Fundamental Limits of Timing Resolution for Scintillation Detectors

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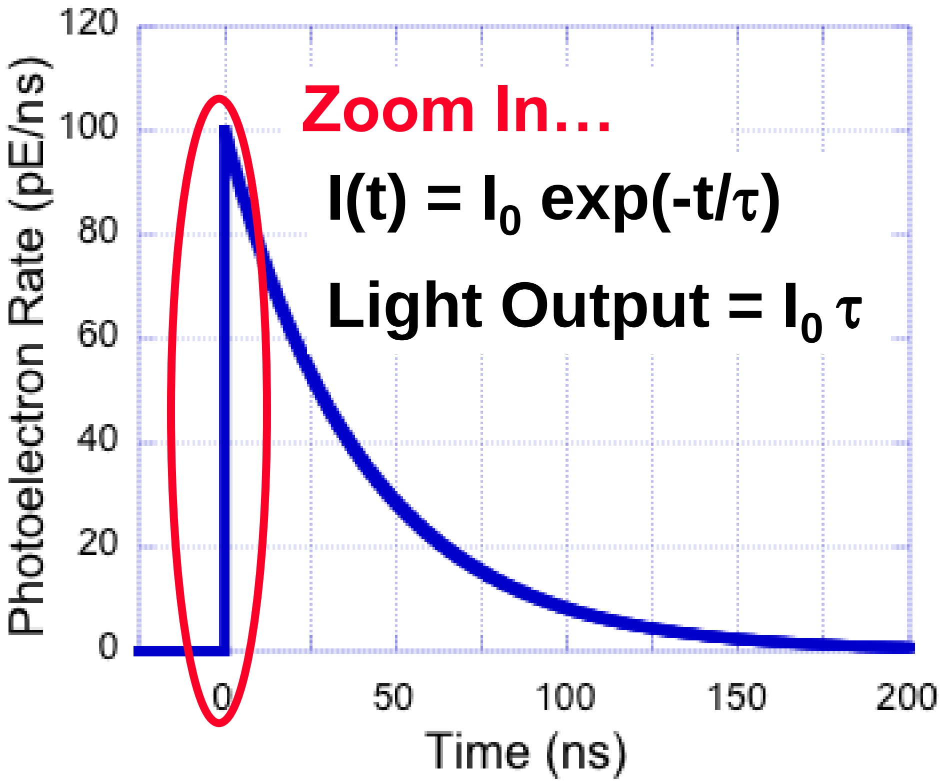
March 13, 2013

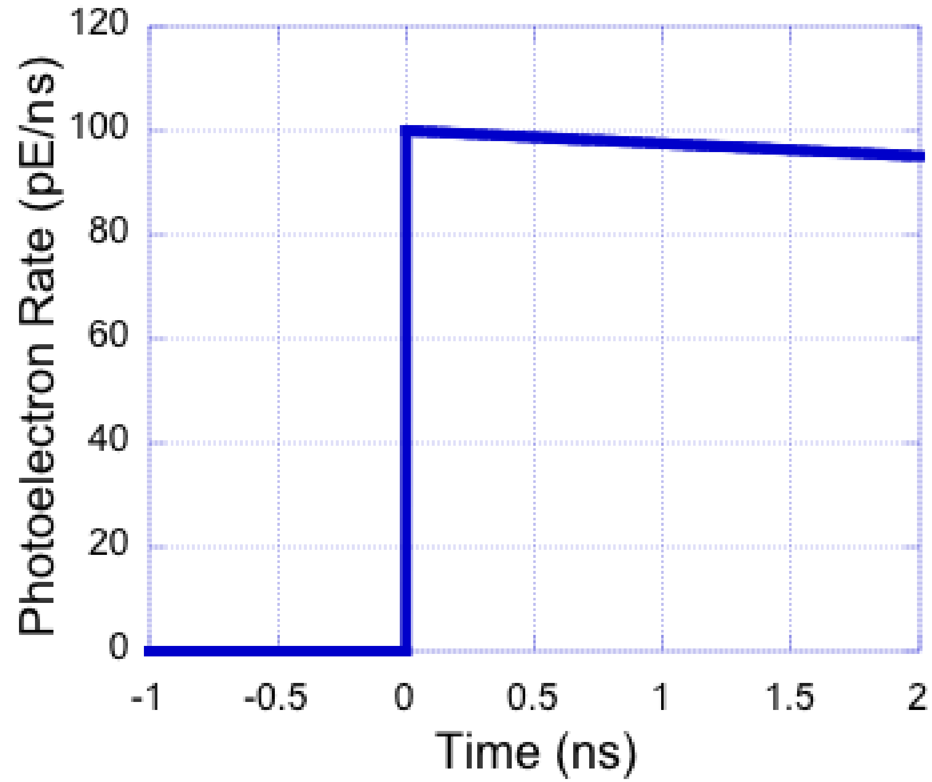
Outline:

- **Basic Theory**
- **Real World Effects**
- **Optical Photon Propagation**

- **This work was supported in part by the U.S. DOE (contract No. DE-AC02-05CH11231) and in part by the NIH (NIBIB grant No. R01-EB006085).**

The Fundamentals…

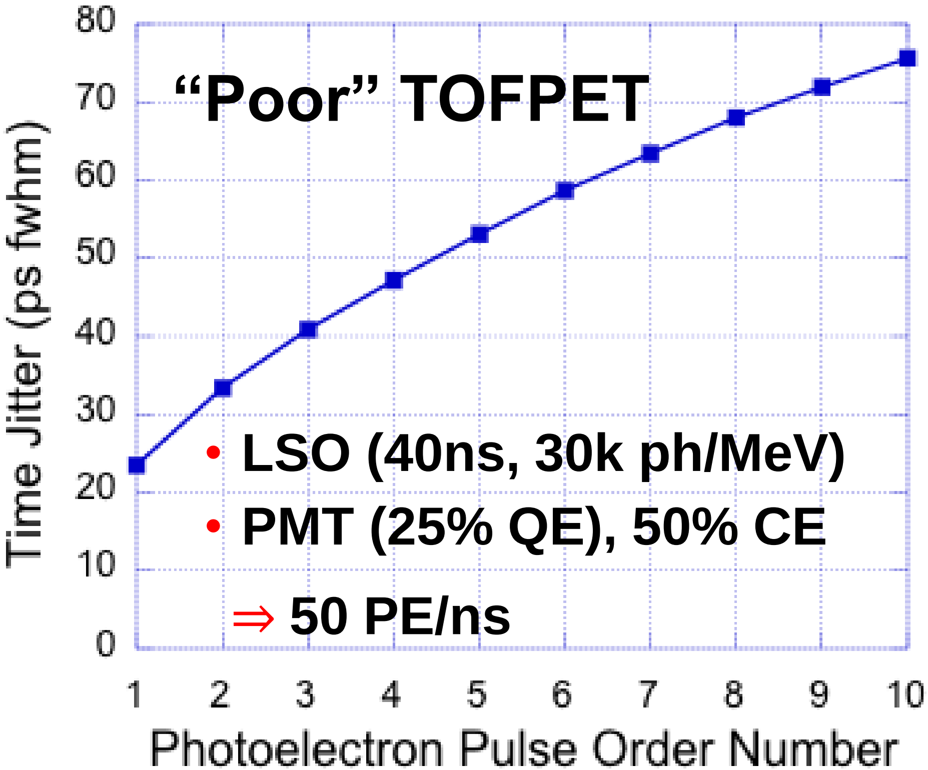




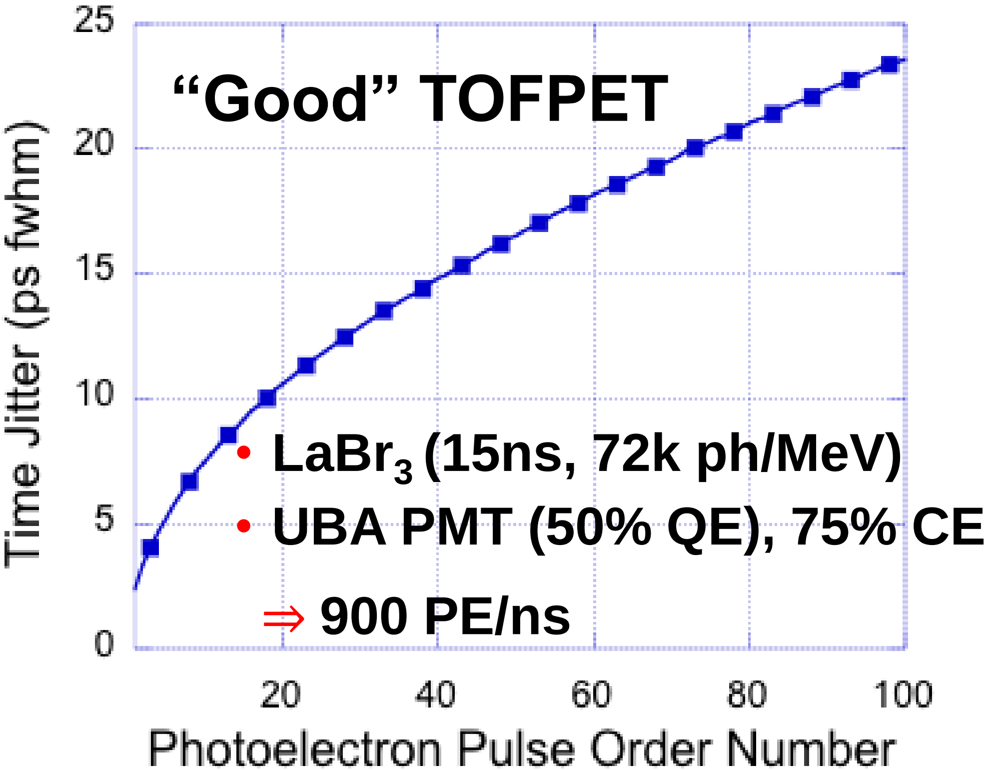
- **Timing Determined by I_0 (Initial PE Rate)**
- **$I_0 = E_\gamma$ □ (Light Output / τ) □ Collect_Eff □ Quantum_Eff**
- **Look at Arrivial Times of Individual Photoelectrons**

Time Jitter of n^{th} Photoelectron

PE Rate = 100 PE/ns

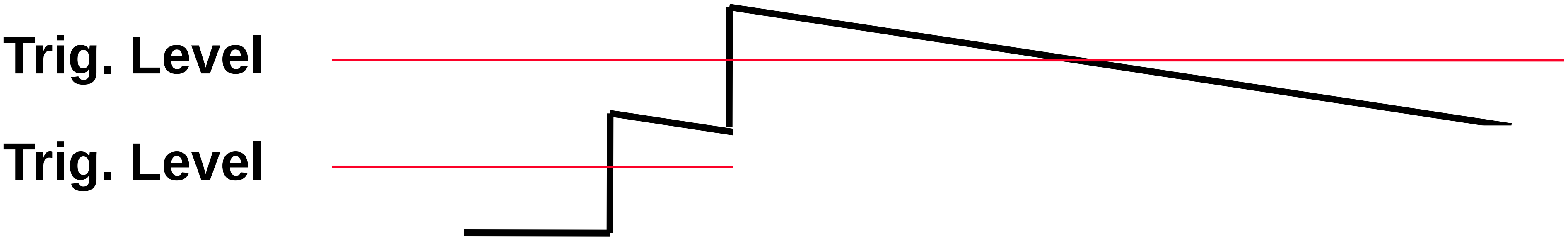


PE Rate = 1000 PE/ns



- **Jitter (fwhm) = 2.35 □ sqrt(n) / I_0**
- **First Photoelectron has Least Jitter**

Timing Pickoff



- Leading Edge Discriminator Often Used
- If the trigger level is at n photoelectrons, timing resolution ≈ jitter of n^{th} photoelectron
- Valid if PD_Rise ≪ PE Jitter ≪ PPD_Fall (individual photoelectrons resolved)

- **First PE Traditionally Favored**
- **Described by Post & Schiff in 1960**

Problem: Predicts *Incredible* Timing

“Poor” TOFPET

- LSO (40ns, 30k ph/MeV)
- Standard PMT (25% QE)
- 50% CE

⇒50 PE/ns

⇒ 66 ps fwhm PET Timing

“Good” TOFPET

- $LaBr_3$ (15ns, 72k ph/MeV)
- UBA PMT (50% QE)
- 75% CE

⇒900 PE/ns

⇒ 3.7 ps fwhm PET Timing

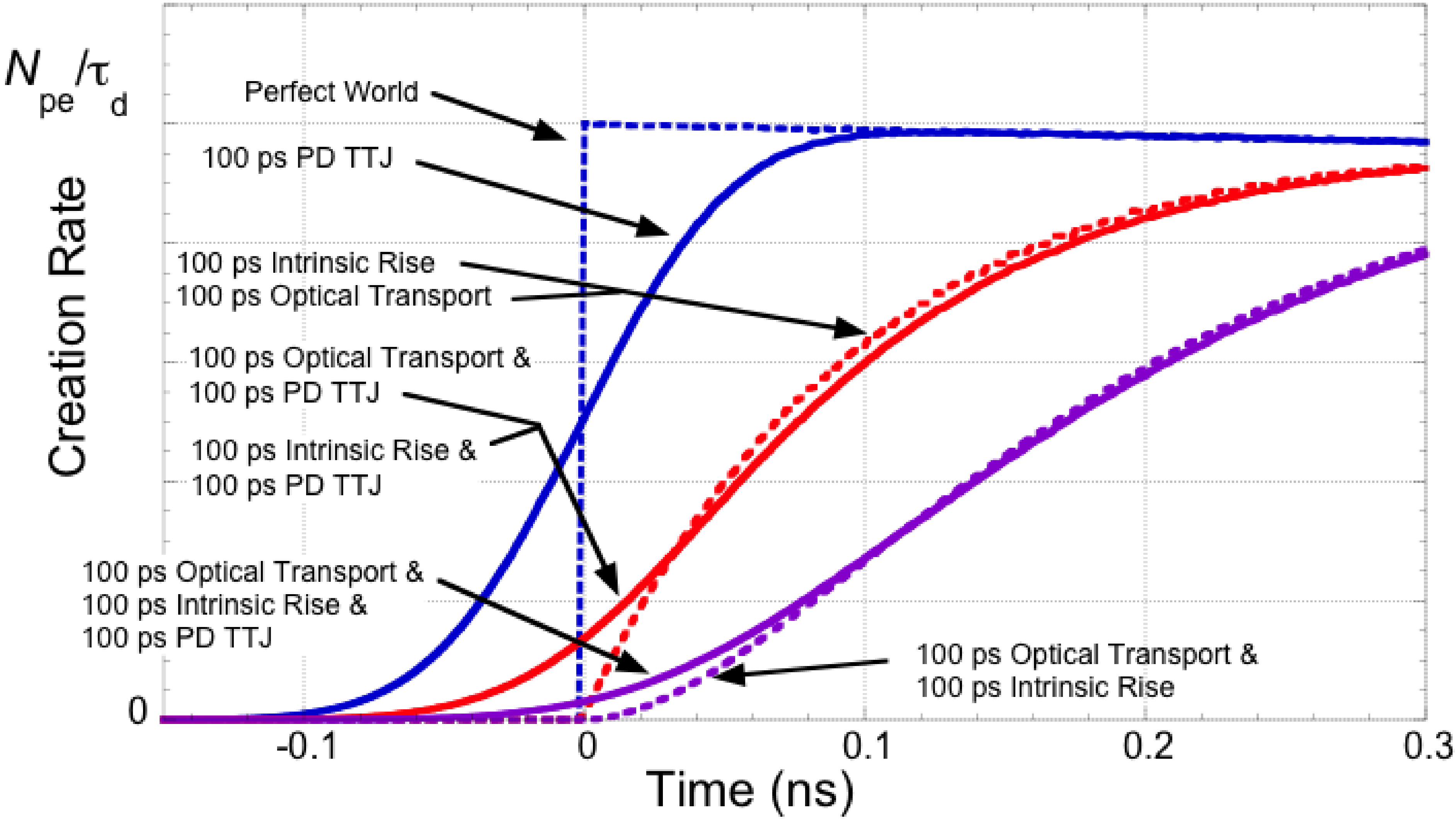
Why? Real World Effects

- Overlap of photoelectron pulses
- Photodetector transit time jitter (Gaussian)
- Photodetector rise & fall time (Bi-Exponential)
- Intrinsic scintillator rise time (Exponential)
- Optical transport in scintillator (Exponential)

Our Contribution

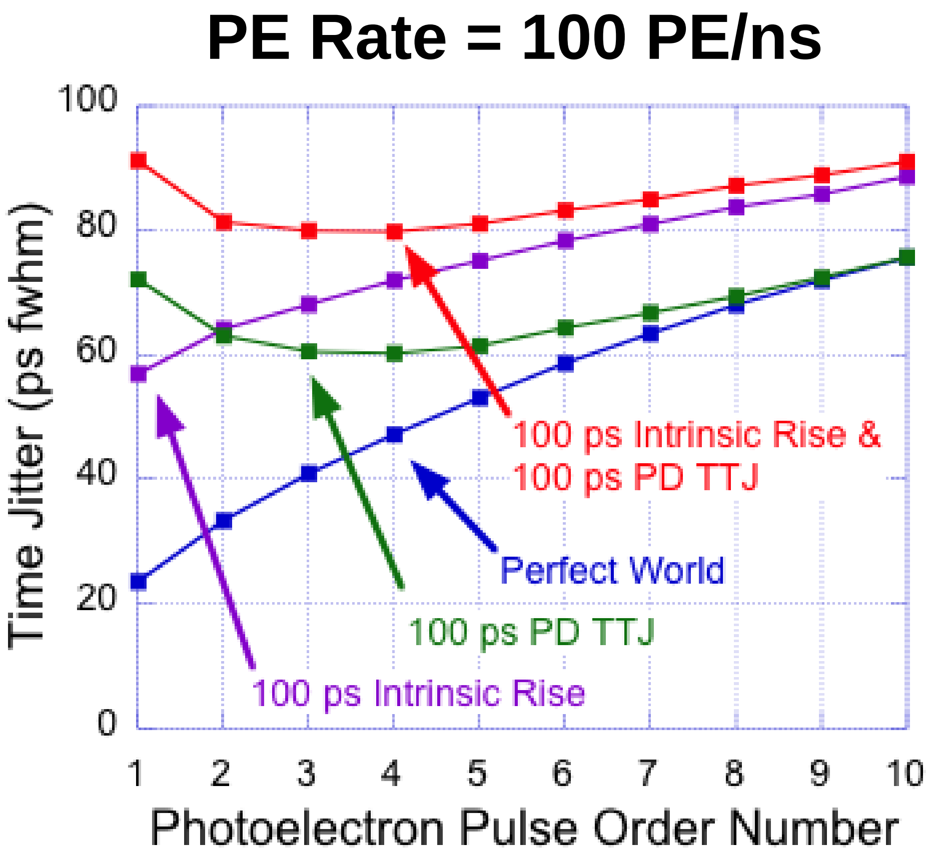
- Estimated Through Monte Carlo Simulation
- Described by Hyman in 1964 & 1965

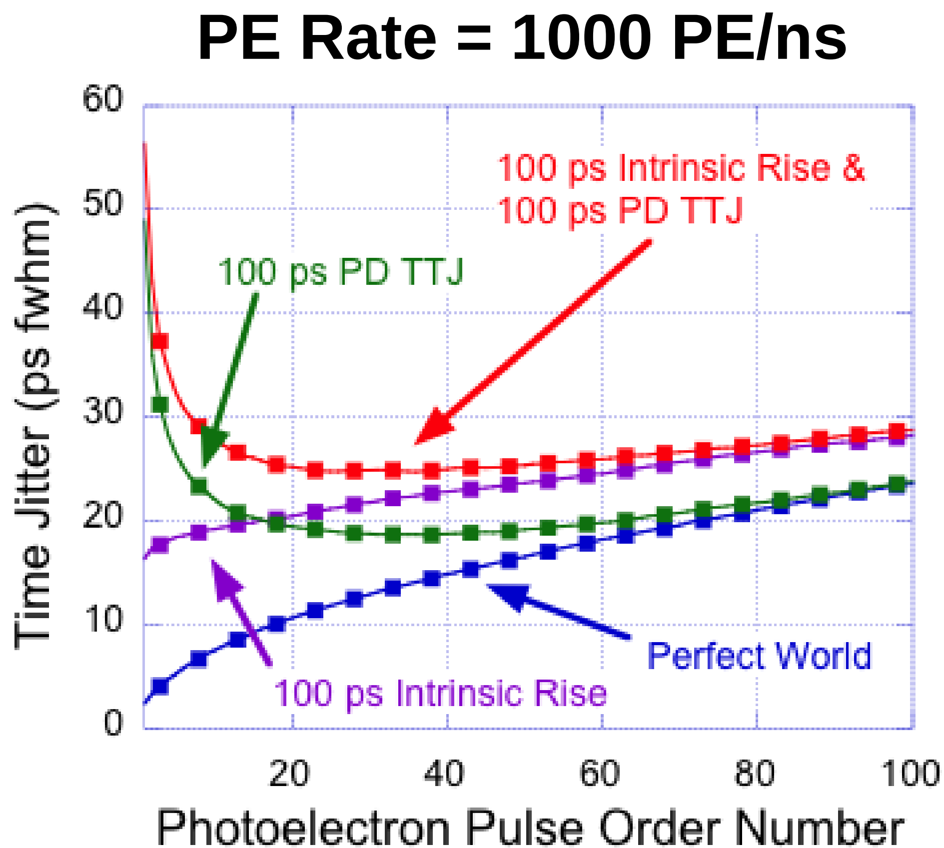
"Real World" Probability Density Function



"Abrupt" Rising Edge Disappears…

"Real World" Jitter of n^th Photoelectron





- First PE No Longer Has Lowest Jitter
- Broad Minimum – Many PEs Have Similar Jitter

Run *Lots* of Monte Carlo

820 Combinations

- Initial Photoelectron Rate I_0: 10 – 10,000 PE/ns
- Intrinsic Scintillator Rise Time: 0 – 1 ns (exponential)
- Photodetector Transit Time Jitter: 0 – 0.5 ns fwhm (Gauss)
- Optical Photon Propagation Time: 0 – 0.2 ns (exponential)

Convolve w/ Photodetector Response to Simulate Analog Out (results insensitive to PD response, including rise time)

Simulate Leading Edge Discriminator with Optimized Threshold for Each Combination

- **Get Tables of Simulated Timing Resolution**
- **Fit Tables to Analytic Formula**

Analytic Formula for Timing Resolution

Timing Resolution (ns fwhm) = B / sqrt(I_0)

$B(\tau_r, d, J, I_0) =$

$$\mathrm{sqrt}[\, 5.545\, I_0^{-1} + 2.424\,(\tau_r + d) + 2.291\, J + 4.938\, \tau_r d + 3.332\, J^2 + 8.969\, J^2 I_0^{-1/2} + 9.821(\tau_r^2 + d^2)\, I_0^{-1/2} - 0.6637(\tau_r + d)\, I_0^{-1/2} - 3.305\, J(\tau_r + d) - 6.149\, J I_0^{-1/2} - 0.3232\,(\tau_r^2 + d^2) - 3.530\, J^3 - 5.361\, J \tau_r d - 9.287\, \tau_r d\, I_0^{-1/2} - 5.814\, J\tau(\tau_r^3 + d^3)\,]$$

I_0 = E_γ □ (Light Output / τ) □ Collect_Eff □ Quantum_Eff (photons/MeV/ns)

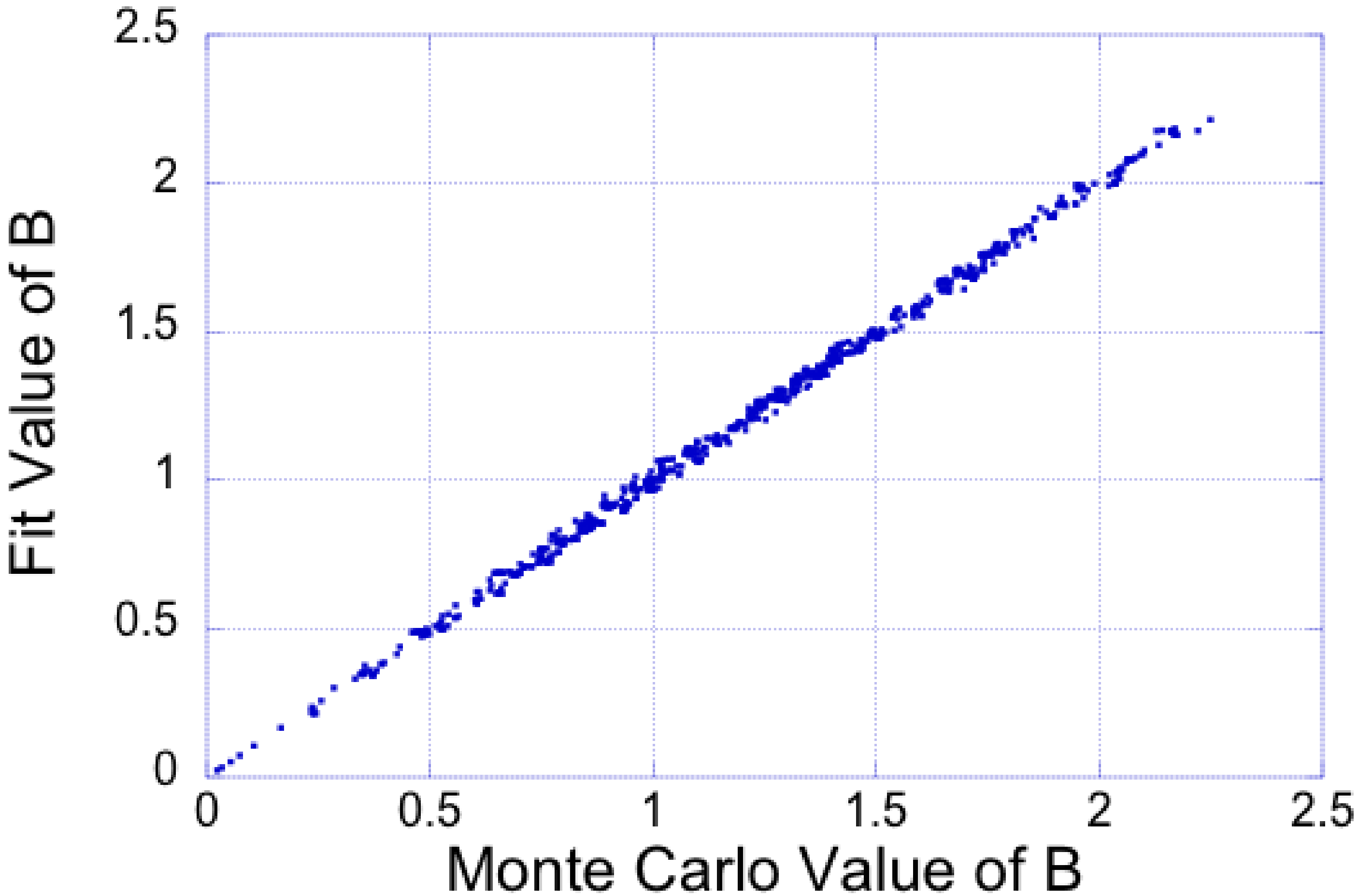
τ_r = Intrinsic Scintillator Rise Time (ns)

J = Photodetector Transit Time Jitter (ns)

d = Optical Photon Propagation "Decay Time" (ns)

Formula Obtained Using "Eureqa" Software

Fit vs. True Value for *B*



- **Excellent Fit (RMS Deviation ~ 2%)**
- **For Virtually All Conditions, 0.5 < *B* < 2**

Other Observations

- **No Other Significant Dependencies**
  - **Photodetector Rise Time**
  - **Photodetector Fall Time**
- **Timing Resolution Using “Optimal” Estimator that Uses *All* Detected Photoelectrons is ≤15% Better than Leading Edge Timing (for τ_r = 0.5 ns, *d = 0.1 ns*, *J = 0.2 ns*)**
- **Paper submitted to *Phys. Med. Biol.***

Timing Resolution Predicted If You Know:

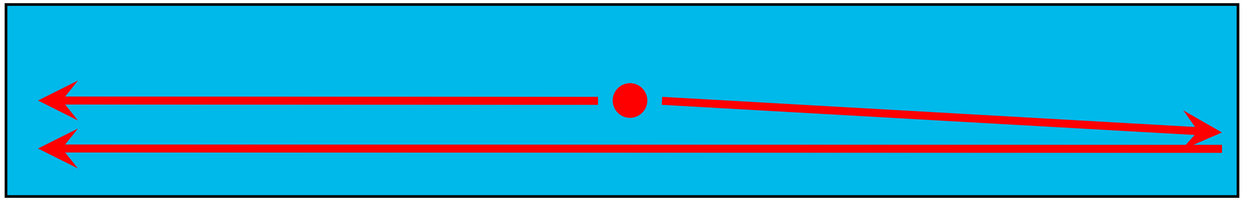
- ✔ Initial Photoelectron Rate I_0
  - ✔ Gamma Ray Energy
  - ✔ Scintillator Luminosity
  - ✔ Scintillator Decay Time
  - ✔ Collection Efficiency
  - ✔ Quantum Efficiency
- ✔ Intrinsic Scintillator Rise Time
- ✔ Photodetector Transit Time Jitter
- Optical Photon Propagation Time

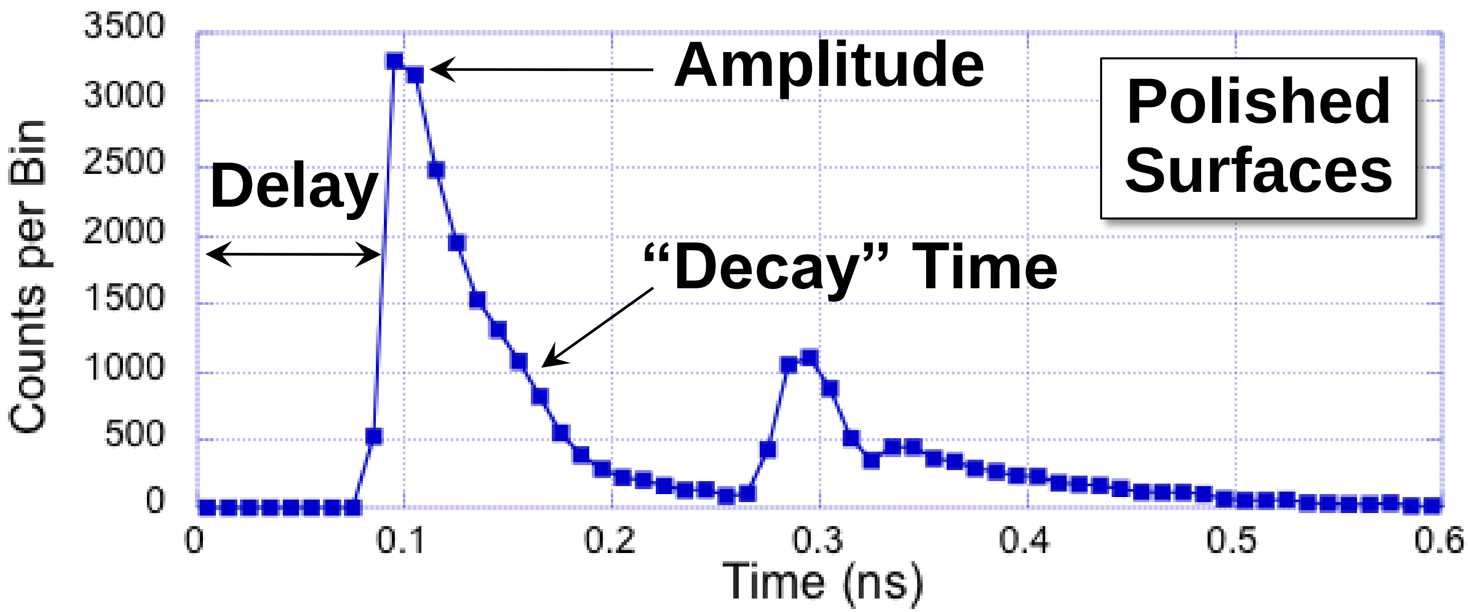
What Value to Use for Propagation Time???

Estimate Using GEANT / GATE / DETECT

- Index of Refraction *n* = 1.82 (same as LSO)
- Light Generated as a Delta Function in Time at Random Positions in the Scintillator
- Three Surface Finishes Simulated
  - Polished
  - Chemically Etched
  - Rough
- Two Simulation Types for Each Surface
  - “Unified Model” (σ_α = 0°, 6°, and 12°)
  - “RealReflector” (measured values in LUT)

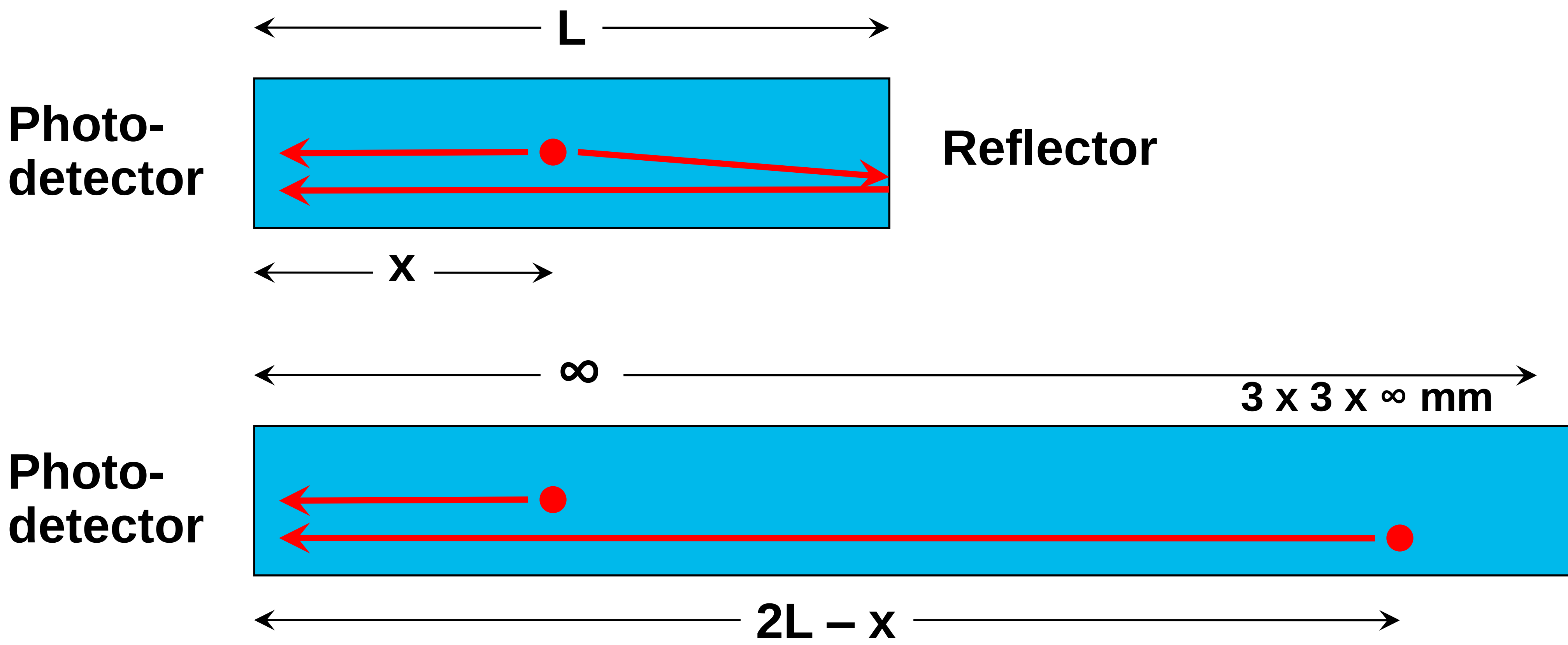
Basic Pulse Shape





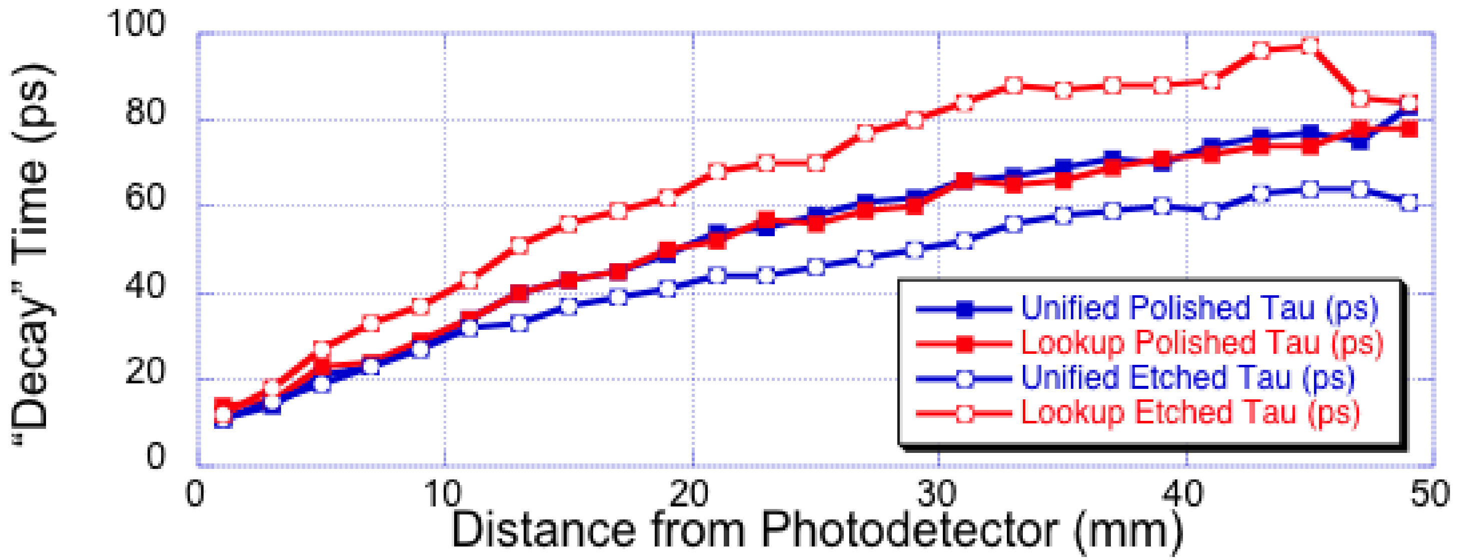
Well-Described By Exponential Decay

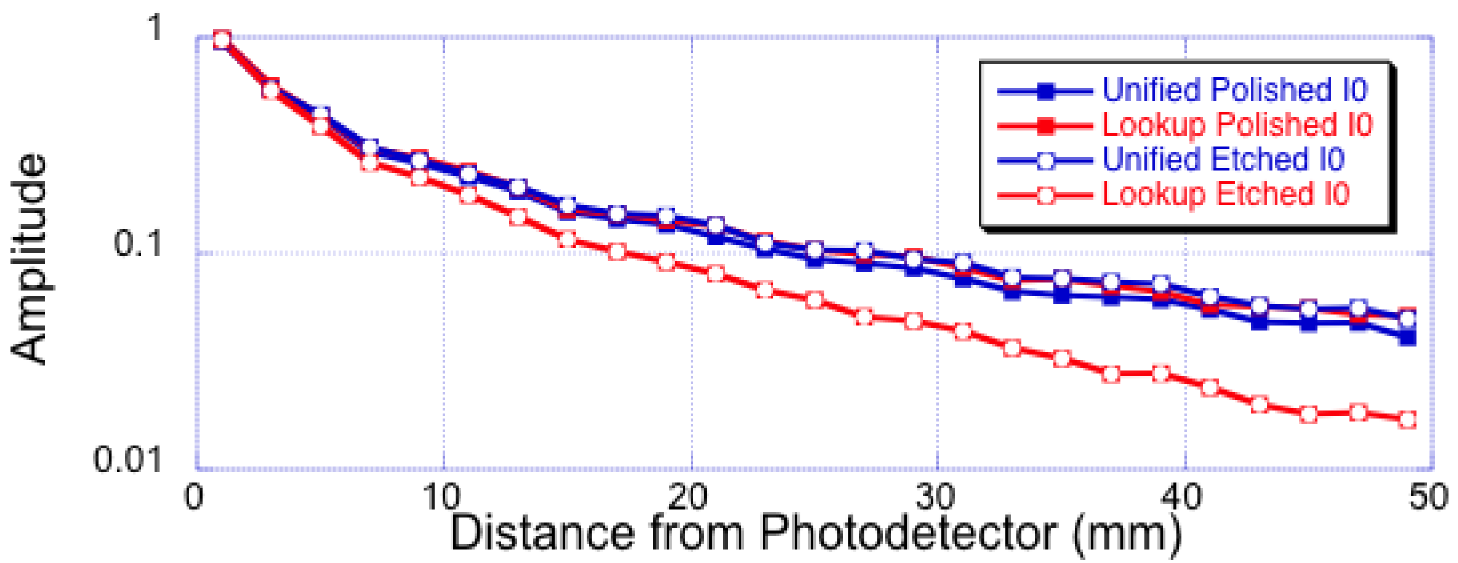
Simplifying the Simulation



- Run Single Simulation w/ Infinitely Long Crystal
- Superpose Signals from Positions x and 2L–x
- Reflected Signal Less Important for Timing…

Simulation Results (Polished & Etched)

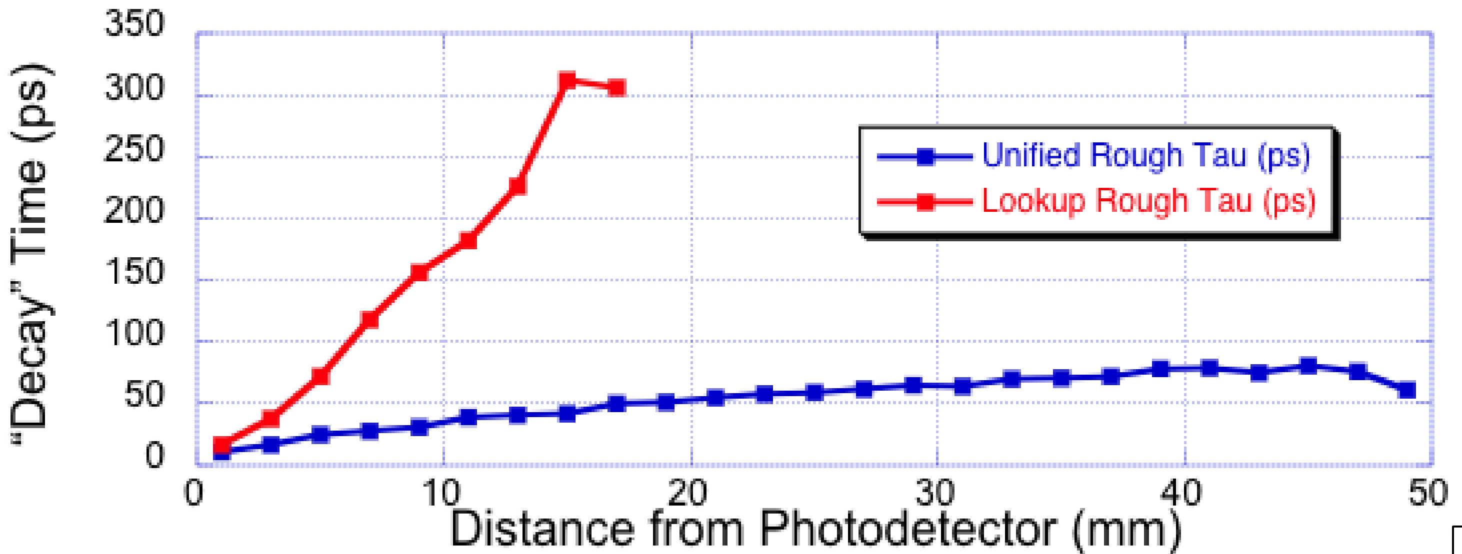


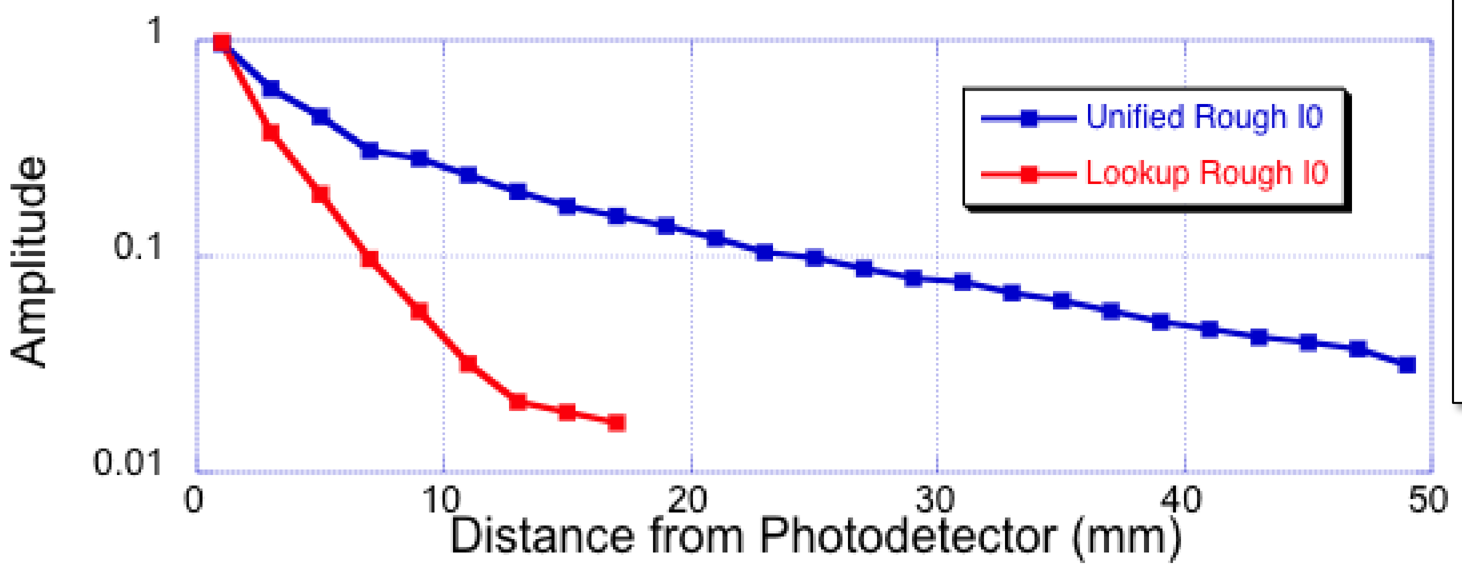


Delay =

$$\textbf{Distance} * \frac{n}{c}$$

Simulation Results (Rough)

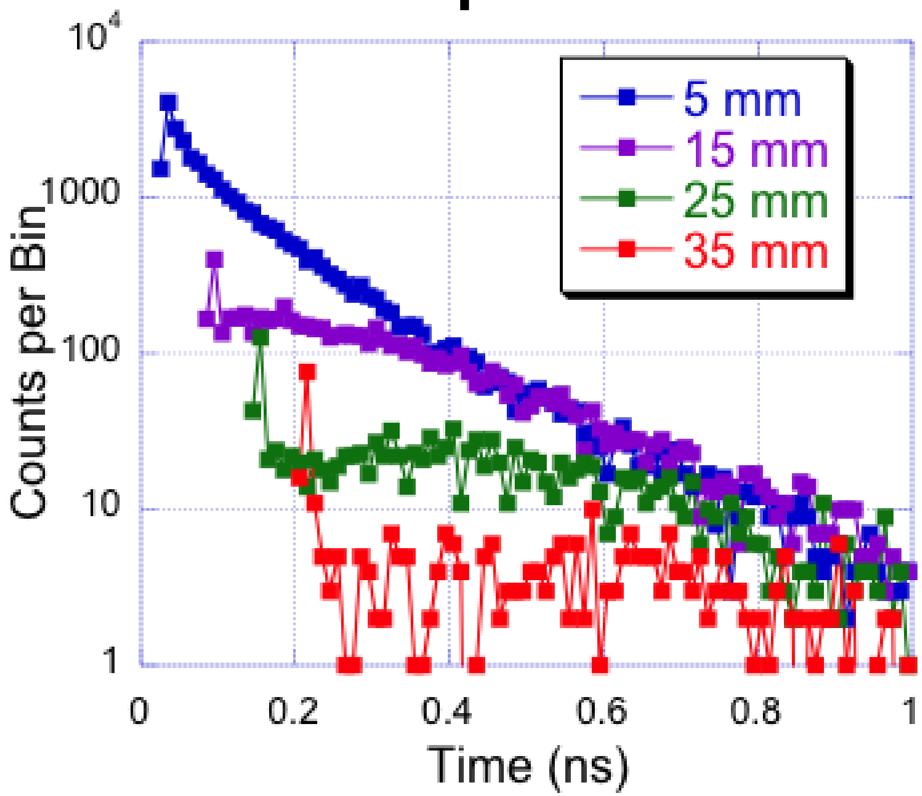




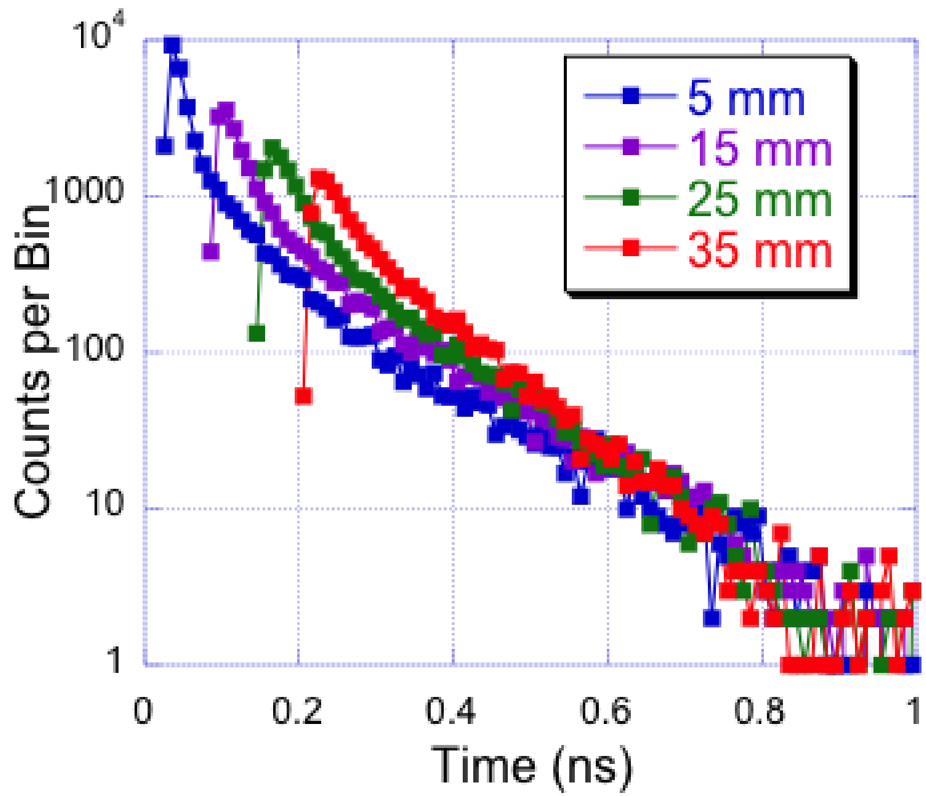
- **Unified "Rough" Very Similar to "Etched" and "Polished"**
- **Lookup "Rough" is *VERY* Different**

Simulation Results (Rough Surface)

Lookup Table

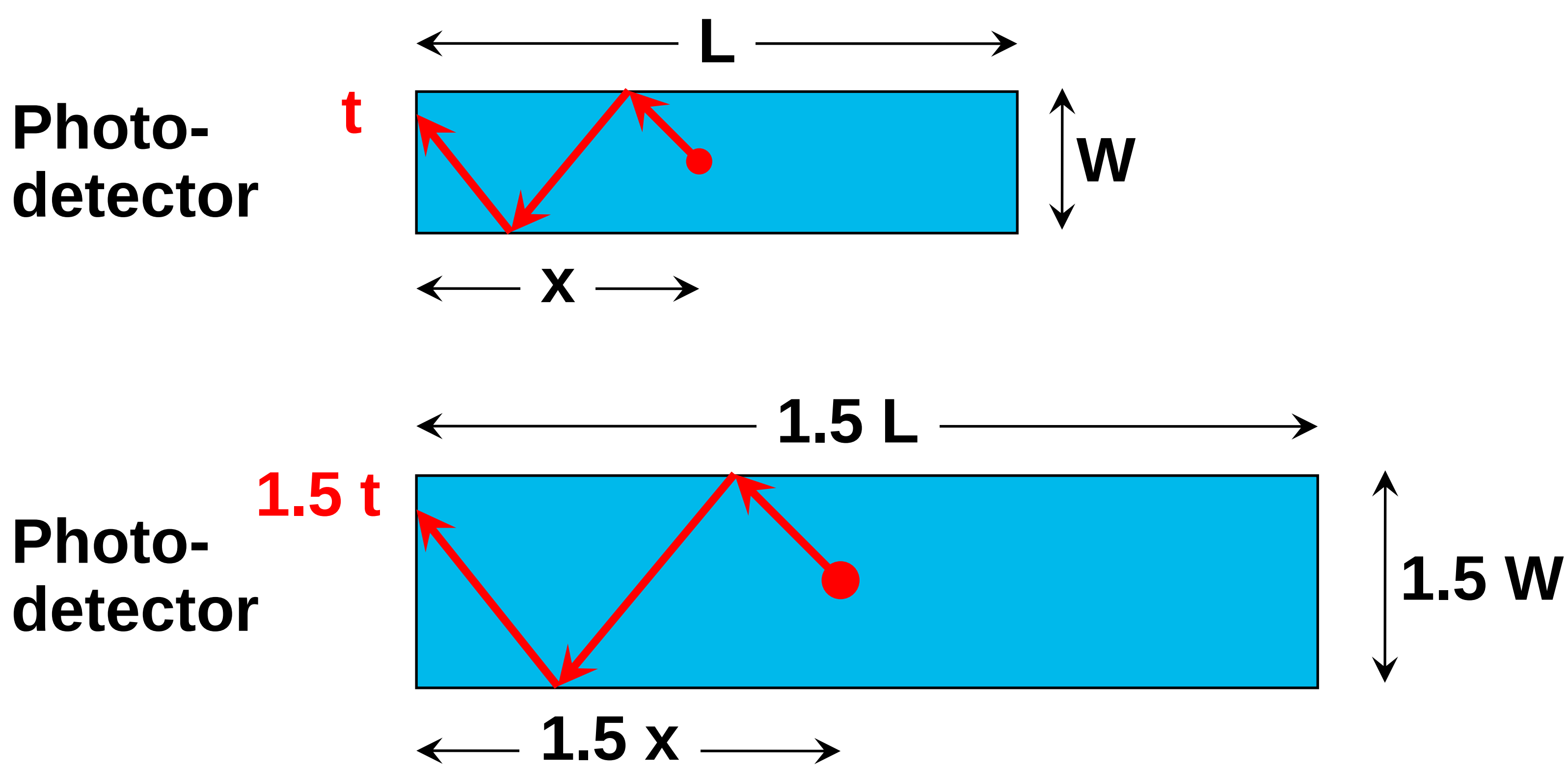


Unified Model



Different Models Predict Very Different Behavior

Scaling Crystal Dimensions



- Simulations Run on 3 mm x 3 mm Crystal
- Results for a W x W Crystal Can Be Found By Multiplying All Distances & Times on Graphs by W/3

Conclusions

- **Simulation used to estimate timing resolution for many combinations of scintillation detector parameters**
- **Includes virtually all known effects**
- **Results encapsulated in “simple” formula that predicts timing resolution**
- **All inputs to formula readily obtained (including optical dispersion in crystal)**
- **Needs experimental validation!**