5/2/15

CERN Winter School

Lecture 4

1) Towards the end, let me describe a few objections to this proposal.

The first is the "Frozen vacuum" objection. (Bousso)

This is the simple point that we clearly should not have

$$N_a |\psi\rangle = 0$$

For all states since some states are excited.

2) So we have to be careful to apply our construction only in "equilibrium states".

One necessary, and almost sufficient, feature of equilibrium states is that correlators should be time-translationally invariant

$$\langle\psi| e^{iHT} A_\alpha e^{-iHT} |\psi\rangle - \langle\psi|A_\alpha|\psi\rangle \approx e^{-S/2}.$$

We can see this through the eigenstate thermalization hypothes

$$\langle E_i | A_\alpha | E_j \rangle = A(E_i)\delta_{ij} + e^{-S/2} R_{ij}$$

where R_{ij} has "erratic phases"

3) Now for a generic state drawn from a range of energies with e^S states

$$|\psi\rangle = c_i |E_i\rangle \quad ; \quad |\Sigma|c_i|^2 = 1$$

we find

$$\langle \psi | A_\alpha | \psi \rangle = \sum_i |c_i|^2 A_i + \sum_{i,j} e^{-S/2} c_i c_j^* R_{ij}$$

Consider the second term. It involves a sum over e^{2S} terms. Each term is of typical size $e^{-3S/2}$ but the terms contribute incoherently. So the typical off-diagonal term is $e^{-S/2}$.

But this is the only term that leads to time-dependence. So this gives the advertised time-independence for generic states.

4) Given an equilibrium state $|\psi\rangle$, and

$$|\psi'\rangle = U|\psi\rangle$$

where $U = e^{iA_\alpha}$.

then $|\psi'\rangle$ is not equilibrium. We call it near equilibrium.

By measuring correlators of A_α, we can detect U.

5) For example take.

$$U = e^{i\lambda(O_\omega + O_\omega^\dagger)}.$$

Then

$$\langle\psi'|\,(O_\omega - O_\omega^\dagger)\,|\psi'\rangle.$$

$$= \langle\psi|\, e^{-i\lambda(O_\omega + O_\omega^\dagger)}\,(O_\omega - O_\omega^\dagger)\, e^{i\lambda(O_\omega + O_\omega^\dagger)}\,|\psi\rangle.$$

$$\approx 2i\lambda.$$

This correlator would be $e^{-S/2}$ in equilibrium state. because it clearly evolves under time evolution.

Using this, we can undo the unitary.

So, if you just give me the state $|\psi'\rangle$, then I can detect it is out of eq. and then construct the tildes as:

$$\tilde{O}_\omega\,|\psi'\rangle = U\, e^{-\beta\omega/2}\, O_\omega^\dagger\, U^\dagger\,|\psi'\rangle.$$

We see this just gives the correct tildes on the base equilibrium state.

So, while

$$N_a\,|\psi'\rangle \neq 0. \quad \text{[as expected]}$$

the tildes still obey all EFT predictions

6) van Raamsdonk and Harlow have described some other more tricky potential ambiguities.

Given an eq. state $|\psi\rangle$, construct the tildes and then consider

$$|\psi'\rangle = e^{i\tilde{A}_\alpha}|\psi\rangle$$

Note the ~ on A

This is like a beam coming from behind the horizon lurking to hit the infalling observer.

However, we can fix this ambiguity by using correlators involving H, which does not commute with $\tilde{O}$

Exercise

Consider $|\psi'\rangle = e^{i(\tilde{O}_\omega + \tilde{O}_\omega^+)}|\psi\rangle$.

It can be detected through

$$\langle\psi'| H (O_\omega - O_\omega^+)|\psi'\rangle$$

which would be exponentially small in a bona-fide eq. state but is not here

7) Absolute Locality Objection (Mathur)

Is there some independent evidence for breakdown of locality over large distances in N-pt correlators?

Yes!

First, we should examine why locality holds at all in Q.G.. The path integral

$$Z = \int e^{-S} dg$$

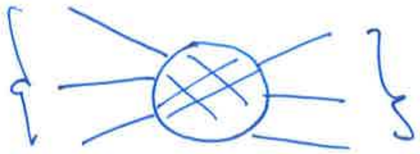
integrates over all metrics. But sometimes we can do a series expansion about a particular metric, and then we can speak of locality w.r.t. that metric.

In AdS/CFT, this asymptotic series expansion is controlled by $\frac{1}{N}$.

So locality is tied to the $\frac{1}{N}$ expansion.

If there is no $\frac{1}{N}$ expansion, for some quantity, then it is difficult to say this is local.

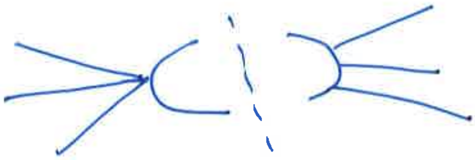
8) Consider a N-pt scattering amplitude in the bulk.



where $p \sim N$.

The number of Feynman diagrams scales like $p!$ at tree-level. $M_{tree} = g^{p-2}\, p!$

Now, consider a 1-loop amplitude, which we can cut



total number of diagrams is

$$\sum_m (m+2)!\,(p-m+2)!\,\binom{p}{m}\, g^p$$

ways of selecting m particles.

$$= \sum_m \frac{(m+2)!\,(p-m+2)!}{m!\,(p-m)!} \cdot p!\; g^p$$

$$\gtrsim p^t\, p!\, g^p$$

The one-loop to tree ratio is

$$\frac{M_{1\ell}}{M_{tree}} = g^2 p^t$$

if $g \sim \frac{1}{N}$ and $p \sim N$, then $M_{1\ell} \sim M_{tree}$

Breakdown of $\frac{1}{N}$!

a) General Remarks about State-Dependence

State-dependent operators are designed to make quantum effective field theory work. So absolutely no observable violation of Q.M. But requires one to think carefully about what can be observed, and when ~~an~~ observable may depend in a subtle way on the state that is being used.

Very interesting topic for further work.