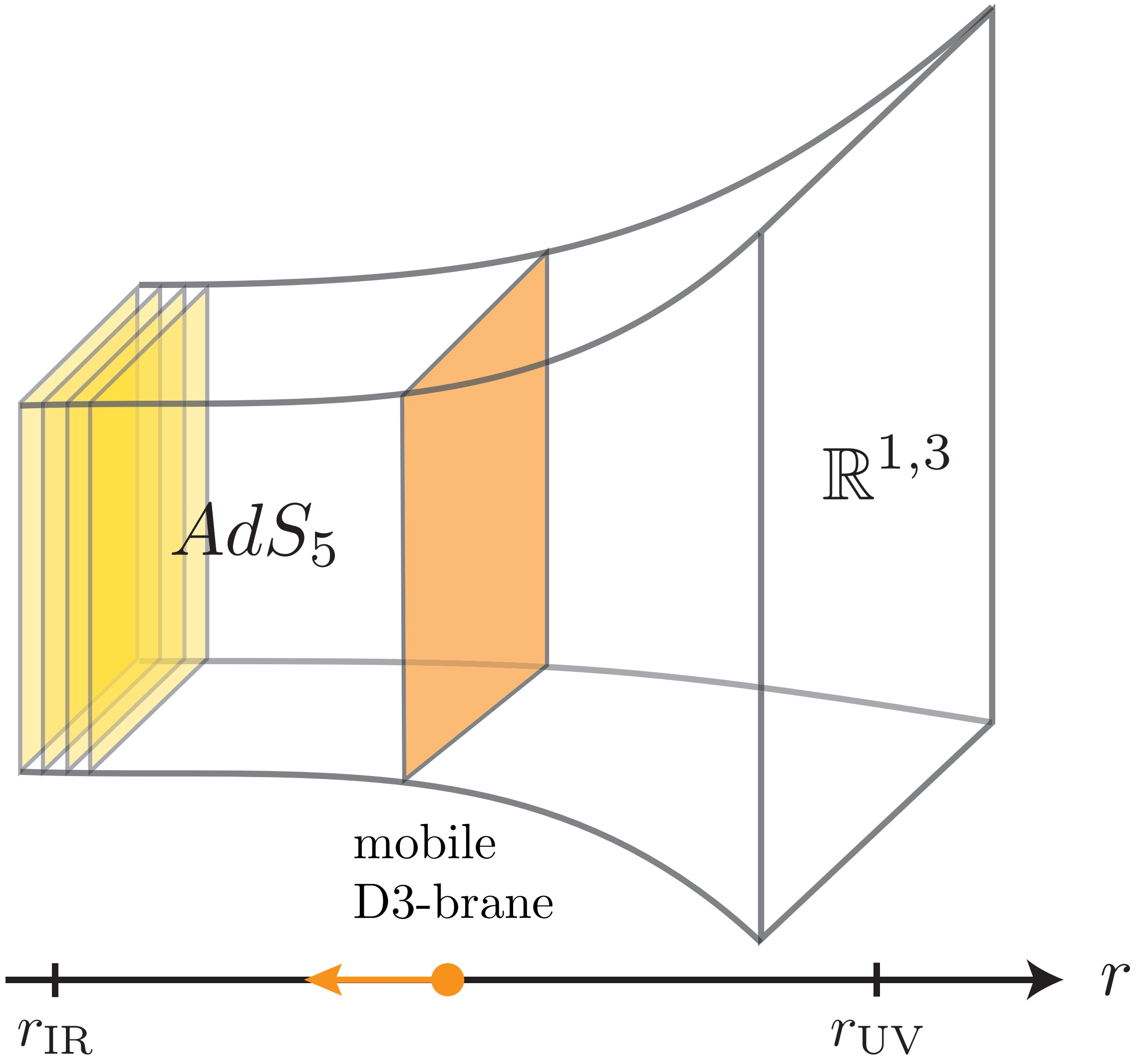
Inflation in String Theory



Outline

- String Inflation as an EFT
- Moduli Stabilization
- Examples of String Inflation
  - Inflating with Branes
  - Inflating with Axions
  - (Inflating with Volume Moduli)

Reference: DB and Liam McAllister, *Inflation and String Theory*

String Inflation as an Effective Theory

moduli

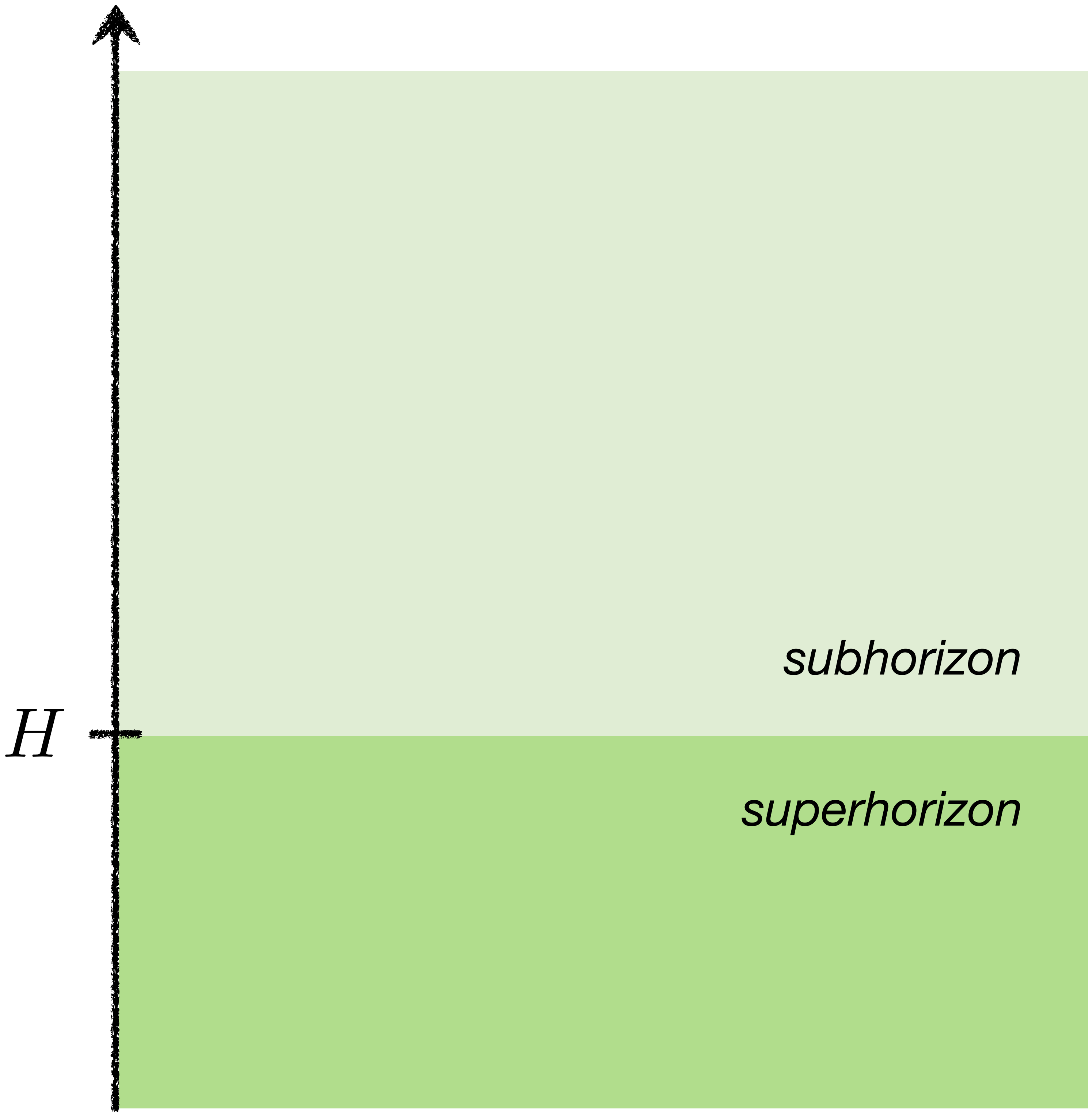
$$S_{10}[\mathcal{C}] \xrightarrow[\mathrm{M}_4 \times \mathrm{X}_6]{\textit{compactification}} S_4[\Phi(t)]$$

4D Effective Action

- topology, geometry
- fluxes
- local sources
  (D-branes, O-planes)

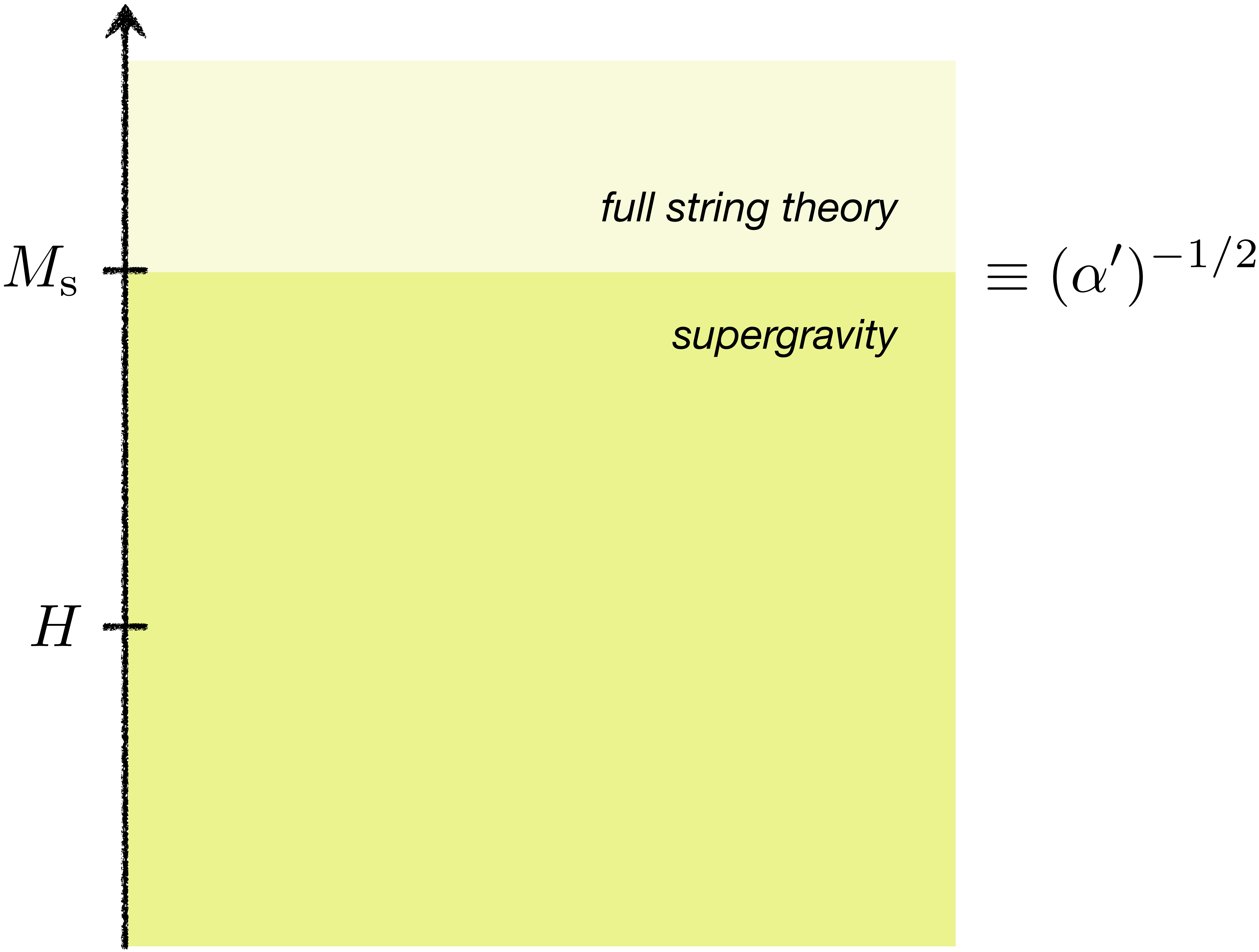
Inflation?

Energy Scales



scale of the experiment

Energy Scales



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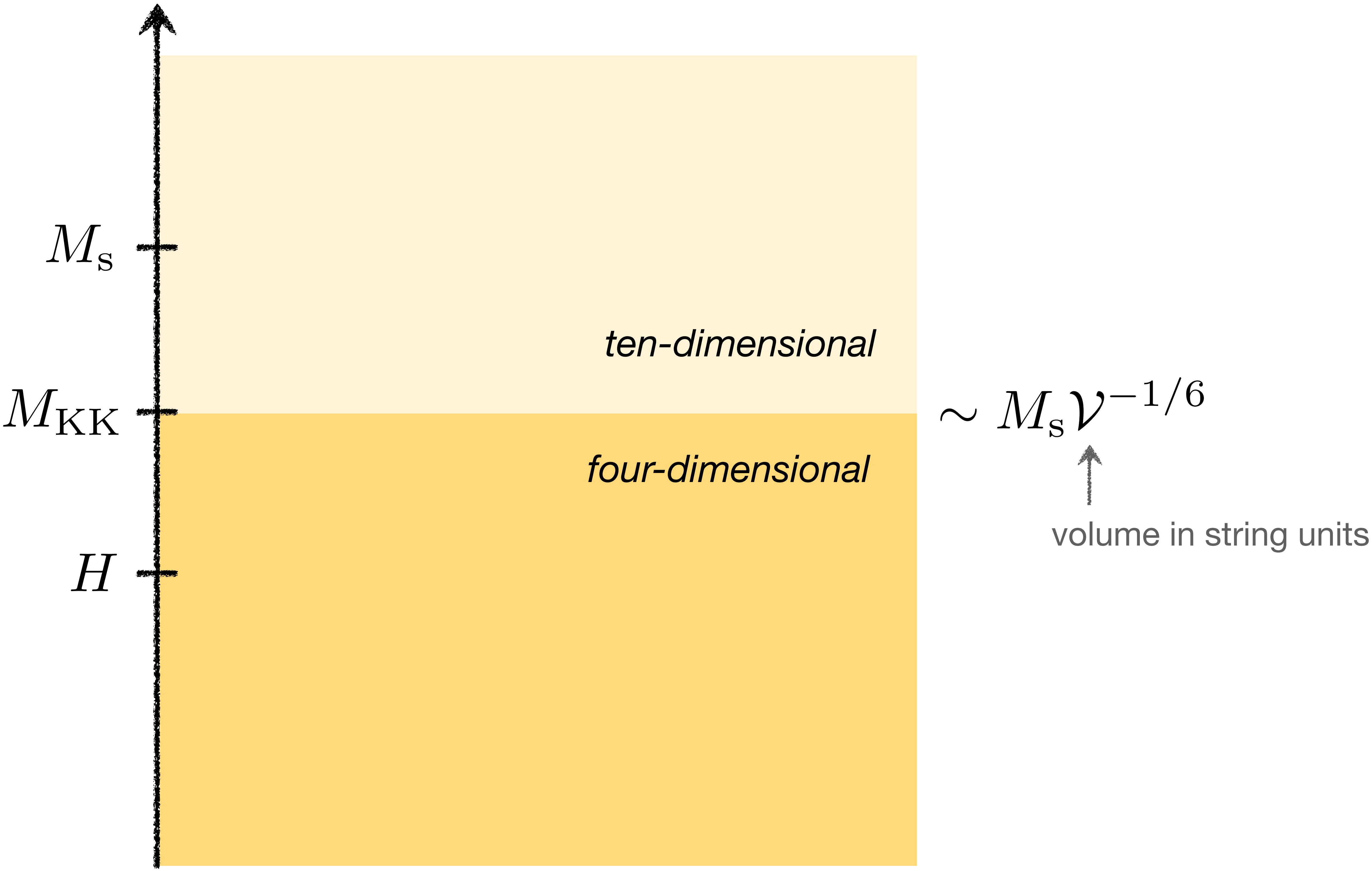

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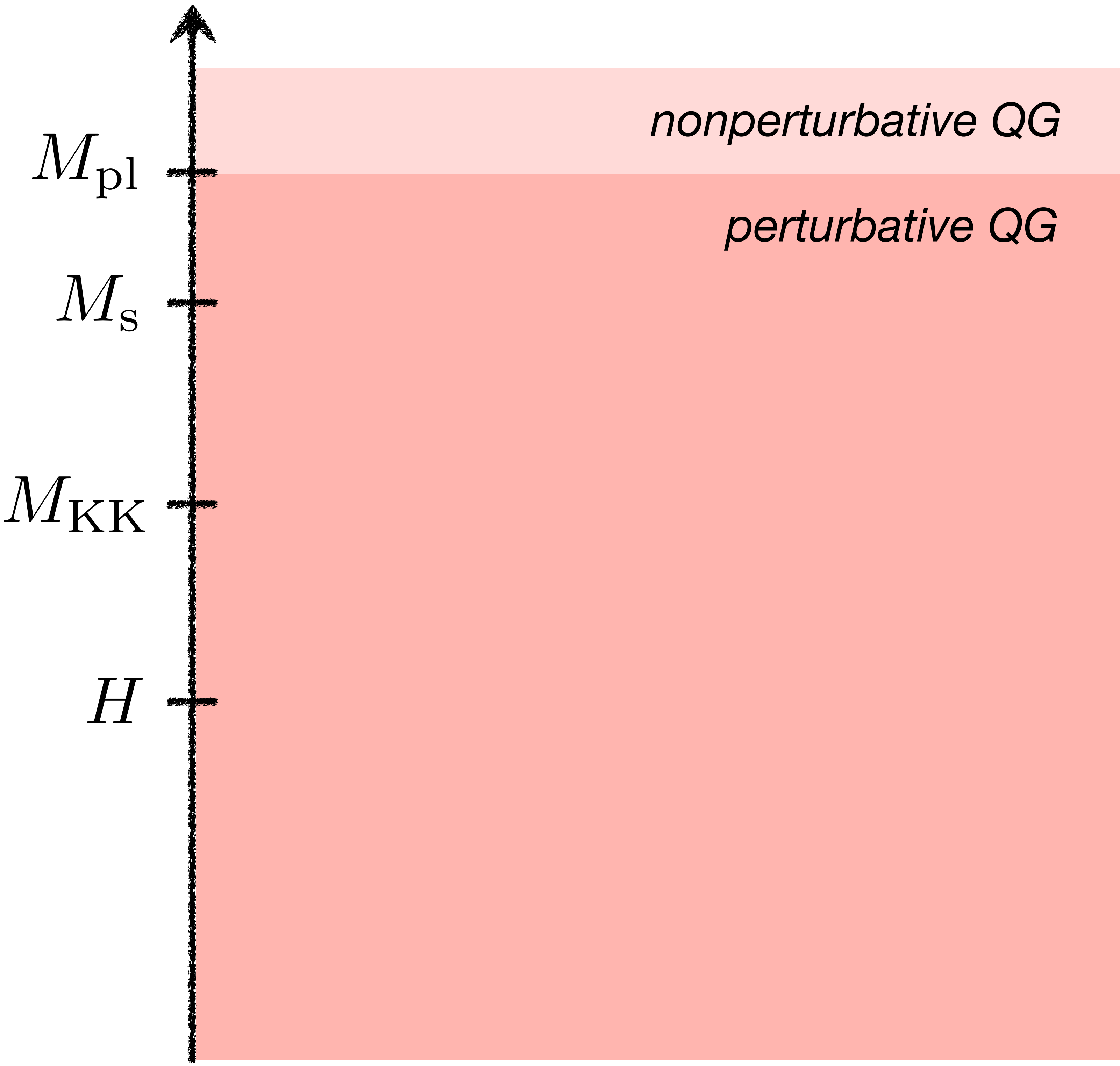
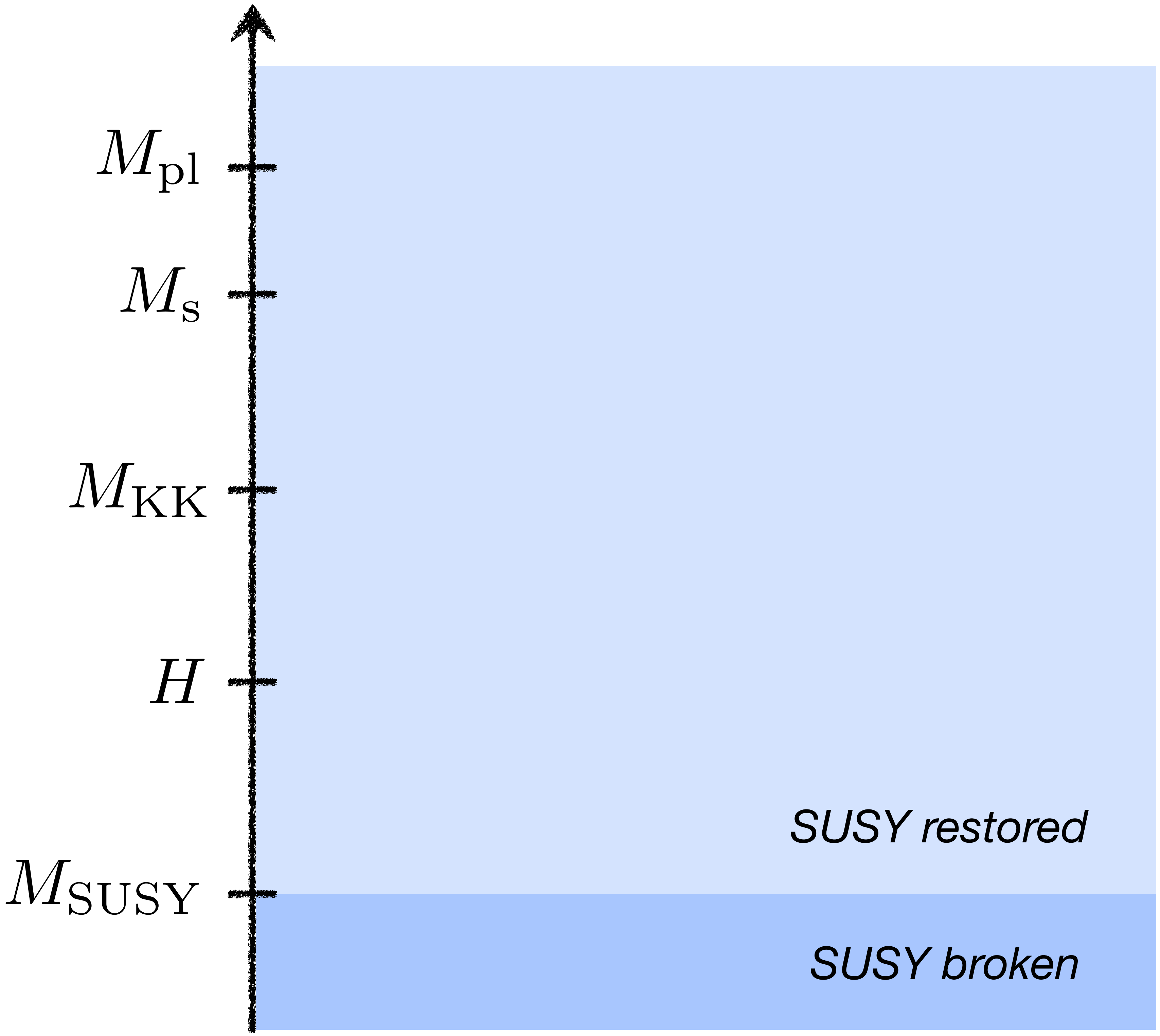


$$\sim \frac{M_{\mathrm{s}}}{g_{\mathrm{s}}} \left(\frac{M_{\mathrm{s}}}{M_{\mathrm{KK}}} \right)^3$$

Energy Scales



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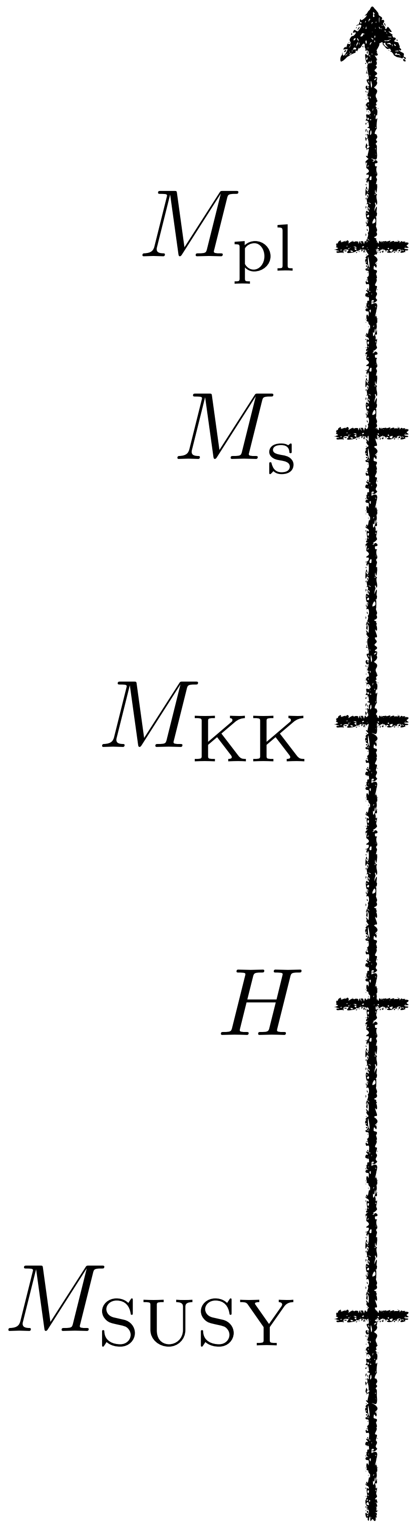


All controlled models of string inflation work with this hierarchy of scales.

Moduli

= zero energy deformations of X_6

= massless fields on M_4

- volume

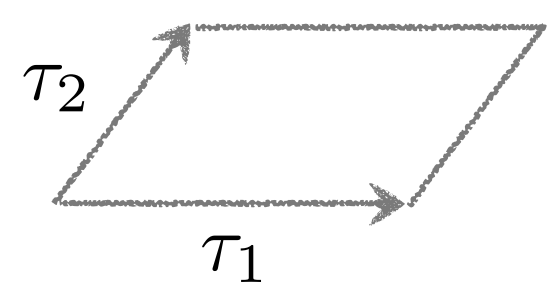
  vacuum Einstein equations are scale-invariant

- dilaton

  even a modulus in 10D

- metric moduli

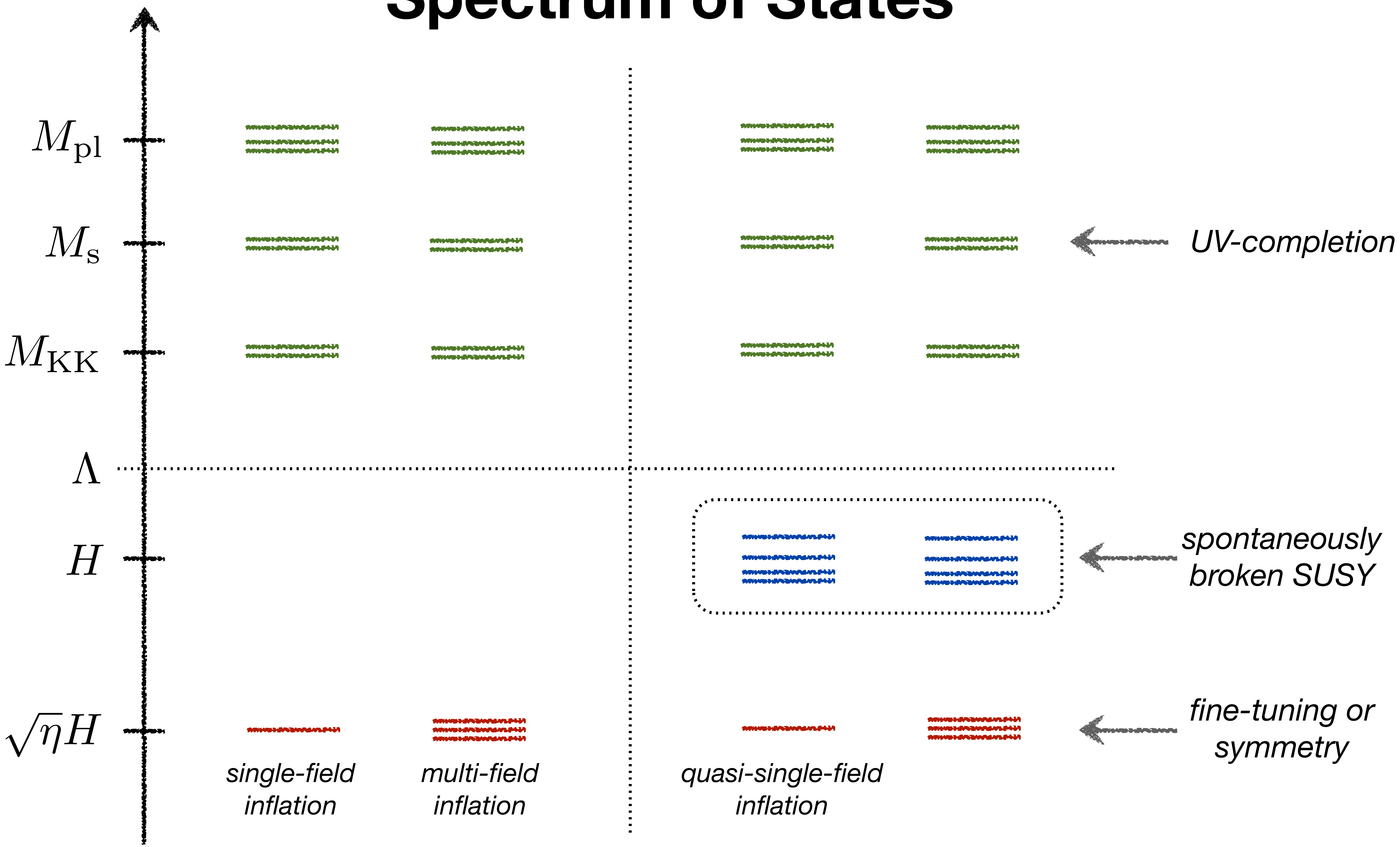
  - *Kähler moduli*
  - *complex structure moduli*



- axions

- brane moduli

Spectrum of States



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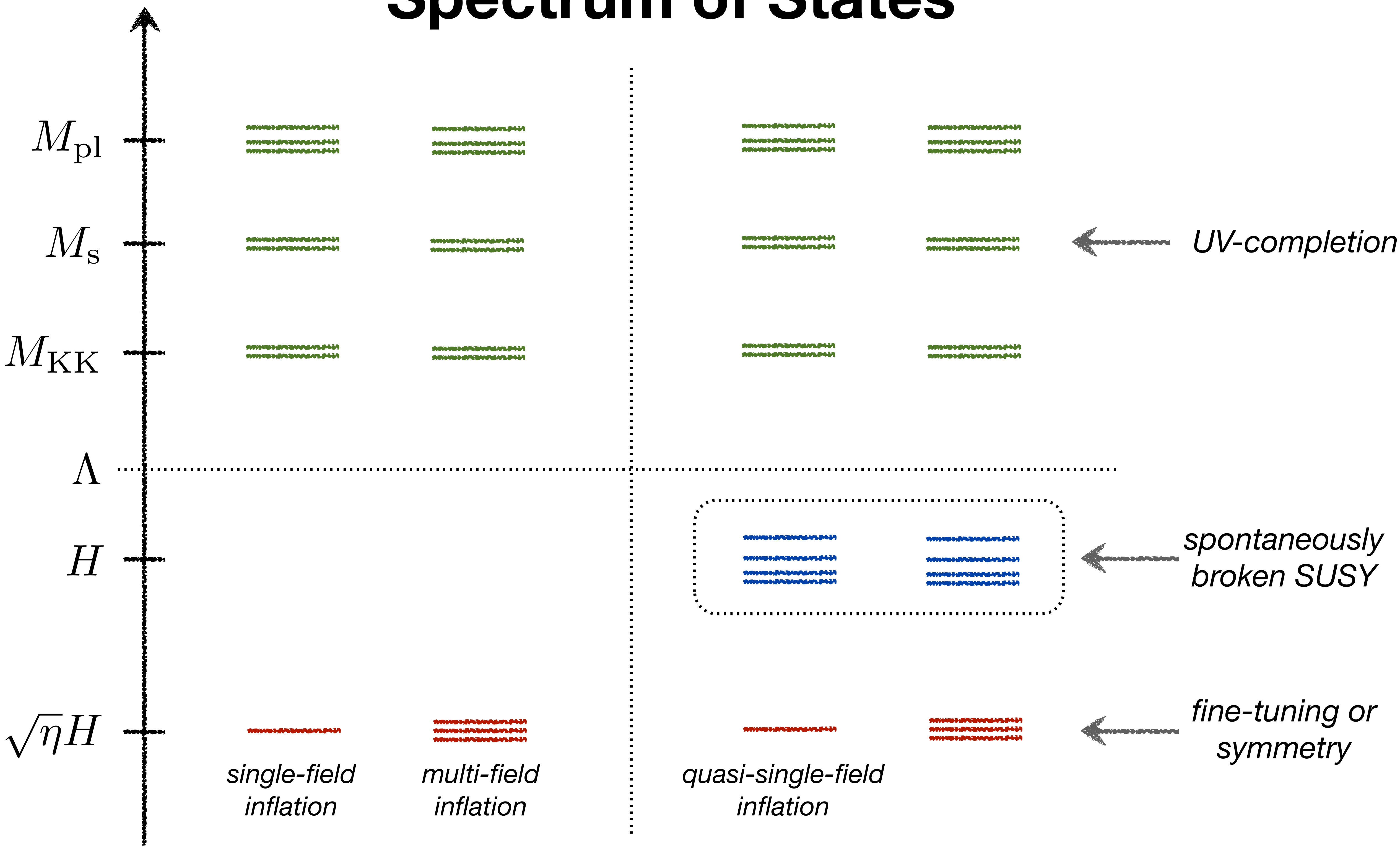

To derive the spectrum of states and their interactions,
we need to study ***compactification*** and ***moduli stabilization***.

Common Challenges

- Moduli stabilization does ***not*** decouple from the inflationary dynamics.

  It is hard to keep the inflaton light.

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- Moduli stabilization does ***not*** decouple from the inflationary dynamics.

  It is hard to keep the inflaton light.

- If one field is light, why not many?

  Fields with $m < H$ are quantum-mechanically active and determine the inflationary phenomenology.

Common Challenges

- To compute the effective action one has to make **approximations**:

  - α' expansion
  - String loop expansion
  - Probe approximation
  - Large charge approximation
  - Smeared approximation
  - Linear approximation
  - Noncompact approximation
  - Large volume approximation
  - Adiabatic approximation
  - Truncation
  - Moduli space approximation

It is rare that all approximations are well-controlled.

Why string vacua are dirty

Denef, *Les Houches Lectures on Constructing String Vacua*

Dine-Seiberg Problem

Let ρ be a modulus and $\rho \to \infty$ correspond to the weakly-coupled region.

compactification volume

(inverse) string coupling

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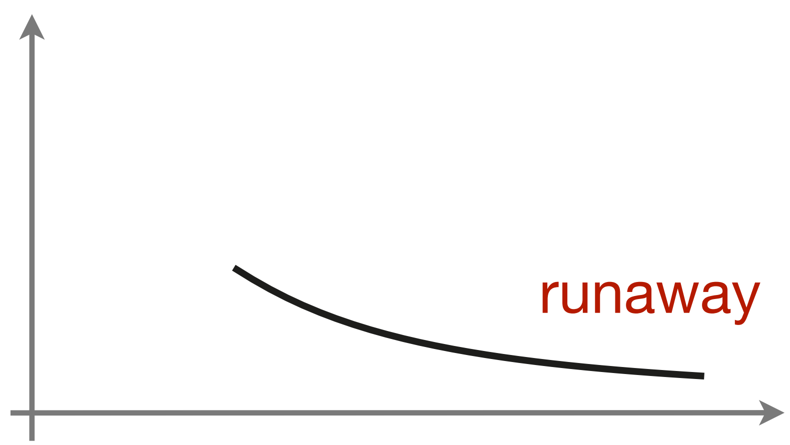

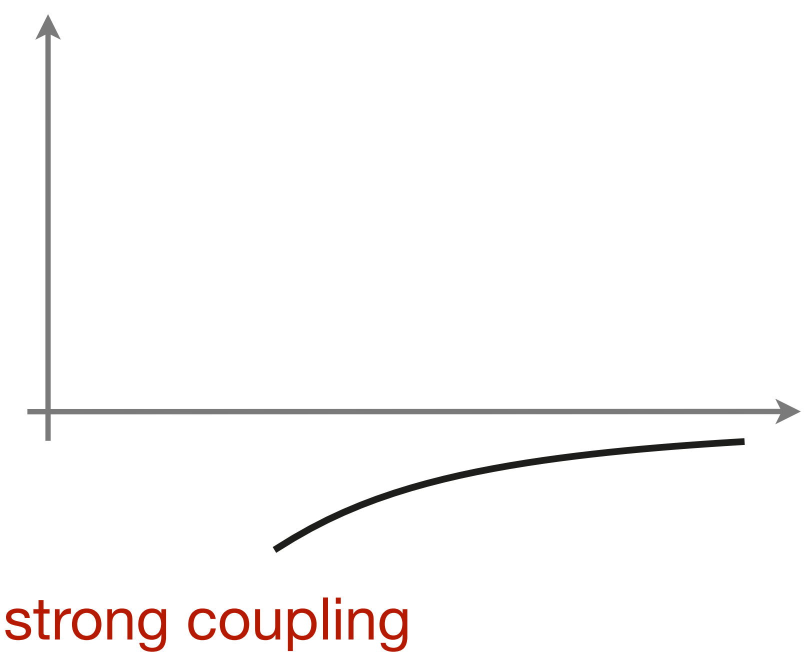

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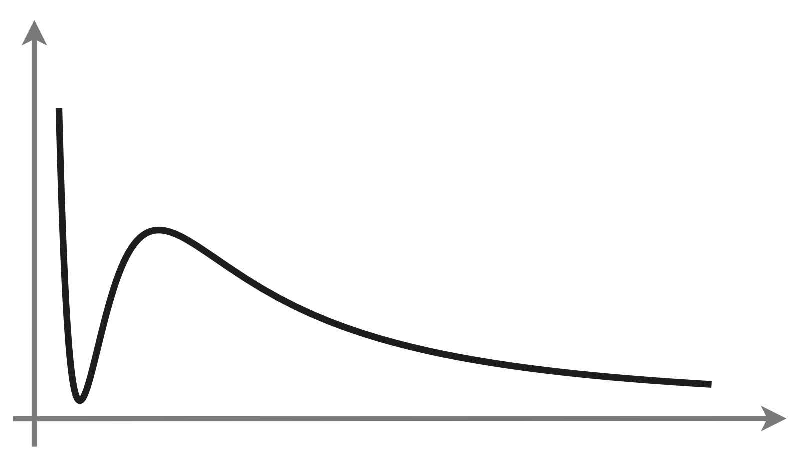

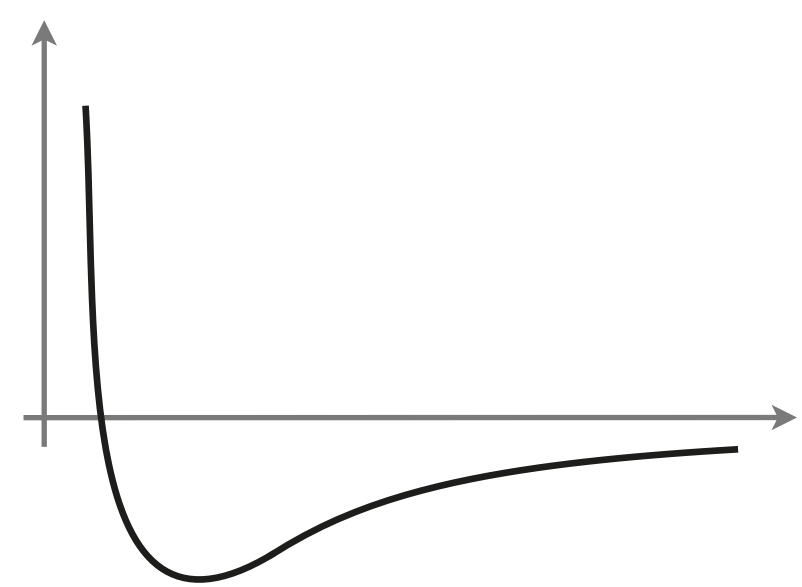

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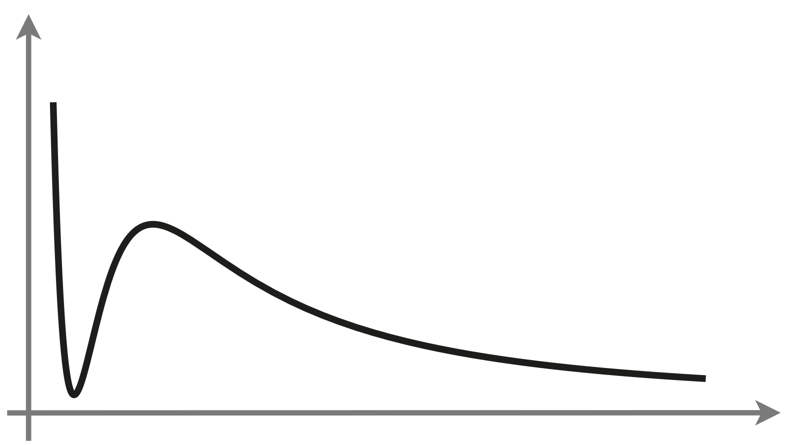

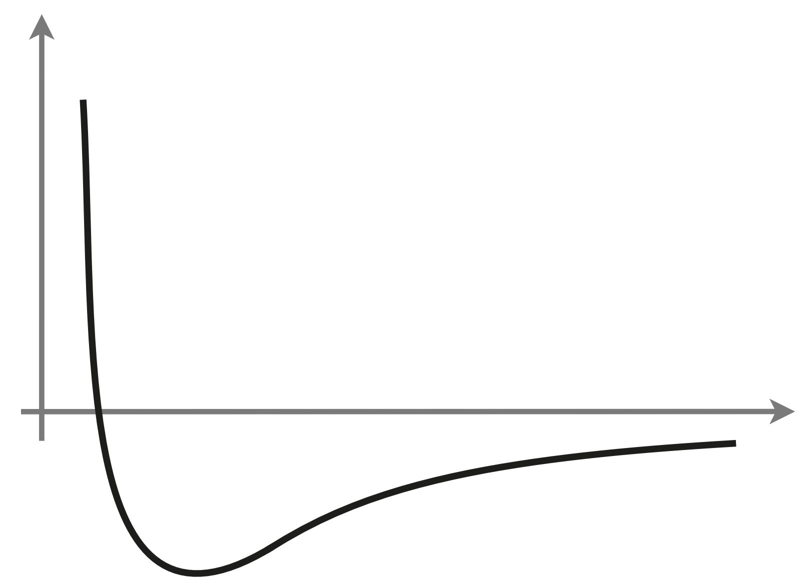

Hard to maintain parametric control.

"The string vacuum we live in is probably strongly coupled."
Dine and Seiberg

"When corrections can be computed, they are not important,
and when they are important, they cannot be computed."
Denef

Maldacena-Nunez No-Go

Assumptions:

1. Leading order 10D SUGRA + brane actions.
2. Finite Newton's constant.
3. Positivity requirements for T_{MN}.

Theorem: No de Sitter solutions.

Maldacena and Nunez

String theory evades the theorem by having higher-order curvature corrections (violates 1) and singular negative tension O-planes (violates 3).

Case Study: Type IIB Vacua

Douglas and Kachru, *Flux Compactifications*

Kachru, Kallosh, Linde and Trivedi, *De Sitter Vacua in String Theory*

Giddings, Kachru, and Polchinski, *Hierarchies from Fluxes in String Compactifications*

Type IIB Supergravity

The bosonic fields of type IIB supergravity are:

- metric G_{MN}
- dilaton Φ
- NS 3-form $H_3 = \mathrm{d}B_2$
- RR p-forms $F_p = \mathrm{d}C_{p-1}$

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- metric $G^{(E)}_{MN} \equiv e^{-\frac{1}{2}\Phi} G_{MN}$
- axiodilaton $\tau \equiv C_0 - ie^{-\Phi}$
- 3-form flux $G_3 \equiv F_3 - \tau H_3$
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The type IIB action is

$$\boxed{S_{\mathrm{IIB}} = \frac{1}{2\kappa^2} \int \mathrm{d}^{10}X \sqrt{-G} \left[R - \frac{|\partial\tau|^2}{2(\mathrm{Im}\,\tau)^2} - \frac{|G_3|^2}{12 \mathrm{Im}\,\tau} - \frac{1}{4}|\tilde{F}_5|^2 \right] + \mathrm{CS}}$$

$+\,\alpha'$ corrections

$+\,g_s$ corrections

where $\kappa^2 \equiv \frac{1}{2}(2\pi)^7(\alpha')^4$

Type IIB Supergravity

In addition, we have localized sources: D-brane and O-planes.

The action of a Dp-brane is

DBI CS

$$\boxed{S_{\mathrm{Dp}} = -T_p \int \mathrm{d}^{p+1}\sigma \sqrt{-\det(G_{ab} + F_{ab})} + \mu_p \int C_{p+1}}$$

tension

$$T_p \equiv \frac{1}{(2\pi)^p g_s (\alpha')^{(p+1)/2}}$$

worldvolume flux

charge

$$\mu_p = g_s T_p$$

induced metric

$$G_{ab} \equiv \frac{\partial X^M}{\partial \sigma^a} \frac{\partial X^N}{\partial \sigma^b} G_{MN}$$

Dimensional Reduction

Consider the following ansatz for the metric:

$$\mathrm{d}s^2 = e^{-6u(x)} g_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu + e^{2u(x)} \hat{g}_{mn} \mathrm{d}y^m \mathrm{d}y^n$$

breathing mode

reference metric with fixed volume

and substitute it into the gravity part of the IIB action

$$S_{\mathrm{EH}}^{(10)} = \frac{1}{2\kappa^2} \int \mathrm{d}^{10} X \sqrt{-G}\, R_{10}$$

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$$S_{\mathrm{EH}}^{(10)} = \frac{1}{2\kappa^2} \int \mathrm{d}^{10} X \sqrt{-G}\, R_{10}$$

$$= \frac{M_{\mathrm{pl}}^2}{2} \int \mathrm{d}^4 x \sqrt{-g} \left[R_4 + e^{-8u} \hat{R}_6 + \frac{12}{u^2} \partial_\mu u \, \partial^\mu u \right]$$

where $\quad M_{\mathrm{pl}}^2 \equiv \dfrac{\mathrm{Vol}(X_6)}{g_s^2 \kappa^2} \quad .$

Dimensional Reduction

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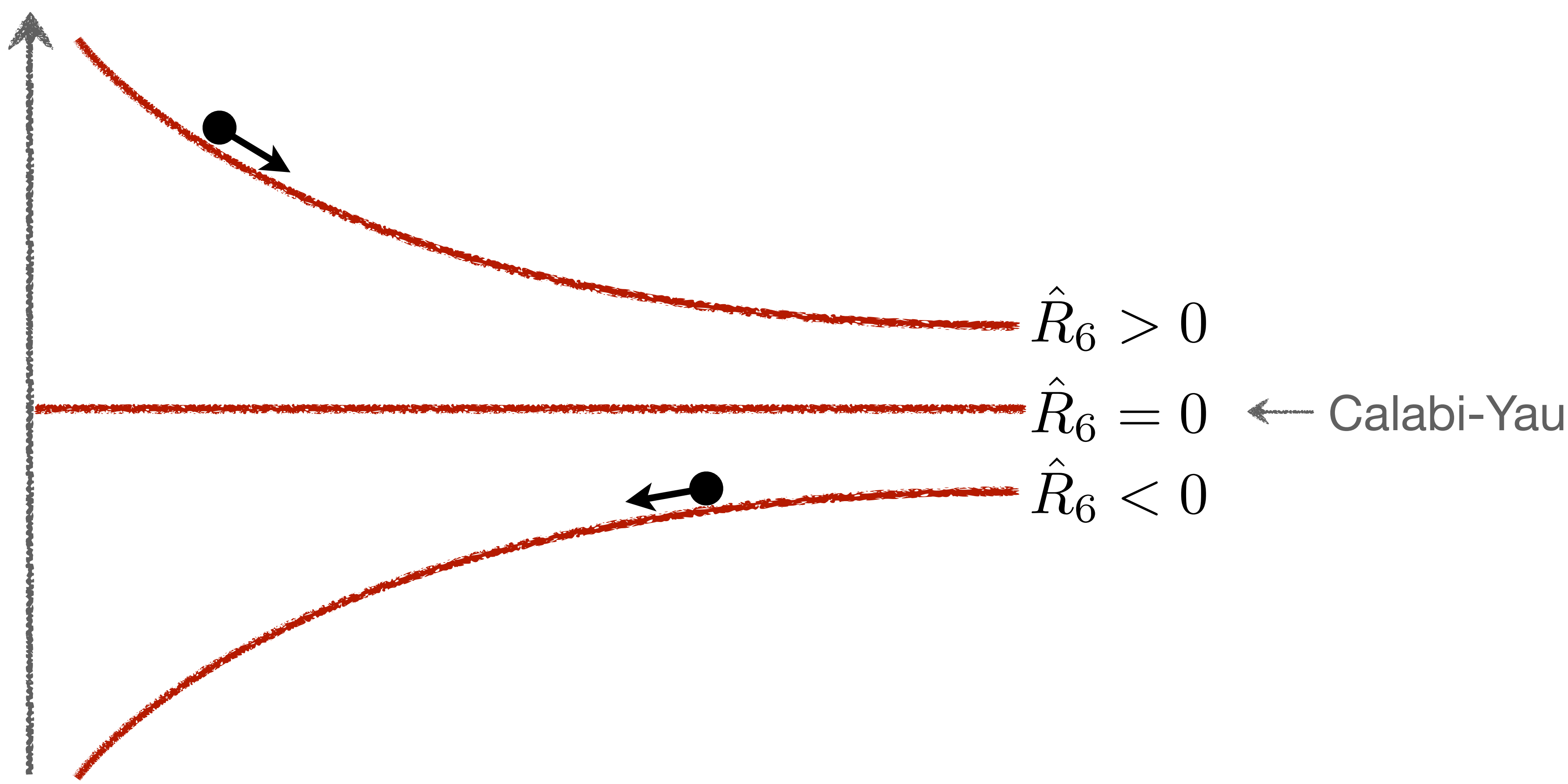

At least the breathing mode is unfixed in vacuum compactifications.

Therefore consider compactifications with ***branes*** and ***fluxes***.

Moduli Stabilization I: Classical Effects

Consider the ansatz:

warp factor

$$\mathrm{d}s^2 = e^{2A(y)}\mathrm{d}x^\mu \mathrm{d}x_\mu + e^{2u(x)}e^{-2A(y)}\hat{g}_{mn}\mathrm{d}y^m\mathrm{d}y^n$$

$$\tilde{F}_5 = (1+\star)\,\mathrm{d}\alpha(y) \wedge \mathrm{d}x^0 \wedge \mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}x^3$$

GKP

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This satisfies the supergravity equations of motion if

$$\boxed{\star G_3 = i\,G_3} \quad \text{and} \quad \boxed{e^{4A} = \alpha}$$

imaginary self-dual (ISD)

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Ex: *Show that a D3-brane feels no force in an ISD compactification, i.e. show that the potential for a D3-brane is*

$$V_{\mathrm{D3}} = T_3(e^{4A} - \alpha)$$

Moduli Stabilization I: Classical Effects

Turning on $G_3 = F_3 - \tau H_3$ creates a potential for the complex structure moduli and the dilaton.

recall: $\mathcal{L} \subset G_3 \wedge \star G_3$

metric

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metric

It is convenient to describe this in **4D N=1 supergravity**:

- The flux superpotential is

holomorphic (3,0) form

$$\boxed{W = \int G_3 \wedge \Omega}$$

Gukov, Vafa, and Witten

- The Kähler potential is

$$\boxed{K = -3\ln(T + \bar{T}) + \cdots}$$

where $T \equiv \underbrace{\int_{\Sigma_4} \sqrt{G}}_{\propto e^{4u}} + i \int_{\Sigma_4} C_4$ is the complexified Kähler modulus.

Moduli Stabilization I: Classical Effects

The scalar potential is $V_F = e^K \left[K^{A\bar{B}} D_A W \overline{D_B W} - 3|W|^2 \right]$

Ex: *Show that* $K^{T\bar{T}} \partial_T K \partial_{\bar{T}} K = 3$ *, so that*

$$V_F = e^K \sum_{A,B \neq T} K^{A\bar{B}} D_A W \bar{(} D_B W)$$

no-scale potential

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when $D_A W = 0$ for all $A \neq T$.

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The volume is still unfixed.

Moduli Stabilization I: Classical Effects

In perturbation theory, T does not appear in W.

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Reason: $T = \cdots + i \underbrace{\int_{\Sigma_4} C_4}_{\equiv \theta}$

axion

with shift symmetry $\theta \mapsto \theta + const.$

So, $W = T^{\alpha}$ is forbidden to all orders in α' and g_s .

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There are then two options for generating a potential for T :

1) Perturbative corrections to K . $\longrightarrow$ LVS

2) Nonperturbative corrections to W . $\longrightarrow$ KKLT

We will look at the second.

Moduli Stabilization II: Quantum Corrections

Consider N_c D7-branes wrapping a 4-cycle Σ_4 in X_6 .

Ex: *Show that the dimensional reduction of the D7-brane action gives*

$$S = \frac{1}{4g^2} \int \mathrm{d}^4x \sqrt{-g}\ \mathrm{Tr}[F_{\mu\nu} F^{\mu\nu}]$$

gauge coupling

$$\boxed{\frac{1}{g^2} = \frac{T_3 \mathrm{Vol}(\Sigma_4)}{8\pi^2}}$$

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Moduli Stabilization II: Quantum Corrections

Since $\mathrm{Vol}(\Sigma_4) \propto T + \bar{T}$, we have

$$\boxed{W_{\langle\lambda\lambda\rangle} = \mathcal{A} e^{-aT}} \qquad \text{where} \quad a \equiv \frac{2\pi}{N_c}$$

This breaks no-scale and stabilizes the volume.

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At low energy, complex structure and dilaton can be integrated out and the EFT for T is:

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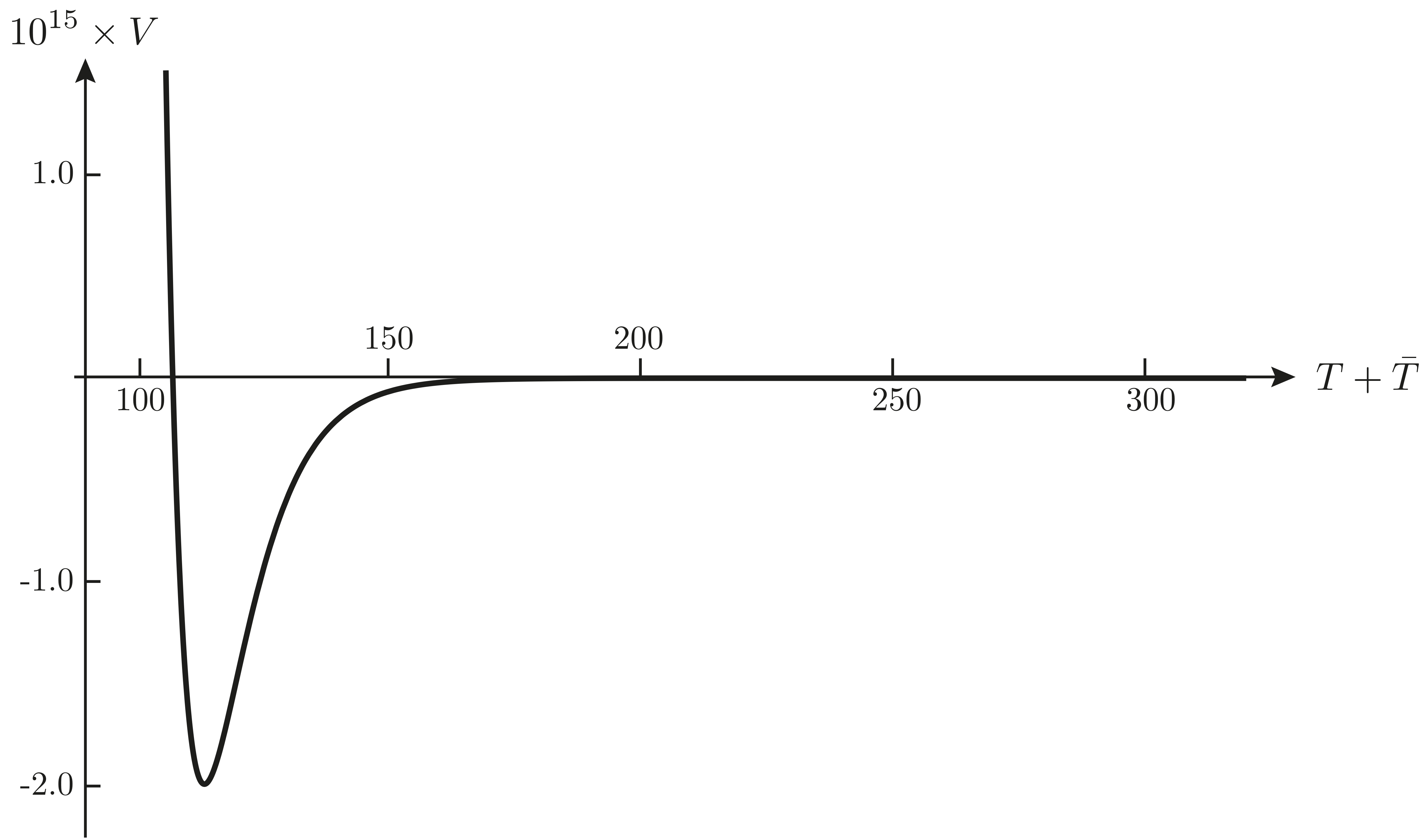
Ex: *Show that*

$$V_F = \frac{a\mathcal{A} e^{-a(T+\bar{T})}}{2(T+\bar{T})^2}\left[\left(1+\frac{T+\bar{T}}{3}\right) a\mathcal{A} e^{-a(T+\bar{T})} + W_0\right]$$

Plot this for $\mathcal{A} = 1$ *,* $a = 0.1$ *and* $W_0 = -10^{-4}$ *.*

AdS Vacua



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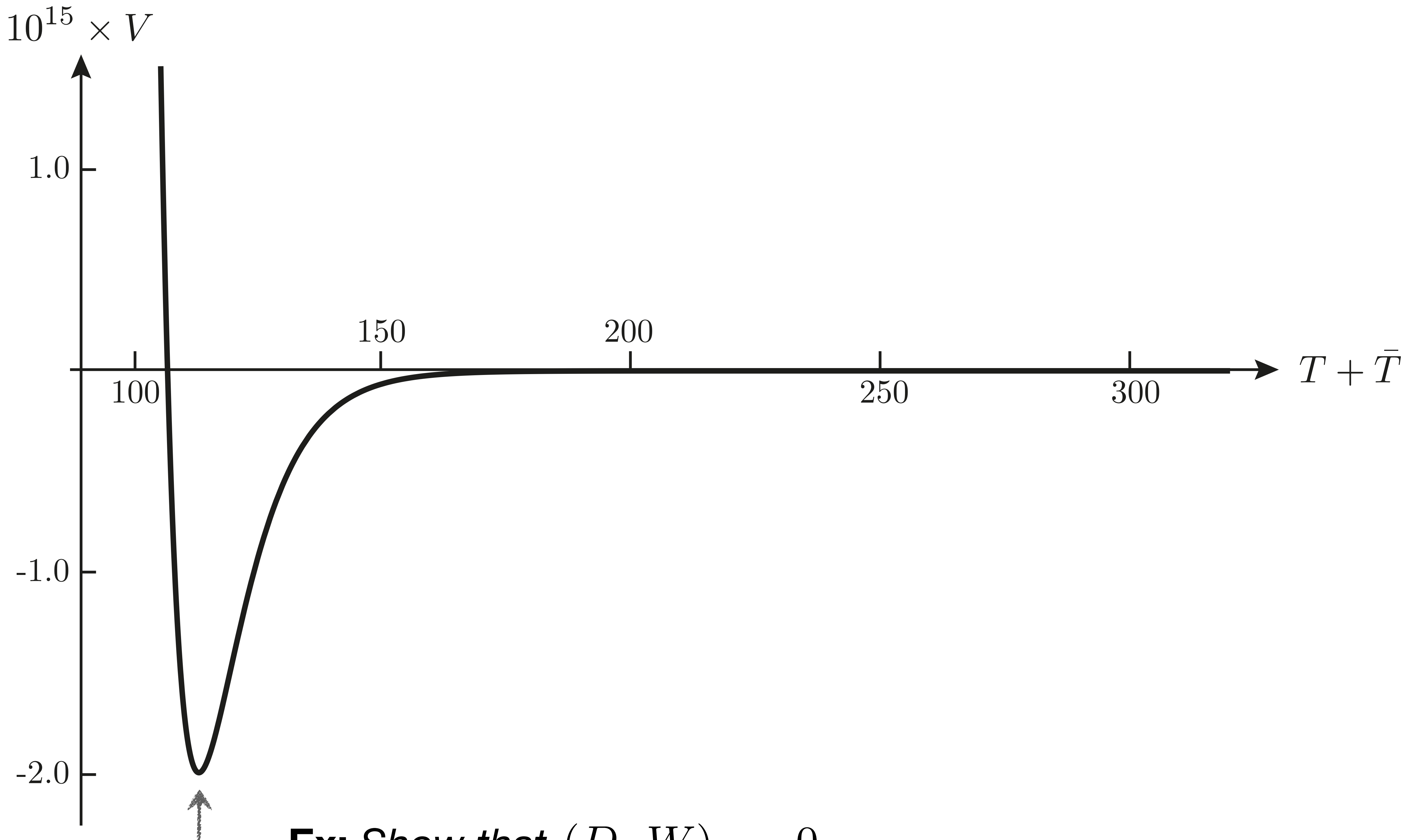


Ex: *Show that* $(D_T W)_\star = 0$

i.e. the vacuum is ***SUSY AdS.***

De Sitter Vacua

There are various proposals for ***uplifting*** to de Sitter:

For example, we may add an anti-D3-brane:*

Recall $\mathcal{L}_{\rm D3} = \mathcal{L}_{\rm DBI} + \mathcal{L}_{\rm CS} = 0$ in ISD compactifications.

So, $\mathcal{L}_{\overline{\rm D3}} = \mathcal{L}_{\rm DBI} - \mathcal{L}_{\rm CS} \neq 0$

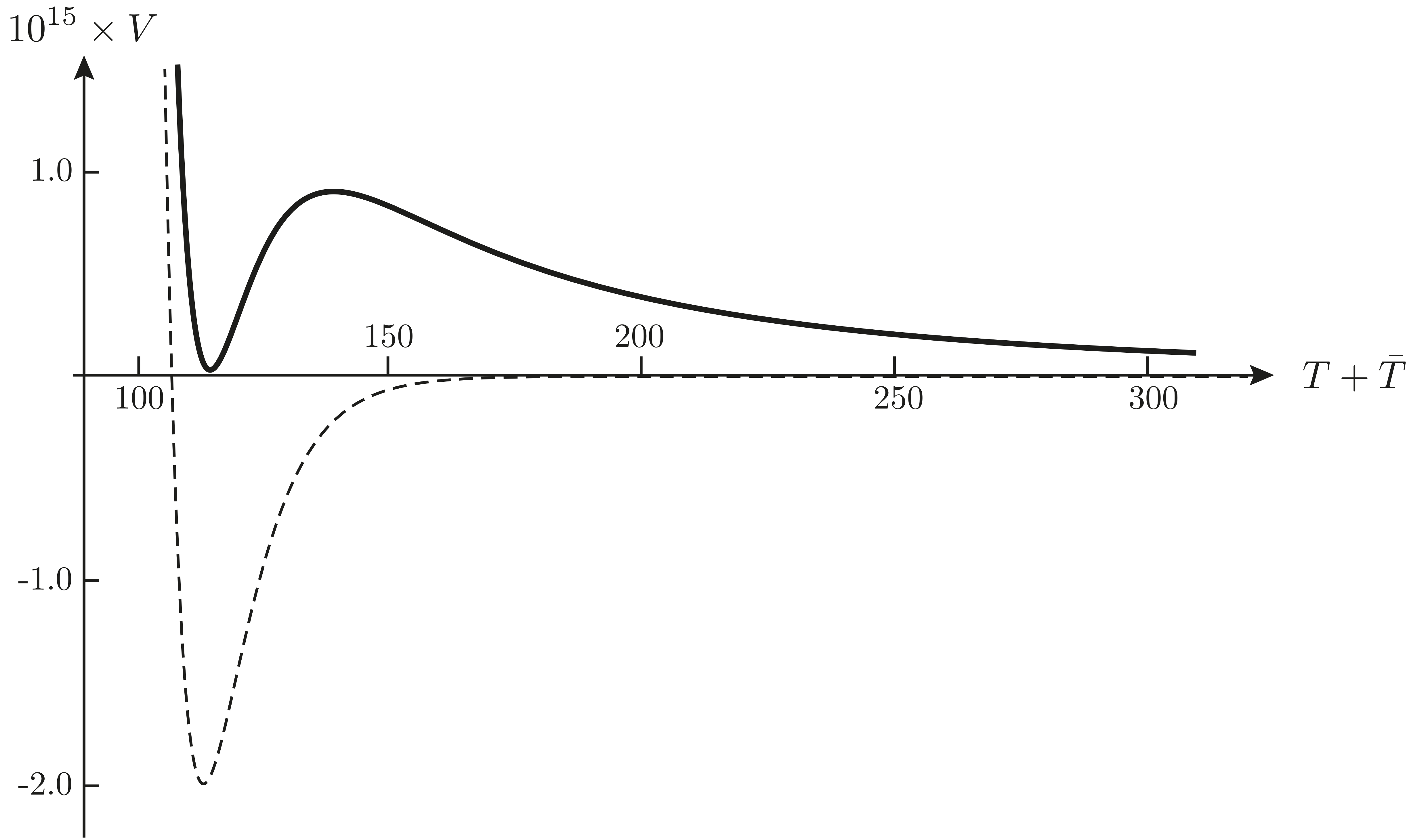
In Einstein frame, the anti-D3 contribution is

$$\boxed{\delta V_{\overline{\rm D3}} = e^{-12u} D}$$

This leads to metastable dS vacua.

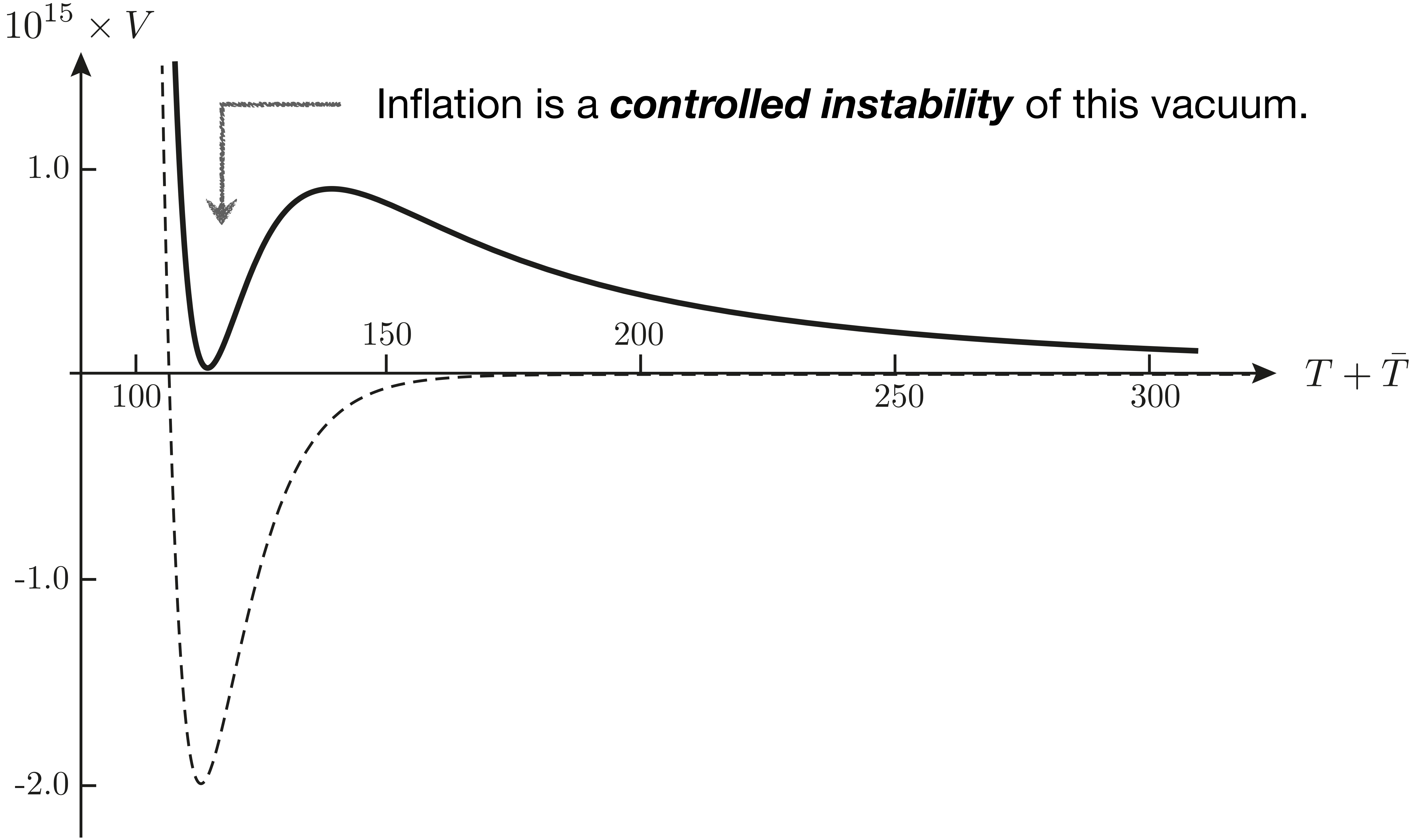
* For a debate on the consistency of the metastable anti-D3-solution see: Bena et al., Van Riet et al. vs Polchinski et al., Harnett

De Sitter Vacua



De Sitter Vacua

Inflation is a ***controlled instability*** of this vacuum.



Examples of String Inflation

Inflating with Branes

Warped D3-brane Inflation

DBI Inflation

Inflating with Axions

N-flation

Lattice Alignment

Kinetic Alignment

Monodromy

Inflating with Volume Moduli

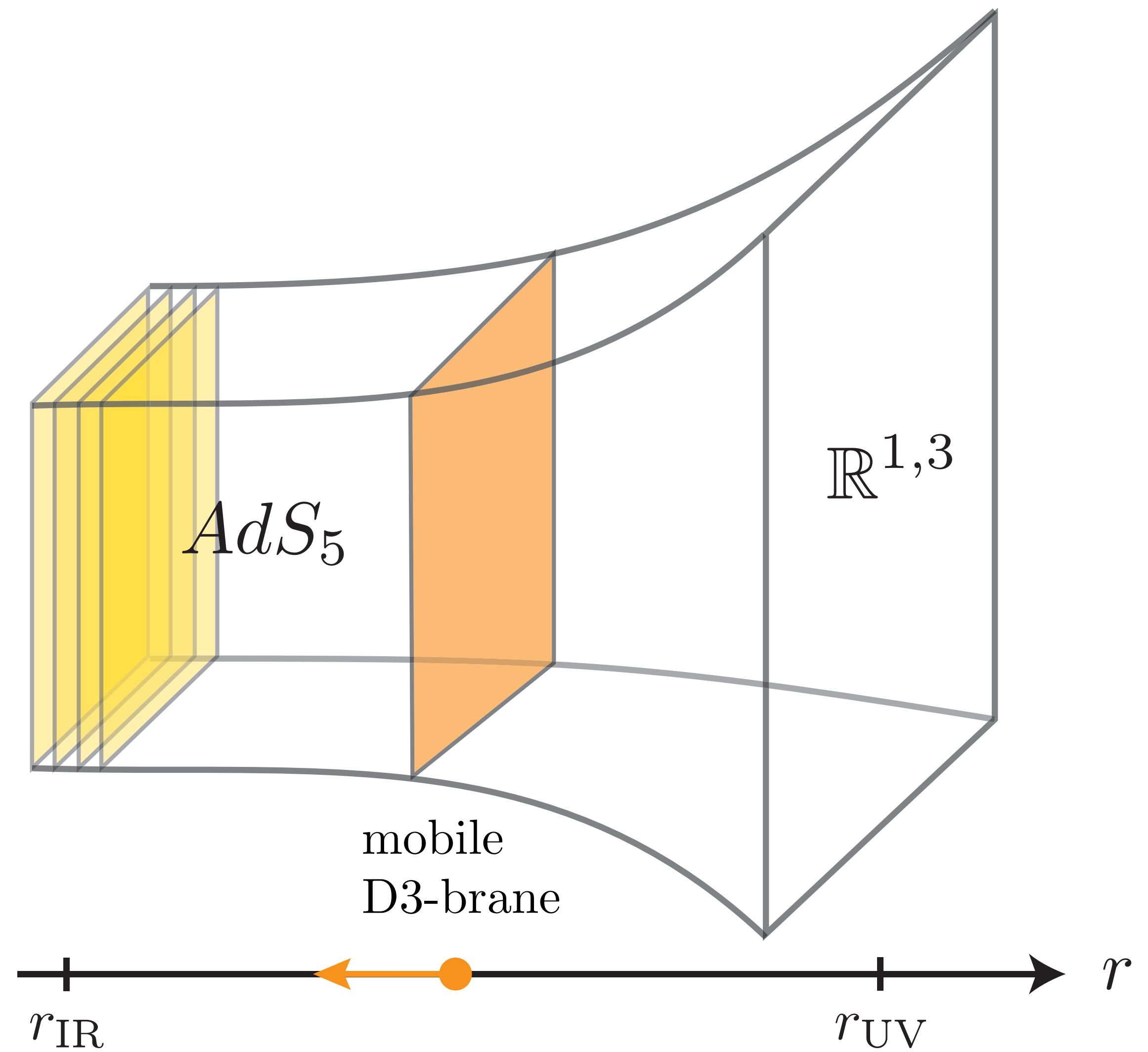
Inflating with Branes

Warped Brane Inflation

KKLMMT, *Towards Inflation in String Theory*

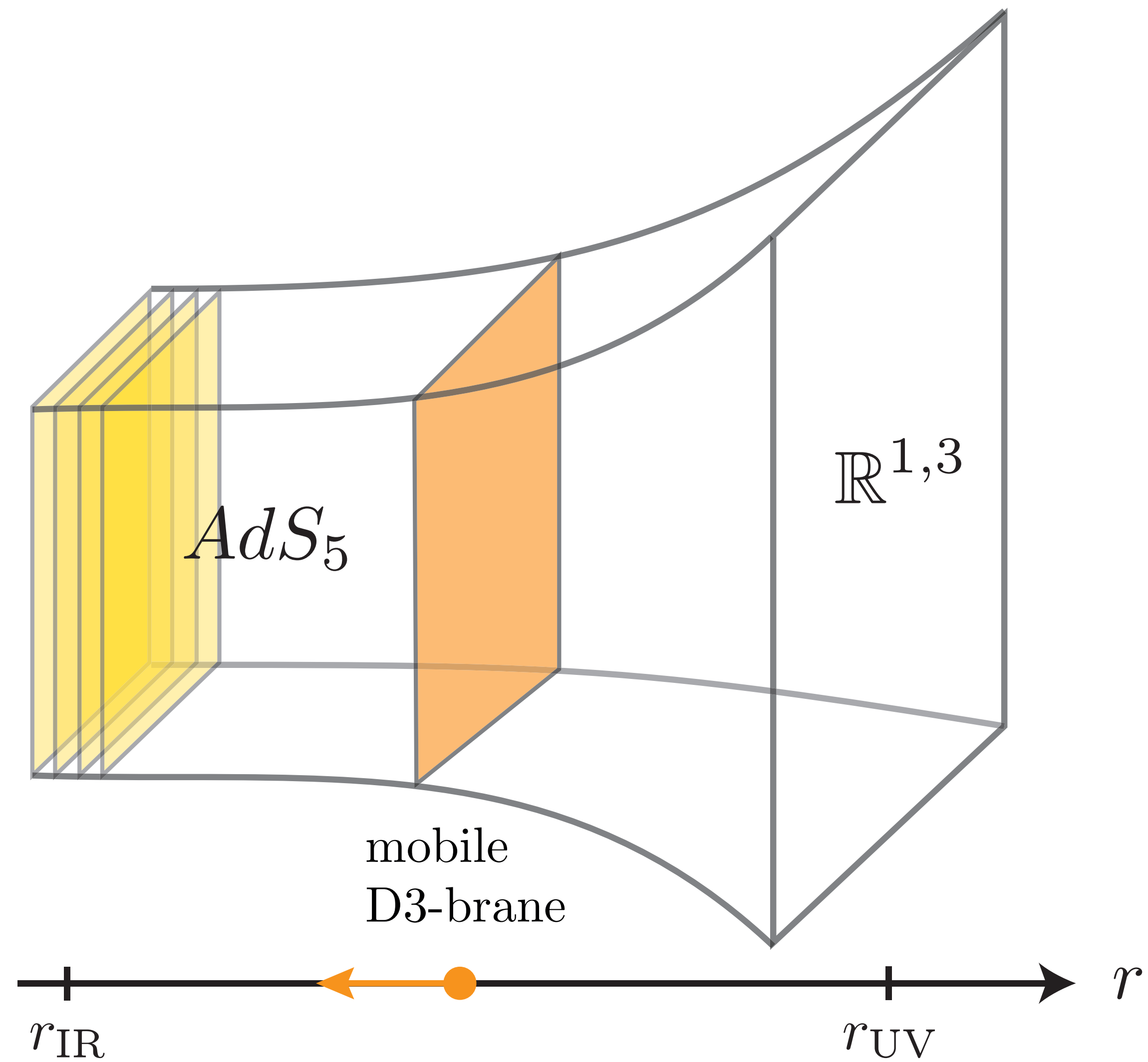
DB, Dymarsky, Klebanov, and McAllister, *Towards an Explicit Model of D-brane Inflation*

Consider a **spacetime-filling D3-brane**:



Can the position in the extra dimension play the role of the inflaton?

Consider a **spacetime-filling D3-brane**:



Can the position in the extra dimension play the role of the inflaton?

Recall that a **D3-brane in an ISD compactification feels no force**.

A good starting point for inflation?

ISD is broken in stabilized compactifications.

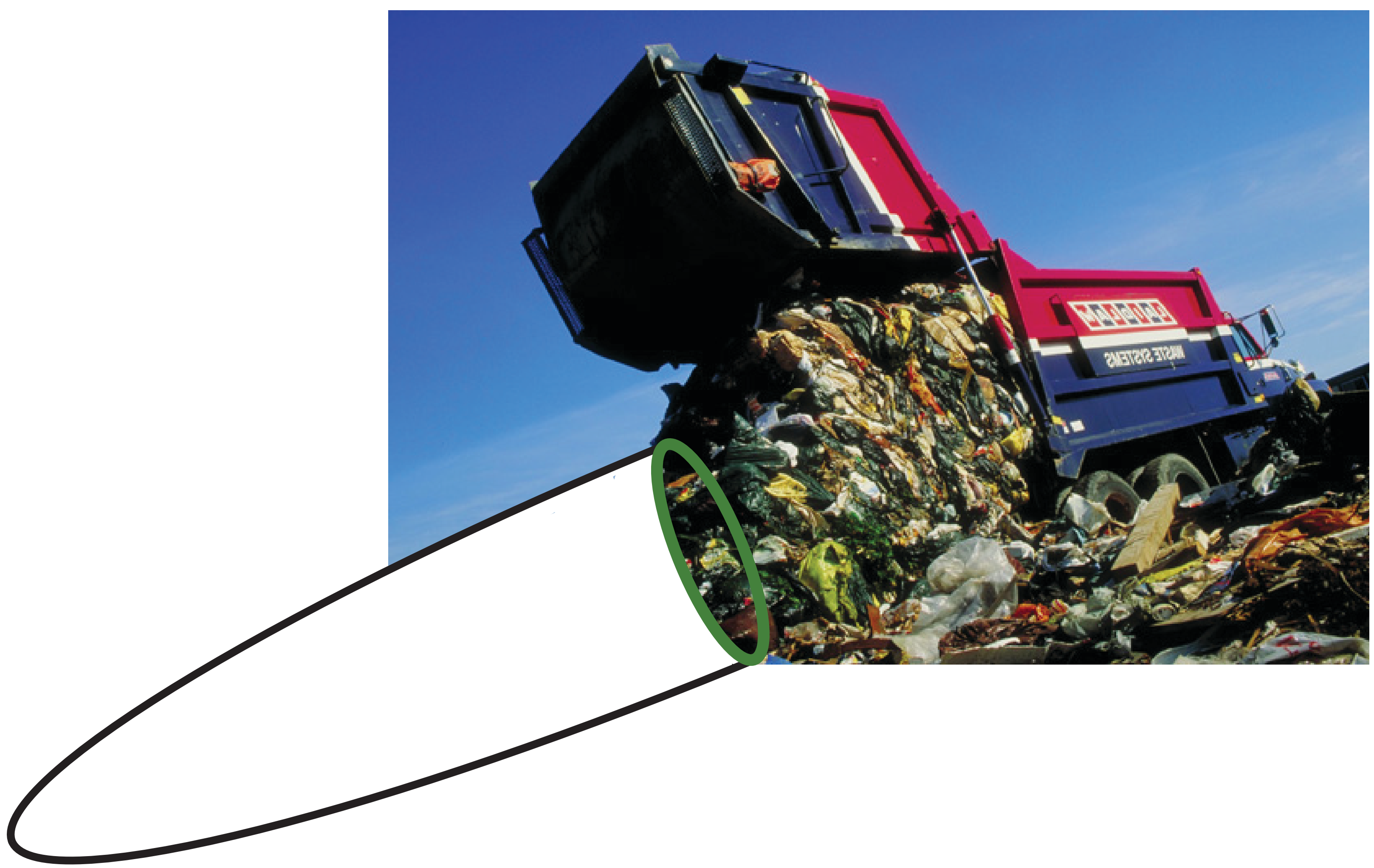
What is the D3-brane potential?

ISD is broken in stabilized compactifications.

What is the D3-brane potential?

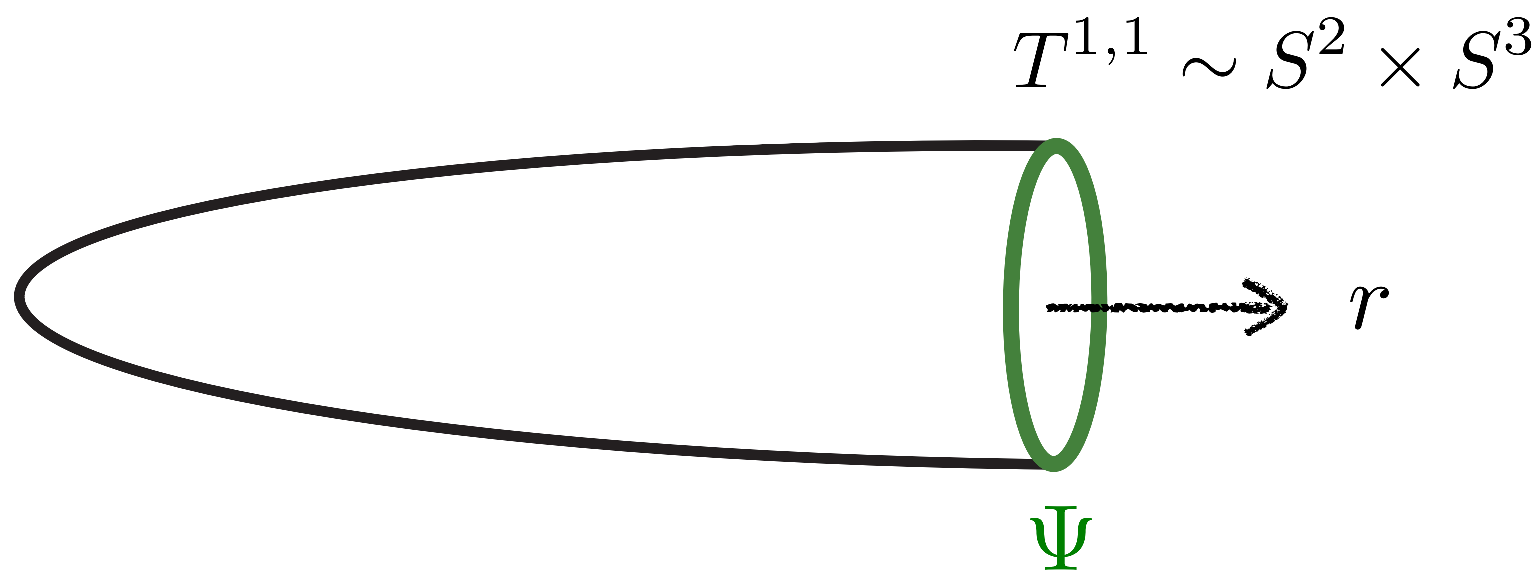
Problem: We don’t know the metric of compact CYs.

Idea: Study D3 in a non-compact CY cone and systematically incorporate effects from the compact bulk:



Warped Deformed Conifold

Consider the six-dimensional CY cone defined by $\sum_{A=1}^{4} z_A^4 = 0$:

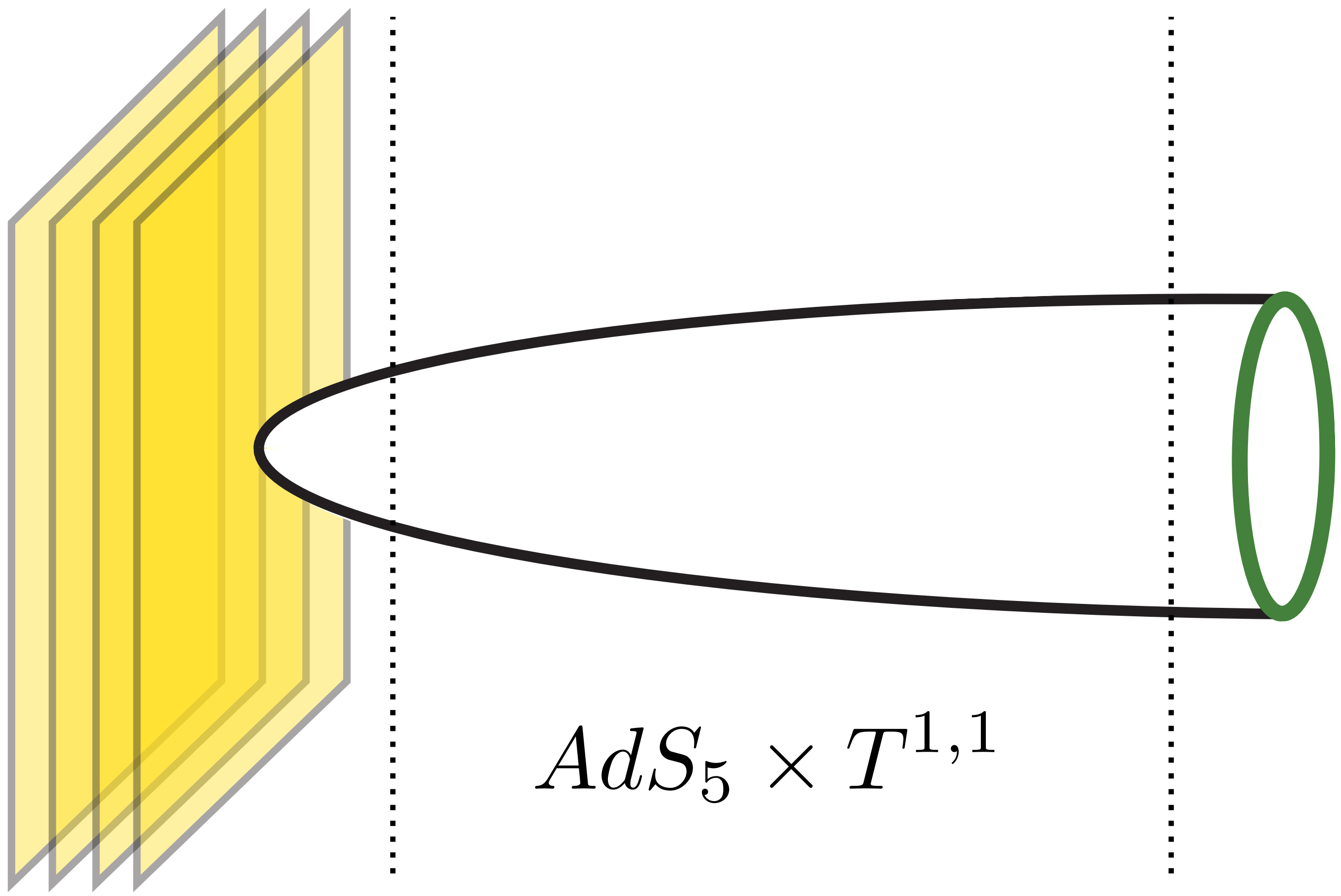


$$\mathrm{d}s^2 = \mathrm{d}r^2 + r^2 \mathrm{d}\Omega^2_{T^{1,1}}$$

Klebanov and Strassler

Warped Deformed Conifold

Add N D3-branes at the tip of the cone:



The backreacted solution is:

$$\mathrm{d}s^2 = e^{2A(r)}\mathrm{d}x^\mu \mathrm{d}x_\mu + e^{-2A(r)}\left(\mathrm{d}r^2 + r^2 \mathrm{d}\Omega^2_{T^{1,1}}\right)$$

$$e^{2A(r)} \equiv \left(\frac{r}{R}\right)^2 \quad , \quad \frac{R^4}{(\alpha')^2} = \frac{27\pi}{4} g_s N \; .$$

Klebanov and Strassler

Digression: A Field Range Bound

This canonical field range available to a D3-brane is

$$\Delta\phi^2 < T_3 r_{\rm UV}^2 = \frac{r_{\rm UV}^2}{(2\pi)^3 g_s (\alpha')^2}$$

We wish to compare this to the four-dimensional Planck scale: $M_{\rm pl}^2 = \dfrac{\mathcal{V}}{g_s^2 \kappa^2}$

Using the volume of the throat

$$\mathcal{V}_{\rm T} \equiv \int {\rm d}\Omega^2_{T^{1,1}} \int_{r_{\rm IR}}^{r_{\rm UV}} r^5 {\rm d}r\, e^{-4A(r)} \; = \; 2\pi^4 g_s N (\alpha')^2 r_{\rm UV}^2$$

allows us to put a lower bound on the Planck mass

$$M_{\rm pl}^2 > \frac{N}{4} \frac{r_{\rm UV}^2}{(2\pi)^3 g_s (\alpha')}$$

Comparing this to the field range, we find

$$\boxed{\frac{\Delta\phi}{M_{\rm pl}} < \frac{2}{\sqrt{N}} \;\ll\; 1}$$

Lyth bound

No observable gravitational waves from brane inflation.

DB and McAllister

Dynamics

The D3-brane action is

$$S_{\rm D3} \;=\; -T_3 \int \mathrm{d}^4\sigma \sqrt{-\det[g^{\rm ind}]} \;+\; T_3 \int C_4$$

$$(g^{\rm ind})_{ab} \equiv \frac{\partial X^M}{\partial \sigma^a}\frac{\partial X^N}{\partial \sigma^b} G_{MN} \qquad M, N \equiv 0, \dots, 9$$

$$a, b \equiv 0, \dots, 3$$

The brane moves in the extra dimensions $y^m(t) \quad m = 4, \dots, 9$

The induced metric on the brane worldvolume is

$$(g^{\rm ind})_{ij} = G_{ij} = e^{2A} g_{ij}$$

$$(g^{\rm ind})_{00} = G_{00} + G_{mn}\dot{y}^m \dot{y}^n = e^{2A} g_{00} + e^{-2A}\tilde{g}_{mn}\dot{y}^m \dot{y}^n$$

$$-\det[g^{\rm ind}] = -e^{4A}\det[g]\left[1 - e^{-4A}\tilde{g}_{mn}\dot{y}^m\dot{y}^n\right]$$

so the 4D effective Lagrangian becomes

$$\boxed{\mathcal{L}_{\rm D3} \;=\; -T_3 e^{4A}\sqrt{1 - e^{-4A}\tilde{g}_{mn}\dot{y}^m\dot{y}^n} \;+\; T_3\alpha(y)}$$

Dynamics

DBI inflation: $|\dot{y}|^2 \to e^{4A}$ the dynamics is driven by the non-linearities of the DBI action.

Slow-roll inflation: $|\dot{y}|^2 \ll e^{4A}$ the dynamics is driven by the shape of the potential.

$$\mathcal{L}_{\rm D3} \approx \frac{1}{2} T_3\, \tilde{g}_{mn} \dot{y}^m \dot{y}^n - T_3 (e^{4A} - \alpha)$$

$$\equiv \frac{1}{2} \dot{\phi}^2 - V(\phi)$$

$$\boxed{V(\phi) \equiv T_3 (e^{4A} - \alpha)} \quad = 0$$

for ISD compactification.

Eta Problem

Tr(Einstein) - Bianchi:

$$\boxed{\nabla^2(e^{4A} - \alpha) = R_4 + |G_-|^2 + \text{local}}$$

Laplacian on X_6

4D Ricci

IASD flux $G_- \equiv (\star - i)G_3$

(sourced e.g. by gaugino condensation)

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In quasi-de Sitter, we have $R_4 \approx 12H^2$.

This induces a dangerous mass term for the inflaton

$$V \equiv T_3(e^{4A}-\alpha) \;=\; V_0 + H^2\phi^2 + \cdots$$

and hence a large contribution to the eta parameter

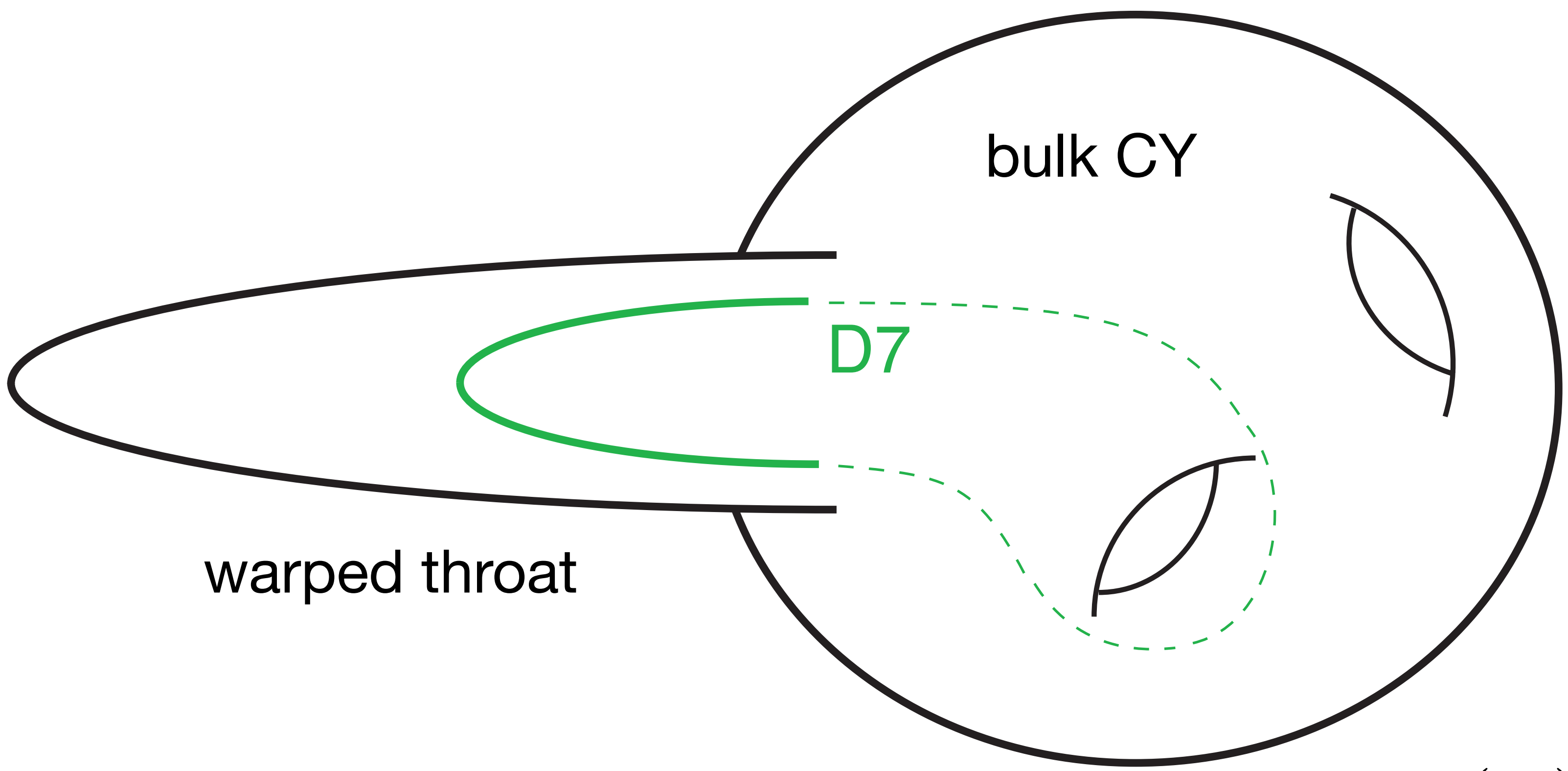
$$\eta \;=\; \frac{2}{3} + \cdots$$

Inflation requires tuning. Is it possible?

Need to compute the additional contributions to the potential.

D3-brane Potential from 4D

Consider a compactification with volume-stabilizing D7-branes reaching into the warped throat:



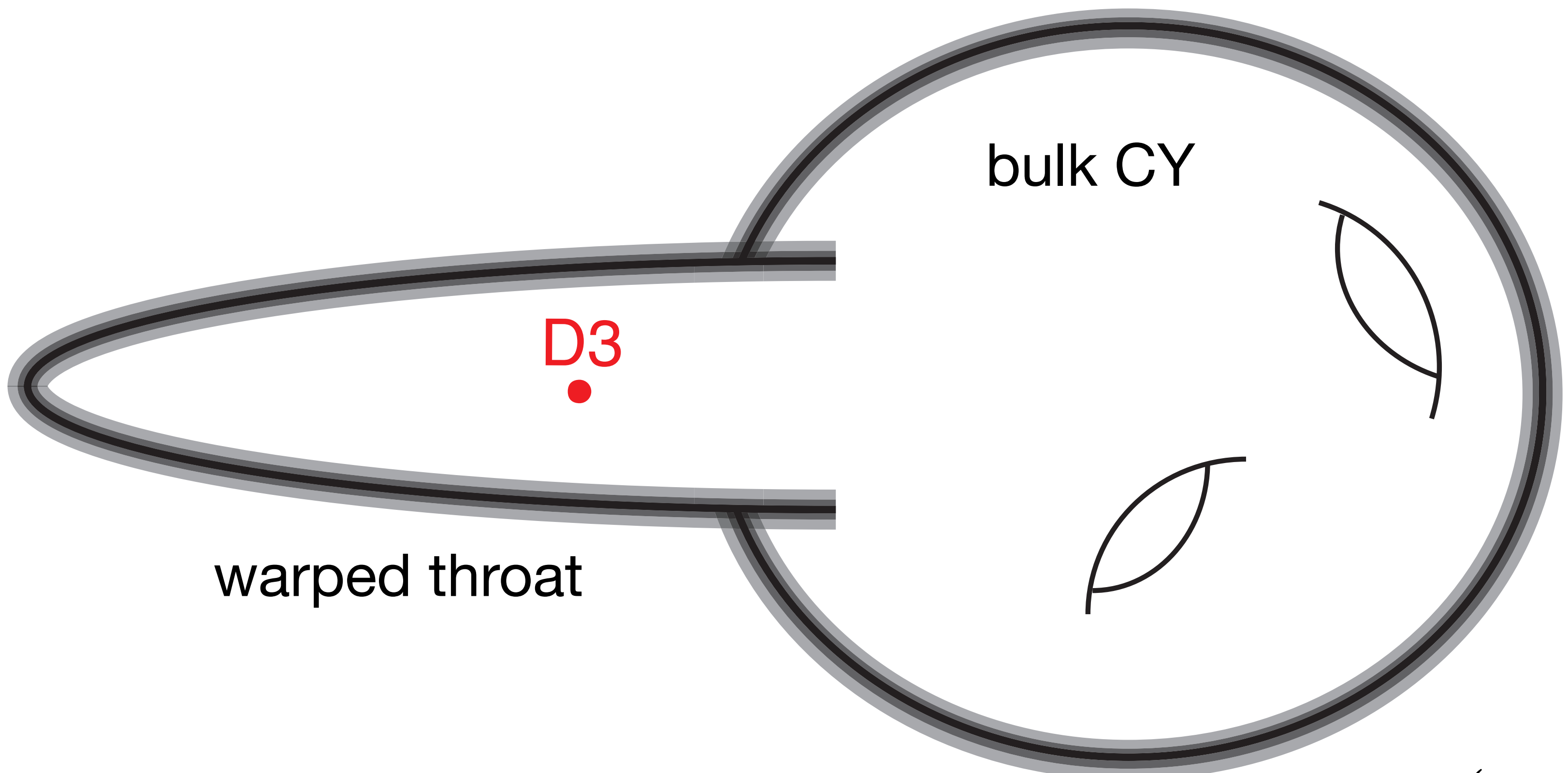
$$\mathcal{V}(T) = \left(T + \bar{T}\right)^{3/2}$$

compactification volume

D3-brane Potential from 4D

D3-brane backreacts on the compactification volume:

De Wolfe-Giddings



$$\mathcal{V}(T,\phi) = \left(T + \bar{T} - \phi\bar{\phi}\right)^{3/2}$$

$$K = -2\ln\mathcal{V}(T,\phi)$$

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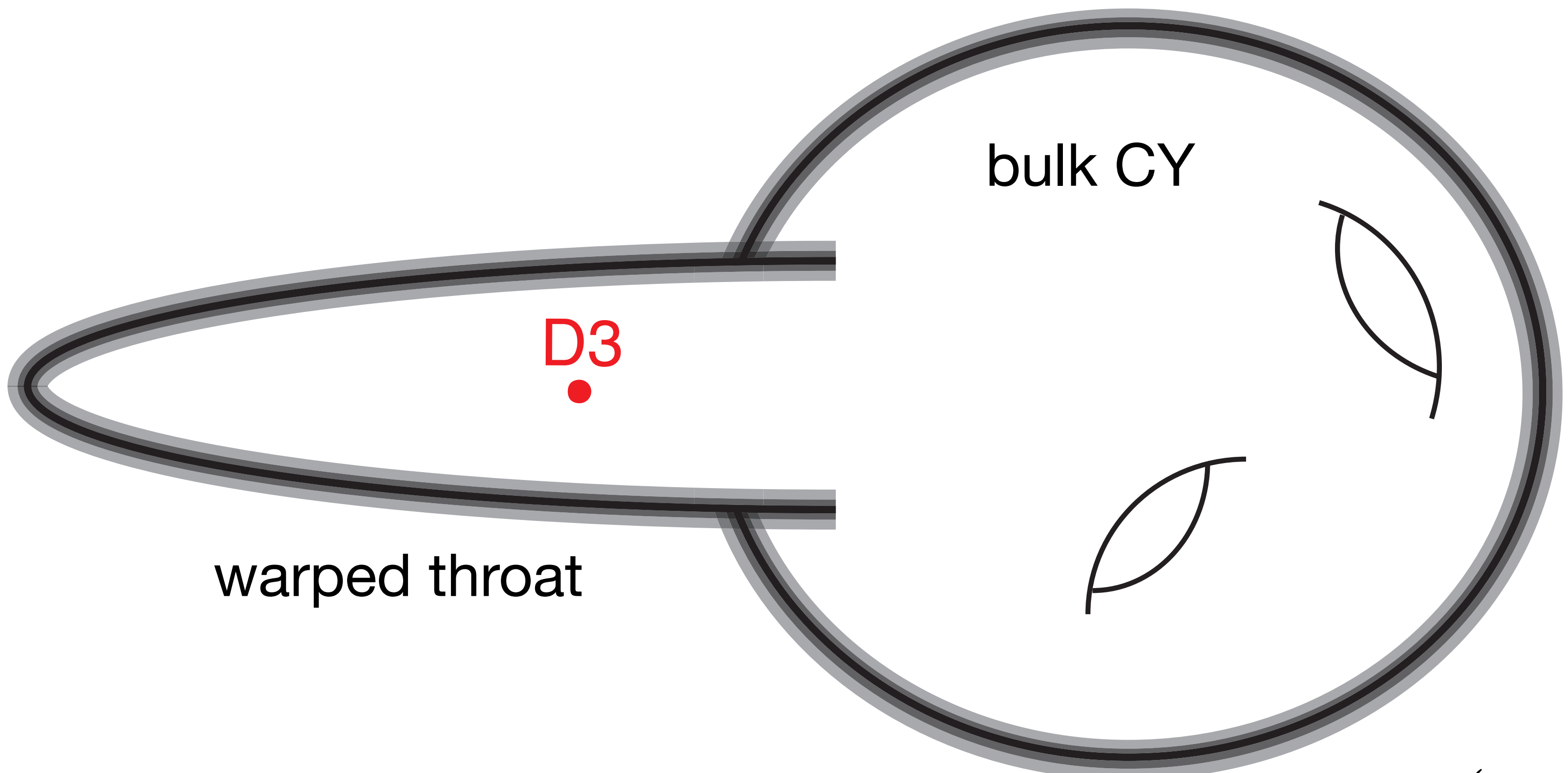


$$\mathcal{V}(T,\phi) = \left(T + \bar{T} - \phi\bar{\phi}\right)^{3/2}$$

$$K = -2\ln \mathcal{V}(T,\phi)$$

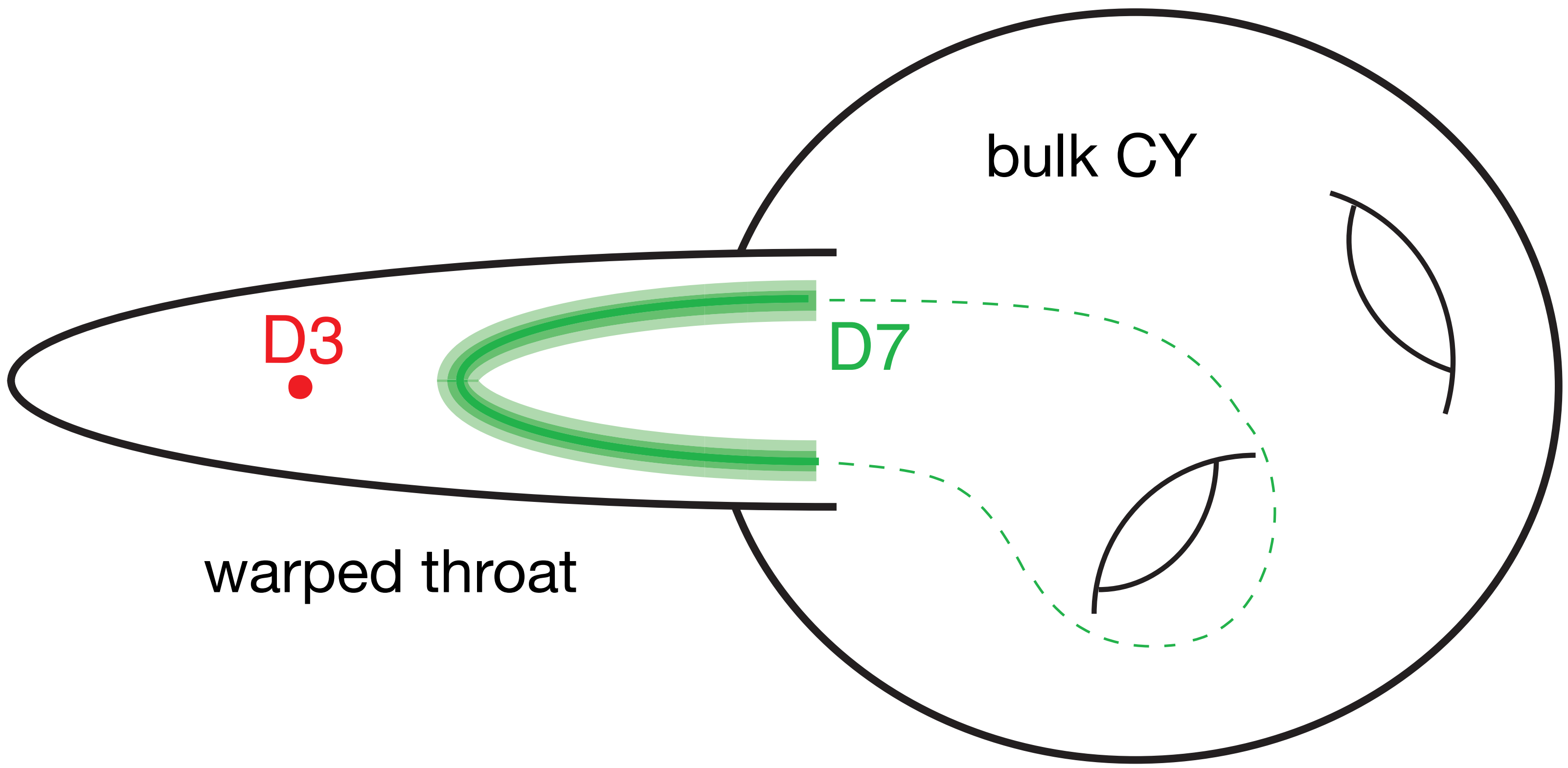
This implies the eta problem

$$V_F = e^K \left[\,\ldots\,\right] \;=\; \frac{C}{\mathcal{V}^2(\phi)} + \cdots$$

$$= V_0 + H^2\phi^2 + \cdots$$ as before.

D3-brane Potential from 4D

D3-brane backreacts on the 4-cycle wrapped by the 7-branes:



This changes the gauge coupling on the 7-brane, which changes the strength of the gauging condensation:

$$W_{\rm np}(T,\phi) \propto \exp\left(-\frac{T_3\mathcal{V}_4(\phi)}{N_c}\right) \;\equiv\; \mathcal{A}(\phi)e^{-aT}$$

computable

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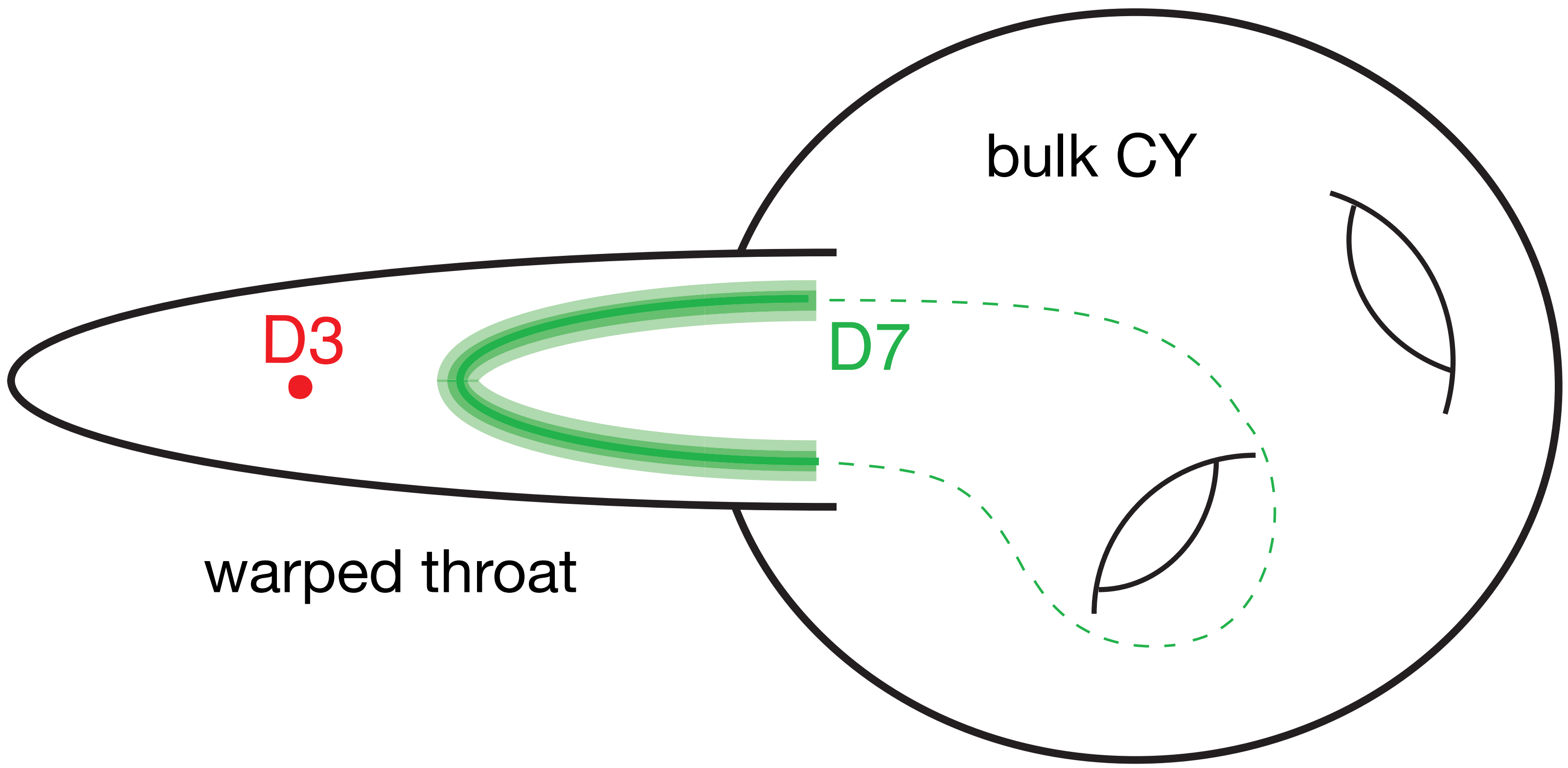

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Successful inflation can be achieved by fine-tuning this against the curvature coupling.

Although this example was very explicit, the geometry of the compactification was rather special.

Furthermore, distant sources often do not decouple and critically influence the inflationary dynamics.

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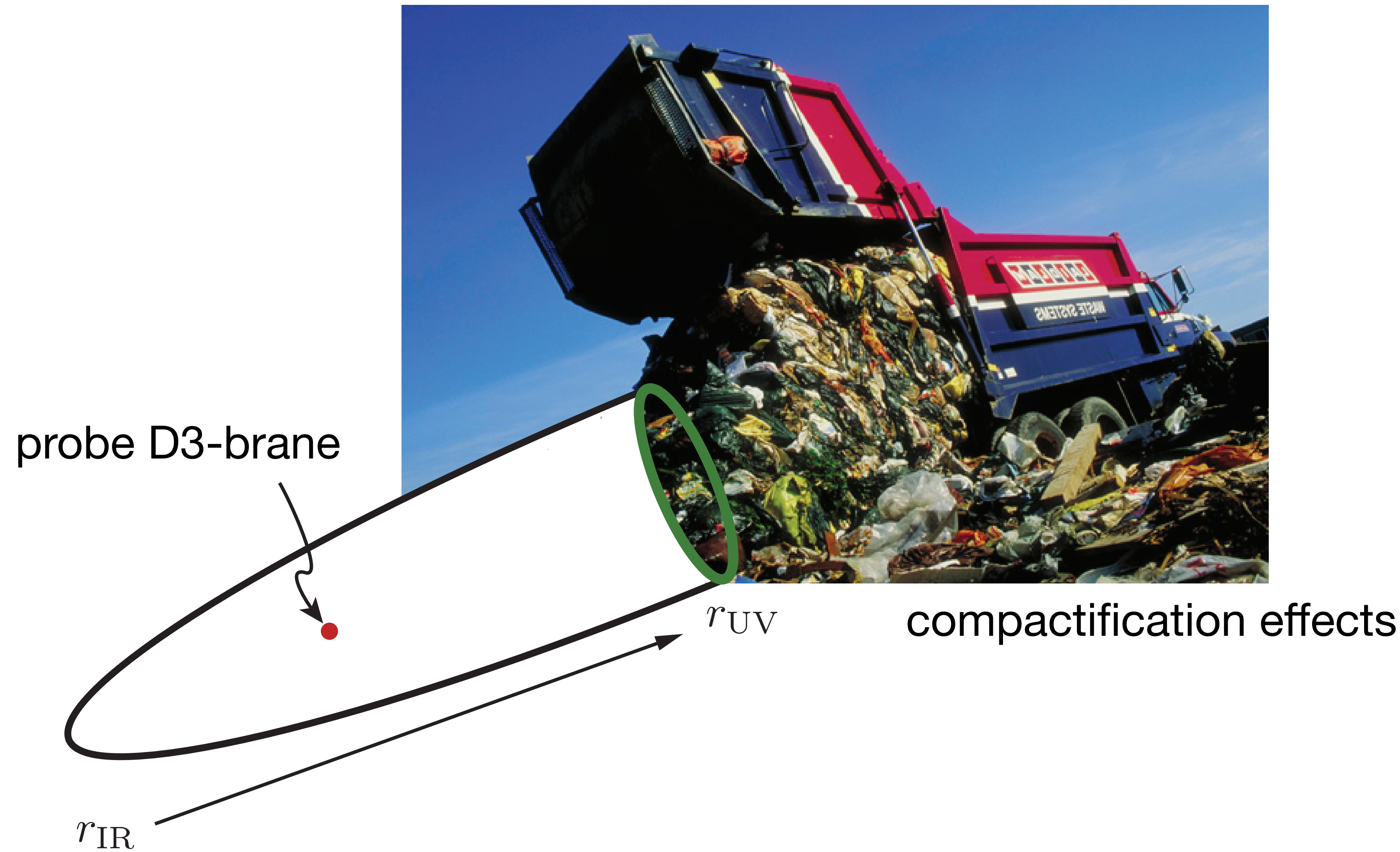
and critically influence the inflationary dynamics.

How do we characterize more general compactification effects?

D3-brane Potential from 10D



We wish to solve the following master equation:

$$\boxed{\nabla^2(e^{4A} - \alpha) = R_4 + |G_-|^2 + \text{local}}$$

+ equations of motion for fluxes and dilaton.

D3-brane Potential from 10D

The solution can be written as:

scaling dimensions

$$V(\phi, \Psi) = \sum_L c_L \left(\frac{\phi}{\Lambda}\right)^{\Delta(L)} j_L(\Psi)$$

harmonic functions on X_5

After some work, one finds

eta problem

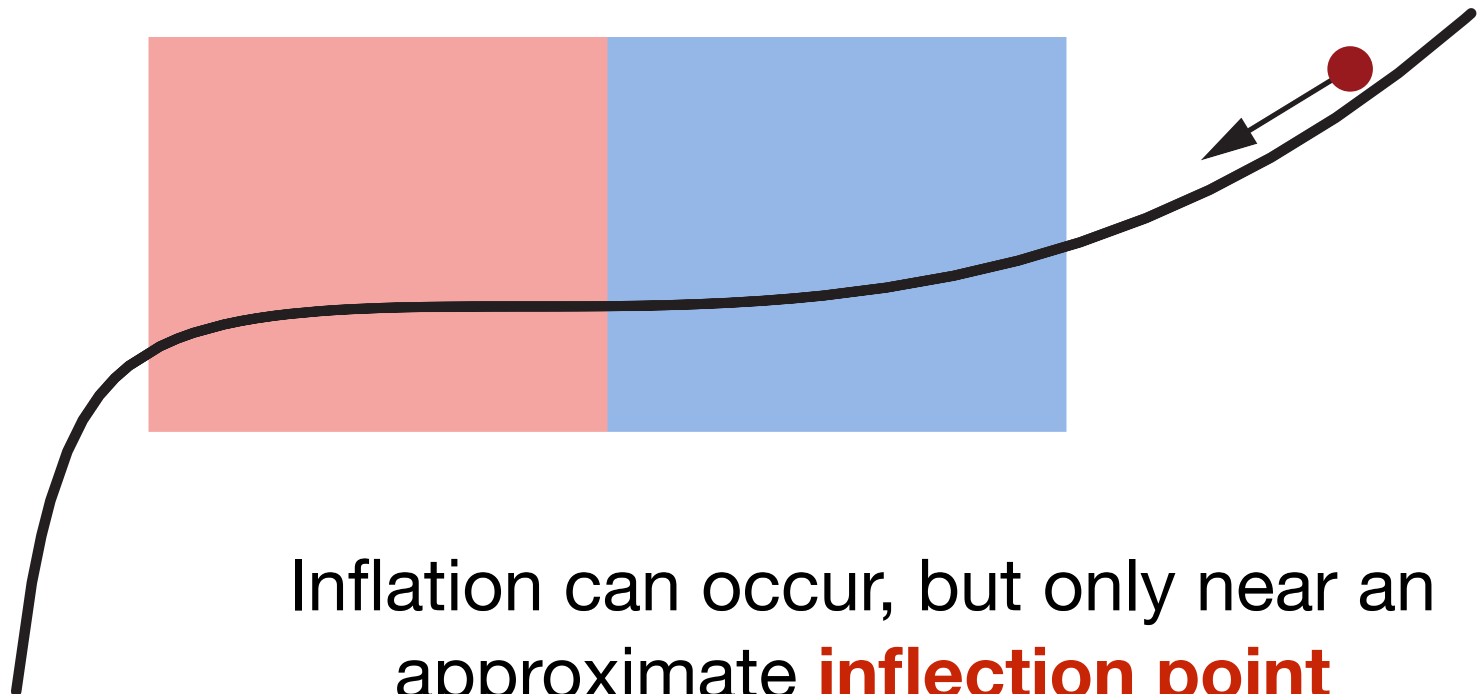
Δ_{R_4}	$\boxed{2}$	3	$\frac{7}{2}$	4	$\cdots$	
Δ_{G_-}	$\boxed{1}$	2	$\frac{5}{2}$	$\sqrt{28}-\frac{5}{2}$		$\cdots$
Δ_0	$\frac{3}{2}$	2	3	$\cdots$		

gaugino condensation

DB, Dymarsky, Kachru, Klebanov, and McAllister

Phenomenology

Successful inflation can be achieved by fine-tuning:



Inflation can occur, but only near an approximate **inflection point**

$$V(\phi) = V_0 \left[1 + c_1\, \phi^1 + c_{3/2}\, \phi^{3/2} + c_2\, \phi^2 + \ldots \right]$$

However, unless $N_{tot} > 120$, the spectrum is **too blue**!

DB, Dymarsky, Kachru, Klebanov, and McAllister

DBI Inflation

Silverstein and Tong, *Acceleration from D-celeration*

Non-Slow-Roll Dynamics

$$\mathcal{L}_{\mathrm{D3}} = T(\phi)\sqrt{1-(\partial\phi)^2/T(\phi)} - V(\phi)$$

In the limit $\dot{\phi} \to T(\phi)$, inflation may occur even for a step potential.

However, before successful inflation happens, the probe approximation for the brane breaks down.

Maldacena
Chen
DB and McAllister

No explicit example for DBI inflation in string theory is known.

Inflating with Axions

Could the inflaton be an axion?

Axions have a shift symmetry, $a \mapsto a + const.$, broken by nonperturbative effects to $a \mapsto a + 2\pi$.

The axion Lagrangian is

decay constant

$$\mathcal{L} = -\frac{1}{2} f^2 (\partial a)^2 - \Lambda^4 \cos(a)$$

$$= -\frac{1}{2}(\partial\phi)^2 - \Lambda^4 \cos\left(\frac{\phi}{f}\right)$$

Freese, Frieman, Olinto
Natural Inflation

Ex: *Show that successful inflation requires* $f \gg M_{\rm pl}$.

Is this reasonable?

Axions in String Theory

In string theory, we get axions from the dimensional reduction of p-forms:

e.g. $$B_2 = \sum_{I=1}^{h^{1,1}} b_I(x)\,\omega^I \qquad\qquad b_I \equiv \frac{1}{\alpha'}\int_{\Sigma_I} B_2$$

harmonic 2-forms

4D axions

2-cycle

What is the decay constant?

Digression: Axion Decay Constant

Svrcek and Witten
Banks, Dine, Fox and Gorbatov

Examine the axion kinetic term:

$$S_{10} \supset \frac{1}{2(2\pi)^7 g_s^2 (\alpha')^4} \int \mathrm{d}^{10}X \, |\mathrm{d}B_2|^2 \qquad B_2 \equiv \sum_I b_I(x)\omega_I$$

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$$= \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\,\gamma^{IJ}\partial^\mu b_I \partial_\mu b_J$$

$$\uparrow$$

$$\gamma^{IJ} \equiv \frac{1}{6(2\pi)^7 g_s^2 (\alpha')^4} \int_{X_6} \omega^I \wedge \star\omega^J \quad \mapsto f_I^2\,\delta_{IJ}$$

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For an isotropic compactification, with $\mathcal{V}(\alpha')^3 = L^6$, we have

$$\int \omega \wedge \star\omega \sim \sqrt{g_6}\, g_6^{\cdot\cdot} g_6^{\cdot\cdot} \sim L^2$$

Using $\alpha' M_{\mathrm{pl}}^2 = \frac{2}{(2\pi)^7} \frac{\mathcal{V}}{g_s^2}$, we get .

$$\boxed{\frac{f^2}{M_{\mathrm{pl}}^2} \sim \frac{(\alpha')^2}{L^4} \sim \frac{1}{\mathcal{V}^{2/3}}}$$

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A systematic exploration of other axions shows:

$$\boxed{\frac{f^2}{M_{\mathrm{pl}}^2} \sim \frac{g_s^\alpha}{\mathcal{V}^\beta}} \qquad \alpha \geq 0\,, \;\; \beta > 0$$

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What is the decay constant?

$$\boxed{\frac{f}{M_{\rm pl}} \sim \frac{g_s^\alpha}{\mathcal{V}^\beta}} \qquad \alpha \geq 0\,,\ \beta > 0$$

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Banks, Dine, Fox and Gorbatov

$f \ll M_{\mathrm{pl}}$ in computable limits of string theory!

Natural inflation with $V(\phi) = \Lambda^4 \cos\left(\frac{\phi}{f}\right)$ seems hard to achieve.

Proposals for Evading the No-Go

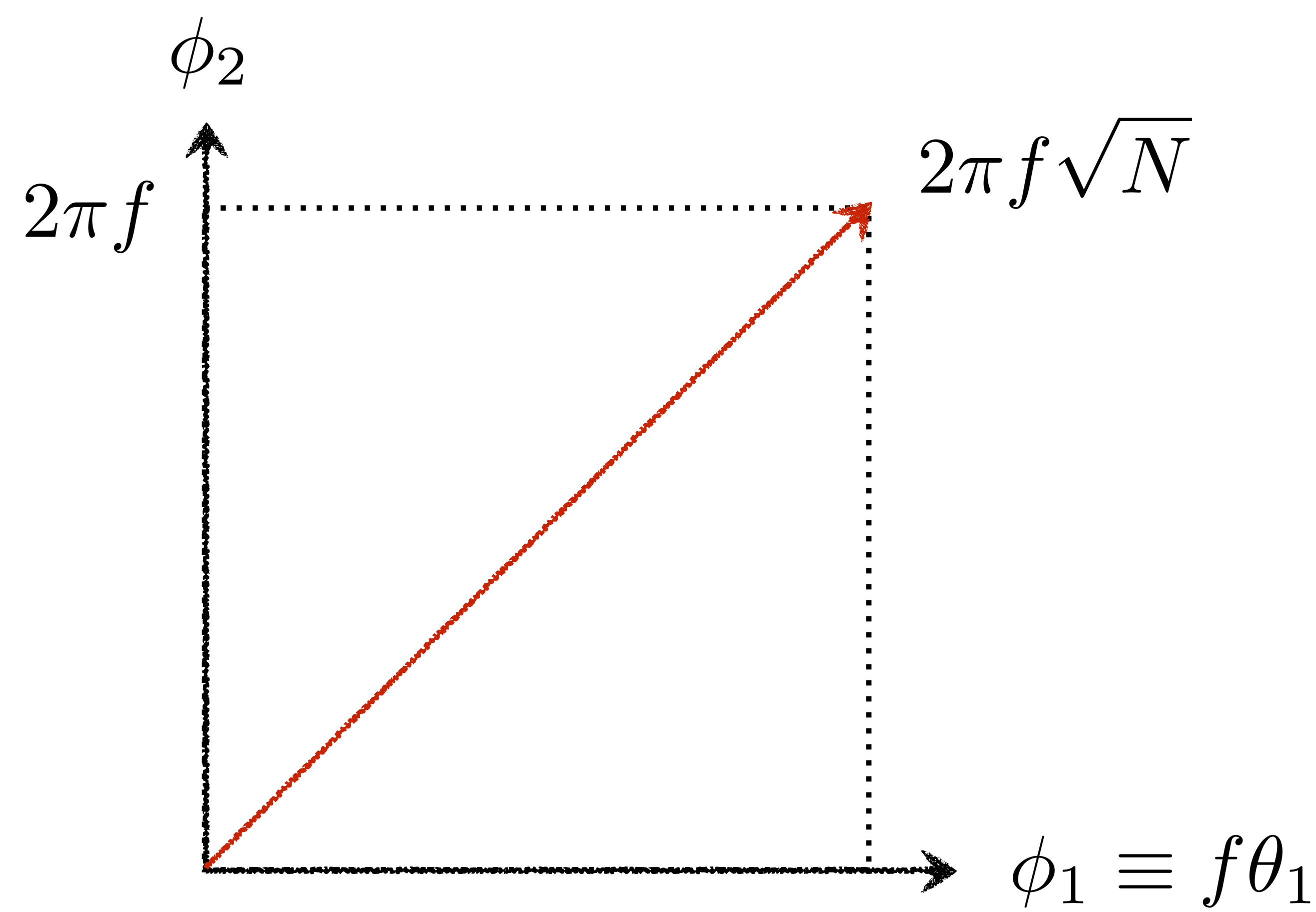
- Multiple axions
  - Assisted Inflation
    Dimopoulos, Kachru, McGreevy and Wacker
  - Lattice Alignment
    Kim, Nilles and Peloso
  - Kinetic Alignment
    Bachlechner et al.
    Burgess and Roest
- Monodromy
  McAllister, Silverstein and Westphal
- Strong Coupling
  Grimm

N-flation

Dimopoulos, Kachru, McGreevy and Wacker

Saved by Pythagoras

Consider $$\mathcal{L} = \sum_{I=1}^{N} \left[\frac{1}{2} f^2 (\partial\theta_I)^2 - \Lambda^4 \cos(\theta_I) \right]$$



$$\Delta\Phi^2 \equiv \sum_I (\Delta\phi_I)^2 > M_{\rm pl}^2 \quad \xleftrightarrow{\textbf{N-flation}} \quad N \gg 1$$

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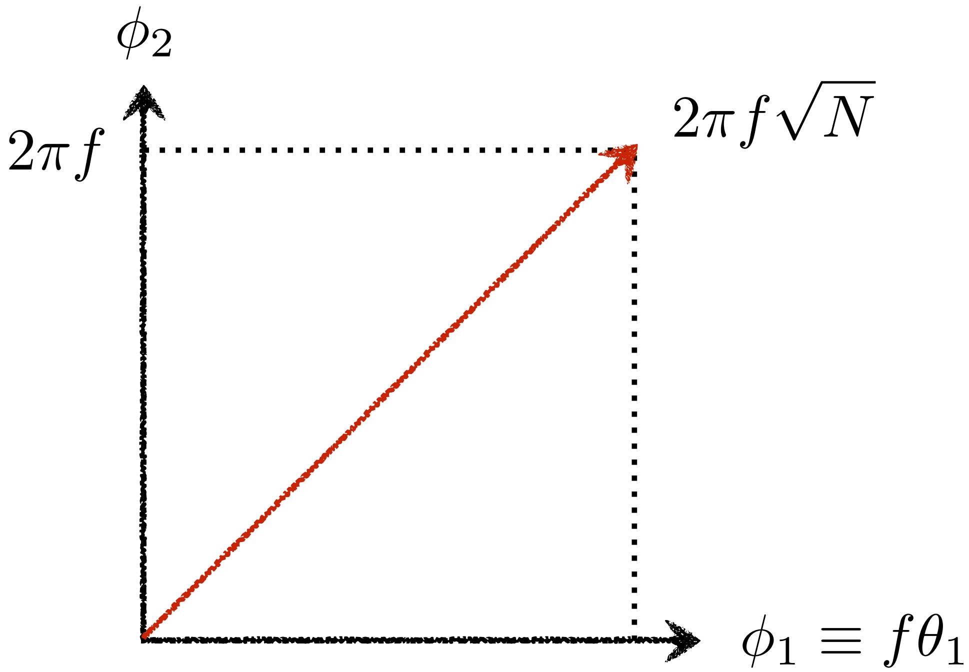

$$\Delta\Phi^2 \equiv \sum_I (\Delta\phi_I)^2 > M_{\rm pl}^2 \longleftrightarrow N \gg 1$$

Problems:

- Successful inflation requires: $N > 10^4$
- Light Kähler moduli
- Renormalization of the Planck mass: $\Delta M_{\rm pl}^2 \sim \dfrac{N}{16\pi^2}\Lambda^2$

Digression: Kinetic Alignment

Bachlechner et al.

Problems are alleviated by consider the more generic axion Lagrangian

$$\mathcal{L} = \frac{1}{2} K^{IJ} \partial^\mu \theta_I \partial_\mu \theta_J - \sum_{I=1}^{N} \Lambda_I^4 \cos(Q^{IJ} \theta_J)$$

We can diagonalize the kinetic term (*lattice basis*) or the potential (*kinetic basis*), but typically not both.

Let's work in the lattice basis:

$$\mathcal{L} = \frac{1}{2} K^{IJ} \partial^\mu \theta_I \partial_\mu \theta_J - \sum_{I=1}^{N} \Lambda_I^4 \cos(\theta_I)$$

The field range is maximal if the largest eigenvalue of the kinetic matrix points along a diagonal:

$$\Delta\Phi \sim 2\pi f_{\max} \sqrt{N}$$

This is **likely**: 2^N diagonals vs. N edges

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$$\Delta\Phi \sim 2\pi f_{\max}\sqrt{N}$$

Using arguments from **random matrix theory**, one can show that: $\sqrt{N}f < f_{\max} < Nf \longrightarrow \boxed{2\pi f N < \Delta\Phi < 2\pi f N^{3/2}}$

Bachlechner et al.

Axion Monodromy

McAllister, Silverstein and Westphal

Digression: The Axion Shift Symmetry

Wen and Witten
Dine and Seiberg

Consider the worldsheet coupling of B_2 :

$$S_\sigma \subset -\frac{1}{2\pi\alpha'}\int_{\Sigma_2} B_2 \equiv -\frac{b}{2\pi}$$

$$= -\frac{1}{4\pi\alpha'}\int_{\Sigma_2} \mathrm{d}^2\sigma\, \epsilon^{ab}\partial_a X^M \partial_b X^N B_{MN}(X)$$

Write $B_{MN}(X) = B_{MN}(0) + X^P \partial_P B_{MN}(0) + \cdots$

leads to $\partial_\mu b$

only this term can violate the shift symmetry

to get

In the absence of D-branes, $V(b)$ must vanish to all orders in α' and g_s .

On the other hand, the shift symmetry can be broken by ***D-branes*** and ***nonperturbative effects*** (instantons).

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$$S_\sigma \supset -\frac{1}{4\pi\alpha'}\int_{\Sigma_2} \mathrm{d}^2\sigma\; \partial_a \left(\epsilon^{ab} X^M \partial_b X^N B_{MN}(0)\right)$$

total derivative: vanishes if the worldsheet has ***no boundary***.

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On the other hand, the shift symmetry can be broken by ***D-branes*** and ***nonperturbative effects*** (instantons).

Axions Monodromy

McAllister, Silverstein and Westphal

- ***Idea:*** Take a compactification without D-branes ($V = 0$), then slightly lift the flat axion direction.

Axions Monodromy

McAllister, Silverstein and Westphal

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- ***Setup:*** IIB on CY_3 O3/O7

A D5-brane (NS5-brane) wrapping a two-cycle Σ_2 generates a potential for the b-axion (c-axion).

To satisfy Gauss' law an anti-D5-brane wraps a homologous 2-cycle Σ_2' .

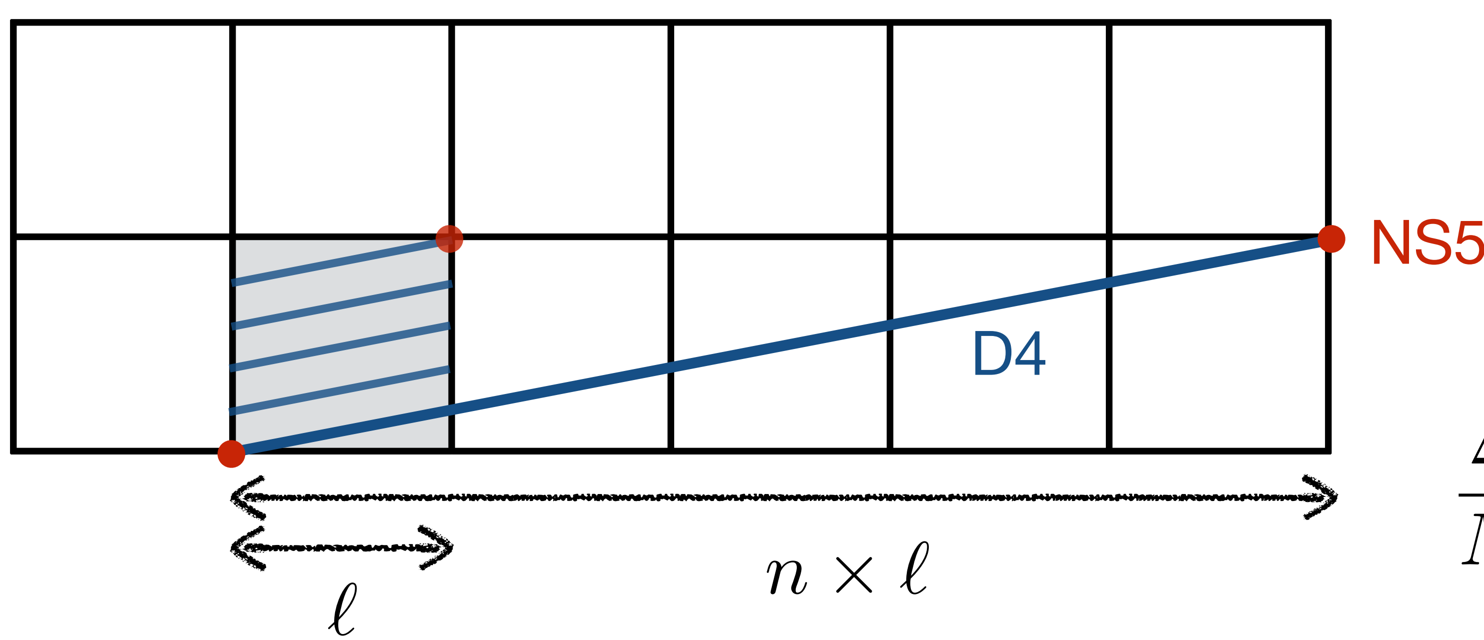
Axions Monodromy

McAllister, Silverstein and Westphal

We find the axion potential from the dimensional reduction of the 5-brane action:

$$V = 2T_5 \int_{\Sigma_2} \mathrm{d}^2\sigma \sqrt{-\det(G+B)}$$

The extended field range becomes transparent in the T-dual description:



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$$V = 2T_5 \int_{\Sigma_2} \mathrm{d}^2\sigma \sqrt{-\det(G+B)}$$

Ex: *Show that*

$$V = 2T_5 \sqrt{\ell^2 + b^2}$$ *where* ℓ *is the size of* Σ_2 *.*

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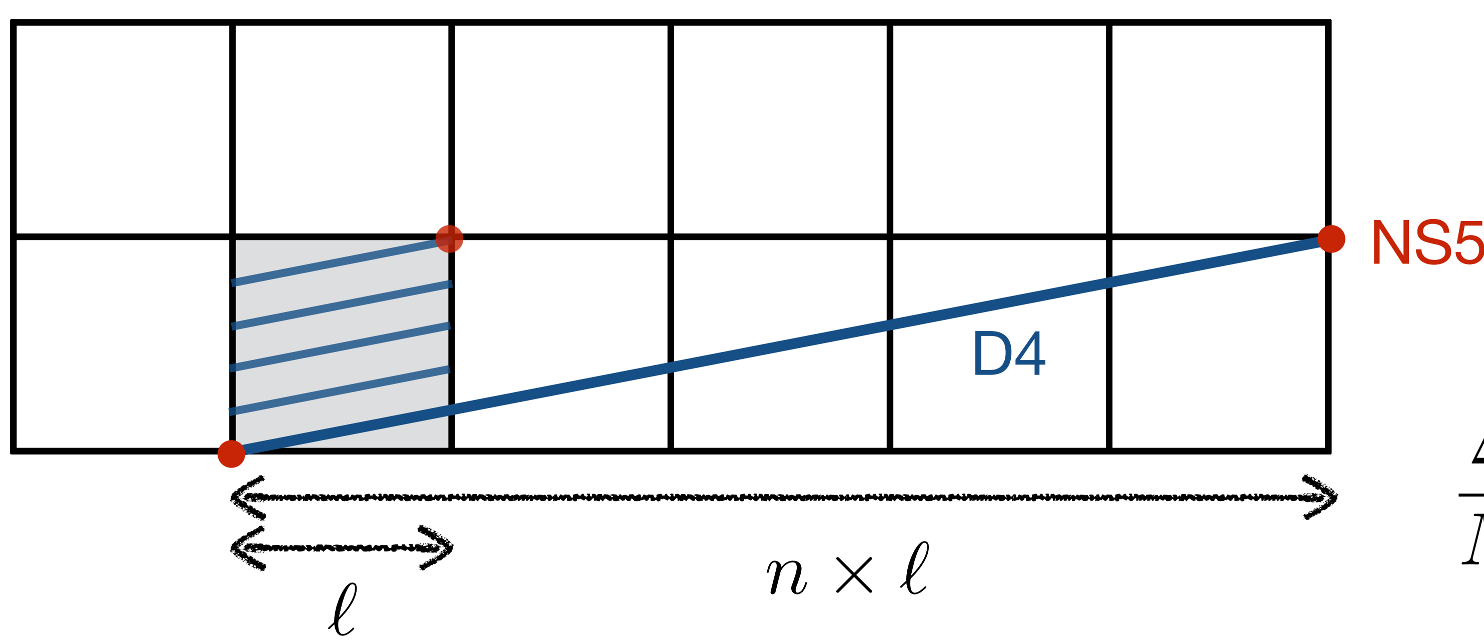

$$\frac{\Delta\phi}{M_{\mathrm{pl}}} \sim \frac{n}{\ell^2} > 1$$

Axions Monodromy Inflation

For $b \gg \ell$, we have $V \approx 2T_5\, b \equiv \mu^3 f\, b$.

The canonically-normalized inflaton action is

$$\boxed{\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \mu^3\phi}$$

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Generalization to other axions:

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \mu^{4-p}\phi^p \qquad \text{with} \quad p < 2$$

Consistency Checks

- Symmetry breaking from nonperturbative effects
  - ✳ universal eta problem for the b-axion
  - ✳ success of the c-axion model-dependent

- Symmetry breaking from backreaction
  - ✳ induced D3-brane charge on NS5-brane

→ backreacts on the geometry
changes the strengths of nonperturbative effects
modifies the inflaton potential

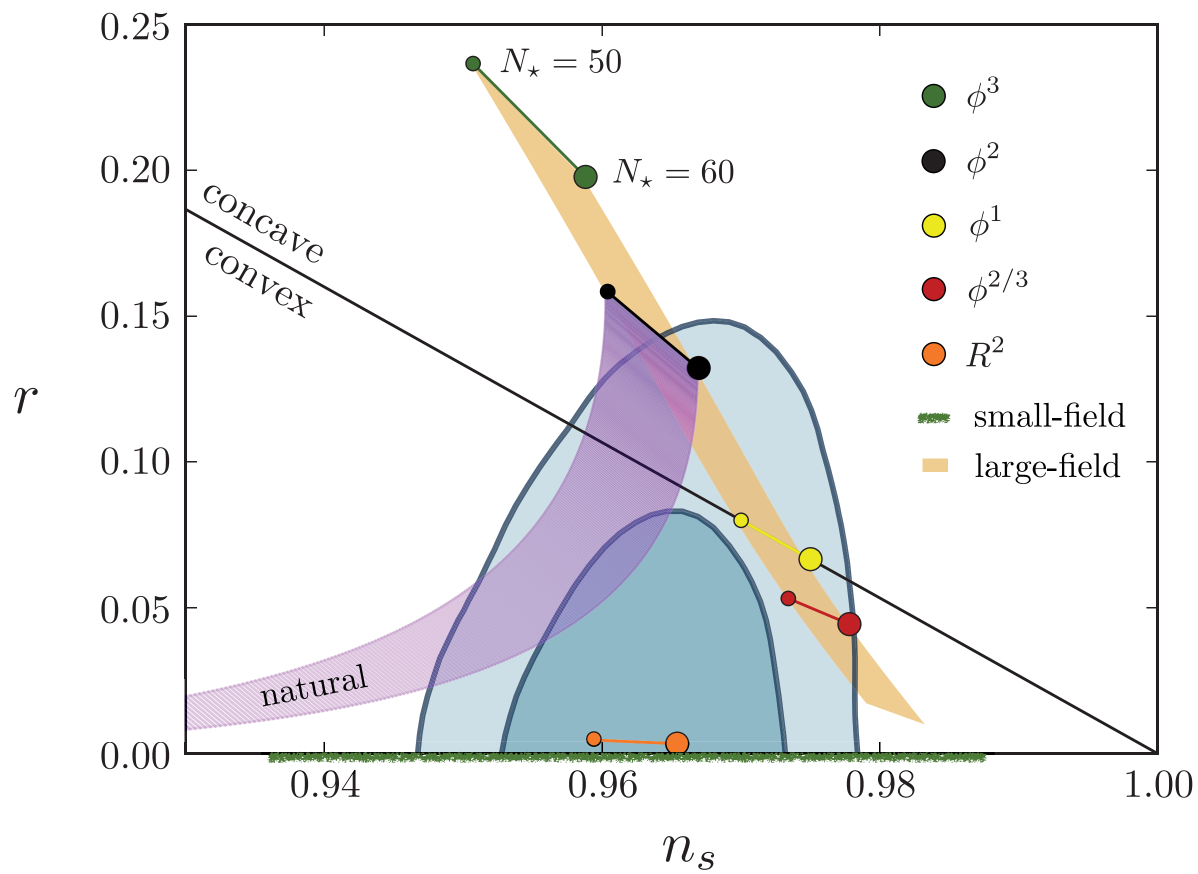
↕

geometry must be arranged to have
an *additional* approximate symmetry

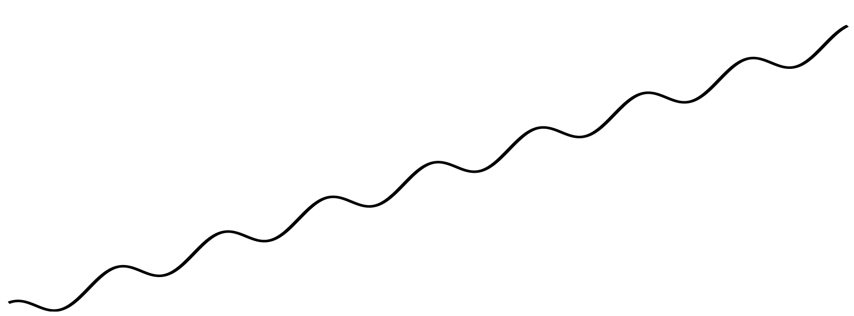
Please see our review or the original papers for the details.

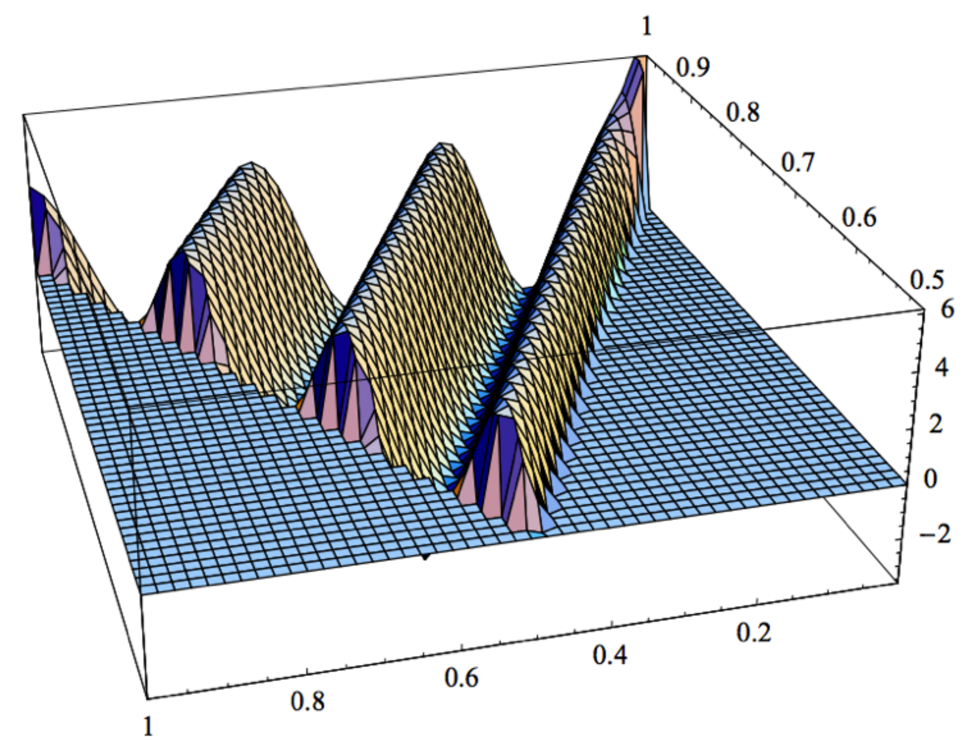
Phenomenology

- Large tensors



- (Maybe) resonant non-Gaussianity





Summary and Conclusions

- The Planck data is incredible!
- Inflation successfully describes the data.
- The inflationary mechanism is UV-sensitive.
- Inflationary models in string theory exist
  ... but should be explored further.

Thanks for your attention!