Progress in Lattice QCD: a new landscape

Aida X. El-Khadra
(University of Illinois)

15th International Conference on B-Physics at Frontier Machines (Beauty 2014), Edinburgh, 14-18 July 2014

Progress in Lattice QCD: a new landscape

Thanks to better methods (algorithms, formalism/theoretical understanding) and significant increases in computational resources we now have a growing number of results for

- ☆ simple quantities with unprecedented precision
- ☆ new quantities (two hadron systems, resonances, ...) with control over systematic errors

Farah Willenbrock

Progress in Lattice QCD:
a new landscape

highlights of

Thanks to better methods (algorithms, formalism/theoretical understanding) and significant increases in computational resources we now have a growing number of results for

- ☆ simple quantities with unprecedented precision
- ☆ new quantities (two hadron systems, resonances, ...) with control over systematic errors

Lattice 2014 in NYC, June 23-28 2014

Farah Willenbrock

Outline

- Motivation and introduction
- Simple quantities with single, (almost) stable hadrons
  - ☆ low-lying QCD spectrum
  - ☆ weak decays (leptonic, semileptonic, mixing)
    → CKM, BSM phenomenology
  - ☆ high precision → including QED
- Beyond simple quantities
  - ☆ $K \to \pi\pi$ amplitudes and Δm_K
  - ☆ resonances, ...
- Conclusions & Outlook

Why Lattice QCD?

generic weak process involving hadrons:

(experiment) = (known) x (**CKM elements**) x (had. matrix element)

$$\Delta m_{d(s)}$$

$$\frac{d\Gamma(B\to\pi\ell\nu)}{dq^2}, \frac{d\Gamma(D\to K\ell\nu)}{dq^2}, \ldots$$

$$\frac{d\Gamma\left(B_{(s)}\to D^{(*)}_{(s)}\ell\nu\right)}{d\omega},$$

$$\Gamma_{K\ell3}, \Gamma_{K\ell2}, \ldots$$

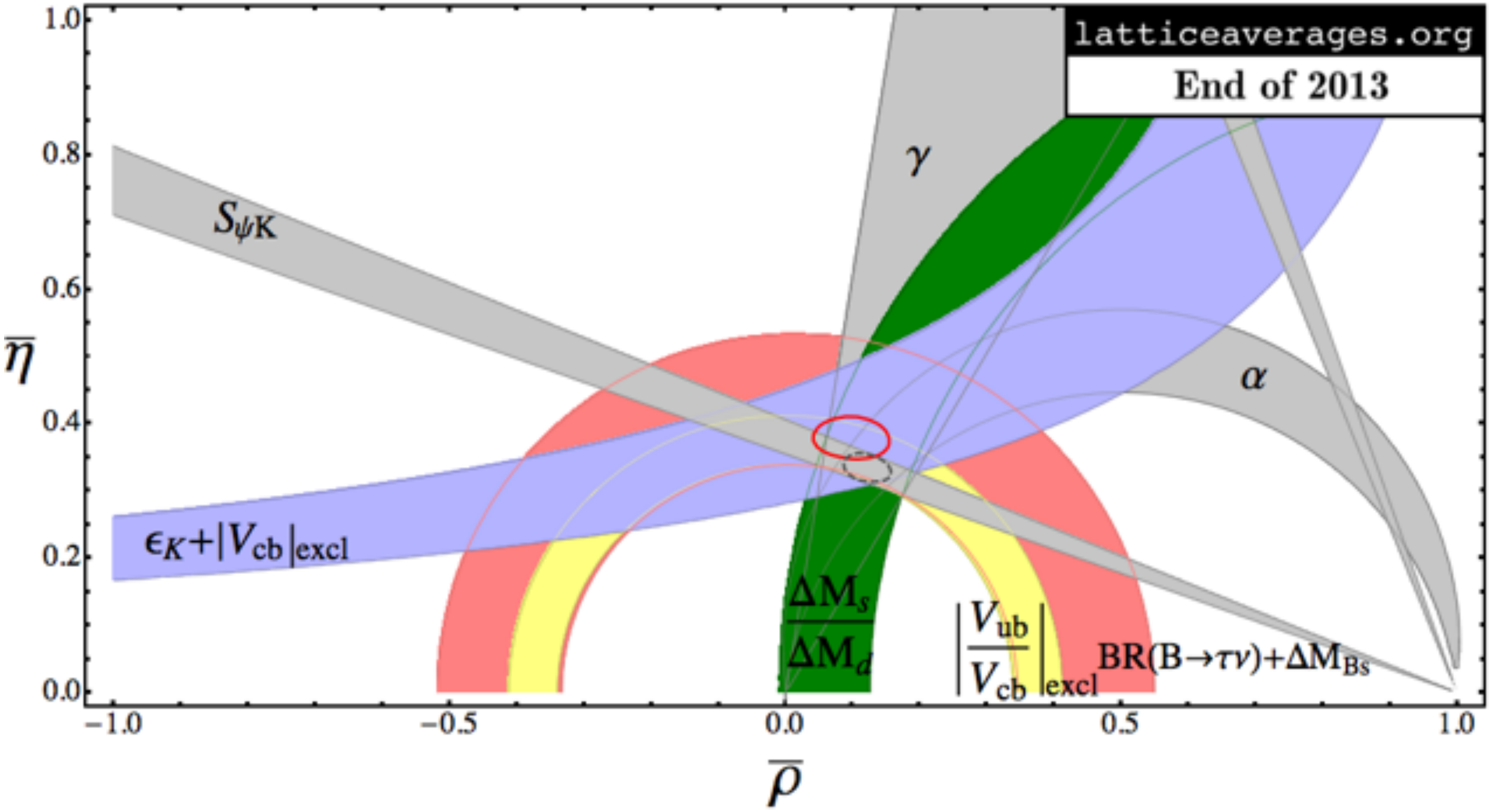
$$\vdots$$

Lattice QCD

parameterize the ME in terms of form factors, decay constants, bag parameters, ...

Why Lattice QCD?

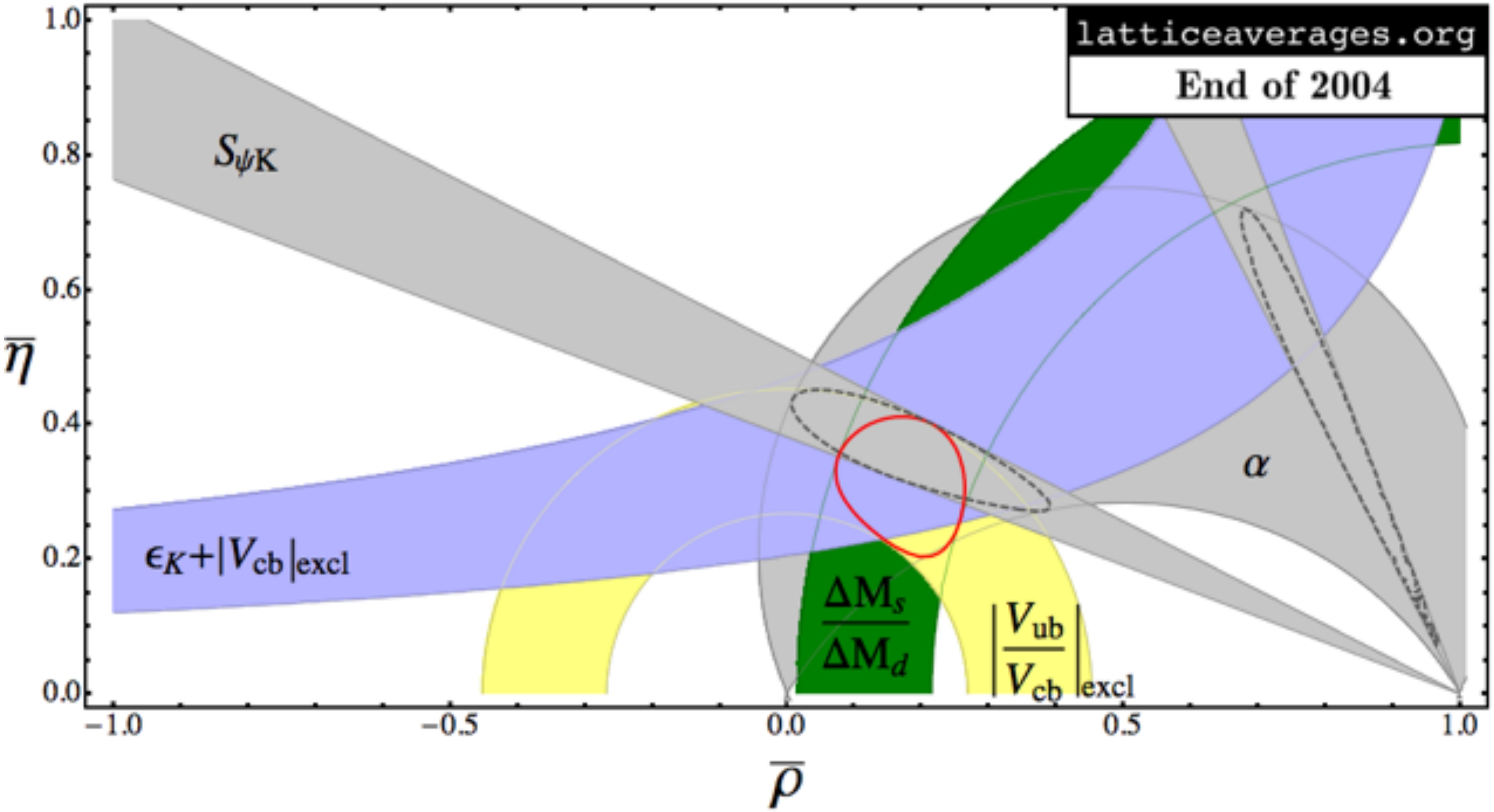
Laiho, Lunghi & Van de Water (Phys.Rev.D81:034503,2010), E. Lunghi, private comm.



The (red, yellow, green and blue) error bands are (still) dominated by theory errors, in particular by errors on hadronic matrix elements calculated in LQCD.

Why Lattice QCD?

Laiho, Lunghi & Van de Water (Phys.Rev.D81:034503,2010), E. Lunghi, private comm.



The (red, yellow, green and blue) error bands are (still) dominated by theory errors, in particular by errors on hadronic matrix elements calculated in LQCD.

Introduction to Lattice QCD

$$\langle \mathcal{O} \rangle \sim \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A \, \mathcal{O}(\psi, \bar{\psi}, A) \, e^{-S} \qquad S = \int d^4x \left[\bar{\psi}(\not{D} + m)\psi + \frac{1}{4}(F^a_{\mu\nu})^2 \right]$$

use monte carlo methods (importance sampling) to evaluate the integral.

Note: Integrating over the fermion fields leaves $\det(\not{D} + m)$ in the integrand. The correlation functions, $\mathcal{O}$, are then written in terms of $(\not{D}+m)^{-1}$ and gluon fields.

steps of a lattice QCD calculation:

1. generate gluon field configurations according to $\det(\not{D}+m)\, e^{-S}$
2. calculate quark propagators, $(\not{D}+m_q)^{-1}$, for each valence quark flavor and source point
3. tie together quark propagators into hadronic correlation functions (usually 2 or 3-pt functions)
4. statistical analysis to extract hadron masses, energies, hadronic matrix elements, …. from correlation functions
5. systematic error analysis

systematic error analysis

...of lattice spacing, chiral, and finite volume effects is based on EFT (Effective Field Theory) descriptions of QCD ➙ ab initio

The EFT description:

- provides functional form for extrapolation (or interpolation)
- can be used to build improved lattice actions/methods
- can be used to anticipate the size of systematic effects

To control and reliably estimate the systematic errors

- repeat the calculation on several lattice spacings, spatial volumes, light quark masses

systematic error analysis

...of lattice spacing, chiral, and finite volume effects is based on EFT (Effective Field Theory) descriptions of QCD ➙ ab initio

The EFT description:

- provides functional form for extrapolation (or interpolation)
- can be used to build improved lattice actions/methods
- can be used to anticipate the size of systematic effects

To control and reliably estimate the systematic errors

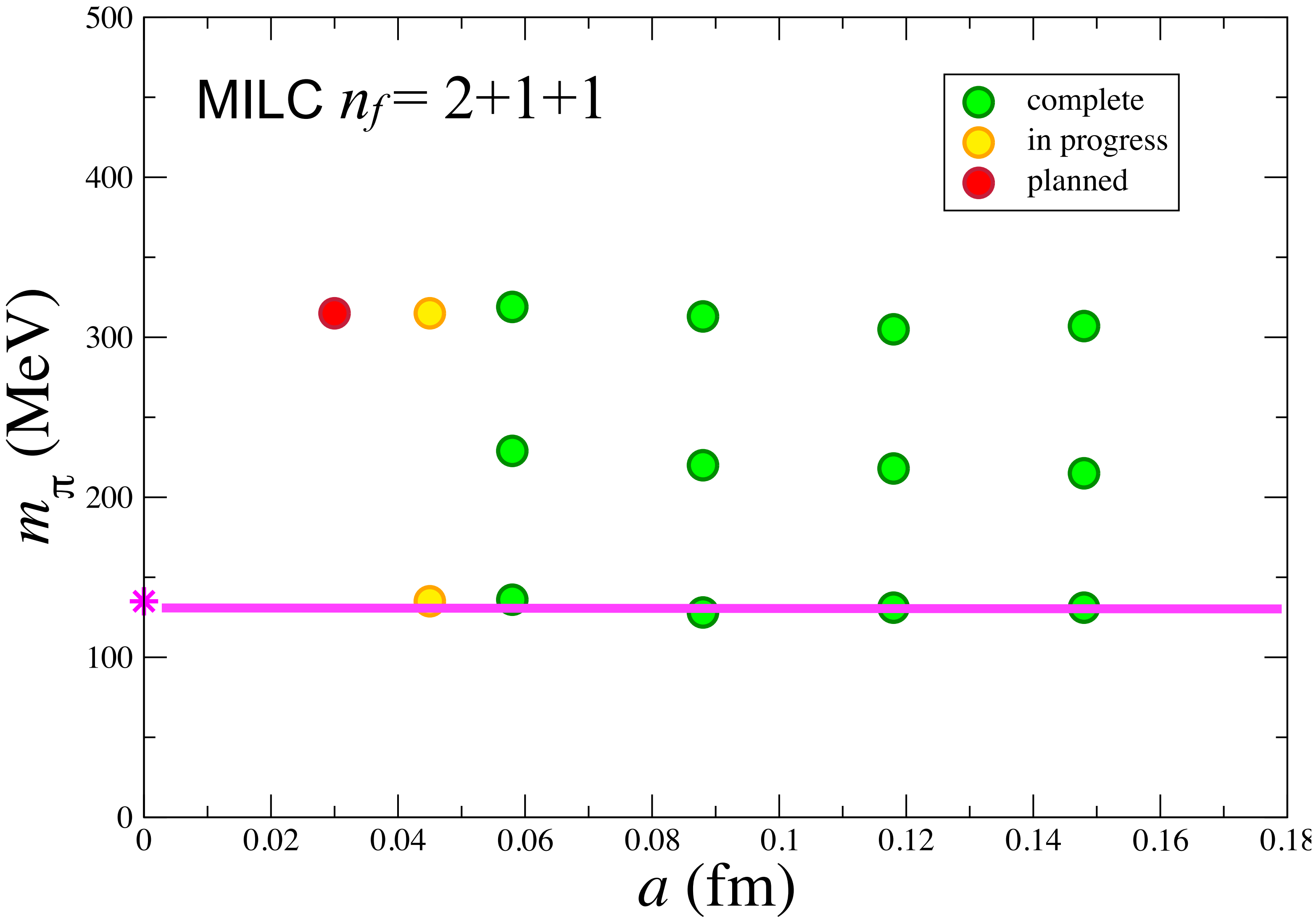
- repeat the calculation on several lattice spacings, spatial volumes, light quark masses

see the backup slides for more details on:

- ✦ **EFT description of discretization effects**
- ✦ **strategies for heavy quark methods**
- ✦ **light quark mass effects, a.k.a chiral extrapolation**
- ✦ **finite volume effects**

systematic error analysis

For example, set of ensembles by MILC collaboration



Five collaborations have now generated sets of ensembles that include sea quarks with physical light-quark masses:

PACS-CS, BMW, MILC, RBC/UKQCD, ETM

Strategy

- Lattice QCD action has the same free parameters as continuum QCD:
  quark masses and α_s

- use experimentally measured hadron masses as input, for example:
  π, K, D_s, B_s mesons for u, d, s, c, b quark masses

- need an experimental input to determine the lattice spacing (a) in GeV:
  2S-1S splitting in Υ system, f_π, Ω, Ξ mass, …
  this also determines α_s

- lattice QCD calculations of all other quantities should agree with experiment …

Simple quantities in LQCD

Stable (or almost stable) hadrons, masses and amplitudes with no more than one initial (final) state hadron, for example:

- π, K, D, D_s, B, B_s mesons

  masses, decay constants, weak matrix elements for mixing, semileptonic and rare decay form factors

- charmonium and bottomonium (η_c, J/ψ, h_c, …, η_b, $\Upsilon(1S)$, $\Upsilon(2S)$, ..) states below open D/B threshold

  masses, leptonic widths, electromagnetic matrix elements

This list includes low-lying hadron spectrum and most of the important quantities for CKM physics.

Excluded are ρ, K^* mesons and other resonances.

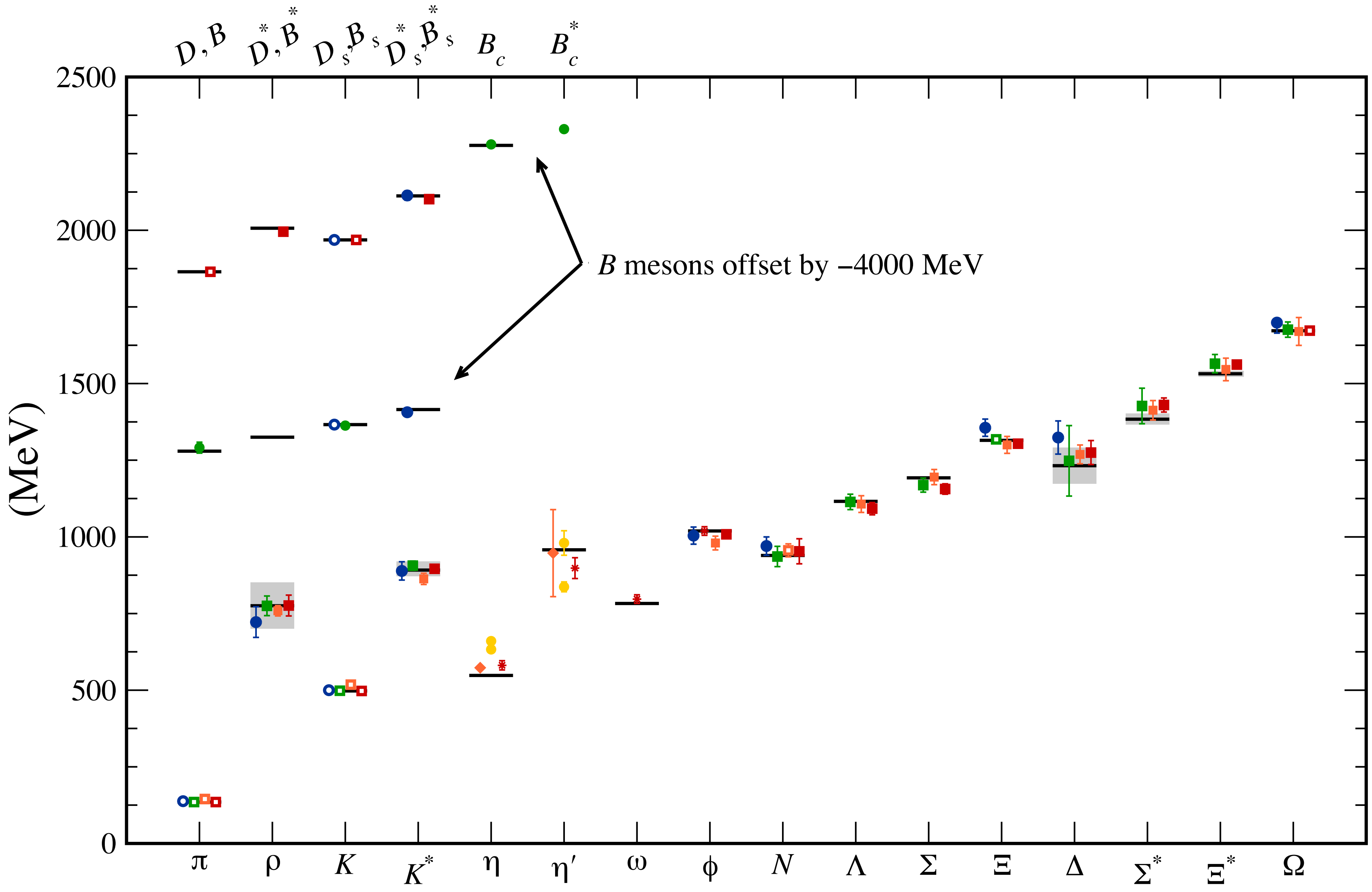
Simple quantities in LQCD

Focus on results with complete error budgets and reliable systematic error estimates.

- ☆ low-lying hadron spectrum ➙ quark masses, α_s
- ☆ weak decays (leptonic, semileptonic, mixing) ➙ CKM, BSM phenomenology
- ☆ high precision ➙ including QED

Low-lying hadron spectrum

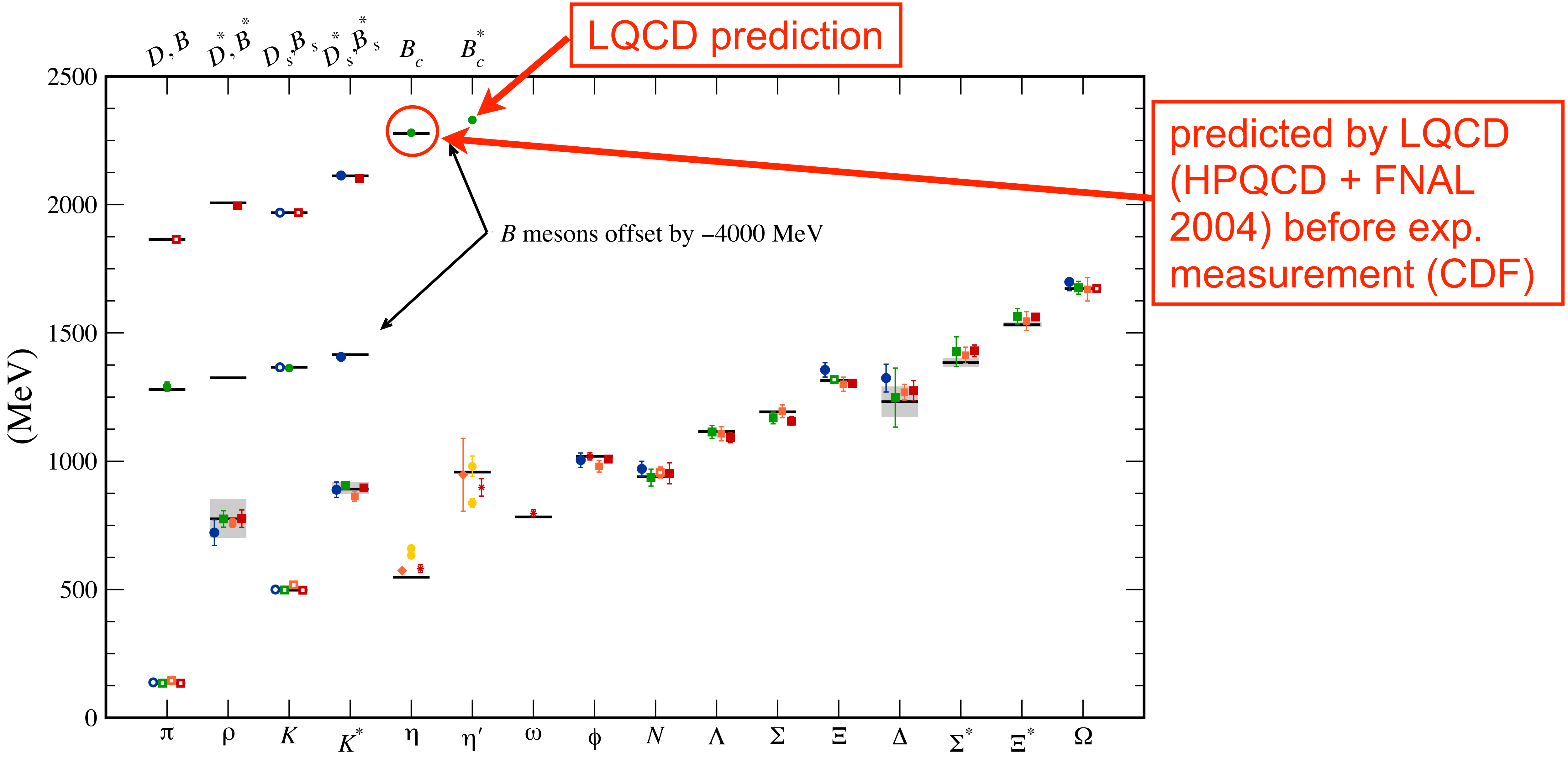
A. Kronfeld (Annu. Rev. Part. & Nucl. Sci, arXiv:1203.1204, updated)



$\pi\ldots\Omega$: BMW, MILC, PACS-CS, QCDSF; η-η': RBC, UKQCD, Hadron Spectrum (ω);
D, B: Fermilab, HPQCD, Mohler-Woloshyn

Low-lying hadron spectrum

A. Kronfeld (Annu. Rev. Part. & Nucl. Sci, arXiv:1203.1204, updated)



LQCD prediction

predicted by LQCD (HPQCD + FNAL 2004) before exp. measurement (CDF)

π…Ω: BMW, MILC, PACS-CS, QCDSF; η-η′: RBC, UKQCD, Hadron Spectrum (ω);
D, *B*: Fermilab, HPQCD, Mohler-Woloshyn

Simple quantities in LQCD

Focus on results with complete error budgets and reliable systematic error estimates.

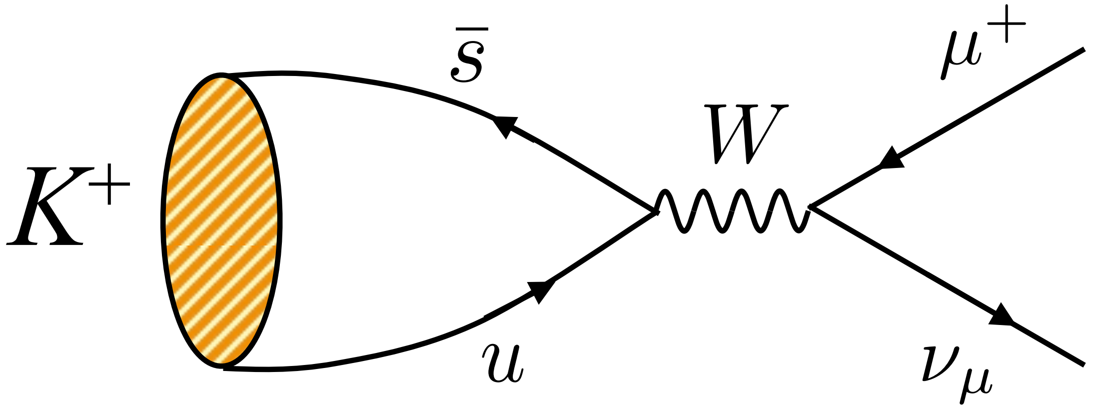
- ☆ low-lying hadron spectrum ➙ quark masses, α_s
- ☆ weak decays - leptonic, semileptonic, mixing
  - ✦ Kaons
  - ✦ D mesons
  - ✦ B mesons

➙ CKM, BSM phenomenology

- ☆ high precision ➙ including QED

Leptonic K, D, B decays

example: $K^+ \to \mu^+ \nu_\mu$

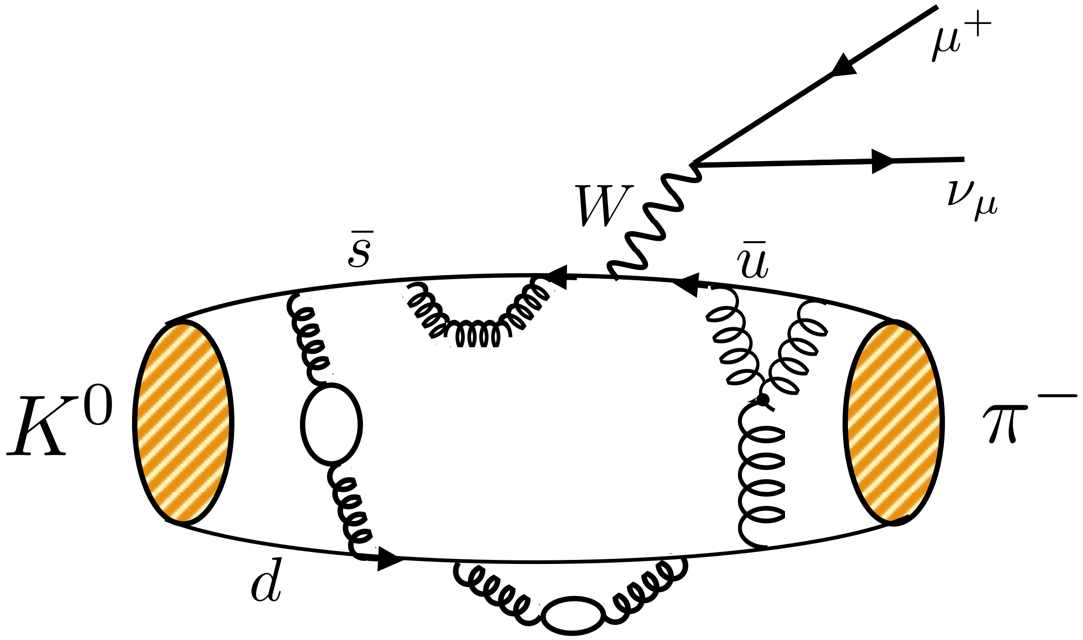


$$\Gamma(K^+ \to \mu^+ \nu_\mu) = (\text{known}) \times |V_{us}|^2 \times f_{K^+}^2$$

- use experiment + LQCD input for determination of CKM element
- same for B ($|V_{ub}|$) and $D_{(s)}$ ($|V_{cd(s)}|$) mesons
- **ratios** for example f_{K^+}/f_{π^+}: statistical and systematic errors tend to cancel.

semileptonic K, D, B decays

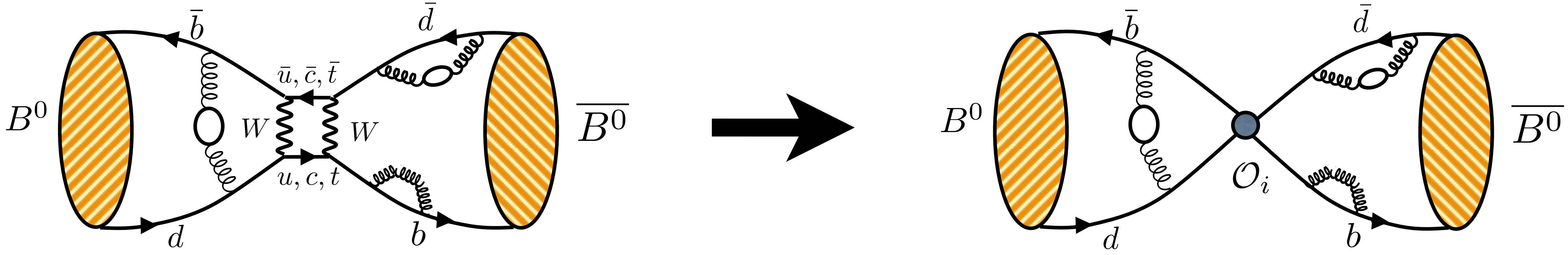
example: $K^0 \to \pi^- \ell^+ \nu_\ell$



$$\Gamma_{K\ell 3} = (\text{known}) \times \begin{pmatrix} \text{phase} \\ \text{space} \end{pmatrix} \times \left(1 + \delta_{\text{EM}}^{K\ell} + \delta_{\text{SU(2)}}^{K\pi}\right) \times |V_{us}|^2 \times |f_+^{K^0\pi^-}(0)|^2$$

Neutral K, B mixing

Standard Model



SM: $$\Delta M_q = (\text{known}) \times |V_{tq}^* V_{tb}|^2 \times \langle \bar{B}_q^0 | \mathcal{O}_1 | B_q^0 \rangle$$

also:

$$\frac{\Delta M_s}{\Delta M_d} = \frac{m_{B_s}}{m_{B\,d}} \times \left| \frac{V_{ts}}{V_{td}} \right|^2 \times \xi^2$$ with $$\xi \equiv \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}$$

$$\Delta\Gamma_q = \left[G_1 \, \langle \bar{B}_q^0 | \mathcal{O}_1 | B_q^0 \rangle + G_3 \, \langle \bar{B}_q^0 | \mathcal{O}_3 | B_q^0 \rangle \right] \cos\phi_q + O(1/m_b)$$

$$\epsilon_K = (\text{known}) \times B_K \, \kappa_\epsilon \times |V_{cb}|^2 \times \bar{\eta} \times f(\bar{\rho}, \bar{\eta}, V_{cb}, \eta_i)$$

Simple LQCD quantities for CKM elements

$$\begin{pmatrix} \begin{array}{c} V_{ud} \\ \pi\to\mu\nu \end{array} & \begin{array}{c} V_{us} \\ K\to\pi\ell\nu \\ K\to\mu\nu \end{array} & \begin{array}{c} V_{ub} \\ B\to\pi\ell\nu,\ B_s\to K\ell\nu \\ \Lambda_b\to p\,\ell\nu \end{array} \\ \begin{array}{c} V_{cd} \\ D\to\pi\ell\nu \\ D\to\ell\nu \end{array} & \begin{array}{c} V_{cs} \\ D\to K\ell\nu \\ D_s\to\ell\nu \end{array} & \begin{array}{c} V_{cb} \\ B_{(s)}\to D_{(s)}, D^*_{(s)}\,\ell\nu \end{array} \\ \begin{array}{c} V_{td} \\ B^0-\overline{B^0} \end{array} & \begin{array}{c} V_{ts} \\ B_s^0-\overline{B_s^0} \end{array} & V_{tb} \end{pmatrix}$$

$$(\rho,\eta)\quad K^0-\overline{K^0}$$

Simple quantities in LQCD

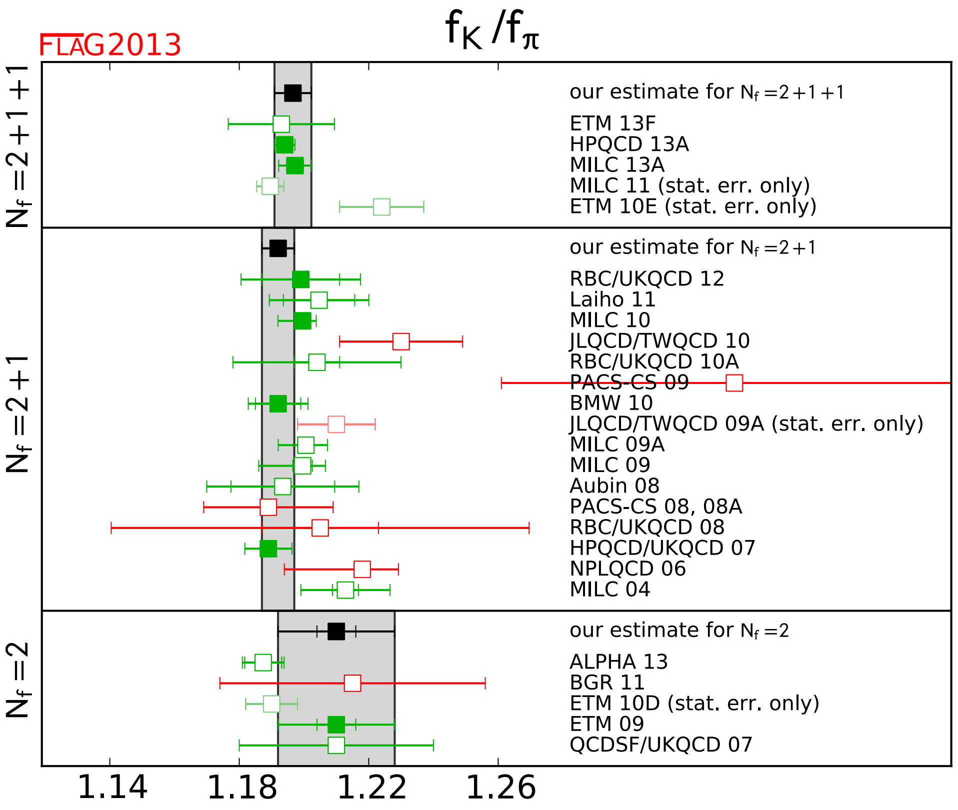
- ☆ low-lying hadron spectrum ➙ quark masses, α_s
- ☆ weak decays - leptonic, semileptonic, mixing
  - ✦ Kaons
  - D mesons
  - B mesons

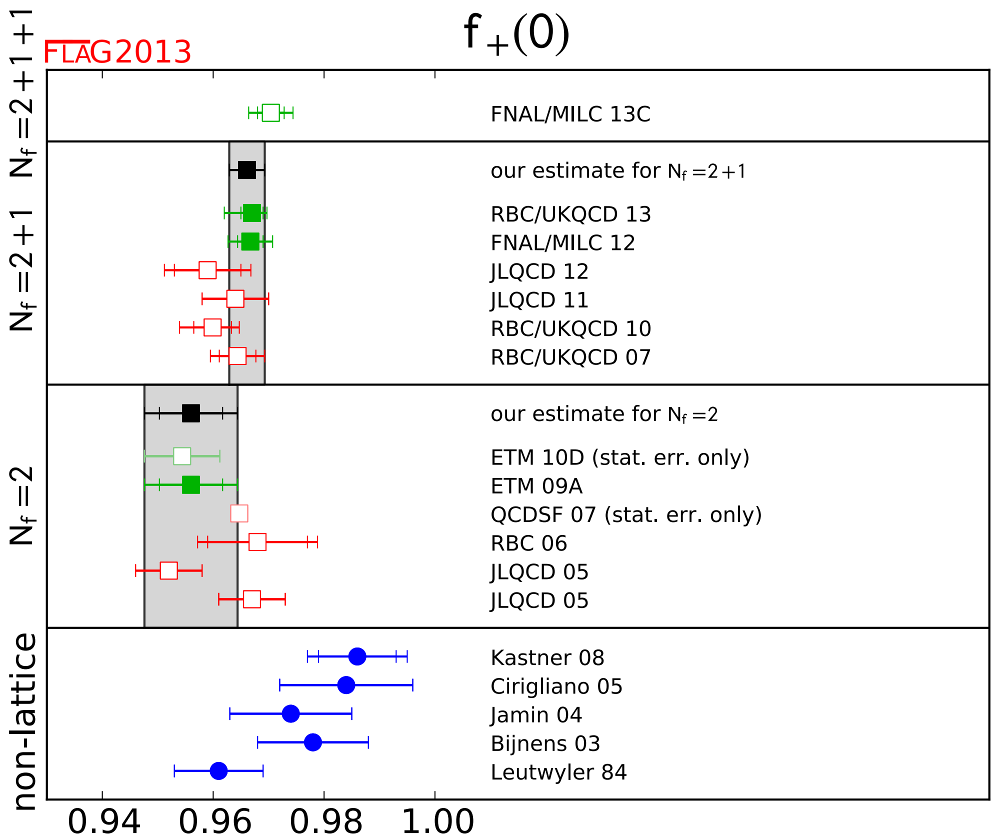
➙ CKM, BSM phenomenology

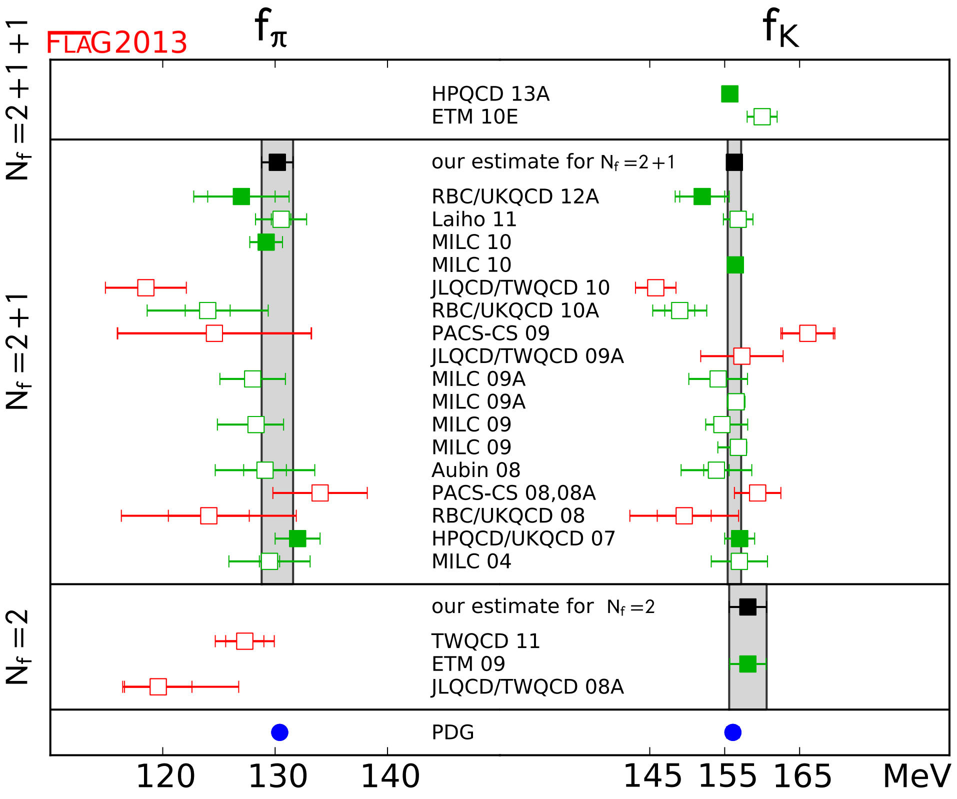
- ☆ high precision ➙ including QED

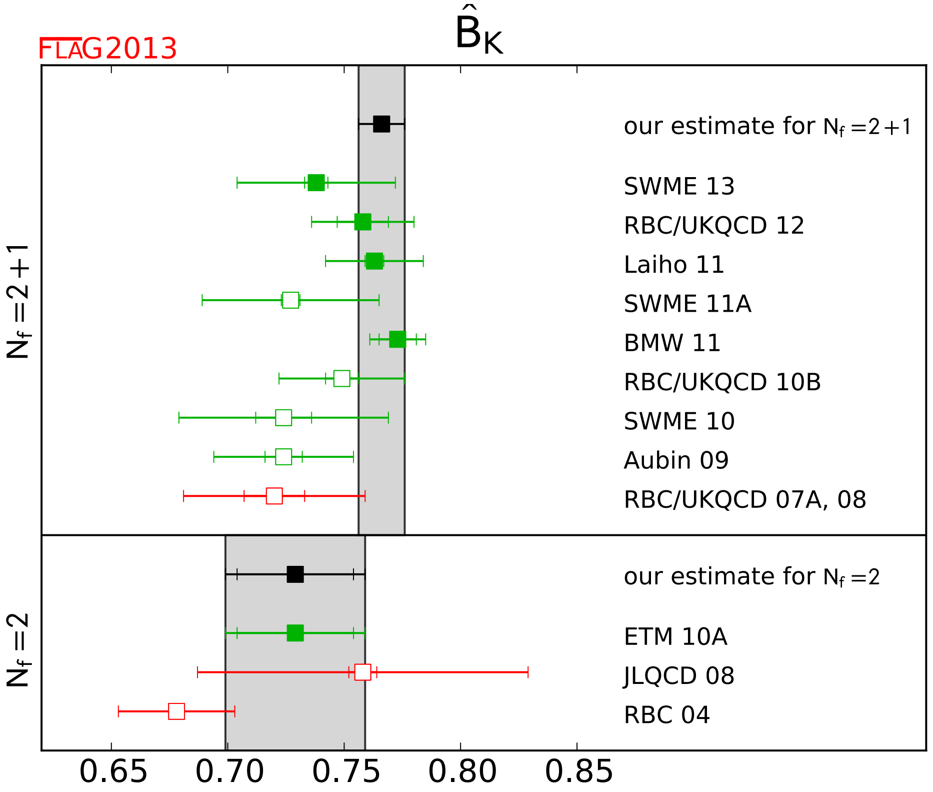
Kaon summary

S. Aoki et al (FLAG-2 review, arXiv:1310.8555)





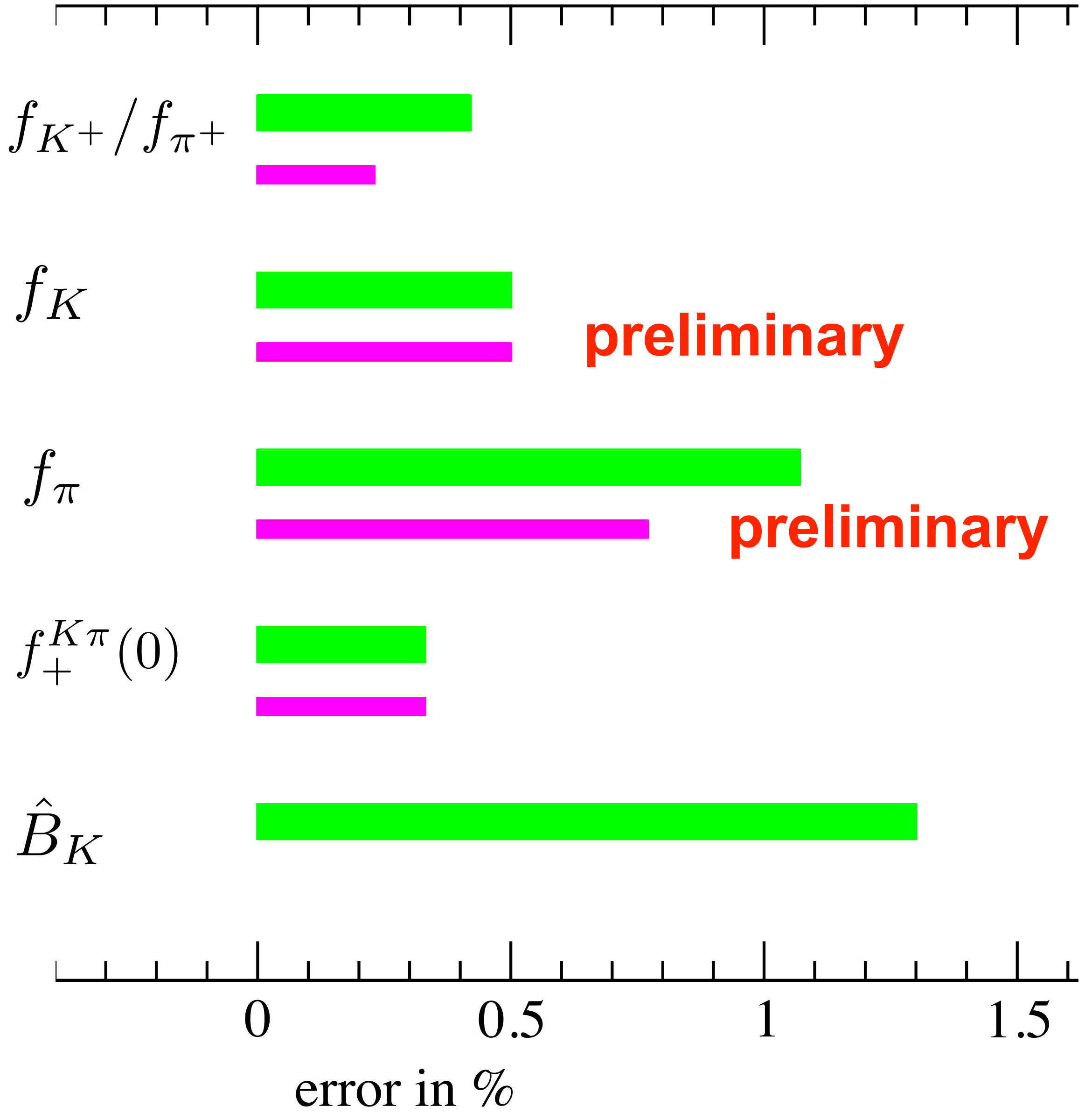




Kaon summary

For all quantities there are results that use **physical mass** ensembles

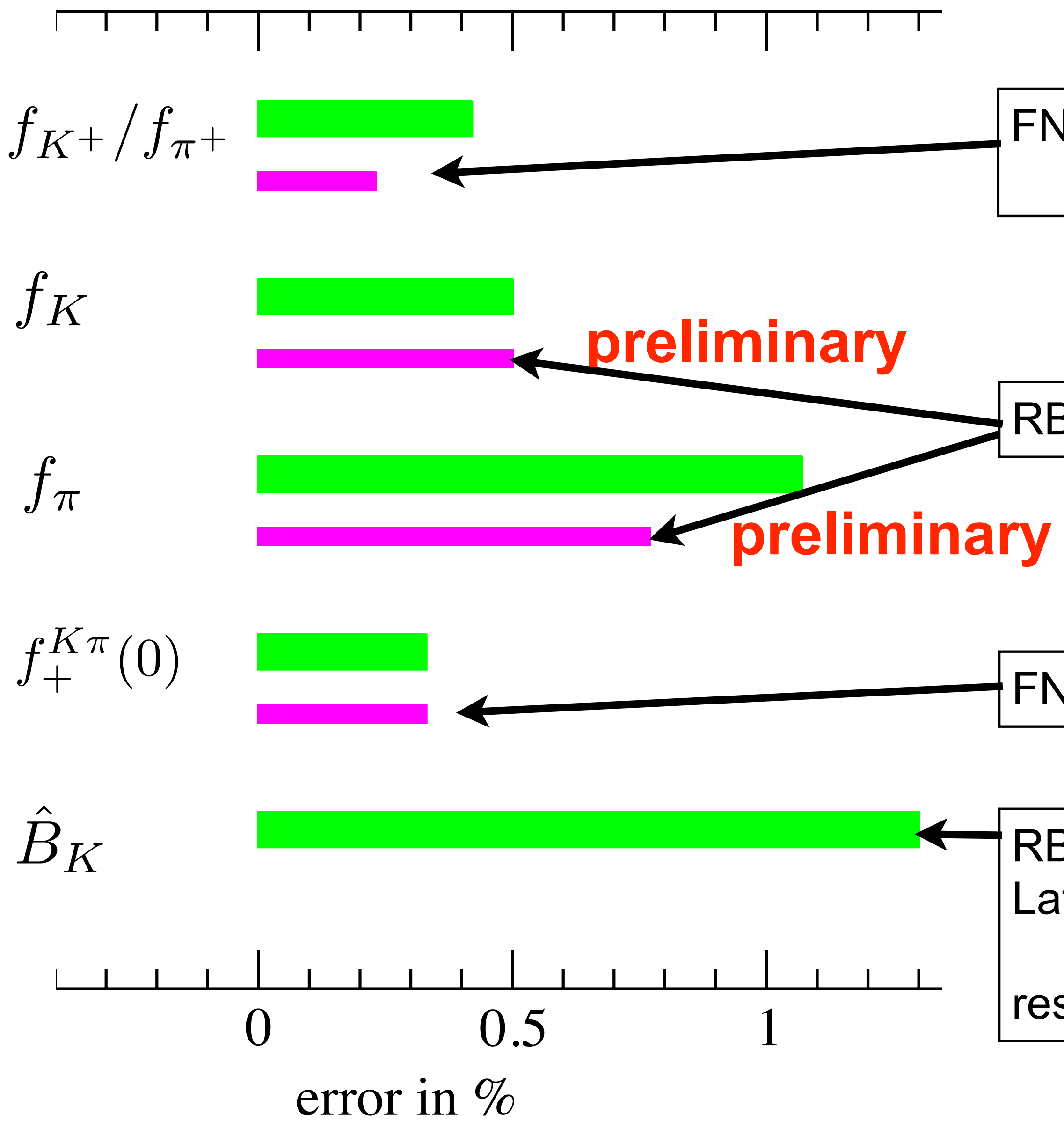
errors (in %) comparison: **FLAG-2 averages** vs. **new results**



Kaon summary

For all quantities there are results that use **physical mass** ensembles

errors (in %) comparison: **FLAG-2 averages** vs. **new results**



FNAL/MILC (arXiv:1407.3772) also: RBC/UKQCD (Lattice 2014)
ETM (Lattice 2014)

RBC/UKQCD (Lattice 2014) also: BMW 13, HPQCD 13

FNAL/MILC 13 (arXiv:1312.1228, PRL 2014)

RBC/UKQCD, SWME, ETM, ALPHA: new preliminary results at Lattice 2014

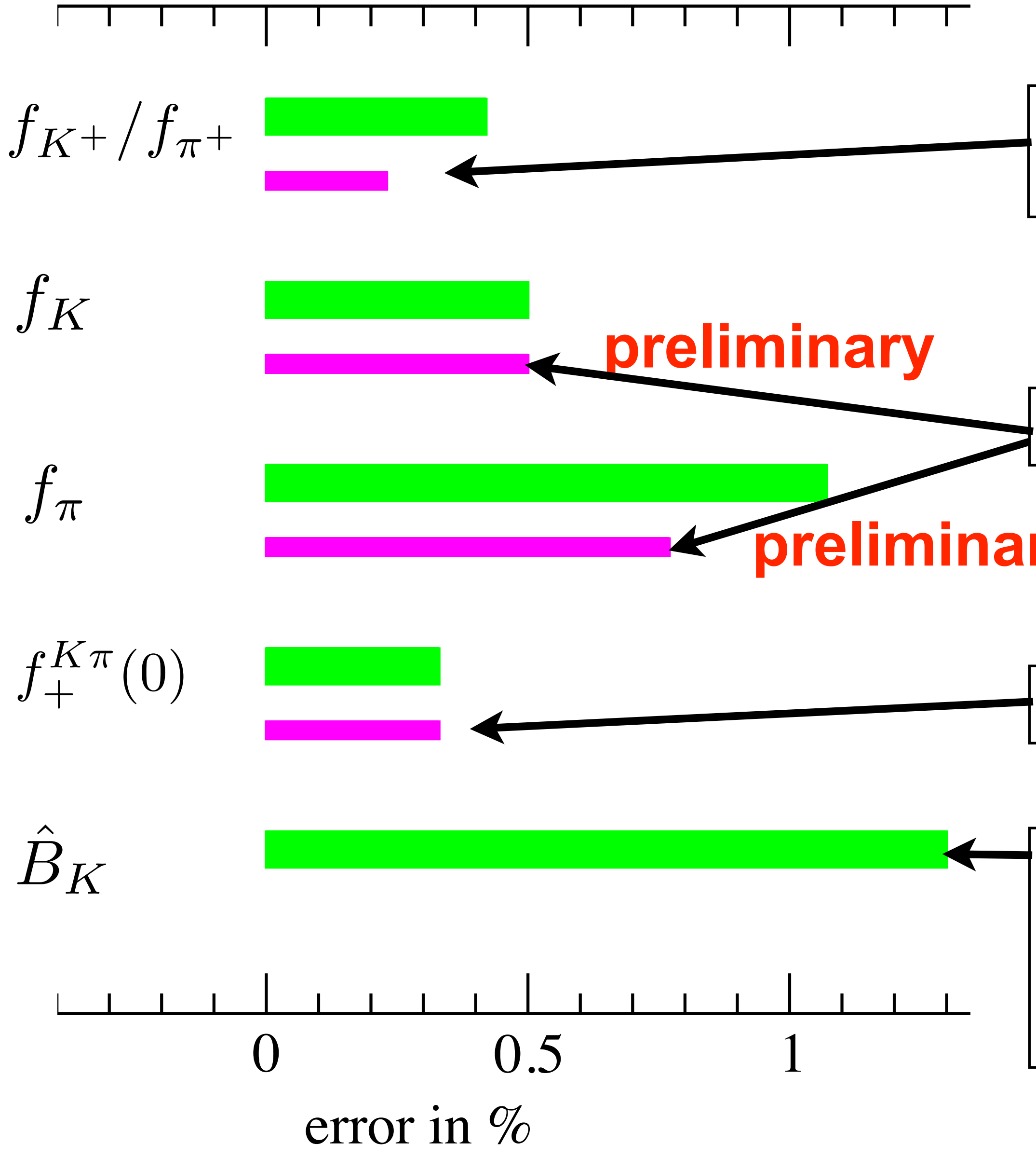
results for all 5 bag parameters (including BSM)

review by N. Garron @ Lattice 2014

Kaon summary

For all quantities there are results that use **physical mass ensembles**

errors (in %) comparison: **FLAG-2 averages** vs. **new results**



preliminary

preliminary

FNAL/MILC (arX

RBC/UKQCD (La

FNAL/MILC 13 (a

RBC/UKQCD, SWME, ETM, ALPHA: new preliminary results at Lattice 2014

results for all 5 bag parameters (including BSM)

small errors due to

- ✦ **physical light quark masses**
- ✦ improved light-quark actions
- ✦ NPR or no renormalization
- ✦ ensembles with small lattice spacings
- ✦ stat. error dominates $f_+(0)$

review by N. Garron @ Lattice 2014

see back up slides for more details

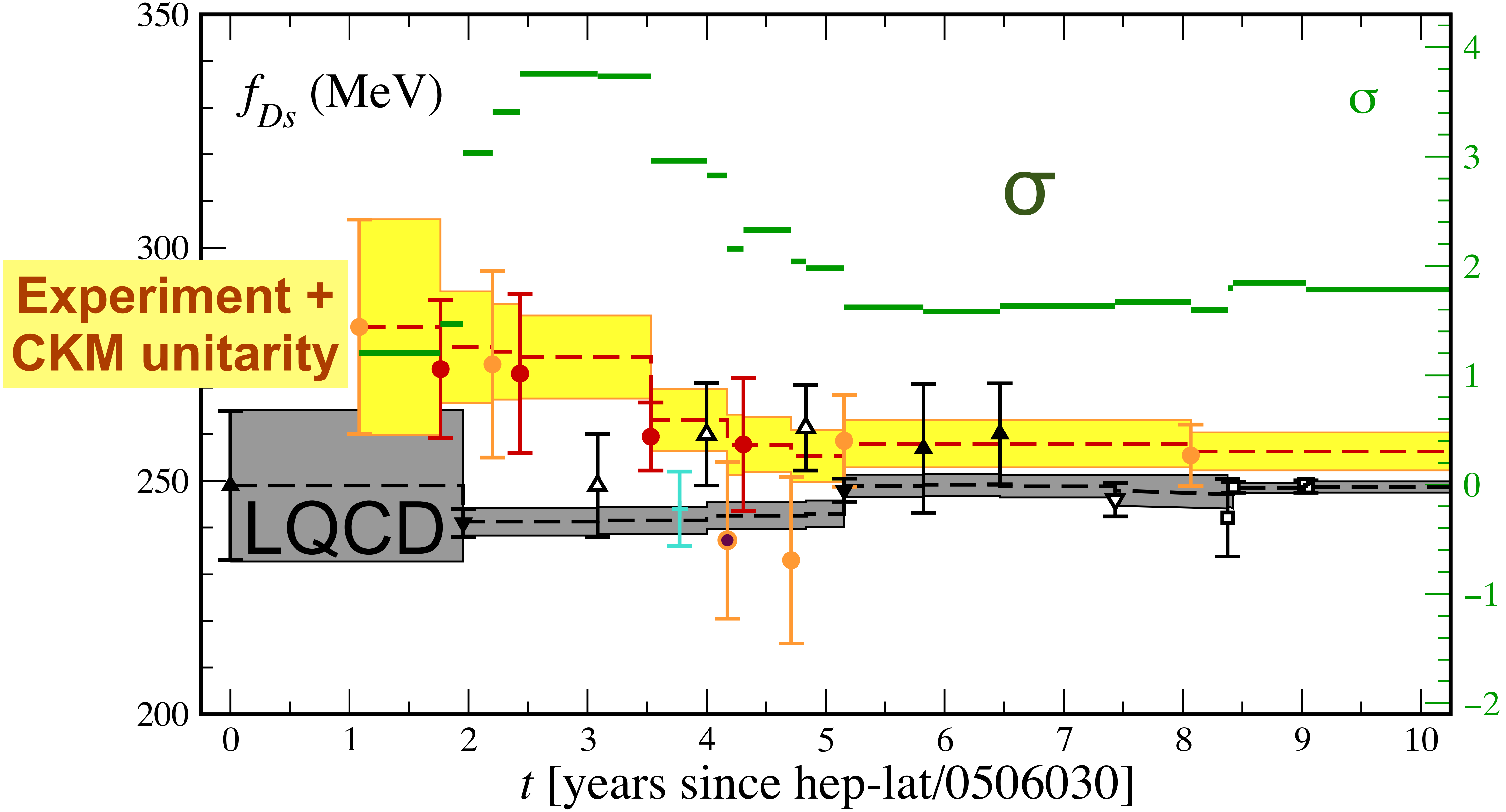
Simple quantities in LQCD

- ☆ low-lying hadron spectrum ➙ quark masses, α_s
- ☆ weak decays - leptonic, semileptonic, mixing
  - Kaons
  - ✦ D mesons
  - B mesons

  ➙ CKM, BSM phenomenology
- ☆ high precision ➙ including QED

LQCD Achievements: f_{Ds} time history

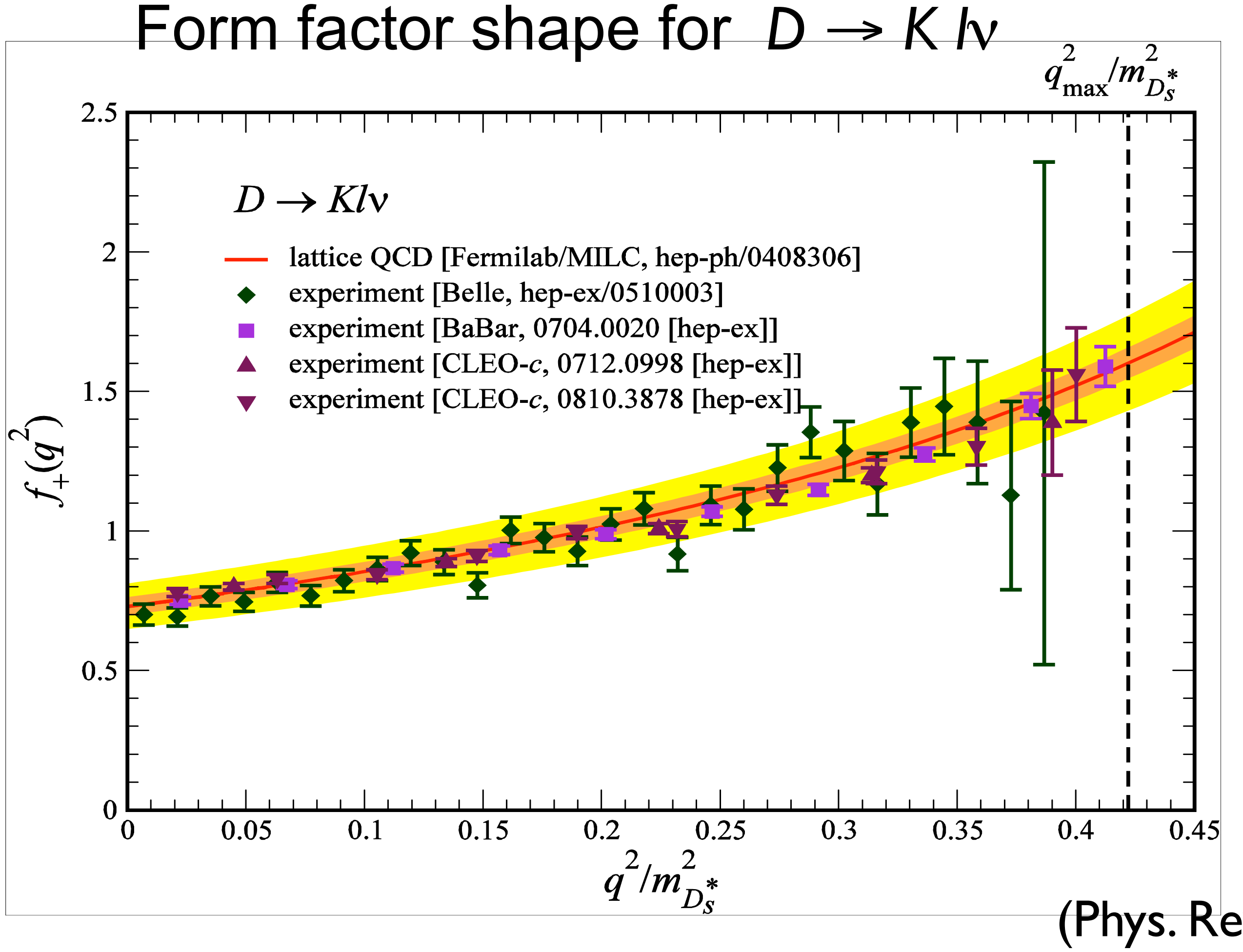
A. Kronfeld (Annu. Rev. Part. & Nucl. Sci, arXiv:1203.1204)



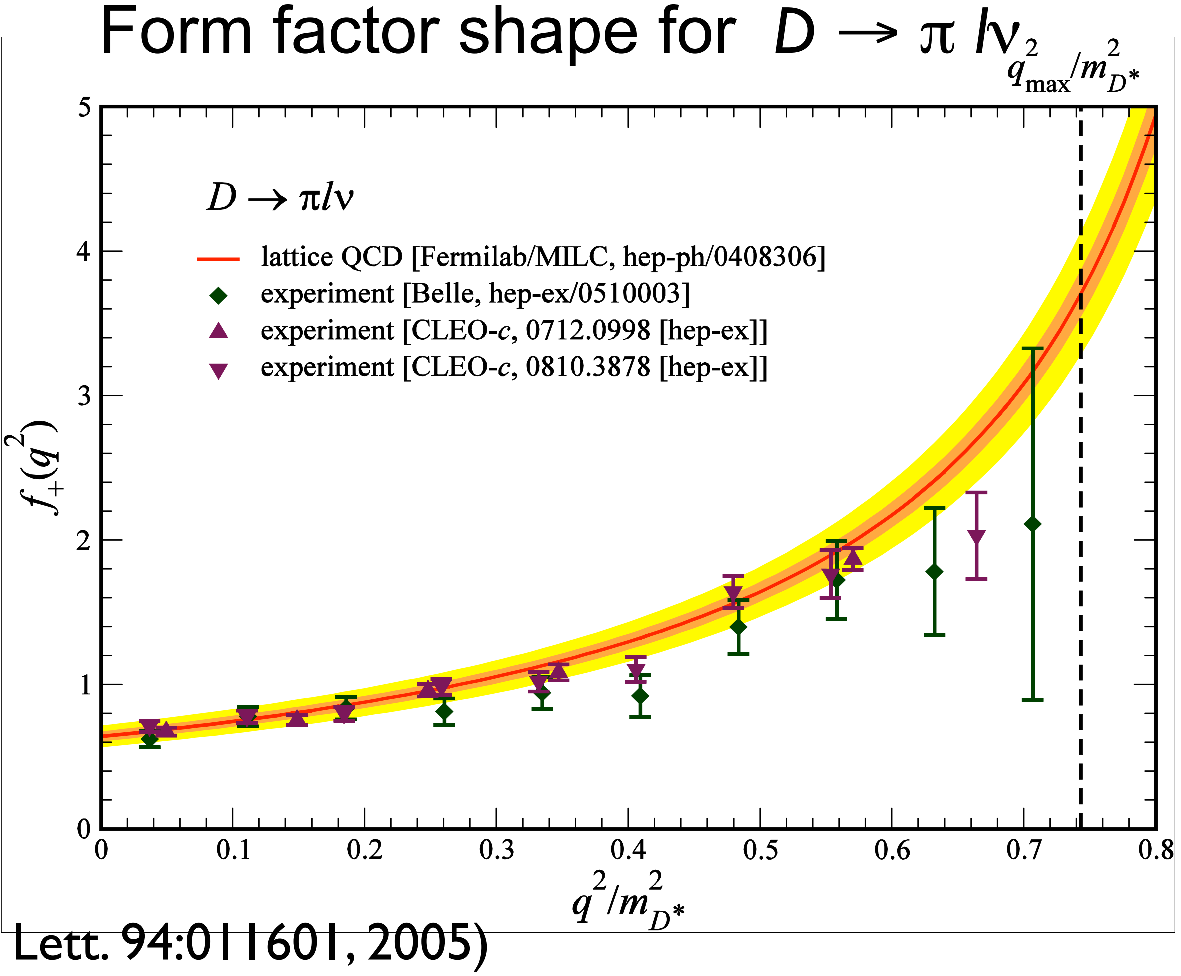
year

LQCD Achievements: Predictions

Form factor shape for $D \to K\,l\nu$



Form factor shape for $D \to \pi\,l\nu$



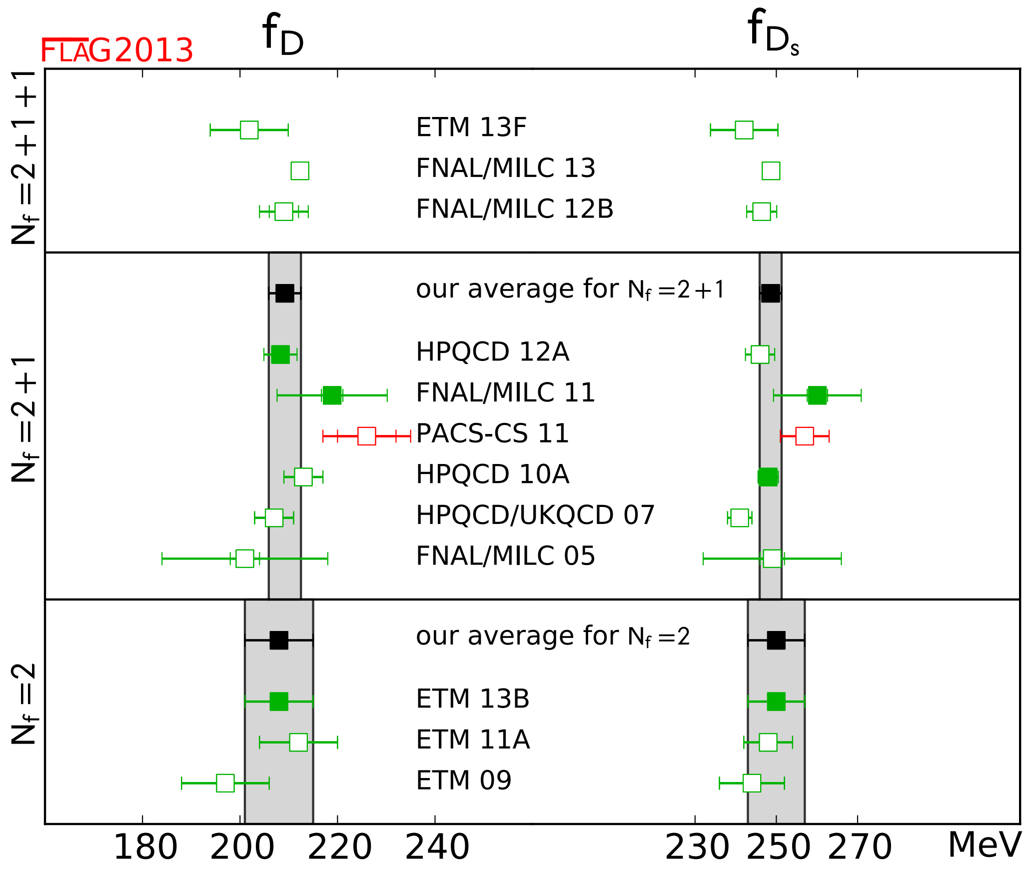
(Phys. Rev. Lett. 94:011601, 2005)

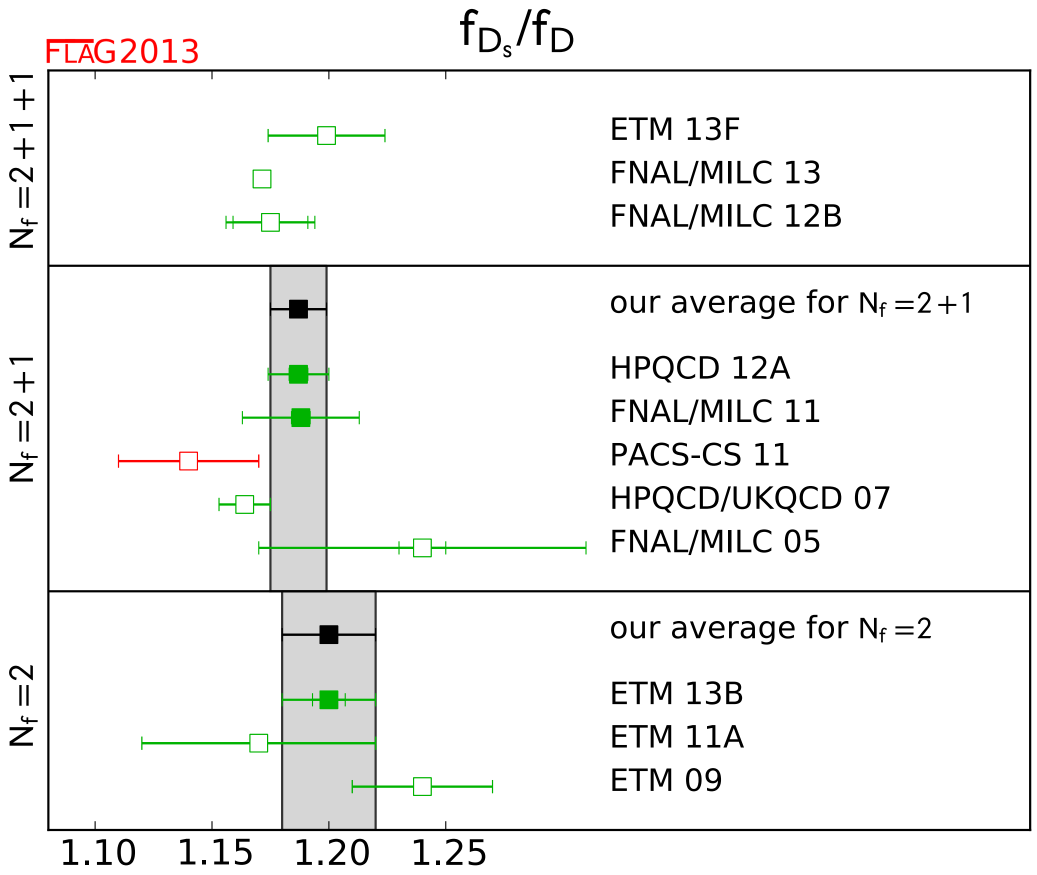
- Normalization agrees with experiment plus CKM unitarity
- *Prediction* of the shape

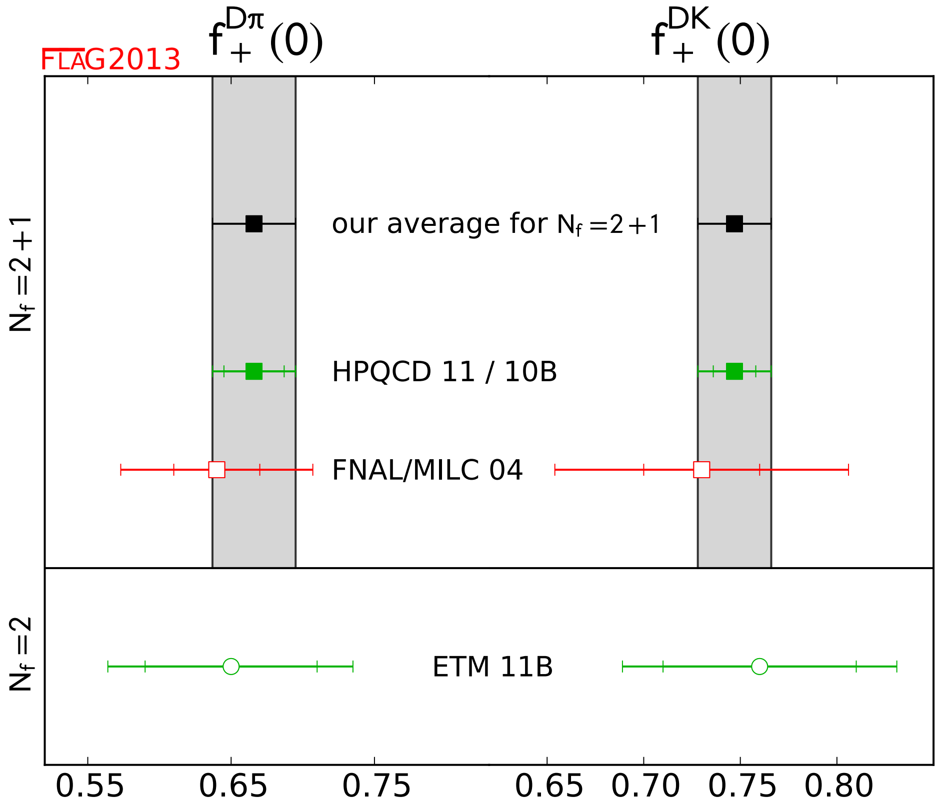
also: B_c mass prediction (HPQCD+FNAL PRL 2005, hep-lat/0411027)

D meson summary

S. Aoki et al (FLAG-2 review, arXiv:1310.8555)

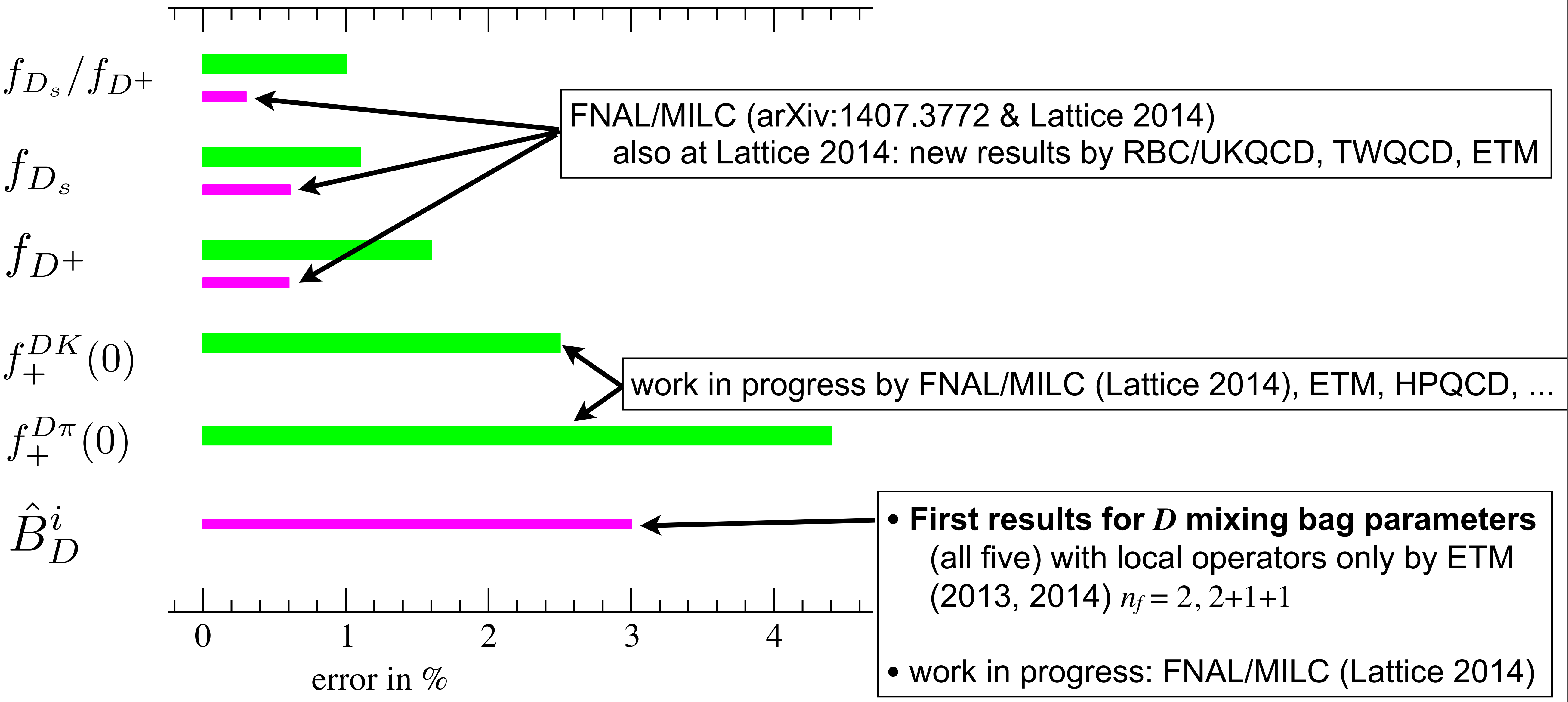






D meson summary

errors (in %) comparison: **FLAG-2 averages** vs. **new results**



FNAL/MILC (arXiv:1407.3772 & Lattice 2014)
also at Lattice 2014: new results by RBC/UKQCD, TWQCD, ETM

work in progress by FNAL/MILC (Lattice 2014), ETM, HPQCD, ...

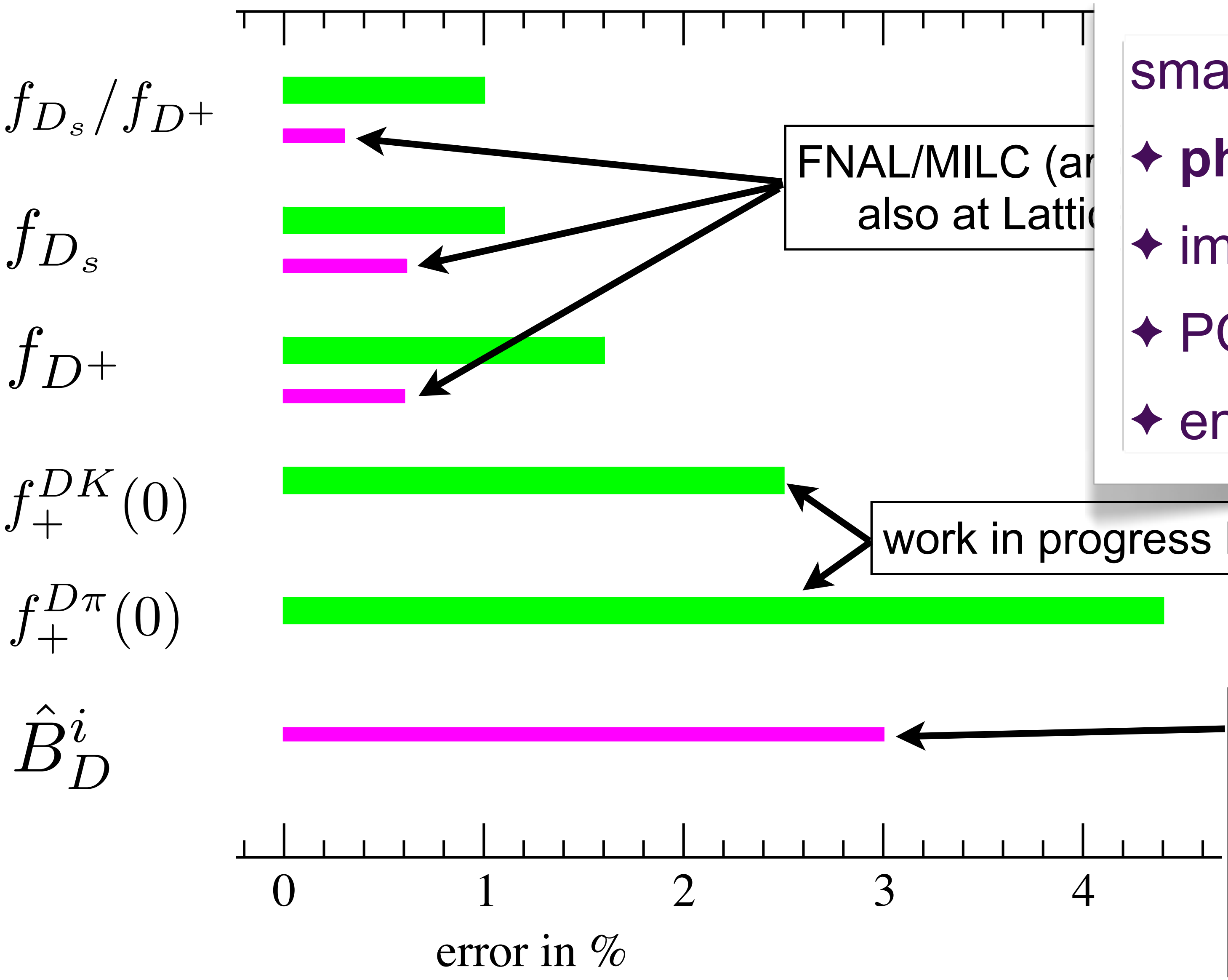
- **First results for *D* mixing bag parameters** (all five) with local operators only by ETM (2013, 2014) $n_f = 2, 2{+}1{+}1$

- work in progress: FNAL/MILC (Lattice 2014)

review by C. Bouchard @ Lattice 2014

D meson summary

errors (in %) comparison: **FLAG-2 averages** vs. **new results**



FNAL/MILC (ar
also at Lattic

small errors due to

- ✦ **physical light quark masses**
- ✦ improved charm-quark action (HISQ)
- ✦ PCAC (no renormalization)
- ✦ ensembles with small lattice spacings

work in progress by FNAL/MILC (Lattice 2014), ETM, HPQCD, ...

- **First results for D mixing bag parameters** (all five) with local operators only by ETM (2013, 2014) $n_f = 2, 2{+}1{+}1$
- work in progress: FNAL/MILC (Lattice 2014)

review by C. Bouchard @ Lattice 2014

see back up slides for more details

Neutral *D*-meson mixing

N. Carrasco
@ ICHEP 2014

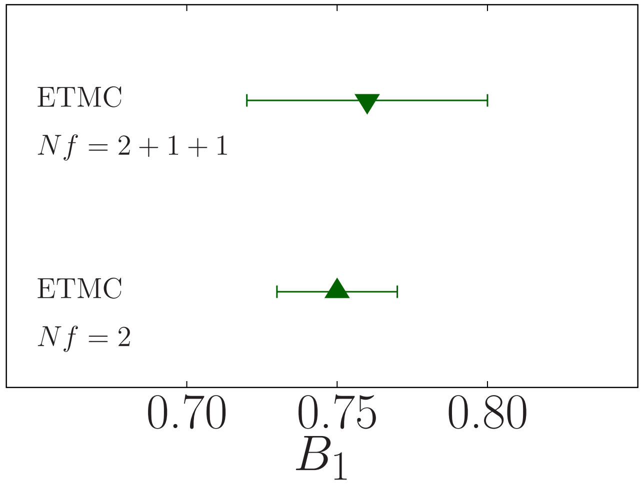
First unquenched LQCD calculation by ETM in 2013

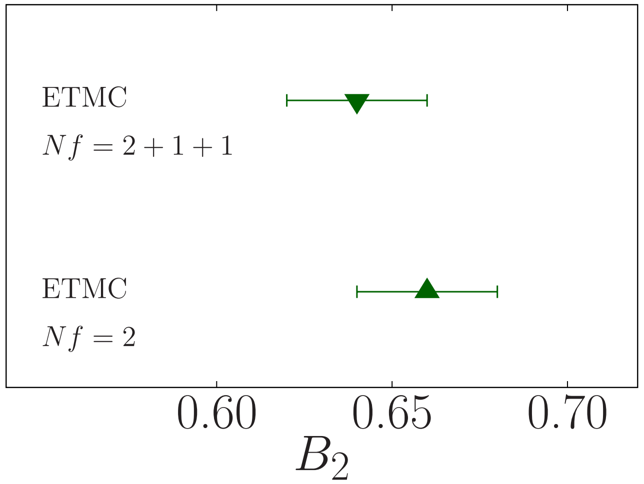
short-distance operators only

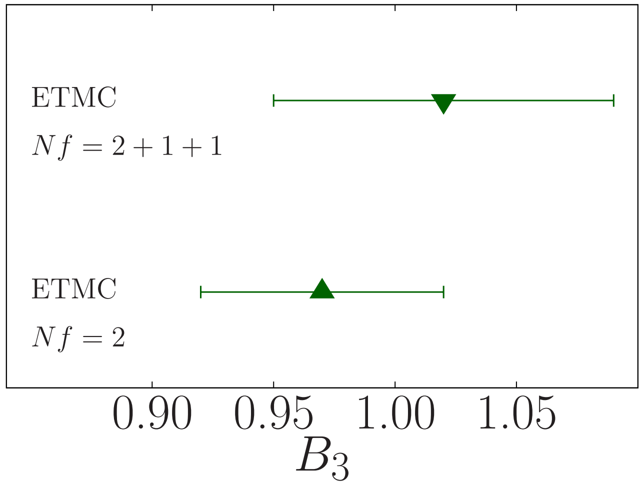
- **ETMC:** OS/MTM Mixed action

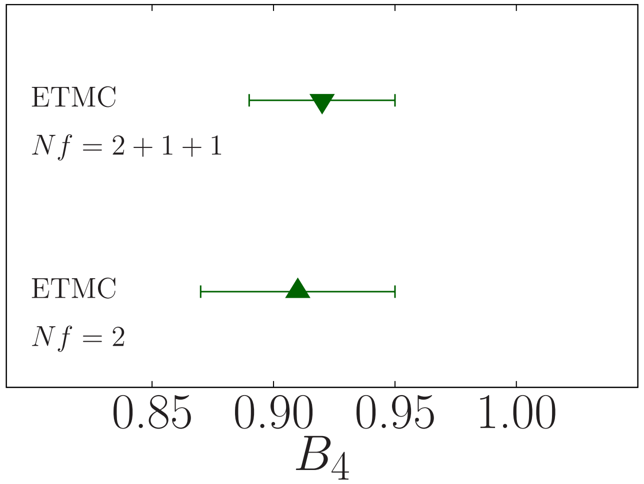
$N_f = 2$, (N. Carrasco et al. arxiv 1403.7302, To be published in Phys. Rev. D)

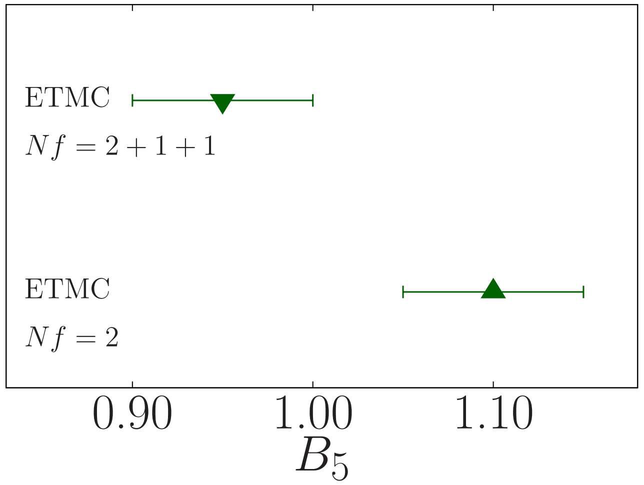
$N_f = 2+1+1$ (N. Carrasco et al. PoS LATTICE2013 393, arxiv 1310:5461)











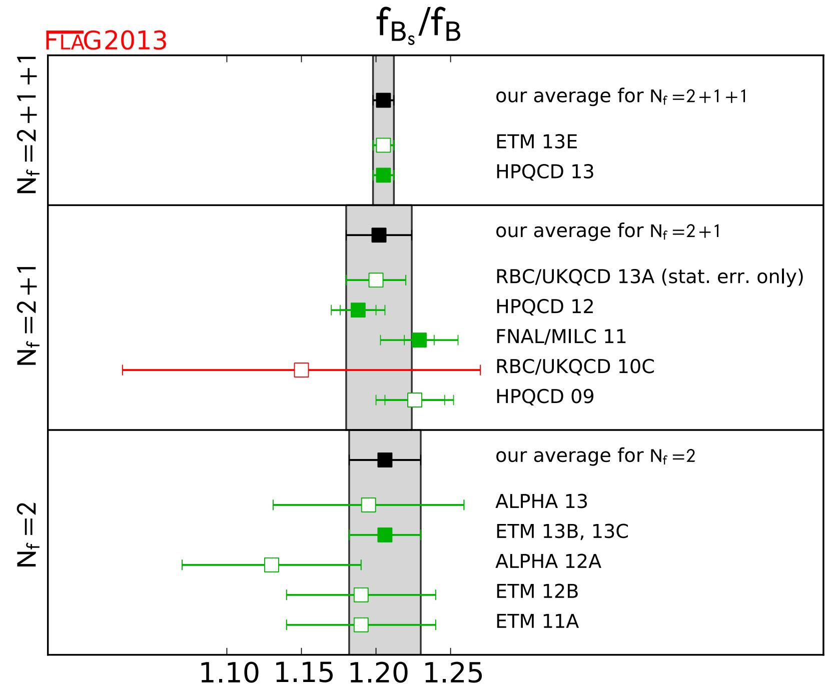
3-5% precision

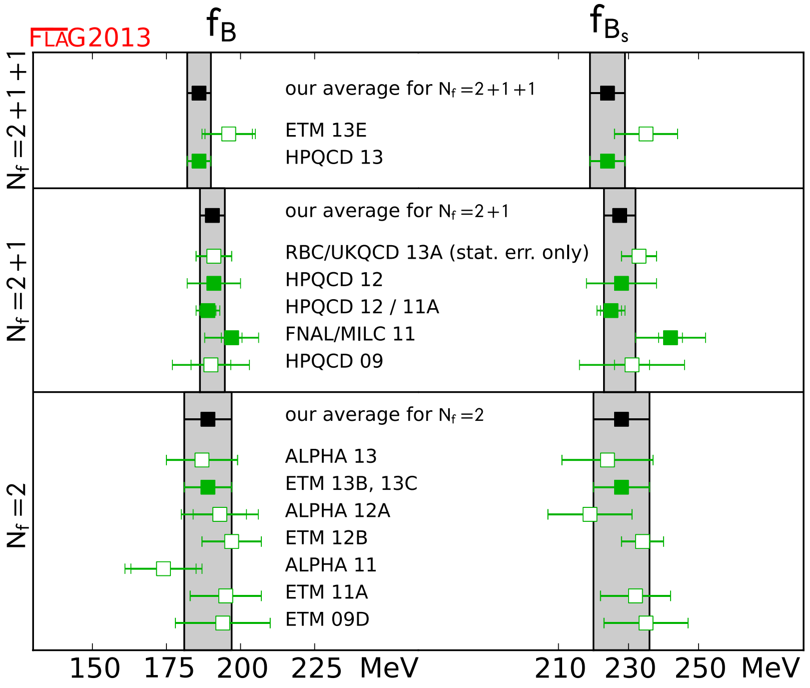
Simple quantities in LQCD

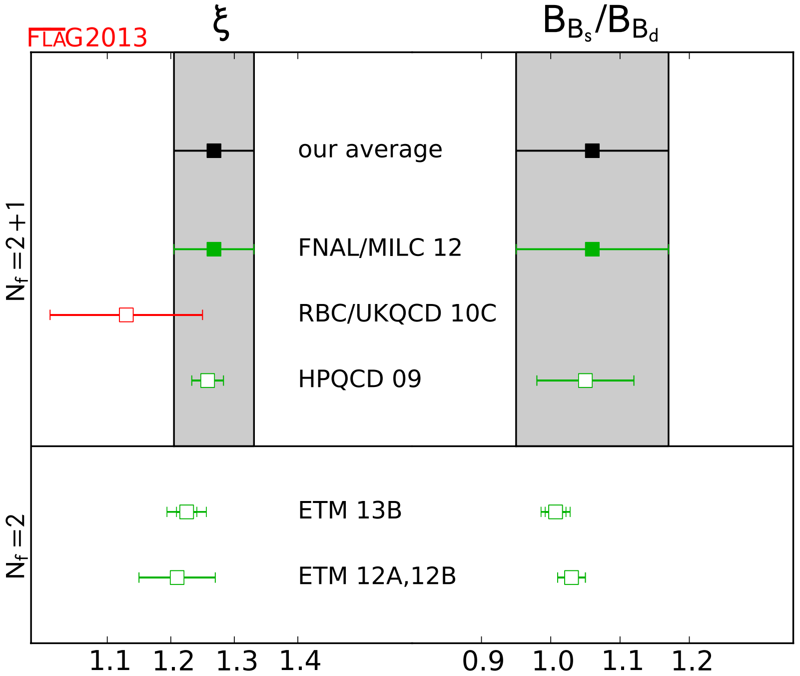
- ☆ low-lying hadron spectrum → quark masses, α_s
- ☆ weak decays - leptonic, semileptonic, mixing
  - Kaons
  - D mesons
  - ✦ B mesons
- → CKM, BSM phenomenology
- ☆ high precision → including QED

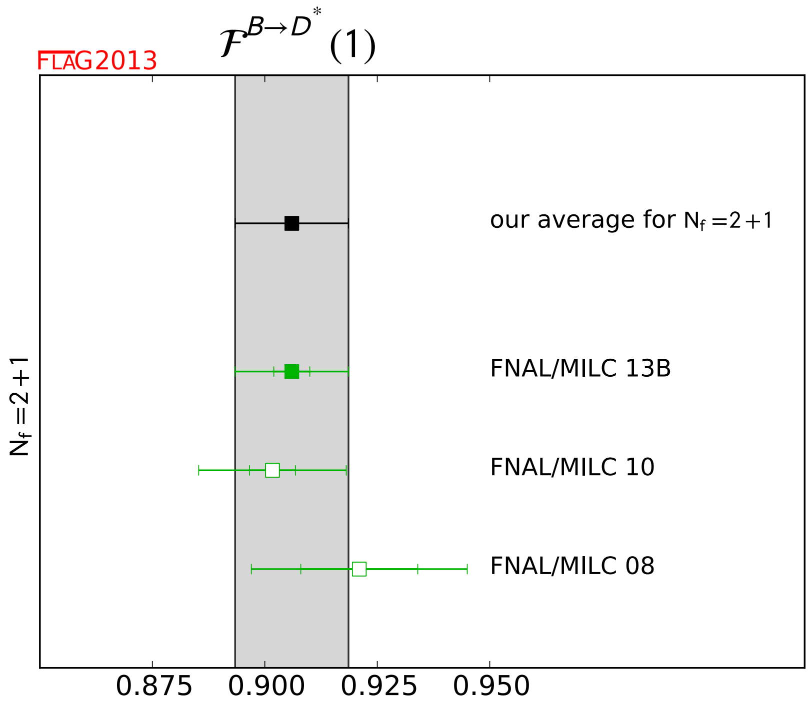
B meson summary

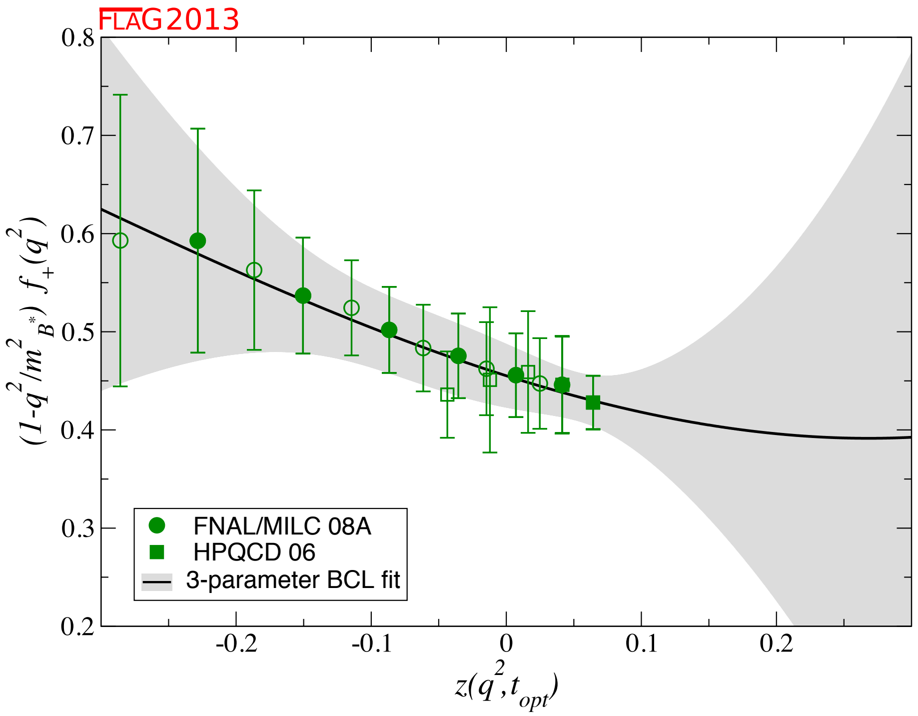
S. Aoki et al (FLAG-2 review, arXiv:1310.8555)





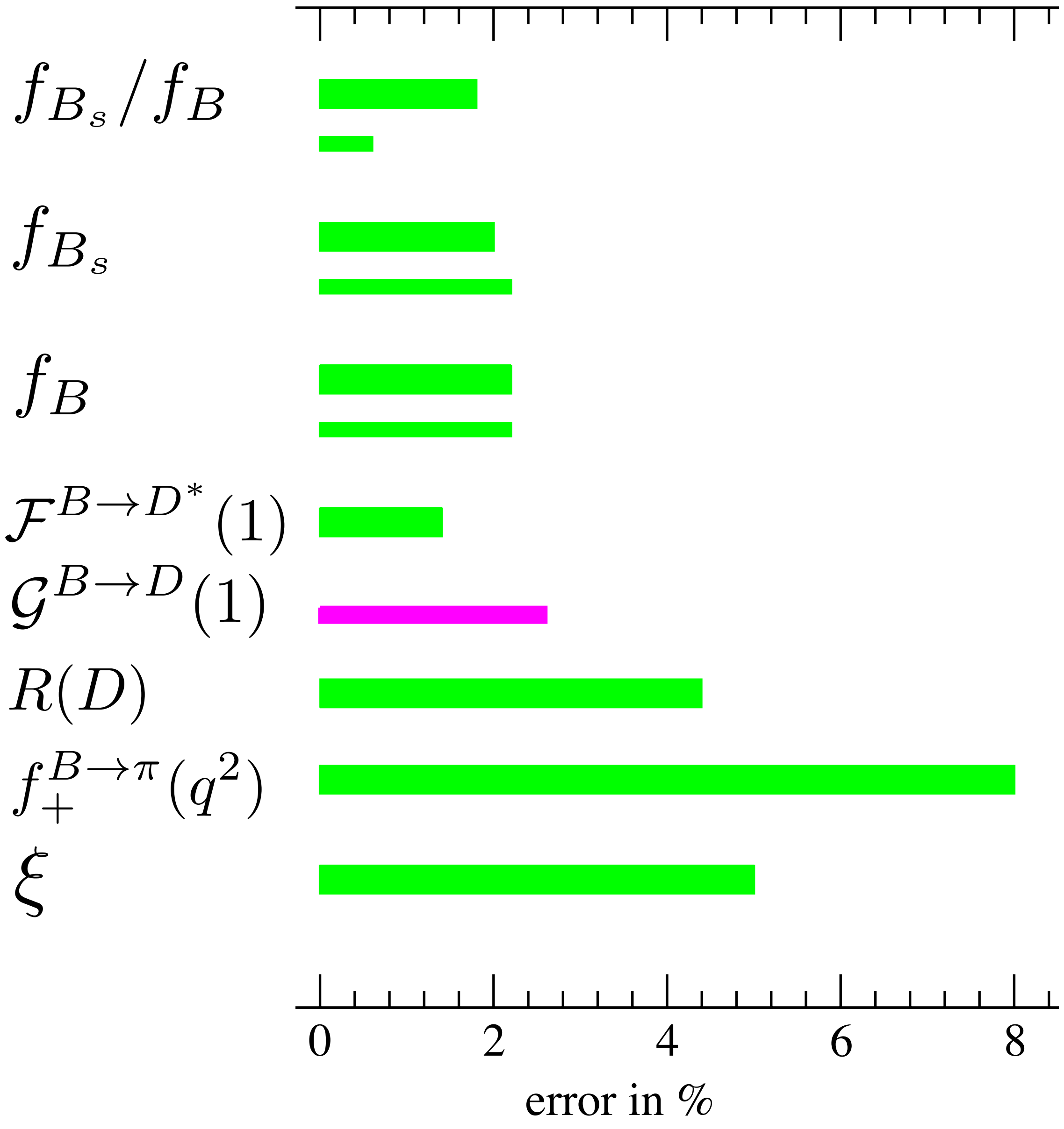






B meson summary

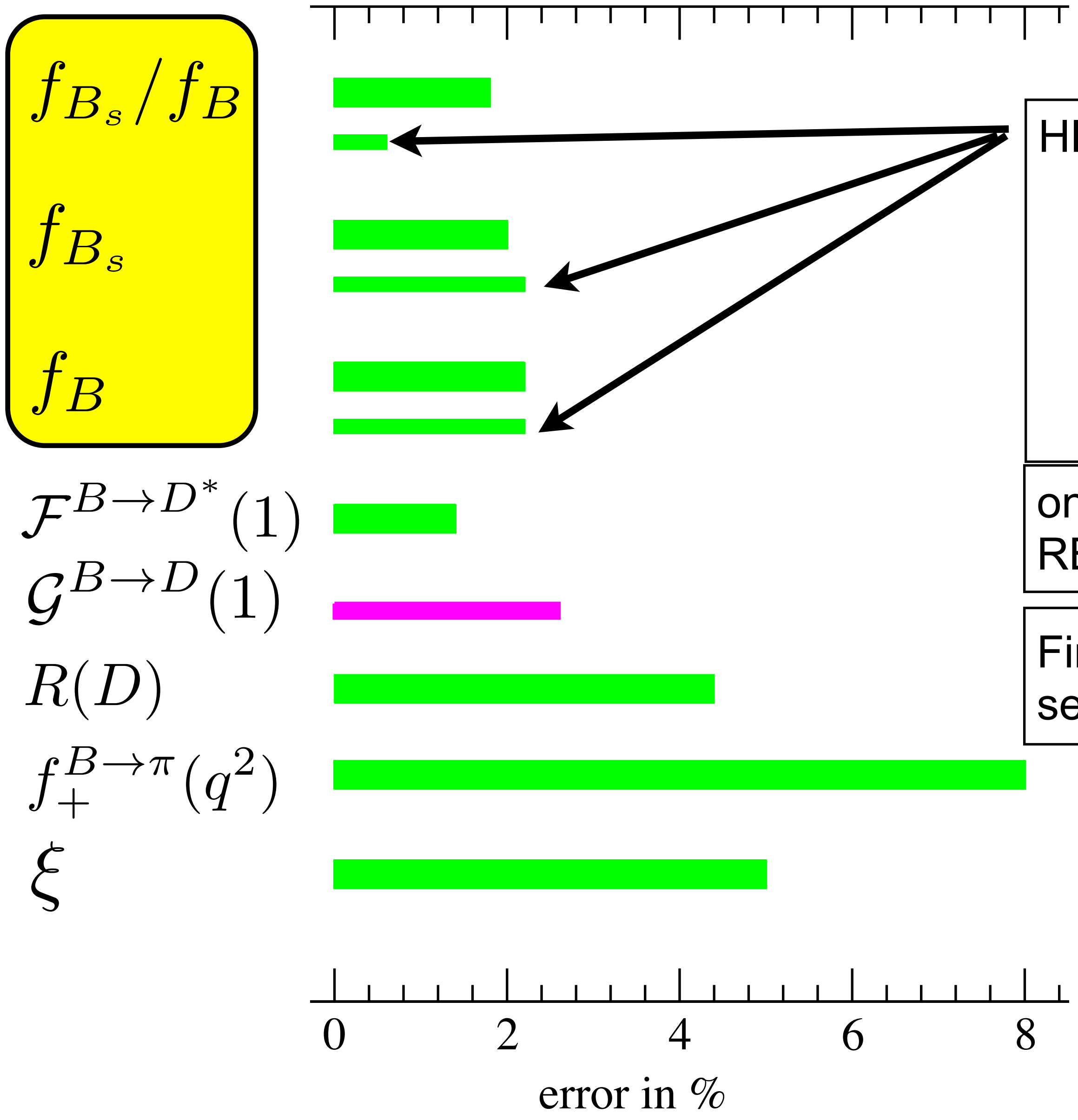
errors (in %) comparison: **FLAG-2 averages** vs. **new results**



review by C. Bouchard @ Lattice 2014

B meson summary

errors (in %) comparison: **FLAG-2 averages** vs. **new results**



HPQCD 13 (arXiv:1302.2644, PRL 2013):

- ✦ $n_f = 2+1+1$
- ✦ physical pion mass
- ✦ renormalization: 1-loop PT (dominates error budget for f_B, f_{Bs})
- ✦ f_{Bs}/f_B: renormalization, discretization errors cancel: small error due to physical mass ensembles

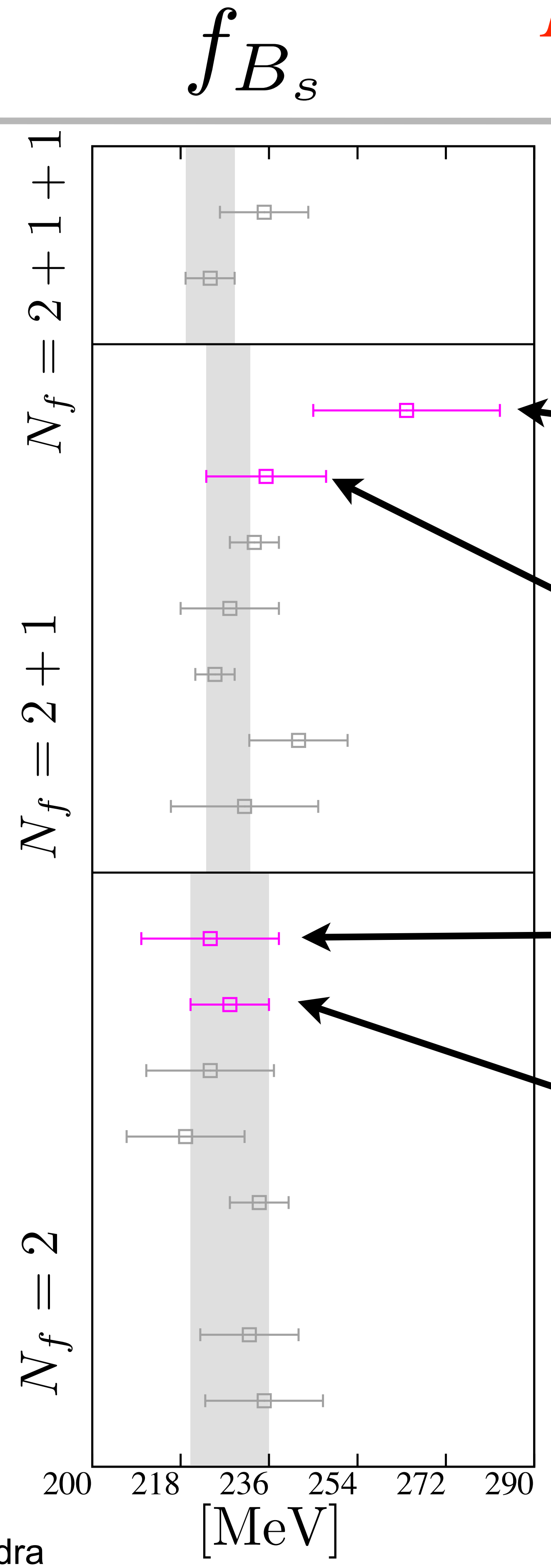
ongoing work @ Lattice 2014:
RBC/UKQCD (2 separate projects), ETM, FNAL/MILC, ALPHA, ...

First results for f_{B^*}/f_B by ETM/Orsay group:
see A. Oyanguren talk (ICHEP, Flavor physics session, Saturday)

review by C. Bouchard @ Lattice 2014

see backup slides for more details

B meson summary



review by C. Bouchard @ Lattice 2014

reviewed by FLAG-2 (arXiv:1310.8555)

new (since LAT'13):

RBC/UKQCD (arXiv:1406.6192):
uses static limit action for b quark, focus on SU(3) ratios
2 lattice spacings, min. $m_\pi \sim 289$ MeV
combined chiral continuum extrapolation, no $1/m$ correction

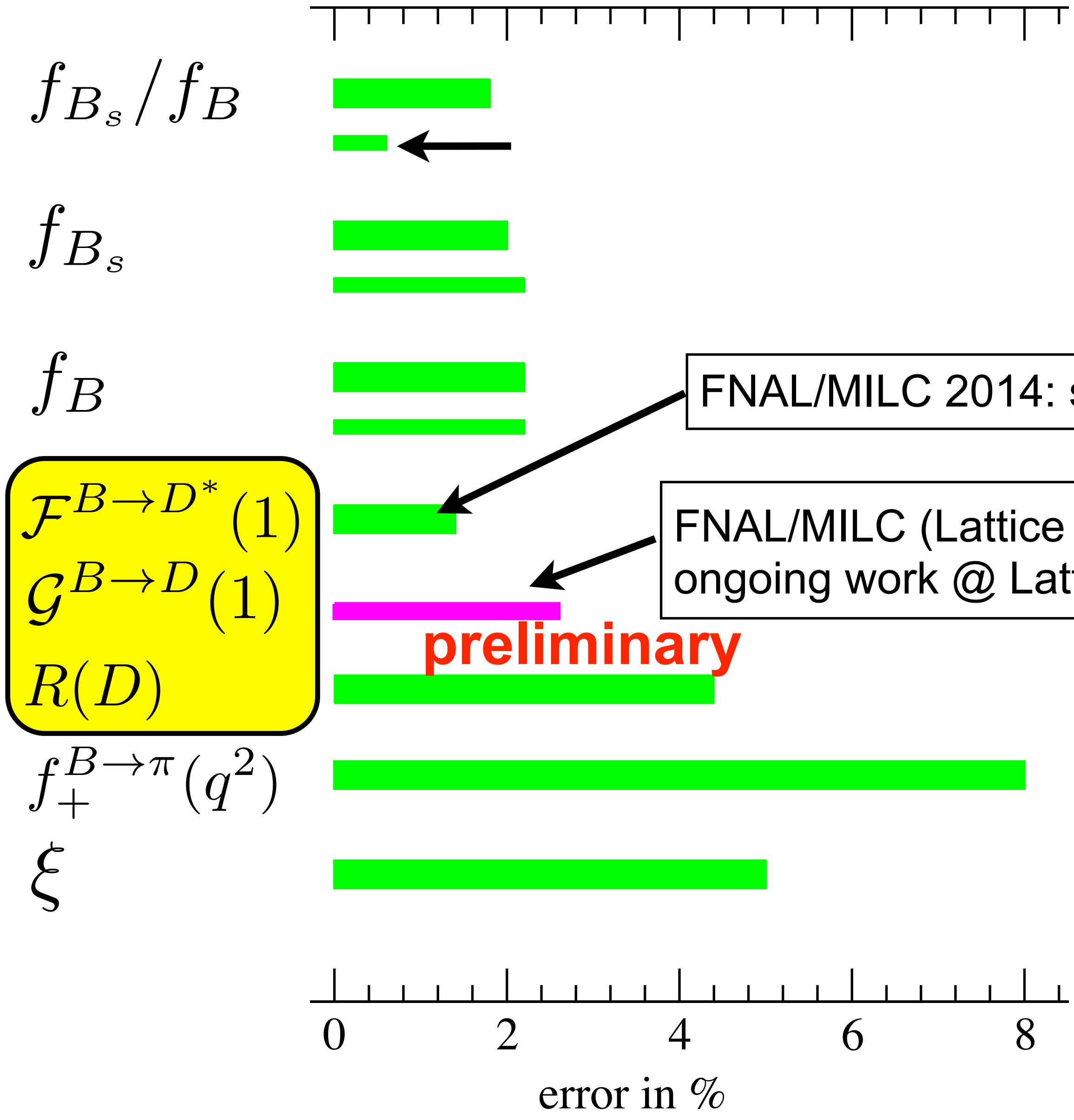
RBC/UKQCD (arXiv:1404.4670):
uses RHQ action at physical b-quark mass,
2 lattice spacings, min. $m_\pi \sim 289$ MeV
combined chiral-continuum extrapolation

ALPHA (arXiv:1404.3590):
HQET action, NP 1/m improved
3 lattice spacings, min. $m_\pi \sim 190$ MeV
combined chiral-continuum extrapolation

ETM (arXiv:1308.1851, JHEP 2014):
twisted mass Wilson, ratio method to extrapolate to b-quark
4 lattice spacings, min. $m_\pi \sim 280$ MeV
combined chiral-continuum extrapolation

B meson summary

errors (in %) comparison: **FLAG-2 averages** vs. **new results**



FNAL/MILC 2014: small errors due to ratios (error cancellation), small lattice spacings

FNAL/MILC (Lattice 2013) also: results for B_s D_s by Orsay group
ongoing work @ Lattice 2014 by HPQCD, SWME, ...

review by C. Bouchard @ Lattice 2014

see backup slides for more details

Form factors for $B \to D^{(*)}\ell\nu$ & V_{cb}

$$\frac{d\Gamma(B\to D^*\ell\nu)}{d\omega} = (\text{known}) \times |V_{cb}|^2 \times (\omega^2 - 1)^{1/2} |\mathcal{F}(\omega)|^2$$

$$\frac{d\Gamma(B\to D\ell\nu)}{d\omega} = (\text{known}) \times |V_{cb}|^2 \times (\omega^2 - 1)^{3/2} |\mathcal{G}(\omega)|^2$$

at zero recoil (HFAG 2011):

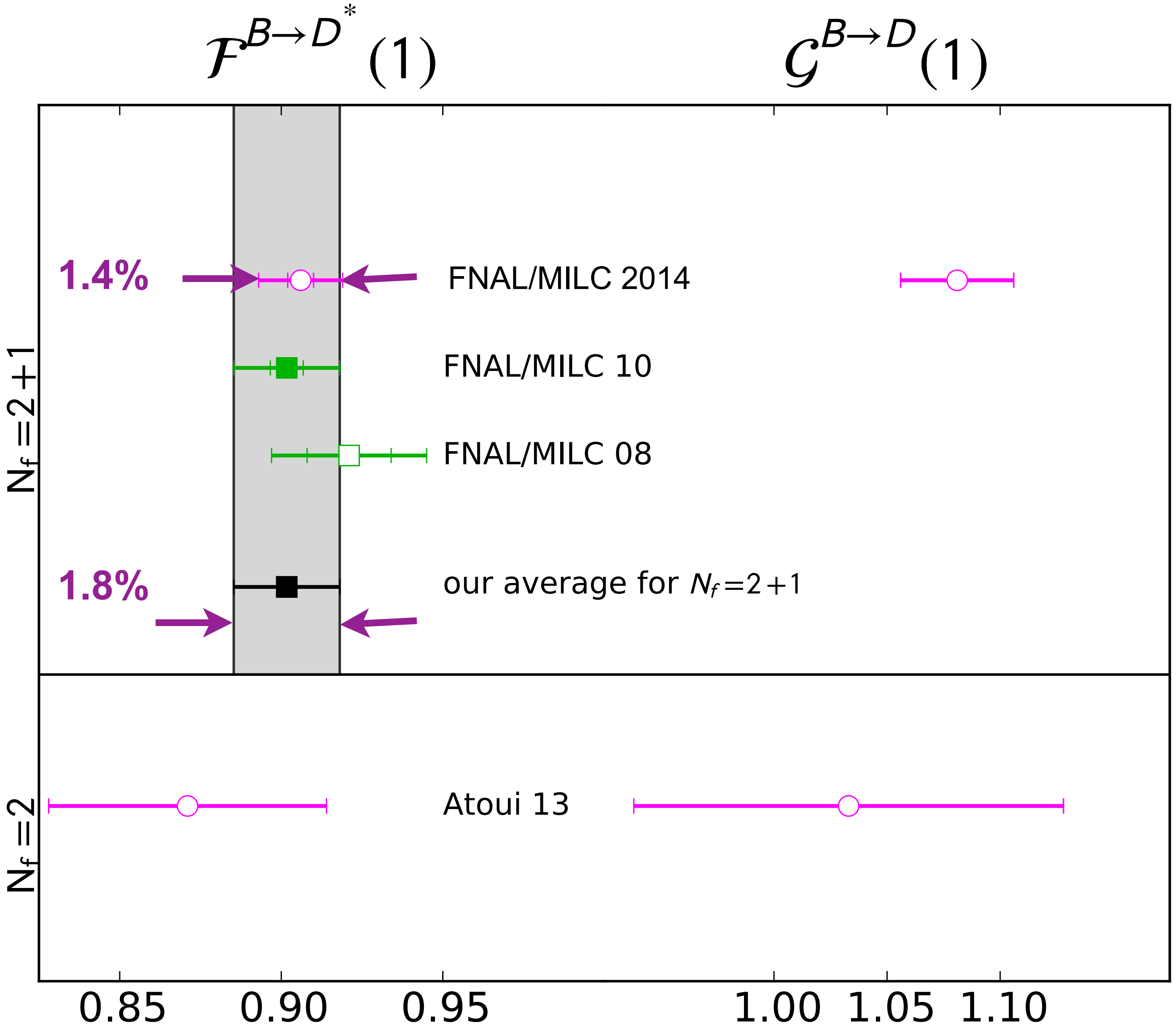
$$B \to D^*\ell\nu : |V_{cb}|\mathcal{F}(1) = (35.90 \pm 0.45) \times 10^{-3}$$

$$B \to D\ell\nu : |V_{cb}|\mathcal{G}(1) = (42.6 \pm 1.5) \times 10^{-3}$$

⇒ need form-factors at non-zero recoil for V_{cb} determination from $B \to D\ell\nu$

Note: the experimental average doesn't include Coulomb correction
(~1%) for the neutral meson decay

Form factors for $B \to D^{(*)}\ell\nu$ & V_{cb}



FNAL/MILC:
small errors due to

- ✦ use of ratios
- ✦ 2013:
  5 a's, 12 ensembles

- ✦ new results by Orsay group using ETM ratio method

- ✦ work in progress:
  HPQCD (NRQCD-HISQ)
  Bailey (OK action)

Also recent work on $B_s \to D_s^{(*)}$ form factors

Form factors for $B_{(s)} \to D_{(s)}\ell\nu$ & V_{cb}

review by C. Bouchard @ Lattice 2014

$B_{(s)} \to D_{(s)}\ell\nu$

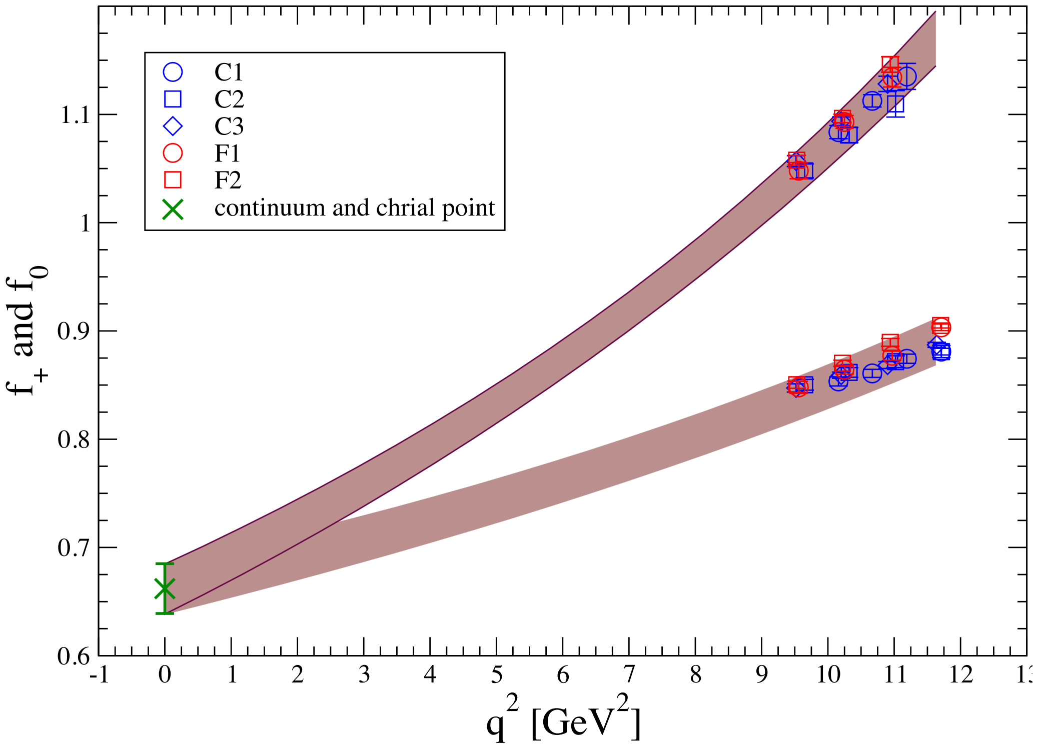
Heechang Na; 27th @ 17:30; sess. 6

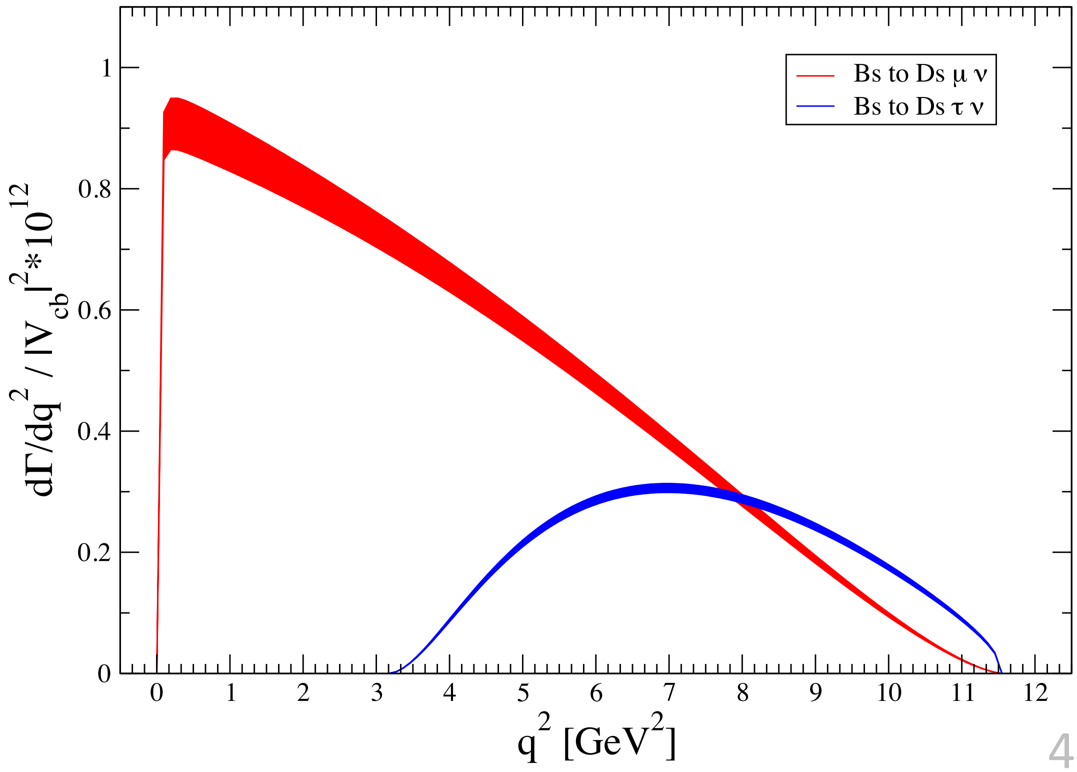
HPQCD

MILC 2+1 asqtad gauge cfgs
NRQCD b with HISQ light valence
a: 0.09, 0.12 fm
Mpi: 260 – 500 MeV

calculating:

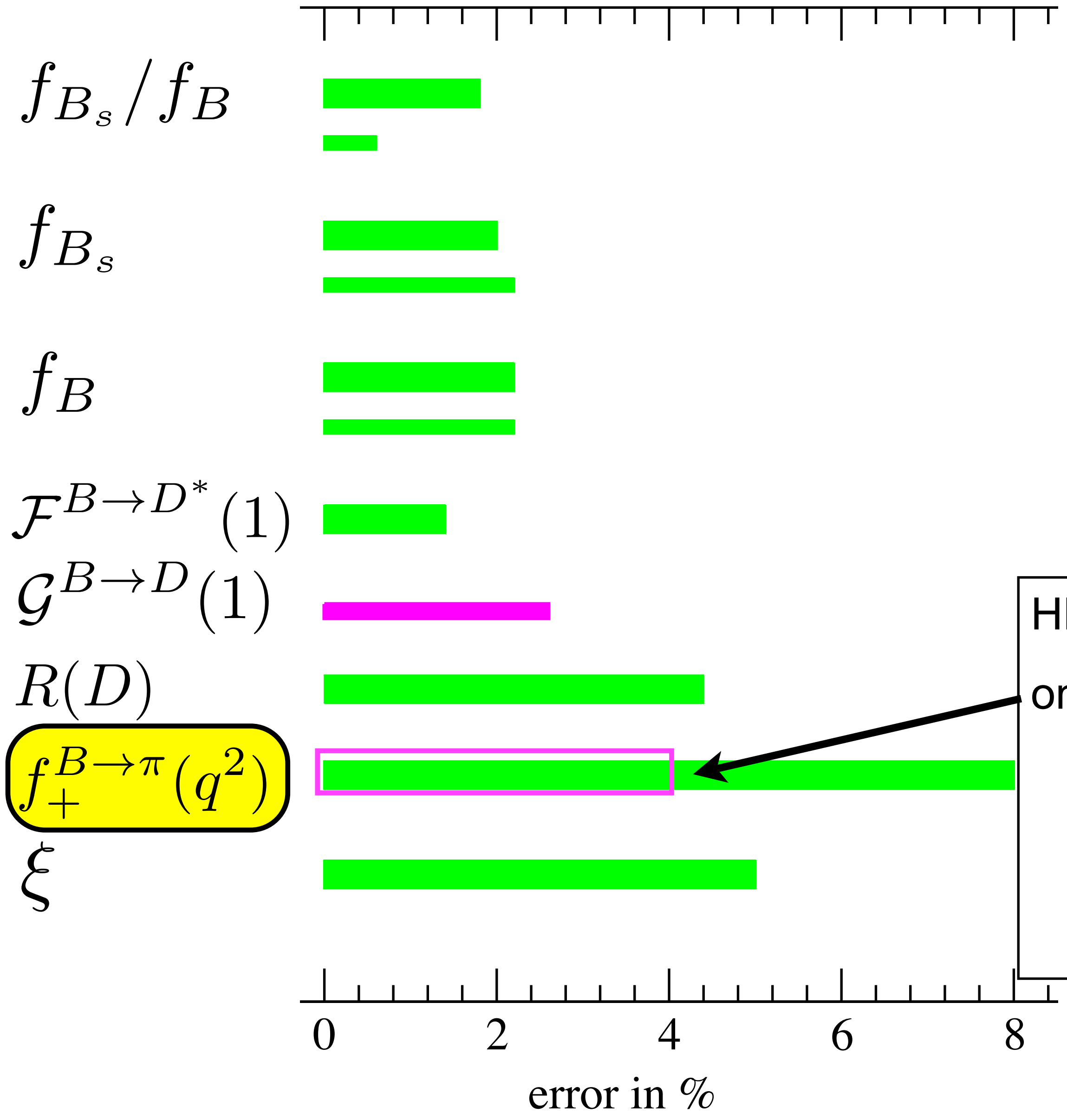
- shape of form factors for all q^2
- ratio of branching fractions: R(D), R(Ds)





B meson summary

errors (in %) comparison: **FLAG-2 averages** vs. **new results**



HPQCD: first results for $B \to K\ell\ell$ (2013) and $B_s \to K\ell\nu$ (2014)

ongoing work @ Lattice 2014:

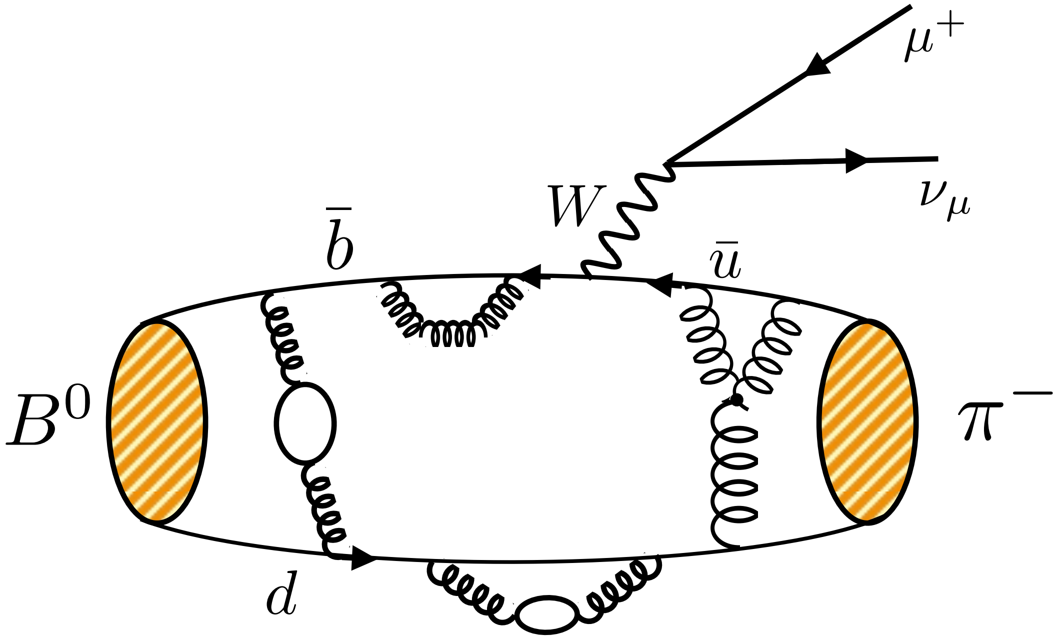
- HPQCD, FNAL/MILC, RBC/UKQCD, ...
- also for $B \to \pi\ell\ell$
- also for Λ_b decay (talk by A. Barucha @ Beauty 2014)

review by C. Bouchard @ Lattice 2014

see backup slides for more details

Semileptonic B-meson decay to light hadrons

Example: $B \to \pi \ell \nu$



$$\frac{d\Gamma(B\to\pi\ell\nu)}{dq^2} = (\text{known}) \times |V_{ub}|^2 \times |f_+(q^2)|^2$$

★ shape for semileptonic B decays:

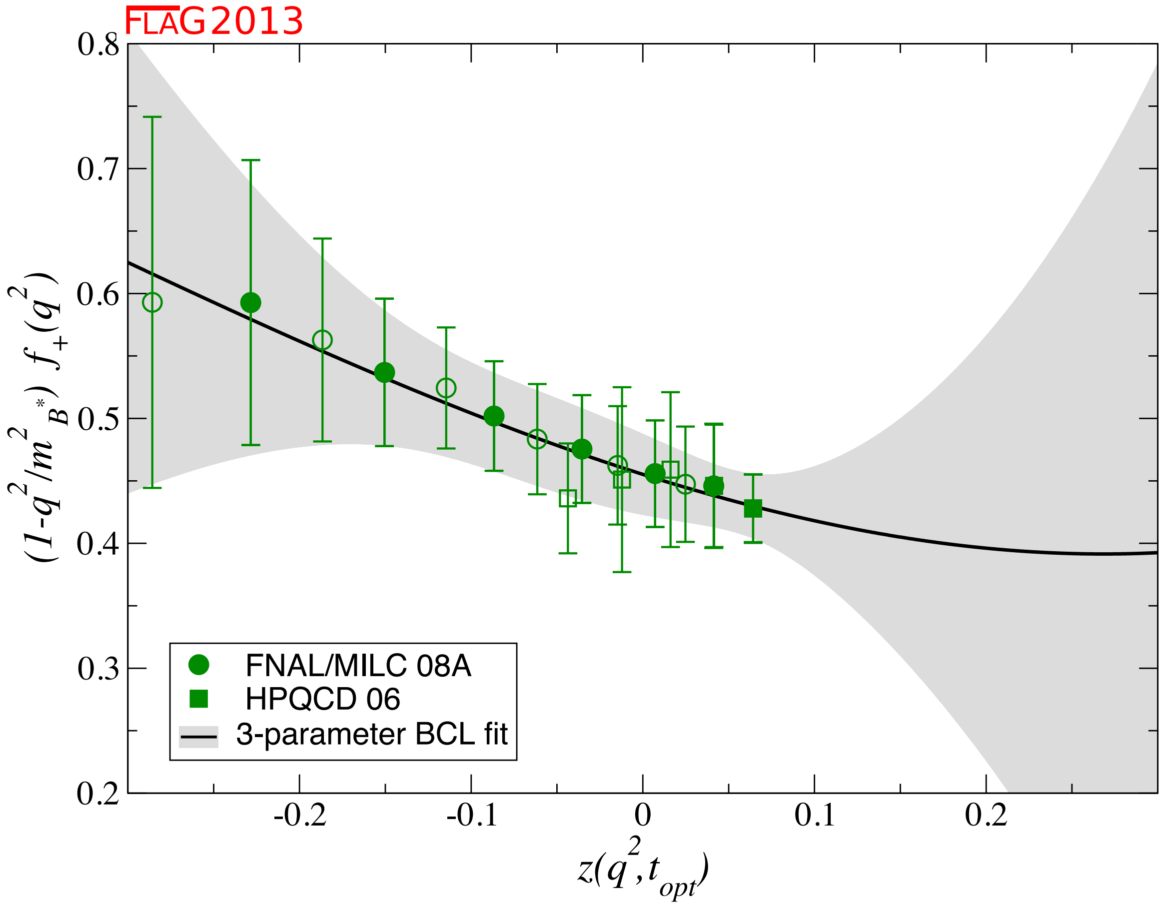
use z-expansion for model-independent parameterization of q^2 dependence

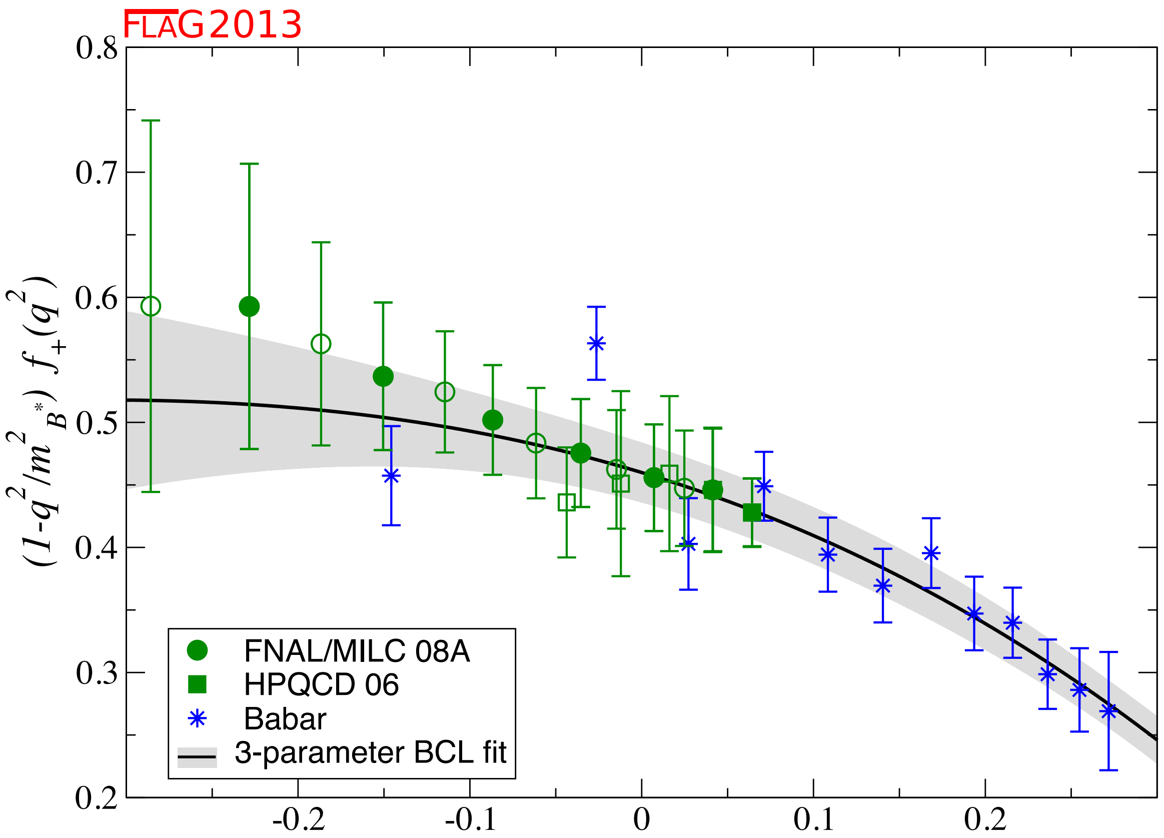
★ calculate all form factors, $f_+(q^2)$, $f_0(q^2)$(and $f_T(q^2)$ for the corresponding rare decay)

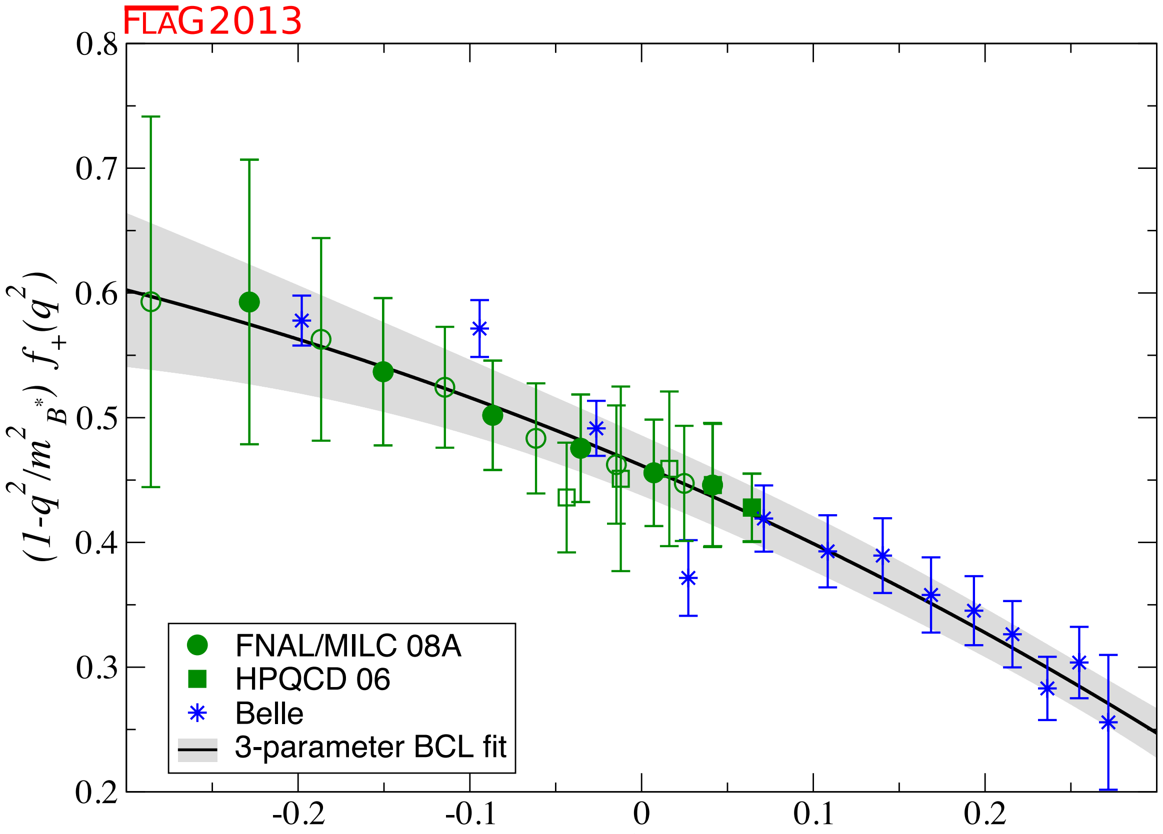
★ LQCD predictions of $B_s \to K\ell\nu$ form factors exist (HPQCD) and more are in progress (FNAL/MILC, RBC/UKQCD)

Form factor for $B \to \pi \ell \nu$ & V_{ub}

S. Aoki et al (FLAG-2 review, arXiv:1310.8555)







Form factor for $B \to \pi\ell\nu$ & V_{ub}

review by C. Bouchard
@ Lattice 2014

$B \to \pi\ell\nu$

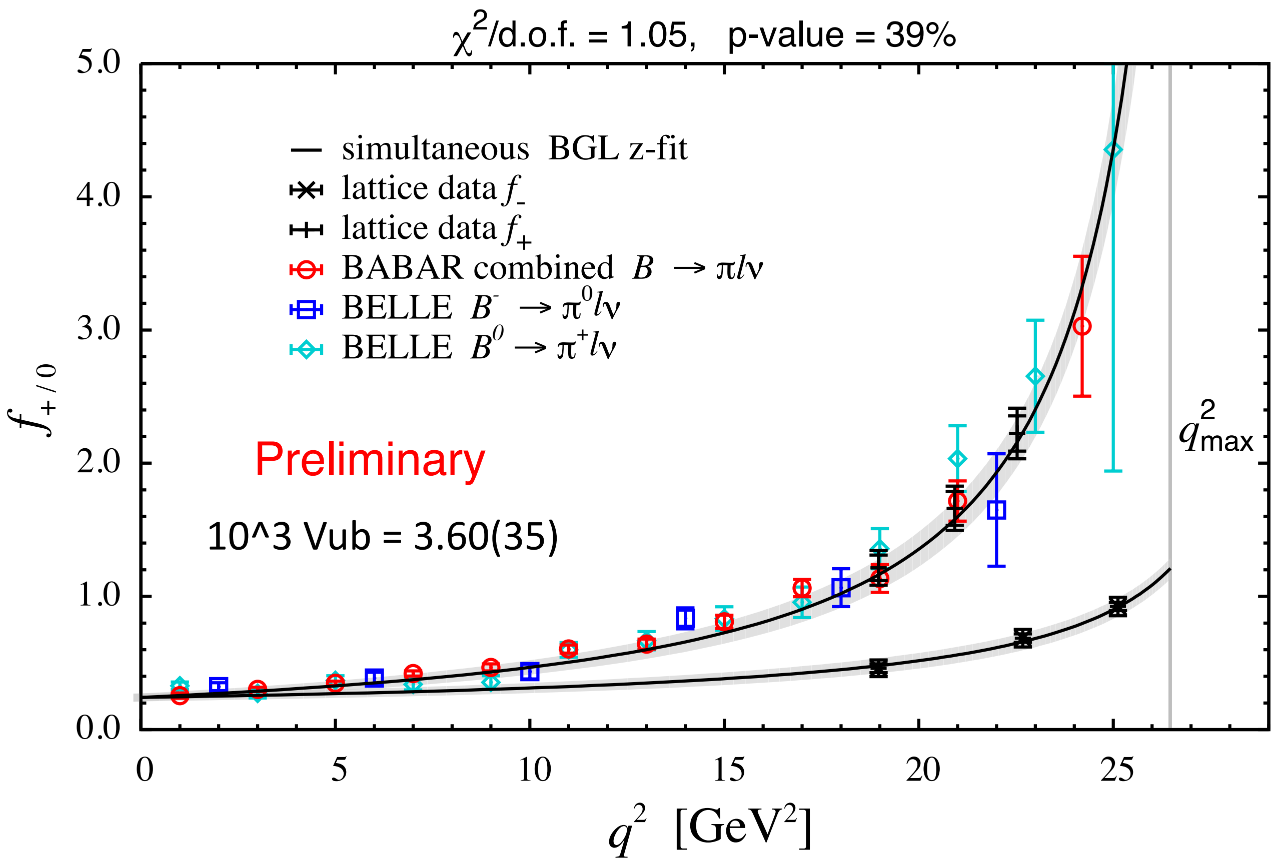
RBC-UKQCD

2+1 flavor DW + Iwasaki gauge fields
DW light and non-pert tuned RHQ b valence
a: 0.08, 0.11 fm
Mpi: 289 – 422 MeV

Taichi Kawanai; 27th @ 16:50; sess. 6

combined chiral/continuum extrapolation with SU(2) Hard Pion ChPT
kinematic extrapolation via z-expansion

χ^2/d.o.f. = 1.05, p-value = 39%



Preliminary

10^3 Vub = 3.60(35)

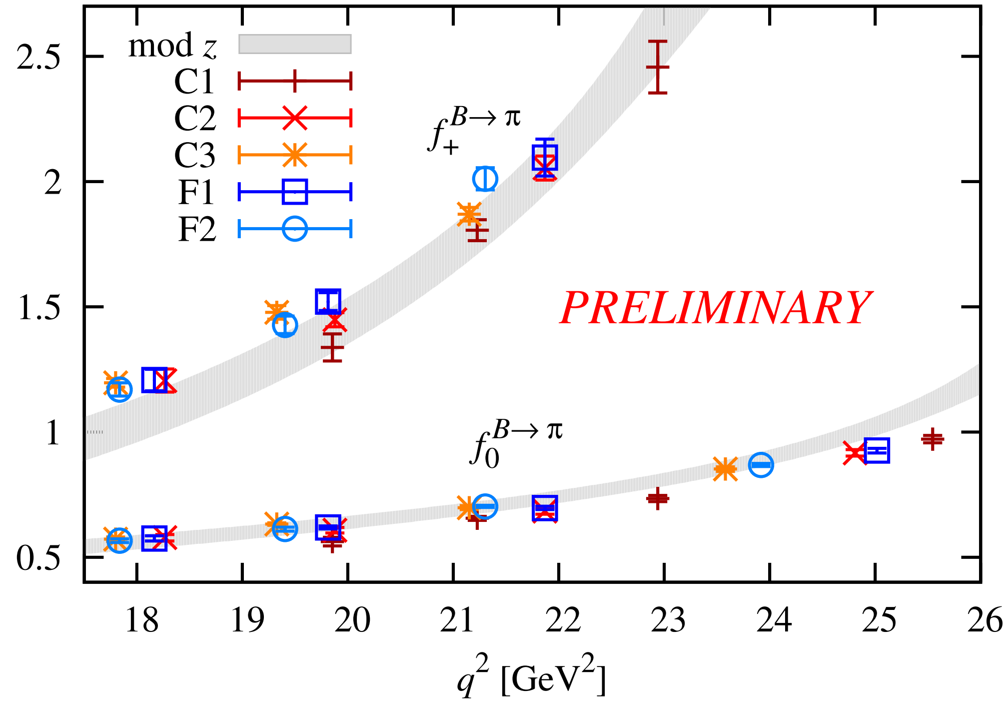
Form factor for $B \to \pi\ell\nu$ & V_{ub}

review by C. Bouchard @ Lattice 2014

$B \to \pi\ell\nu$

HPQCD

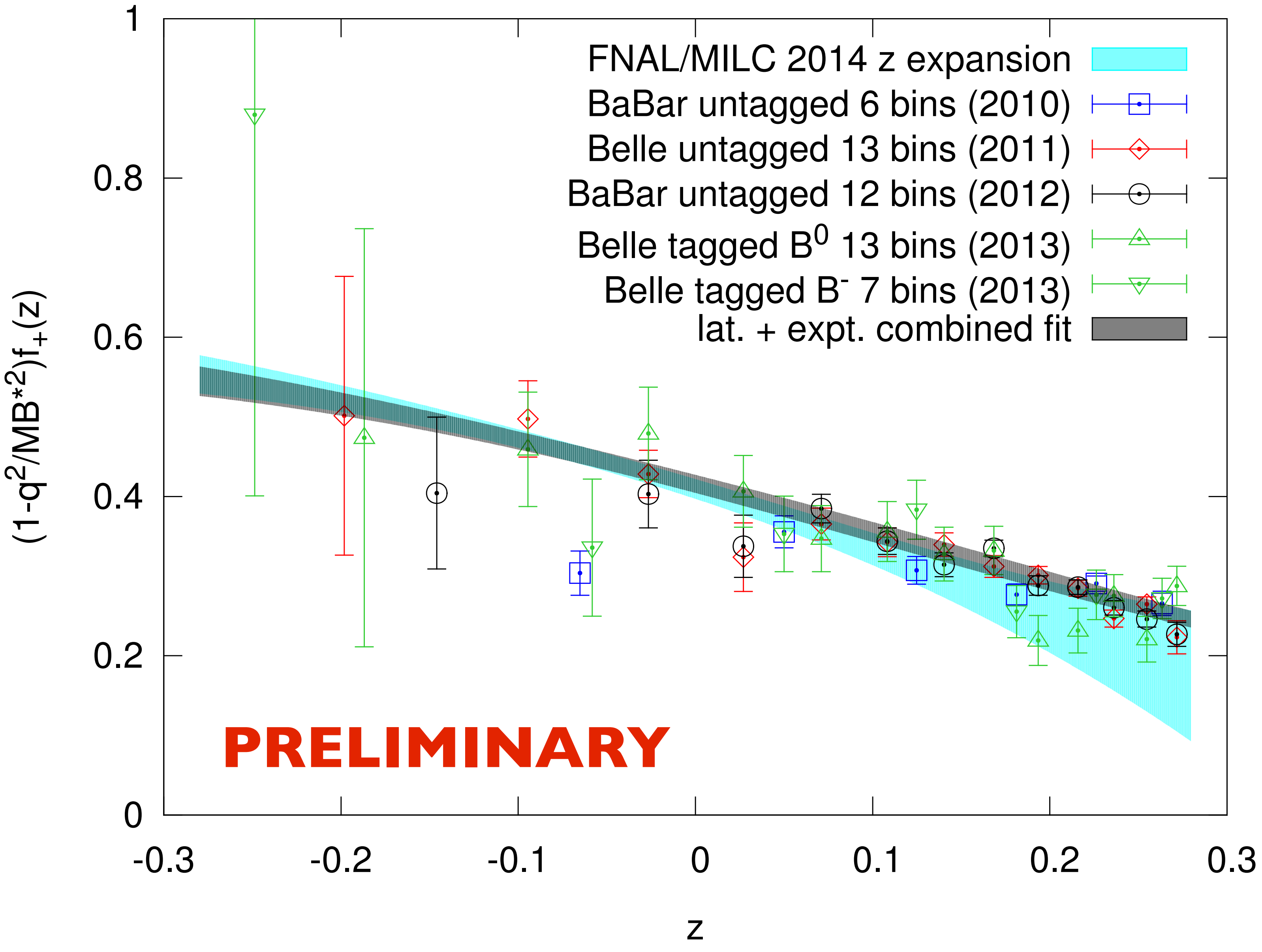
MILC 2+1 asqtad sea with NRQCD b and HISQ light valence
a: 0.09, 0.12 fm
Mpi: 190 – 400 MeV



- adding statistics
- exploring possibility of using Hard Pion ChPT + modified z-expansion to extend range of q^2 and improve overlap with experiment
  - p = 2pi/L (000, 001, 011, 111, 002, 003, 004)
  - would give q2 range: ~6 – 26 GeV^2

Form factor for $B \to \pi \ell \nu$ & V_{ub}

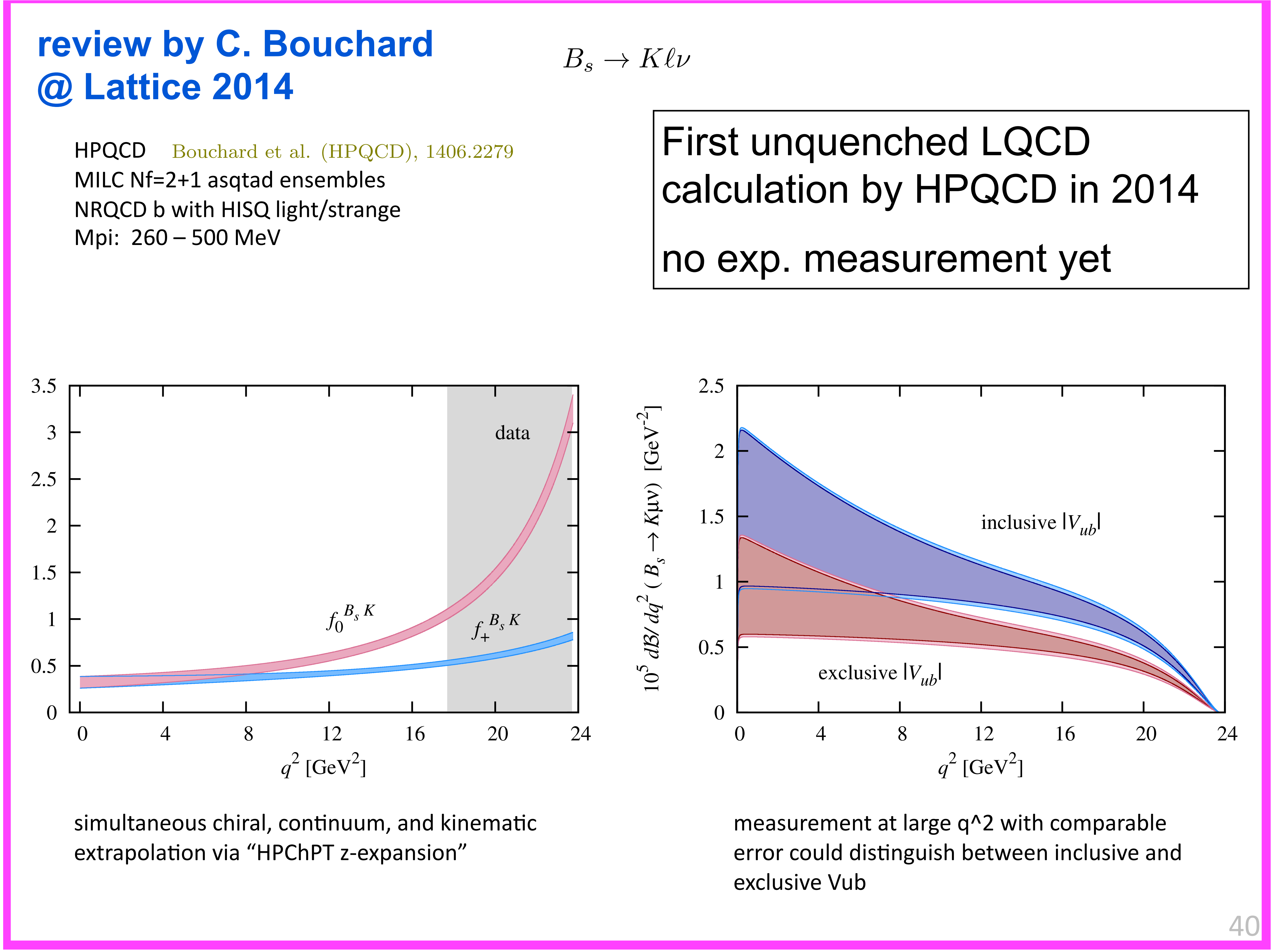
D. Du (FNAL/MILC) @ Lattice 2014



- **blind analysis**
- $N_f = 2{+}1$ (Asqtad)
- 4 a's, 12 ensembles
- min. $m_\pi \sim 174$ MeV
- Fermilab b quarks
- new functional method for z-expansion fit after chiral extrapolation.
- complete systematic error budget

➙ error on $|V_{ub}| \sim 4.1\%$

Form factors for $B_s \to K\ell\nu$ & V_{ub}



review by C. Bouchard @ Lattice 2014

$B_s \to K\ell\nu$

HPQCD Bouchard et al. (HPQCD), 1406.2279
MILC Nf=2+1 asqtad ensembles
NRQCD b with HISQ light/strange
Mpi: 260 – 500 MeV

First unquenched LQCD calculation by HPQCD in 2014

no exp. measurement yet

simultaneous chiral, continuum, and kinematic extrapolation via "HPChPT z-expansion"

measurement at large q^2 with comparable error could distinguish between inclusive and exclusive Vub

Form factors for $B_s \to K\ell\nu$ & V_{ub}

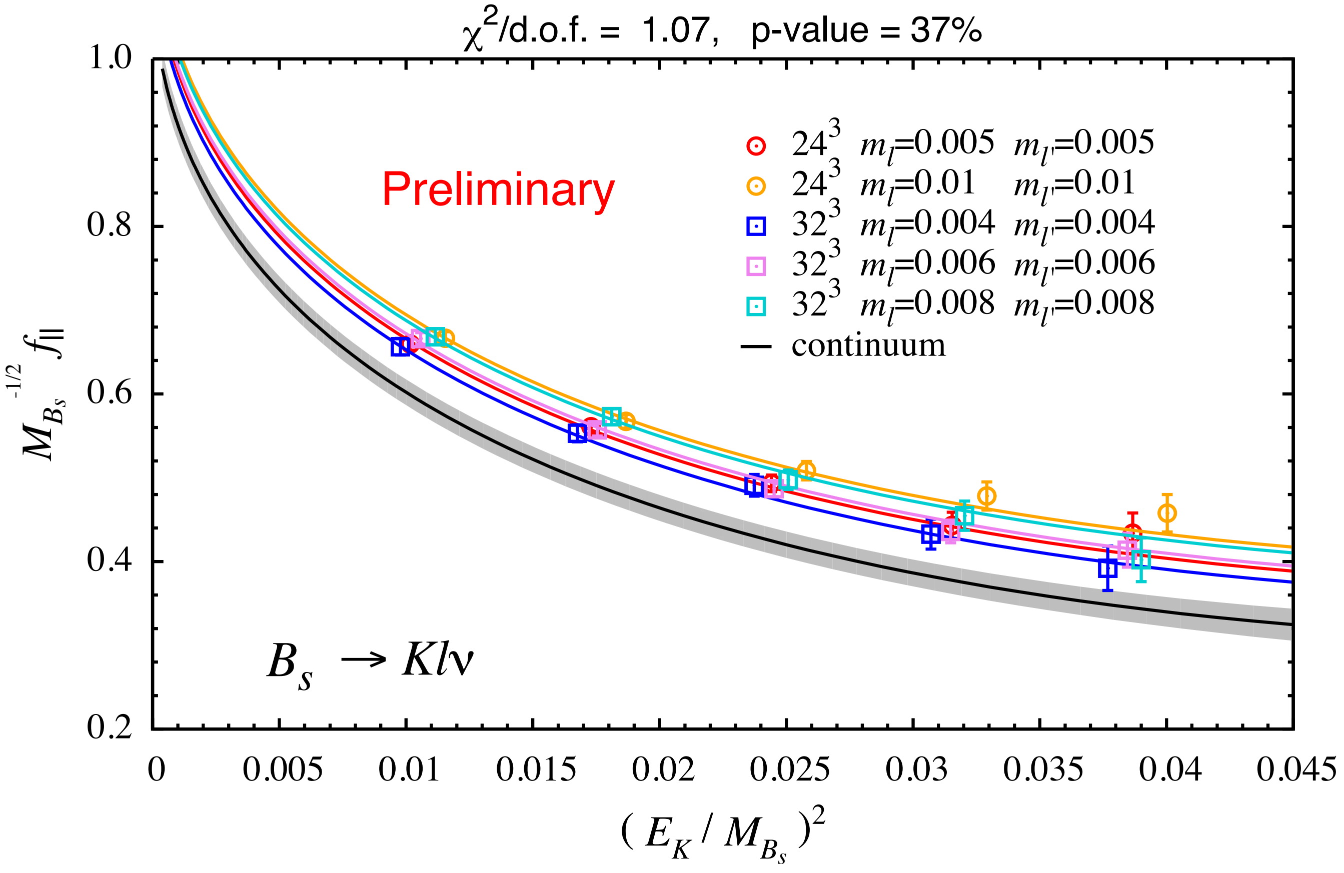
review by C. Bouchard @ Lattice 2014

$B_s \to K\ell\nu$

Taichi Kawanai; 27th @ 16:50; sess. 6

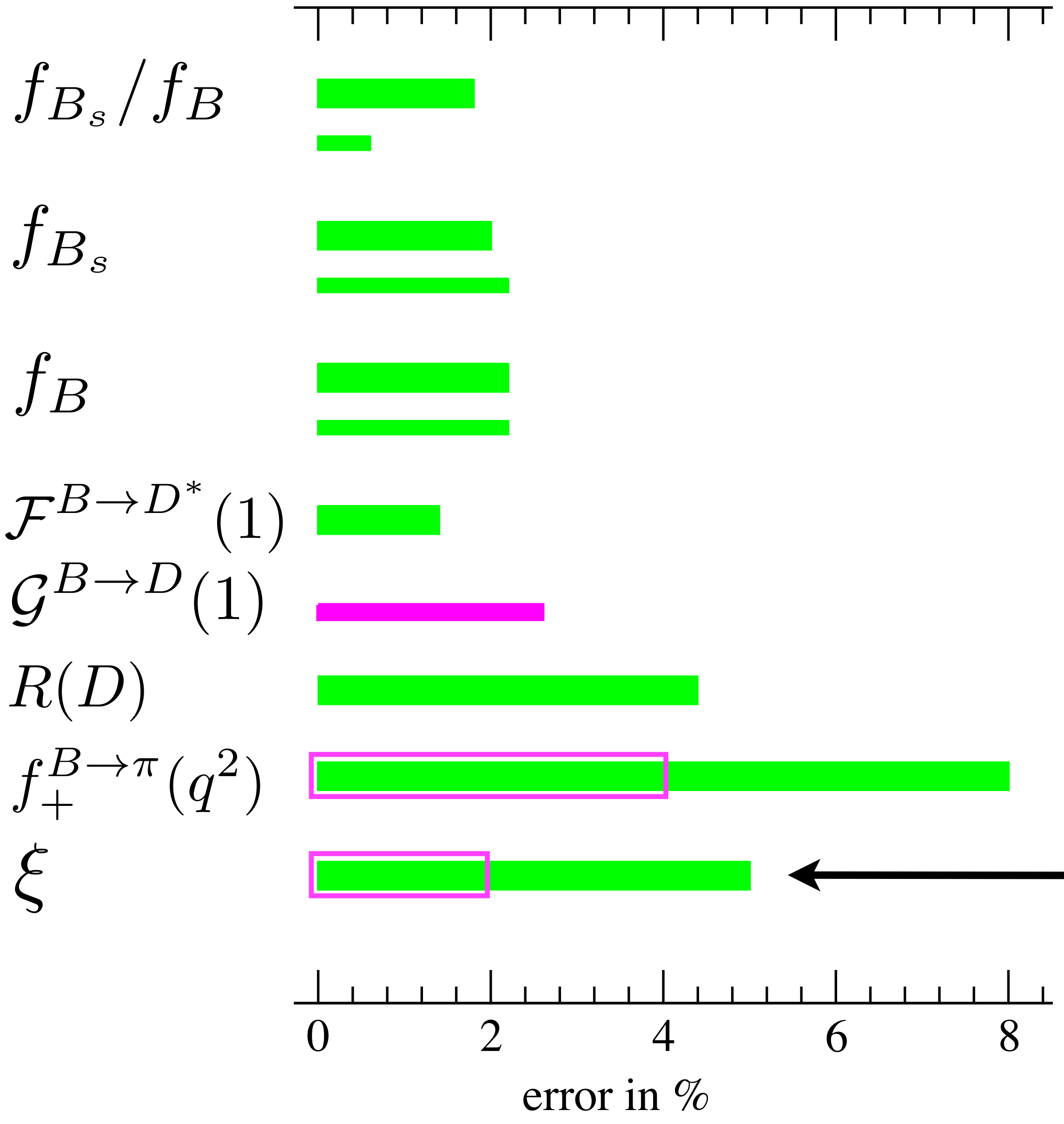
RBC-UKQCD

2+1 flavor DW + Iwasaki gauge fields
DW light/strange and non-pert tuned RHQ b valence
a: 0.08, 0.11 fm
Mpi: 289 – 422 MeV



B meson summary

errors (in %) comparison: **FLAG-2 averages** vs. **new results**



ongoing work for all five matrix elements @ Lattice 2014:
RBC/UKQCD, HPQCD, FNAL/MILC, ETM, ...

review by C. Bouchard @ Lattice 2014

see backup slides for more details

Simple quantities in LQCD

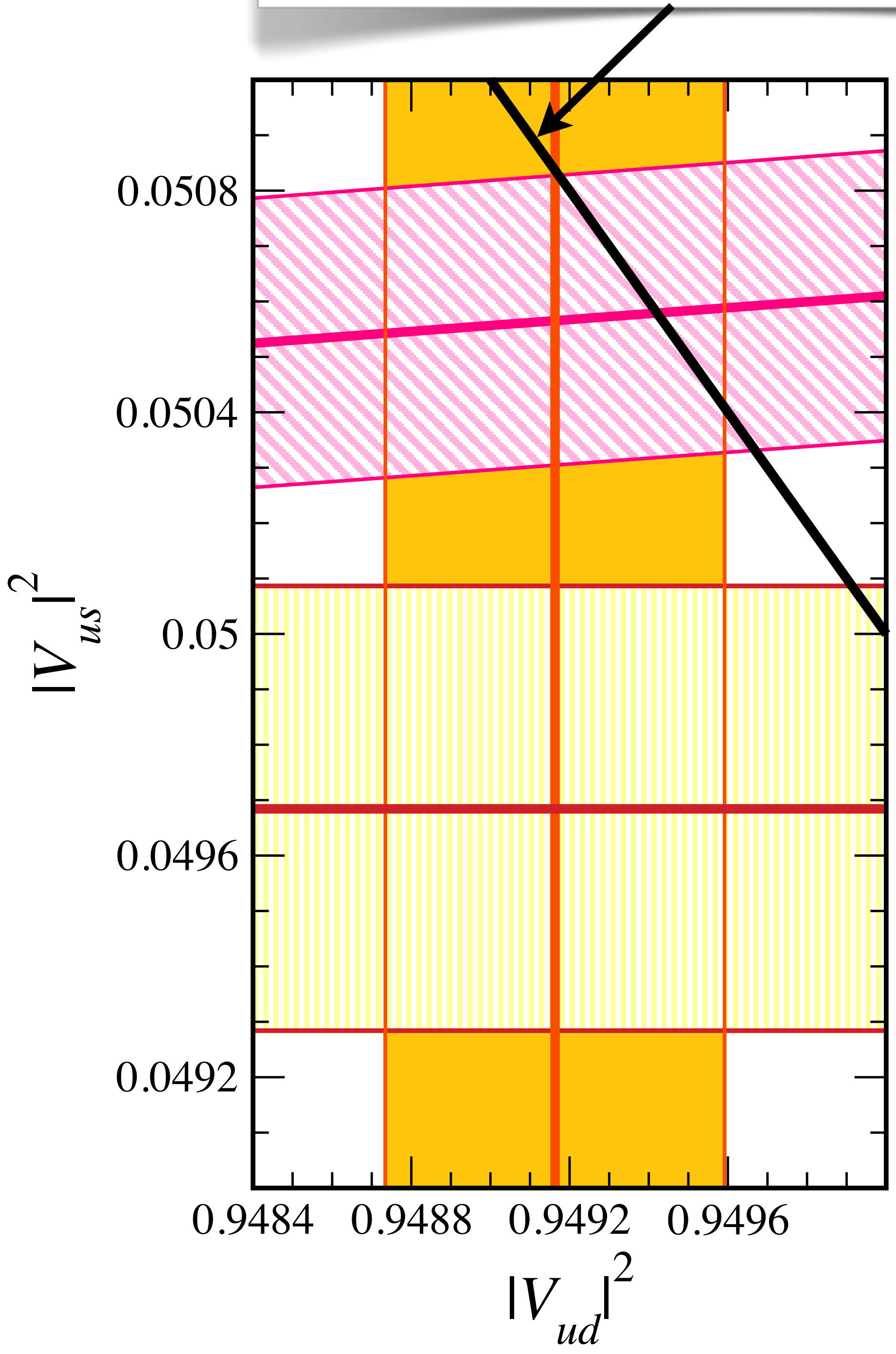
- ☆ low-lying hadron spectrum ➙ quark masses, α_s
- ☆ weak decays - leptonic, semileptonic, mixing
  - Kaons
  - D mesons
  - B mesons

  ➙ CKM, BSM phenomenology
- ☆ high precision ➙ including QED

Implications for the 1st row of the CKM Matrix

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

$|V_{ub}| \approx 4 \times 10^{-3} \approx 0$



Constraining $|V_{us}|$ using FNAL/MILC 13 (K_{l3}) or FNAL/MILC 2014 (K_{l2}):

The uncertainty on $|V_{us}|^2$ is the same/smaller compared to the uncertainty on $|V_{ud}|^2$

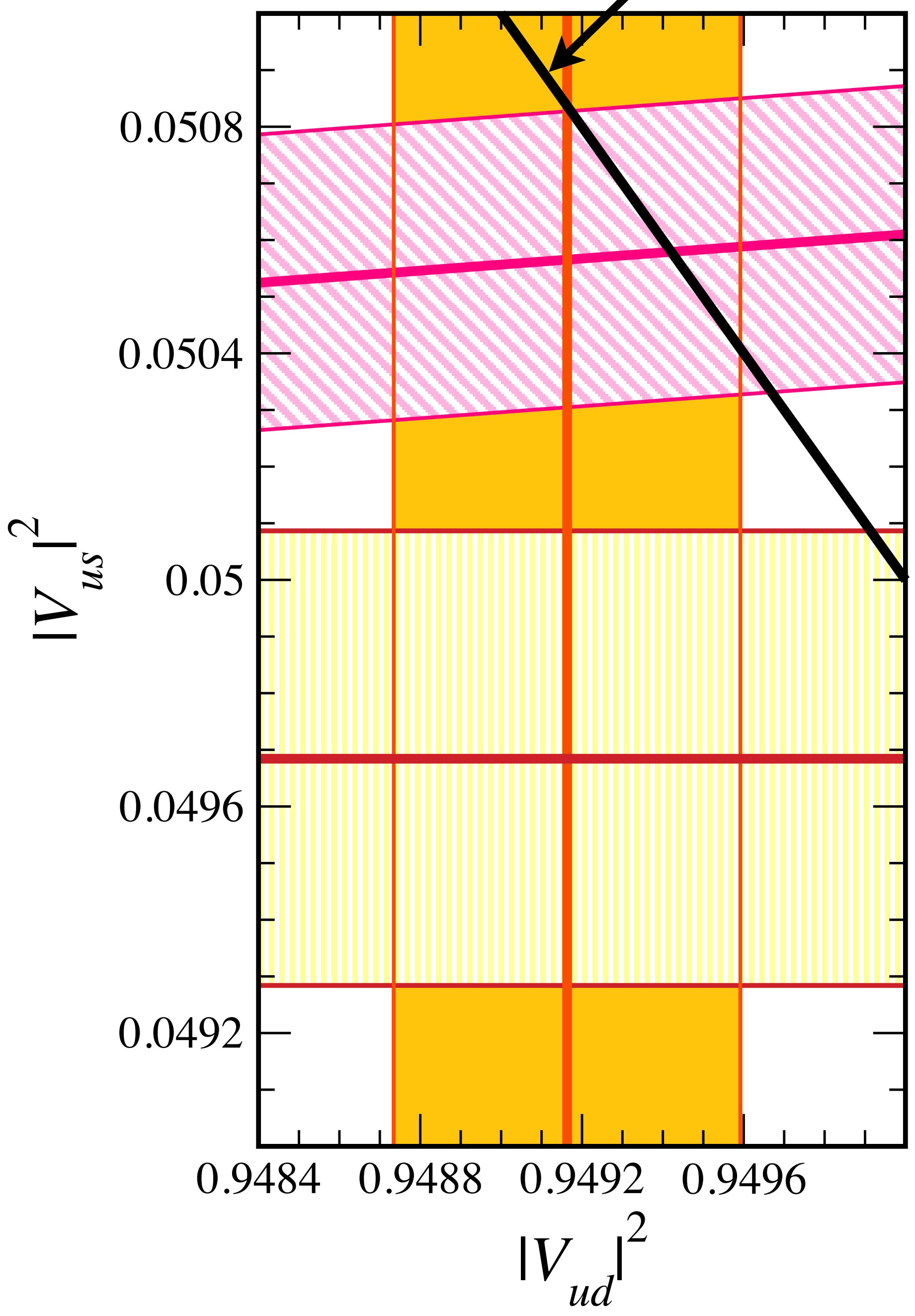
Time to revisit the uncertainty on $|V_{ud}|$?

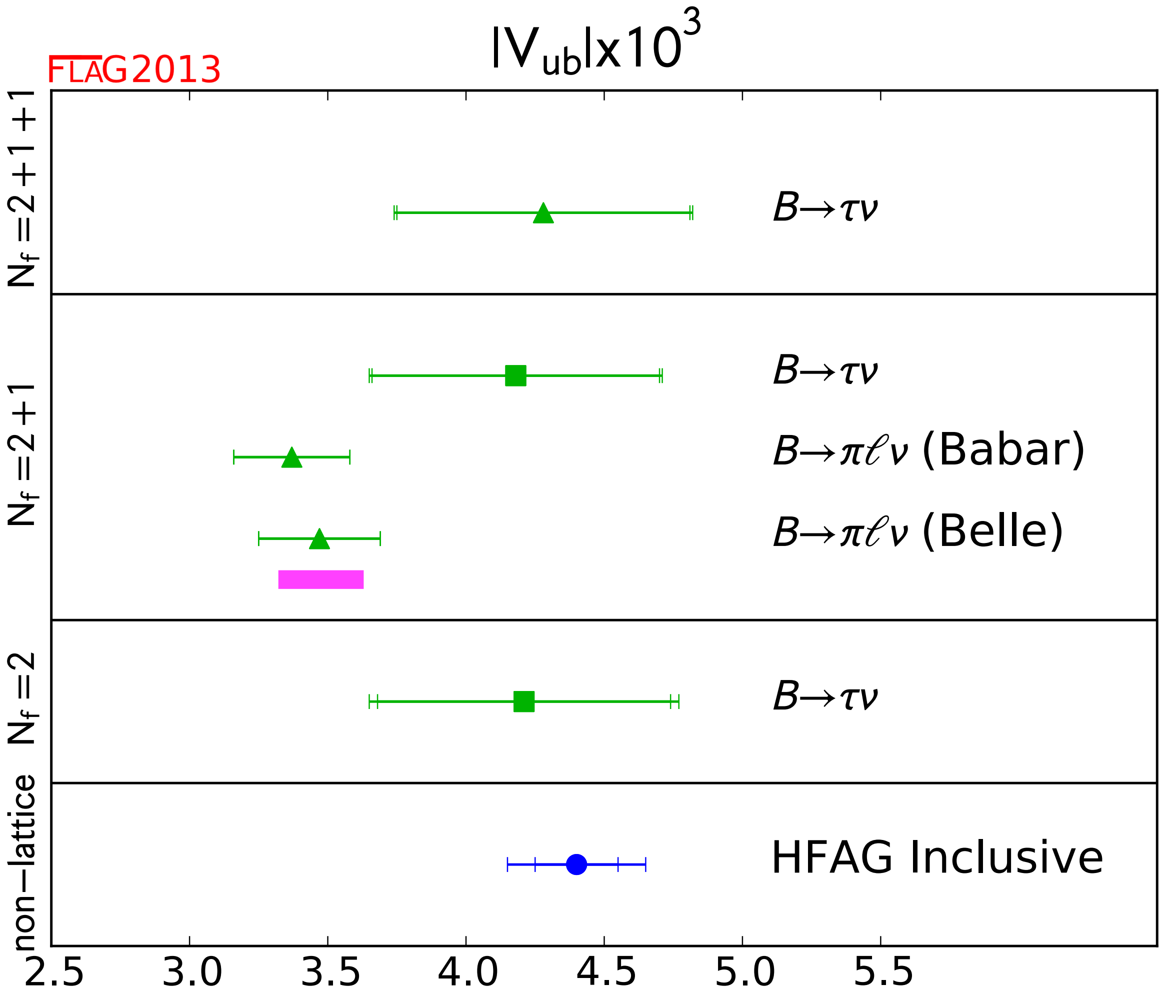
Slight tension between K_{l2} and K_{l3} and for K_{l3} with unitarity prediction.

Implications for the 1st row of the CKM Matrix

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

$$|V_{ub}| \approx 4 \times 10^{-3} \approx 0$$

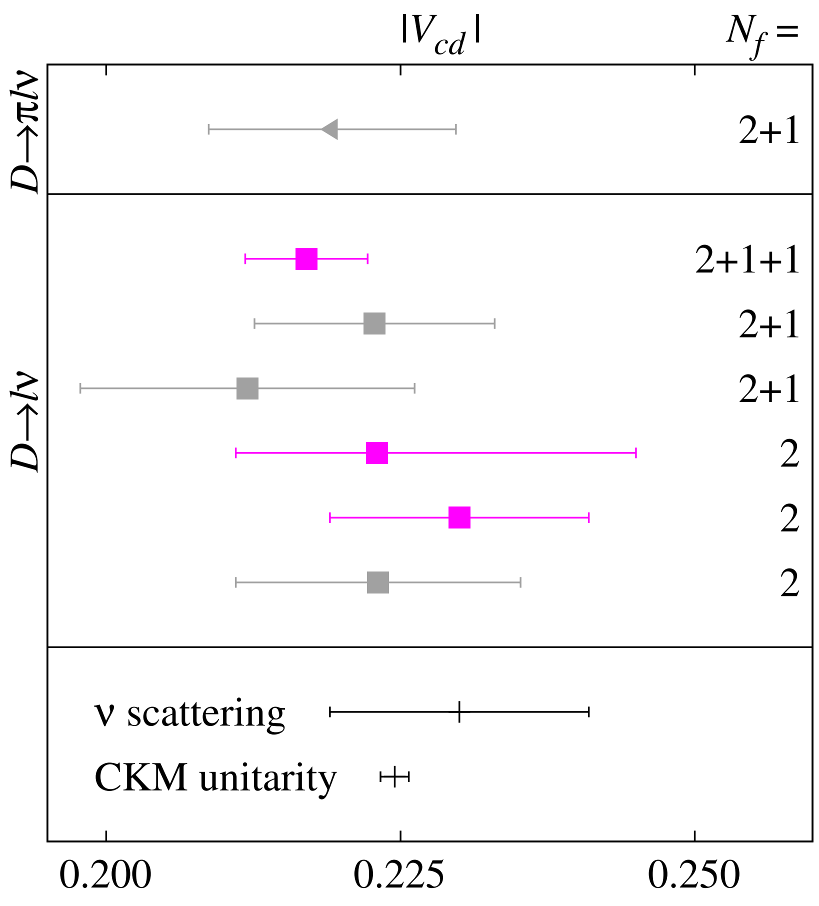


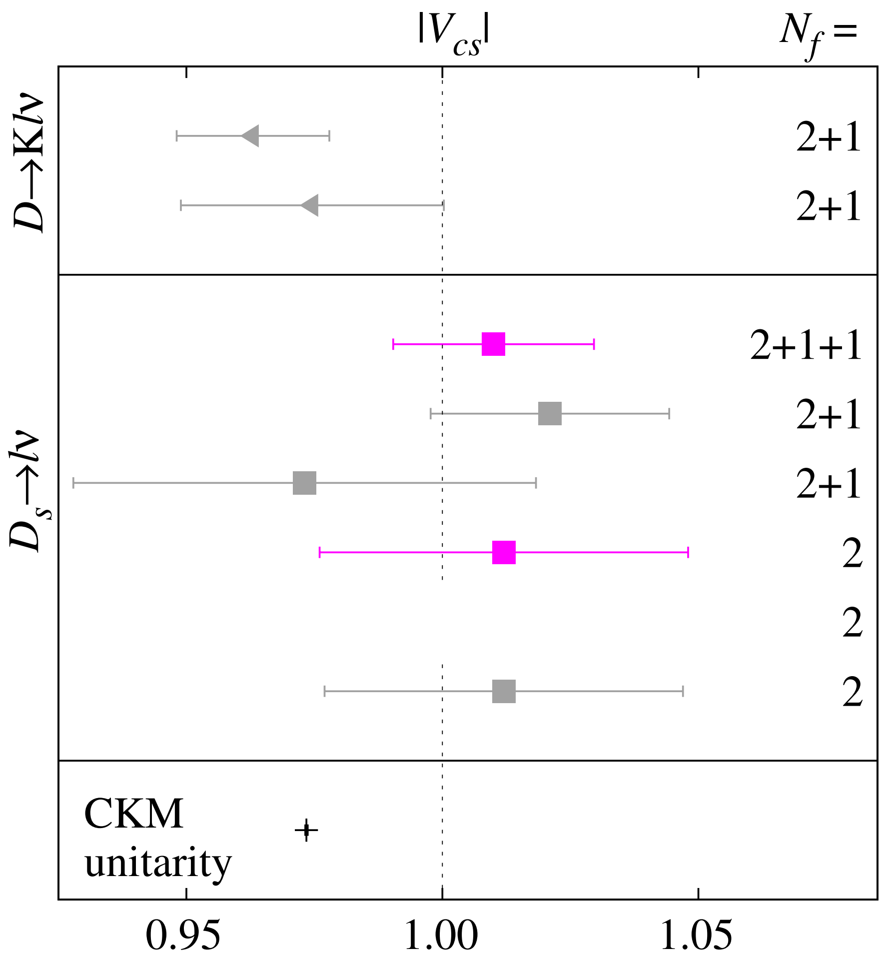


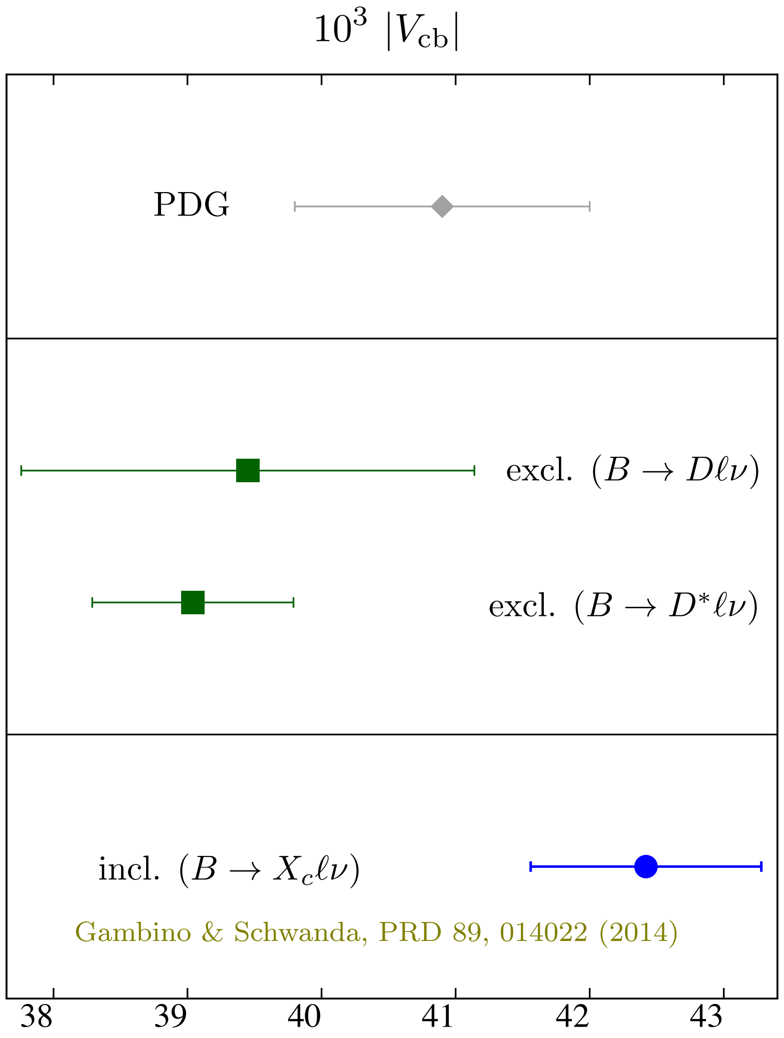
$\sim3\sigma$ tension between exclusive and inclusive determinations

Implications for the 2nd row of the CKM Matrix

review by C. Bouchard @ Lattice 2014



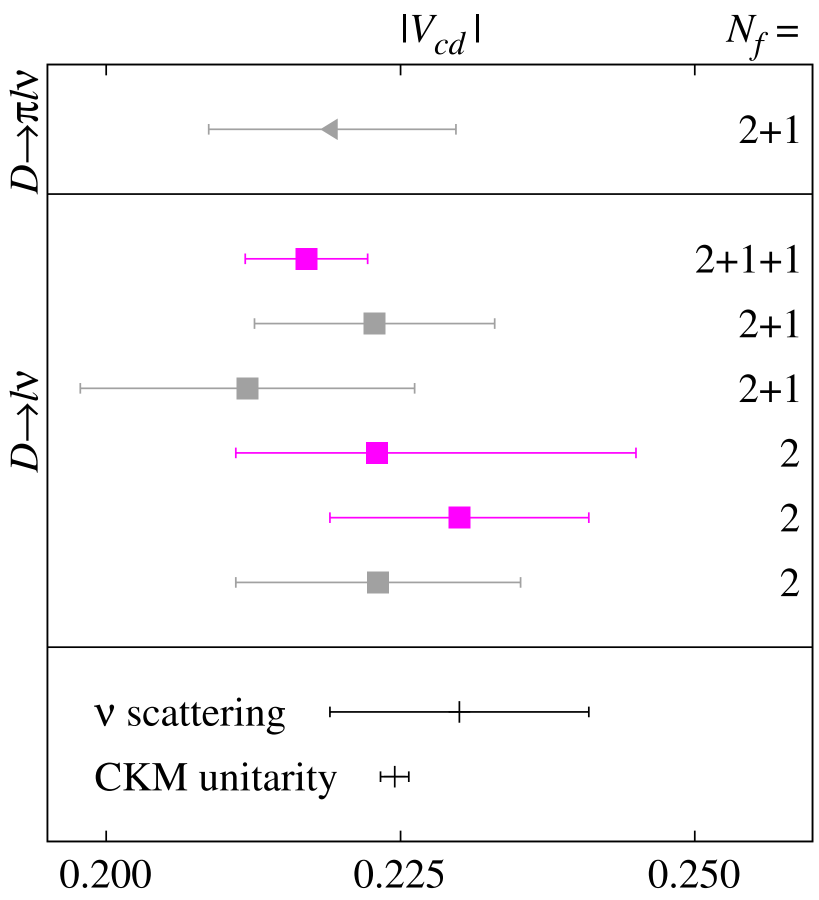


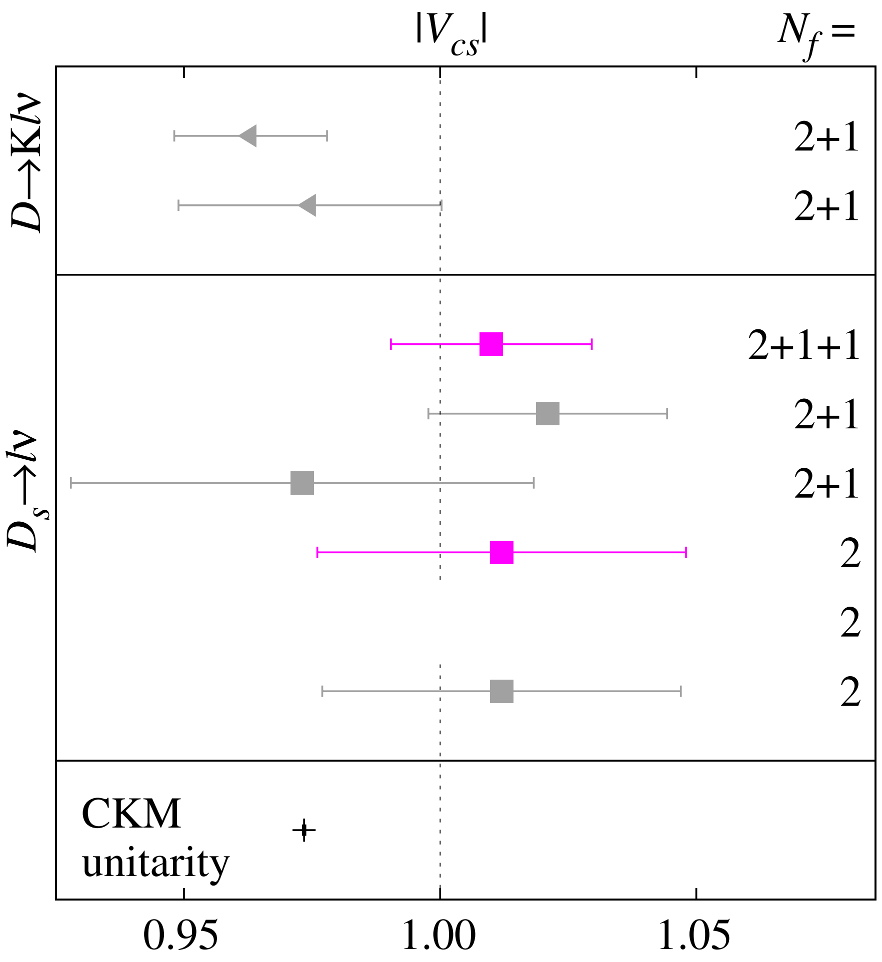


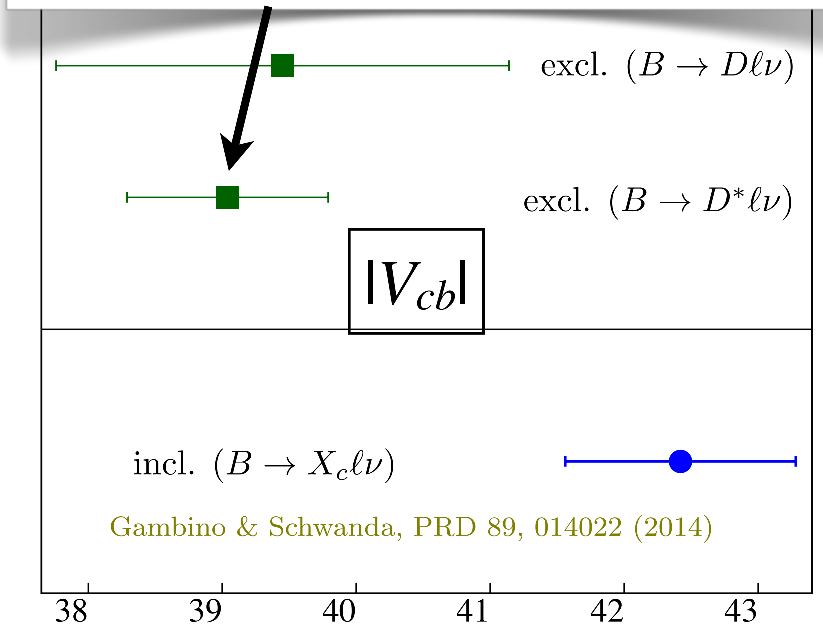
Implications for the 2nd row of the CKM Matrix

review by C. Bouchard @ Lattice 2014

- LQCD error commensurate with exp.
- a 0.5% error due to Coulomb, EM effects is included in the total error for $|V_{cb}|$





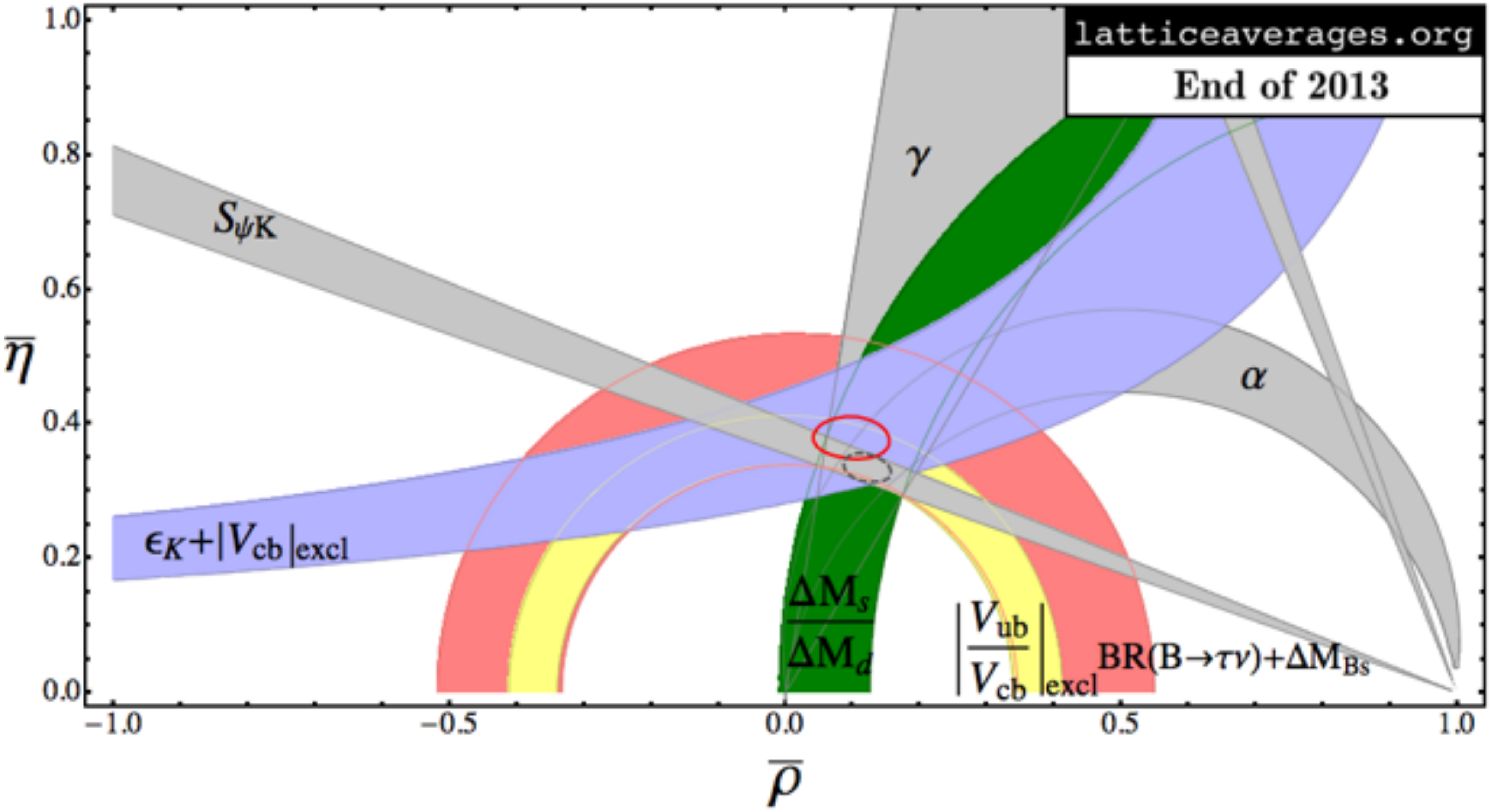


Slight tension between leptonic channel and CKM unitarity

$\sim 3\sigma$ tension between exclusive and inclusive determinations

UT analysis

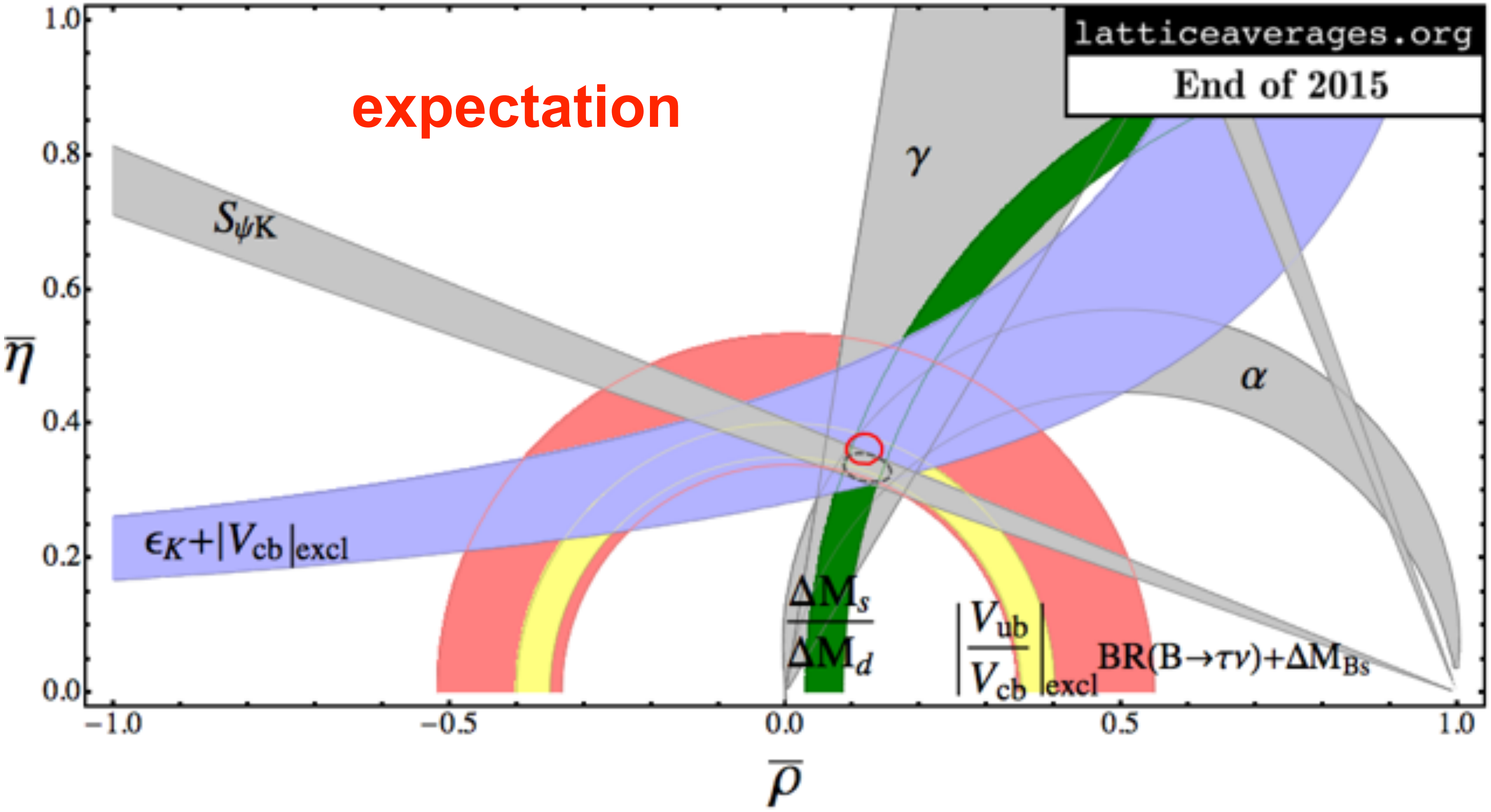
Laiho, Lunghi & Van de Water (Phys.Rev.D81:034503,2010), E. Lunghi, private comm.



The (red, yellow, green and blue) error bands are (still) dominated by theory errors, in particular by errors on hadronic matrix elements calculated in LQCD.

UT analysis

Laiho, Lunghi & Van de Water (Phys.Rev.D81:034503,2010), E. Lunghi, private comm.



New bands for $|V_{ub}/V_{cb}|_{excl}$ and $\Delta m_s/\Delta m_d$ (yellow, green) assuming a 4% error on $|V_{ub}|_{excl}$ and a 2% error on ξ.

Exclusive $|V_{cb}|$, $|V_{ub}|$ only

BSM phenomenology

S. Hansmann-Menzemer @ EPS 2013

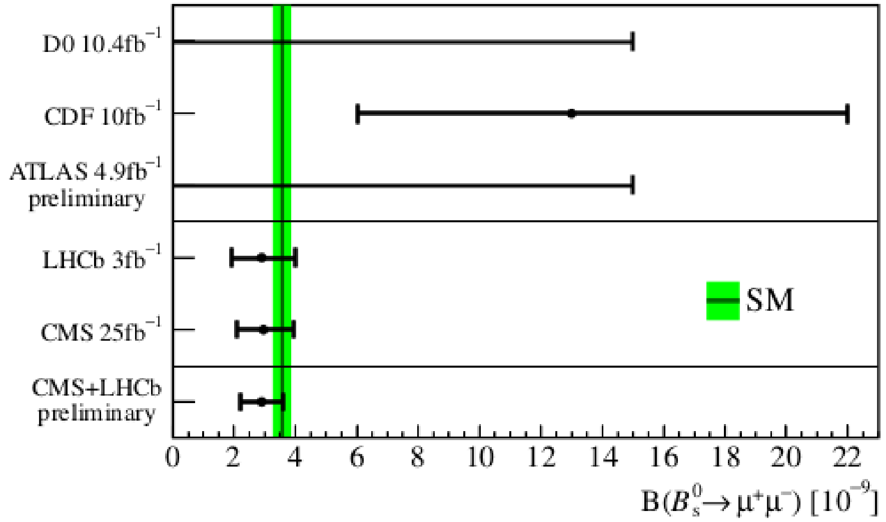
Combined LHCb + CMS Result

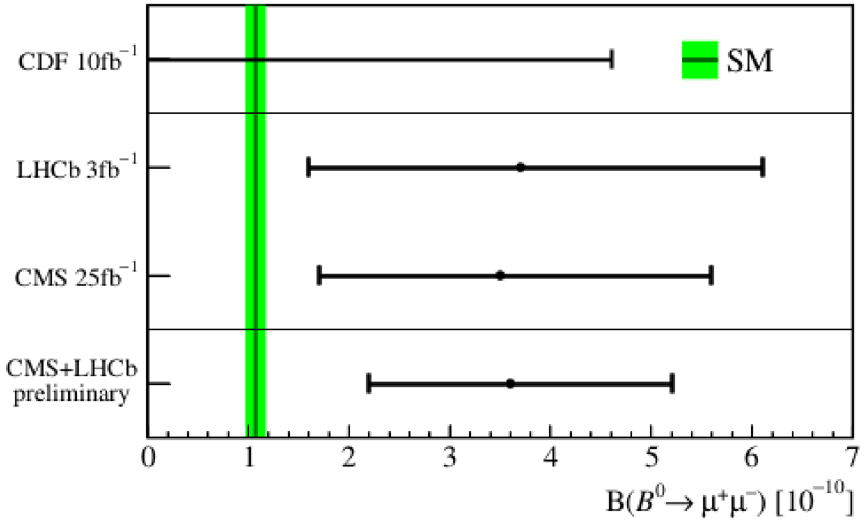
new @ EPS2013

Observation:

BR($B_s \to \mu^+\mu^-$) = (2.9 $\pm$ 0.7) $\times$ 10^{-9}

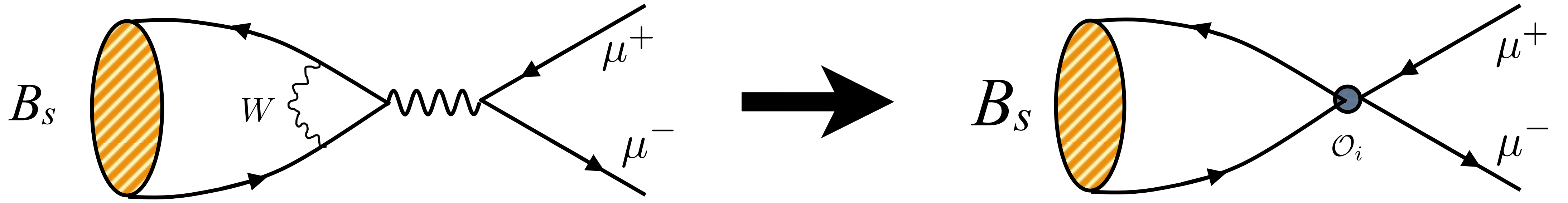
BR($B^0 \to \mu^+\mu^-$) = $3.6^{+1.6}_{-1.4}$ $\times$ 10^{-10}



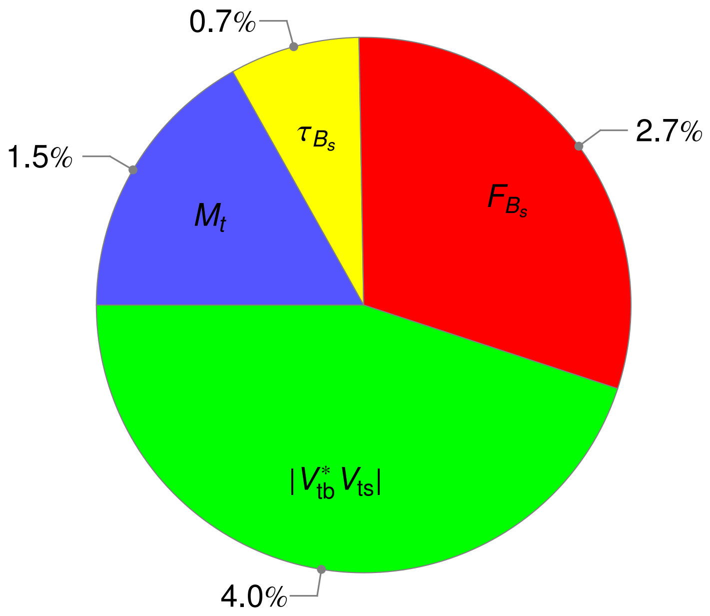


SM prediction depends on f_{Bs} or $\hat{B}_{B_s}$

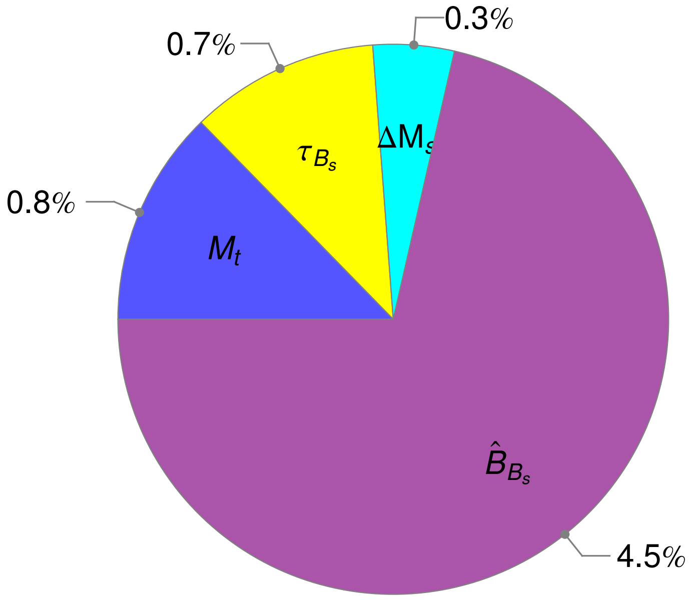
BSM phenomenology $B_s \to \mu^+\mu^-$



Standard Model prediction: Buras, et al (arXiv:1303.3820, JHEP 2013), Bobeth, et al (arXiv:1311.0903, PRL 2014)

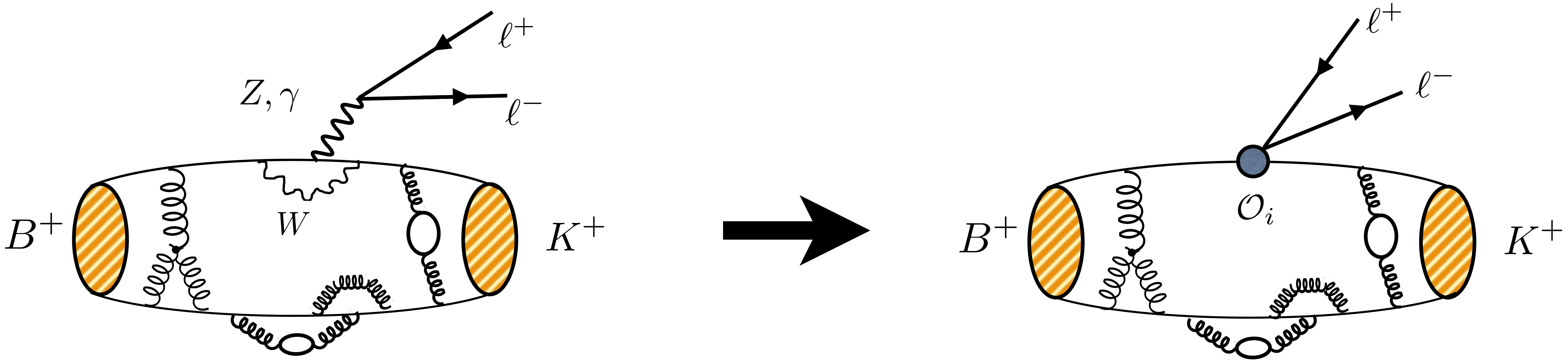


uses f_{B_s} from HPQCD 13



uses $\hat{B}_{B_s}$ from HPQCD 09

Form factors for $B \to K\ell^+\ell^-$



$$\mathcal{H}_{\rm eff} = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_i \left(C_i O_i + C_i' O_i' \right)$$

- SM GIM, loop, and Cabibbo suppressed
- $O_i^{(\prime)}$ are local operators
- $C_i^{(\prime)}$ are Wilson coefficients (model specific)
- hadronic matrix elements $\langle K|\mathcal{O}_i^{(\prime)}|B\rangle$
- observed rate constrains $C_i^{(\prime)}$

e.g.

$$O_7^{(\prime)} = \frac{e\, m_b}{16\pi^2}\ \bar{s}\sigma_{\mu\nu}P_{R(L)}b\ F^{\mu\nu}$$

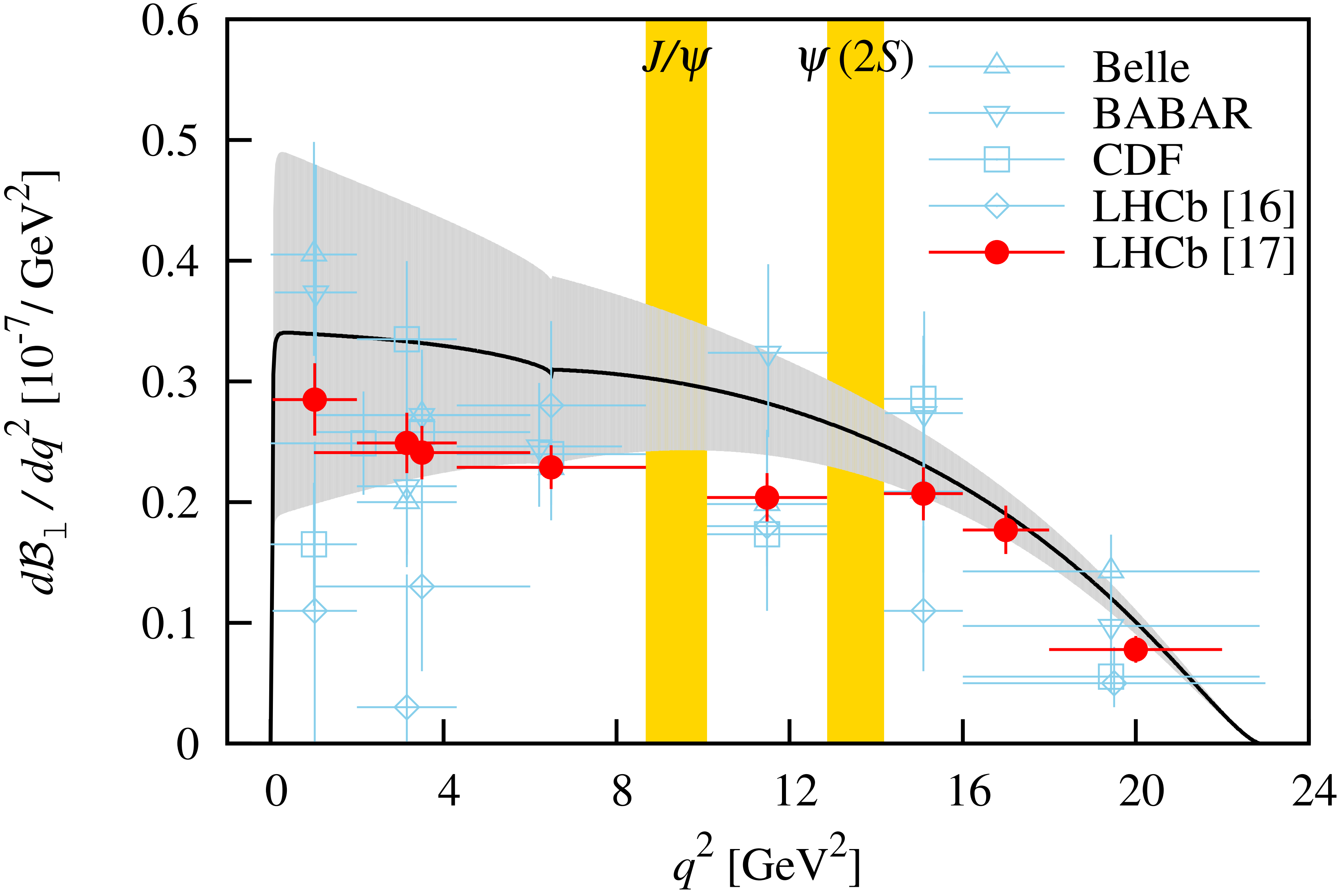
$$O_9^{(\prime)} = \frac{e^2}{16\pi^2}\ \bar{s}\gamma_\mu P_{L(R)}b\ \bar{\ell}\gamma^\mu\ell$$

$$O_{10}^{(\prime)} = \frac{e^2}{16\pi^2}\ \bar{s}\gamma_\mu P_{L(R)}b\ \bar{\ell}\gamma^\mu\gamma_5\ell$$

$\vdots$

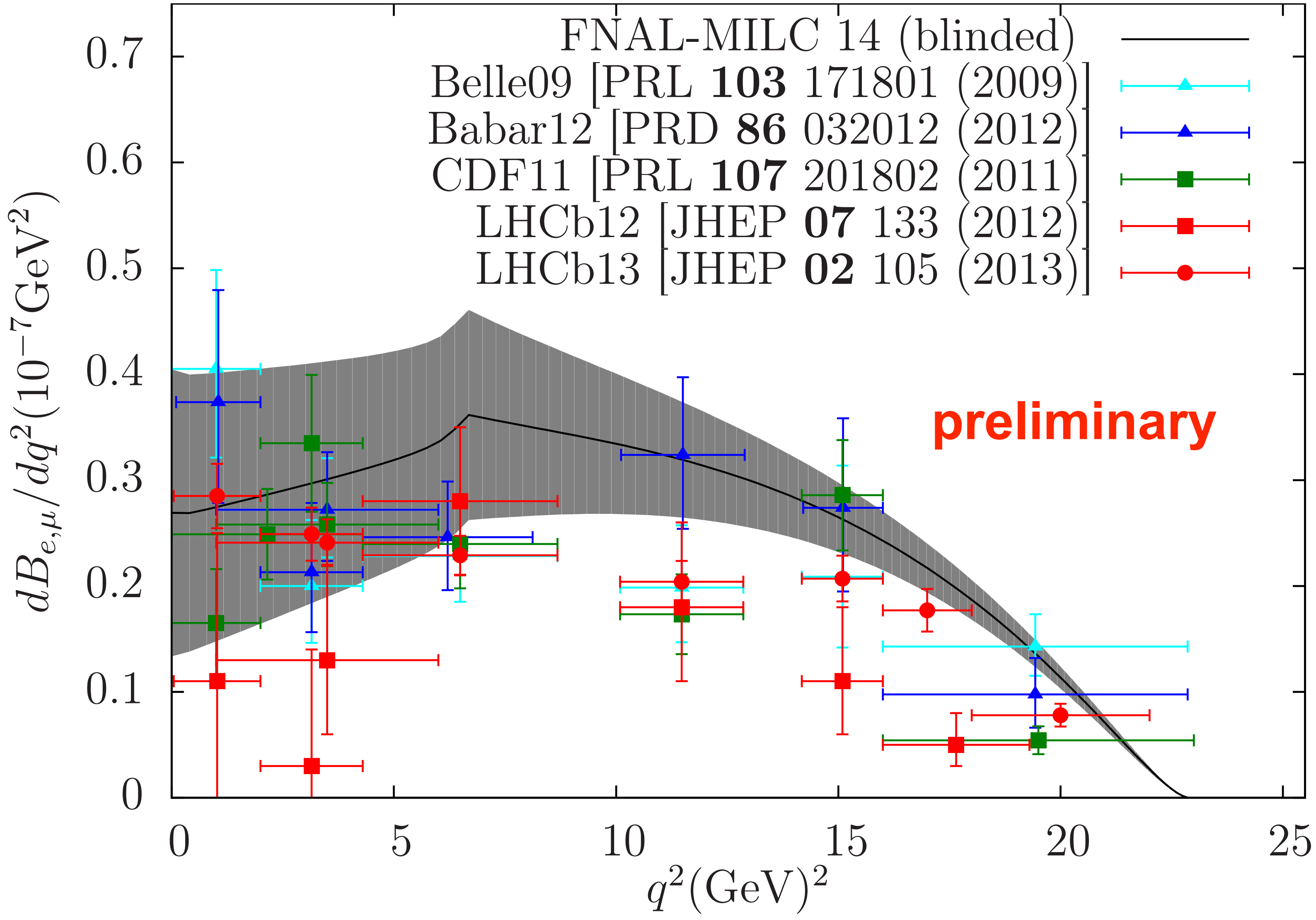
Form factors for $B \to K\ell^+\ell^-$

C. Bouchard (HPQCD, based on 1306.0434, 1306.2384)



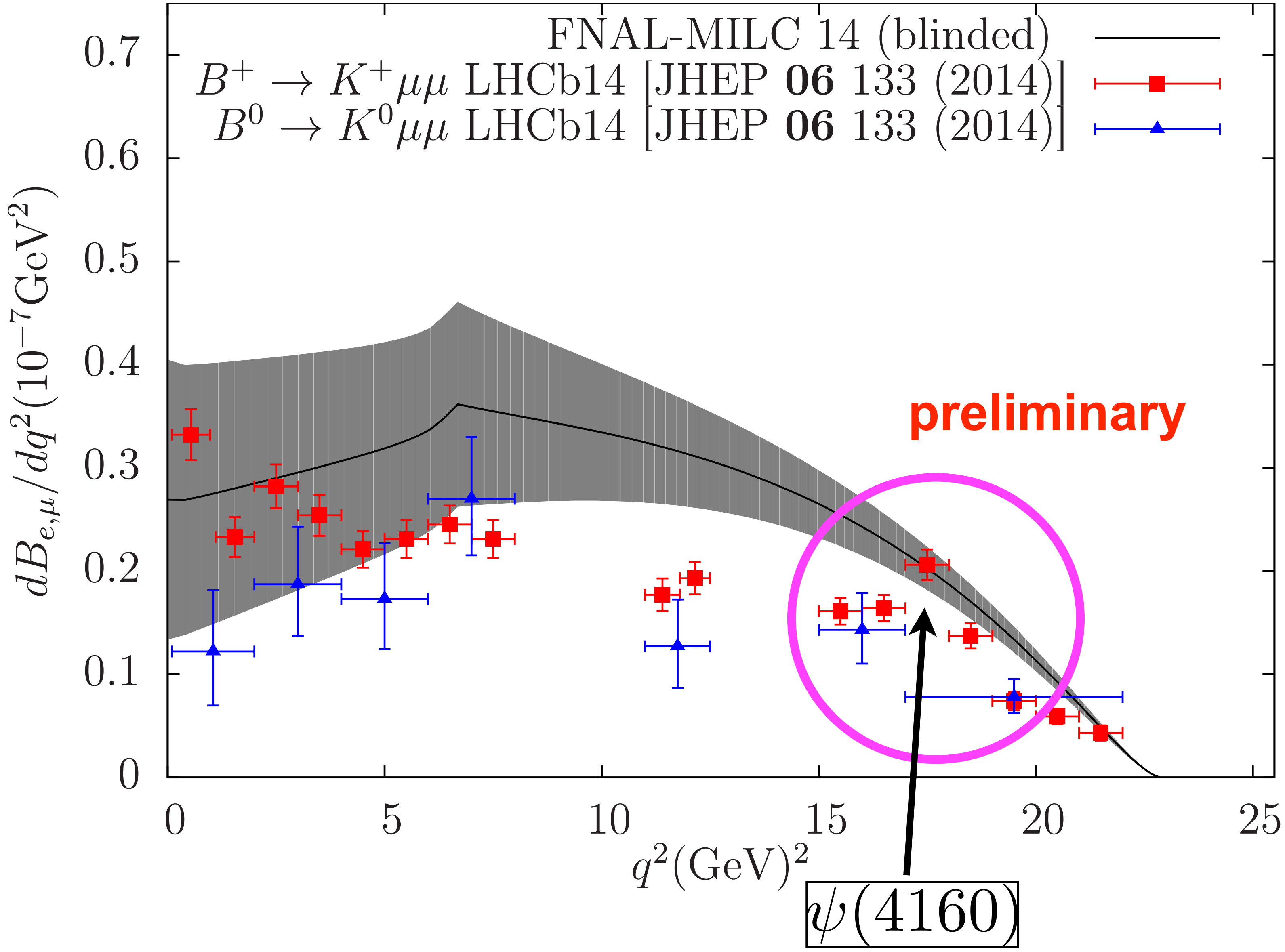
Form factors for $B \to K\ell^+\ell^-$

R. Zhou (FNAL/MILC 2014, preliminary)



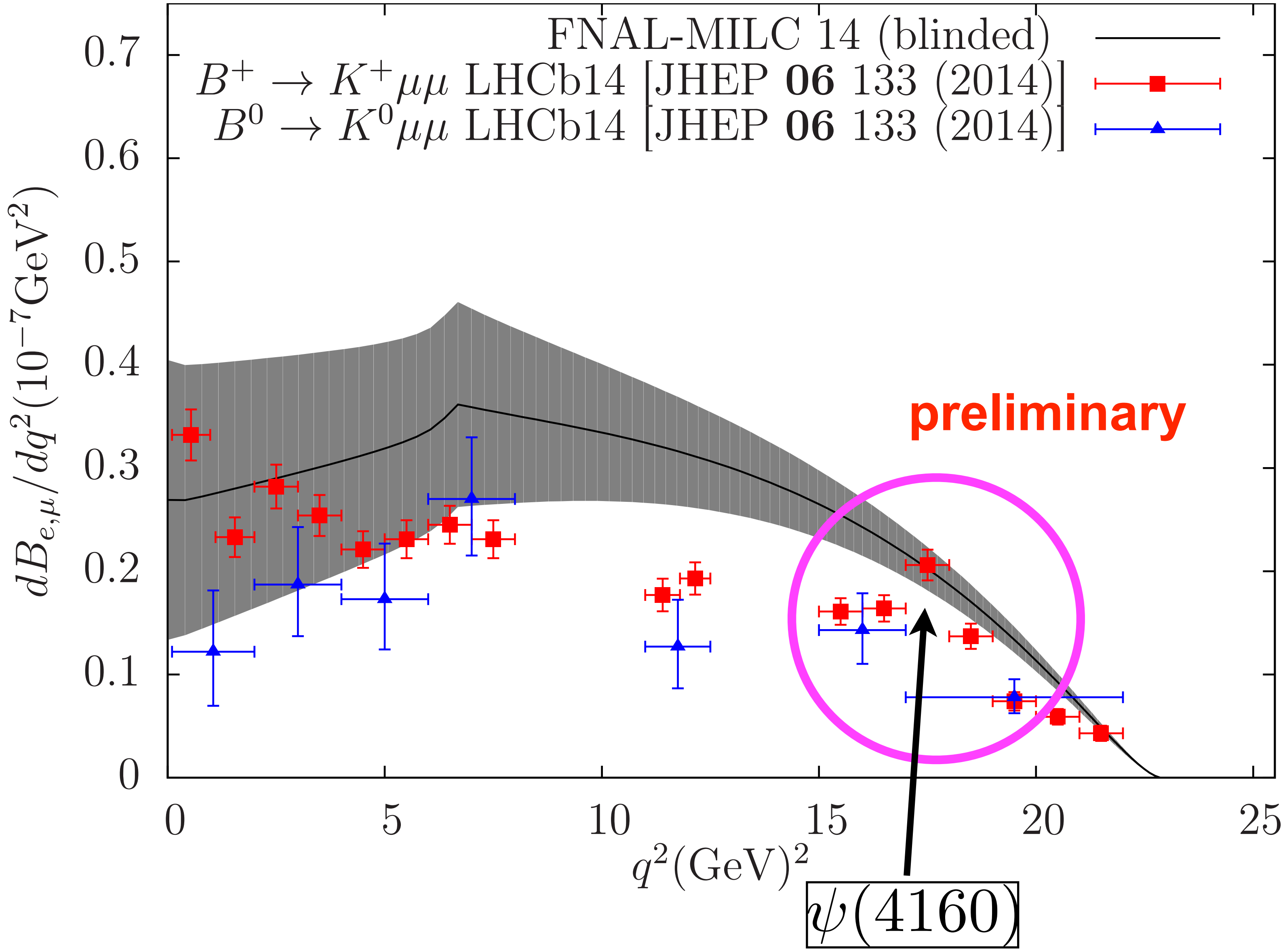
Form factors for $B \to K\ell^+\ell^-$

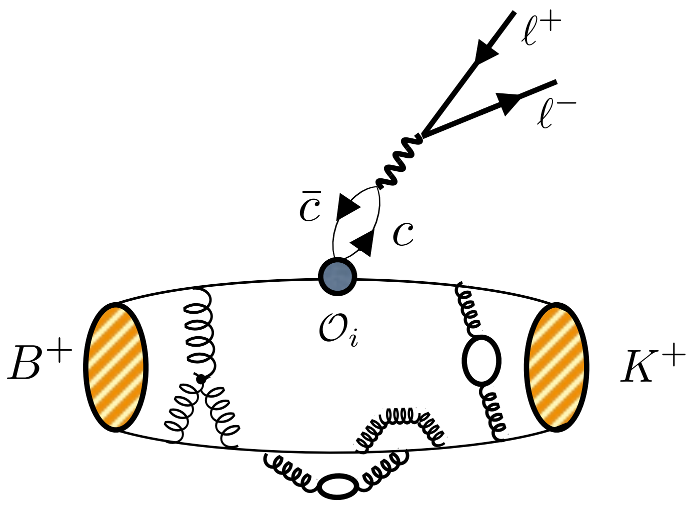
R. Zhou (FNAL/MILC 2014, preliminary)



Form factors for $B \to K\ell^+\ell^-$

R. Zhou (FNAL/MILC 2014, preliminary)



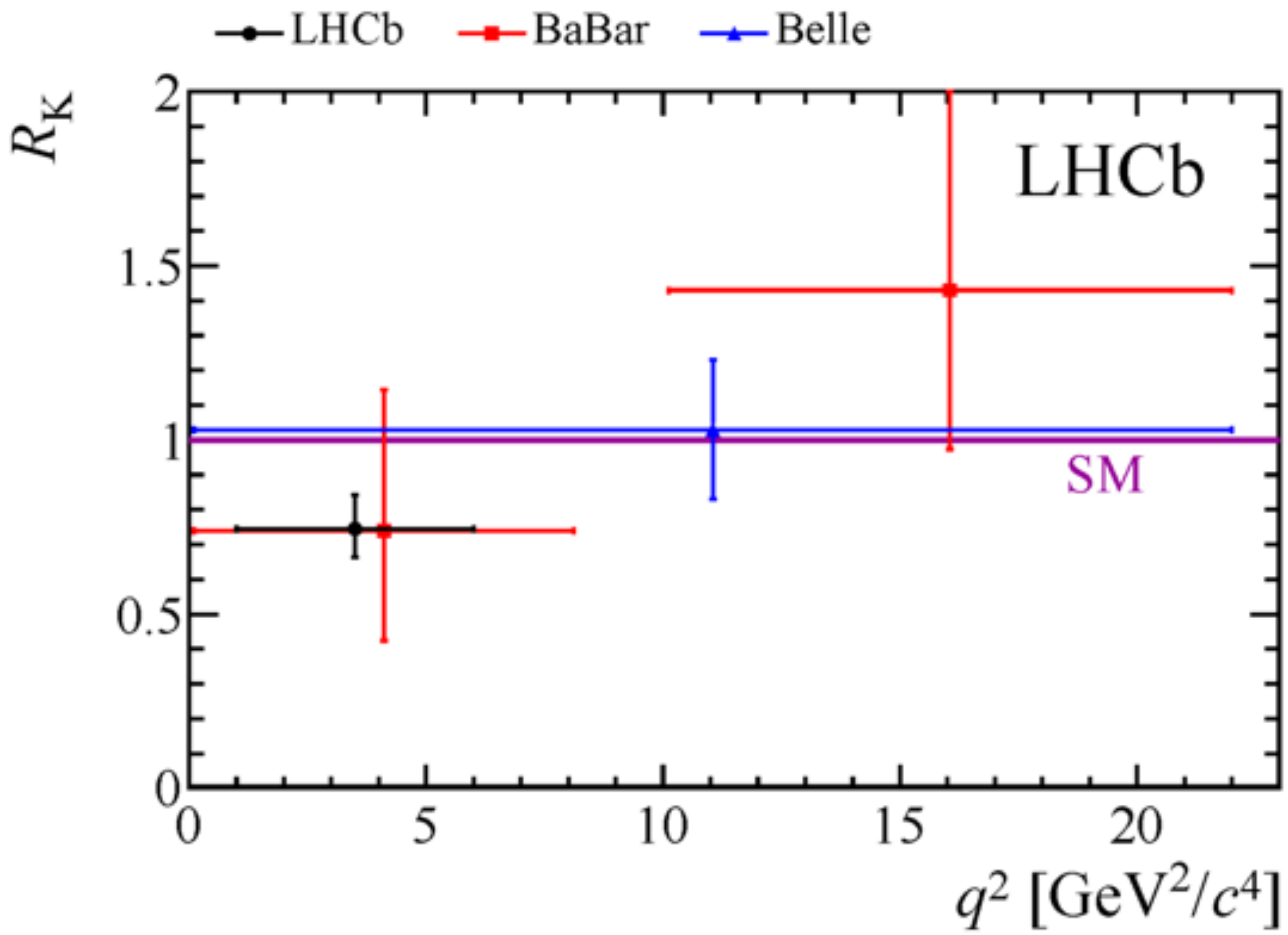


Include non-local operators in LQCD calculation?

not a simple quantity

BSM phenomenology

Lepton universality test: $B \to K\mu^+\mu^-/B \to Ke^+e^-$



LHCb (arXiv:1406.6482):

$$\boxed{R_K = 0.745\,\left({}^{90}_{74}\right)(36)}$$

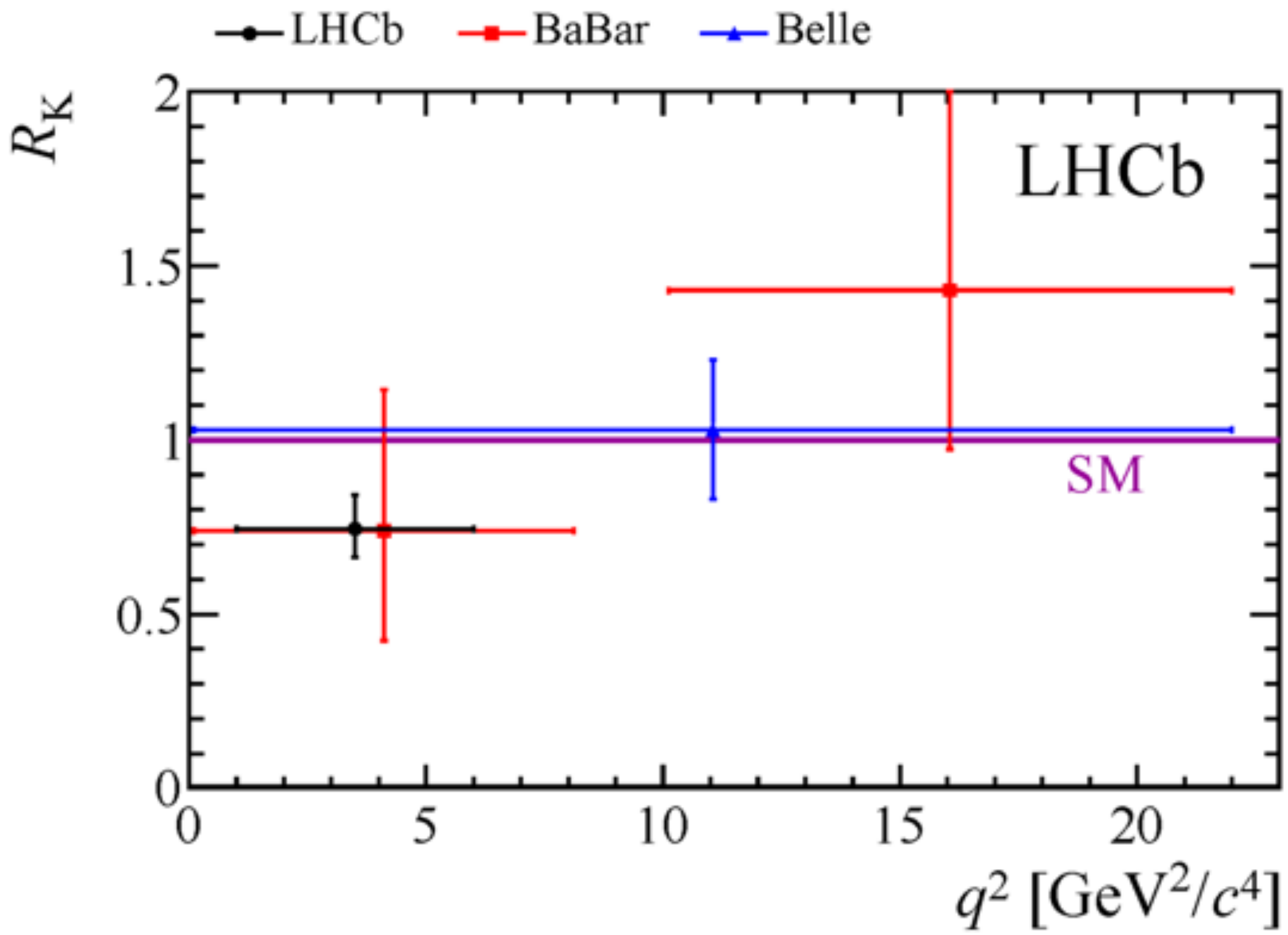
SM prediction using LQCD form factors calculated by HPQCD

(C. Bouchard et al, arXiv:1303.0434, PRL 2013):

$$\boxed{R_K(1\,\mathrm{GeV}^2, 6\,\mathrm{GeV}^2) = 1.00081(38)}$$

BSM phenomenology

Lepton universality test: $B \to K\mu^+\mu^- / B \to K e^+ e^-$



LHCb (arXiv:1406.6482):

$$\boxed{R_K = 0.745\,\left({}^{90}_{74}\right)(36)}$$

SM prediction using LQCD form factors calculated by HPQCD

(C. Bouchard et al, arXiv:1303.0434, PRL 2013):

$$\boxed{R_K(1\,\mathrm{GeV}^2, 6\,\mathrm{GeV}^2) = 1.00081(38)}$$

~2.6 σ tension between LHCb measurement and SM prediction

BSM phenomenology

review by C. Bouchard @ Lattice 2014

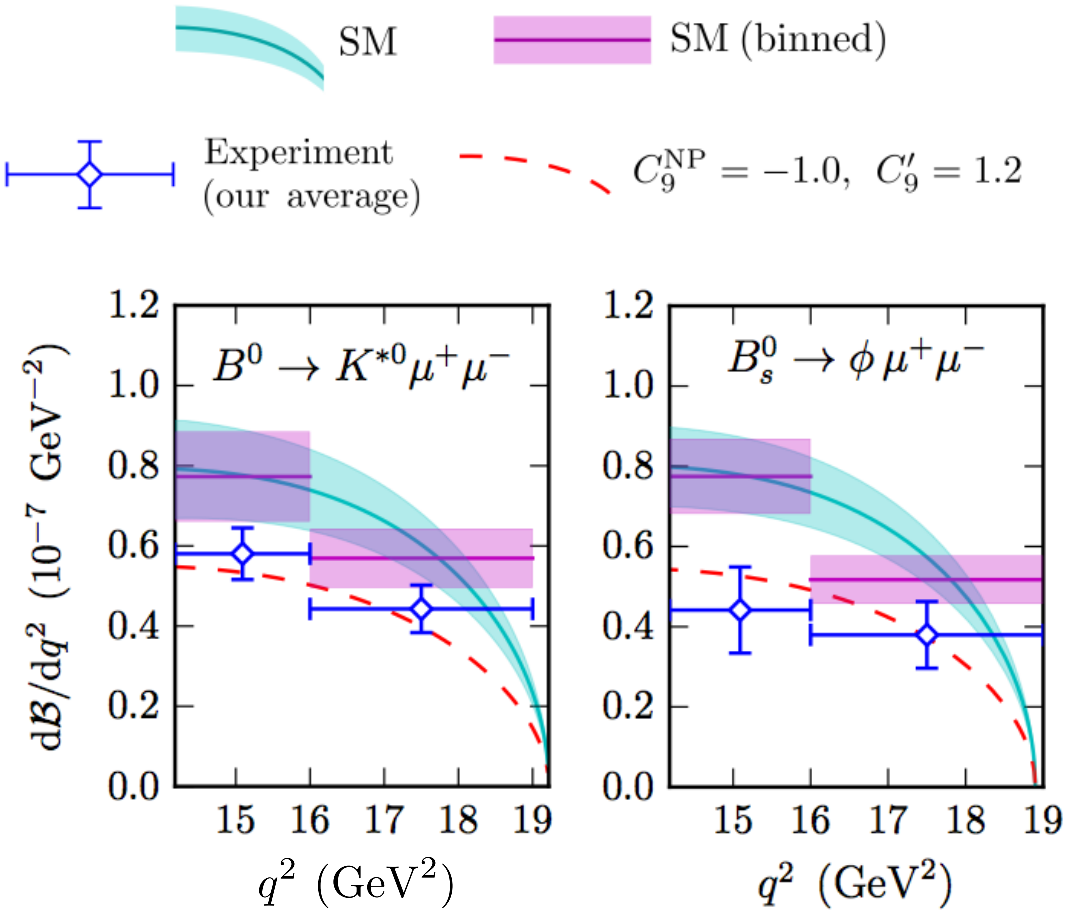
$B \to K^{(*)}\ell\ell,\ B_s \to \phi\ell\ell$

Horgan et al., PRL 112, 212003 (2014); PRD 89, 094501 (2014)

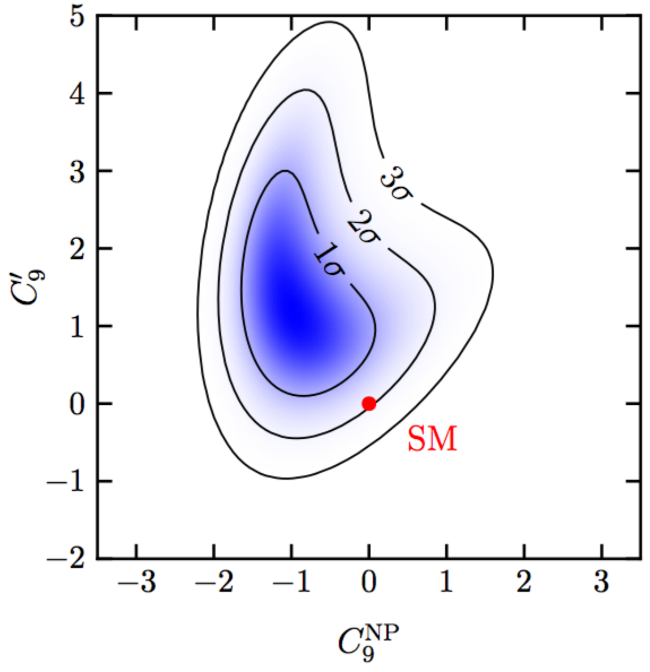
MILC 2+1 asqtad gauge fields
NRQCD b with asqtad light/strange valence
a: 0.09, 0.12 fm
Mpi: 313 – 519 MeV

caveat:
K^*, ϕ treated as stable
(narrow width approximation)

unstable K^*, ϕ: beyond simple



Combined fit to $B \to K^*\mu\mu$ and $B_s \to \phi\mu\mu$ data.

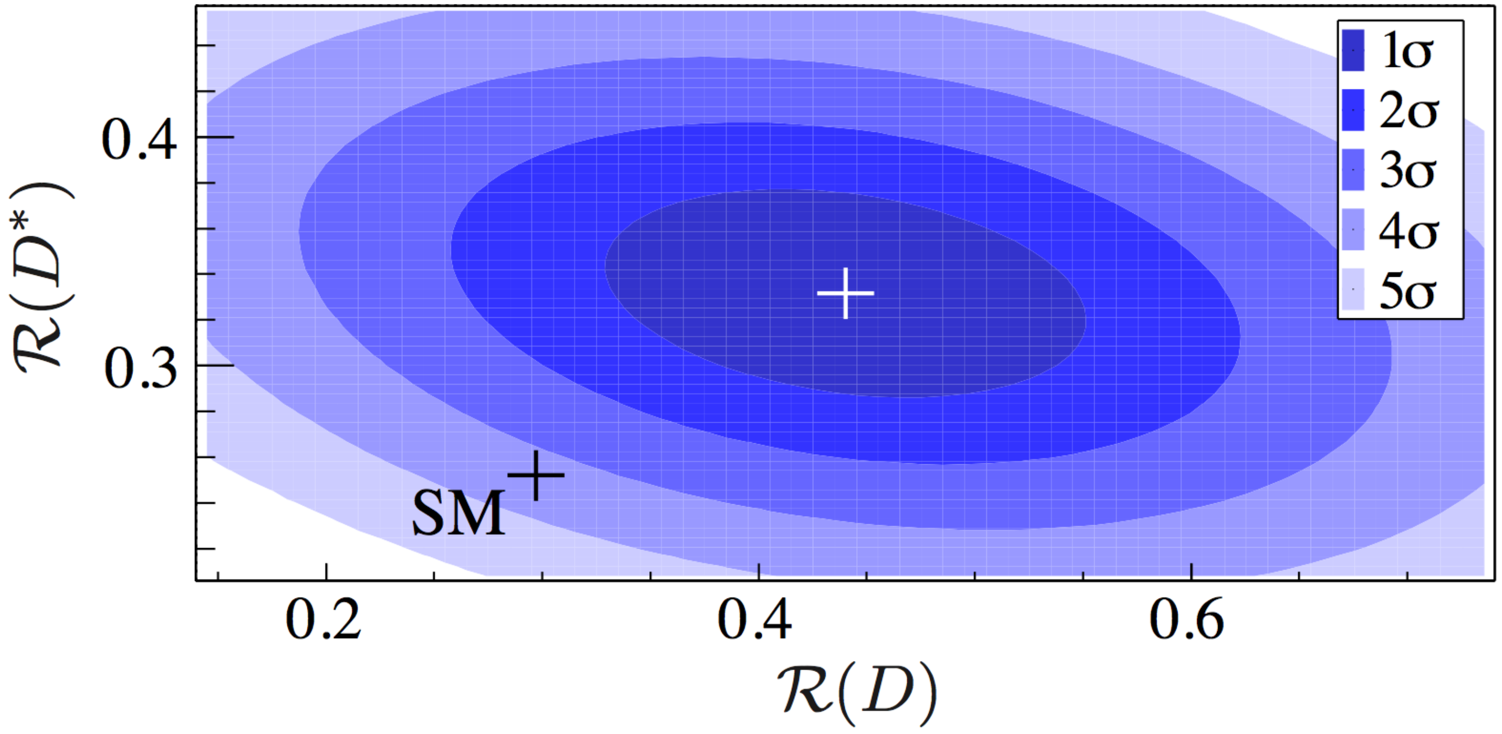


BSM phenomenology

review by C. Bouchard
@ Lattice 2014

$$\mathcal{R}(D^{(*)}) = \frac{\mathcal{B}(B \to D^{(*)}\tau\nu)}{\mathcal{B}(B \to D^{(*)}\mu\nu)}$$

BaBar, PRD 88, 072012 (2013)



$\mathcal{R}(D)_{\mathrm{SM}}$ from lattice FNAL/MILC, PRL 109, 071802 (2012)

$\mathcal{R}(D^*)_{\mathrm{SM}}$ needs lattice Fajfer et al., PRD 85, 094025 (2012)

Simple quantities in LQCD

- ☆ low-lying hadron spectrum ➙ quark masses, α_s
- ☆ weak decays - leptonic, semileptonic, mixing
  - Kaons
  - D mesons
  - B mesons

➙ CKM, BSM phenomenology

- ☆ high precision ➙ including QED

Including QED

- current strategy: isospin symmetric u,d sea: $m_u = m_d$
- QCD + quenched QED (electro quenched):
  sea quarks neutral, valence quarks charged
- can use results from QCD + quenched QED in pure QCD calculations by adjusting the valence quark masses to include strong and EM isospin breaking effects,
  $m_u \neq m_d$

- strong and EM isospin breaking are subdominant effects in the sea
- to connect LQCD calculations of weak matrix elements to experiment, need to account for EM radiative corrections:
  K, π decay: estimated phenomenologically using CHPT
  (see for example, Cirigliano, et al, arXiv:1107.6001)

- We now need similar phenomenological estimates for weak D and B decays

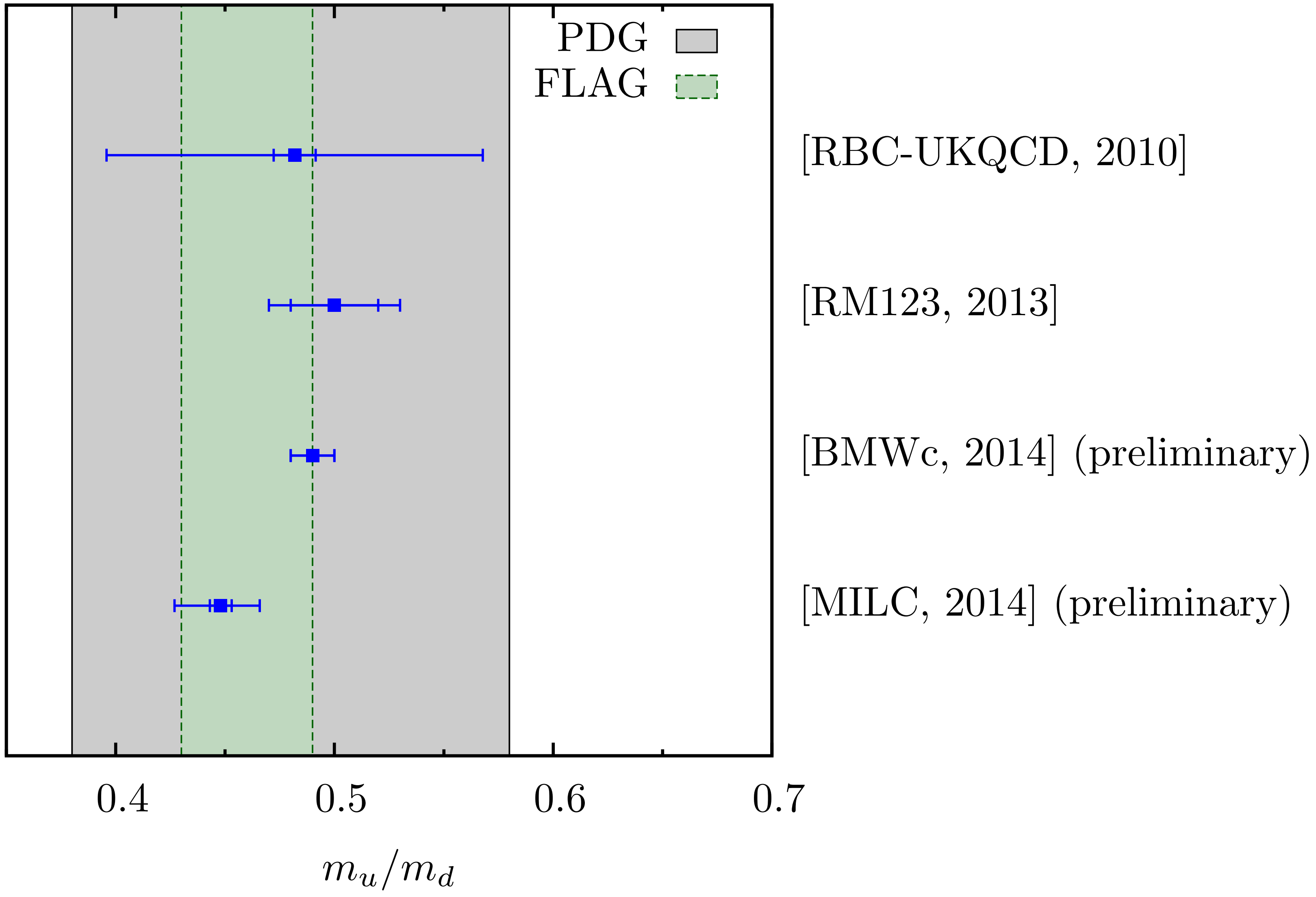
Including QED

review by A. Portelli @ Lattice 2014 and ICHEP

- **new**: full QCD+QED simulations used in spectrum calculations:
  - BMW ($n_f = 1+1+1+1$) at multiple lattice spacings, light quark masses
  - QCDSF ($n_f = 1+1+1$)
  - RBC/UKQCD ($n_f = 2+1$)
  - PACS-CS ($n_f = 1+1+1$)
  - similar plans by other groups (MILC, RBC/UKQCD, ...)

Including QED

review by A. Portelli @ Lattice 2014



see backup slides for more spectrum results

Including QED

review by A. Portelli @ Lattice 2014 and ICHEP

- **new**: full QCD+QED simulations used in spectrum calculations:
  - BMW ($n_f = 1{+}1{+}1{+}1$) at multiple lattice spacings, light quark masses
  - QCDSF ($n_f = 1{+}1{+}1$)
  - RBC/UKQCD ($n_f = 2{+}1$)
  - PACS-CS ($n_f = 1{+}1{+}1$)
  - similar plans by other groups (MILC, RBC/UKQCD, ...)

- Will eventually need to calculate EM radiative corrections in full QCD+QED, for example:

$$\Gamma\left(\pi^+ \to \ell^+\nu_\ell(\gamma)\right) = \Gamma\left(\pi^+ \to \ell^+\nu_\ell\right) + \Gamma\left(\pi^+ \to \ell^+\nu_\ell\gamma\right)$$

Proposal by RBC/UKQCD (see talk by C. Sachrajda @ Lattice 2014)

Beyond simple quantities

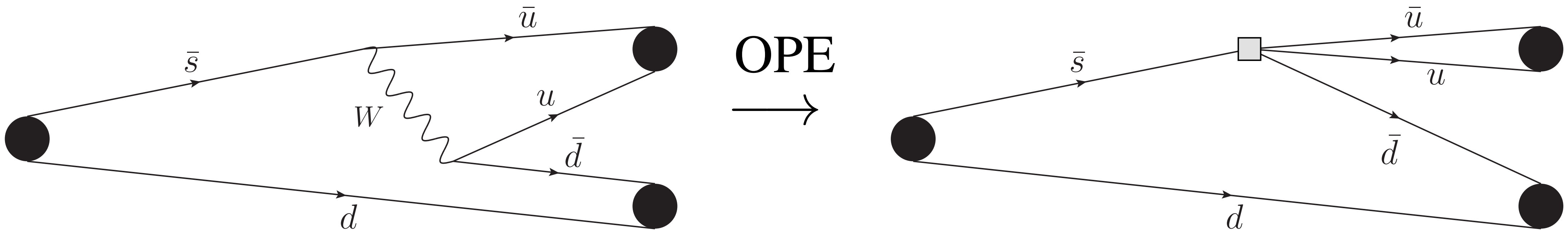
Note: When there are two (or more) hadrons in the initial or final state we need additional formalism to relate the quantites calculated in the Euclidean box to physical observables in Minkowski space.

see review talk by R. Briceño @ Lattice 2014

- ☆ $K \to \pi\pi$ amplitudes and Δm_K
- ☆ resonances, ...

$K \to \pi\pi$

review by N. Garron @ Lattice 2014



Describe $K \to (\pi\pi)_{\mathrm{I}=0,2}$ with an effective Hamiltonian

$$H^{\Delta s=1} = \frac{G_F}{\sqrt{2}} \left\{ \sum_{i=1}^{10} \left(V_{ud} V_{us}^* z_i(\mu) - V_{td} V_{ts}^* y_i(\mu) \right) Q_i(\mu) \right\}$$

Short distance effects factorized in the Wilson coefficients y_i, z_i

Long distance effects factorized in the matrix elements

$$\langle \pi\pi | Q_i | K \rangle \longrightarrow \text{Lattice}$$

$K \to \pi\pi$

review by N. Garron @ Lattice 2014

$$\Delta I = 1/2$$

RBC/UKQCD (arXiv:1106.2714, PRD 2011):

- Pilot study on small volume, unphysical pion mass, but complete with all operators, disconnected diagrams and NPR.
- Computation with physical kinematics is in progress
- Emerging understanding of the $\Delta I = 1/2$ rule:
  I=2 amplitude is suppressed due to cancellation between two dominant contributions, while the I=0 amplitude is not.

several other efforts:

- Ishizuki et al (Lattice 2014), improved Wilson fermions, enhancement is observed
- Endress, Pena, role of the charm quark in $\Delta I = 1/2$ rule

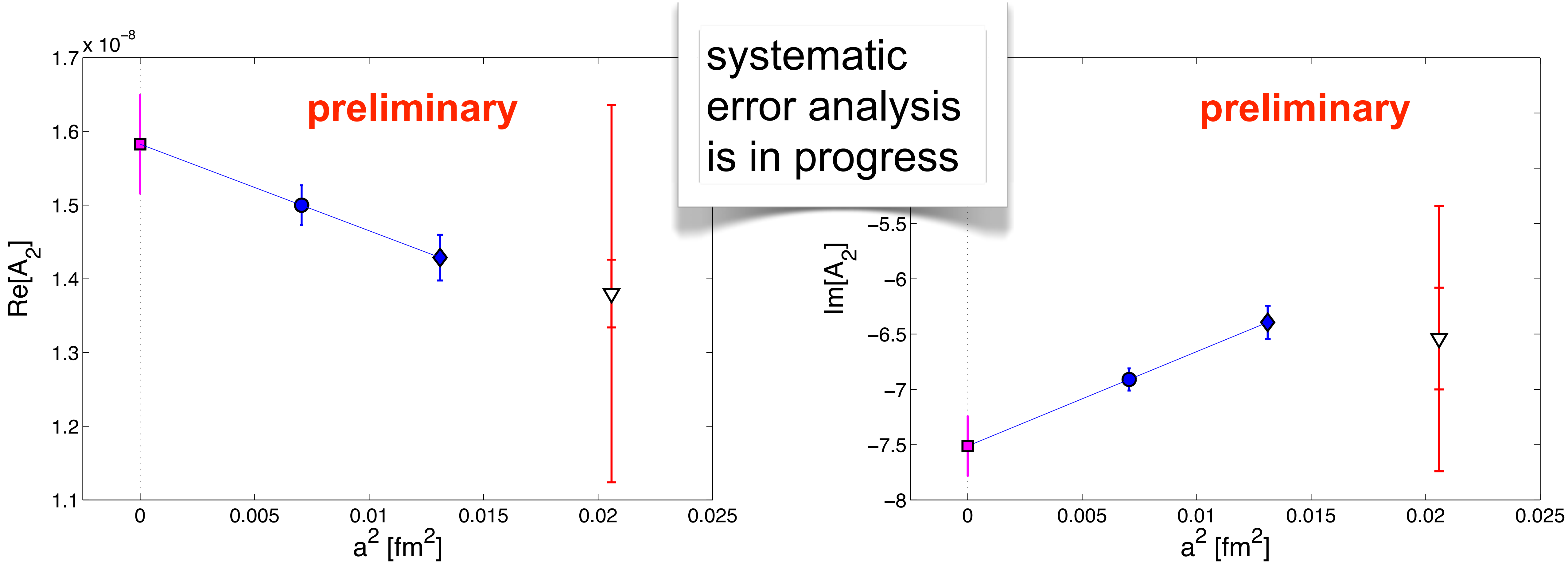
$K \to \pi\pi$

review by N. Garron @ Lattice 2014

$$\Delta I = 3/2$$

RBC/UKQCD (Lattice 2014):

- calculation with physical mass pions, large volumes, two lattice spacings
- **first result with continuum extrapolation**, complete error budget coming soon!

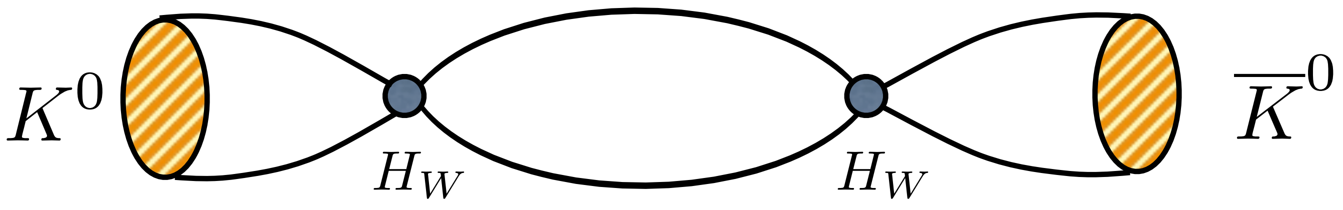


systematic error analysis is in progress

The goal of this effort is to eventually calculate ϵ' to ~15% accuracy

Δm_K

plenary talk by C. Sachrajda @ Lattice 2014



Finite volume dependence more complicated than for $K \to \pi\pi$
(N. Christ et al, arXiv:1401.1362)

RBC/UKQCD (arXiv:1406.0916):

complete calculation with unphysical parameters, $m_K < 2\, m_\pi$

$$\Delta m_K = 3.19(41)(96) \times 10^{-12}\,\mathrm{MeV}$$

Z. Bai (RBC/UKQCD, Lattice 2014):

preliminary results at near physical mass with $m_K > 2\, m_\pi$

stat. errors only

Work has also started on rare K decays, such as $K_L \to \pi^0 \ell^+ \ell^-$
(RBC/UKQCD, ETM)

Beyond simple quantities

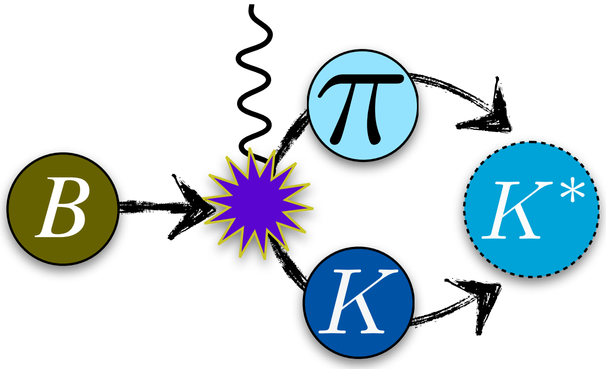
- ☆ $K \to \pi\pi$ amplitudes and Δm_K
- ☆ resonances

Resonances

Returning to the calculation of the $B \to K^* \ell\ell$ and the $B_s \to \phi\ell\ell$ form factors (R. Horgan et. al, arXiv:1310.3722, arXiv:1310.3887, PRDs 2014), a first calculation of the K^* width was reported by Prelovsek et al (arXiv: 1307.0736, PRD 2013).

The **formalism** for treating vector mesons as resonances in weak decay transitions was only very recently (!) developed (see review talk by R. Briceño, and arXiv:1406.5965)

No numerical LQCD calculation of a weak transition amplitude to a final state resonance has been done yet.



There are now a number of calculations of the ρ width, excited charmed meson widths, ... (see the review talks by S. Prelovsek, T. Yamazaki, R. Briceño @ Lattice 2014).

Summary

- simple quantities:
  kaons: $< 0.5\%$ for SU(3) breaking ratios
  $\sim 1\%$ for other quantities
  D,D_s-mesons: $< 0.5\%$ for SU(3) breaking ratio f_{Ds}/f_D
  $< 1\%$ for decay constants
  $\sim 3\text{-}5\%$ for other quantities
  B,B_s-mesons: $< 1\%$ for SU(3) breaking ratio f_{Bs}/f_B
  $\sim 2\%$ for decay constants, $B \to D^*$
  $\sim 3\text{-}8\%$ for other quantities → $\lesssim 5\%$
- precision will continue to improve with better simulations (especially for D,B mesons)
- for B: leverage high precision D results with B/D ratios
- not-simple: 10-30% with complete error budget

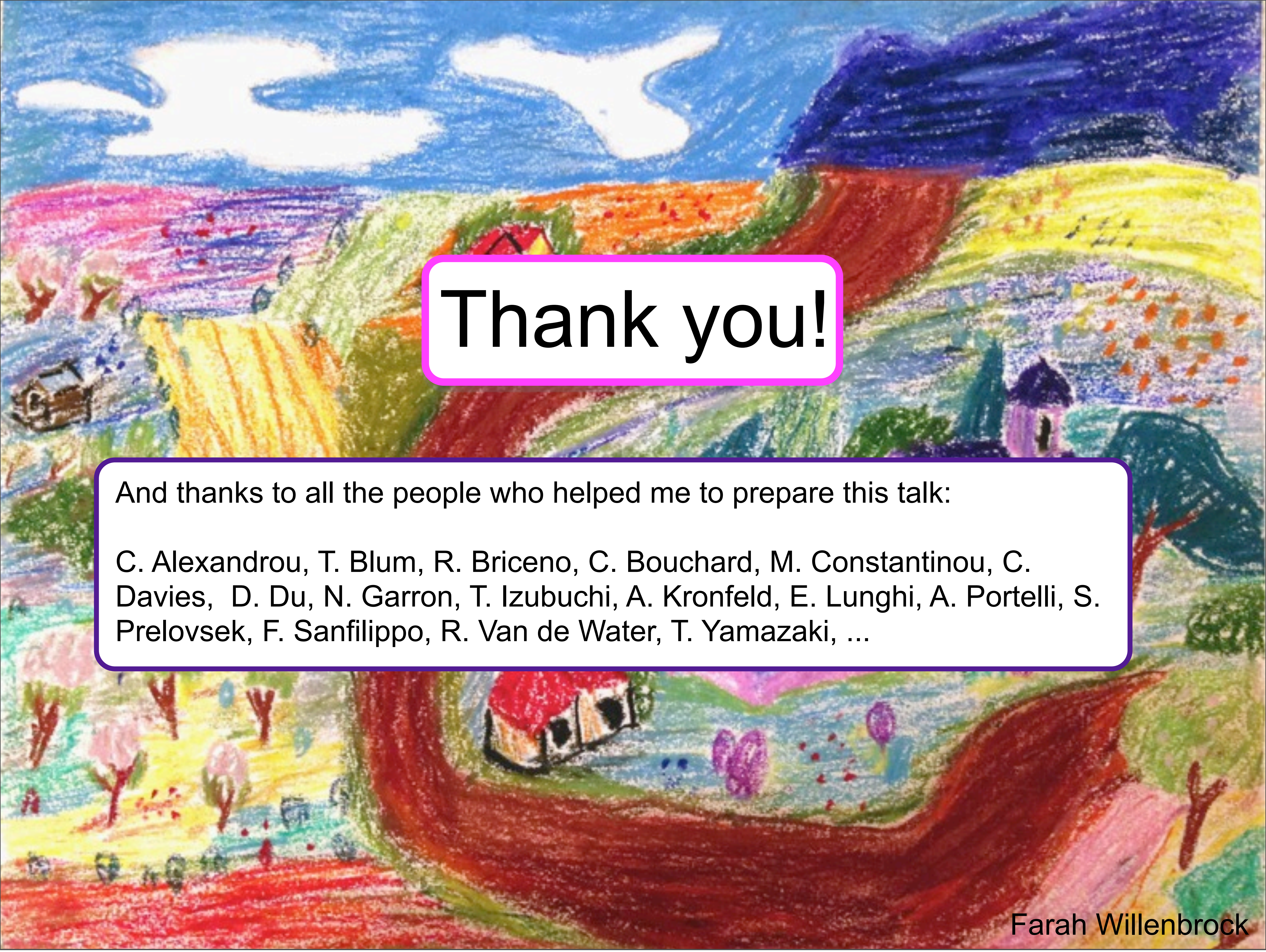
Conclusions and Outlook

- LQCD (Lattice Field Theory, more generally) is an idea driven area of research
- progress made (especially recently) would not be possible without innovative ideas (and a lot of courage)
- we will see an increasing number of very precise results for an increasing number of simple quantities
- at the same time we will see reliable results for an increasing number of new (not simple) quantities
- **sufficient computational resources are absolutely essential**
- ambitious program is in place to provide (much needed) theoretical support for all three frontiers (the same can be said for Nuclear physics)

Thank you!

And thanks to all the people who helped me to prepare this talk:

C. Alexandrou, T. Blum, R. Briceno, C. Bouchard, M. Constantinou, C. Davies, D. Du, N. Garron, T. Izubuchi, A. Kronfeld, E. Lunghi, A. Portelli, S. Prelovsek, F. Sanfilippo, R. Van de Water, T. Yamazaki, ...



Farah Willenbrock

Omitted Topics

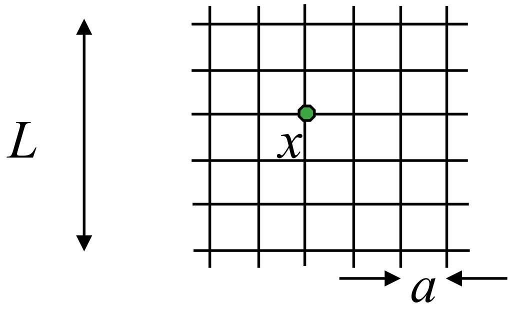
- QFT at finite temperature (review by A. Bazavov @ Lattice 2014)
- lattice calculations of BSM theories (review by Y. Aoki @ Lattice 2014)
- QCD at finite density (review by D. Sexty @ Lattice 2014)
- hadron spectrum studies of exotica, states near threshold, hadron structure calculations, ...
  (review talks by S. Prelovsek, T. Yamazaki, M. Constantinou, R. Briceño @ Lattice 2014)
- ....

Lattice 2014 in NYC, June 23-28 2014

Backup slides

systematic error analysis

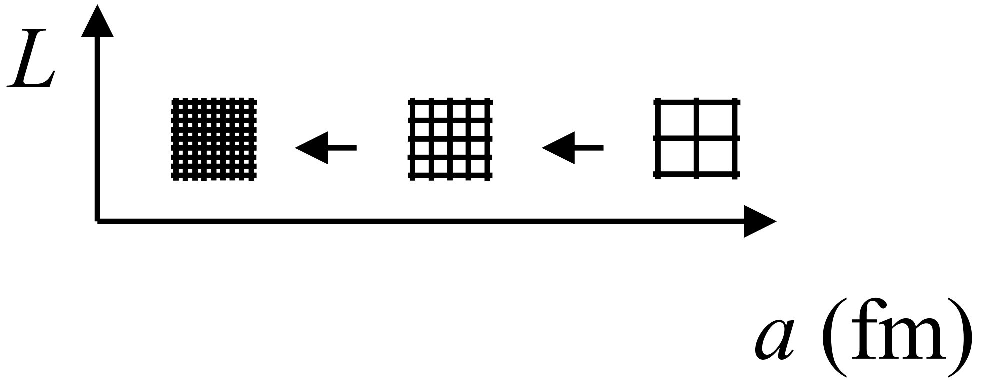
discretization effects



discrete space-time → discrete QCD action

Symanzik EFT: $\langle \mathcal{O} \rangle^{\mathrm{lat}} = \langle \mathcal{O} \rangle^{\mathrm{cont}} + O(ap)^n$

p is the typical momentum scale associated with $\langle \mathcal{O} \rangle$
for light quark systems, $p \sim \Lambda_{\mathrm{QCD}}$

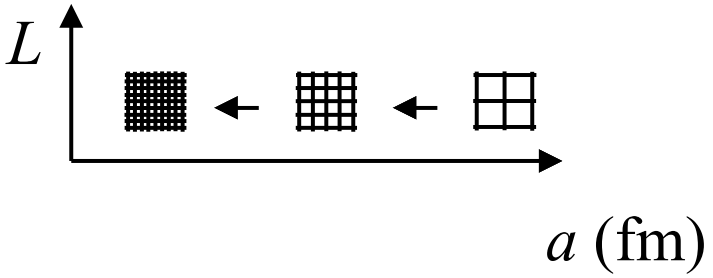


The form of $O(ap)^n$ depends on the details of the lattice action.

All modern light-quark actions start at $n = 2$
(improved Wilson, twisted-mass Wilson, asqtad, HISQ, Domain Wall, Overlap, ...).

systematic error analysis

discretization effects for b quarks



- If we use light quark actions for heavy quarks, discretization errors ~ $O(am_h)^2$, with currently available lattice spacings
  for charm $am_c \sim 0.15\text{-}0.6$ and for b: $am_b > 1$

➡ need effective field theory methods for b quarks
for charm lattice spacings are sufficiently small so that we can use improved light quark methods

- avoid errors of $(am_b)^2$ by using EFT in the formulation/matching of lattice action/currents:
  - ✦ relativistic HQ actions (Fermilab, Columbia, Tsukuba)
  - ✦ HQET
  - ✦ NRQCD

or

- use the same improved light quark action as for charm (HISQ, twisted mass Wilson, NP imp. Wilson, Overlap, ...)
  - ✦ keep $am_h < 1$
  - ✦ use HQET and/or static limit to extrapolate/interpolate to the physical b quark mass

systematic error analysis

light quark mass effects

Simulations with $m_{\text{light}} = 1/2\,(m_u + m_d)$ **at the physical u/d quark masses are now available**, but many results still have

$$m_{\text{light}} > 1/2\,(m_u + m_d)_{\text{phys}}$$

χPT can be used to extrapolate/interpolate to the physical point.

- Can include discretization effects (for example, staggered χPT)
- It is now common practice to perform a combined continuum-chiral extrapolation/interpolation

systematic error analysis

finite volume effects

One hadron (meson) in initial/final state:

If L is large enough, FV error $\sim e^{-m_\pi L}$

- keep $m_\pi L \gtrsim 4$

To quantify residual error:

- include FV effects in CPT
- compare results at several Ls (with other parameters fixed)

The story changes completely with two or more hadrons in initial/final state!
(more later)

review of few-body systems by R. Briceño @ Lattice 2014

systematic error analysis

other effects

✓ statistical errors: from monte carlo integration
consider/include systematic errors from correlator fit procedure

✓ n_f dependence: realistic sea quark effects: use $n_f = 2+1$ or $n_f = 2+1+1$
Note: $n_f = 2$ (quenched strange quark effects appear to be small)

❖ renormalization (and matching):
- ⇒ with lattice perturbation theory: need to include PT errors
- ⇒ nonperturbative methods
- ⇒ use nonrenormalized operators where possible

Simple quantities in LQCD

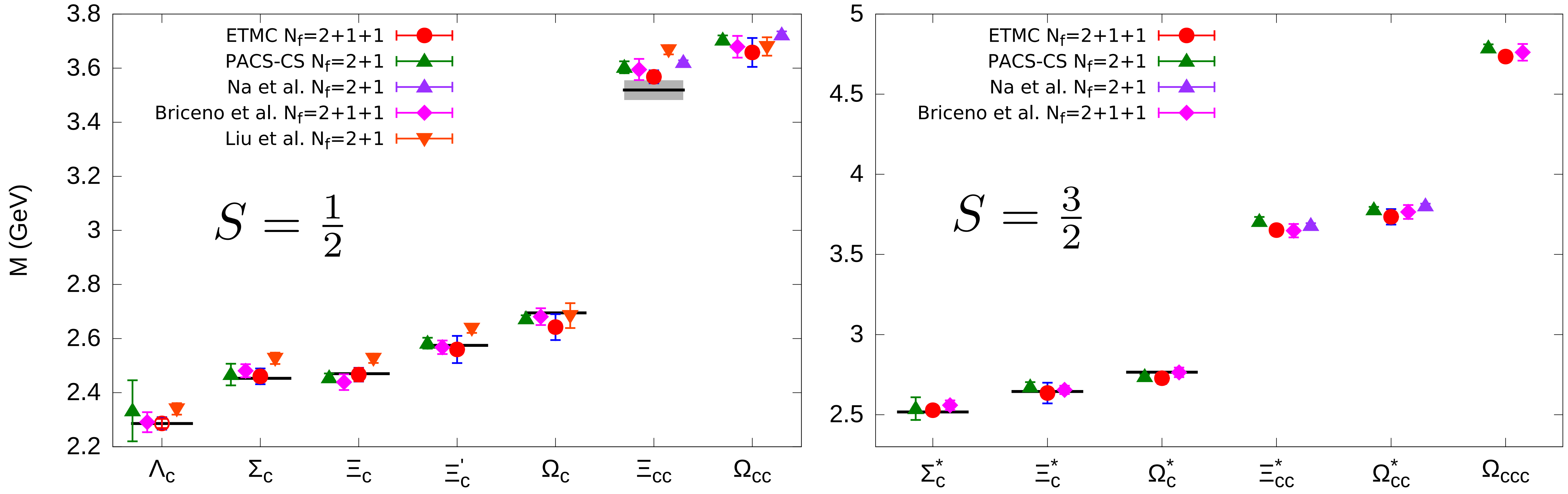
Focus on results with complete error budgets and reliable systematic error estimates.

- ☆ low-lying hadron spectrum ➙ quark masses, α_s
- ☆ weak decays (leptonic, semileptonic, mixing)
  ➙ CKM, BSM phenomenology
- ☆ high precision ➙ including QED

Low-lying hadron spectrum

new results for the charmed baryon spectrum:

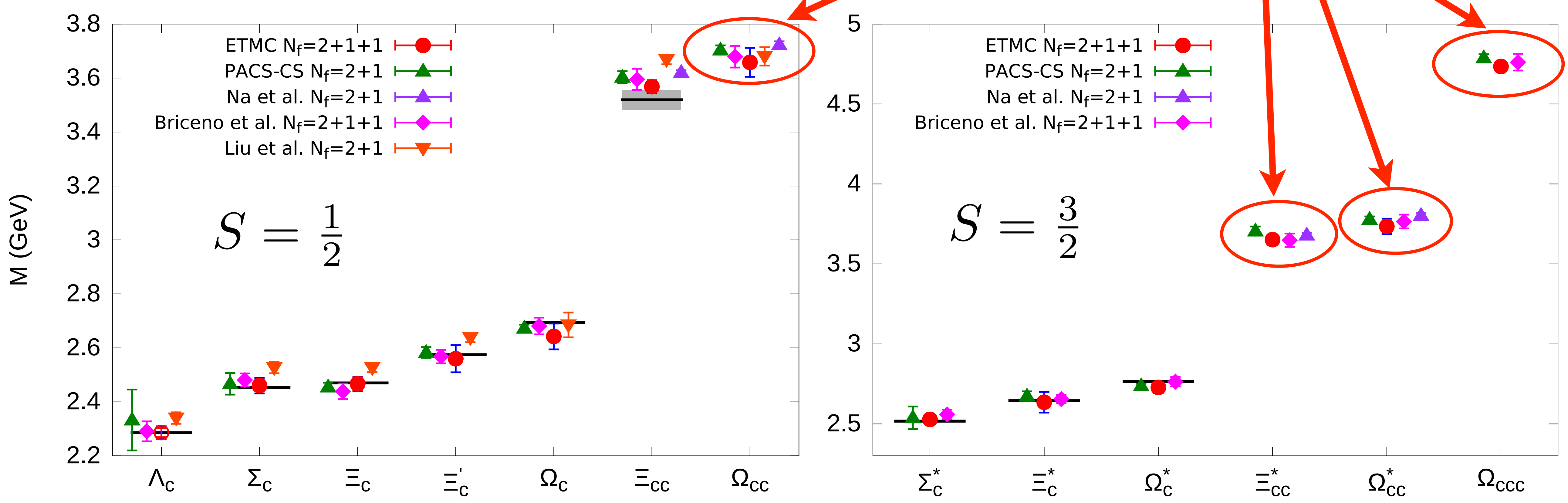
C. Alexandrou (ETM collaboration, arXiv:1406.4310)



Low-lying hadron spectrum

new results for the charmed baryon spectrum:

C. Alexandrou (ETM collaboration, arXiv:1406.4310)



LQCD predictions

quark masses and α_s

- with experimental inputs (m_π, m_K, etc..) we obtain the bare lattice masses and lattice spacing in physical units.

- need additional work to determine renormalized quark masses and α_s:

  for α_s:

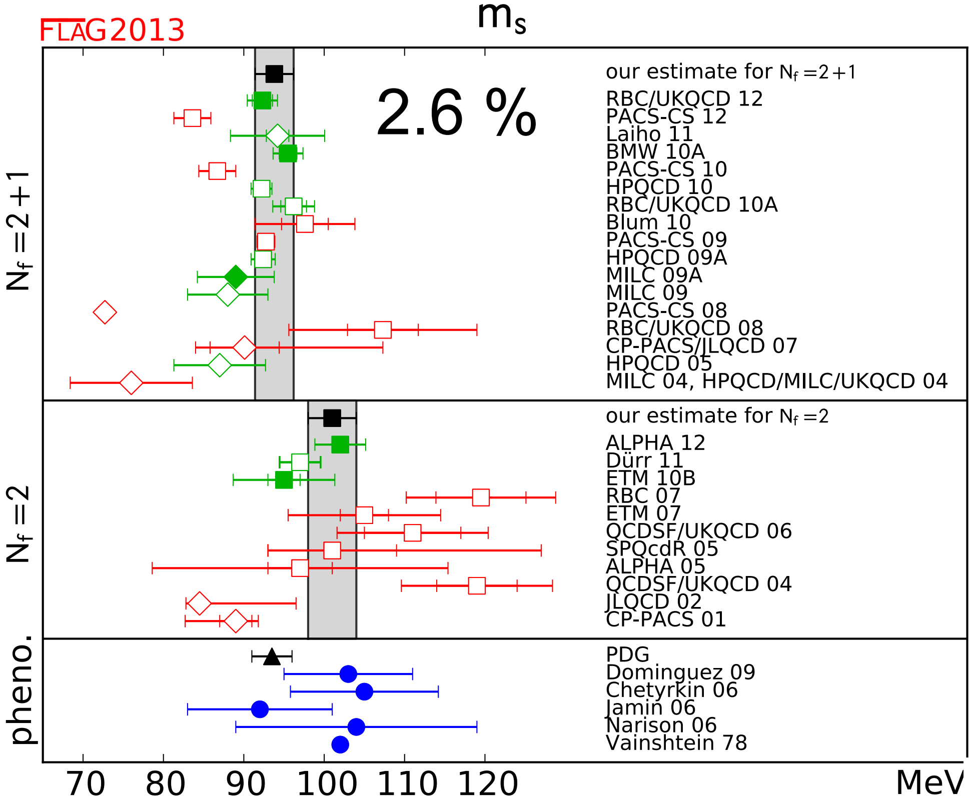
  calculate additional short distance quantities (Wilson loops, step-scaling functions, short distance potential, QCD vertices, current-current correlators, …)

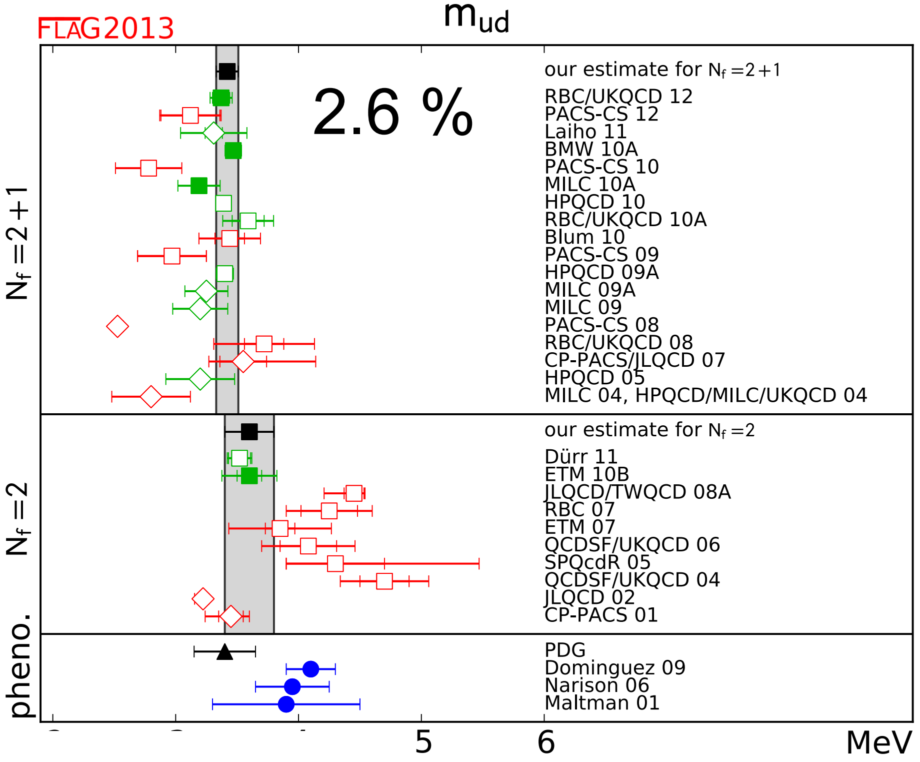
  for quark masses and α_s:

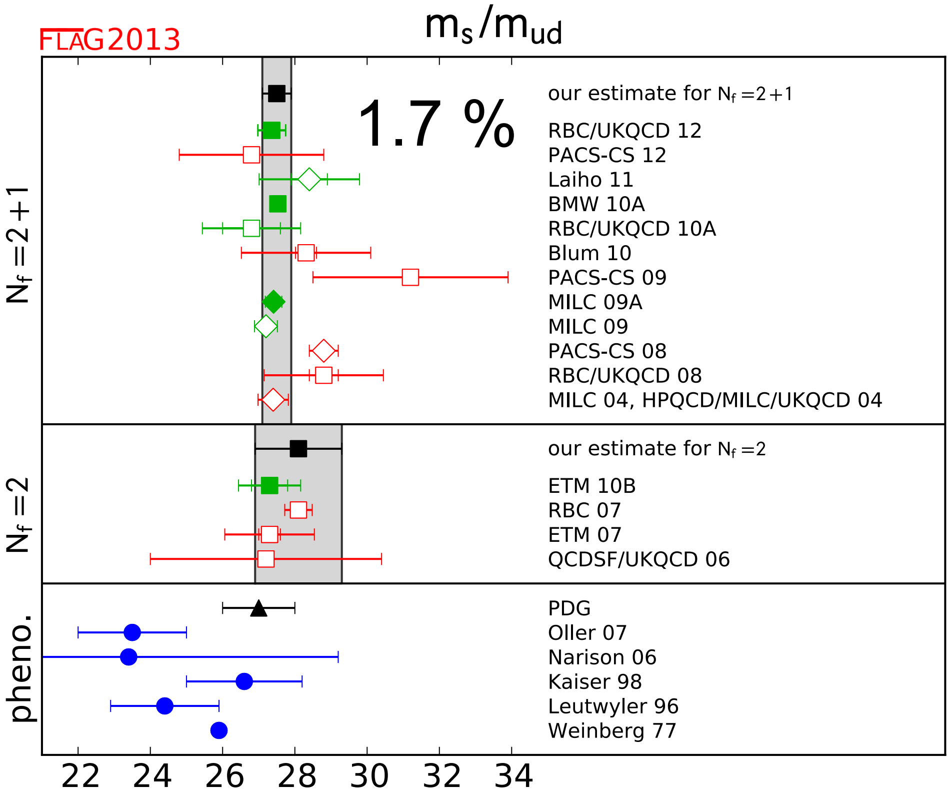
  ➢ define a renormalization scheme
  nonperturbative schemes: RI-MOM, Schrödinger functional, ...
  ➢ match to $\overline{MS}$ scheme

quark masses and α_s summary

S. Aoki et al (FLAG-2 review, arXiv:1310.8555)

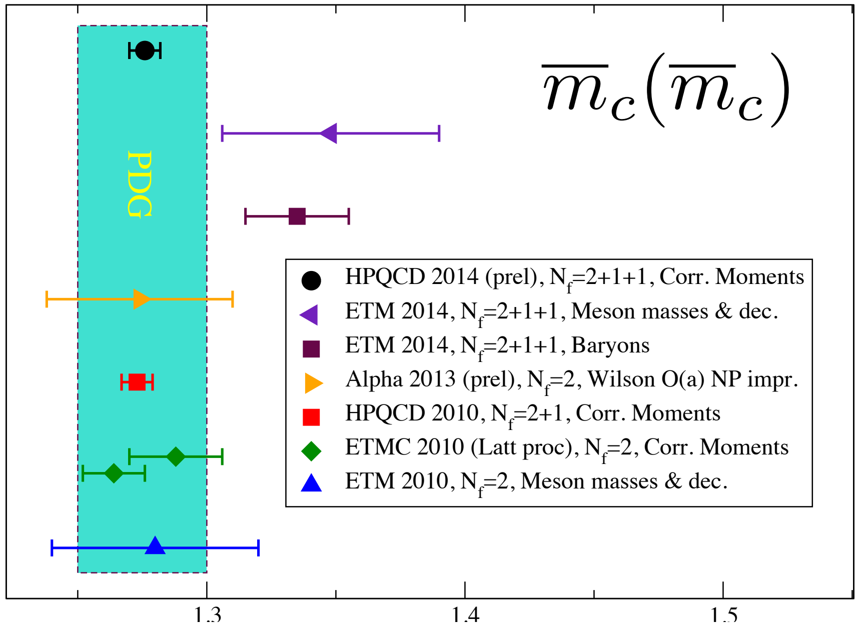


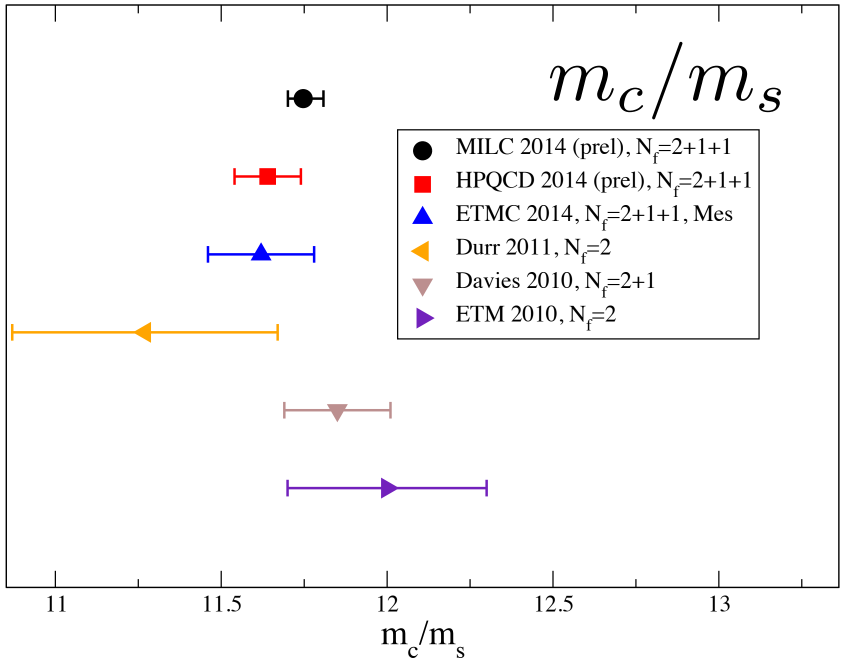




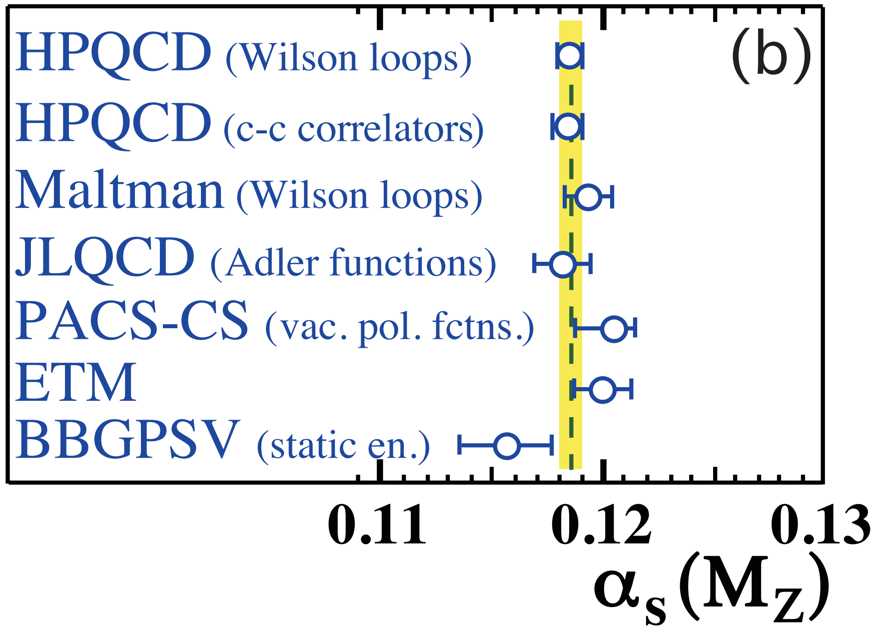
quark masses and α_s summary

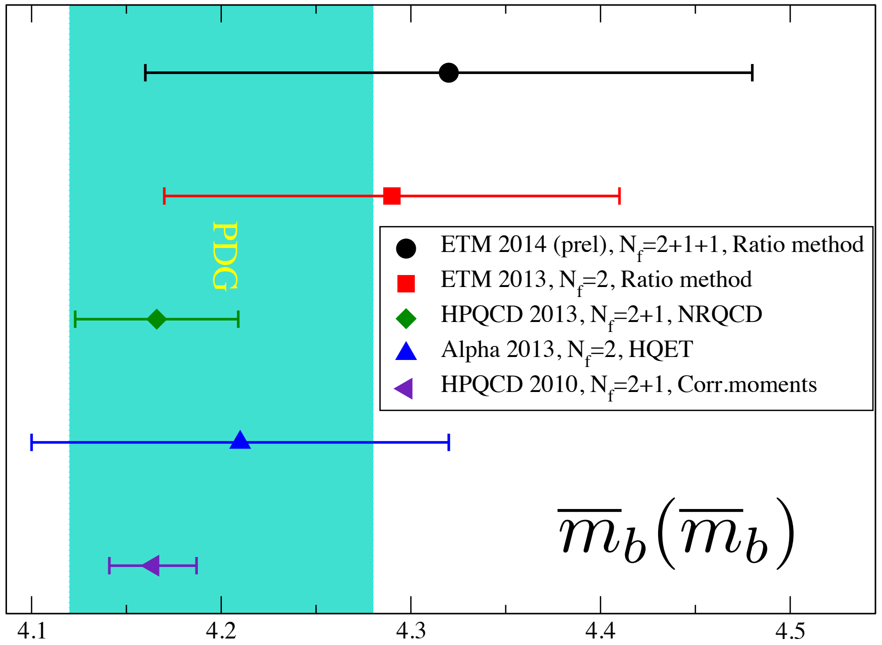
review by F. Sanfilippo @ Lattice 2014

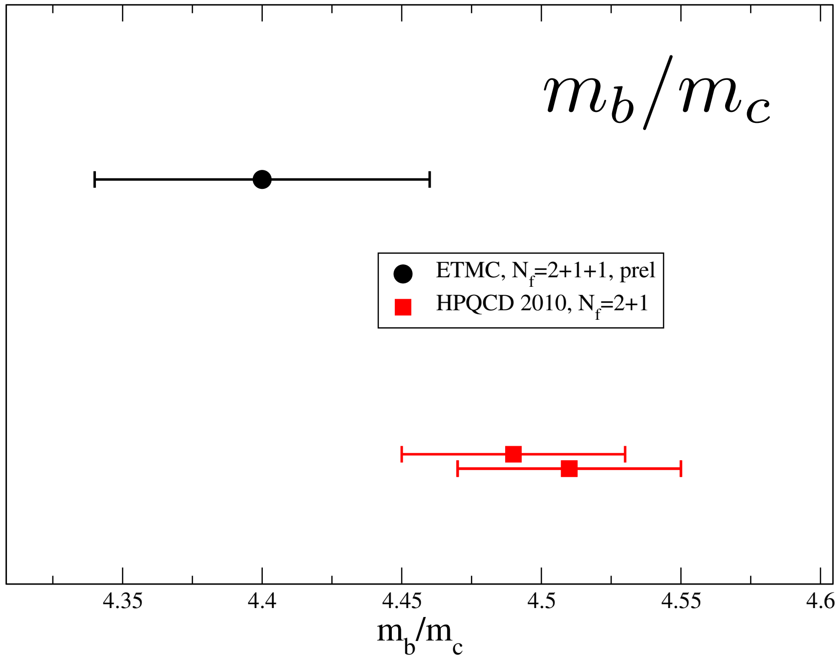




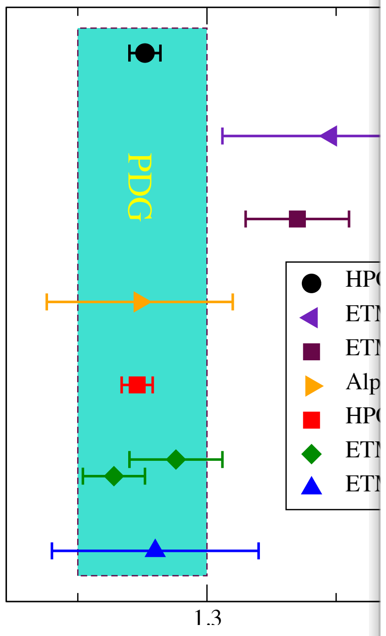
PDG 2014

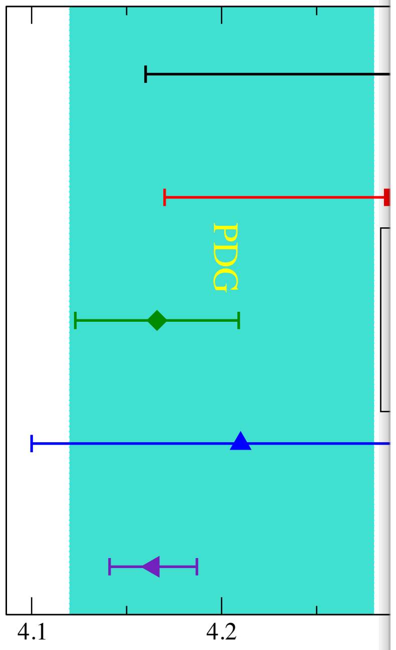


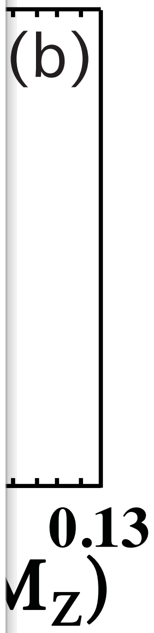




quark masses and α_s summary







- mass ratios can be determined very accurately

- FLAG plans to add the heavy quark masses to their averages in coming year.

- uncertainty in the SM prediction of Higgs partial widths is dominated by parametric uncertainties due to m_b, m_c, and α_s.

  ⇒ need masses, strong coupling with ~0.1-0.4% precision for testing SM Higgs couplings.

- Lepage, Mackenzie, Peskin (arXiv:1401.0319)

  using the HPQCD 10 determinations of m_b and m_c and the PDG average for α_s:

$$\boxed{\delta_b = 0.77 \quad \delta_c = 0.89 \quad \delta_g = 0.78}$$

cf. ILC goals: $\delta_b = 0.3 \quad \delta_c = 0.7 \quad \delta_g = 0.6$

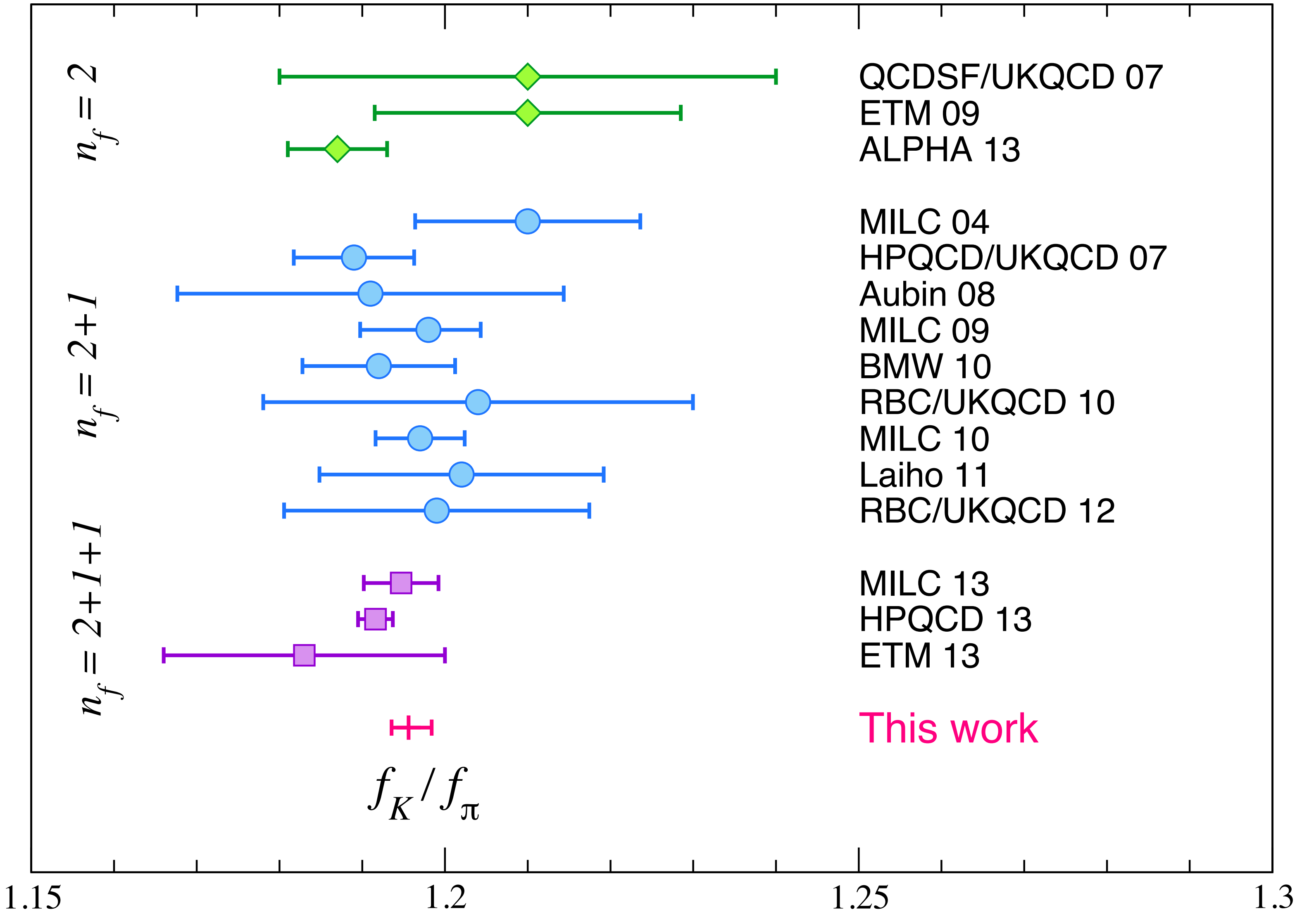
Note: $\delta_b = \frac{1}{2}\delta\Gamma(h \to b\bar{b})$

- improving the precision of lattice quark mass and α_s determinations is straightforward

Kaon summary

For all quantities there are results that use **physical mass** ensembles

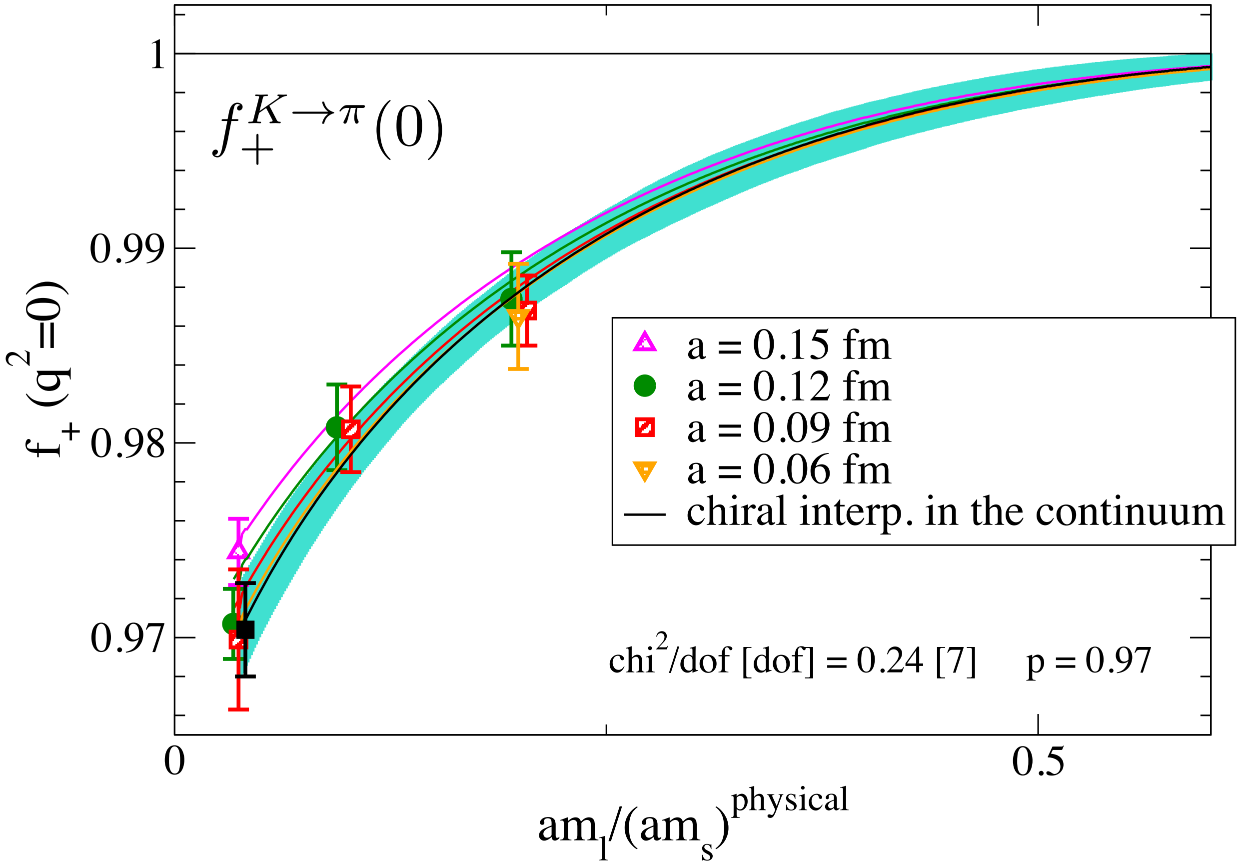
J. Komijani @ Lattice 2014 (FNAL/MILC, arXiv:1407.3772)



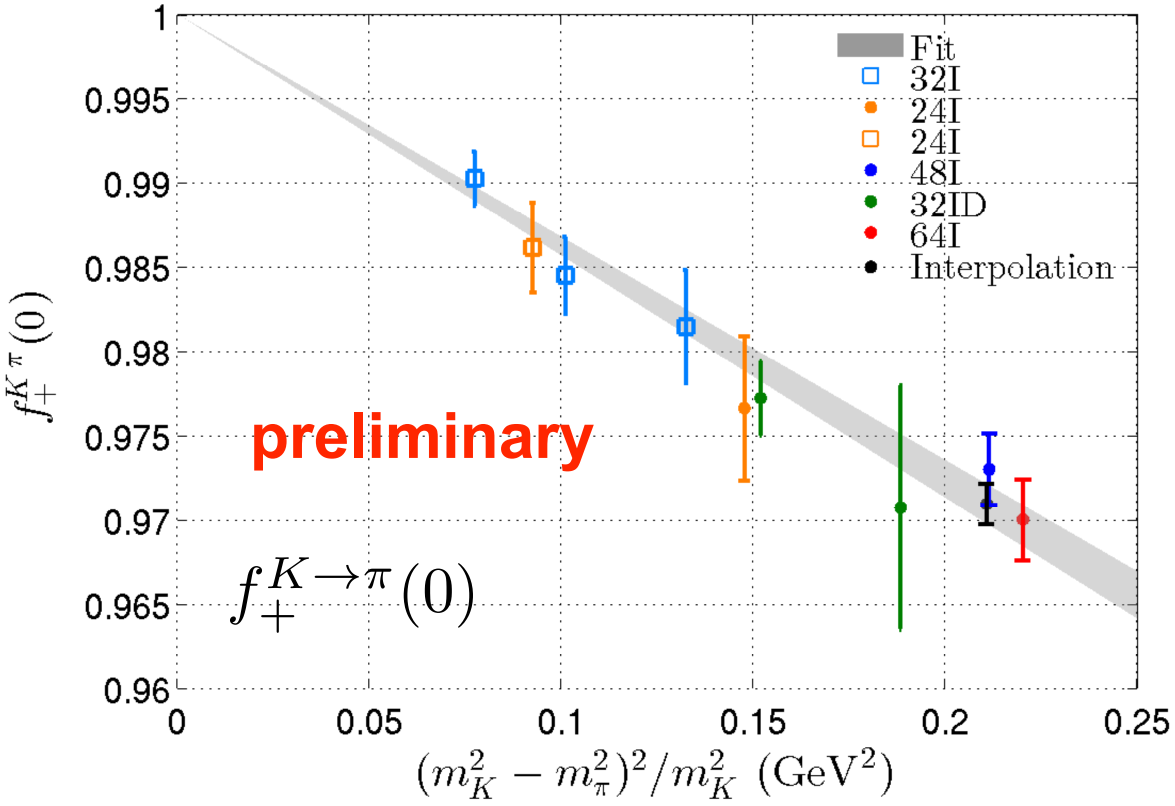
Kaon summary

For all quantities there are results that use **physical mass** ensembles

FNAL/MILC (arXiv:1312.1228, PRL 2014, T. Primer @ Lattice 2014)



D. Murphy @ Lattice 2014 (RBC/UKQCD)



Kaon summary

For all quantities there are results that use **physical mass** ensembles

FNAL/MILC (arXiv:1312.1228, PRL 2014, T. Primer @ Lattice 2014)

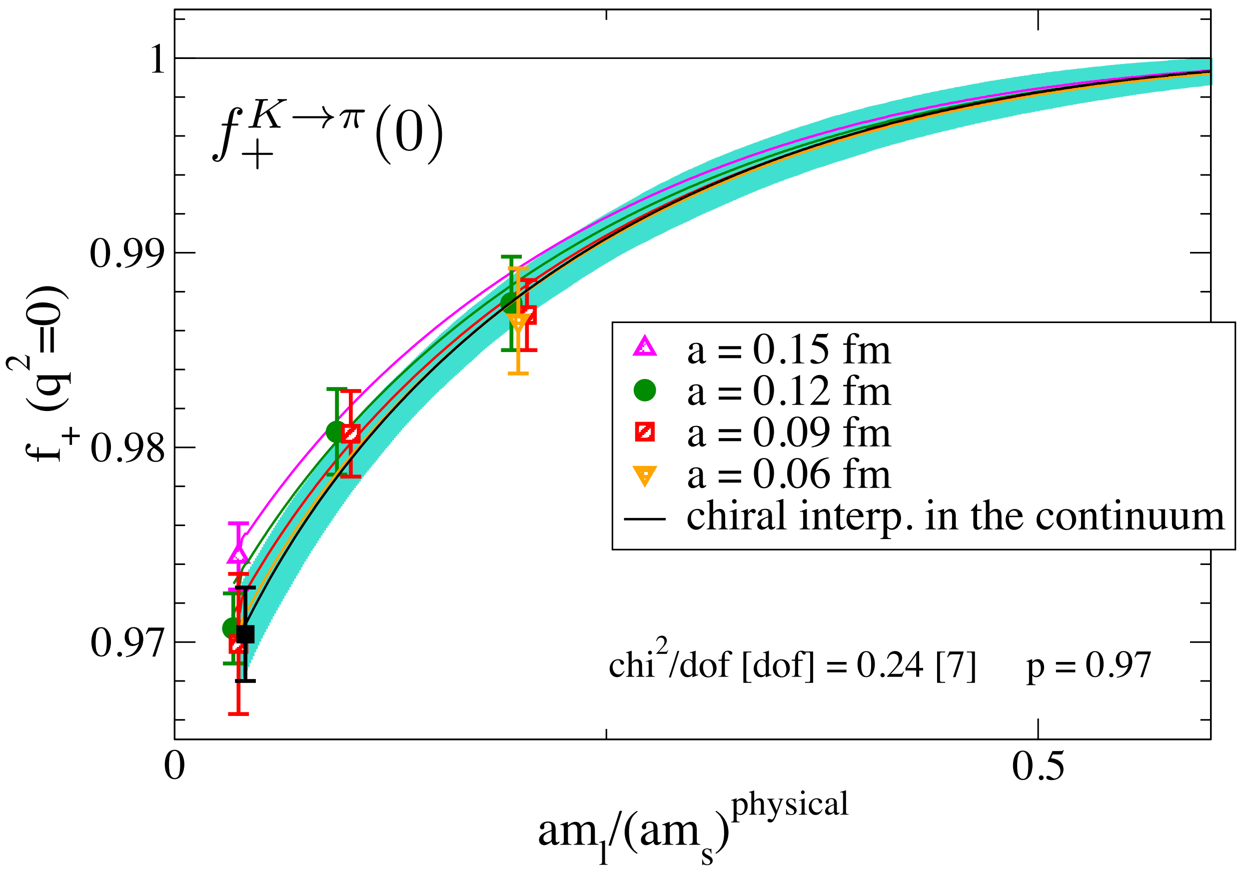
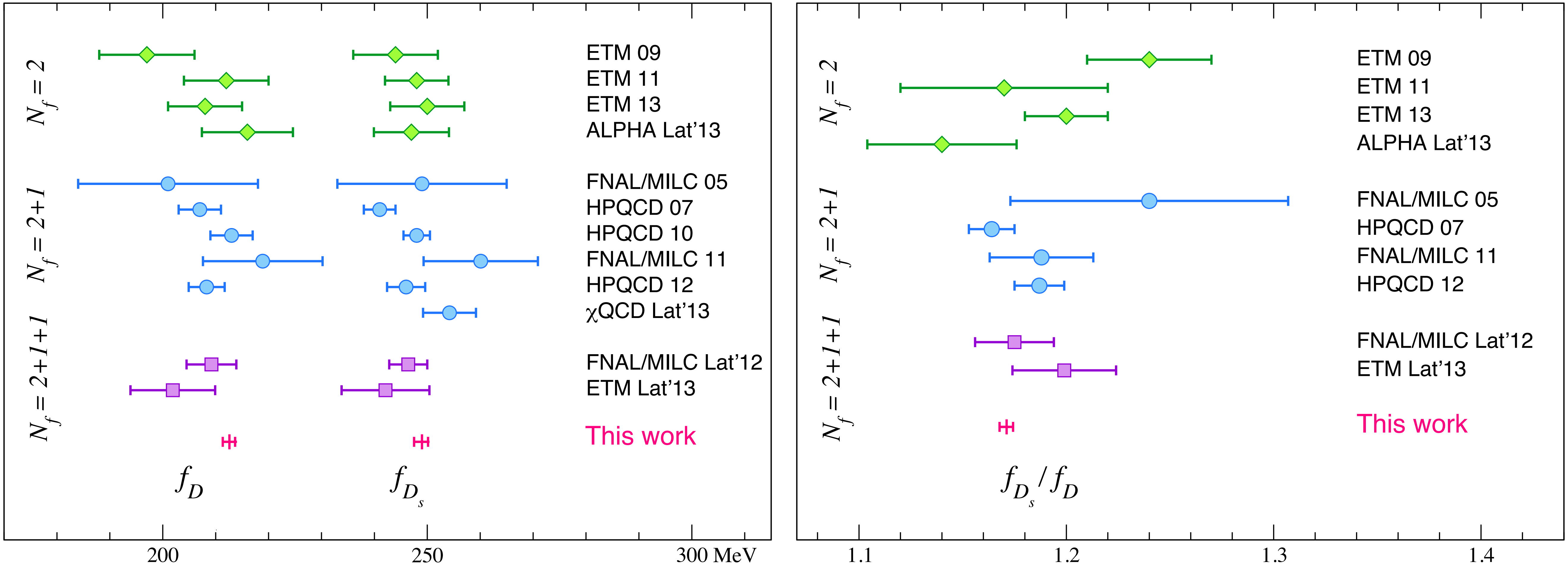


TABLE III. Error budget for $f_+(0)$ in percent.

Source of uncertainty	Error $f_+(0)$ (%)
Stat. + disc. + chiral inter.	0.24
$m_s^{\rm val} \neq m_s^{\rm sea}$	0.03
Scale r_1	0.08
Finite volume	0.2
Isospin	0.016
Total Error	0.33

D meson summary

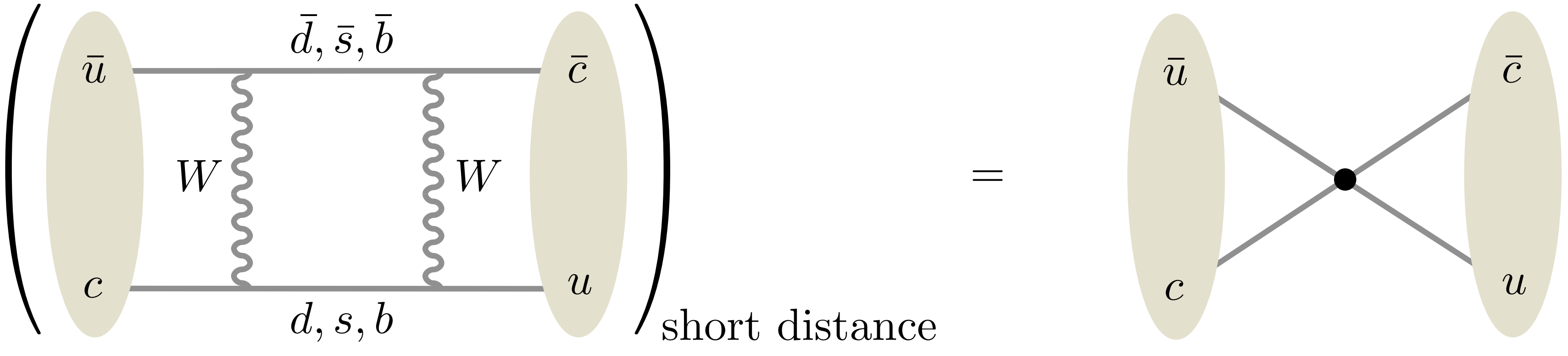
J. Komijani @ Lattice 2014 (FNAL/MILC, arXiv:1407.3772)



Neutral *D*-meson mixing

review by C. Bouchard @ Lattice 2014

Short Distance D^0 Mixing ($c \to u$ FCNCs)



$$\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)_{\text{short distance}} = \sum_i C_i^{\Delta_c=2}\langle \bar{D}^0|\mathcal{O}_i^{\Delta_c=2}|D^0\rangle$$

Now: UTfit, 1402.1664
LHCb, PRL 111, 251801 (2013)

$$|M_{12}| = (4.4 \pm 2.0) \times 10^{-3}\ \text{ps}^{-1}$$

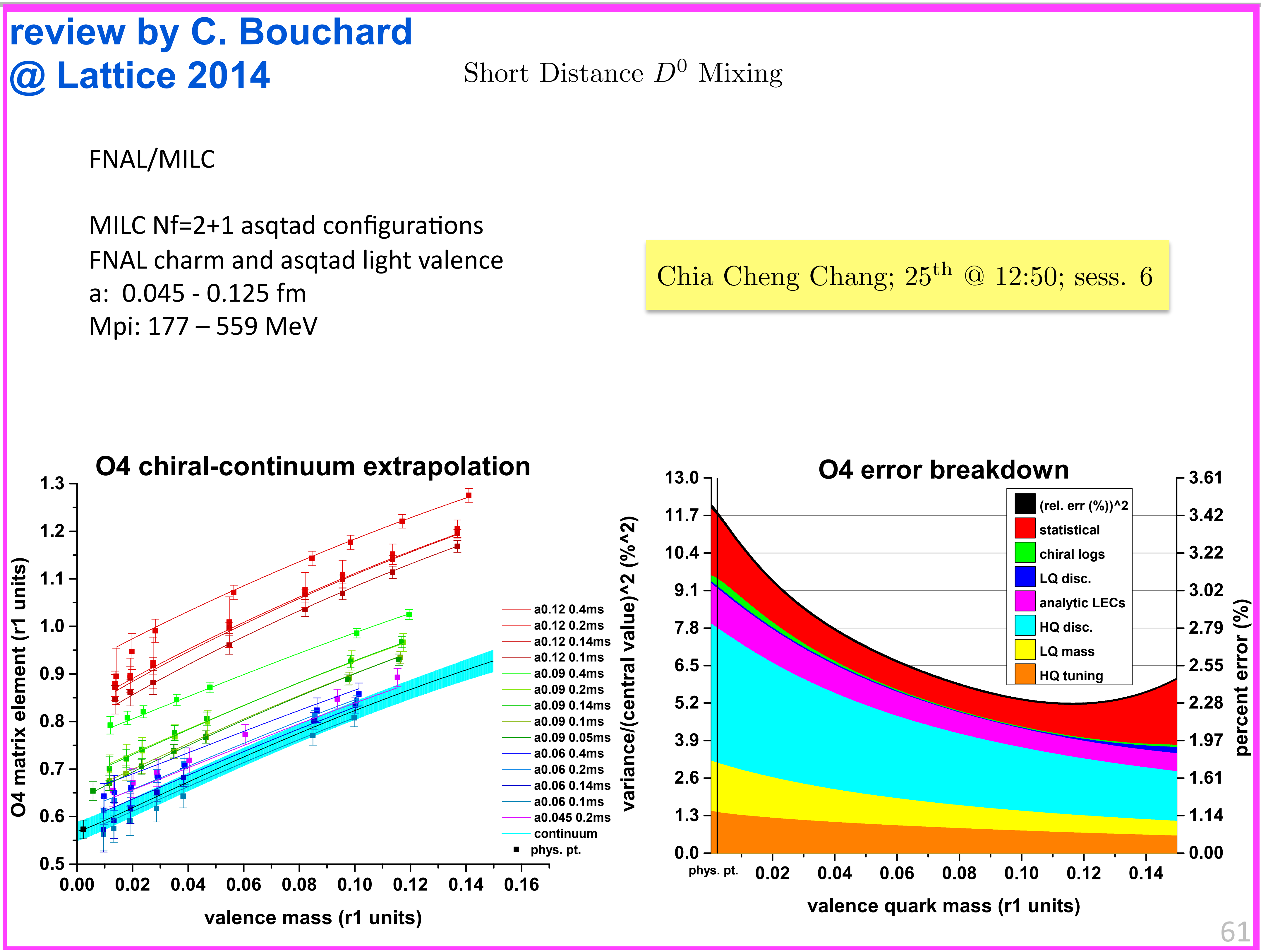
$$|\Gamma_{12}| = (14.9 \pm 1.6) \times 10^{-3}\ \text{ps}^{-1}$$

$$\arg\left(\frac{\Gamma_{12}}{M_{12}}\right) = (2.0 \pm 2.7)^{\text{o}}$$

2020: Briere, ANL Intensity Frontier (2013)

$\sim 5\times$ current precision

Neutral D-meson mixing



review by C. Bouchard
@ Lattice 2014

Short Distance D^0 Mixing

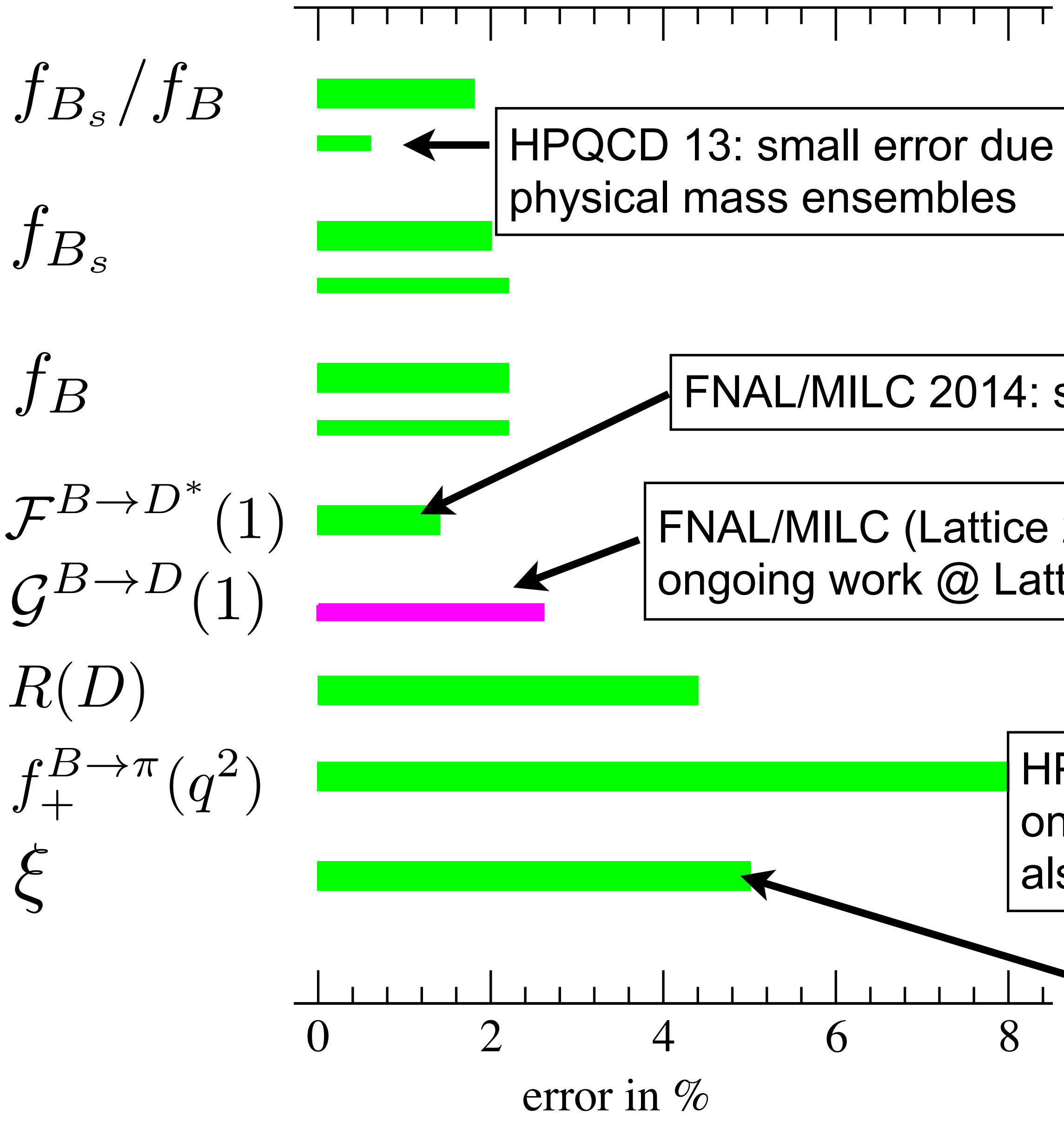
FNAL/MILC

MILC Nf=2+1 asqtad configurations
FNAL charm and asqtad light valence
a: 0.045 - 0.125 fm
Mpi: 177 – 559 MeV

Chia Cheng Chang; 25th @ 12:50; sess. 6

B meson summary

errors (in %) comparison: **FLAG-2 averages** vs. **new results**



HPQCD 13: small error due to physical mass ensembles

ongoing work @ Lattice 2014:
RBC/UKQCD (2 separate projects), ETM, FNAL/MILC, ALPHA, ...

FNAL/MILC 2014: small errors due to ratios (error cancellation), small lattice spacings

FNAL/MILC (Lattice 2013) also: results for B_s D_s by Orsay group
ongoing work @ Lattice 2014 by HPQCD, SWME, ...

HPQCD: first results for $B \to K\ell\ell$ (2013) and $B_s \to K\ell\nu$ (2014)
ongoing work @ Lattice 2014: HPQCD, FNAL/MILC, RBC/UKQCD, ...
also for $B \to \pi\ell\ell$

ongoing work for all five matrix elements @ Lattice 2014:
HPQCD, FNAL/MILC, RBC/UKQCD, ETM, ...

see N. Carrasco and P. Dimopoulos talks (ICHEP Lattice session, Friday)

review by C. Bouchard @ Lattice 2014

First results for f_{B^*}/f_B by ETM/Orsay group , see A. Oyanguren talk (ICHEP, Flavor physics session, Saturday)

Form factors for $B \to D^{(*)}\ell\nu$ & V_{cb}

review by C. Bouchard
@ Lattice 2014

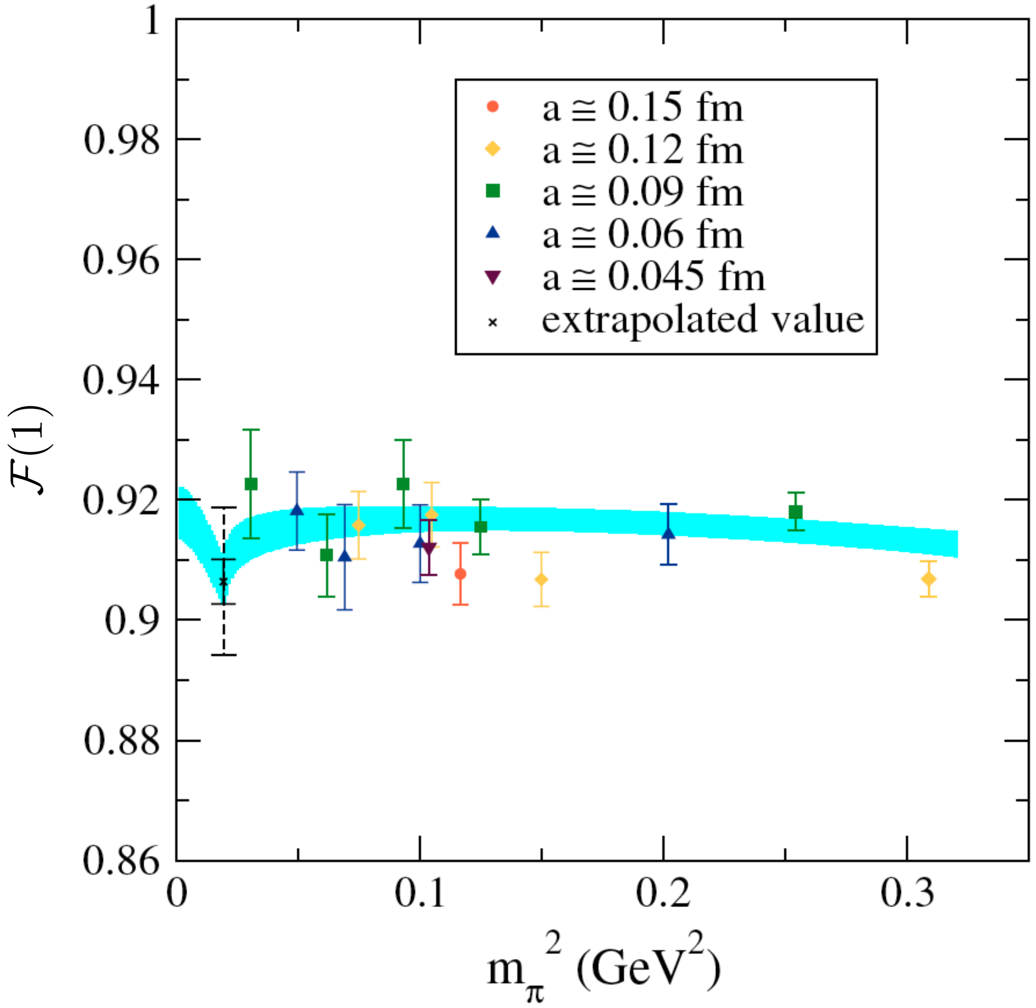
$B \to D^{(*)}\ell\nu$

FNAL/MILC Bailey et al. (FNAL/MILC), 1403.0635

MILC Nf=2+1 asqtad
FNAL b and c with asqtad light valence
a: 0.045 – 0.15 fm
Mpi: 174 – 520 MeV

work at zero recoil, calc F(1)

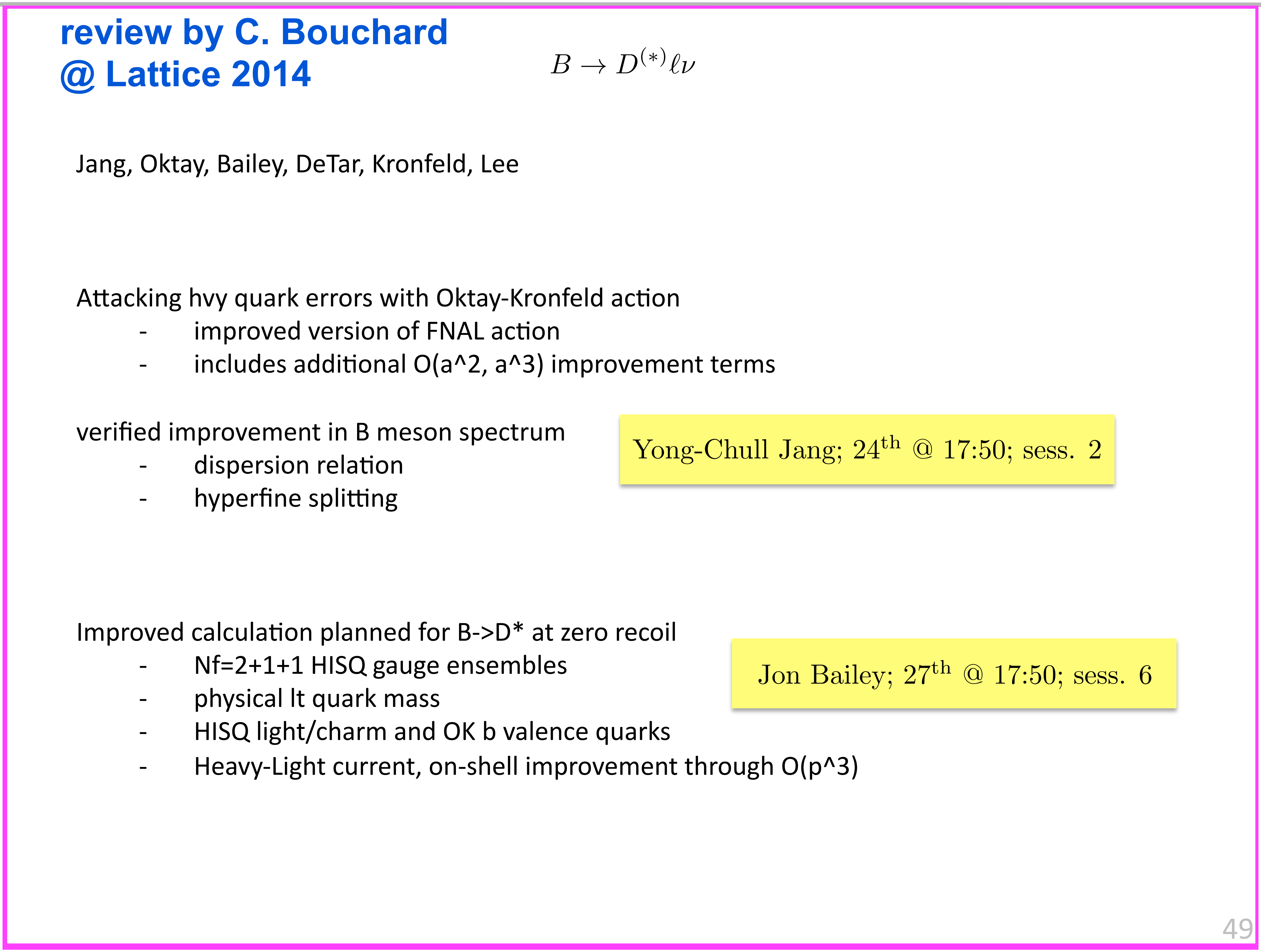
leading source of error is hvy q disc effects



small errors due to ratios
(error cancellation),
many ensembles,
small lattice spacings

* Lattice error now equal to experimental error.

Form factors for $B \to D^{(*)}\ell\nu$ & V_{cb}



**review by C. Bouchard
@ Lattice 2014**

$B \to D^{(*)}\ell\nu$

Jang, Oktay, Bailey, DeTar, Kronfeld, Lee

Attacking hvy quark errors with Oktay-Kronfeld action
- improved version of FNAL action
- includes additional O(a^2, a^3) improvement terms

verified improvement in B meson spectrum
- dispersion relation
- hyperfine splitting

Yong-Chull Jang; 24th @ 17:50; sess. 2

Improved calculation planned for B->D* at zero recoil
- Nf=2+1+1 HISQ gauge ensembles
- physical lt quark mass
- HISQ light/charm and OK b valence quarks
- Heavy-Light current, on-shell improvement through O(p^3)

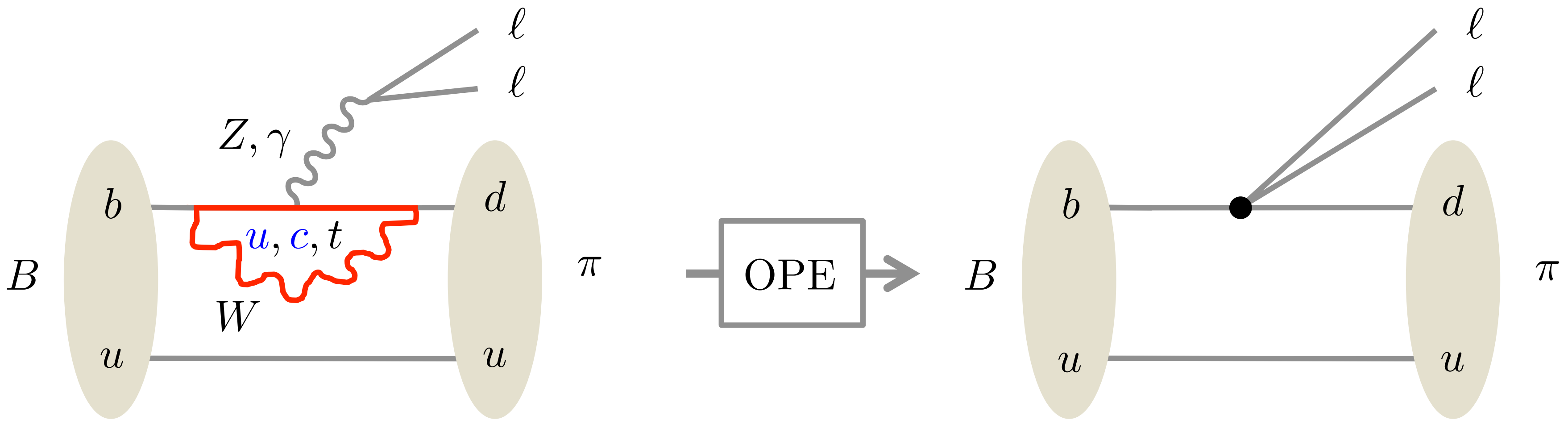
Jon Bailey; 27th @ 17:50; sess. 6

49

B meson summary

review by C. Bouchard
@ Lattice 2014

$B \to \pi \ell \ell$

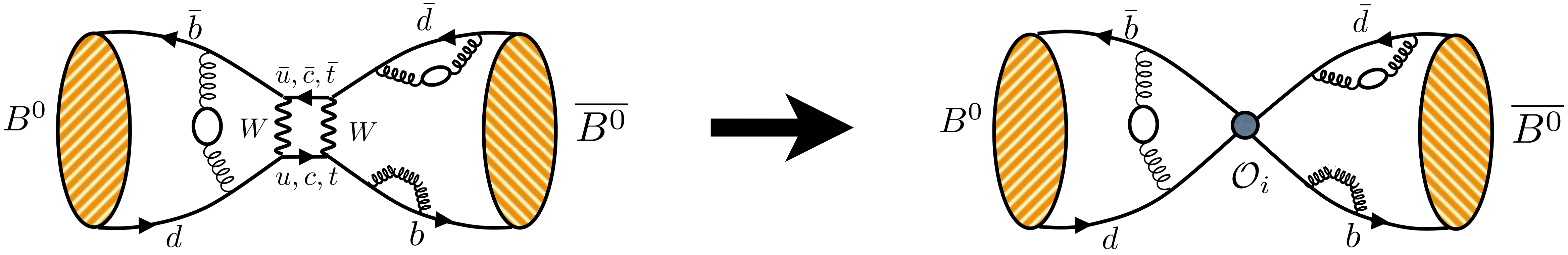


$$10^8 \mathcal{B}(B \to \pi\mu\mu) = 2.3(6)_{\text{stat}}(1)_{\text{sys}}$$ LHCb, JHEP 12 (2012) 125

FNAL/MILC	HPQCD
MILC 2+1 asqtad ensembles	MILC 2+1 asqtad ensembles
FNAL b, asqtad light/strange valence	NRQCD b, HISQ light/strange valence
a: 0.045 – 0.12 fm	a: 0.09, 0.12 fm
Mpi: 174 – 520 MeV	Mpi: 174 – 520 MeV

Neutral B-meson mixing

Standard Model



In general :

$$\mathcal{H}_{\text{eff}} = \sum_{i=1}^{5} c_i(\mu)\mathcal{O}_i(\mu)$$

SM:

$$\mathcal{O}_1 = (\bar{b}^\alpha \gamma_\mu L q^\alpha)\ (\bar{b}^\beta \gamma_\mu L q^\beta)$$

$$\mathcal{O}_2 = (\bar{b}^\alpha L q^\alpha)\ (\bar{b}^\beta L q^\beta)$$

$$\mathcal{O}_3 = (\bar{b}^\alpha L q^\beta)\ (\bar{b}^\beta L q^\alpha)$$

BSM:

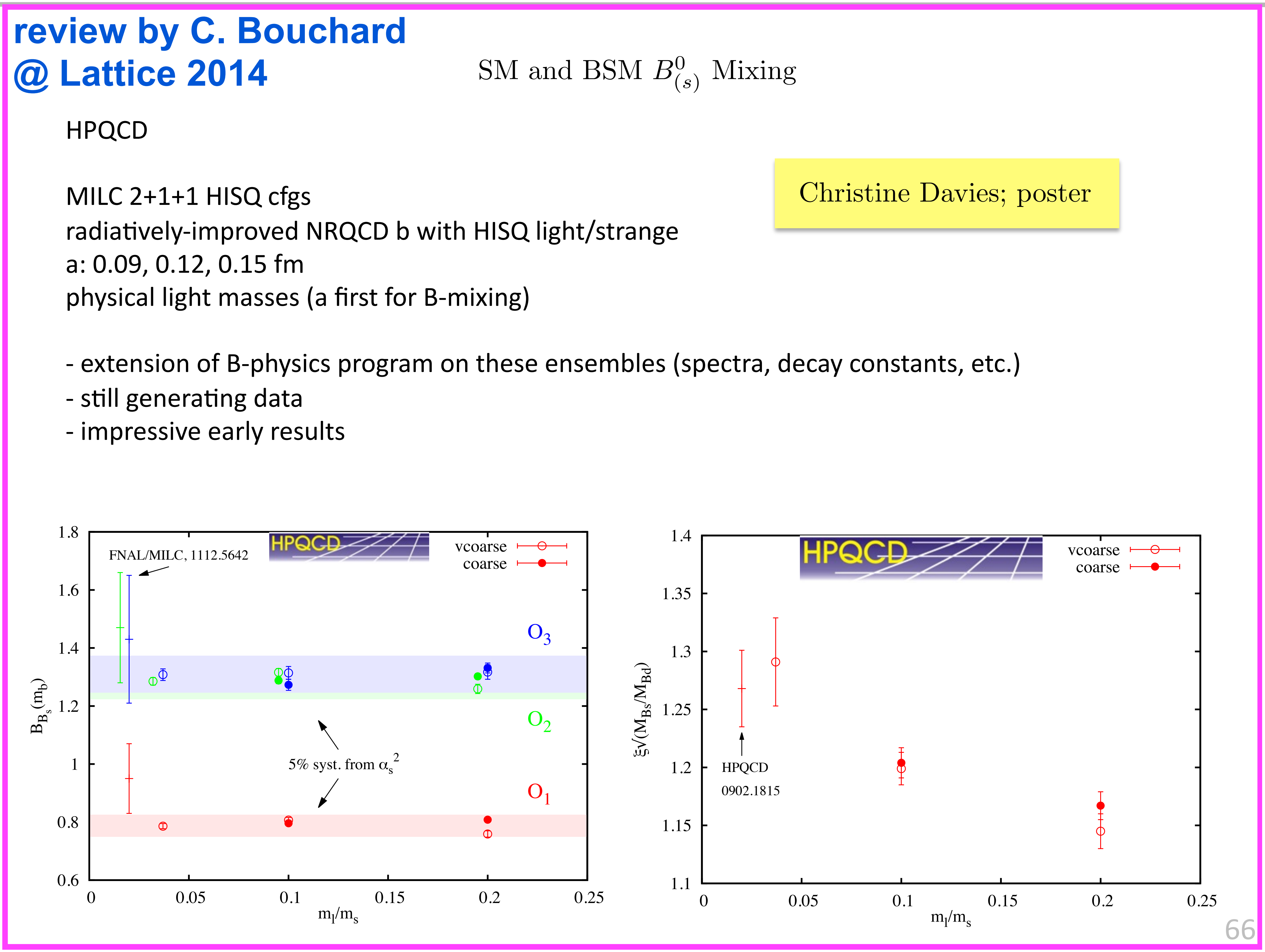
$$\mathcal{O}_4 = (\bar{b}^\alpha L q^\alpha)\ (\bar{b}^\beta R q^\beta)$$

$$\mathcal{O}_5 = (\bar{b}^\alpha L q^\beta)\ (\bar{b}^\beta R q^\alpha)$$

$$\langle \mathcal{O}_i \rangle \equiv \langle \bar{B}_q^0 | \mathcal{O}_i | B_q^0 \rangle(\mu) = e_i\ m_{B_q}^2\ f_{B_q}^2\ B_{B_q}^{(i)}(\mu)$$

We calculate all five matrix elements.

Neutral B-meson mixing



review by C. Bouchard
@ Lattice 2014

SM and BSM $B^0_{(s)}$ Mixing

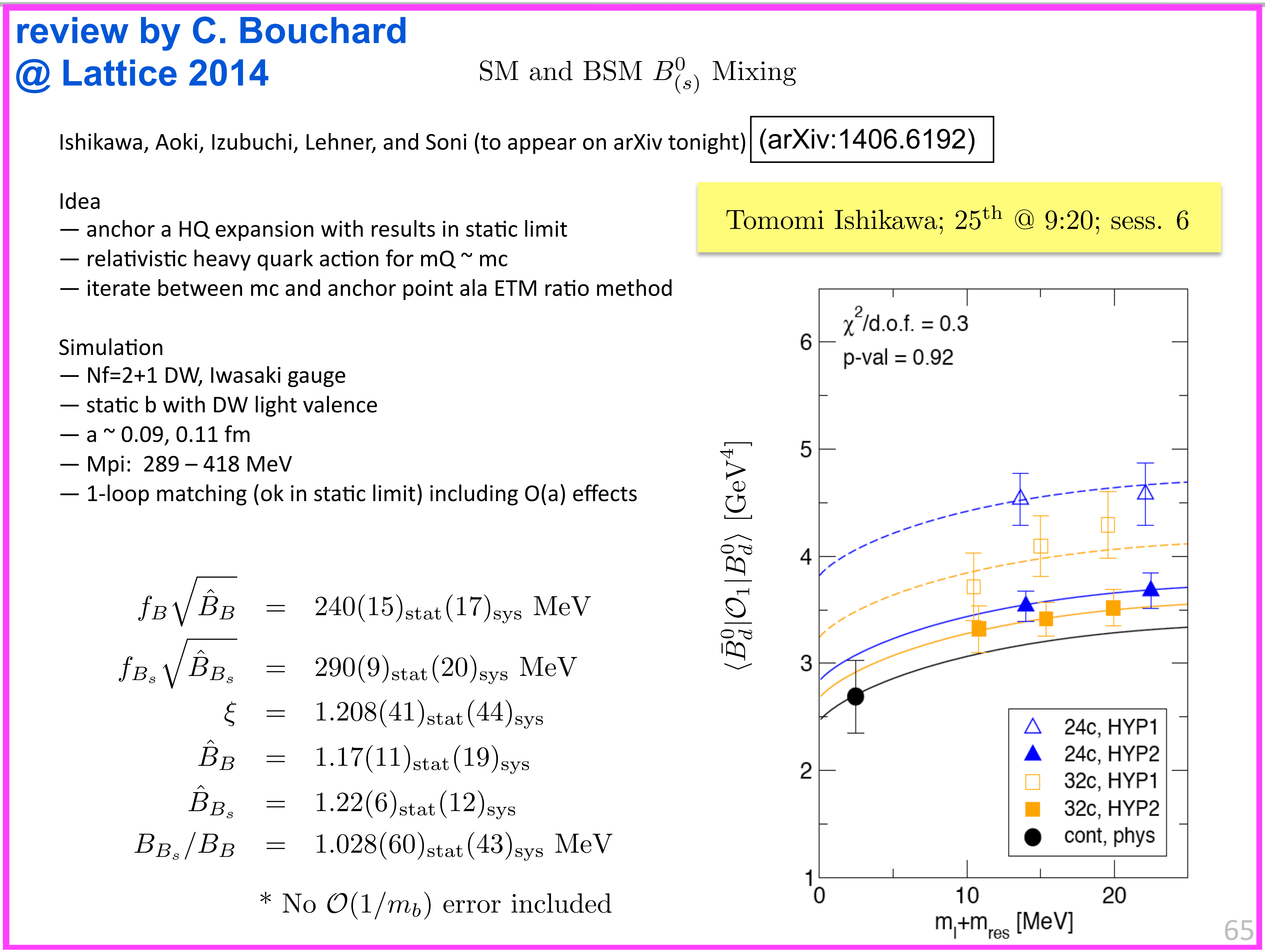
HPQCD

MILC 2+1+1 HISQ cfgs
radiatively-improved NRQCD b with HISQ light/strange
a: 0.09, 0.12, 0.15 fm
physical light masses (a first for B-mixing)

- extension of B-physics program on these ensembles (spectra, decay constants, etc.)
- still generating data
- impressive early results

Christine Davies; poster

Neutral B-meson mixing



review by C. Bouchard
@ Lattice 2014

SM and BSM $B^0_{(s)}$ Mixing

Ishikawa, Aoki, Izubuchi, Lehner, and Soni (to appear on arXiv tonight) (arXiv:1406.6192)

Tomomi Ishikawa; 25th @ 9:20; sess. 6

Idea
— anchor a HQ expansion with results in static limit
— relativistic heavy quark action for mQ ~ mc
— iterate between mc and anchor point ala ETM ratio method

Simulation
— Nf=2+1 DW, Iwasaki gauge
— static b with DW light valence
— a ~ 0.09, 0.11 fm
— Mpi: 289 – 418 MeV
— 1-loop matching (ok in static limit) including O(a) effects

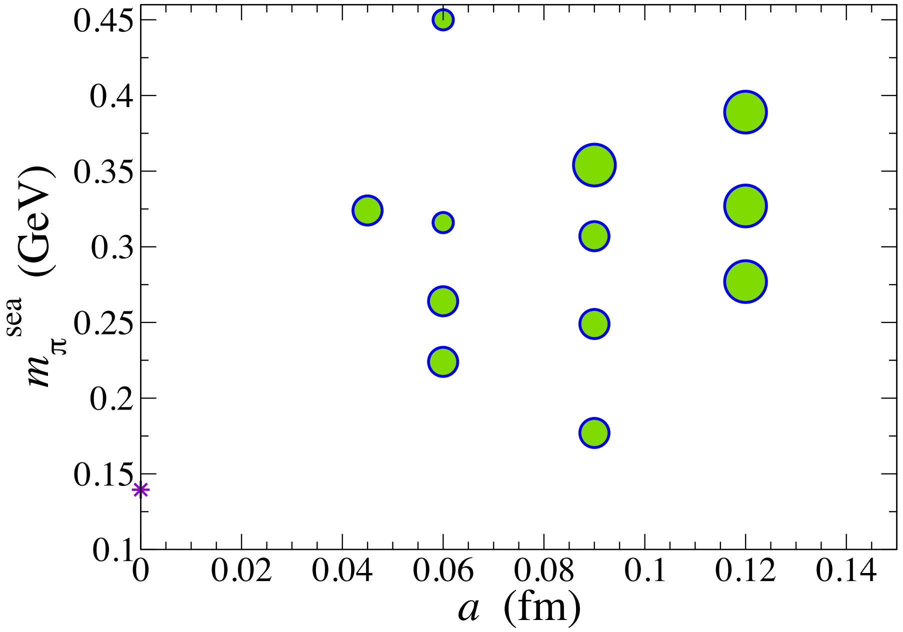
$$\begin{aligned}
f_B\sqrt{\hat{B}_B} &= 240(15)_{\rm stat}(17)_{\rm sys}\ \text{MeV}\\
f_{B_s}\sqrt{\hat{B}_{B_s}} &= 290(9)_{\rm stat}(20)_{\rm sys}\ \text{MeV}\\
\xi &= 1.208(41)_{\rm stat}(44)_{\rm sys}\\
\hat{B}_B &= 1.17(11)_{\rm stat}(19)_{\rm sys}\\
\hat{B}_{B_s} &= 1.22(6)_{\rm stat}(12)_{\rm sys}\\
B_{B_s}/B_B &= 1.028(60)_{\rm stat}(43)_{\rm sys}\ \text{MeV}
\end{aligned}$$

* No $\mathcal{O}(1/m_b)$ error included

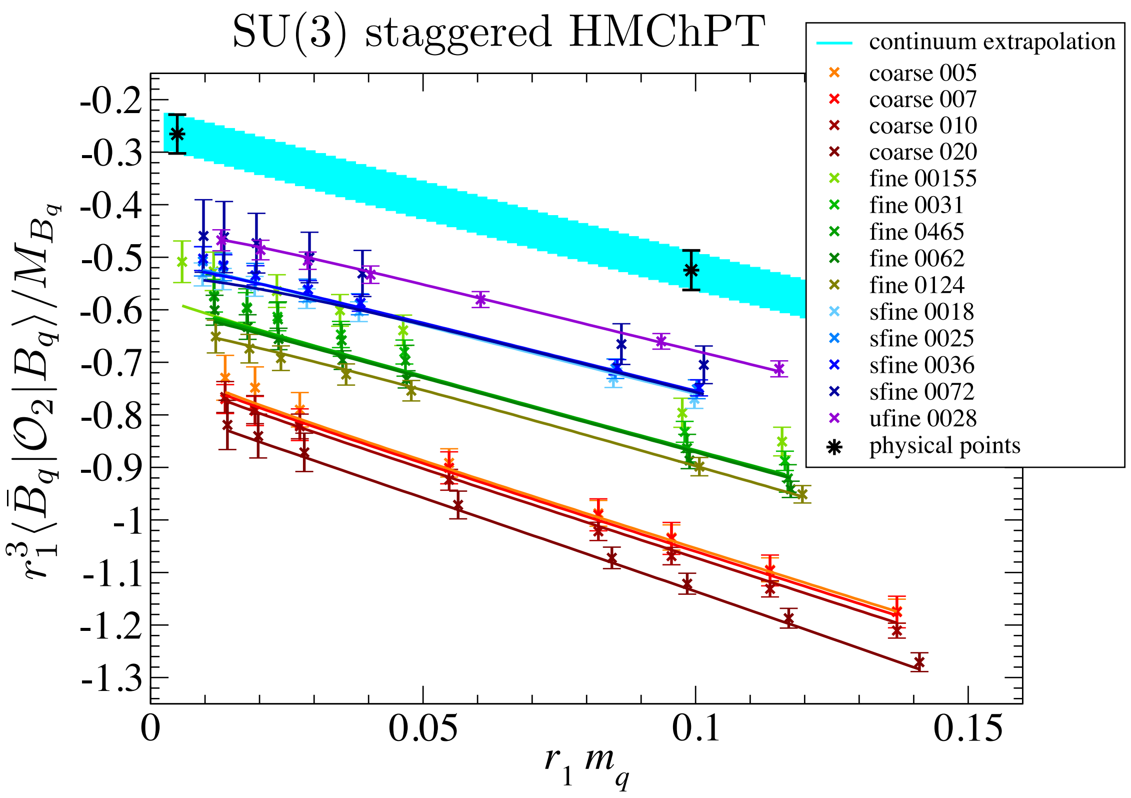
Neutral B-meson mixing

FNAL/MILC @ Lattice 2014

review by C. Bouchard @ Lattice 2014



- 14 MILC asqtad ensembles
  4 lattice spacings
  ~ 4 sea quark masses per lattice spacing
  ~ 600 - 2000 configurations
  × 4 time-sources per ensemble
- asqtad light valence quarks
  ~ 7 light valence masses per ensemble
- Fermilab b quarks
- $O(a)$ improved four-quark operators



no results quoted yet

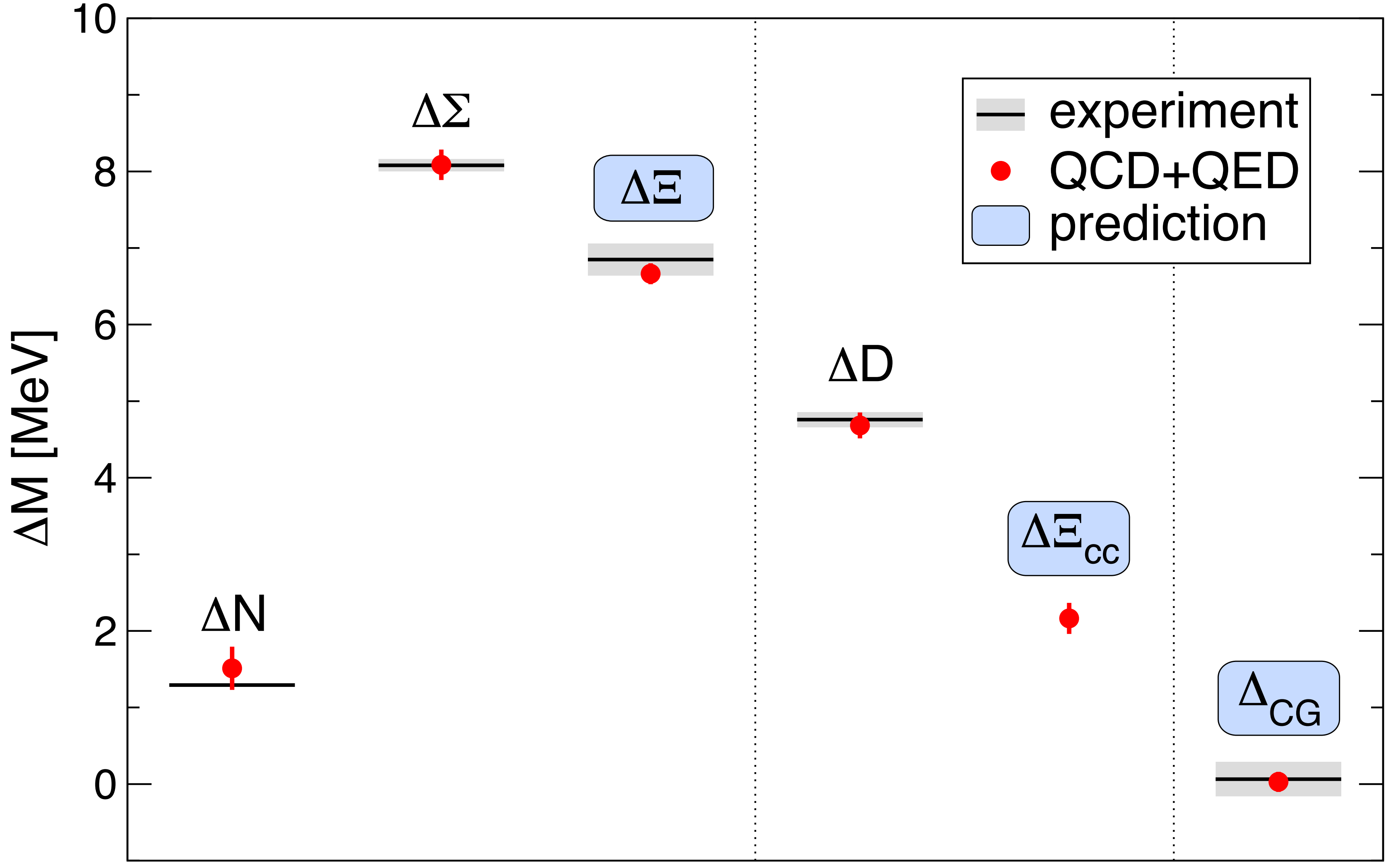
expected errors:

- $\langle \bar{B} | \mathcal{O}_1 | B \rangle$: $\sim 9\%$
- $\langle \bar{B} | \mathcal{O}_{2,3,4,5} | B \rangle$: $10 - 15\%$
- ξ: $< 2\%$
- $\langle \bar{B} | \mathcal{O}_3 | B \rangle / \langle \bar{B} | \mathcal{O}_1 | B \rangle$: $\sim 10\%$

Including QED

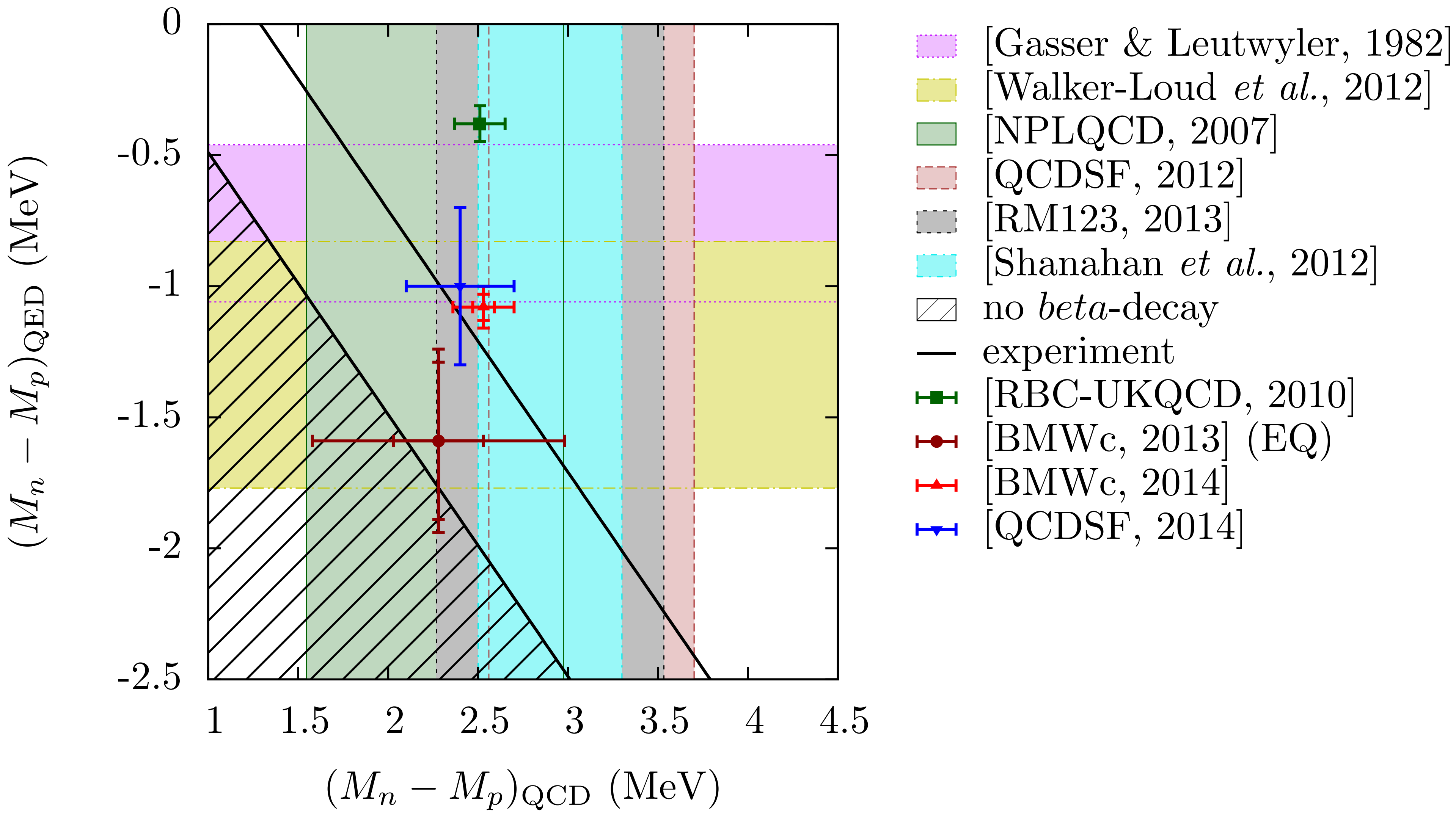
review by A. Portelli @ Lattice 2014 and ICHEP (in Lattice session, Saturday)

Sz. Borsanyi et al (BMW, arXiv:1406.4088)



Including QED

review by A. Portelli @ Lattice 2014 and ICHEP (in Lattice session, Saturday)

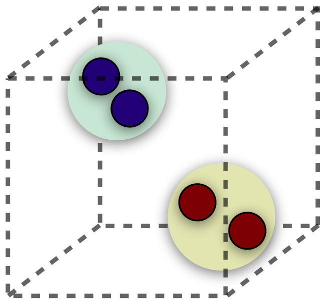


Few body systems in a box

review by R. Briceño
@ Lattice 2014

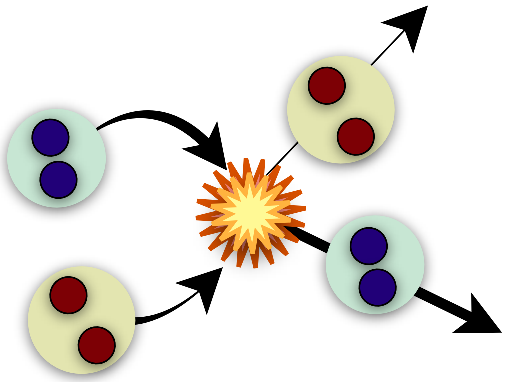
A roadmap towards physics

① Calculate finite volume spectrum



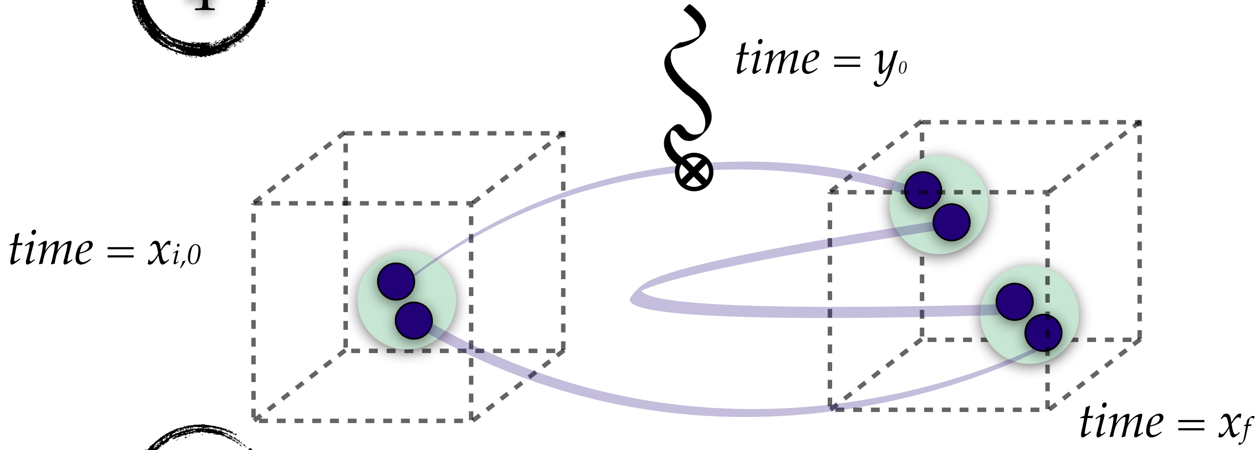
② Plug into formalism

③ Out goes elastic & inelastic QCD scattering amplitudes



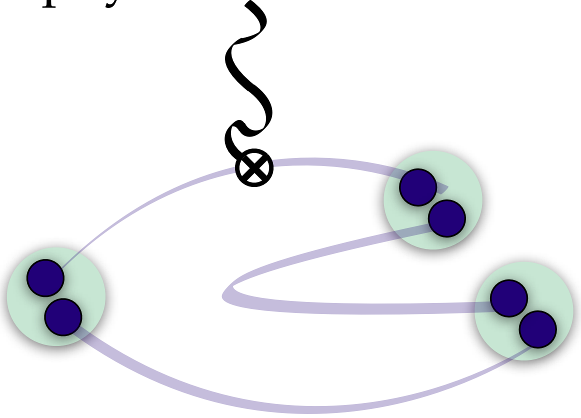
à la mode de **Lüscher (1986)**

④ Calculate finite volume form factor



⑤ Plug spectrum, scattering parameters and finite volume form factor into formalism

⑥ Out go physical form factors

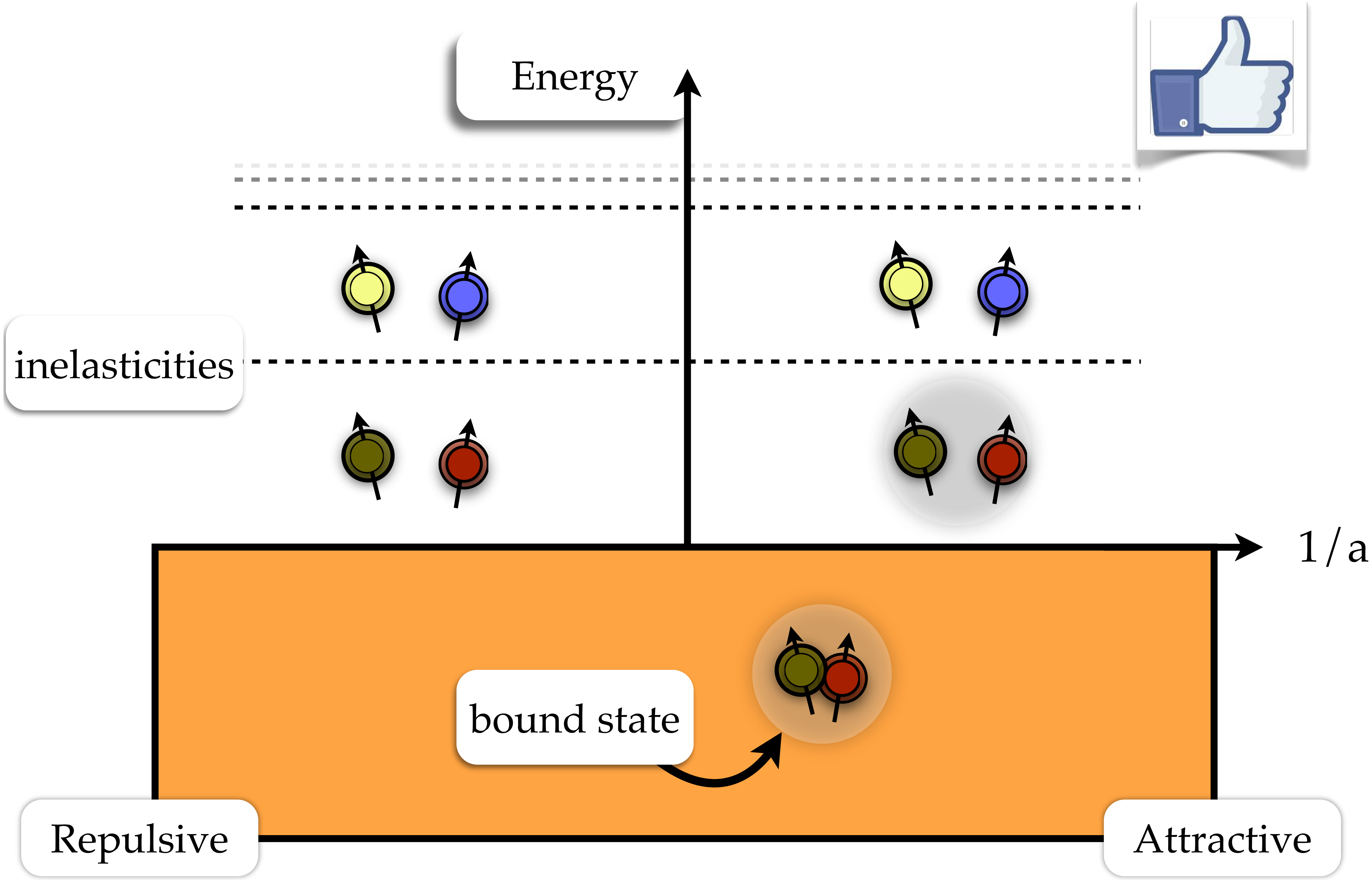


à la mode de **Lellouch & Lüscher (2000)**

Few body systems in a box

review by R. Briceño
@ Lattice 2014

Spectrum 2-body system in a box



Few body systems in a box

review by R. Briceño @ Lattice 2014

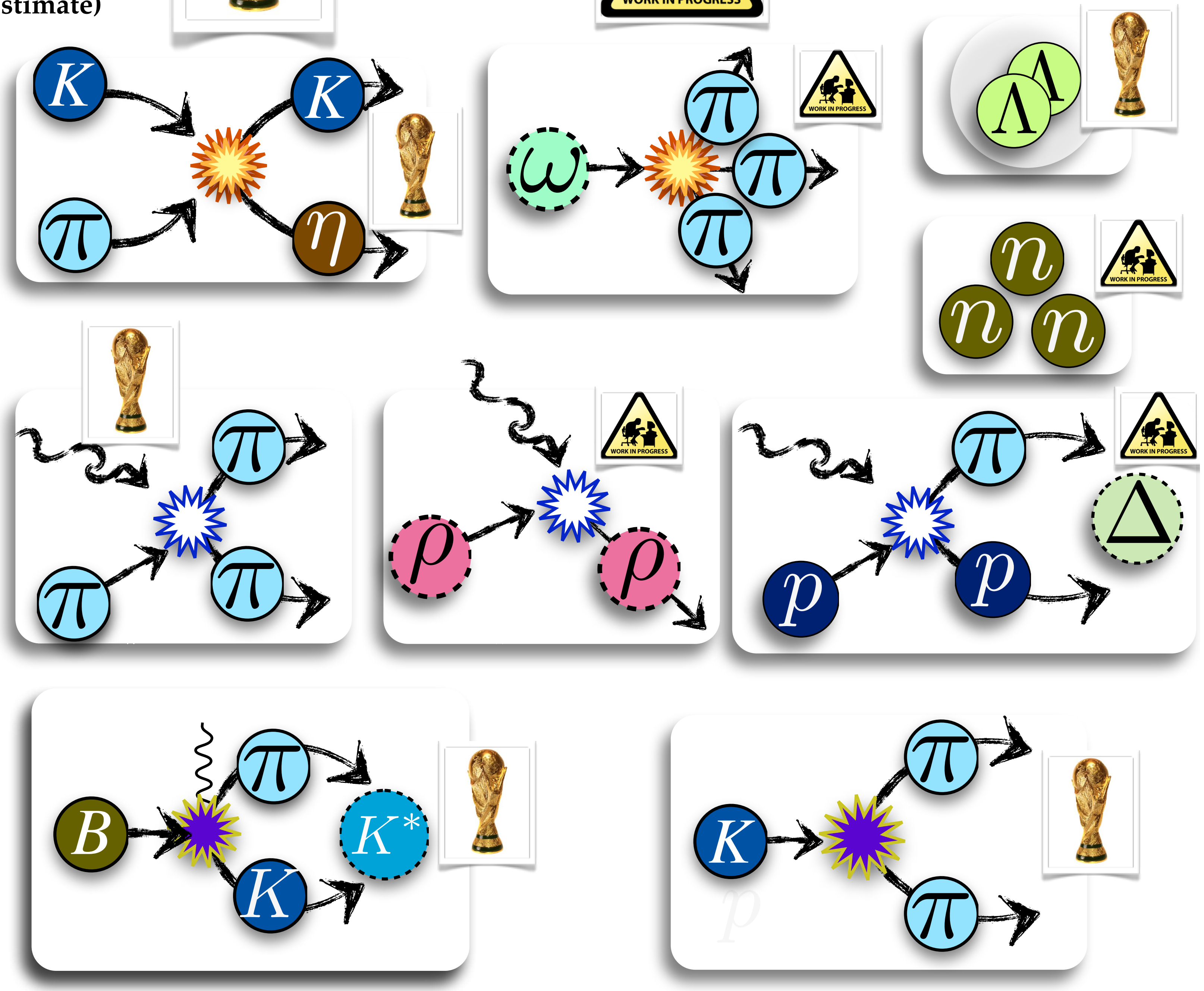
Status of formalism

(somewhat bias estimate)

🏆 : Under control

⚠ : progress made/ more to come

- Spectroscopy/ scattering:
- Electromagnetic form factors:
- Fundamental symmetries:



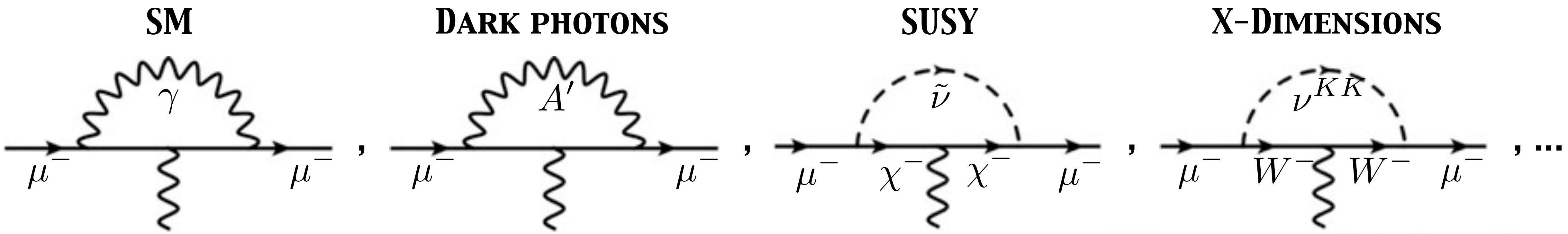
Beyond simple quantities

- ☆ $K \rightarrow \pi\pi$ amplitudes and Δm_K
- ☆ hadronic corrections to muon g-2
- ☆ hadron structure, resonances, ...

hadronic contributions to muon g-2

review by B. Casey @ Lattice 2014

The experimental measurement (BNL-E821) of the muon g-2 disagrees with the SM prediction by $> 3\sigma$.

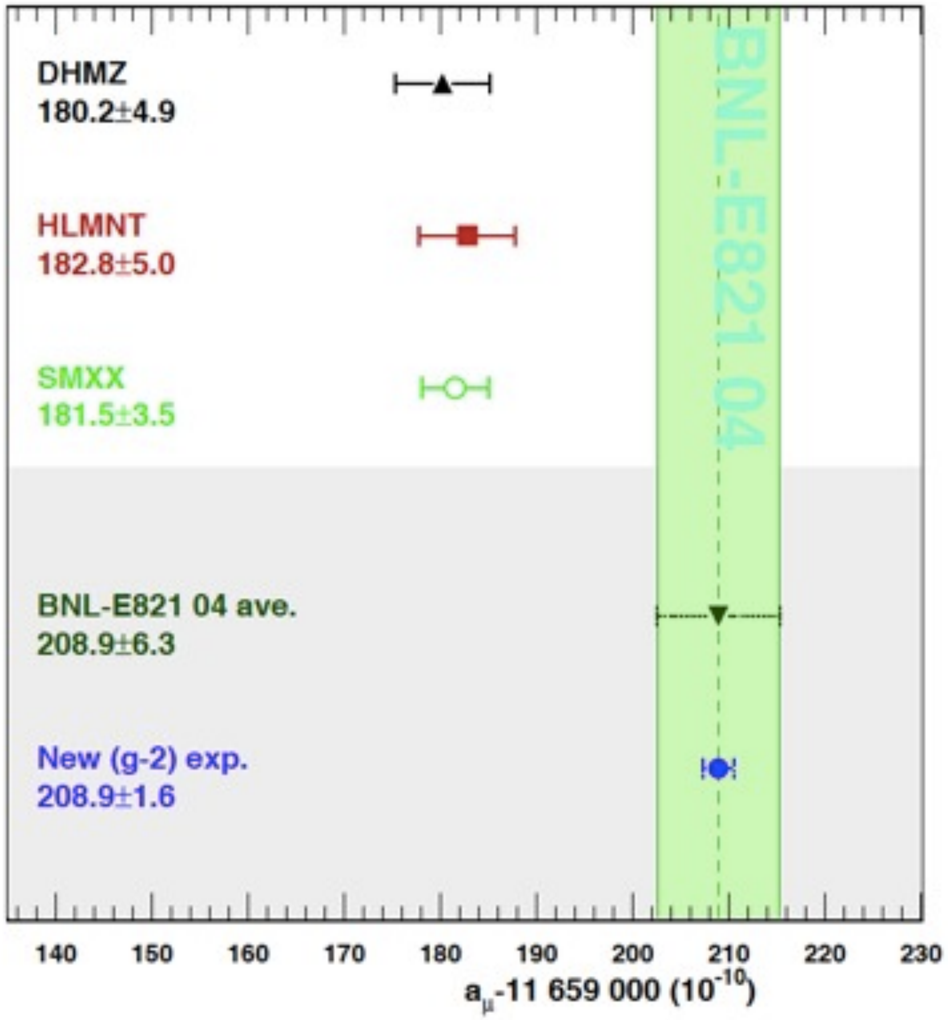


muon g-2 is a sensitive probe of new physics.

The goal of the Fermilab muon g-2 experiment is to reduce the experimental uncertainty by a factor of 4.

The uncertainty of the SM prediction is dominated by the error on the hadronic corrections (HVP and HLbL):

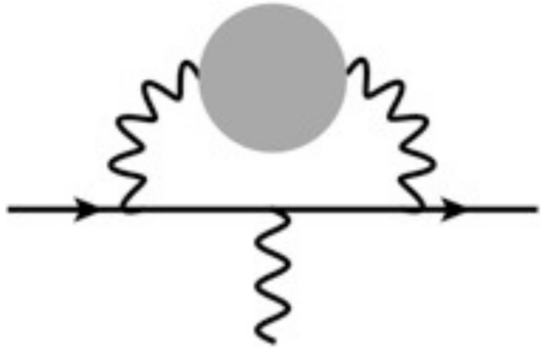
$$\delta(a_\mu^{\mathrm{HVP}}) = 0.6\% \quad \delta(a_\mu^{\mathrm{HLbL}}) = 25\%$$



[Blum *et al.*, arXiv:1311.2198]

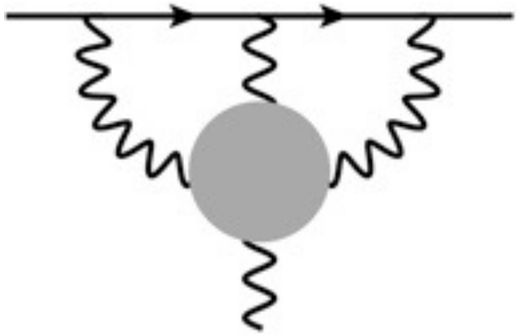
hadronic contributions to muon *g*-2

Hadronic vacuum polarization (HVP):



from experimental result for $e^+e^- \to$ hadrons plus dispersion relation

Hadronic light-by-light (HLbL):

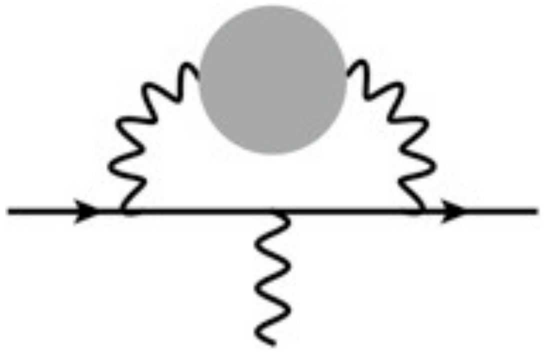


estimated from models such as large N_c, χPT, vector meson dominance, etc...

- Both quantities are calculable, in principle, with LQCD methods.
- For HVP there are already methods in place, with a lot of activity in the last 6 months, and first results have been reported.
- The calculation of the HLbL correction is very difficult, but methods for it are also being developed and tested.

hadronic contributions to muon *g*-2

Status of HVP calculations

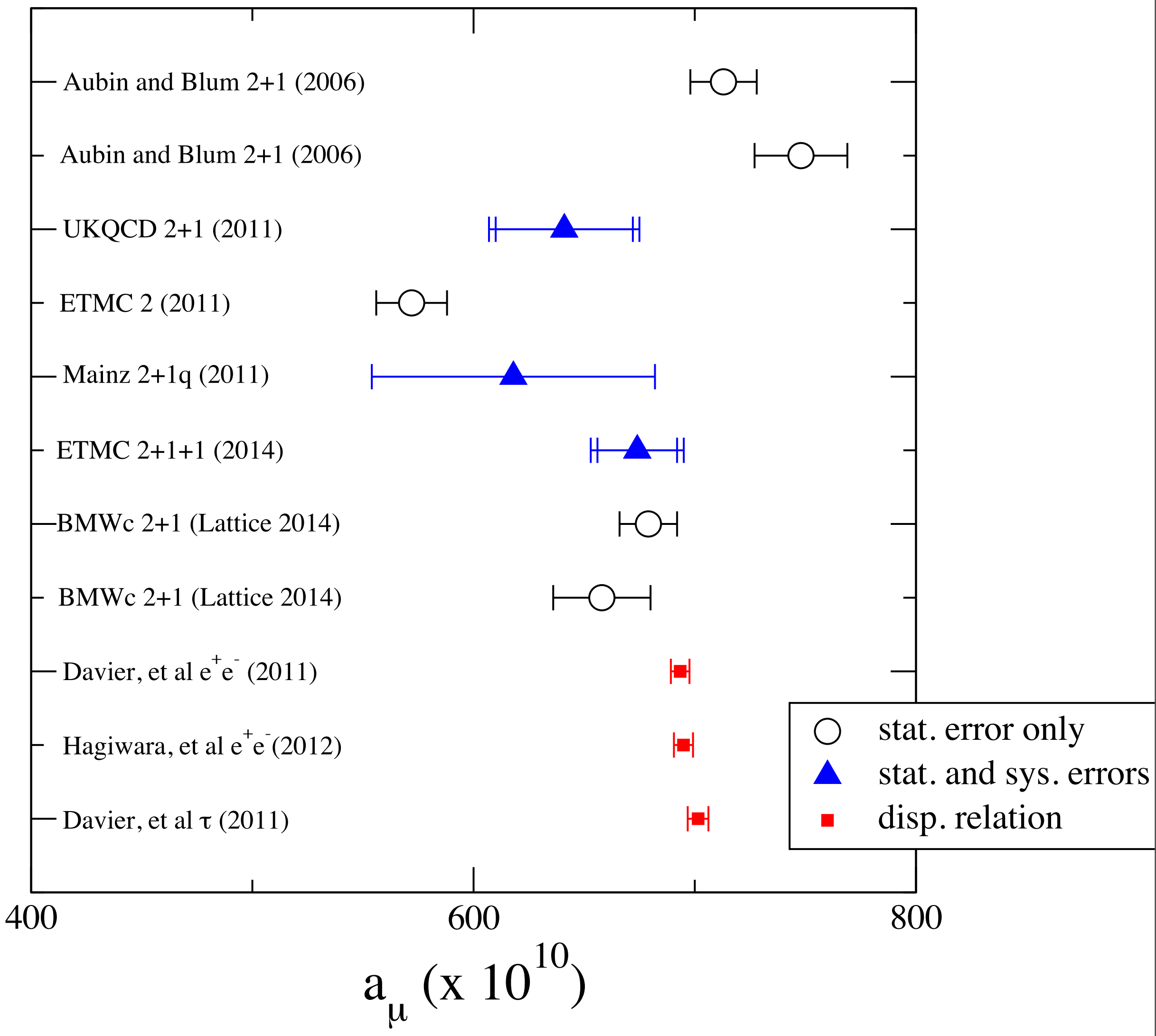


A lot of progress in method development:

- statistical noise reduction techniques (AMA, ...)
- methods for controlling q^2 extrapolation (twisted boundary conditions, Pade approximants, mixed time time- and space-like calculations, position-space moments, ...)
- use of physical mass ensembles (BMW, RBC/UKQCD, ETM)
- disconnected contributions (Mainz group)

See talks by G. Herdoiza, J. Koponnen, P. Santiago @ ICHEP (Lattice session, Saturday)

compiled by T. Blum + T. Izubuchi

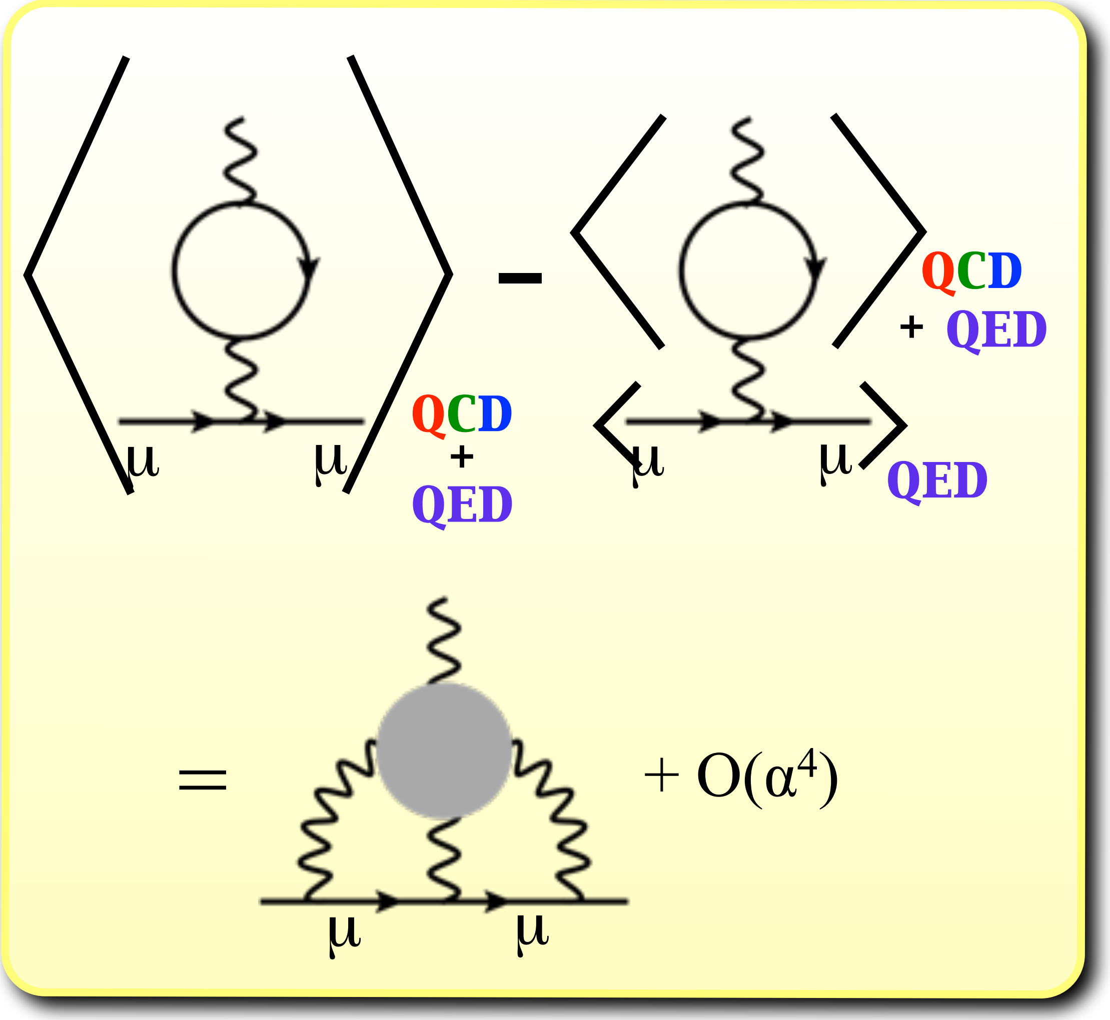


hadronic contributions to muon g-2

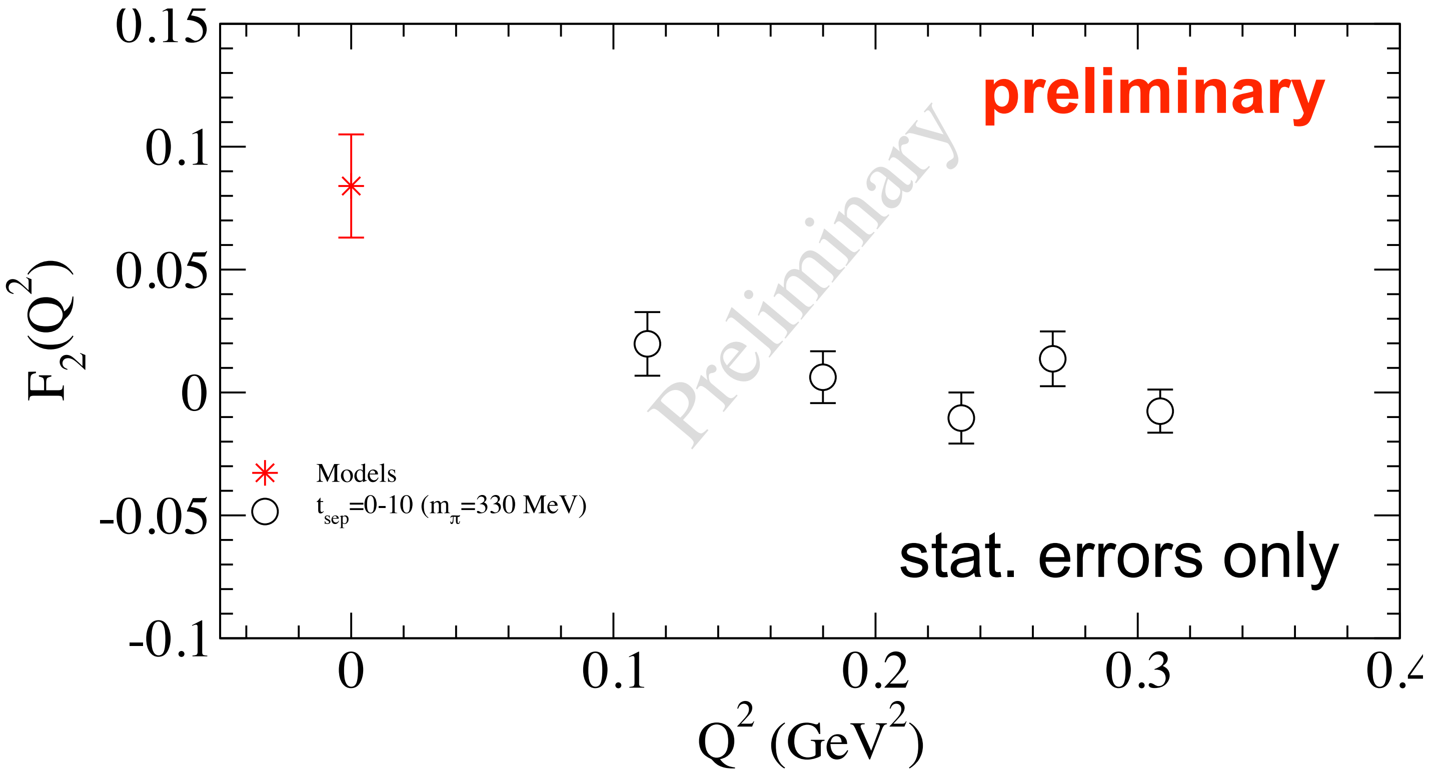
Status of Hadronic light-by-light (HLbL)

T. Blum, T. Izubuchi, M. Hayawaka
(paper in preparation)

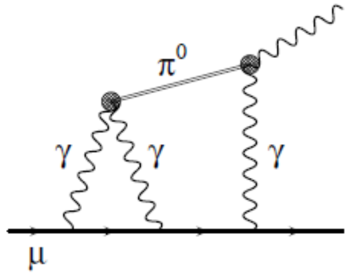
pilot study of direct method
needs systematic error analysis



T. Blum, T. Izubuchi, priv. comm.



- alternate approach: calculate dominant contribution (pion transition form factor)

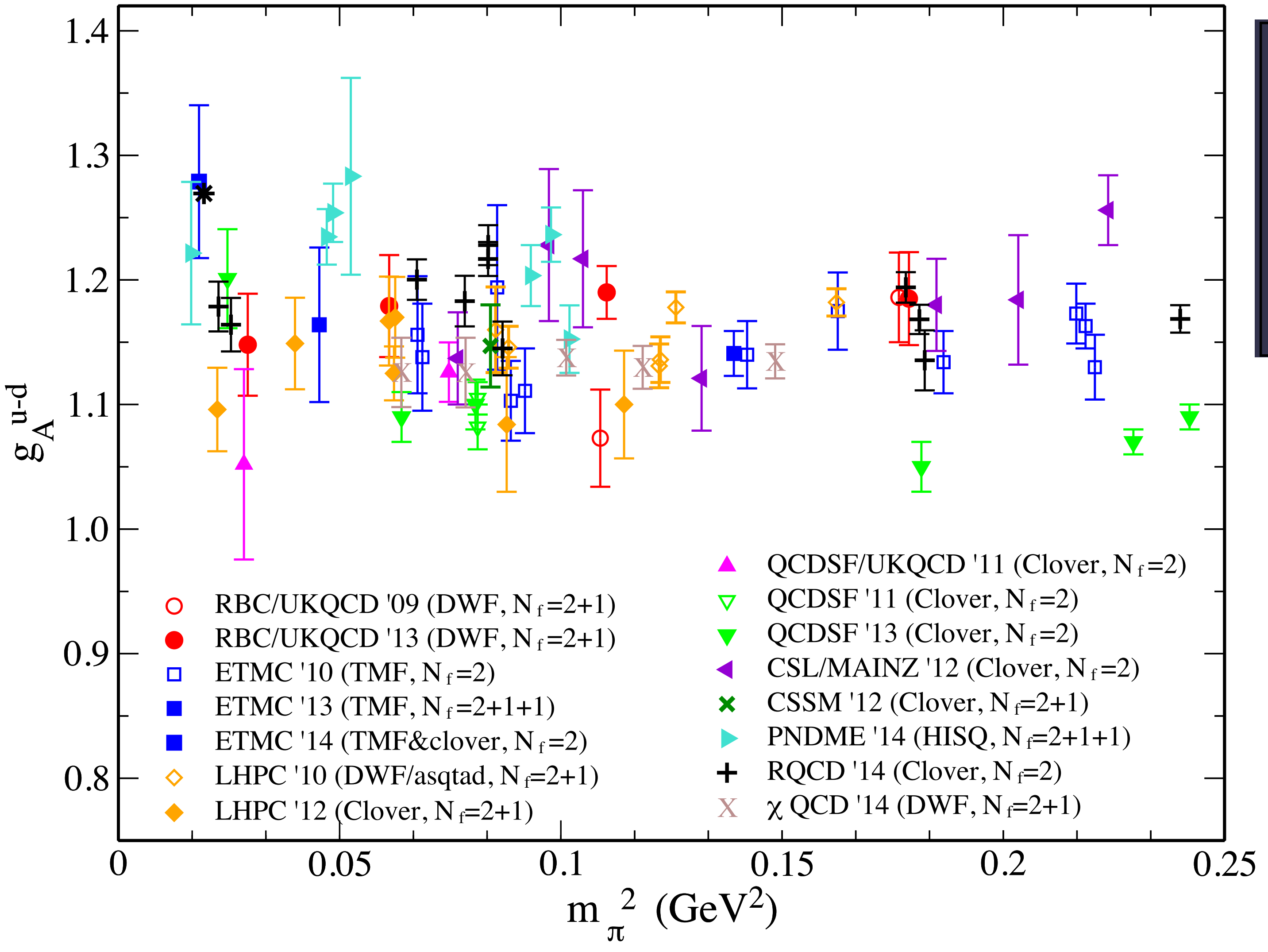


see talk by E. Shintani @ ICHEP (Lattice session, Saturday)

Hadron structure

review by M. Constantinou @ Lattice 2014

Nucleon axial charge g_A



- ★ Lattice data from 'plateau' methods
- ★ Latest achievement: lattice results at physical m_π
- ★ No necessity of chiral extrapolation
- ★ Different strategies for addressing systematic uncertainties

Finite volume effects are an important source of systematic error

Hadronic interactions

review by T. Yamazaki @ Lattice 2014

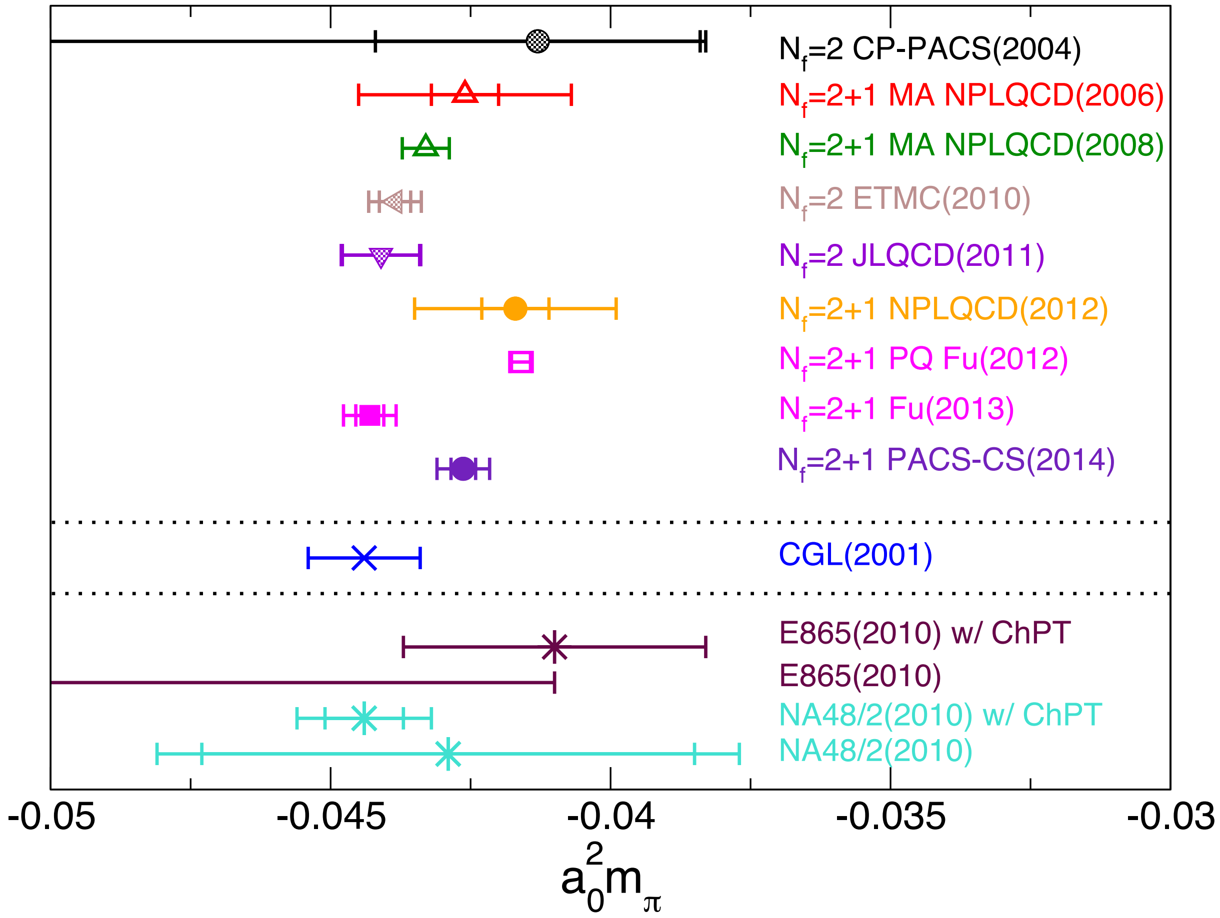
Scattering length a_0^I

$$a_0 = \lim_{p \to 0} \frac{\tan \delta(p)}{p}$$

$I = 2$ $\pi\pi$ Simplest scattering system

Comparison of dynamical calculations at physical m_π

$I = 2$ $\pi\pi$ a_0^2 and $I = 1/2$ $K\pi$ $a_0^{1/2}$



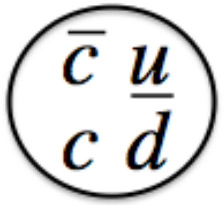
Most (but not all) results displayed include systematic error budgets

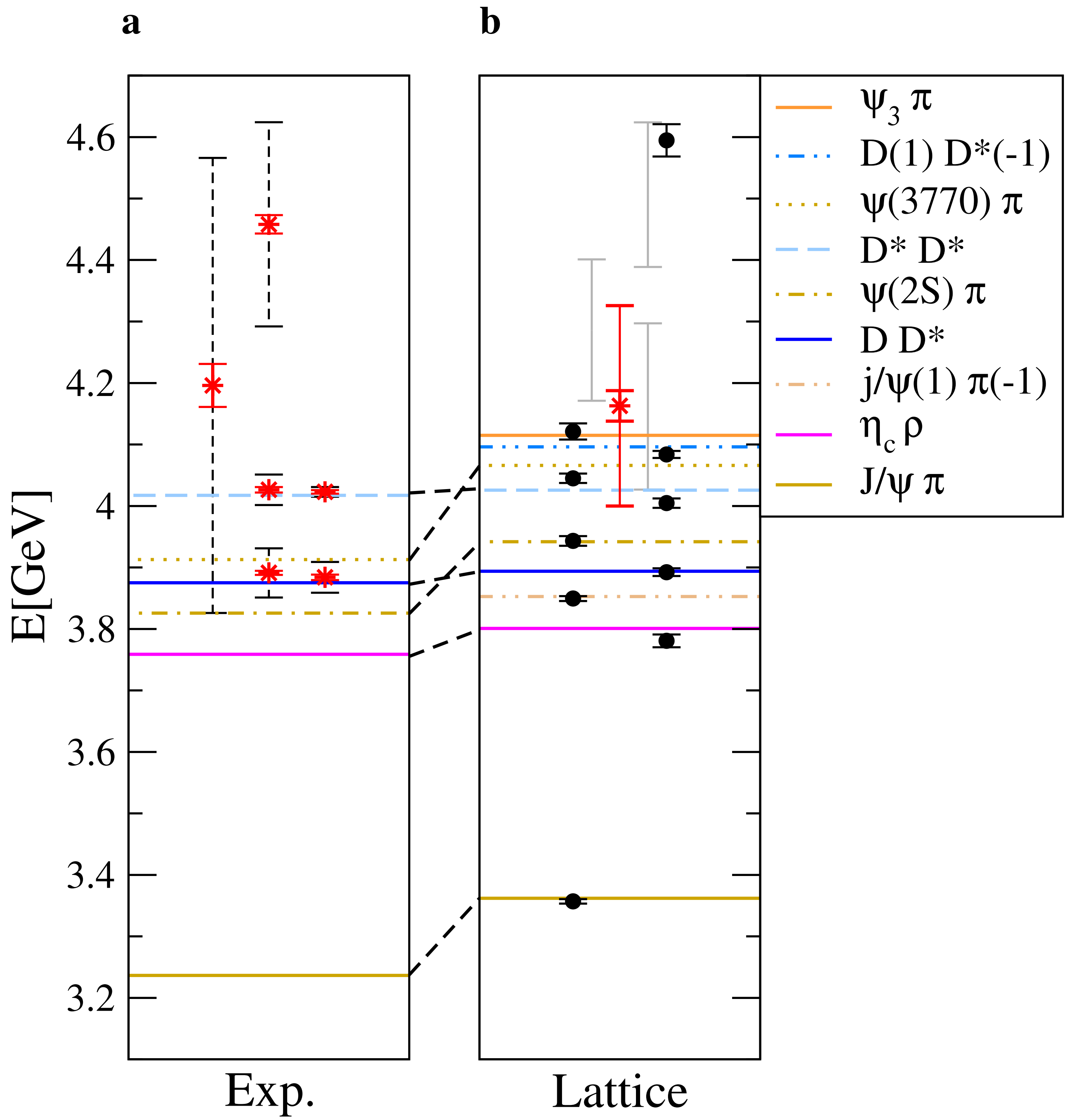
most are consistent with each other

Resonances

review by S. Prelovsek @ Lattice 2014

Evidence for Z_c^+ from lattice: $I^G=1^+$, $J^{PC}=1^{+-}$





- Black circles: two-meson states
- Red asterix: candidate for Z_c^+

  (the smaller error is statistical,

  the larger corresponds to systematics)

- 9 two meson states below 4.3 GeV
- an additional state found
- since we exhausted all two meson-states below 4.3 GeV, it is a candidate for an exotic Z_c^+.