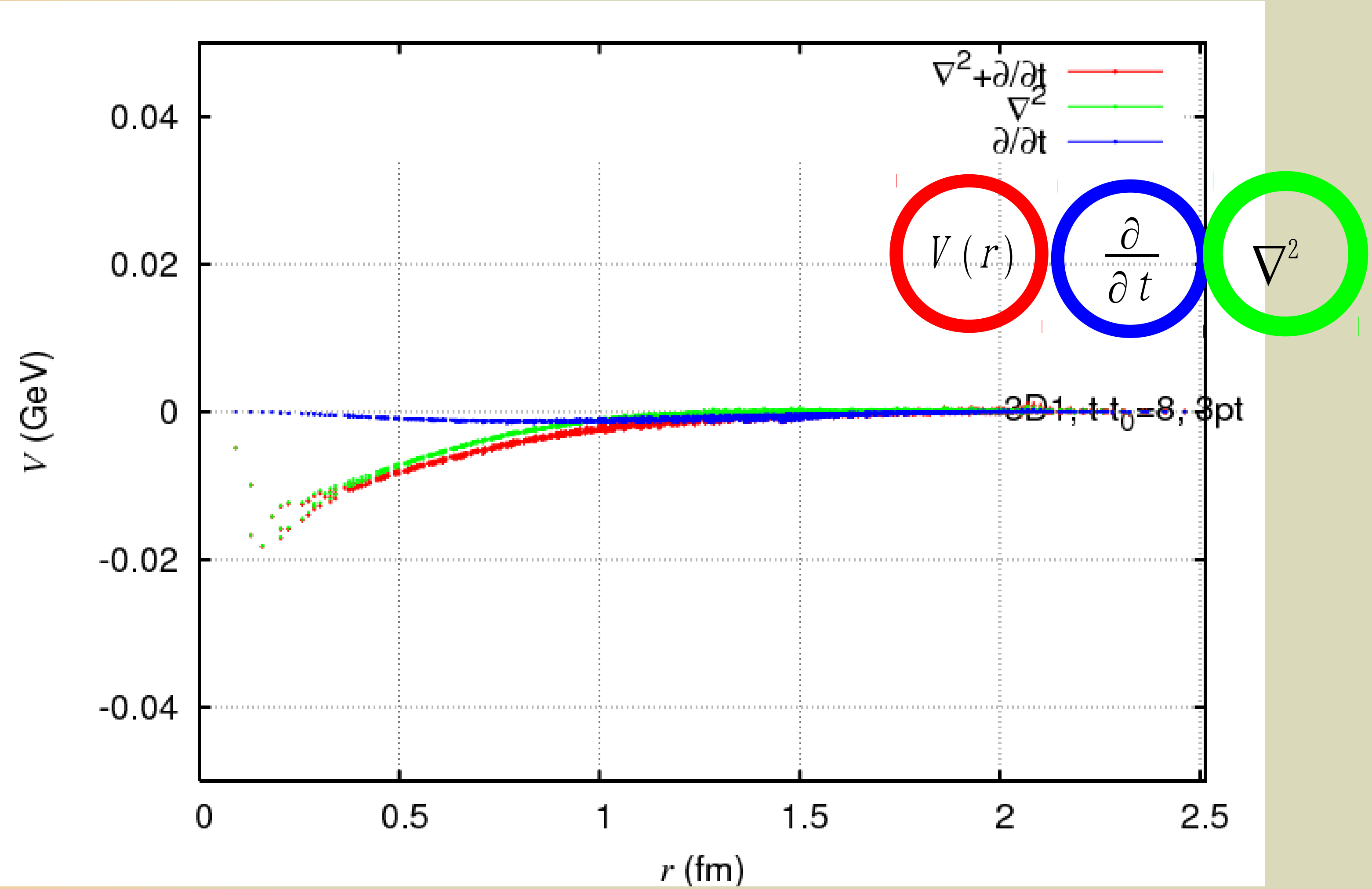
- {27}+{8s}
- Similar to NN (1S0)
- Sizable contribution from time-derivative part

$V_C(\Lambda N;\ 3S1\text{-}3D1)$



- {10*}+{8a}
- Sizable attractive contribution from time-derivative part

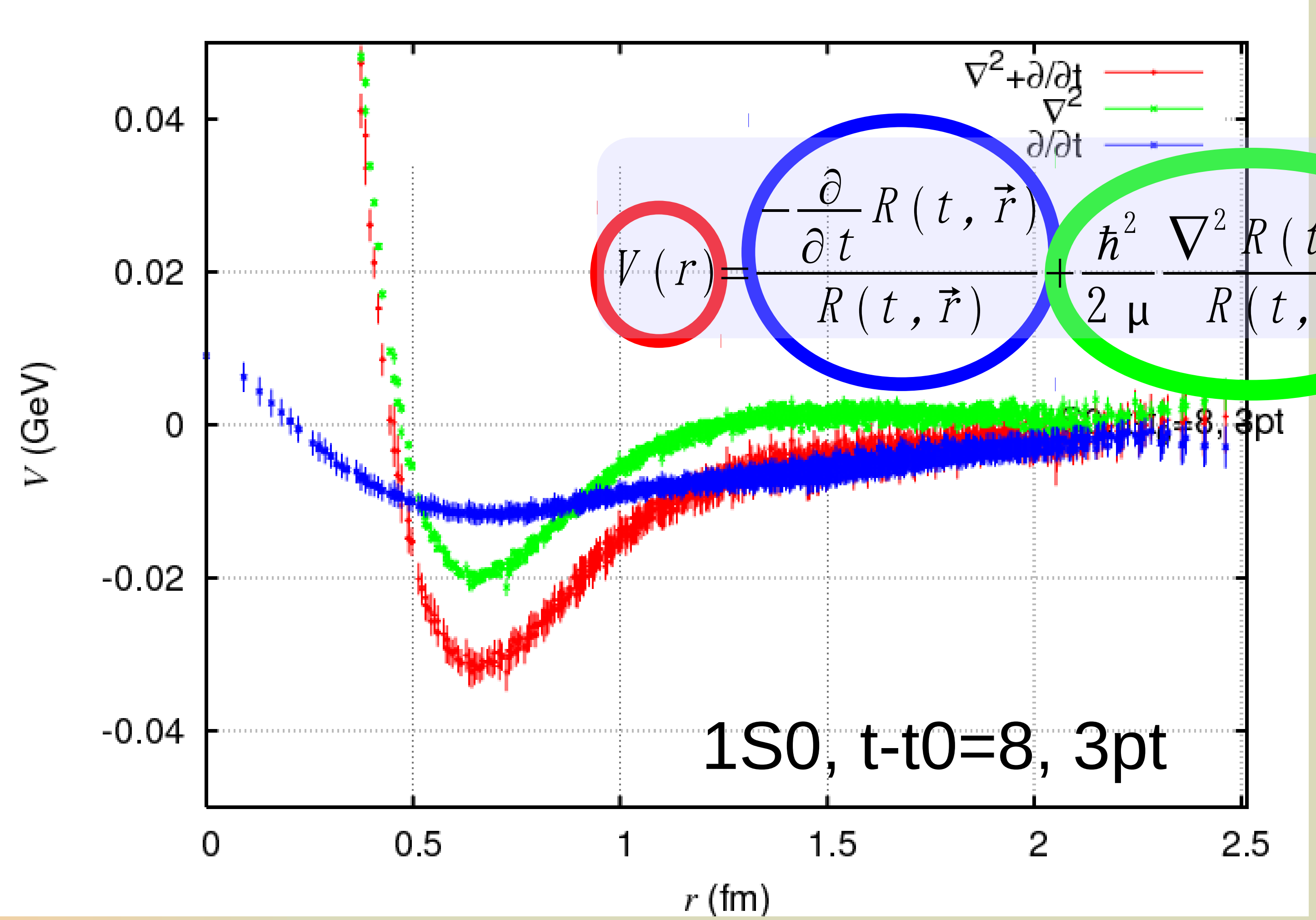
$V_T(\Lambda N;\ 3S1\text{-}3D1)$



- Weaker tensor force than NN
- Small contribution from time-derivative part

$\Sigma N(I=3/2)$ potential

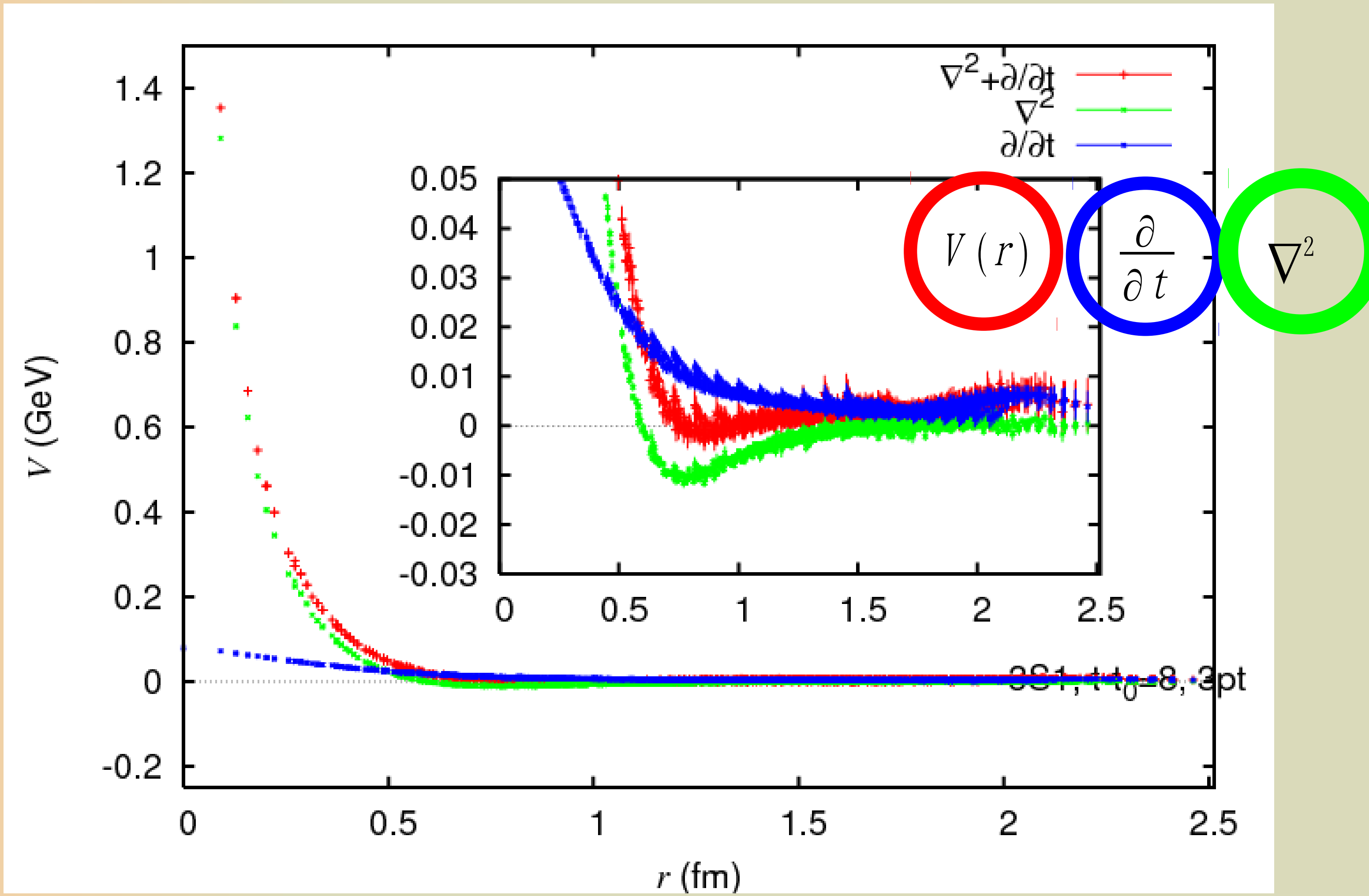
$V_C(\Sigma N(I=3/2);\ 1S0)$



$$V(r) = \frac{-\frac{\partial}{\partial t} R(t,\vec{r})}{R(t,\vec{r})} + \frac{\hbar^2}{2\mu} \frac{\nabla^2 R(t,\vec{r})}{R(t,\vec{r})}$$

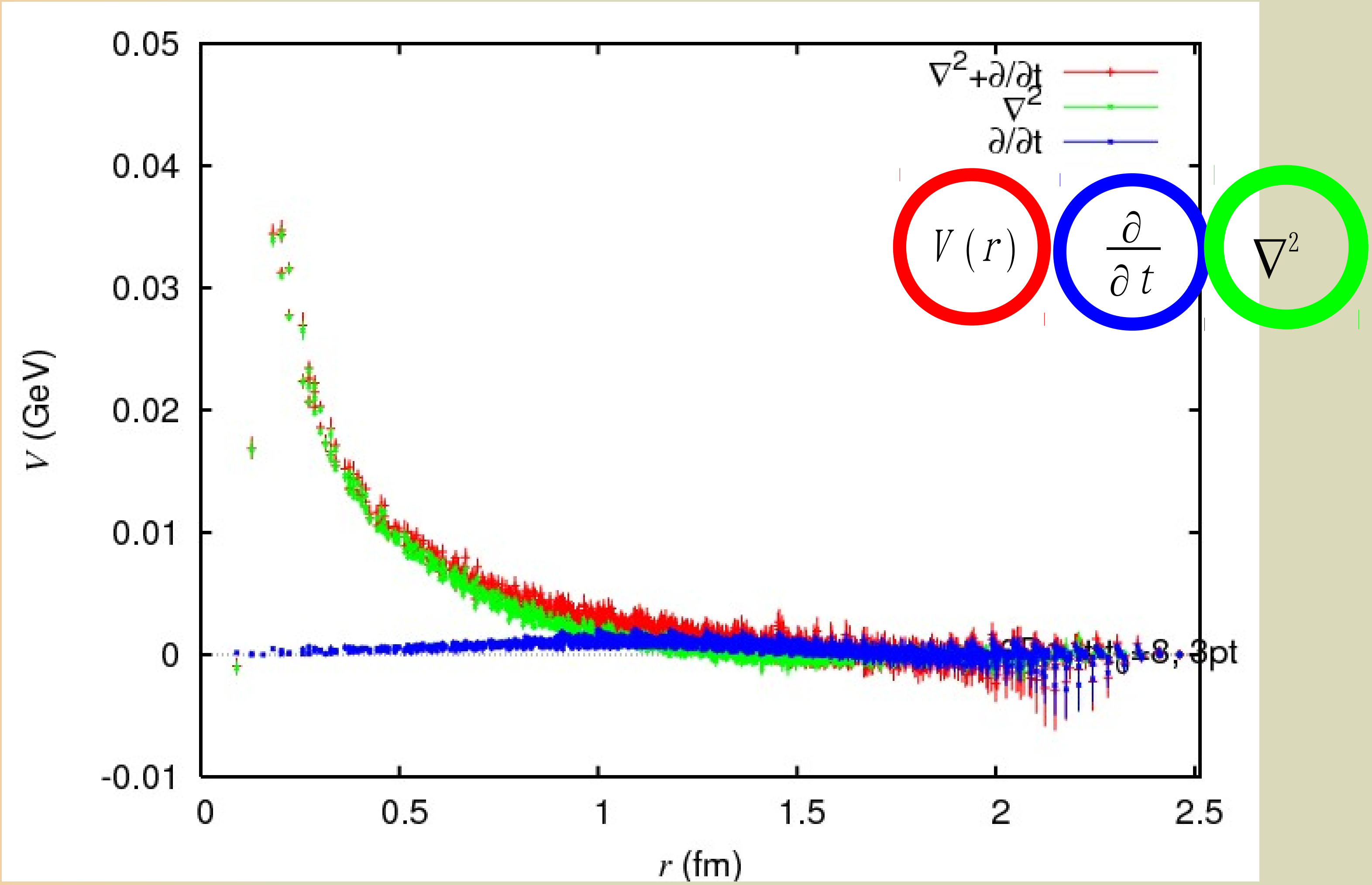
- {27}
- Similar to NN (1S0) (as well as Lambda-N (1S0))
- Sizable contribution from time-derivative part

V_C(ΣN(I=3/2); 3S1-3D1)



- {10}
- Repulsive potential (consistent with quark model)
- sizable repulsive contribution from time-derivative part

V_T(ΣN(I=3/2); 3S1-3D1)



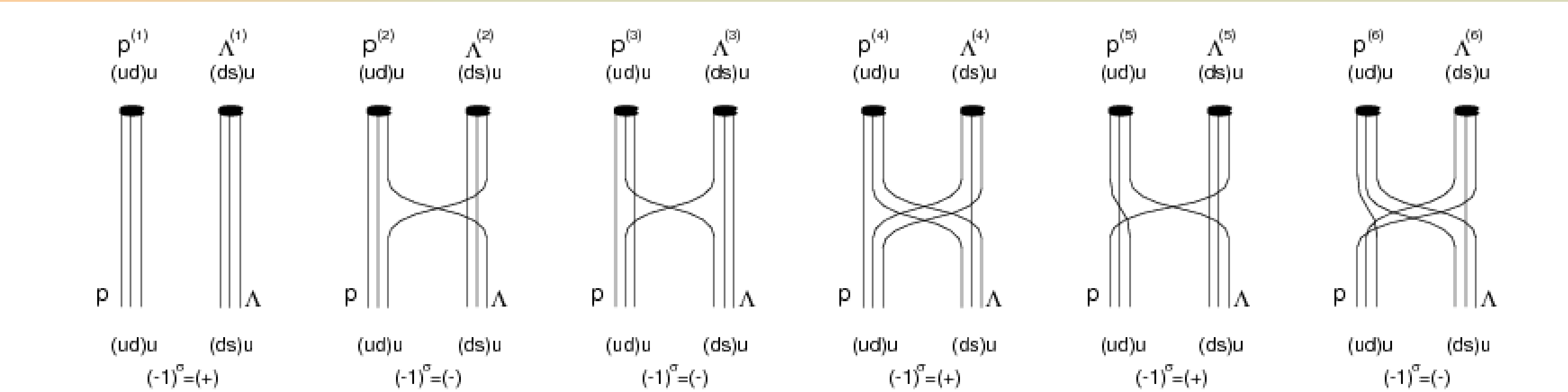
- Weak tensor force
- Small contribution from time-derivative part

Scattering phase shifts

Proton-Lambda scattering (preliminary)

Parametrized potential → Phase shift

Effective block algorithm for various baryon-baron calculations

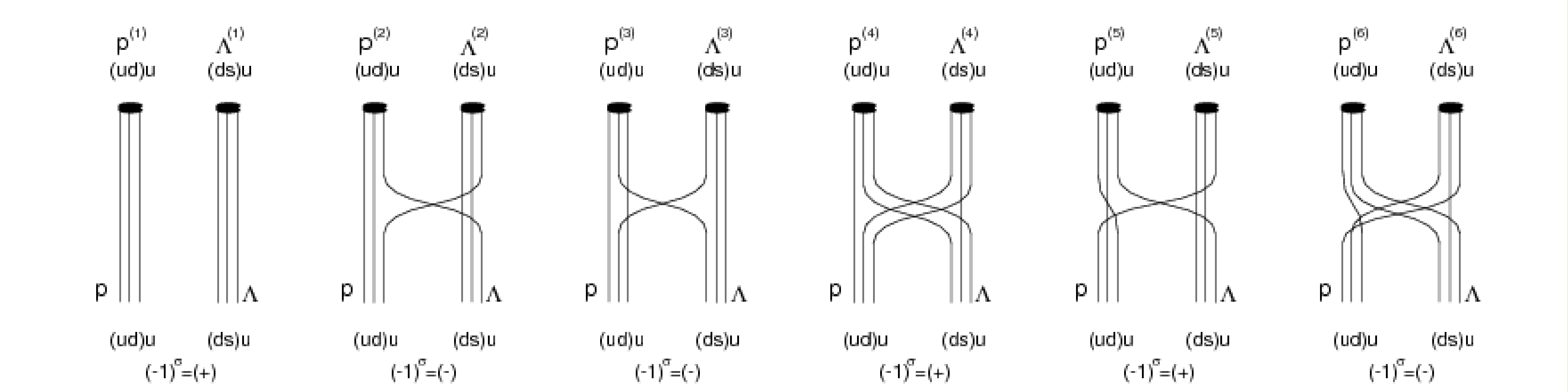


$$
\begin{aligned}
p_\alpha(x) &= \varepsilon(c_1,c_2,c_3)(C\gamma_5)(\alpha_1,\alpha_2)\delta(\alpha,\alpha_3)u(\xi_1)d(\xi_2)u(\xi_3), \qquad (\xi_i = x_i\alpha_i c_i)\\
&= \varepsilon(1,2,3)(C\gamma_5)(1,2)\delta(\alpha,3)u(1)d(2)u(3).
\end{aligned} \tag{11}
$$

$$
\begin{aligned}
&\sum_{\vec{X}} \left\langle 0 \left| p_\alpha(\vec{X}+\vec{r},t)\Lambda_\beta(\vec{X},t)\overline{\mathcal{J}_{p_{\alpha'}\Lambda_{\beta'}}(t_0)} \right| 0 \right\rangle \\
= \ & \sum_{\vec{X}} \tfrac{1}{6}\varepsilon(1,4,2)\varepsilon(5,6,3)\varepsilon(1',4',2')\varepsilon(5',6',3')(C\gamma_5)(1,4)\delta(\alpha,2)(C\gamma_5)(1',4')\delta(\alpha',2') \\
& \times\{(C\gamma_5)(5,6)\delta(\beta,3)+(C\gamma_5)(6,3)\delta(\beta,5)-2(C\gamma_5)(3,5)\delta(\beta,6)\} \\
& \times\{(C\gamma_5)(5',6')\delta(\beta',3')+(C\gamma_5)(6',3')\delta(\beta',5')-2(C\gamma_5)(3',5')\delta(\beta',6')\} \\
& \times\langle u(1)d(4)u(2)d(5)s(6)u(3)\bar{u}(3')\bar{s}(6')\bar{d}(5')\bar{u}(2')\bar{d}(4')\bar{u}(1')\rangle.
\end{aligned} \tag{12}
$$

$$
\sum_{c_1,\ldots,c_6}\ \sum_{\alpha_1,\ldots,\alpha_6}\ \sum_{c_1',\ldots,c_6'}\ \sum_{\alpha_1',\ldots,\alpha_6'}
$$

$$
(N_c!\,N_\alpha)^{2B} \times N_u!\,N_d!\,N_s!
$$



Performed these manipulations based on the diagrammatic classification, most of the summations can be carried out prior to evaluating the FFT so that the number of iterations significantly reduces; The numbers of iteration are $\{1, 9, 144, 144, 36, 36\}$ for the baryon blocks $\{([p_\alpha^{(i)}] \times [\Lambda_\beta^{(i)}]); i = 1, \cdots, 6\}$. Therefore only 370 iterations should be explicitly performed to obtain the four-point correlation function of the $p\Lambda$ system when we take the operator $\overline{X}_u$ in $\overline{\Lambda}_{\beta'}$ in the source. For the sake of completeness, the total number of iterations does not change when we take the operator $\overline{X}_s$ in $\overline{\Lambda}_{\beta'}$ in the source whereas the numbers of iteration are $\{1, 36, 36, 144, 144, 36\}$ when we consider the contribution from the operator $\overline{X}_d$ in $\overline{\Lambda}_{\beta'}$ in the source which slightly differ from the former cases and the total number of iterations is 397.

Effective block algorithm to calculate the 52 channels of 4pt correlator

$$\langle pn\overline{pn}\rangle, \tag{4.1}$$

$$\begin{aligned}
&\langle p\Lambda\overline{p\Lambda}\rangle, \quad \langle p\Lambda\overline{\Sigma^+ n}\rangle, \quad \langle p\Lambda\overline{\Sigma^0 p}\rangle, \\
&\langle \Sigma^+ n\overline{p\Lambda}\rangle, \quad \langle \Sigma^+ n\overline{\Sigma^+ n}\rangle, \quad \langle \Sigma^+ n\overline{\Sigma^0 p}\rangle, \\
&\langle \Sigma^0 p\overline{p\Lambda}\rangle, \quad \langle \Sigma^0 p\overline{\Sigma^+ n}\rangle, \quad \langle \Sigma^0 p\overline{\Sigma^0 p}\rangle,
\end{aligned} \tag{4.2}$$

$$\begin{array}{llllll}
\langle \Lambda\Lambda\overline{\Lambda\Lambda}\rangle, & \langle \Lambda\Lambda\overline{p\Xi^-}\rangle, & \langle \Lambda\Lambda\overline{n\Xi^0}\rangle, & \langle \Lambda\Lambda\overline{\Sigma^+\Sigma^-}\rangle, & \langle \Lambda\Lambda\overline{\Sigma^0\Sigma^0}\rangle, & \\
\langle p\Xi^-\overline{\Lambda\Lambda}\rangle, & \langle p\Xi^-\overline{p\Xi^-}\rangle, & \langle p\Xi^-\overline{n\Xi^0}\rangle, & \langle p\Xi^-\overline{\Sigma^+\Sigma^-}\rangle, & \langle p\Xi^-\overline{\Sigma^0\Sigma^0}\rangle, & \langle p\Xi^-\overline{\Sigma^0\Lambda}\rangle, \\
\langle n\Xi^0\overline{\Lambda\Lambda}\rangle, & \langle n\Xi^0\overline{p\Xi^-}\rangle, & \langle n\Xi^0\overline{n\Xi^0}\rangle, & \langle n\Xi^0\overline{\Sigma^+\Sigma^-}\rangle, & \langle n\Xi^0\overline{\Sigma^0\Sigma^0}\rangle, & \langle n\Xi^0\overline{\Sigma^0\Lambda}\rangle, \\
\langle \Sigma^+\Sigma^-\overline{\Lambda\Lambda}\rangle, & \langle \Sigma^+\Sigma^-\overline{p\Xi^-}\rangle, & \langle \Sigma^+\Sigma^-\overline{n\Xi^0}\rangle, & \langle \Sigma^+\Sigma^-\overline{\Sigma^+\Sigma^-}\rangle, & \langle \Sigma^+\Sigma^-\overline{\Sigma^0\Sigma^0}\rangle, & \langle \Sigma^+\Sigma^-\overline{\Sigma^0\Lambda}\rangle, \\
\langle \Sigma^0\Sigma^0\overline{\Lambda\Lambda}\rangle, & \langle \Sigma^0\Sigma^0\overline{p\Xi^-}\rangle, & \langle \Sigma^0\Sigma^0\overline{n\Xi^0}\rangle, & \langle \Sigma^0\Sigma^0\overline{\Sigma^+\Sigma^-}\rangle, & \langle \Sigma^0\Sigma^0\overline{\Sigma^0\Sigma^0}\rangle, & \\
 & \langle \Sigma^0\Lambda\overline{p\Xi^-}\rangle, & \langle \Sigma^0\Lambda\overline{n\Xi^0}\rangle, & \langle \Sigma^0\Lambda\overline{\Sigma^+\Sigma^-}\rangle, & & \langle \Sigma^0\Lambda\overline{\Sigma^0\Lambda}\rangle,
\end{array} \tag{4.3}$$

$$\begin{aligned}
&\langle \Xi^-\Lambda\overline{\Xi^-\Lambda}\rangle, \quad \langle \Xi^-\Lambda\overline{\Sigma^-\Xi^0}\rangle, \quad \langle \Xi^-\Lambda\overline{\Sigma^0\Xi^-}\rangle, \\
&\langle \Sigma^-\Xi^0\overline{\Xi^-\Lambda}\rangle, \quad \langle \Sigma^-\Xi^0\overline{\Sigma^-\Xi^0}\rangle, \quad \langle \Sigma^-\Xi^0\overline{\Sigma^0\Xi^-}\rangle, \\
&\langle \Sigma^0\Xi^-\overline{\Xi^-\Lambda}\rangle, \quad \langle \Sigma^0\Xi^-\overline{\Sigma^-\Xi^0}\rangle, \quad \langle \Sigma^0\Xi^-\overline{\Sigma^0\Xi^-}\rangle,
\end{aligned} \tag{4.4}$$

$$\langle \Xi^-\Xi^0\overline{\Xi^-\Xi^0}\rangle. \tag{4.5}$$

- Elapse times to calculate the 52 matrix correlators (MPI+OpenMP)
- [tasks_per_node] x [OMP_NUM_THREADS]

	64x1	32x2	16x4	8x4	4x8	2x16	1x32
Step-1	0:14	0:16	0:09	0:09	0:07	0:06	0:06
Step-2	0:10	0:11	0:12	0:12	0:12	0:13	0:14

Summary

(1) Lattice QCD calculation for hyperon potentials toward the physical point calculation.
Lambda-N, Sigma-N: central, tensor

(2) Effective hadron block algorithm for the various baron-baryon interaction

A hybrid parallel C++ program is implemented by using MPI and OpenMP.

Reasonable performances at various hybrid parallel execution on the supercomputer (BlueGene/Q)