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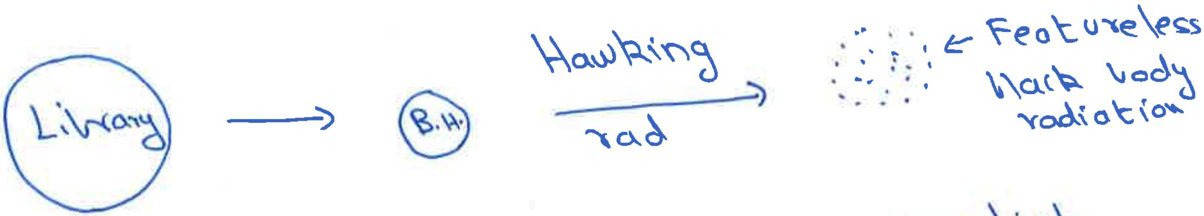
Information Paradox in AdS/CFT

Lectures at CERN Winter School.

1) Introduction :-

The Info paradox is an old problem in quantum gravity, and is usually phrased as follows.

Consider a black hole formed from the collapse of matter



The classical no-hair theorems predict that very soon the B.H. becomes feature-less. Then Hawking's computation suggests that the B.H. starts to radiate via Hawking radiation. and turns into feature-less black body radiation.

But this is in conflict with unitarity of Q.M.

2) At its heart, the information paradox is a conflict between

effective field theory vs Q.M.

The usual answer to the old information paradox was that effective field theory is only effective. It is not precise enough to really set up a contradiction. For example, information about the initial matter distribution may be stored in exponentially small correlations between different Hawking quanta.

3) Over the past few years, there have been attempts starting with Mathur, Almheiri, Marolf, Polchinski, Sully, Stanford to refine the contradiction between EFT and Q.M.

4) In these lectures, we will explore these paradoxes, and our resolution to them. The central object in E.F.T. is the local perturbative field. So, a lot of our attention will be focussed on reconstructing this object in AdS/CFT

5) An overview of how I plan to proceed is as follows.

<u>Lecture</u> 1: Bulk local fields about an AdS black hole, two point functions and commutators, construction of operators outside the horizon.

<u>Lecture</u> 2: Operators inside the horizon, the $N_a \neq 0$ argument, the counting argument, the strong subadditivity the commutor and

<u>Lecture</u> 3: Construction of operators inside the horizon, resolving all AMPS paradoxes, near-equilibrium states, resolving various ambiguities.

<u>Lecture</u> 4: State dependence in the eternal black hole, other examples of state-dep, possible semi-classical origins.

6) Let us consider the AdS black brane metric given by

$$ds^2 = \frac{\ell^2}{z^2}\left[-h(z)\,dt^2 + \frac{dz^2}{h(z)} + d\vec{x}^2\right]$$

$$h(z) = 1 - \frac{z^d}{z_0^d}$$

here the boundary is at $z=0$. The horizon is at $z = z_0$. We are in AdS_{d+1}, so the flat directions are $d-1$ in number. ℓ is the AdS radius. The temperature is

$$\beta = \frac{4\pi z_0}{d}.$$

It will turn out to be very useful for us to study the near horizon region. Introduce tortoise coordinates

$$\frac{dz^2}{h(z)} = h(z)\, dz_*^2.$$

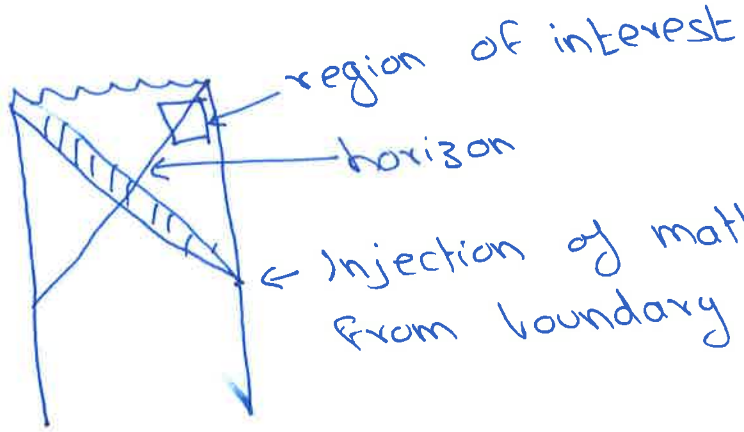
so

$$\frac{dz_*}{dz} = \frac{-1}{h(z)}.$$

with the condition $z_* = 0$ as $z = 0$

[The general solution for z_* involves hypergeometrics and is not relevant here.]

7) It is useful to keep in mind a physical picture of the situation we are considering



we are interested in the late time geometry after details of the initial shock have gone away.

8) To describe the region behind the horizon we need some new coordinates. Note that

$$h(z) \xrightarrow[z \to z_0^-]{} \frac{d(z_0 - z)}{z_0}$$

So,

$$z_* = \frac{z_0}{d} \ln(z_0 - z) + \text{const.}$$

and

$$h \to \alpha \frac{d}{z_0} e^{\frac{d z_*}{z_0}} = \alpha \frac{d}{z_0} e^{(4\pi/\beta) z_*}$$

where α is a constant set by the precise relation betwee z_* and z that we are uninterested in.

a) Now change to coordinates

$$U = r e^{2\pi/\beta\,(z_* - t)} \; ; \quad V = r e^{2\pi/\beta\,(z_* + t)}$$

in these coordinates, the metric is

$$ds^2 = \frac{h(z)}{z^2}\left(-dt^2 + dz_*^2\right) + \frac{dx_i^2}{z^2}.$$

$$= \frac{h(z)}{z^2}\,\frac{dU\,dV}{UV}\left(\frac{\beta}{2\pi}\right)^2 + \frac{dx_i^2}{z^2}$$

$$= \frac{dU dV}{z_0^2} + \frac{dx_i^2}{z_0^2}$$

with an appropriate choice of the constant r.

The horizon is at $U = 0$, but V finite.

In region II, we set

$$U = -\,r e^{2\pi/\beta\,(z_* - t)}$$

$$V = r e^{2\pi/\beta\,(z_* + t)}$$

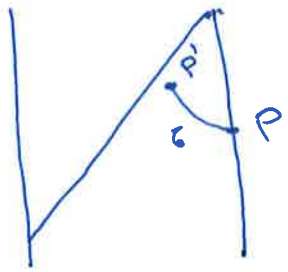
to introduce a second Schwarzschild patch.

10) Now, I turn from a consideration of the geometry to fields propagating on it.

We want to construct a scalar field $\phi(x)$.

The first issue to understand is: what does "x" mean? The moment we include gravity this becomes important because there are no local gauge invariant operators. Under a diff: $\phi \to \phi - \varepsilon^{\mu} \partial_{\mu} \phi \quad (x^{\mu} \to x^{\mu} + \varepsilon^{\mu})$

So we define points relationally using the boundary



Start from a point P on the boundary and jump in a particular direction for a proper time τ.

We will discuss this in detail in the last lecture, but use it briefly now, so the intuitive picture is useful to keep in mind.

These observables transform only under "large gauge transformations", which is fine.

11) Now what do we want from this object $\Phi(x)$.

a) If it is perturbative, it satisfies

$$\mathcal{B}\Phi = 0$$

b) If it is perturbative, its correlators factorize

$$\langle \Phi(x_1) \dots \Phi(x_{2n}) \rangle = \langle \Phi(x_1)\Phi(x_2)\rangle \dots \langle \Phi(x_{2n-1})\Phi(x_{2n})\rangle + \frac{1}{N} .$$

This is because there is no hierarchy in AdS/CFT and all interactions are controlled by $\frac{l_{pl}}{l_{ads}} \sim \frac{1}{N}$.

12) We see that the important object is the two point function. We can impose some additional constraints on this guy:

$$[\Phi(t,x), \dot{\Phi}(t,x')] = \frac{i}{\sqrt{-g}} \delta(x-x').$$

Second, its "short distance" behaviour is fixed

$$\lim_{|x_1-x_2| \to 0} \langle \Phi(x_1) \Phi(x_2) \rangle = \frac{1}{|g_{\mu\nu}(x_1^\mu - x_2^\mu)(x_1^\nu - x_2^\nu)|^{(d-1)/2}} .$$

These are very simple requirements, but turn out to play a very important role. We will analyze them further in the bulk and then turn to the boundary.

11 c) Since this is AdS/CFT, we must have

$$\lim_{z \to 0} z^{n\Delta} \langle \Phi(x_1) \dots \Phi(x_n) \rangle = \langle O(\tilde{x}_1) \dots O(\tilde{x}_n) \rangle$$

where Φ is dual to the op. O and $x_i = (\tilde{x}_i, z)$.

12) It turns out to be very useful to consider the near-horizon region.

The wave equation is

$$\frac{1}{\sqrt{-g}}\partial_\mu \sqrt{-g}\, g^{\mu\nu}\partial_\nu \phi = 0$$

In the t, z_* coordinates

$$\sqrt{-g} = h(z).$$

But $g^{tt} = g^{z_* z_*} = \frac{1}{h(z)}$

So we get

$$\frac{1}{h(z)}\left[\partial_t^2 \phi - \partial_{z_*}^2 \phi\right] - \partial_{x_i}^2 \phi = 0$$

If we consider a soln $e^{i\omega t} e^{i\vec{k}\cdot\vec{x}_i} f(z)$ we see that the $\vec{k}$ term becomes irrelevant near the horizon. So our solutions are just

$$e^{i\vec{k}_i\cdot\vec{x}_i}\, e^{i\omega(t-z_*)}, \quad e^{i\omega(t+z_*)}\, e^{i\vec{k}\cdot\vec{x}}$$

The dispersion relation in the near horizon region does not involve k!

13) Normalizability at $z=0$ tells us we must consider pairs:

$$e^{i\vec{k}\cdot\vec{x}}\left(e^{i\delta_\omega} e^{i\omega(t-z_*)} + e^{+i\omega(t+z_*)} e^{-i\delta_\omega}\right)$$

14) So, in the near horizon region, we expand the field as

$$\Phi = \int \frac{d^{d-1}k}{(2\pi)^{d/2}} \frac{d\omega}{\sqrt{2\omega}}\, \hat{a}_{\omega,k}\, e^{-i\omega t} e^{i\vec{k}\cdot\vec{x}} \left(e^{i\delta} e^{+i\omega z_*} + e^{-i\delta} e^{-i\omega z_*} \right) + h.c.$$

15) From the canonical commutation relations we immediately see

$$[\hat{a}_{\omega,k}, \hat{a}^{\dagger}_{\omega',k'}] = \delta(\omega-\omega') \cdot \delta^{d-1}(k-k').$$

It will be very convenient for us to consider slightly smeared operators

$$b_{\omega,k} = \int a_{\omega,k} f_{\omega,k}\, d\omega\, dk$$

so that

$$[b, b^{\dagger}] = 1.$$

where we can take f to be strongly peaked about some ω_0, k_0.

16) Now lets consider the near horizon region <u>behind</u> the horizon. Once again locally, we find two independent solutions and we can write

$$\Phi = \int \frac{d^{d-1}k}{(2\pi)^{d/2}} \frac{d\omega}{\sqrt{2\omega}} \Big[a_{\omega,k}\, e^{-i\delta} e^{-i\omega(t+z_*)+ik\cdot\vec{x}} + \tilde{a}_{\omega,k}\, e^{-i\omega(z_*-t)-i\vec{k}\cdot\vec{x}} \cdot e^{+i\delta} + h.c. \Big].$$

Notice - sign on x as well in $\tilde{a}$ term

17) We immediately notice a peculiarity. Behind the horizon, it is z_* that plays the role of a time coordinate. So the destruction operator $\tilde{a}_{\omega,k}$ multiplies $e^{i\omega t}$.

So, more specifically

$$[H, a_{\omega,k}] = -\omega\, a_{\omega,k}$$

but

$$[H, \tilde{a}_{\omega,k}] = \omega\, \tilde{a}_{\omega,k}.$$

[Here, the alert reader may object that H translates boundary time, and not Schwarzschild time behind the horizon. But in the relational definition, we can check that translating the boundary time also translates the Schwarzschild time.]

18) In U, V coordinates, in front of the horizon we have

$$\phi = \sum_{\omega,k} a_{\omega,k}\left(U^{+i\beta\omega/2\pi} e^{i\delta} + e^{-i\delta} V^{-i\beta\omega/2\pi}\right)$$

behind the horizon we have

$$\phi = \sum_{\omega,k} \left(a' e^{-i\delta} V^{-i\beta\omega/2\pi} + \tilde{a}\, U^{-i\beta\omega/2\pi} e^{-i\delta}\right).$$

Since the horizon extends for a range of V, by continuity we immediately see that $a' = a$.

1a) Using the equal time commutation relations, at equal z^*, we now find

$$[\tilde{a}_\omega, \tilde{a}_{\omega'}] = \delta(\omega-\omega')$$
$$[\tilde{a}_\omega, a_\omega] = 0$$

[Technically we may get a non-zero commutator between $\tilde{a}_\omega$ and a_ω^+ but we will ignore that possibility for now]

The reason is, if we set

$$[\tilde{a}_\omega \; a_\omega^+] = G$$

then under time-translations

$$e^{iHT}[\tilde{a}_\omega, a_\omega^+]e^{-iHT} = G e^{2i\omega T}$$

Therefor G cannot be a constant and so we will set it to 0.

2a) We have already derived a lot of properties. The final ones we want to derive are

$$\langle a_\omega a^+_{\omega'} \rangle = \frac{\delta(\omega-\omega')}{1-e^{-\beta\omega}}.$$

$$\langle a_\omega \tilde{a}_{\omega'} \rangle = \frac{\delta(\omega-\omega')}{1-e^{-\beta\omega}} e^{-\beta\omega/2}.$$

$$\langle a^+_{\omega'} a_\omega \rangle = \frac{\delta(\omega-\omega') e^{-\beta\omega}}{1-e^{-\beta\omega}}$$

$$\langle \tilde{a}_\omega \tilde{a}^+_{\omega'} \rangle = \frac{\delta(\omega-\omega')}{1-e^{-\beta\omega}}.$$

$$\langle \tilde{a}^+_\omega \tilde{a}_\omega \rangle = \frac{\delta(\omega-\omega') e^{-\beta\omega}}{1-e^{-\beta\omega}}$$

21) To derive this, we consider the short distance behaviour of the two-point function.

Near the horizon where the metric is flat, we have

$$\langle \phi(U_1, V_1, x_1)\, \phi(U_2, V_2, x_2)\rangle = \frac{1}{\left(\frac{\delta U \delta V}{z_0^2} + \frac{\delta x^2}{z_0^2}\right)^{(d-1)/2}}$$

Let us differentiate w.r.t. V_1, V_2 and take the limit $V_1 = V_2 \to 0$ [The reason will become clear shortly]

$$\lim_{\delta V \to 0} \langle \partial_U \phi(U_1, V_1, x_1)\, \partial_U \phi(U_2, V_2, x_2)\rangle$$

$$= \lim_{\delta V \to 0}\left[\left(1 - \frac{d^2}{4}\right)\frac{1}{z_0^4} \frac{\delta V^2}{\left(\frac{\delta U \delta V}{z_0^2} + \frac{\delta x^2}{z_0^2}\right)^{(d-3)/2}}\right]$$

to analyze the limit consider the integral over $d^{d-1}x$. Changing variables to

$$\alpha = \frac{\delta x}{\sqrt{\delta U \delta V}}$$

we find

$$I \propto \int d^{d-1}\alpha \, \frac{(\delta U \delta V)^{(d-1)/2}\, \delta V^2}{(1 + \vec{\alpha}^2)\, (\delta U\, \delta V)^{(d+3)/2}}$$

$$= \frac{1}{(\delta U)^2} \int \frac{d^{d-1}\alpha}{(1 + \vec{\alpha}^2)}.$$

On the other hand, as we take $\delta V \to 0$ the integrand clearly vanishes for $\delta \vec{x} \neq 0$.

22) So we have concluded that

$$\lim_{v_1 - v_2 \to 0} \langle \partial_U \phi(U_1, V_1, x_1)\, \partial_U \phi(U_2, V_2, x_2) \rangle \propto \frac{1}{(\delta U)^2} \delta^{d-1}(\vec{\delta x})$$

This is what is sometimes called <u>ultralocality</u> in light cone physics.

23) Now, let us take the two points on opposite sides of the horizon, and try and recover this expansion. We focus on terms involving only U.

$$\langle \phi(U_1, V_1, x_1)\, \phi(U_2, V_2, x_2) \rangle$$

$$= \int \frac{d^{d-1}k\, d\omega}{(2\pi)^d (2\omega)} \langle a_{\omega, k}\, \tilde{a}_{\omega', k'} \rangle\, U_1^{i\beta\omega/2\pi}\, U_2^{-i\beta\omega'/2\pi}\, e^{ik\cdot x_1} e^{-ik'\cdot x_2} + h.c. + \ldots$$

Now, we use the claimed 2-pt function and immediately find we get the $\delta^{d-1}(x_1 - x_2)$ from the integral over k.

We can also use the h.c. to extend the integral over ω to $-\infty$ to ∞. Note now why the derivatives are important. Without the derivatives the integral has a singularity at $\omega = 0$

24) $\langle \partial_1 \phi_1 \, \partial_2 \phi_2 \rangle$

$$= \delta(x_1 - x_2) \int \left(+\frac{\beta}{2\pi}\right)^2 \omega \frac{d\omega}{4\pi} \frac{e^{-\beta\omega/2}}{1-e^{-\beta\omega}} \left(\frac{U_1}{U_2}\right)^{i\beta\omega/2\pi - 1} \cdot \frac{1}{U_2^2}$$

For $|U_2| > |U_1|$, we complete the contour in the ~~upper~~ lower half plane and pick up poles at

$\beta\omega = -2n\pi i$

We find

$$\langle \partial_1 \phi_1 \partial_2 \phi_2 \rangle \propto \delta(x_1 - x_2) \frac{1}{U_2^2} \sum_{n=0}^{\infty} n \left(\frac{U_1}{U_2}\right)^{n-1}$$

$$= \delta(x_1 - x_2) \frac{1}{U_2^2} \frac{1}{\left(1 - \frac{U_1}{U_2}\right)^2}$$

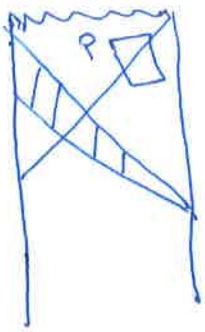
$$= \frac{\delta(x_1 - x_2)}{(U_2 - U_1)^2}$$

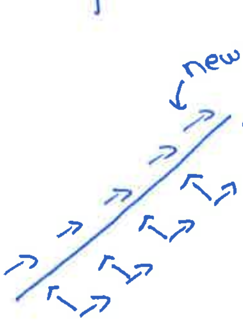
which is exactly what we need.

So we see that the claimed two point function betwee $\langle a \, \tilde{a} \rangle$ is necessary to get a smooth two point function across the horizon

25) Using a similar analysis, we can derive the other two point functions as well.

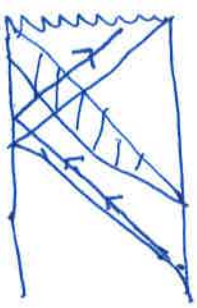
25) Let us recap what we have achieved.



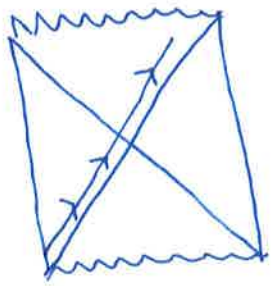


We have left and right movers outside. The left movers continue smoothly past the horizon. There we find new right movers, which look like they are coming from the surface of the star but are entangled with the right movers outside the horizon.

26) There are other ways to think about these right movers. Hawking originally thought of these as modes reflected from $r=0$



Hawking understanding of right movers



Eternal black hole understanding of right movers

26) contd We can also think of these modes as follows. At late times, the single sided black hole approaches the geometry of the eternal black hole. These modes can be understood as modes with initial data ~~on~~ the other boundary.

Both these derivations have some difficulties. In the Hawking computation, we have a trans-Planckian issue. In the EBH derivation, one may object that the other asymptotic region does not exist.

So here we have achieved a <u>local</u> and <u>unambiguous</u> derivation of the necessity of the new right movers and their entanglement with the right movers outside the horizon.

27) The last remark is that effectively the state of the B.H. is

$$|\psi\rangle = e^{\sum e^{-\beta\omega/2} a^{\dagger}_{\omega,k} \tilde{a}^{\dagger}_{\omega,-k}} |\Omega\rangle.$$