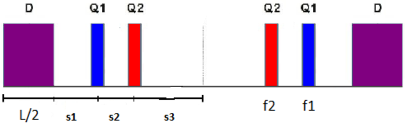
Stefania Papadopoulou, CERN/ University of Crete

ALERT workshop 2014, Valencia

Contents



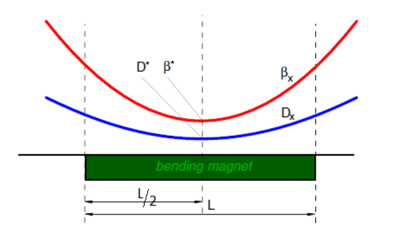
Schematic layout of the TME cell

The balance between radiation damping and quantum excitation results in the equilibrium betatron emittance. Using a theoretical minimum emittance TME cell low emittance values can be achieved. The horizontal emittance of the beam in an iso-magnetic ring is:

$$\epsilon_x = \frac{C_q \gamma^2}{J_x \rho_x} \langle H_x \rangle$$

An improved design for a further emittance reduction when using **variable dipole field strengths** results in the maintenance of the integrals inside the emittance equation ($C_q[m] = 3.84 \times 10^{-13}$ and $J_x \approx 1$):

$$\epsilon_x = \frac{C_q \gamma^2}{J_x} \frac{\oint \frac{H_x(s)}{|\rho_x|^3} ds}{\oint \frac{1}{\rho_x^2} ds}$$



Considering the *symmetry condition* that should be satisfied for a TME cell ($\eta'_0 = 0$, α_0=0 and $\gamma_0 = \frac{1}{\beta_0}$) and taking as initial point the dipole center indicated by index "*cd*" results in:

$$H_x(s) = \beta_{(s)} \cdot {\eta'_{(s)}}^2 + 2 \cdot \alpha_{(s)} \cdot \eta_{(s)} \cdot \eta'_{(s)} + \gamma_{(s)} \cdot \eta^2_{(s)}$$

$$\beta(s) = \beta_{cd} + s^2 \gamma_{cd}$$

$$\alpha(s) = -s\gamma_{cd}$$

$$\gamma(s) = \gamma_{cd}$$

$$\eta(s) = \eta_{cd} + \tilde{\theta}(s)$$

$$\eta'(s) = \theta(s)$$

If the dipole field varies along the magnet the equation for the betatron emittance retains its integrals as neither ρ_x nor $\mathcal{H}_x$ are invariant inside the bending magnet. Then, the emittance can be described using some integrals I_i (I_1, I_2, I_3 and I_4):

$$I_1 = \oint \frac{\theta^2}{|\rho(s)|^3} ds,\ I_2 = \oint \frac{(-s\cdot\theta + \tilde{\theta})^2}{|\rho(s)|^3} ds,\ I_3 = \oint \frac{-2\cdot s\cdot\theta + 2\cdot\tilde{\theta}}{|\rho(s)|^3} ds,\ I_4 = \oint \frac{1}{|\rho(s)|^3} ds$$

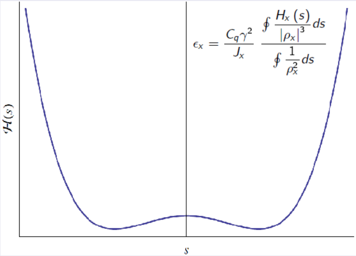
$$\epsilon_x = G\left(I_1\cdot\beta_{cd} + \frac{I_2 + I_3\cdot\eta_{cd} + I_4\cdot\eta_{cd}^2}{\beta_{cd}}\right)$$

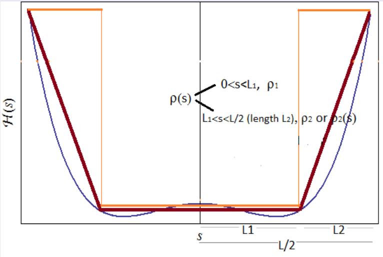
The beta and dispersion functions at the center of the dipole that give the TME, using the integrals I_i (where $G \approx 0.003$):

$$\beta_{xTME} = \frac{\sqrt{-I_3^2 + 4\cdot I_2\cdot I_4}}{2\sqrt{I_1\cdot I_4}} \quad , \quad \eta_{xTME} = -\frac{I_3}{2\cdot I_4}$$

$$\epsilon_{xTME} = \frac{G\sqrt{I_1}\sqrt{-I_3^2 + 4\cdot I_2\cdot I_4}}{\sqrt{I_4}}$$

Ref.:J. Guo, T. Raubenheimer (EPAC'02) , Y. Papaphilippou, P. Elleaume (PAC'05) , R. Nagaoka, A.F. Wrulich (NIM, 2007), C.-x Wang (PRST-AB, 2009)





The evolution of the dispersion invariant $\mathcal{H}_x(s)$ along the dipole and the possible dipole profiles approaching it for a variable bending radius.

In order to minimize the emittance ϵ_x the bending radius should follow the evolution of the dispersion invariant, along the dipole . Considering only the *half dipole* for simplicity (from 0 till $L/2$), as the other is symmetric, and then divide it into two parts:

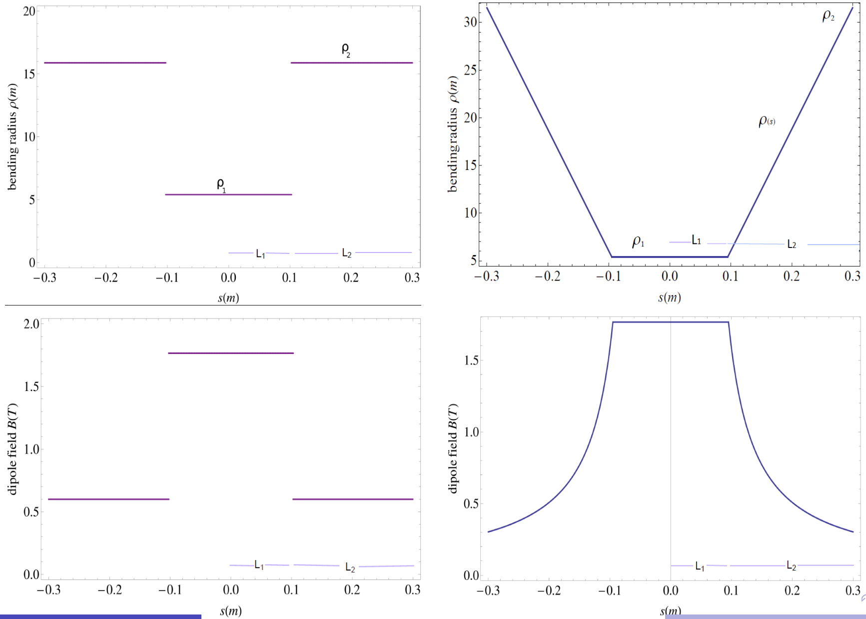
- **length L_1 with bending radius ρ_1**
- **length L_2 with bending radius ρ_2 or $\rho_{(s)}$**

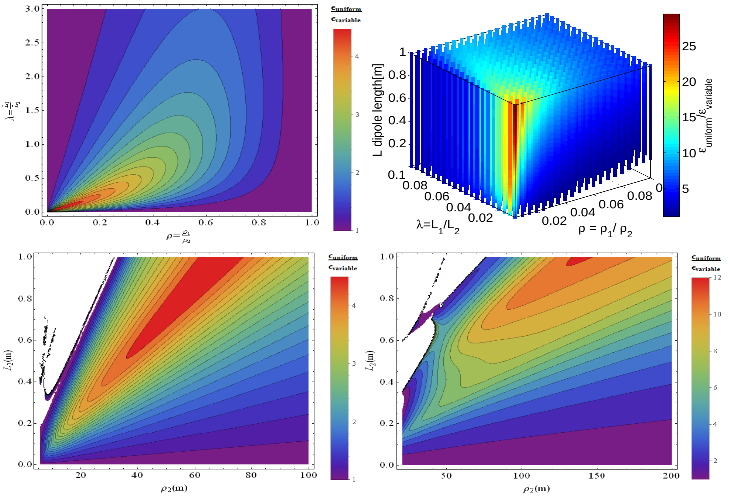
provides a satisfactory approach of the dispersion invariant evolution along the dipole.

So the chosen profiles to be presented here are the step shape and the trapezium shape.

- If the total bending magnet angle is $\theta_{tot} = \frac{2\pi}{Nd}$ (where Nd the number of dipoles) , the bending angle for the half dipole is $\theta = \frac{\pi}{Nd} = \left(\frac{L_1}{\rho_1} + \frac{L_2}{\rho_2}\right)$.
- Lengths and bending radii ratios: $\lambda = \frac{L_1}{L_2}$ *and* $\rho = \frac{\rho_1}{\rho_2}$ where ρ_1=minimum bending radius, ρ_2=maximum bending radius ($\rho < 1$ as $\rho_2 > \rho_1$ and $\lambda > 0$ as $L_1, L_2 > 0$)
- **The reduction factor** $f_r = \frac{\epsilon_{uniform}}{\epsilon_{variable}}$ that shows how much the "emittance of a uniform dipole" differs from the "emittance of a dipole with variable field", when having the same bending angle. The ratios λ *and* ρ are parameterized with the emittance reduction factor fr in order to find the best dipole profile. Whenever $f_r > 1$ it means that an emittance reduction is achieved.
- The **emittance detuning factor** shows how much the emittance deviates from its theoretical minimum $\epsilon_r = \frac{\epsilon_x}{\epsilon_{xTME}}$
- Constraints and design parameters:

$Bmax$ $[T]$	θ_{tot}	L $[m]$	$\epsilon_{uniform}$ $[nm]$
1.8	$2\pi/100$	0.6	360


bending radius ρ(m)
s(m)
ρ1
ρ2
L1
L2
bendingradius ρ(m)
ρ(s)
dipole field B(T)

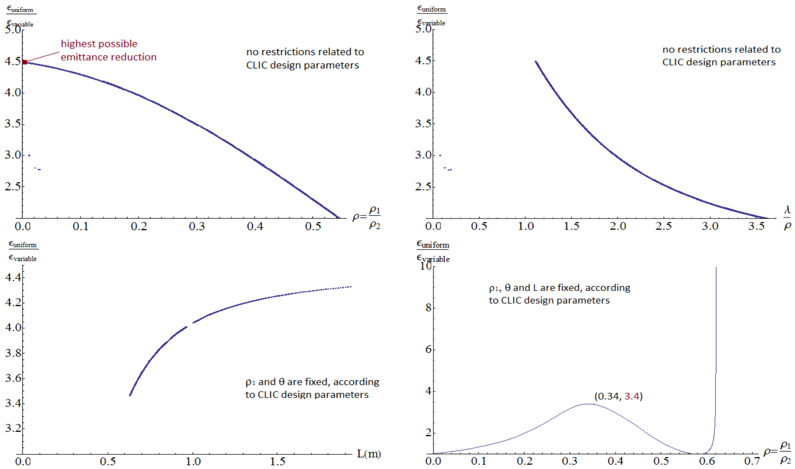


These plots (right:step profile and left:trapezium profile) show the parameterization of the emittance reduction factor with respect to:

Top: ρ,λ for the step profile and ρ,λ,L for the trapezium profile, without restrictions

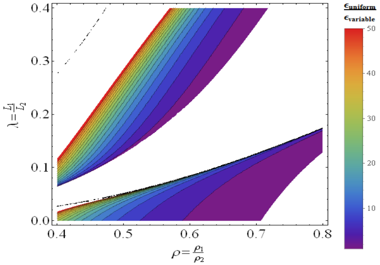
Bottom: ρ_2,L_2(when ρ_1 and θ are fixed).

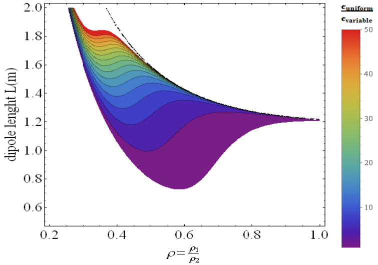
The red areas are the ones where the emittance for the non uniform dipole is higher than the uniform's one.

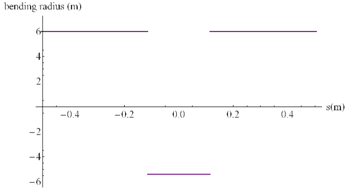


These plots show the emittance minimization ($\epsilon_{uniform}/\epsilon_{variable}$ = maximum) with respect to:
Top: the bending radii ratio ρ and the lengths-bending radii ratio λ/ρ
Bottom: the total dipole length L and d) the bending radii ratio ρ.

There is a limit for the maximum possible emittance reduction for any (λ, ρ) and is found to be $\epsilon_{uniform}/\epsilon_{variable} = 4.5$. For the CLIC design parameters it is $\rho = 0.34$ for the maximum reduction $\epsilon_{uniform}/\epsilon_{variable} = 3.4$.

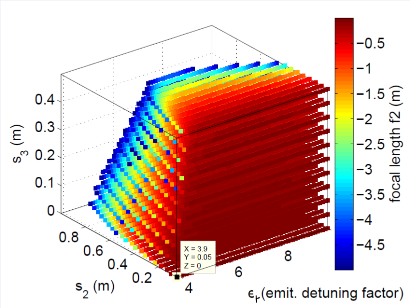


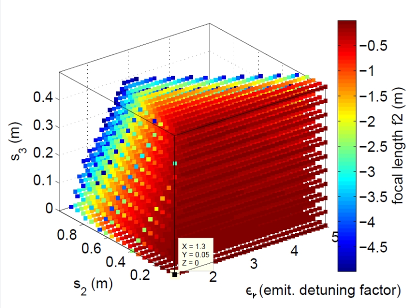




for CLIC design parameters->no reduction (fr=0.06)
restriction(L>0)->$\lambda < \rho$
$\epsilon_{uniform}/\epsilon_{variable}$->$10^6$, but then L»0
attention to quadrupole focal lengths f1,f2

For a negative bending radius in the central dipole part, the emittance reduction factor is parameterized with ρ,λ and ρ,L. Two areas can be compared; i)$L < 1$m $\rightarrow$ $\epsilon_{uniform}/\epsilon_{variable}$<10 and f1,f2>0 and ii)$L > 1$m $\rightarrow$ $\epsilon_{uniform}/\epsilon_{variable}$>10 and f1>0,f2<0



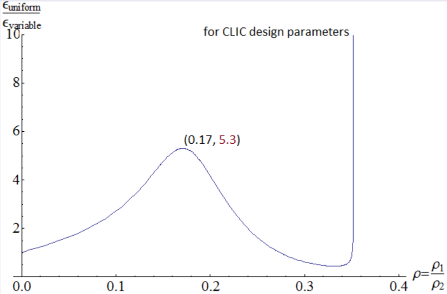


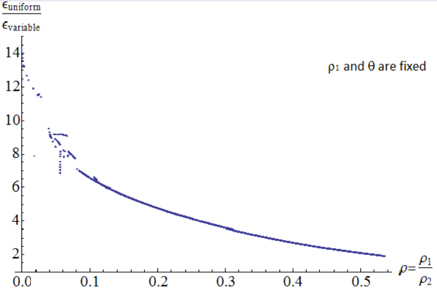
Parameterization of the focal length f2 with the drift spaces lengths s_2, s_3 and the emittance detuning factor ($\epsilon_r = \frac{\epsilon_x}{\epsilon_{xTME}}$).

The minimum bending radius ρ_1, the bending angle θ, the dipole length L and the drift space s_3 are the same for these plots, allowing a comparison of their emittance detuning factors ($\rho_1 = 5.4m$, $L = 1.6m$, $s_3 = 0.3m$). The focal length f1 is always positive for any detuning factor.

Right: for ($\rho = 0.49, \lambda = 0.045$) $\rightarrow$ $\epsilon_{uniform}/\epsilon_{variable}$=35.3 and $\epsilon_r > 3.9$.

Left: for ($\rho = 0.43, \lambda = 0.01$) $\rightarrow$ $\epsilon_{uniform}/\epsilon_{variable}$=21.4 and $\epsilon_r > 1.3$.





Right: fixing the minimum bending radius $\rho_1 = 5.4m$, the bending angle θ and the dipole length $L = 0.6$m (CLIC design parameters) it is $\rho = 0.17$ for the maximum $\epsilon_{uniform}/\epsilon_{variable} = 5.3$.

Left: with ρ_1 and θ fixed the reduction factor is plotted with respect to the bending radii ratio.

The free parameters left are the drift space lengths s_1, s_2, s_3, and the emittance. The numerical results give the parameterization with respect to the detuning factor that shows how much the emittance deviates from its theoretical minimum $\epsilon_r = \epsilon_x/\epsilon_{xTME}$, taking into account the stability criterion($|cos\varphi_{x,y}| < 1$) and where is needed restrictions for the chromaticity.

Ref.: Fanouria Antoniou, Yannis Papaphilippou, Physical Review Special Topics (18/10/2013).

step profile	
ρ	0.34
λ	0.52
ϵ_{xTME} [nm]	105.9
$\epsilon_{uniform}/\epsilon_{variable}$	3.4

trapezium profile	
ρ	0.17
λ	0.46
ϵ_{xTME} $[nm]$	67.8
$\epsilon_{uniform}/\epsilon_{variable}$	5.3

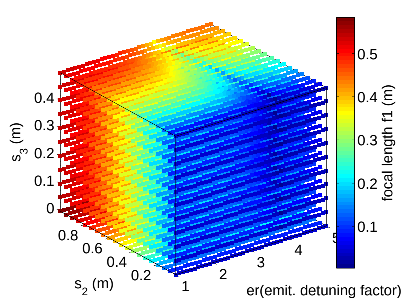
for ρ_1=5.4m(minimum bending radius for normal conducting magnets) and dipole length $L = 0.6m$. The total cell length is $Lcell = L + 2(s_1 + lq_1 + s_2 + lq_2 + s_3)$.

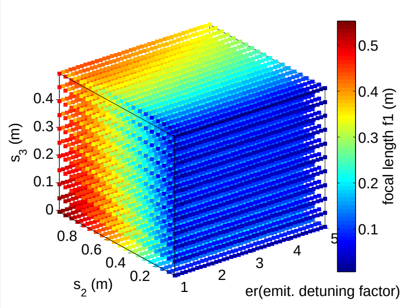
- One of the next steps for the studies of the TME cells with variable bending fields is the comparison of the analytical solutions with the results from the numerical simulation code MADX. Where, for the step profile the results are really close to the numerical values found with MATLAB. However, the trapezium profile needs to be further examined in order for MADX calculations to come to an agreement with the MATLAB ones.
- A further improvement of the final emittance values can be achieved when taking into consideration the collective effects, such as the Intrabeam scattering IBS that in the regime of ultralow emittances with high bunch charge has a significant impact on the emittance limits.

Thank You!

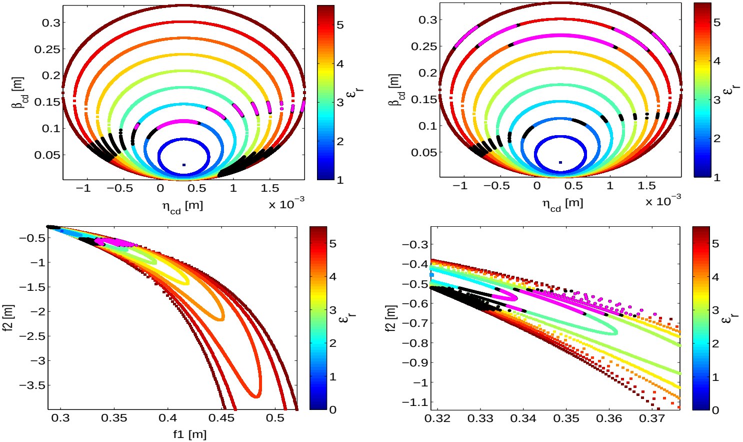
Special thanks to Yannis Papaphilippou and Fanouria Antoniou for their valuable help.

- The Step shape consists of a flat portion with bending radius ρ_2, then a flat portion with bending radius ρ_1 and finally a flat portion with bending radius ρ_2 again, with the entire function symmetric about the center of the magnet.
- The Trapezium shape consists of a straight line sloping downwards, a flat portion with bending radius ρ_1, and a straight line sloping upwards with bending $\rho_{(s)}$ till the maximum bending radius ρ_2, with the entire function symmetric about the center of the magnet.
  $\rho_{(s)} = \frac{s(1+\lambda+(-1-\lambda)\rho)\rho_1}{Ld\rho} + \frac{(-\lambda+(1+\lambda)\rho)\rho_1}{\rho}$
- $B_{max}[T] = \frac{E[GeV]}{0.3\rho_1}$
  and $B_{min}[T] = \frac{E[GeV]}{0.3\rho_2}$

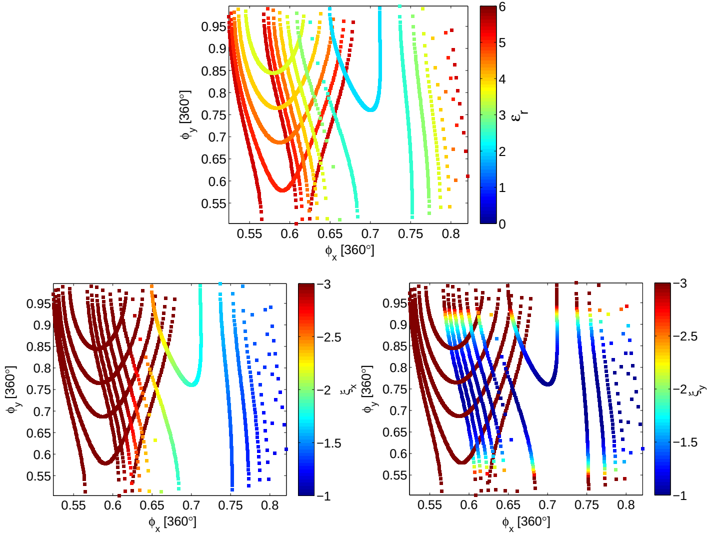




Parameterization of the focal length f1 with respect to the drift spaces lengths s_2, s_3 and the emittance detuning factor. The minimum bending radius ρ_1, the bending angle θ, the dipole length L and the drift space s_3 are the same for these plots, allowing a comparison of their emittance detuning factors ($\rho_1 = 5.4m$ $L = 1.6m$, $s_3 = 0.3m$). The focal length f1 is always positive for any detuning factor.



Parameterization of β_{cd} and η_{cd} with ϵ_r, for different drift lengths triplets [0.4,0.3,0.6] and [0.8,0.55,0.65] respectively (black squares give the stable solutions and pink squares give the stable solutions for $0 < \xi_x, \xi_y < -2$. Parameterization of the focal lengths f1,f2 with ϵ_r and the same with a zoom-in the area of stable solutions(black squares for stability, pink squares for stability and $0 < \xi_x, \xi_y < -2$).



Parameterization of ϵ_r (top) and ξ_x, ξ_y (bottom) with ϕ_x, ϕ_y (low detuning->high chromat.).