5/2/15 (1)

CERN Winter School
Lecture 3.

1) Today, I will describe an explicit construction of the $\tilde{a}_w$ operators in the CFT. Then, I will describe how it fulfills all the properties we demanded and resolves all the AMPS paradoxes.

2). First, we have to re-examine a little more carefully what one can measure in EFT.
So far we have considered correlators

$$\langle \phi(x_1) \ldots \phi(x_n) \rangle$$

but there are constraints on these observables if we want them to respect EFT principles
First, we need

a) $n \ll N^2$; the number of insertions in the correlator cannot scale with N or you insert a huge amount of energy and the background may change completely

b) $|x_i - x_j| \gg \frac{1}{N^2}$ always. If two points are separated by Planck distance then we cannot expect simple EFT predictions to hold.

c) $|x_i - x_j| \ll N^2$: If we take a very long time limit, EFT may break down.

3) To make this precise, think in terms of modes.

$$O^c_{\omega_n} = \frac{1}{T}\int_{-T}^{T} O(t)\, e^{-i\omega_n t}\, dt$$

over some $Or[1]$ time scale $-T$ to T which may be large but smaller than N.

$$1 \ll T \ll N.$$

[We are suppressing the "x" dependence here.]

4) Now we consider polynomials in these coarse grained modes

$$P(O^c_{\omega_n})$$

like

$$P = \alpha_1 O^c_{\omega_1} O^c_{\omega_2} \ldots O^c_{\omega_{n_1}} + \alpha_2 O^c_{\omega_1} O^c_{\omega_2} + \alpha_3 O^c_{\omega_1} O^c_{\omega_2} O^c_{\omega_3} + \ldots$$

This space of polynomials is a linear space and we will call it $\mathcal{A}$, and we will call some representative of this set A_α.

Then, we must also restrict the polynomials so that

a) the total order $\ll N$

b) the total energy in any monomial $\ll N$

With these restrictions $\mathcal{A}$ is the set of "reasonable experiments" that one can do.

5) Now consider a given pure state $|\psi\rangle$.
By acting with elements of $\mathcal{A}$ we get a subspace

$$H_{|\psi\rangle} = \mathcal{A}\,|\psi\rangle.$$

This is the part of the Hilbert space that an infalling observer can measure while falling into a B.H. in the state $|\psi\rangle$.
A basis for this space is given by taking a basis for $\mathcal{A}$, $\hat{A}_1 \dots \hat{A}_D$ and considering

$$\hat{A}_1|\psi\rangle \;\dots\; \hat{A}_D|\psi\rangle.$$

6) For all except a measure zero space of states we have

$$A_\alpha|\psi\rangle = 0 \Rightarrow A_\alpha = 0.$$

States that are annihilated by A_α are pathological; may be firewall states.

8) Now, we will construct appropriately coarse grained $\tilde{a}_w^c$. For sake of consistency with the literature, we just construct $\tilde{O}_w^c$.

To lighten the notation, I will now drop the $\underline{c}$ superscript, and just write O_w and $\tilde{O}_w$, but keep in mind that these are both slightly coarse-grained

9) We simply demand that

$$\tilde{O}_w A_\alpha |\psi\rangle = A_\alpha e^{-\beta w/2} O_w^\dagger |\psi\rangle.$$

$\forall A_\alpha$. By extension $\tilde{O}_w^\dagger A_\alpha |\psi\rangle = A_\alpha e^{\beta w/2} O_w |\psi\rangle$.

This is actually a set of linear equations for a linear operator in $H |\psi\rangle$.

If we satisfy this for all elements of the basis

$$\tilde{O}_w \mathring{A}_i |\psi\rangle = \mathring{A}_i e^{-\beta w/2} O_w^\dagger |\psi\rangle.$$

then it will automatically hold for all elements of $\mathcal{A}$ because $\mathring{A}_i$ appears linearly on both sides.

10) But these linear equations always have a solution, because we argued previously the

$$A^{l}_{1}|\psi\rangle \quad \ldots \quad A^{l}_{D}|\psi\rangle$$

were all linearly independent, and it is possible to arbitrarily specify the action of a linear operator on a linearly independent set of vectors.

More explicitly, define

$$|V_m\rangle = A^{l}_{m}|\psi\rangle.$$

$$|U_m\rangle = A_m \, O^{+}_{\omega}|\psi\rangle e^{-\beta\omega/2}.$$

Now define

$$g_{mn} = \langle V_m|V_n\rangle$$

and then

$$\tilde{O}_{\omega} = g^{mn}|U_m\rangle\langle V_n|.$$

this is an explicit solution to our linear equations

11) So far I have defined $\tilde{O}$ to act on descendants of $|\psi\rangle$ produced by acting with O_{ω_n}. But we also have the operator H, which we have used.

So we impose, in addition,

$$[\tilde{O}_\omega, H] A_\alpha |\psi\rangle$$
$$= -\omega \tilde{O}_\omega A_\alpha |\psi\rangle$$
$$= -\omega A_\alpha O_\omega^\dagger |\psi\rangle e^{-\beta\omega/2}$$

There is no problem in including this additional relation even on energy eigenstates.

12) Now we will check that this obeys all the relations that we need on a given state and its descendants

Let us check

a) Cross Commutator:

$$[\tilde{O}_\omega, O_{\omega'}] A_\alpha |\psi\rangle$$

$$= \tilde{O}_\omega O_{\omega'} A_\alpha |\psi\rangle - O_{\omega'} \tilde{O}_\omega A_\alpha |\psi\rangle$$

$$= (O_{\omega'} A_\alpha O^\dagger_\omega |\psi\rangle - O_{\omega'} A_\alpha O^\dagger_\omega |\psi\rangle) e^{-\beta\omega/2}$$

↑ Because $O_{\omega'} A_\alpha$ is also part of $\cancel{\mathcal{A}}$.

$$= 0$$

b). Canonical Commutator

$$[\tilde{O}_\omega, \tilde{O}^\dagger_{\omega'}] A_\alpha |\psi\rangle$$

$$= \tilde{O}_\omega \tilde{O}^\dagger_{\omega'} A_\alpha |\psi\rangle - \tilde{O}^\dagger_{\omega'} \tilde{O}_\omega A_\alpha |\psi\rangle$$

$$= e^{\beta\omega'/2} \tilde{O}_\omega A_\alpha O_{\omega'} |\psi\rangle - e^{-\beta\omega/2} \tilde{O}^\dagger_{\omega'} A_\alpha O^\dagger_\omega |\psi\rangle$$

$$= e^{\beta\omega'/2} A_\alpha O_{\omega'} e^{-\beta\omega/2} O^\dagger_\omega |\psi\rangle$$

$$- e^{-\beta\omega/2} A_\alpha O^\dagger_\omega e^{\beta\omega'/2} O_{\omega'} |\psi\rangle$$

$$= e^{\beta(\omega'-\omega)/2} A_\alpha [O_{\omega'}, O^\dagger_\omega] |\psi\rangle$$

$$= A_\alpha |\psi\rangle \quad \left[\text{Using } [O_{\omega'}, O^\dagger_\omega] = \delta_{\omega'\omega}\right]$$

13). Define

$$\tilde{a}_\omega = \frac{\tilde{O}_\omega}{\sqrt{G_\omega}}.$$

[Recall $G_\omega = \langle\Psi| [O_\omega, O_\omega^\dagger] |\Psi\rangle.$]

Then $\tilde{a}_\omega$ has just the right two point functions.

$$\langle\Psi| \tilde{a}_\omega \tilde{a}_\omega^\dagger |\Psi\rangle = \langle\Psi| \tilde{a}_\omega a_\omega |\Psi\rangle e^{\beta\omega/2}$$
$$= \langle\Psi| a_\omega a_\omega^\dagger \rangle e^{\beta\omega/2} e^{-\beta\omega/2}$$
$$= \frac{1}{1-e^{-\beta\omega}}$$

which we proved for the ordinary operators.

The cross two point function also works

$$\langle\Psi| \tilde{a}_\omega a_\omega |\Psi\rangle = \langle\Psi| a_\omega a_\omega^\dagger |\Psi\rangle e^{-\beta\omega/2}$$
$$= \frac{e^{-\beta\omega/2}}{1-e^{-\beta\omega}}$$

which is just what we need.

14) We already imposed the right commutator with the Hamiltonian $[H, \tilde{a}_\omega] = \omega\tilde{a}_\omega$ so we have satisfied <u>all</u> the properties we need.

15) For good measure, let us compute the number operator as seen by the infalling observer.

Recall

$$N_a = \frac{1}{1-e^{-\beta\omega}}\Big[(a^+ - e^{-\beta\omega/2}\tilde{a})(a - e^{-\beta\omega/2}\tilde{a}^+) + (\tilde{a}^+ - e^{-\beta\omega/2}a)(\tilde{a} - e^{-\beta\omega/2}a^+)\Big]$$

we see explicitly that

$$N_a|\psi\rangle = 0$$

and so the infalling observer will measure no particles.

16) Let us now examine how this is consistent with the analysis we did earlier.

The key point is that our operators are <u>state-dependent</u>. They act correctly on a given equilibrium state and on its descendants.

The arguments of AMPS were constructed by assuming that the $\tilde{a}$ should have the right properties globally which is too strong.

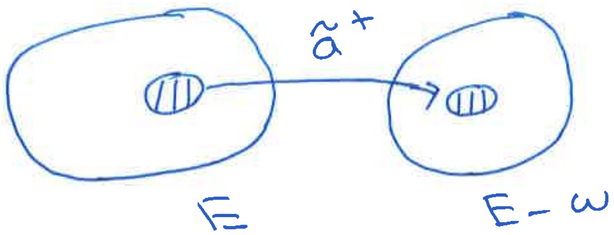
16) Consider the counting argument.

We find that

$$[\tilde{a}, \tilde{a}^+] = 1$$

only on $H|\psi\rangle$.

The picture is as follows



$\tilde{a}^+$ acts on only on a small subspace at energy E and maps it to an equal sized subspace at energy E-ω. The fact that globally there are fewer states at E-ω than at E is unimportant.

That being said, I should point out that the strict counting argument can also be solved by $\frac{1}{N}$ corrections, or "edge effects"

If we take the boundary single trace operator $\int_{-T}^{T} O(t)\, e^{i\omega t}$

then you can check that it contains small positive energy tails we can add the same positive energy tails to $\tilde{a}^+$ and ensure that it does not really have any null vectors.

17) A more robust argument is provided by the "occupancy argument". However, this argument relied on evaluating

$$\mathrm{Tr}\,(e^{-\beta H}\,\tilde{a}_\omega \tilde{a}_\omega^+)$$

and assuming that $\tilde{a}_\omega, \tilde{a}_\omega^+$ do not change as we change the state.
For state dependent operators, we cannot use this trace argument

18) The same objection kills the $N_a \neq 0$ argument.
we cannot change bases for state-dependent operators, because "N_a" depends on the state in which you are measuring it.
If you go to a far-away region of the Hilbert space, you cannot use the same operator to represent what one thinks of as "the same observable"

1a) The answer to the strong subadditivity paradox is also interesting. If we thinks of the C as the $\tilde{a}$, then we seet that although we designed $\tilde{a}$ so that

$$[\tilde{a}, a] A_\alpha |\psi\rangle = 0$$

$[\tilde{a}, a] \neq 0$ as an operator.

So, C and A are not <u>independent</u> degrees of freedom although this interdependence is cleverly hidden.

2a) To see the non-locality, we need to measure a N-pt correlation function. So the claim is that

$$\langle O_1 \cdots O_n [\phi_{P_1}, \phi_{P_2}] O_{n+1} \cdots O_N \rangle \neq 0$$

even if P_1, P_2 are spacelike separated. However we should expect such non-locality in quantum gravity.