In-medium modified energy-momentum tensor form factors in a π-ρ-ω meson model

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Outline

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   b. Energy-momentum tensor form factors of the in-medium nucleons
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Introduction

- Understanding the structure of the nucleon has been one of the fundamental issues of the particle and nuclear physices
- The electro-magnetic and weak properties of the nucleon have been studied intensively (i.e. charge, magnetic moment, axial charge)

$$\langle N | J^{\mu}_{em} | N \rangle \longrightarrow Q, \mu, \ldots$$

$$\langle N | J^{\mu}_{\mathrm{weak}} | N \rangle \longrightarrow g_A, \ldots$$

Introduction

- The energy-momentum tensor form factors (EMTFF) of the nucleon which come from the gravitational interaction have been introduced by Pagels:

H. R. Pagels, Phys. Rev. 144, 1250 (1966)

$$\langle N | T^{\mu\nu}_{\mathrm{grav}} | N \rangle \longrightarrow M_2, J, d_1, \ldots$$

Introduction

$$\left\langle p' \left| \hat{T}_{\mu\nu}(0) \right| p \right\rangle = \overline{u}(p') \left[M_2(t) \frac{P_\mu P_\nu}{M_N} + J(t) \frac{i\left(P_\mu \sigma_{\nu\mu} + P_\nu \sigma_{\mu\nu}\right) \Delta^\rho}{2M_N} + d_1(t) \frac{\Delta_\mu \Delta_\nu - g_{\mu\nu}\Delta^2}{5M_N} \right] u(p)$$

where $P = (p + p')/2,\ \Delta = (p' - p)$ and $t = \Delta^2$

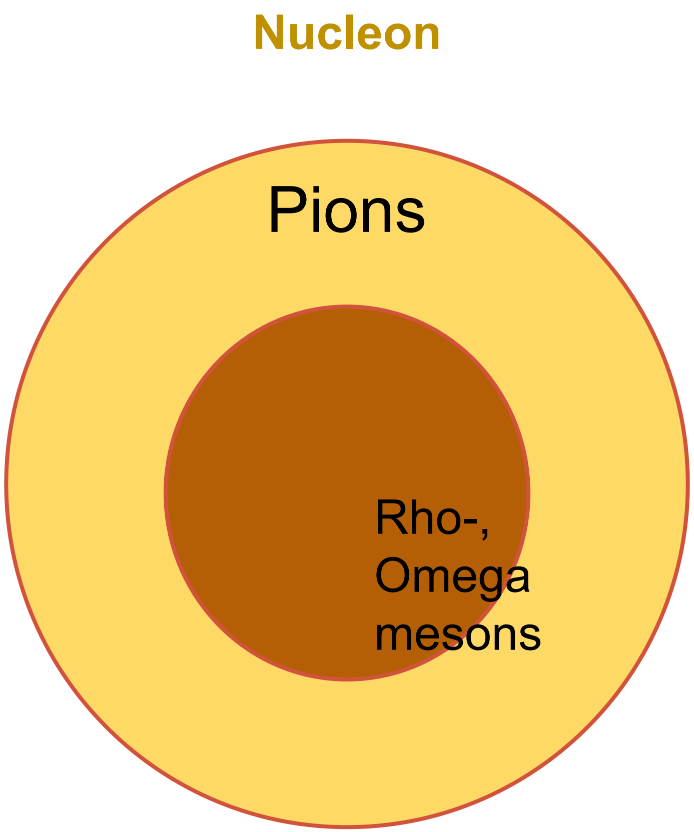
$$\langle p' | p \rangle = 2p^0 (2\pi)^3 \delta^{(3)}(\mathbf{p}' - \mathbf{p}) \qquad \bar{u}(p)u(p) = 2M_N$$

- $M_2(t)$ and $J(t)$ desdribe the nucleon momentum and angular momentum
- Interpretation of $d_1(t)$ less trivial, but also gives information about nucleon structure

Introduction

We will present a chiral solitonic model with the pion, rho and omega mesonic degrees of freedom in order to analyze the EMTFF of the nucleon in nuclear matter

Theoreical framework



- The nucleon has a structure, not a point-like particle
- Outer shell of the nucleon is made of pions and inner core is made of another hadronic degrees of freedom, like mesons and effective pare creations.

The effective Lagrangian for pions, rho- and omega-mesons in nuclear medium

The Effective Lagrangian for rho-, omega mesons and pions in nuclear medium

$$\mathcal{L} = \mathcal{L}_A + \mathcal{L}_{\chi SB} + \mathcal{L}_v + \mathcal{L}_{kin} + \mathcal{L}_{WZ}$$

$$\mathcal{L}_A = \frac{f_\pi^2}{4} Tr\left(\partial_0 U \partial_0 U^\dagger\right) - \alpha_p \frac{f_\pi^2}{4} Tr\left(\partial_i U \partial_i U^\dagger\right)$$

$$\mathcal{L}_{\chi SB} = \alpha_s \frac{f_\pi^2 m_\pi^2}{2} Tr\left(U - 1\right)$$

$$\mathcal{L}_V = \frac{a f_\pi^2}{4} Tr\left[D_\mu \xi \cdot \xi^\dagger + D_\mu \xi^\dagger \cdot \xi\right]^2$$

$$\mathcal{L}_{kin} = -\frac{1}{2g^2\zeta} Tr\left(F_{\mu\nu}^2\right)$$

$$\mathcal{L}_{WZ} = \left(\frac{N_c}{2} g\sqrt{\zeta}\right) \omega_\mu \frac{\epsilon^{\mu\nu\alpha\beta}}{24\pi^2} Tr\left\{\left(U^\dagger \partial_\nu U\right)\left(U^\dagger \partial_\alpha U\right)\left(U^\dagger \partial_\beta U\right)\right\}$$

$$\xi = \sqrt{U}$$

$$\begin{aligned} D_\mu &= \partial_\mu - i V_\mu \\ F_{\mu\nu} &= \partial_\mu V_\nu - \partial_\nu V_\mu - i\left[V_\mu, V_\nu\right] \\ V_\mu &= \frac{g}{2}\left(\tau^a \rho_\mu^a + \omega_\mu\right) \end{aligned}$$

The effective Lagrangian for pions, rho- and omega-mesons in nuclear medium

$$f_\pi \to f_\pi^* = f_\pi\sqrt{\alpha_s} \text{ and } g \to g^* = g\sqrt{\zeta}$$

$$m_\pi^* = m_\pi\sqrt{\alpha_s/\alpha_p}$$

$$m_V^2 = m_\rho^2 = m_\omega^2 = a f_\pi^2 g^2 = 2 f_\pi^2 g^2$$

the KSRF relation is met for $a = 2$.

$$g_{\rho\pi\pi} = \frac{a}{2} g$$

Model I : $2f_\pi^2 g_V^2 \zeta = m_V^{*2}$

Model II : $2f_\pi^2 g_\rho^2 \zeta = m_\rho^{*2}$ $\quad m_\omega^{*2} = m_\omega^2 = 2f_\pi^2 g_\omega^2.$

only ρ meson mass changes in nuclear medium

J. -H. Jung, U. T. Yakhshiev and H. -Ch. Kim, Phys.Lett. B **723** (2013)

- Alpha represent the influence of the surrounding environment on the properties of the single soliton
- We introduce the new density-dependent functional zeta which provides the in-medium dependence of the coupling constant g

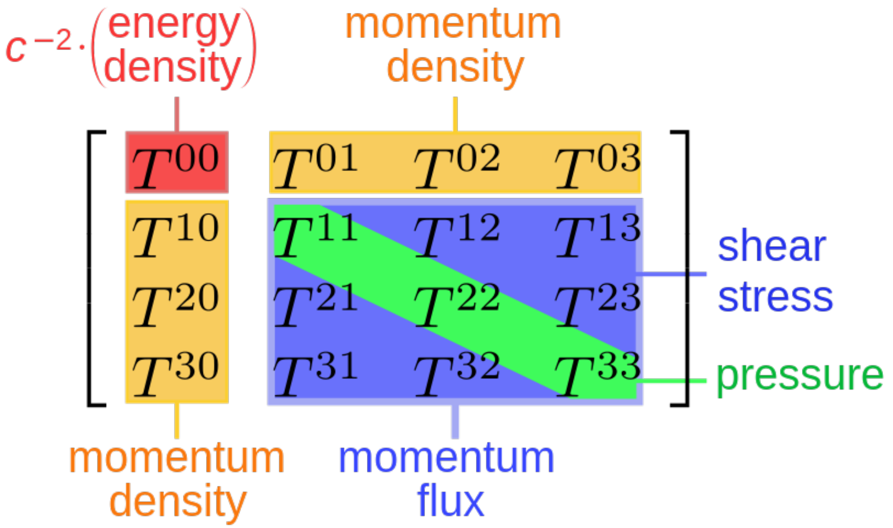
Energy-momentum tensor form factors of Nucleon

What is the EMT?

$$x^{\mu} \rightarrow x^{\mu} - a^{\mu}$$

$$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial \left(\partial_{\mu}\phi_a\right)} \partial^{\nu}\phi_a - g^{\mu\nu}\mathcal{L}$$

$$\partial_{\mu}T^{\mu\nu} = 0$$



Noether's theorem : If a system has a continuous symmetry property, then there are corresponding quantities whose values are conserved in time.

Energy-momentum tensor form factors of Nucleon

$$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial\left(\partial_\mu \phi_a\right)} \partial^\nu \phi_a + \frac{\partial \mathcal{L}}{\partial\left(\partial_\mu \rho_a\right)} \partial^\nu \rho_a + \frac{\partial \mathcal{L}}{\partial\left(\partial_\mu \omega_a\right)} \partial^\nu \omega_a - g^{\mu\nu} \mathcal{L}$$

- Using the Lagrangian, one can calculate the components of the energy-momentum tensor

Energy-momentum tensor form factors of Nucleon

$$T^{00}(r) = \alpha_p \frac{f_\pi^2}{2}\left(2\frac{\sin^2 F}{r^2} + F'^2\right) + \alpha_s f_\pi^2 m_\pi^2 (1-\cos F)$$
$$+\frac{2f_\pi^2}{r^2}(1-\cos F + G)^2 - \zeta g^2 f_\pi^2 \omega^2$$
$$+\frac{1}{2g^2\zeta r^2}\left\{2r^2 G'^2 + G^2 (G+2)^2\right\} - \frac{1}{2}\omega'^2$$
$$+\left(\frac{3}{2}g\sqrt{\zeta}\right)\frac{1}{2\pi^2 r^2}\omega \sin^2 F\, F'$$

$$T^{0i}(\mathbf{r},\mathbf{s}) = \frac{e^{ilm} r^l s^m}{(\mathbf{s}\times\mathbf{r})^2}\rho_J(r)$$

$$T^{ij}(r) = s(r)\left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}\right) + p(r)\,\delta^{ij}$$

Energy-momentum tensor form factors of Nucleon

$$\rho_J(r) = \frac{f_\pi^2}{3\Lambda}\left[\sin^2 F + 8\sin^4\frac{F}{2} + 4\sin^2\frac{F}{2}G - 4\sin^2\frac{F}{2}\xi_1 - 2\xi_1 G\right] + \frac{1}{3g^2r^2\zeta\Lambda}\left[-r^2\xi_1' G' - (\xi_1 G - G - \xi_2)\left(2G + G^2\right)\right] + \frac{g\sqrt{\zeta}}{8\pi^2\Lambda}\Phi\sin^2 F F'$$

Energy-momentum tensor form factors of Nucleon

$$p(r) = -\frac{1}{6}\alpha_p f_\pi^2 \left(F'^2 + 2\frac{\sin^2 F}{r^2} \right) - \alpha_s f_\pi^2 m_\pi^2 (1 - \cos F)$$
$$-\frac{2}{3r^2} f_\pi^2 (1 - \cos F + G)^2 + f_\pi^2 g^2 \zeta \omega^2$$
$$+\frac{1}{6g^2\zeta r^2} \left\{ 2r^2 G'^2 + G^2 (G+2)^2 \right\} + \frac{1}{6}\omega'^2$$

$$s(r) = \alpha_p f_\pi^2 \left(F'^2 - \frac{\sin^2 F}{r^2} \right) - \frac{2f_\pi^2}{r^2} (1 - \cos F + G)^2$$
$$+\frac{1}{g^2 r^2 \zeta} \left\{ r^2 G'^2 - G^2 (G+2)^2 \right\} - \omega'^2$$

Energy-momentum tensor form factors of Nucleon

$$\left\langle p' \left| \hat{T}_{\mu\nu}(0) \right| p \right\rangle = \overline{u}(p') \left[M_2(t) \frac{P_\mu P_\nu}{M_N} + J(t) \frac{i(P_\mu \sigma_{\nu\mu} + P_\nu \sigma_{\mu\nu}) \Delta^\rho}{2M_N} + d_1(t) \frac{\Delta_\mu \Delta_\nu - g_{\mu\nu}\Delta^2}{5M_N} \right] u(p)$$

At the zero momentum transfer $t = 0$,

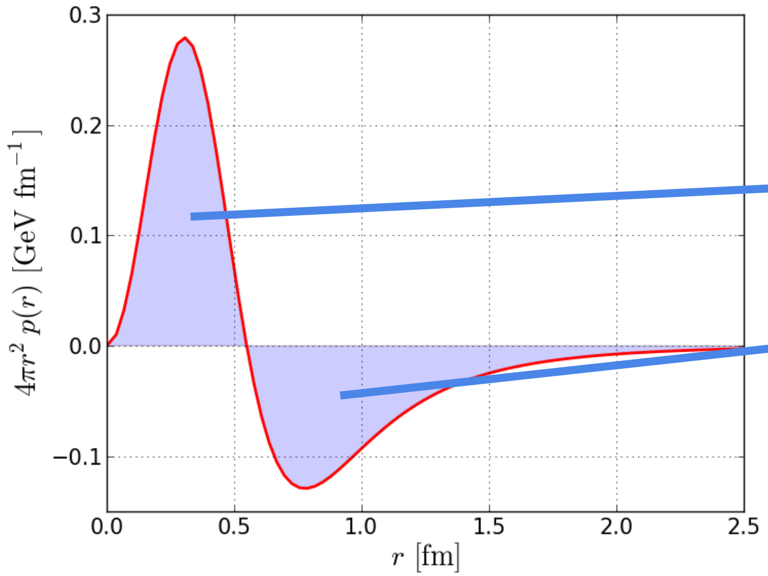
$$M_2^*(0) = \frac{1}{M_N^*} \int \mathrm{d}^3\boldsymbol{r}\, T_{00}^*(r) = 1, \qquad J^*(0) = \int \mathrm{d}^3\boldsymbol{r}\, \rho_J^*(r) = \frac{1}{2}.$$

$$d_1^* = 5\pi M_N^* \int_0^\infty dr\, r^4\, p^*(r) = -\frac{4\pi M_N^*}{3} \int_0^\infty dr\, r^4\, \boldsymbol{s}^*(r)$$

Energy-momentum tensor form factors of Nucleon

stability condition

$$\int_0^\infty dr\, r^2\, p^*(r) = 0\,.$$



$p(r) > 0 \quad \Rightarrow$ **repulsion**

$p(r) < 0 \quad \Rightarrow$ **attraction**

Energy-momentum tensor form factors of Nucleon

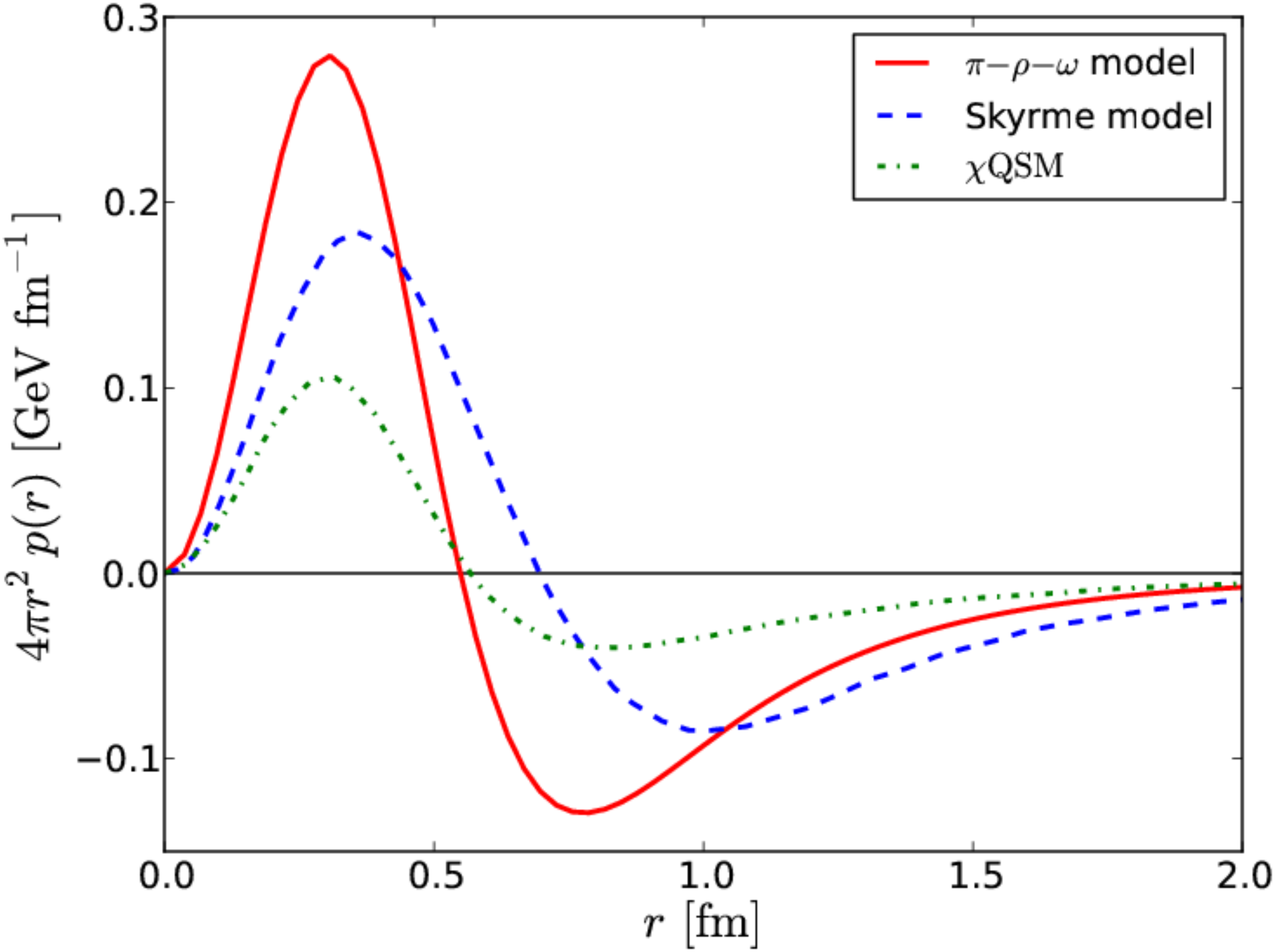
$$\text{stability condition}$$

$$\int_0^\infty dr\, r^2\, p^*(r) = 0\,.$$

We can easily prove analytically that the stability condition

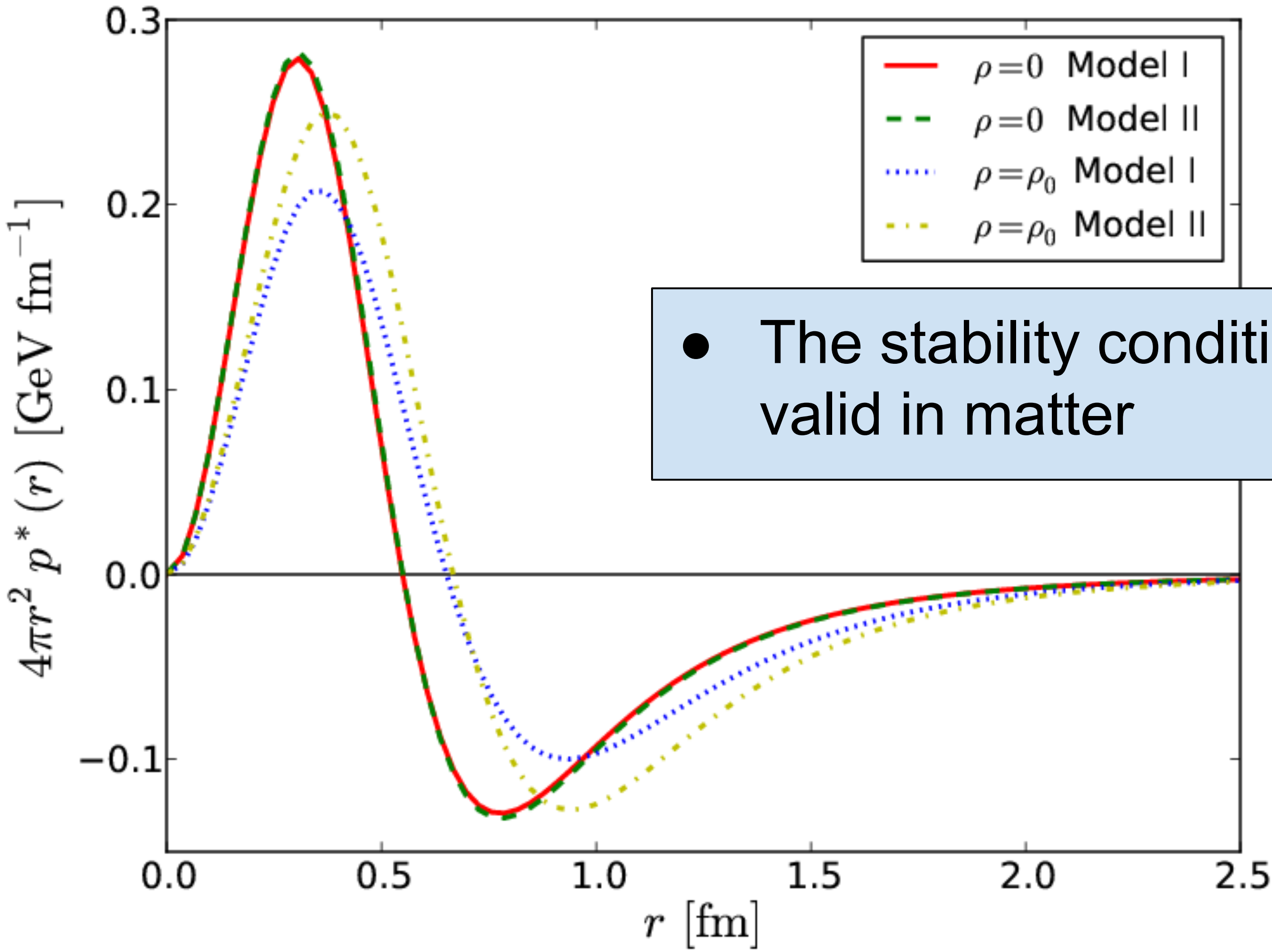
$$\begin{aligned}
r^2 p(r) = \frac{\partial}{\partial r}\Big[& r^3 p - 2r^3\left(-\frac{1}{6}f_\pi^2 F'^2 + \frac{1}{6}\omega'^2\right) \\
& -\frac{2}{3}r^3\left\{f_\pi^2 g^2\omega^2 + \frac{1}{3g^2r^2}\left\{r^2G'^2\right\} - f_\pi^2 m_\pi^2\left(1-\cos F\right)\right\}\Big] \\
& - \frac{f_\pi^2}{3} r F' \times (\text{equations of motion}) \\
& - \frac{1}{3g^2} r G' \times (\text{equations of motion}) \\
& - r\omega' \times (\text{equations of motion})\,.
\end{aligned}$$

Results



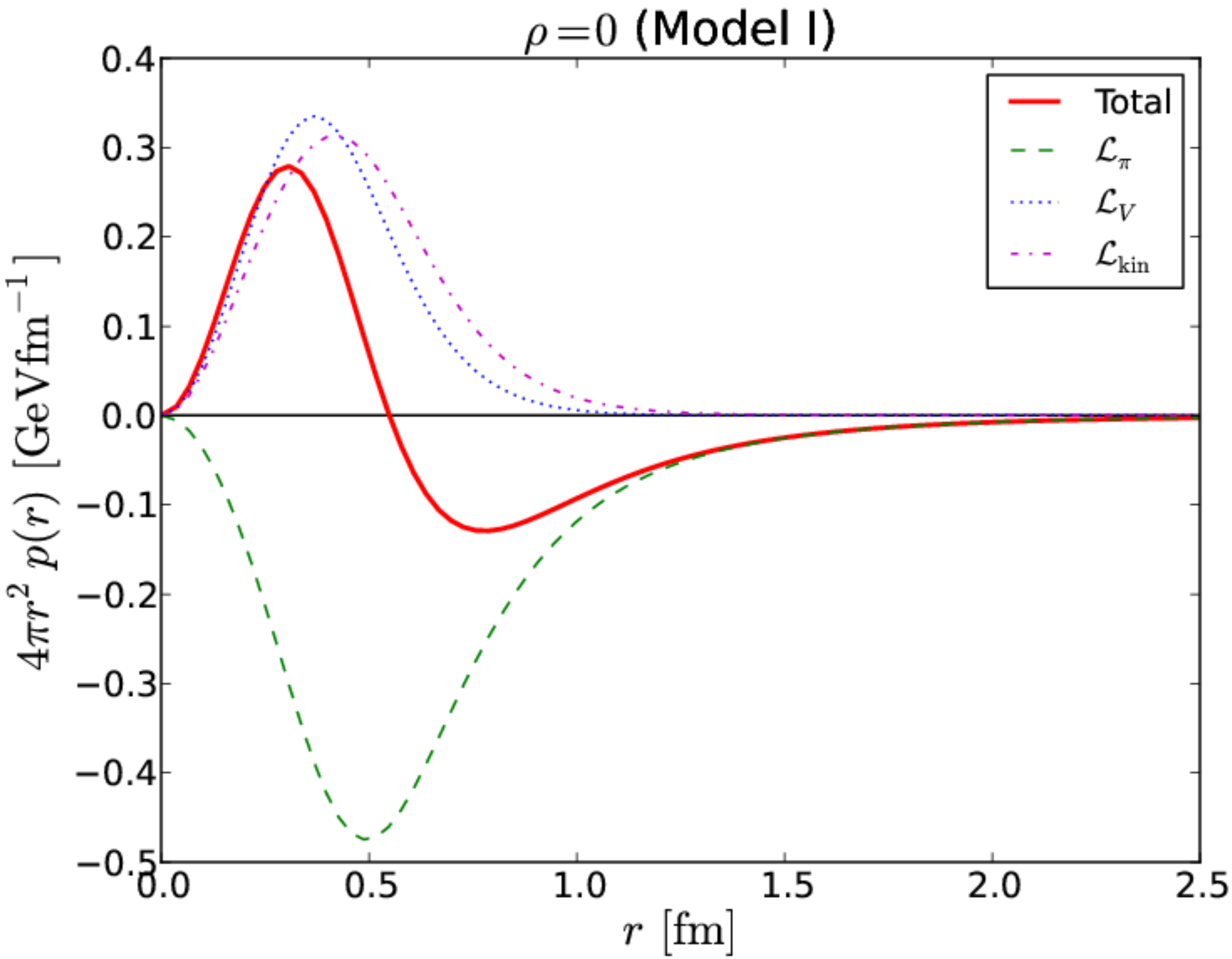
$4\pi r^2\, p(r)$ [GeV fm^{-1}]
r [fm]
0.3
0.2
0.1
0.0
−0.1
0.0
0.5
1.0
1.5
2.0
π−ρ−ω model
Skyrme model
χQSM

Results



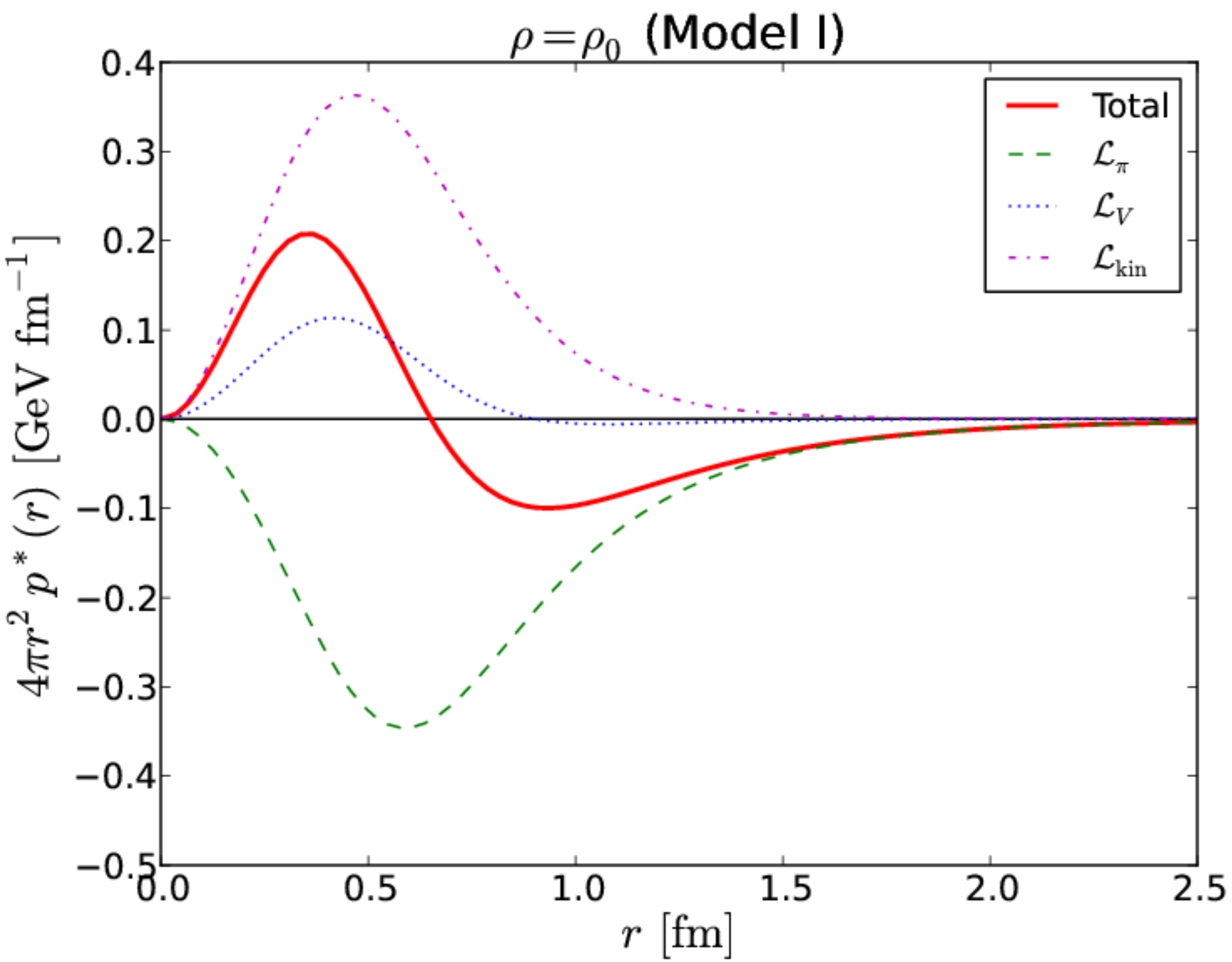
- The stability condition also valid in matter

Results



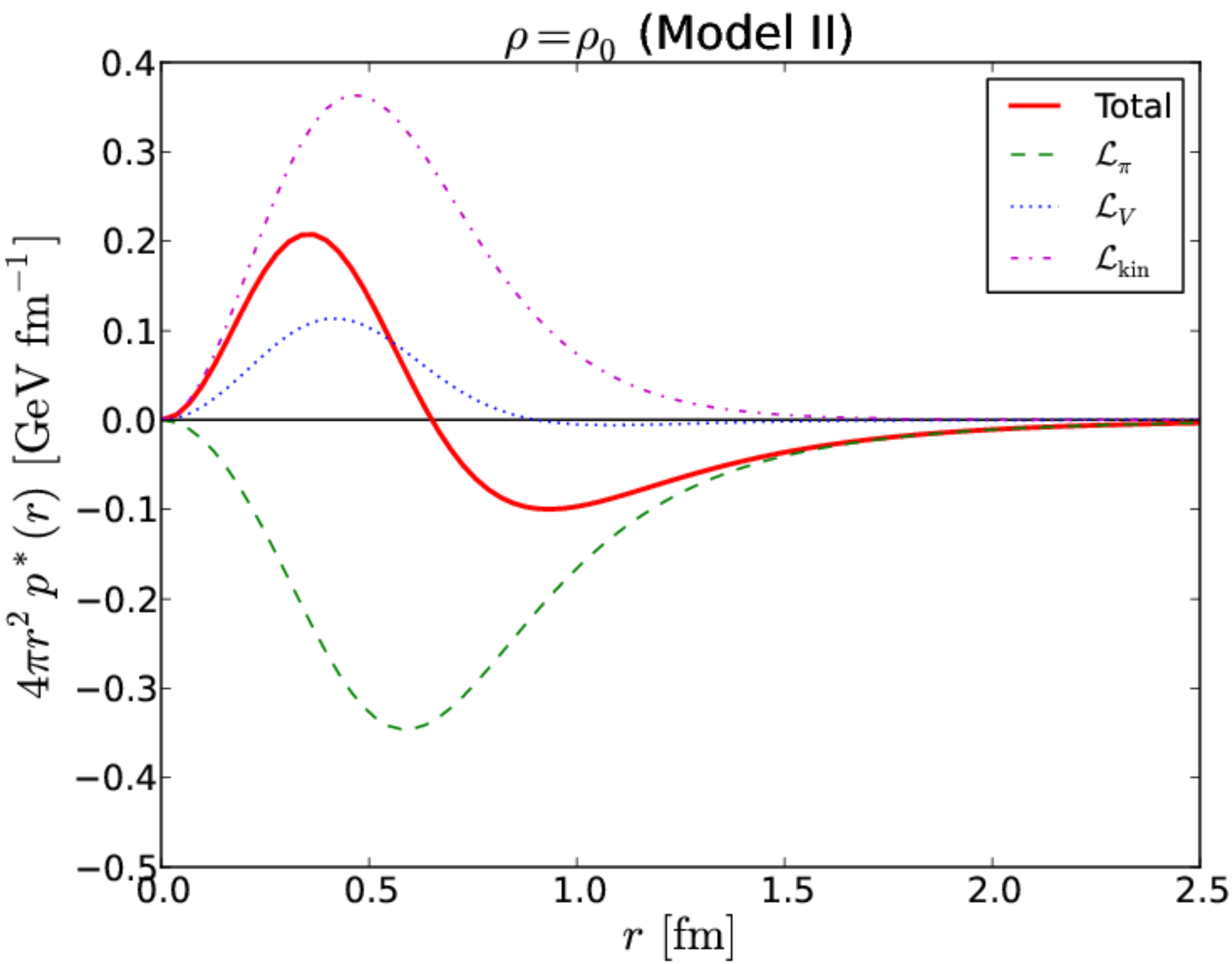
$\rho = 0$ (Model I)
Total
$\mathcal{L}_\pi$
$\mathcal{L}_V$
$\mathcal{L}_{\text{kin}}$
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.4
−0.5
0.0
0.5
1.0
1.5
2.0
2.5
$4\pi r^2\, p(r)$ [GeVfm^{-1}]
r [fm]

Results



$\rho = \rho_0$ (Model I)
Total
$\mathcal{L}_\pi$
$\mathcal{L}_V$
$\mathcal{L}_{\rm kin}$
$4\pi r^2 p^*(r)$ [GeV fm^{-1}]
r [fm]
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.4
−0.5
0.0
0.5
1.0
1.5
2.0
2.5

Results



$\rho = \rho_0$ (Model II)
Total
$\mathcal{L}_\pi$
$\mathcal{L}_V$
$\mathcal{L}_{\rm kin}$
$4\pi r^2 p^*(r)$ [GeV fm^{-1}]
r [fm]
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.4
−0.5
0.0
0.5
1.0
1.5
2.0
2.5

Results

	f_π [MeV]	$T_{00}(0)$ [GeV/fm^3]		$p(0)$ [GeV/fm^3]		$\langle r_H^2 \rangle$ [fm^2]		$\langle r_J^2 \rangle$ [fm^2]		r_0 [fm]		$d_1(0)$	
ρ/ρ_0		0	1	0	1	0	1	0	1	0	1	0	1
Present work (Model I)	93	3.56	2.17	0.58	0.34	0.78	1.06	0.74	0.77	0.55	0.65	−5.03	−5.59
Present work (Model II)	93	3.51	1.82	0.59	0.35	0.79	1.15	0.74	0.81	0.55	0.67	−5.13	−6.82
Skyrme model [17]	54	1.45	0.71	0.26	0.13	0.68	0.95	1.09	1.35	0.71	0.90	−3.54	−4.85
Skyrme model [14]	66	2.28		0.48		0.54		0.92		0.64		−4.48	
Chiral quark soliton model [12]	93	1.70		0.23		0.67		1.32		0.57		−2.35	

TABLE I. The quantities relevant to the nucleon EMT densities and their form factors: $T_{00}(0)$ is the energy density in the center of the nucleon; $p_0(0)$ denote the pressure value at the origin; $\langle r_H^2 \rangle$ is the mean square radii for the energy densities; $\langle r_J^2 \rangle$ represent the squared radii of the angular momentum distribution; r_0 designates the position, where the sign of the pressure is changed; $d_1(0)$ correspond to the $d_1(t)$ form factors at the zero momentum transfer. Each quantities are represented at free space $\rho = 0$ and at the normal nuclear matter density $\rho = \rho_0$. For comparison the results from the works [12, 14, 17] are also presented.

J. -H. Jung, U. T. Yakhshiev and H. -Ch. Kim, J. Phys. G: Nucl. Part. Phys. 41 055107

Conclusion & Outlook

- We studied a nucleon structure within pi-rho-omega soliton model
- The results are consistent with other model (original skyrme model, chiral quark soliton model, etc...)
- The nucleon structure changes in nuclear medium can be easily studied in our framework

Thank you

The effective Lagrangian for pions, rho- and omega-mesons in nuclear medium

$$\mathcal{L}_A = \frac{f_\pi^2}{4} Tr\left(\partial_0 U \partial_0 U^\dagger\right) - \alpha_p \frac{f_\pi^2}{4} Tr\left(\partial_i U \partial_i U^\dagger\right)$$

$$\mathcal{L}_{\chi SB} = \alpha_s \frac{f_\pi^2 m_\pi^2}{2} Tr\left(U - 1\right)$$

- The modified Lagrangian of the non-linear sigma model in nuclear medium.
- $\mathcal{L}_{\chi SB}$ is pion mass term or chiral symmetry breaking term. It satisfies some low energy theorems and also has modified form in nuclear matter.
- This Lagrangian in linear approximation gives well known Lagrangian for the pions in nuclear matter.

The effective Lagrangian for pions, rho- and omega-mesons in nuclear medium

$$\mathcal{L}_V = \frac{a f_\pi^2}{4} Tr\left[D_\mu \xi \cdot \xi^\dagger + D_\mu \xi^\dagger \cdot \xi\right]^2$$

$$\mathcal{L}_{kin} = -\frac{1}{2g^2\zeta} Tr\left(F_{\mu\nu}^2\right)$$

$$\xi = \sqrt{U}$$

$$\begin{aligned} D_\mu &= \partial_\mu - iV_\mu \\ F_{\mu\nu} &= \partial_\mu V_\nu - \partial_\nu V_\mu - i\left[V_\mu, V_\nu\right] \\ V_\mu &= \frac{g}{2}\left(\tau^a \rho_\mu^a + \omega_\mu\right) \end{aligned}$$

- $\mathcal{L}_V$ comes from a local gauge transformation to satisfy gauge-invariance
- This term is coupled with pseudo scalar meson - pion and vector mesons
- The kinetic term of the gauge field are generated by quantum effect

The effective Lagrangian for pions, rho- and omega-mesons in nuclear medium

$$\mathcal{L}_{WZ} = \left(\frac{N_c}{2}g\right)\sqrt{\zeta}\omega_\mu \frac{\epsilon^{\mu\nu\alpha\beta}}{24\pi^2} Tr\left\{\left(U^\dagger\partial_\nu U\right)\left(U^\dagger\partial_\alpha U\right)\left(U^\dagger\partial_\beta U\right)\right\}$$

- $\mathcal{L}_{WZ}$ gives the omega coupling to the topologically conserved baryon current as first discussed by Adkins and Nappi

The effective Lagrangian for pions, rho- and omega-mesons in nuclear medium

Skyrme's spherically symmetric Ansatz for $U(x)$

$$U(x) = \exp\left[i\hat{x}_a \tau_a F(r)\right], \qquad (\hat{x} = x_a/r)$$

- for the pion field, we choose the usual hedgehog configuration which comes from Skyrme's Ansatz

Wu-Yang-t'Hooft-Polyakov Ansatz for $\rho_\mu(r)$

$$\frac{g}{2}\sqrt{\zeta}\tau^a \rho^{i,a}(r) = \frac{\epsilon^{ika}}{2r}\hat{x}^k \tau^a G(r) \qquad \omega_0(r) = \omega(r)$$

- Direction of the isotopic space is related to the direction in usual space by Skyrme's ansatz and Wu-Yang-Thooft-Polyakov ansatz.

The medium functionals

- The medium functionals have the following forms which comes from the fitting pi-N scattering data
- We have tried exponential forms of the dependence on the nuclear density ρ to fit the ground state of nuclear matter

$$\alpha_p = 1 - \frac{4\pi c_0 \rho/\eta}{1 + g_0' 4\pi c_0 \rho)/\eta}, \qquad \alpha_s = 1 - \frac{4\pi\eta b_0 \rho}{m_\pi^2}, \qquad \zeta = \exp\left\{-\frac{\gamma_{\rm num}\rho}{1+\gamma_{\rm den}\rho}\right\}$$

$$\rho = \lambda \rho_0$$

$$\rho_0 = 0.15\,[\mathrm{fm}^{-3}]$$

Lambda is parameter related to nuclear density
λ = 0 : Free space
λ = 1 : Normal nuclear matter

η : Kinematic factor
b_0 And c_0 : Effective pion- nucleon S and P wave scattering lengths
g'_0 : Lorentz-Lorenz or correlation factor

Energy functional

$$
\begin{aligned}
H[F,G,\omega] &= -\int \mathcal{L} d^3r \\
&= -4\pi \int r^2 \left[-\alpha_p \frac{f_\pi^2}{2} \left(F'^2 + 2\frac{\sin^2 F}{r^2} \right) + \alpha_s f_\pi^2 m_\pi^2 (\cos F - 1) \right. \\
&\quad + \frac{a}{2} \zeta g^2 f_\pi^2 \omega^2 - a \frac{f_\pi^2}{r^2} \left[G - (1 - \cos F) \right]^2 \\
&\quad - \frac{1}{2g^2 \zeta r^4} \left\{ 2r^2 G'(r)^2 + G(r)^2 \left(G(r) - 2 \right)^2 \right\} + \frac{1}{2} \omega'^2 \\
&\quad \left. - \left(\frac{N_c}{2} g \right) \sqrt{\zeta} \frac{1}{2\pi^2 r^2} \omega \sin^2 F \, F' \right]
\end{aligned}
$$

- Calculated energy functional for the static fields has the following form
- This static energy gives the hedgehog or skyrmion mass

Equations of motion

$$F'' = \frac{1}{4\alpha_p r^2}\sin F\left[-\frac{3\sqrt{\zeta}g\sin F\omega'}{\pi^2 f_\pi^2} + 8\left\{(\alpha_p - 2)\cos F + 2\right\} - 16G\right] - \frac{2F'}{r} + \frac{\alpha_s m_\pi^2 \sin F}{\alpha_p}$$

$$G'' = 2\zeta f_\pi^2 g^2(\cos F + G - 1) + \frac{(G-2)(G-1)G}{r^2}$$

$$\omega'' = 2\zeta f_\pi^2 g^2 \omega - \frac{3\sqrt{\zeta}gF'\sin^2 F}{4\pi^2 r^2} - \frac{2\omega'}{r}$$

$$F(0) = n\pi = \pi$$

$$F(\infty) = G(\infty) = \omega(\infty) = 0$$

$$g(0) = 1 - (-1)^n = 2$$

$$\omega'(0) = 0$$

$$B = n = 1$$

- Minimizing the functional, we will get the field equations which are presented here.
- The boundary conditions are given in the following form and they satisfy the baryon number 1 solution.
- The solutions of the equations gives the static solitonic configuration.

Quantization

We use rigid body quantization approximation. in this scheme nucleon appears as rotational state of classical skyrmion

$$U(\boldsymbol{r},t) = A(t)U(\boldsymbol{r})A^{\dagger}(t)$$

$$\frac{g}{2}\boldsymbol{\tau}\cdot\boldsymbol{\rho}_0(\boldsymbol{r},t) = \xi_1(\boldsymbol{r})\,A(t)\tau_j K_j A^{\dagger}(t) + \xi_2(\boldsymbol{r})\,K_k\hat{x}_k A(t)\tau_j\hat{x}_j A^{\dagger}(t)$$

$$\boldsymbol{\tau}\cdot\boldsymbol{\rho}_i(\boldsymbol{r},t) = A(t)\boldsymbol{\tau}\cdot\boldsymbol{\rho}_i(\boldsymbol{r})A^{\dagger}(t)$$

$$\boldsymbol{\omega}(\boldsymbol{r},t) = \frac{\Phi(\boldsymbol{r})}{r}\boldsymbol{K}\times\hat{\boldsymbol{x}} \;=\; \Omega(\boldsymbol{x})\,\boldsymbol{K}\times\hat{\boldsymbol{x}}$$

$$A^{\dagger}(t)\,\partial_0 A(t) = A^{\dagger}\dot{A} \;\equiv\; i\boldsymbol{\tau}\cdot\boldsymbol{K}$$

$$H[F,G,\omega,\xi_1,\xi_2,\Phi] = \int \mathcal{L}d^3r = -M_H + \lambda Tr\left(\dot{A}\dot{A}^{\dagger}\right)$$

- In order to ascribe spin and isospin quantum number to soliton, we will consider time-dependent matrix A
- Mesonic excitations are represented by some radial functions satisfying self consistent equation

Quantization

$$\lambda = 4\pi \int dr \left[\frac{2}{3} f_\pi^2 r^2 \sin^2 F - \frac{a}{6} \zeta \left(g f_\pi \right)^2 \Phi^2 \right.$$
$$+ \frac{a}{3} f_\pi^2 r^2 \left(8 \sin^4 \frac{F}{2} - 8 \xi_1 \sin^2 \frac{F}{2} + 3\xi_1^2 + 2\xi_1 \xi_2 + \xi_2^2 \right)$$
$$+ \frac{1}{3 g^2 \zeta} \left\{ 4G^2 \left(\xi_1^2 + \xi_1 \xi_2 - 2\xi_1 - \xi_2 + 1 \right) \right.$$
$$\left. + 2 \left(G^2 - 2G + 2 \right) \xi_2^2 + 3r^2 \left(\xi_1' \right)^2 + r^2 \left(\xi_2' \right)^2 + 2r^2 \xi_1' \xi_2' \right\}$$
$$\left. - \frac{1}{6} \left[\Phi'^2 + \frac{2\Phi^2}{r^2} \right] + g\sqrt{\zeta} \frac{\Phi}{2\pi^2} \sin^2 F F' \right]$$

- Then we quantize solitonic solutions considering zero modes of the model in isotopic space.
- Here Lambda is moment of inertia of the rotating skyrmion. Rotating skyrmion describes the nucleon.