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Measurement of the drift velocities of electrons and holes in high-ohmic <100> silicon

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PHYSICS AT THE TERA SCALE
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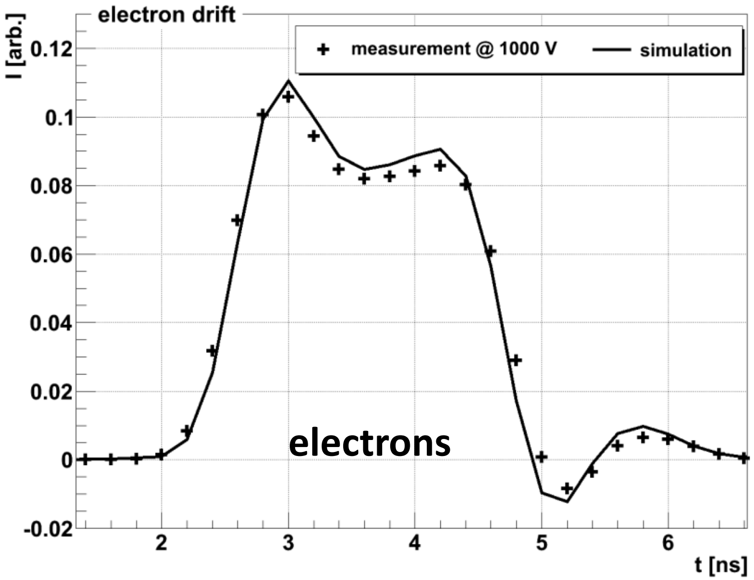
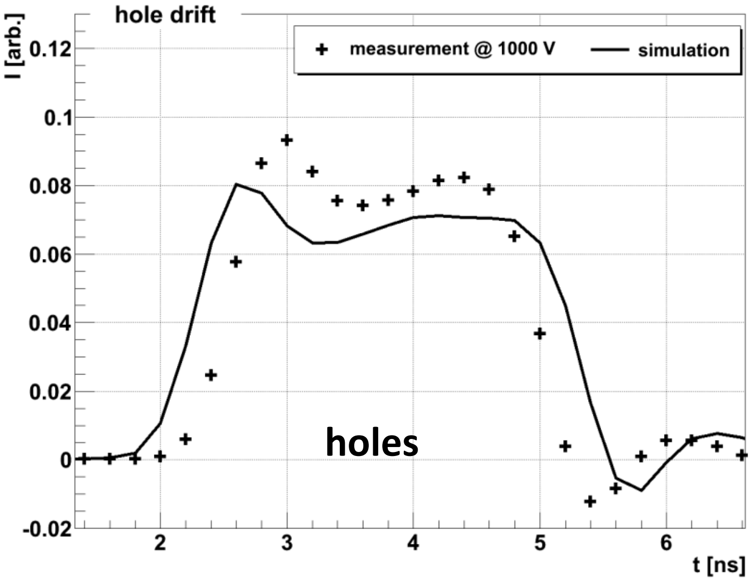
1 – Motivation

Motivation



200 µm
<100> sensor

drift model:
Jacoboni 1977
<111>
↓
often used for
<100>



The <100> drift velocity is a fundamental material parameter!

BUT: The literature mobility models for <100> and <111> direction do not describe measurements of non irradiated sensors

Precise **knowledge of the drift velocities** of electrons and holes is **needed for** precise **simulations** and the **analysis of (edge-)TCT**

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2 – Experimental method

Diodes and measurements

- **Diodes**: High-ohmic **<100>** float-zone silicon p^+nn^+ and n^+pp^+ **pad diodes**

  Three different sensors investigated:

Standard sensor → HPK N-type

Used for cross check: HPK P-type, CiS N-type

Vendor	Bulk	w [µm]	U_{dep} [V]	N_{eff} [cm^{-3}]	ρ [kΩcm]
HPK	N-type	200±2	90	$2.9 \cdot 10^{12}$	1.5
HPK	P-type	200±2	115	$3.8 \cdot 10^{12}$	1.1
CiS	N-type	287±3	50	$0.8 \cdot 10^{12}$	5.5

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Diodes and measurements

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	Vendor	Bulk	w [µm]	U_{dep} [V]	N_{eff} [cm^{-3}]	ρ [kΩcm]
Standard sensor →	**HPK**	**N-type**	**200±2**	**90**	**$2.9 \cdot 10^{12}$**	**1.5**
Used for cross check	**HPK**	**P-type**	**200±2**	**115**	**$3.8 \cdot 10^{12}$**	**1.1**
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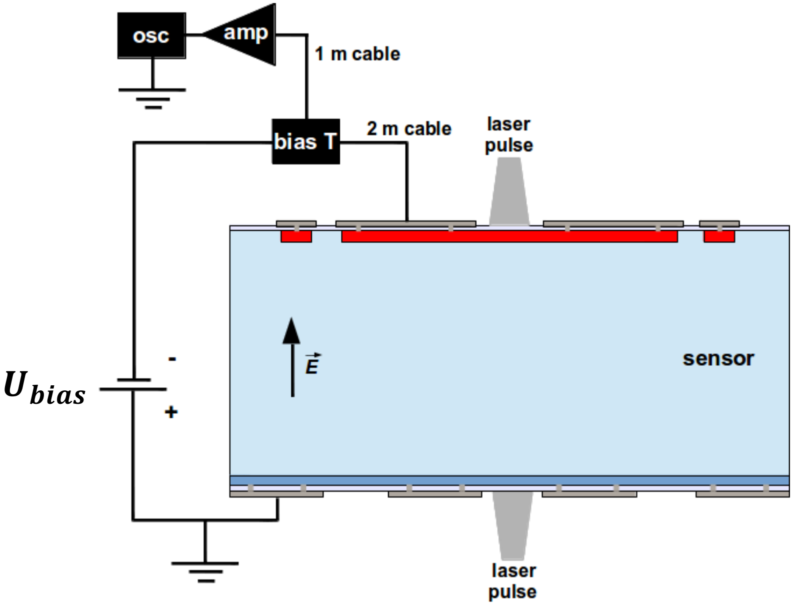
Transient current technique TCT



- Pulsed laser generates e-h pairs in the sensor
- Charge carriers drift in electric field
  → induced signal $\boldsymbol{I} \propto \boldsymbol{v_d}(\boldsymbol{E}(\boldsymbol{x}))$

Measurements

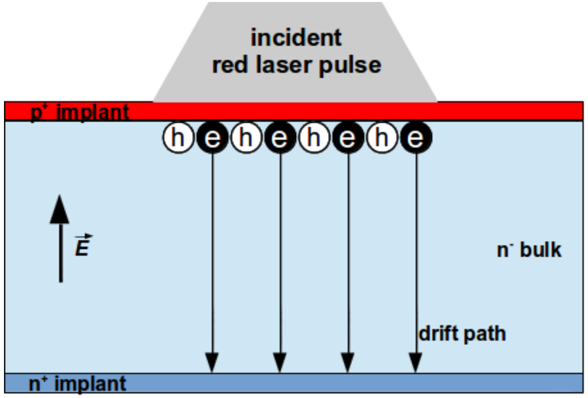
– $U_{dep} < U_{bias} \leq$ **1000 V** steps of $\Delta U_{bias} = 10$ V
– Seven temperatures between **233 K** and **333 K**

Transient Current Technique

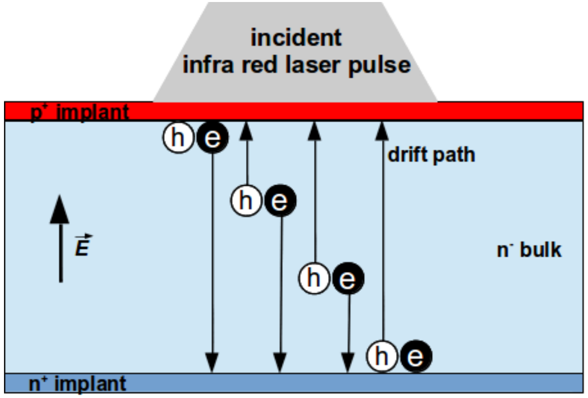
- **Red laser $\lambda = 675$ nm**
  → signal induced by drift of e **or** h

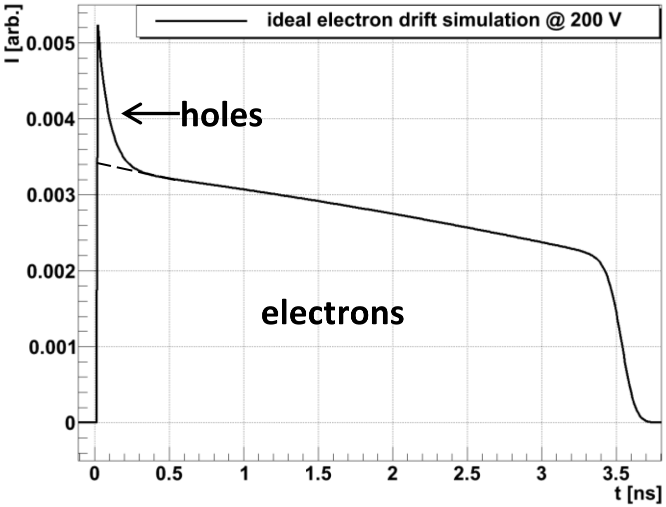
- **Infrared laser $\lambda = 1063$ nm**
  → simultaneous e **and** h drift

$FWHM \approx 50$ ps

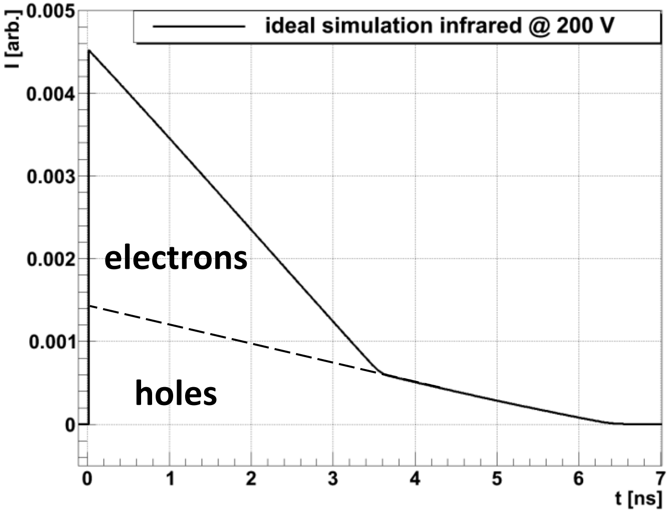


initial charge distribution





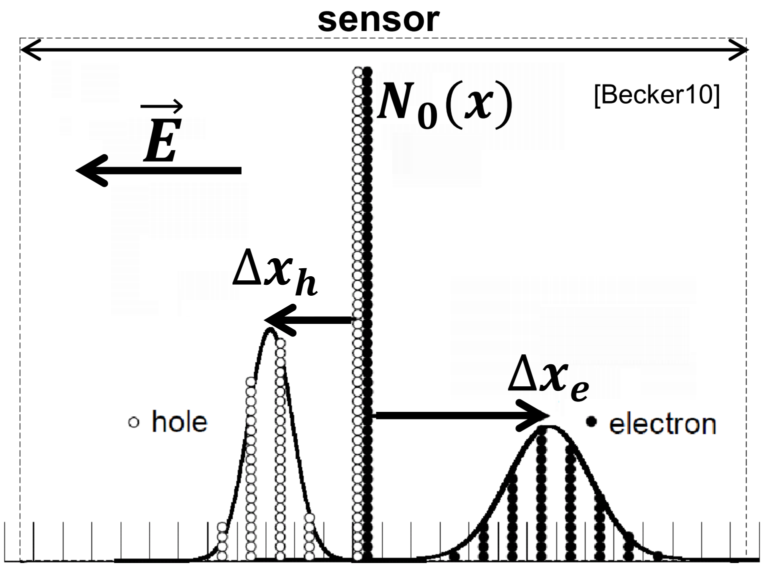
simulation 200 µm sensor @ 200 V



3 – Analysis methods

Transient current simulation

- **Fit of the mobility parameters for red + infrared measurements** at all bias voltages + temperatures at once → up to 622 measurements



- **Grid**

  $\delta x = 100$ nm

- **Charge transport** with $\Delta t = 10$ ps

  → **Drift**

  $$\Delta x_{e,h} = v_{e,h}(x) \cdot \Delta t = \mu_{e,h}(x) E(x) \cdot \Delta t$$

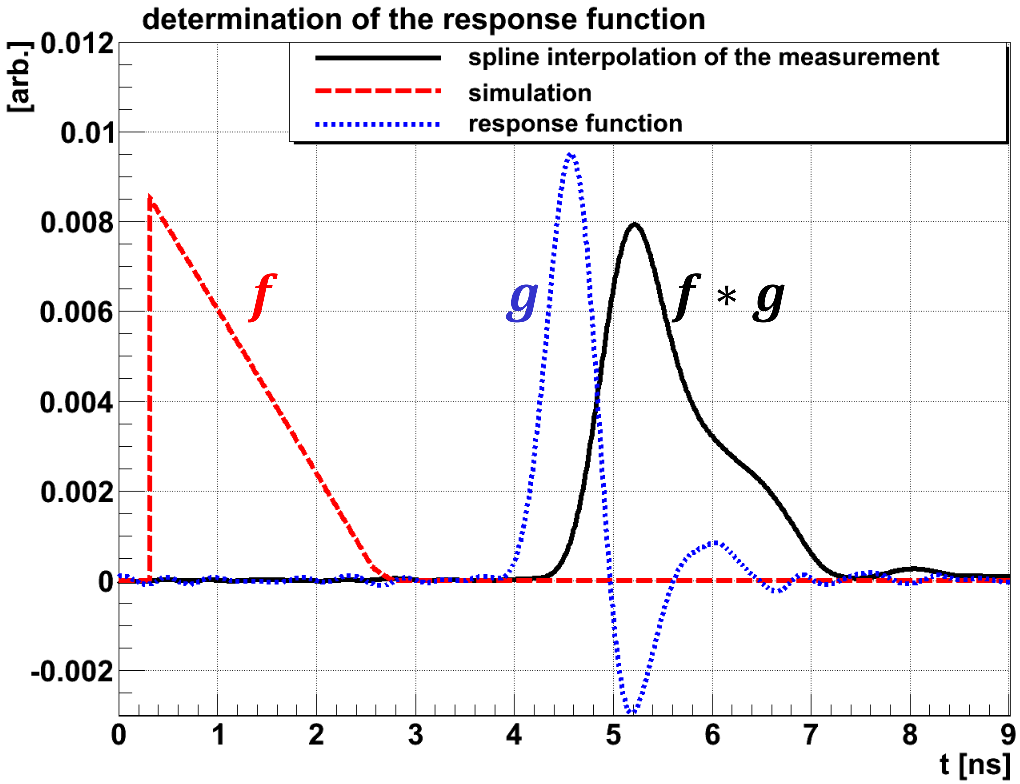
  → **Diffusion** → Gaussian function

  $$\sigma_{e,h}(x) = \sqrt{2D_{e,h}(x) \cdot \Delta t} \qquad D_{e,h}(x) = \frac{\mu_{e,h}(x) k_b T}{e_0}$$

Transient current simulation:

Transfer function

Convolution theorem: $$\mathcal{F}\{f * g\} = \mathcal{F}\{f\} \cdot \mathcal{F}\{g\}$$



infrared laser
200 µm sensor
@ 1000 V and 313 K

Transfer function of the read-out circuit → $$g = \mathcal{F}^{-1}\left(\frac{\mathcal{F}\{measurement\}}{\mathcal{F}\{simulation\}}\right)$$

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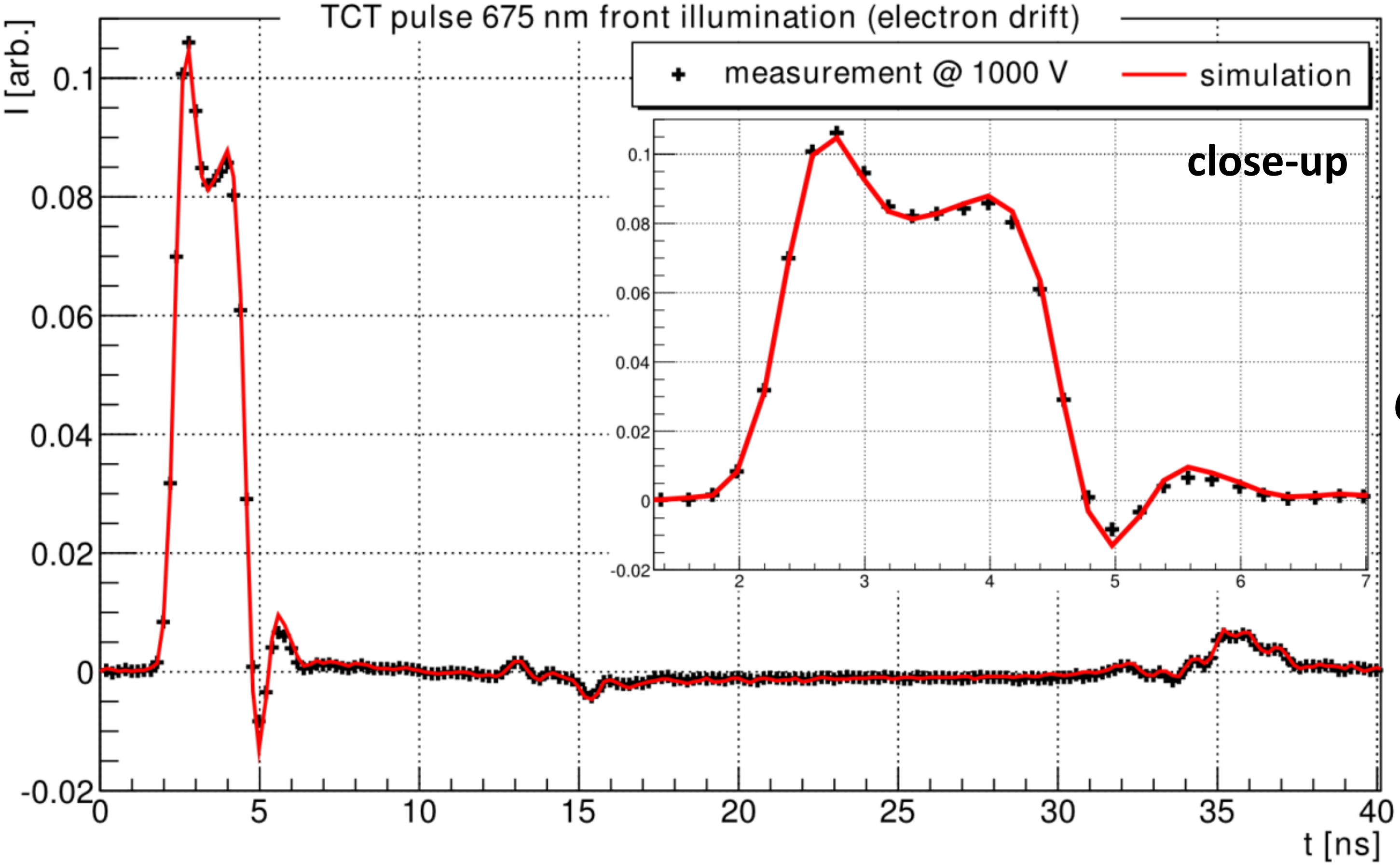
Transient current simulation:

Transfer function



red laser (e drift)
200 µm sensor
@ 1000 V and 313 K

with g for the
infrared laser

- For all simulations g **for the infrared laser @ 1000 V and 313 K** was used
  → fast signal, e and h contribution

The time-of-flight method

- Carrier **time of flight tof**
  → **points of maximum slope**
- Signal sampling 200 ps → spline interpolation

Problem: $E(x) \neq const$

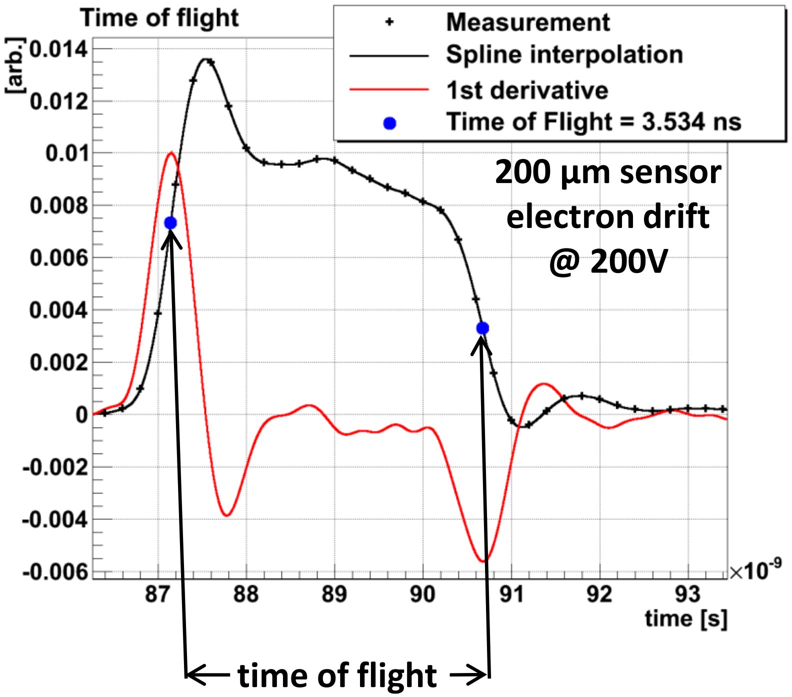
$$tof = \int_0^w \frac{dx}{v_d(E(x))}$$

→ Assume $\frac{1}{v_d(E(x))} \propto \frac{1}{E(x)} + const$

$$\rightarrow tof \propto w \cdot \left(\left\langle {}^{1}/_{E(x)} \right\rangle + const\right)$$

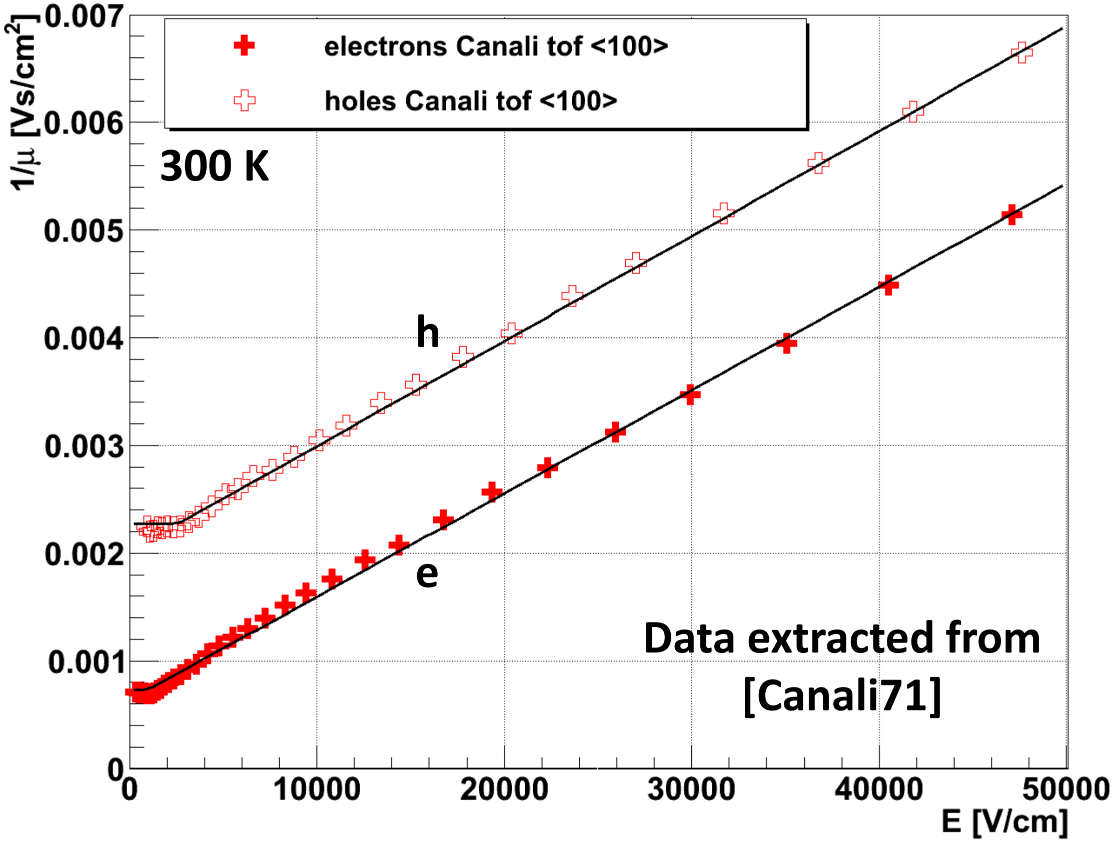
$$\rightarrow v_d\left(\left\langle {}^{1}/_{E(x)} \right\rangle^{-1}\right) = {}^{w}/_{tof}$$

Red laser
→ drift of one charge carrier type



4 – Results

Mobility model



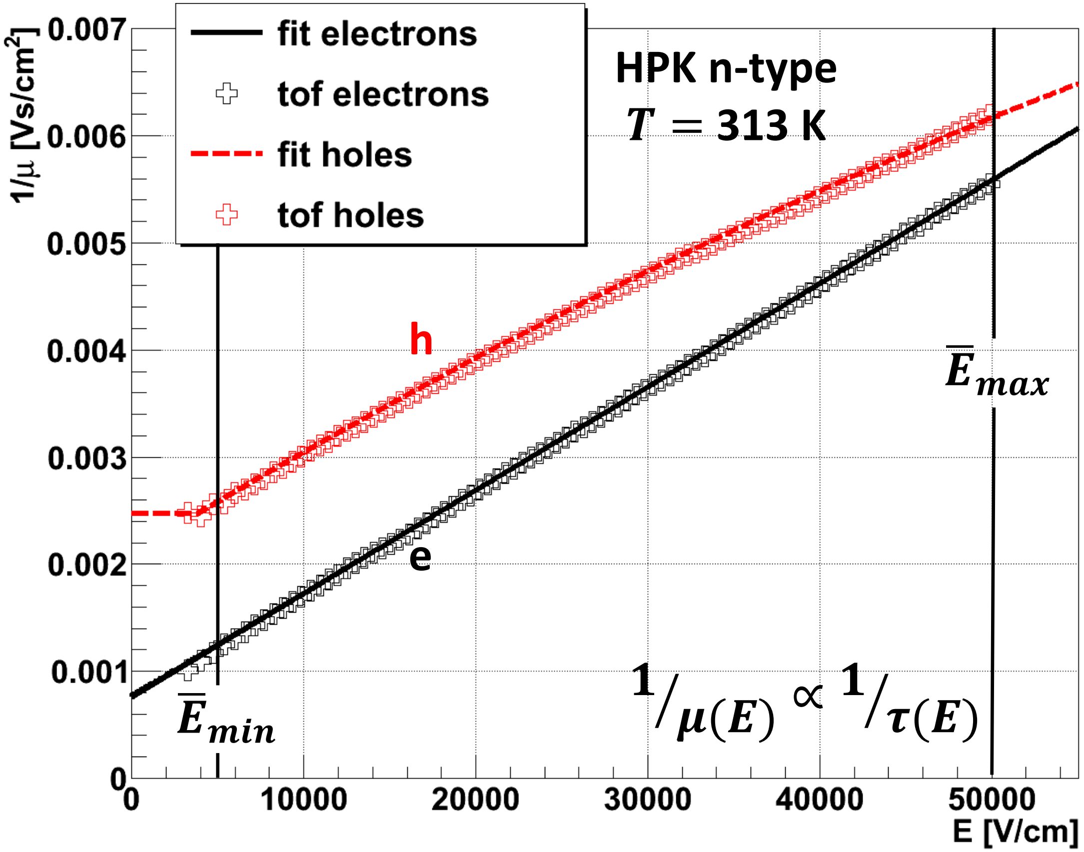
Data extracted from [Canali71]

$$v_d(E) = \frac{qE}{2m^*}\tau(E) = \mu(E)\cdot E$$

$$\frac{1}{\mu(E)} \propto \underset{\substack{\uparrow \\ \text{lattice} \\ \text{scattering}}}{\frac{1}{\tau_{coll}}} + \underset{\substack{\uparrow \\ \text{phonon} \\ \text{emission}}}{\frac{1}{\tau_e(E)}}$$

$$\frac{1}{\mu(E)} = \begin{cases} \frac{1}{\mu_0} & , E < E_0 \\ \frac{1}{\mu_0} + \frac{1}{v_s}\cdot(E - E_0) & , E \geq E_0 \end{cases}$$

Our results



T dependence

$$par_i(T) = par_i^{RT} \cdot \left(\frac{T}{300\ K}\right)^{\alpha_i}$$

New parameterization fitted for

5 kV/cm $< \overline{E} <$ 50 kV/cm
233 K $<$ T $<$ 333 K

→ Global fit with $\sigma_{v_d} = \pm 2.5$ %

- **Electrons** $$^1/_{\mu_e(E)} = {}^1/_{\mu_0^e} + {}^1/_{v_s^e} \cdot E$$
- **Holes** $$^1/_{\mu_h(E)} = max\left({}^1/_{\mu_0^h}\ ,\ {}^1/_{\mu_0^h} + b \cdot (E - E_0) + c \cdot (E - E_0)^2\right)$$

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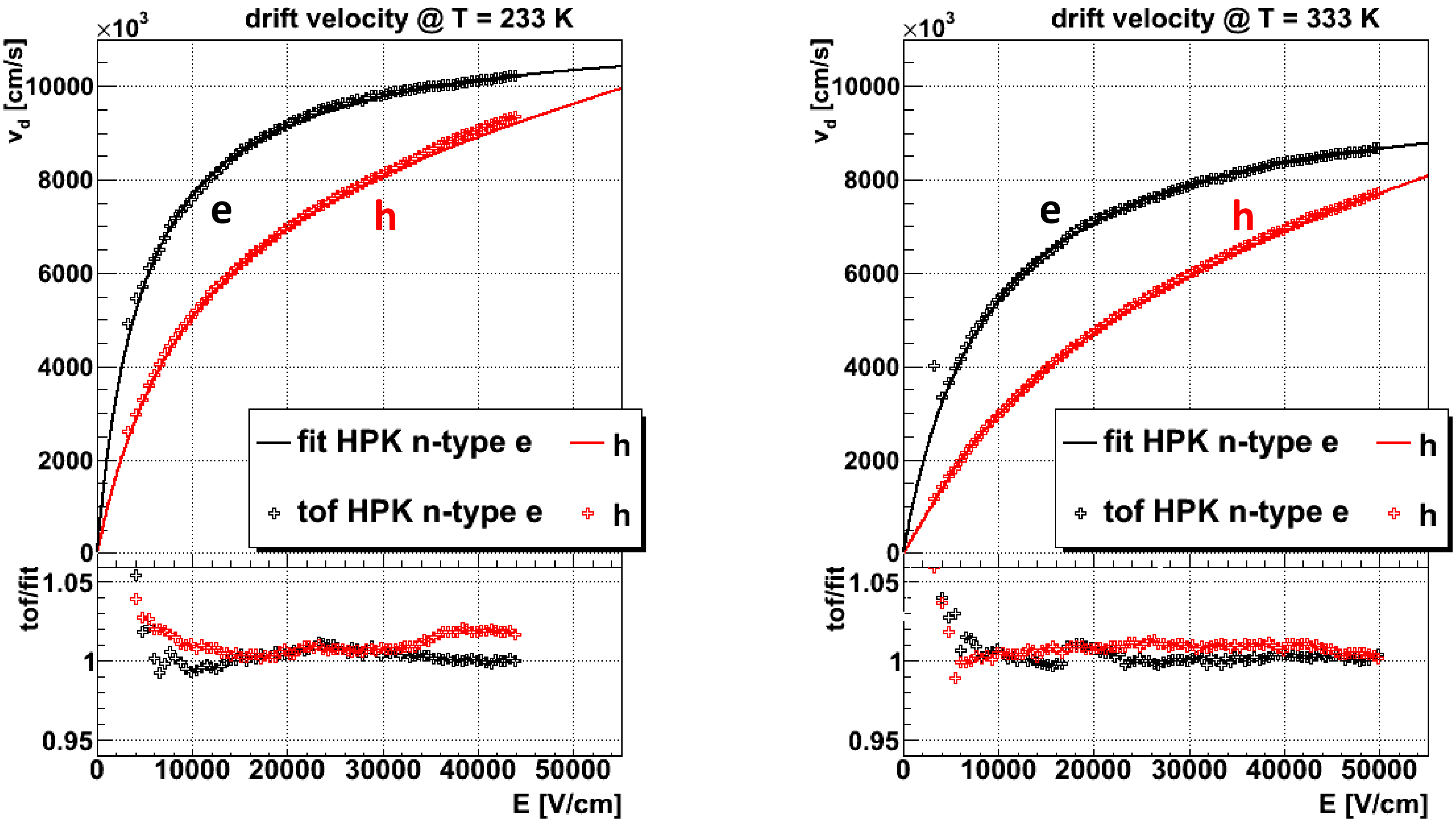
Global fit results

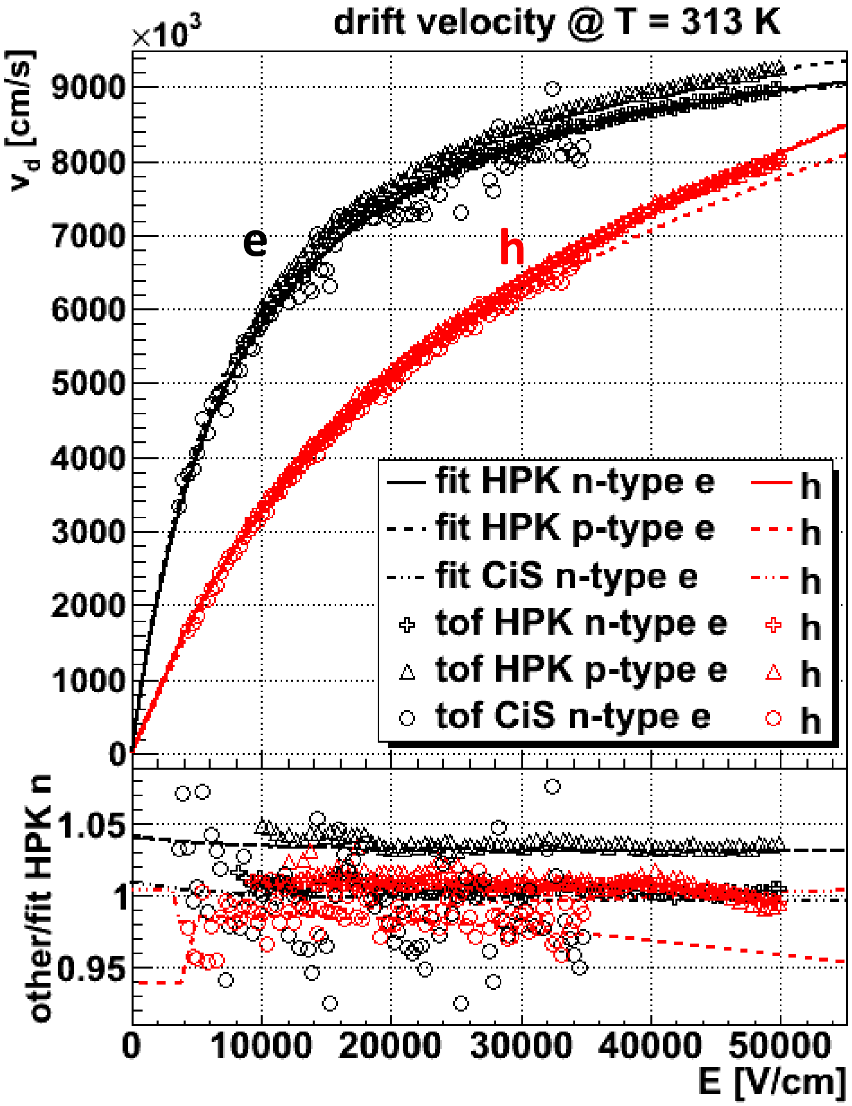


Max tof/fit deviation = **2 %** for $E > 7$ kV/cm

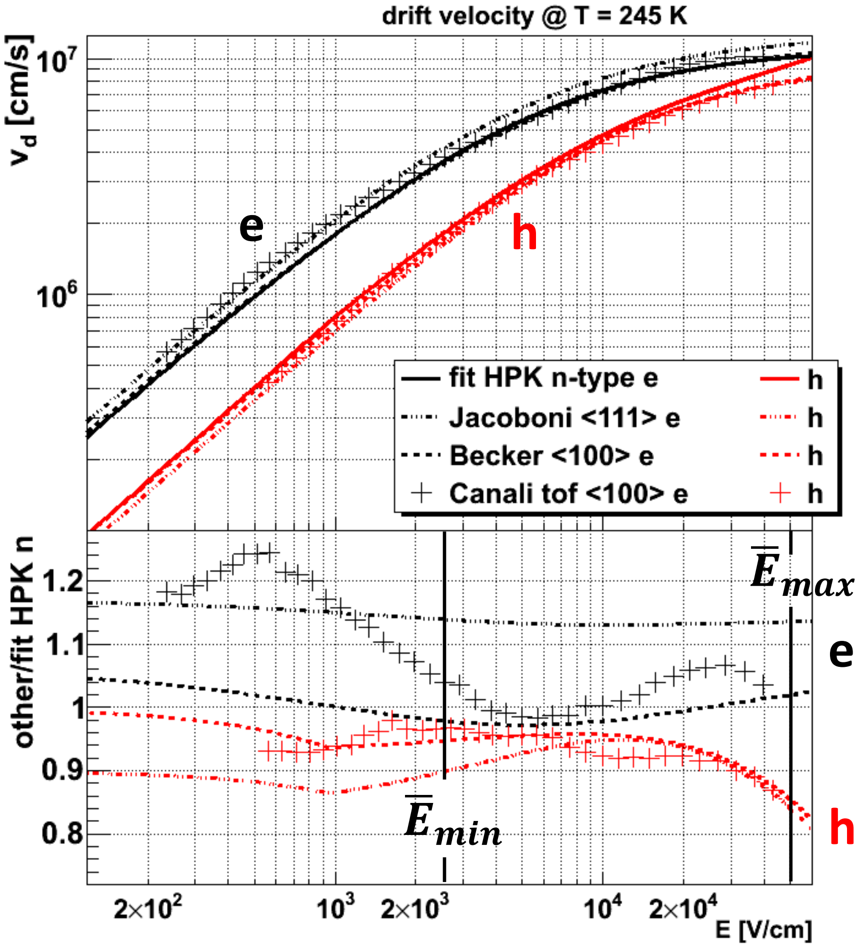
Comparison of different materials

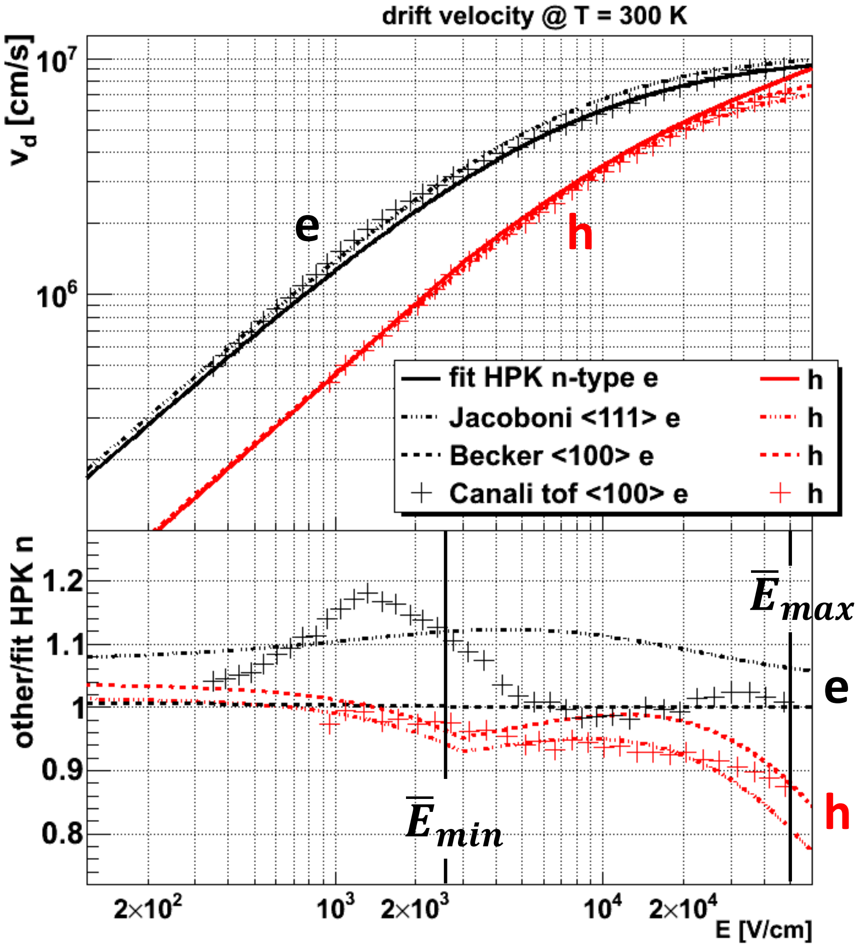
Results of the different sensors @ T=313 K

- The results match within 4 %
- Each point corresponds to one individual measurement!



Comparison with literature





Large differences observed:
- **Jacoboni <111>:** for e and h
- **Becker <100>:** only for h @ high fields
- **Canali <100>:** for h @ high fields and e @ low fields

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5 – Conclusion

Conclusion

- **Analysis**
  - Time-of-flight method ← simple
    - High accuracy despite large field range in the sensor
  - Fit of drift simulation to TCT measurements ← complex
    - Precise determination of electronics response

- **Results**

	max deviation
– Parametrization of the mobility $\mu(E,T)$	
– Results of **different sensors and of fit and tof are consistent**	**4 %**
– Results similar to **<100>** literature values for **electron drift**	**5 %**
– Large differences to **<100> and <111>** literature val. for **hole drift**	**15 %**

- **Impact**

The results will improve simulations and the analysis of TCT and edge-TCT → $\boldsymbol{E(x)},\ \boldsymbol{\tau_e(x)},\ \boldsymbol{\tau_h(x)}$

Thanks for your attention

Special thanks to

J. Becker, J. Erfle, E. Fretwurst , R. Klanner, T. Poehlsen, J. Schwandt

References

J Becker. *Signal development in silicon sensors used for radiation detection*. PhD thesis, University of Hamburg, 2010. DESY-thesis-2010-33.

J Becker, E Fretwurst, and R Klanner. Measurements of charge carrier mobilities and drift velocity saturation in bulk silicon of< 111> and< 100> crystal orientation at high electric fields. *Solid-State Electronics*, 56(1):104110, 2011.

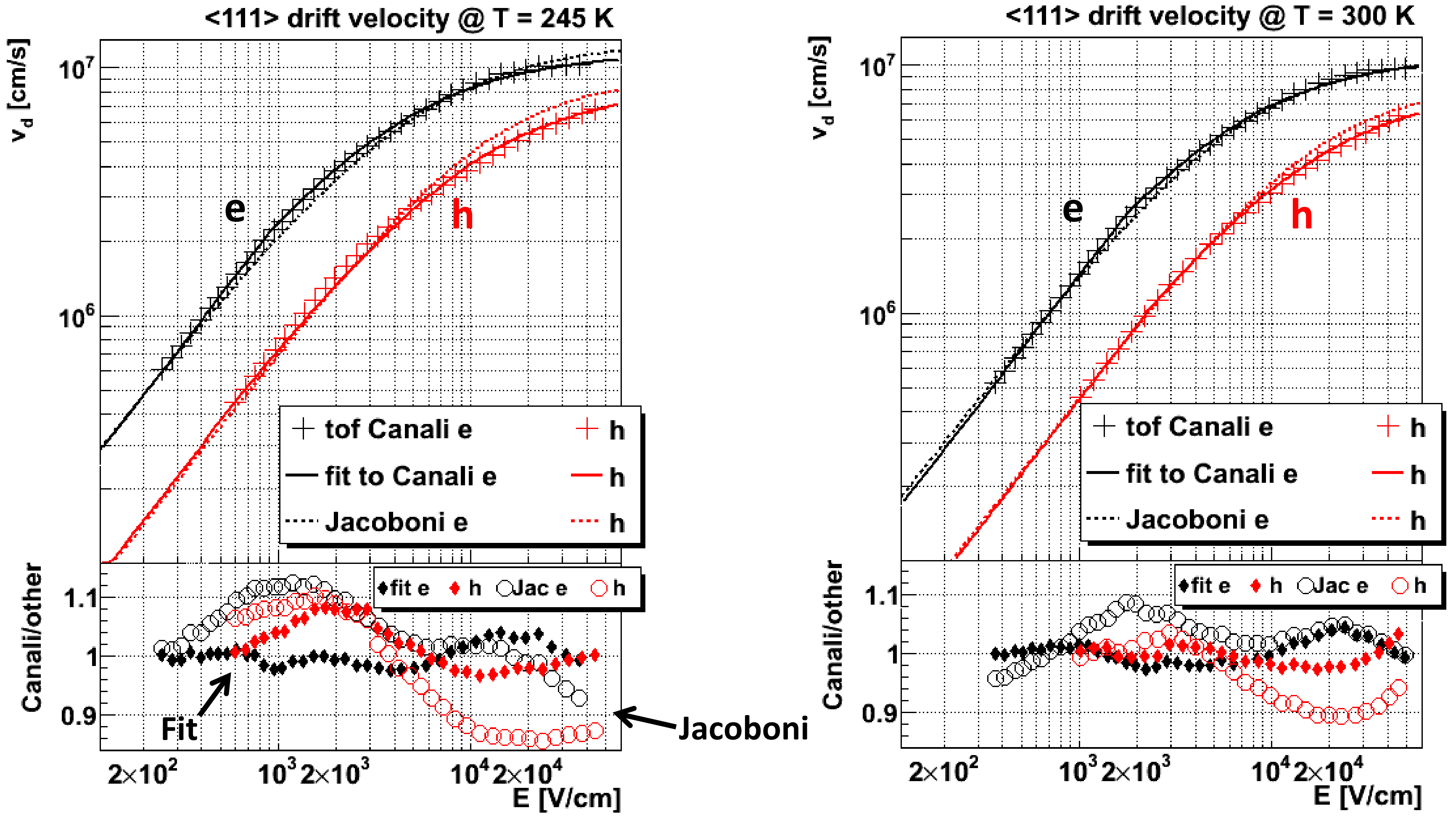
C Canali, G Ottaviani, and A Alberigi Quaranta. Drift velocity of electrons and holes and associated anisotropic effects in silicon. *Journal of Physics and Chemistry of Solids*, 32(8):17071720, 1971.

DM Caughey and RE Thomas. Carrier mobilities in silicon empirically related to doping and field. *Proceedings of the IEEE*, 1967.

C Jacoboni, C Canali, G Ottaviani, and A Alberigi Quaranta. A review of some charge transport properties of silicon. *Solid-State Electronics*, 20(2):77-89, 1977.

Backup

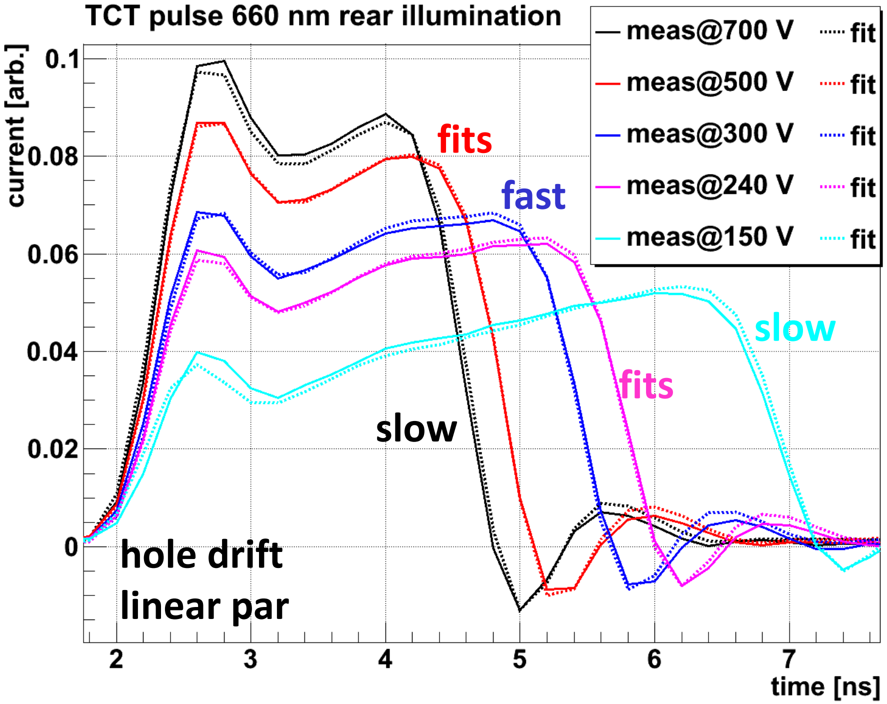
<111> literature mobility

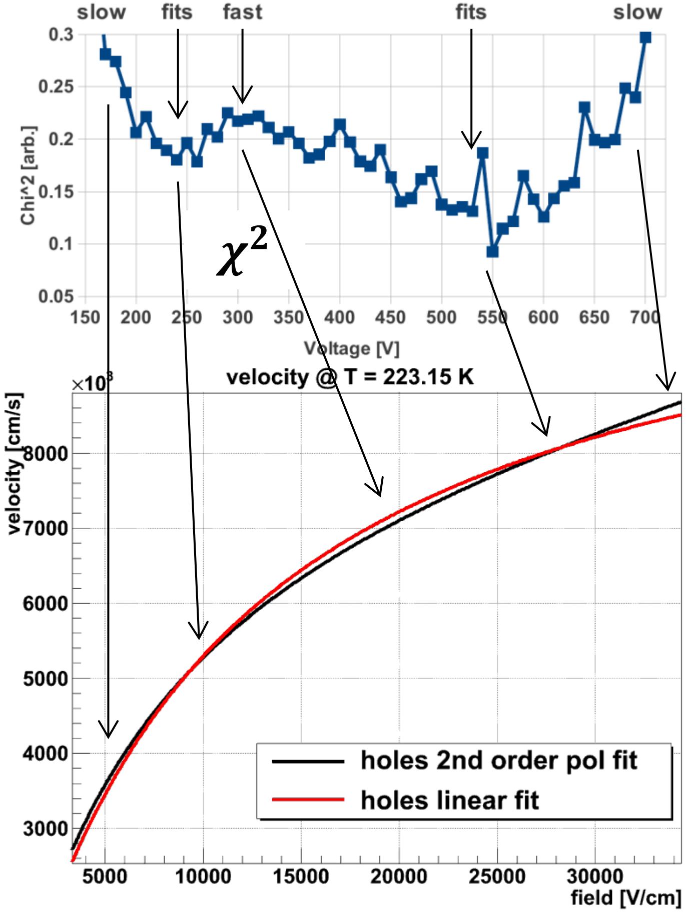


Fit with

$$v_d(E) = E \cdot \mu(E) = E \cdot min\left(\mu_0\,, \frac{\mu_0}{1+{}^{\mu_0}/_{v_s}\cdot(E-E_0)}\right)$$

Mobility model: holes linear @ -40C





- **Holes**

  - Linear par. leads to wrong drift times
    → **2nd degree polynomial**

$$\frac{1}{\mu_h(E)} = max\left(\frac{1}{\mu_0^h}, \frac{1}{\mu_0^h} + b\cdot(E-E_0) + c\cdot(E-E_0)^2\right)$$

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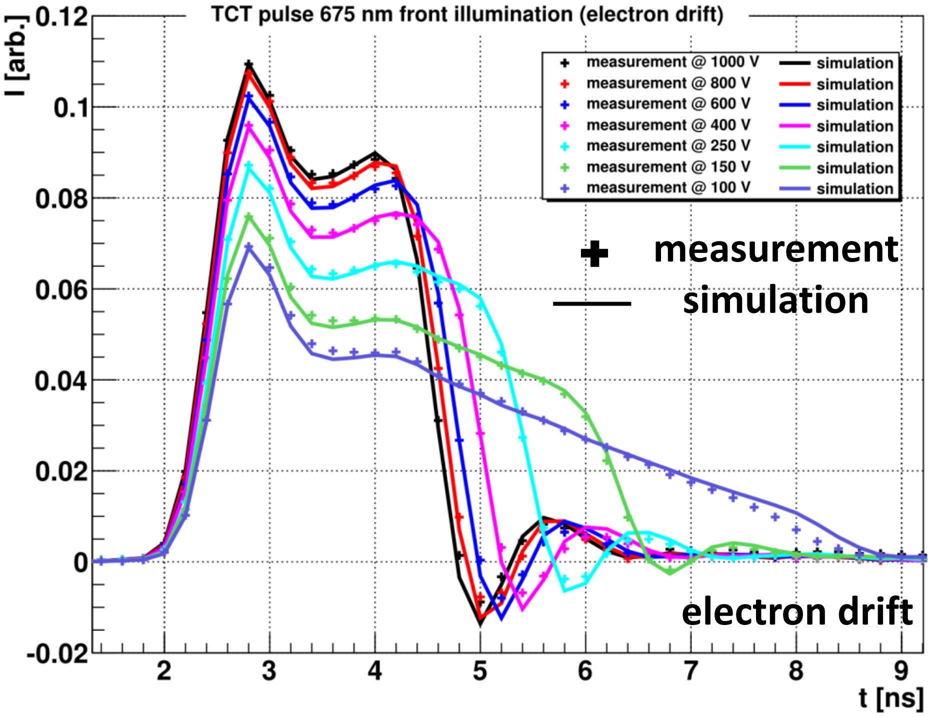
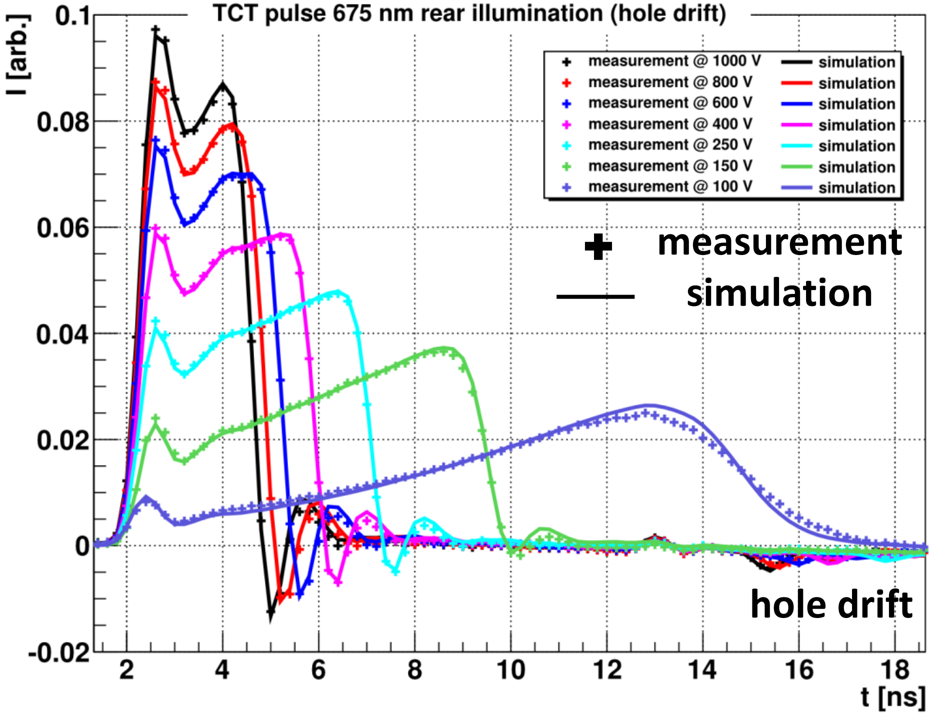
Comparison with measurements

200 µm sensor
293 K



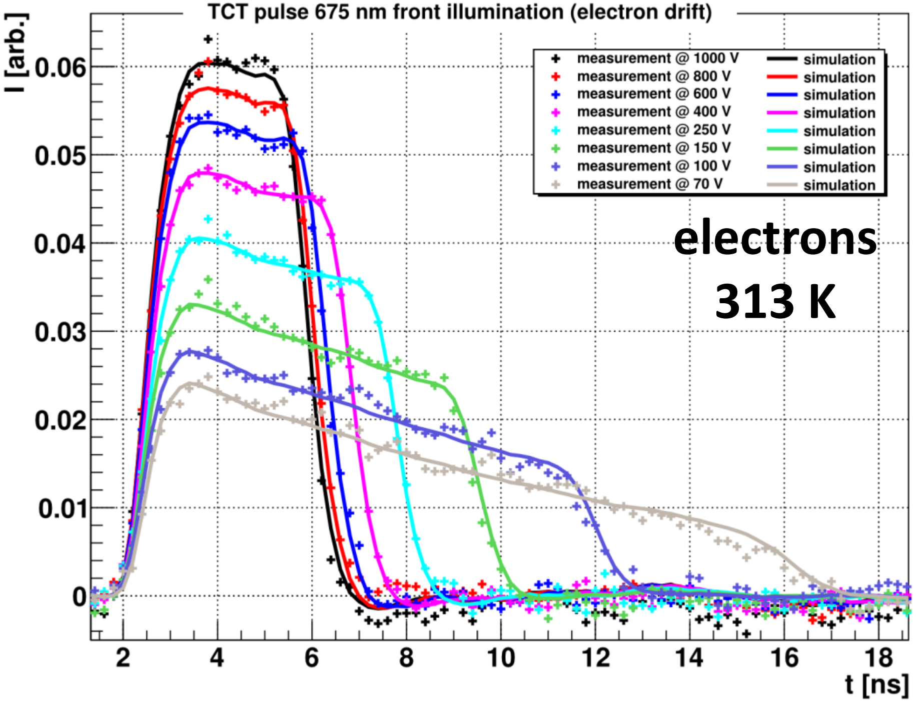


- **Electrons** $$1/\mu_e(E) = 1/\mu_0^e + 1/v_s^e \cdot E$$

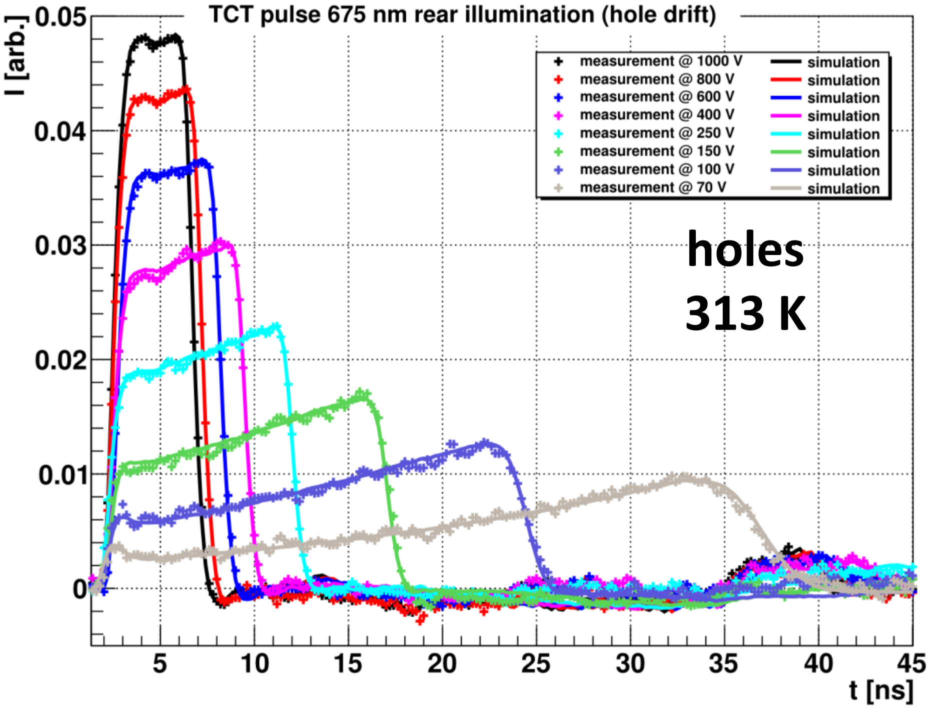
- **Holes** → 2nd degree polynomial

$$1/\mu_h(E) = \max\left(1/\mu_0^h\ ,\ 1/\mu_0^h + b \cdot (E - E_0) + c \cdot (E - E_0)^2\right)$$

Backup CiS sensor

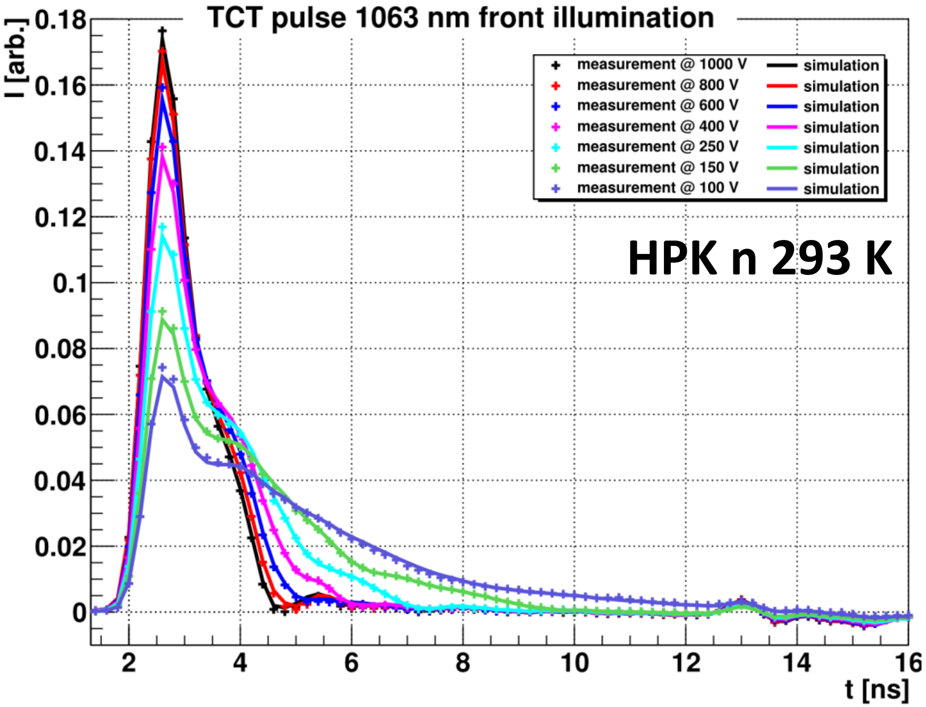


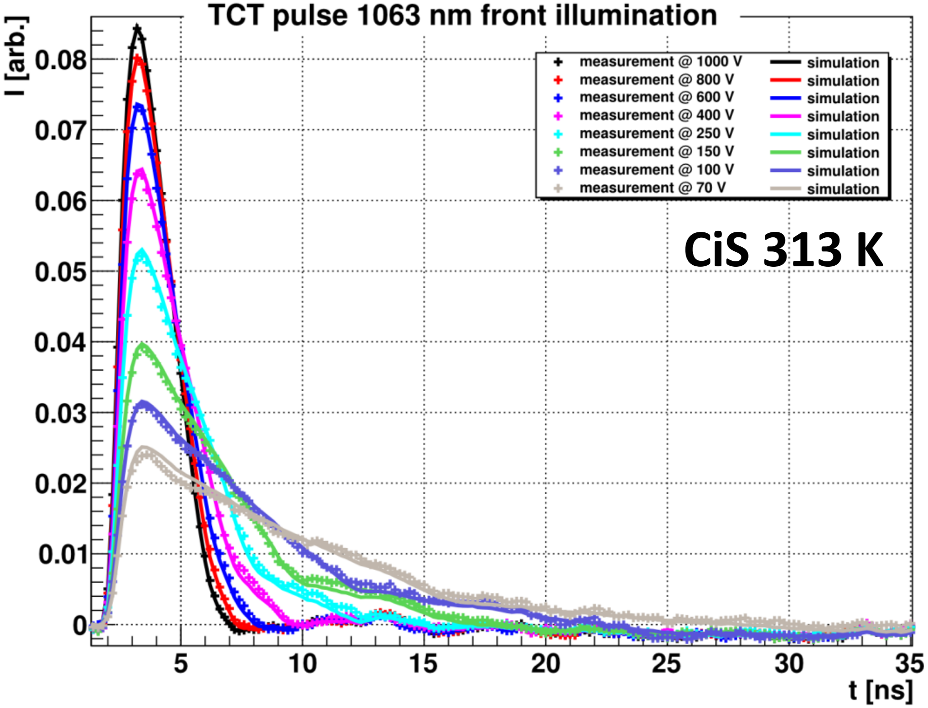
electrons
313 K



holes
313 K

Backup infrared



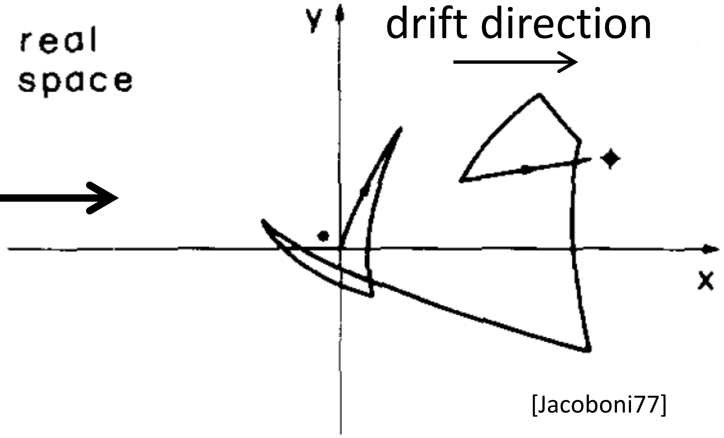


Mobility model

Model based on [Grove67], [Jacoboni77]

Low electric field $E < E_0$

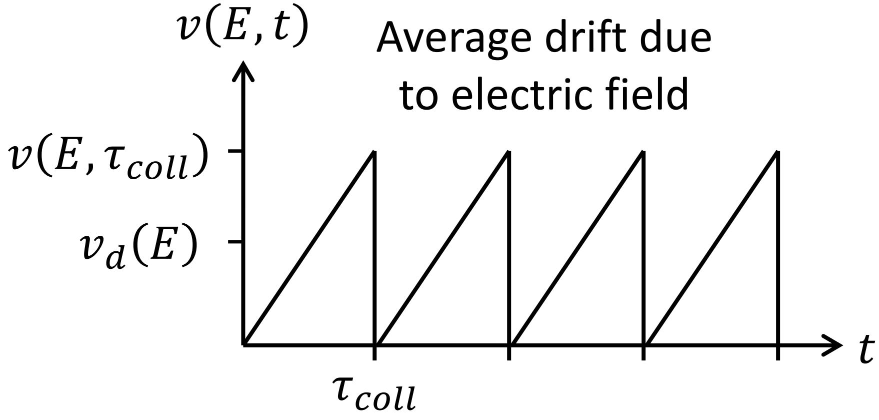
- Random thermal motion ($v_{th} \approx 10^7$ cm/s) superimposed with drift in electric field



[Jacoboni77]

- Mean free time between lattice scattering

  $$\tau_{coll} \propto v_{th} \gg v_d(E)$$



- **Mean drift velocity**

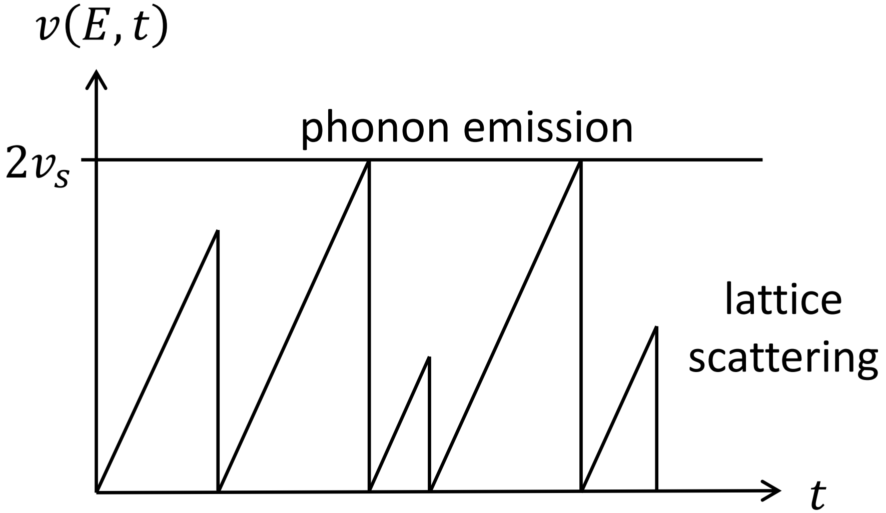
$$\boldsymbol{v_d(E)} = \frac{1}{2} v(E,\tau_{coll}) = \frac{qE}{2m^*}\tau_{coll} = \boldsymbol{\mu_0 E}$$

Mobility model

Model based on [Grove67], [Jacoboni77]

High electric field $\boldsymbol{E \geq E_0}$

- Energy sufficient for optical phonon emission
- Saturation velocity $v_s \propto \sqrt{\hbar\omega_{phonon}}$



Statistical drift due to electric field

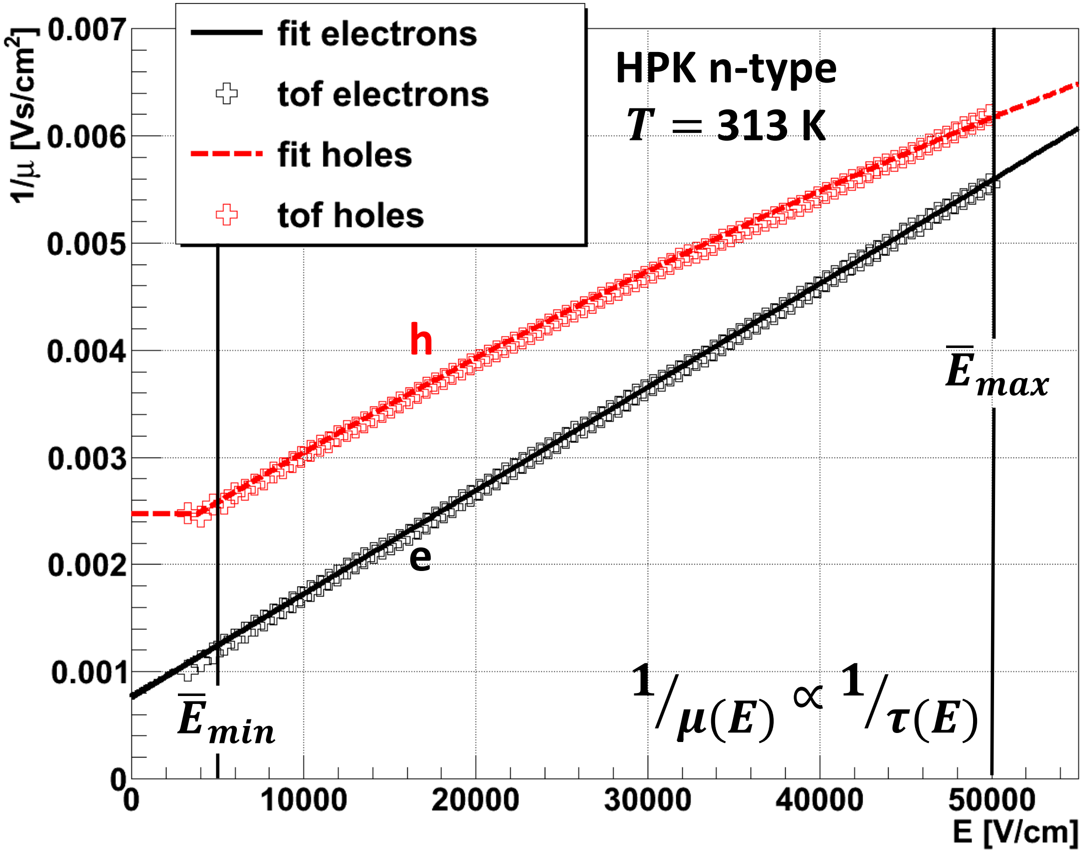
$$\mathrm{v_d(E)} = \frac{qE}{2m^*}\tau(E)$$

$$\boldsymbol{{}^{1}\!/_{\tau(E)} = {}^{1}\!/_{\tau_{coll}} + {}^{1}\!/_{\tau_e(E)}}$$

$\uparrow$ lattice scattering

$\uparrow$ phonon emission

Our results



T dependence

$$par_i(T) = par_i^{RT} \cdot \left(\frac{T}{300\ K}\right)^{\alpha_i}$$

Global fit results ($\sigma_{v_d} = \pm$2.5 %)

		par_i^{RT}	α_i
Electrons	μ_0^e	**1430 cm²/Vs**	**-1.99**
	v_s^e	1.05·10⁷ cm/s	-0.302
Holes	μ_0^h	**457 cm²/Vs**	**-2.80**
	b	9.57·10⁻⁸ s/cm	-0.155
	c	3.24·10⁻¹³ s/V	–
	E_0	2970 V/cm	5.63

- **Electrons** $$^1/_{\mu_e(E)} = {^1/_{\mu_0^e}} + {^1/_{v_s^e}} \cdot E$$
- **Holes** $$^1/_{\mu_h(E)} = max\left({^1/_{\mu_0^h}}\ ,\ {^1/_{\mu_0^h}} + b \cdot (E - E_0) + c \cdot (E - E_0)^2\right)$$