Analytic descriptions of the ITQ fringe fields and progress towards implementation into the HL-LHC lattice

HL-LHC Task 2.3

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1. Analytic descriptions of arbitrary magnetic fields using generalised gradients
2. Benchmarking Field Description
3. Accuracy of Spline Interpolation
4. Transfer Map Calculations
5. Summaries

Analytic descriptions of arbitrary magnetic fields

- Using Dragt's technique of generalised gradients, a general vector potential can be described in terms of 'generalised gradients' [1]

$$A_x^{m,norm} = \frac{\mathcal{R}(x+iy)^{m+1}}{2} \sum_{l=0}^{\infty} (-1)^l \frac{m!}{2^{2l} l!(l+m+1)!)} C_{m,norm}^{[2l+1]}(z)(x^2+y^2)^l$$

where

$$C_{m,c}^{[n]}(z) = \frac{i^n}{2^m m!} \int_{-\infty}^{\infty} dk \exp(ikz) \frac{k^{n+m-1}}{I'_m(kR)} \hat{a}_m$$

are the generalised gradients
Similar expressions exist for $A_y^{m,norm}, A_z^{m,norm}$ and skew components of the vector field A
(See previous talks from B. Dalena)

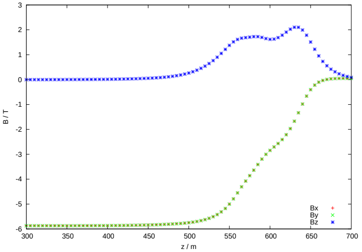
[1] 'Lie Methods for Nonlinear Dynamics with Applications to Accelerator Physics', A. Dragt,http://www.physics.umd.edu/dsat/dsatliemethods.html

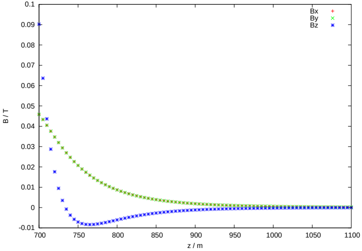
Analytic descriptions of arbitrary magnetic fields

$$C_{m,c}^{[n]}(z) = \frac{i^n}{2^m m!} \int_{-\infty}^{\infty} dk \exp(ikz) \frac{k^{n+m-1}}{I'_m(kR)} \hat{a}_m$$

- $\hat{a}_m$ are the 2D Fourier coefficients of the radial field on the surface of a cylinder
- Using numerical field data on the surface of a cylinder, an analytic description of the field at all interior values can be calculated
- Fully consistent with Maxwell's equations
- The affect of numerical errors in the field data decrease exponentially inside the cylinder
- The generalised gradients describe how the field changes longitudinally (fringe fields)
- The field description is given in terms of skew/normal multipole coefficients

ITQ Numerical Data





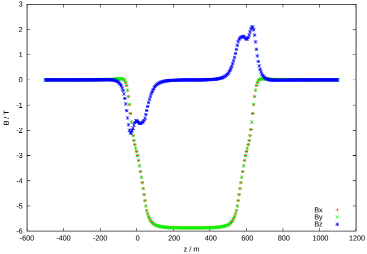
Numerical data files:

`MQXFC_3D_fringefield_z1100_150213.dat`

`MQXFC_3D_fringefield_z700_150213.dat`

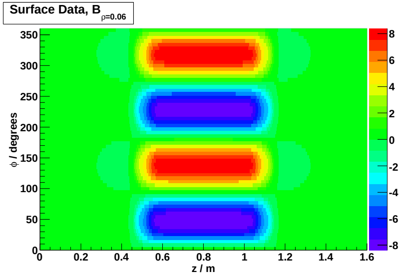
- Numerical data on quarter turn over two maps (300-700 mm and 700-1100 mm)
- Two reflections in ϕ to give 2π coverage
- Two maps concatenated at z=700 mm and reflected to give z-symmetry
- Field Data on grid, 150 mm x 150 mm x 1600 mm, with spacing 3mm:3mm:5mm

ITQ Numerical Data



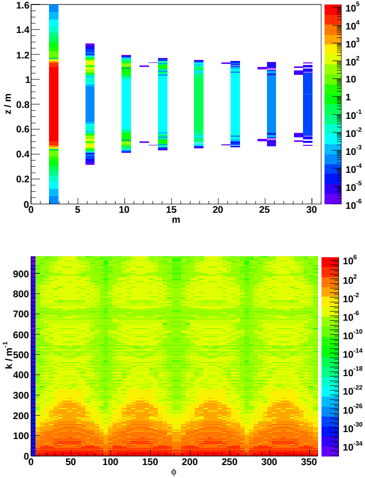
- Interpolation done with a 3D cubic spline library (bspline22)
- Cubic spline interpolation will be approximate close to the edge of the grid
- Don't use data on the edge of the grid

Surface data



- Radial field component calculated on the surface of a cylinder ($\rho = 60$ mm)
- Use 63 points in ϕ and 501 points in z to generate the surface data

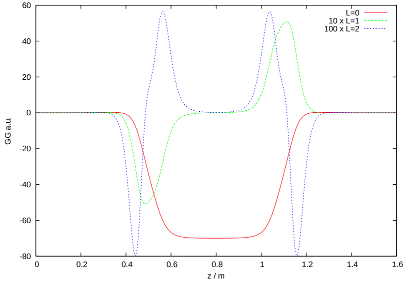
DFT of Surface Data

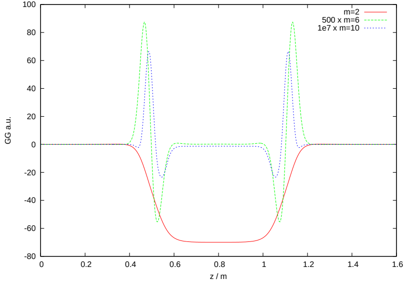


Power in DFT (ϕ top, z bottom)

- Power in azimuthal FT extends beyond 30^{th} multipole
- For pure quadrupole field, symmetry condition only allows multipoles $m = 2(2n+1), n = 0, 1, \ldots$
- By reflecting the field azimuthally we are enforcing quadrupole symmetry (errors in field data also have 4-fold symmetry)
- high m-poles artefact of processing? Possible bias?

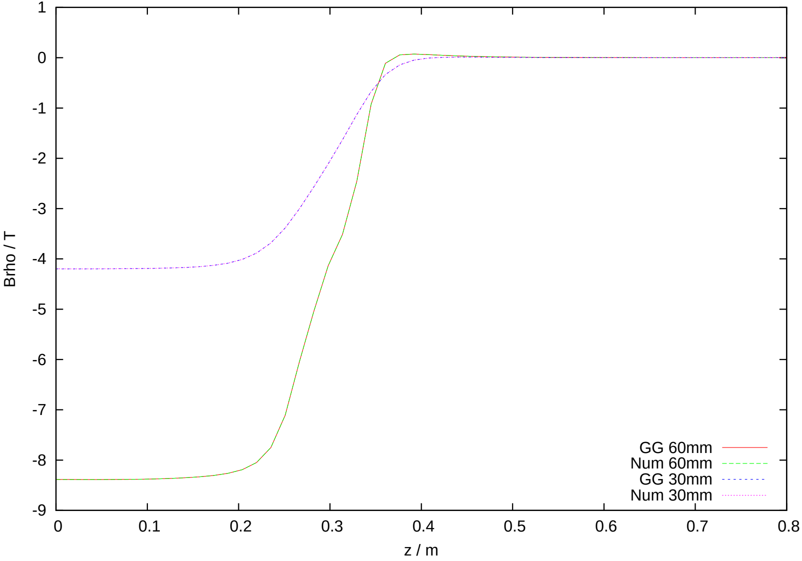
Generalised Gradients



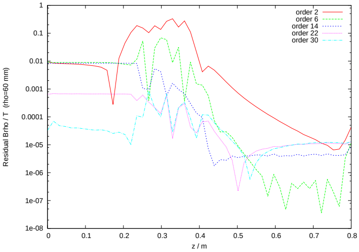


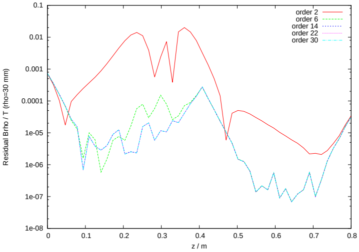
- Generalised gradients calculated numerically using the DFT surface data
- $m = 2$ C_m^n GG shown (top left) for $m = 2$, $n = 0, 1, 2$
- C_m^{n+1} is the gradient (wrt z) of C_m^n
- C_m^n GG shown (bottom left) for $m = 2, 6, 10$, $n = 0$
- Note scaling, amplitude grows rapidly with m

Radial Field Comparison (ρ, $\phi = \pi/4$, z)



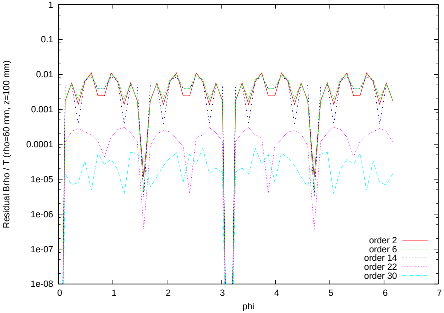
Radial Field Comparison (ρ, $\phi = \pi/4$, z)

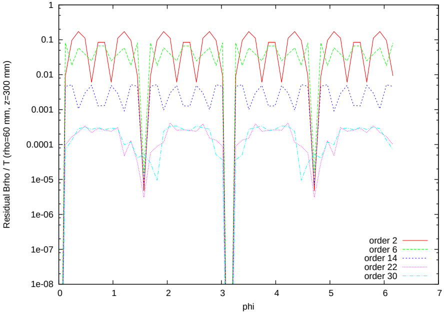




- Field residuals (analytic - spline) as a function of z shown at $\rho = 60$ mm (left) and $\rho = 30$ mm (right)
- Analytic field is truncated at $m = 6, 10, 14, 22, 30$
- Higher order multipole components have a significant effect on the residuals at large ρ
- $B_{max} \sim 8$ T (60 mm) and 4 T (60 mm)
  - Relative accuracy of $\sim 5.10^{-5}$
- Best fit at $z \sim 0.1$ m, worst fit at $z \sim 0.3$ m

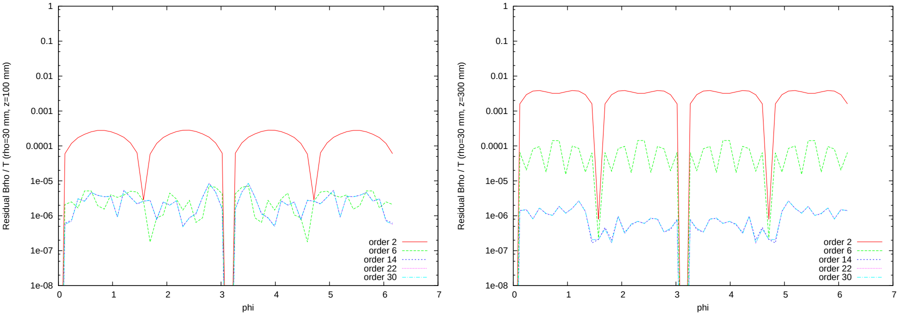
Radial Field Comparison ($\rho = 60$ mm , ϕ, z)





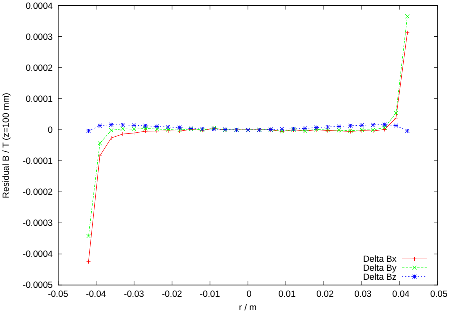
- Field residuals (analytic - spline) as a function of ϕ shown at $z = 0.1$ m (left) and $z = 0.3$ m (right)
- Analytic field is truncated at $m = 6, 10, 14, 22, 30$
- Residuals are significantly larger at $z = 0.3$ m (within fringe field region)

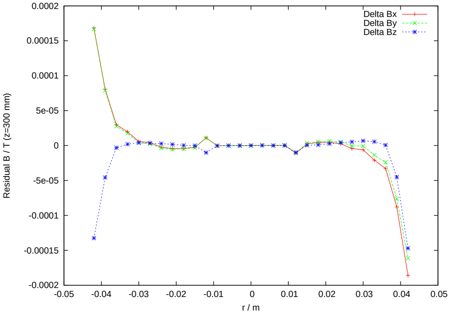
Radial Field Comparison ($\rho = 30$ mm , ϕ, z)



- Field residuals (analytic - spline) as a function of ϕ shown at $z = 0.1$ m (left) and $z = 0.3$ m (right)
- Analytic field is truncated at $m = 6, 10, 14, 22, 30$
- At $\rho = 30$ mm, higher order multipoles have less effect on residuals
- Still important within fringe region

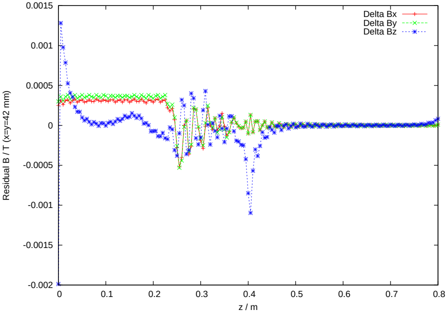
Field comparison: Analytic - Numeric Field Data

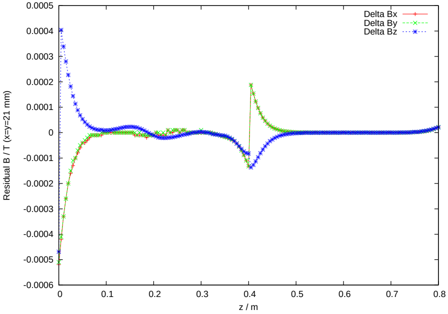




- Analytic field evaluated on the grid points
- Field residuals (analytic - numeric) as a function of ρ ($\phi = \pi/4$) shown at $z = 0.1$ m (left) and $z = 0.3$ m (right)
- Residuals increase exponentially at large ρ (modified Bessel function behaviour)

Field comparison: Analytic - Numeric Field Data



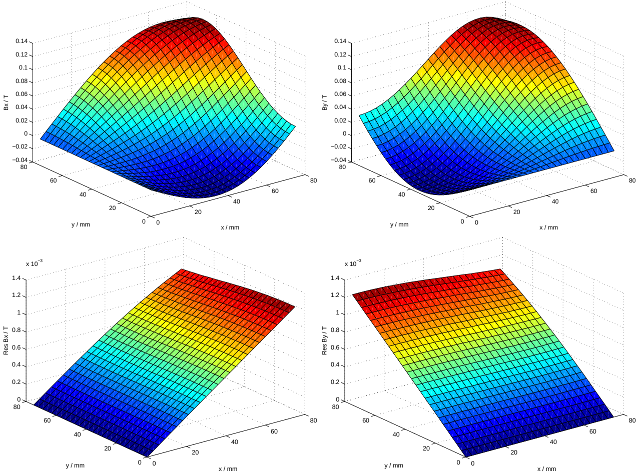


- Analytic field evaluated on the grid points
- Field residuals (analytic - numeric) as a function of z shown at $x = y = 42$ mm (left) and $x = y = 21$ mm (right)
- Generally good agreement: $< 5.10^{-4}$ at $\rho \sim 60$ mm, $< 2.10^{-5}$ at $\rho \sim 30$ mm
- Discontinuity at $z = 0.4$, $z = 0.4$ m)

Field comparison: Analytic - Numeric Field Data

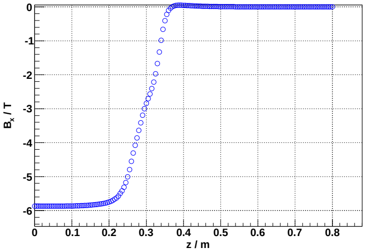
- Discontinuity at $z = 0$ m.
  - Analytic symmetry forces field gradient to zero at $z = 0$ (not quite zero in numeric data)
- Discontinuity at $z = 0.4$ m m where the two data maps were concatenated
  - This is where the two field data maps were concatenated
  - Possible mismatch in field values?

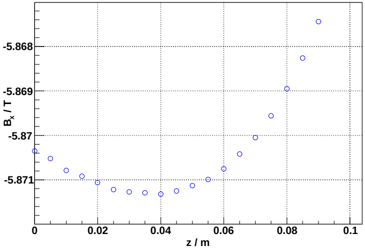
Numerical Field Map Comparison (z=700 mm)



Field values (map 1) B_x and B_y (top). Residuals, map1-map2, (bottom) show a mismatch between the two field maps of $\sim 10^{-3}$ T

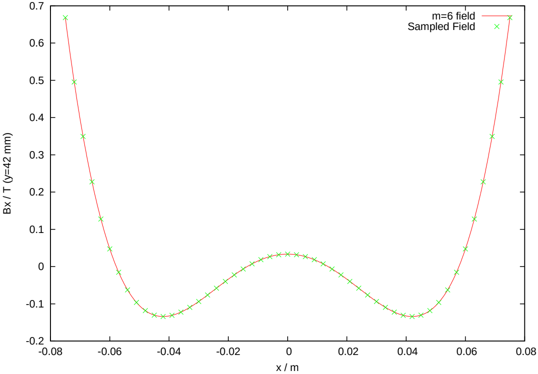
Numerical Field Map Comparison- Discontinuity at z=0





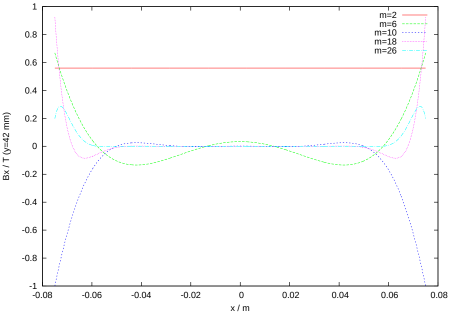
- Numerical field data (B_x) exiting the magnet (top)
- Field increases in strength as it exits the magnet (bottom)
  - Is this real?

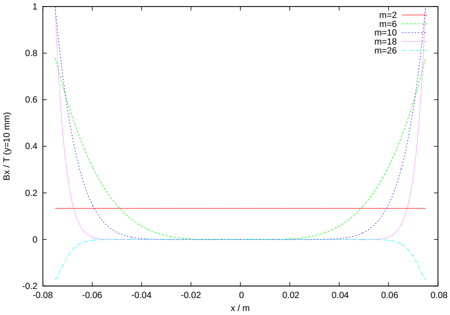
Accuracy of spline interpolation



- If we model an ideal multipole field (B_x, $m = 6$ shown here) and sample it in 3 mm step at some position (x, y=42 mm)
- If the field between points is well described by a cubic equation, a cubic spline should do a reasonable job of interpolation

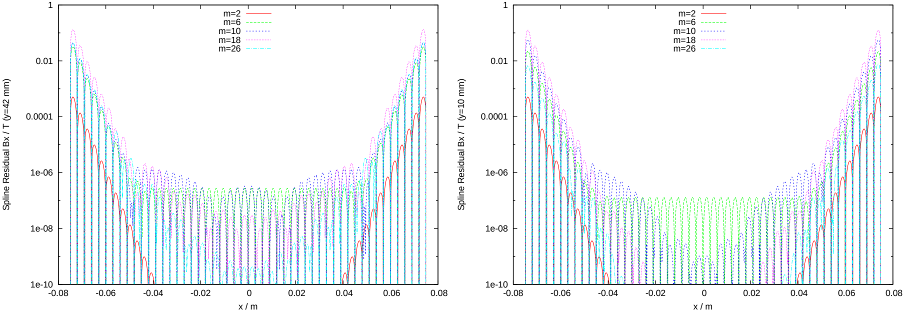
Accuracy of spline interpolation





- Model ideal multipoles field (B_x, $m = 2, 6, 10, 18, 26$ shown here) and sample it in 3 mm step at some position (x, y=42 mm) [left] and (x, y=10 mm) [right]
- Fit splines to the sampled data and inspect the residuals

Accuracy of spline interpolation



- Spline does a reasonable job interpolating between interior points
- Large residuals close to the edge - gradient at end points is set to zero to solve the spline equations
- Could adjust end gradient to give more realistic value in calculating splines
- Shouldn't trust interpolation close to the edge of the numeric grid

Transfer Map Calculations

- Analytic descriptions of the field have been calculated
  - Only up to 6^{th} order in the dynamical variables
  - Used to set up a framework to analyse the maps
  - Work under way to extend the code to handle arbitrary order maps
- Wu-Forest-Robin symplectic integrator used to generate transfer maps (truncated Taylor maps) - see previous talk by B. Dalena
- Exit and entrance fringe field maps have been generated
- Main body map generated by using the analytic field at start of the fringe field region using only C_m^0 terms
  - $C_m^{n>0}$ describe the z-dependence of the field

Transfer Map Calculations

- DA code used to create analytic vector field to 6^{th} order
  - Coulomb gauge ($\nabla \cdot A = 0$)
  - Horizontal free coulomb gauge ($A_x = 0.0$)
- Also require expressions for $\frac{\partial A_z}{\partial x}$, $\frac{\partial A_z}{\partial y}$ and $\int \frac{\partial A_y}{\partial x} dy$ for Wu-Forest-Robin integrator
- Numerical check: $\nabla \cdot A = 0$ (to order 6), $\nabla \cdot B = 0$ (to order 5)

Div A should be zero to order 6

Div A:

n	Coefficient	Order	Exponents
0	8.0779356695e-28	4	40
1	-3.2311742678e-27	4	22
2	-1.6155871339e-27	4	04
3	-4.0389678347e-28	5	50
4	-3.2311742678e-27	5	14
5	-1.6543612251e-24	6	60
6	-5.2939559203e-23	6	24
7	1.6543612251e-24	6	06
8	-7.1240418897e-15	7	70
9	-3.2167892892e-14	7	52
10	5.1608504503e-13	7	34
11	-2.2761484090e-13	7	16
12	-2.5635124472e-07	8	80
13	3.4135614697e-07	8	62
14	2.5648631644e-06	8	44
15	3.0759306141e-06	8	26
16	-4.5167799238e-07	8	08

Div B:

n	Coefficient	Order	Exponents
0	4.2210801765e-20	1	01
1	6.0220468580e-18	2	11
2	-1.6706271756e-16	3	21
3	-4.5459934382e-17	3	03
4	-8.0873499215e-13	4	31
5	-8.0435580238e-13	4	13
6	-1.8114706174e-14	5	41
7	-3.7421463318e-14	5	23
8	8.8598877889e-15	5	05
9	2.6469779602e-23	6	51

Transfer Map Calculations

- A map F_i $(i = 1 \dots 6)$ is symplectic if $M^T S M = S$
- M is the Jacobian Matrix $M_{ij} = \partial_{x_j} F_i$ where S is the skew-symmetric matrix

$$S = \begin{pmatrix} S_2 & & 0 \\ & S_2 & \\ 0 & & S_2 \end{pmatrix}, \quad S_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

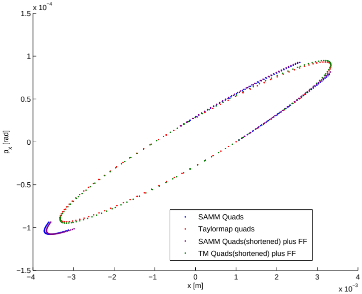
- If M is symplectic, then $M^T S M - S = 0$
- For the quad entrance Taylor map (evaluated at $x = y = 60$ mm):

```
MT S M - S
      -2.30212e-20   1.0299e-12    1.52632e-15   -4.53763e-16   0    1.13745e-18
      -1.0299e-12    -1.4429e-21   9.9068e-16    -3.92293e-16   0    -7.88986e-17
      -1.52631e-15   -9.9068e-16   -1.44559e-21  -9.52876e-12   0    -1.6863e-19
      4.53753e-16    3.9229e-16    9.52876e-12   8.95095e-24    0    -5.53786e-16
      0              0             0             0              0    0
      -1.13745e-18   7.88985e-17   1.68629e-19   5.53786e-16    0    0
```

- Determinant of M: $|M| - 1.0 \sim 10^{-12}$

Summary - Transfer Map Calculation

- ITQ transfer maps have been implemented into an in-house tracking code (SAMM) and work is under way to develop routines to match tunes by adjusting quadrupole component, match lengths, FMA analysis etc, for comparison to hard edge model (Sam Jones)
- HL-LHC Lattice is implemented in SAMM - can import transfer maps when we're satisfied with them



Summary - Transfer Map Calculation

- Sixtrack is being investigated with an eye on future implementation of ITQ maps - so far just run using standard optics files (Emilia Cruz-Alanis)
- Further work:
  - Implement arbitrary order DA code to calculate maps
  - Develop symplectic maps based on generating functions (A. Wolski et.al.)
  - Investigate the best way to implement the map into sixtrack

Summary - Field Descriptions

- Work under way to optimise the analytical descriptions
- Investigation of effect of numerical errors in raw data
  - Optimise spline interpolation and Fourier transform (spline Fourier Transform)
  - Are the high m-pole component real?
  - Generate 'ideal ITQ-like' field with known m-pole amplitudes + Enge function and assess reconstruction
  - Add random errors to the surface data and assess reconstruction (break the pure quadrupole symmetry)
- It would help if we had an idea of the accuracy of the field data (any ideas?)
  - Mismatch between the two maps - is this representative? (error $\sim 10^{-3}$)