

Determinations of γ

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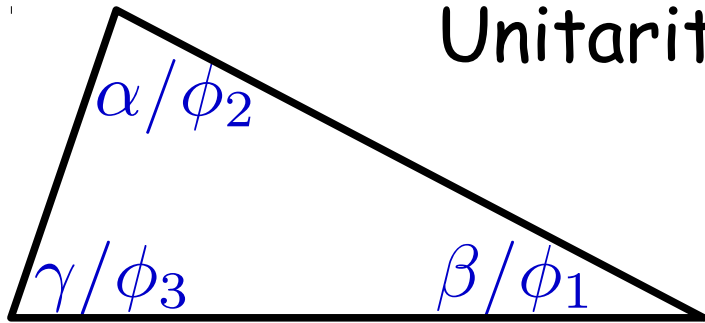


- * γ and the unitarity triangle analysis
- * brief review of the $B \rightarrow D^{(*)}K^{(*)}$ methods
- * a determination based on U-spin symmetry
- * conclusions

Beauty 2014

The 15th International Conference on B-Physics at
the University of Edinburgh, 14th – 18th July 2014

Unitarity Triangle



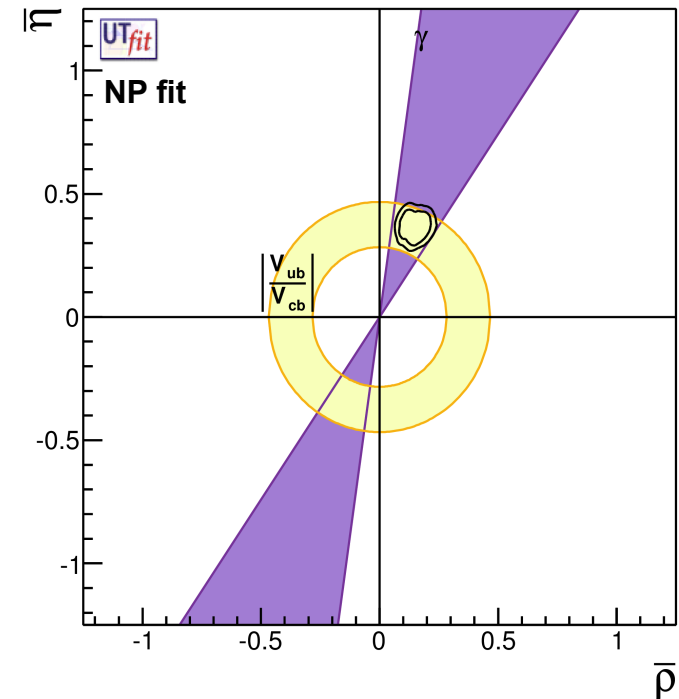
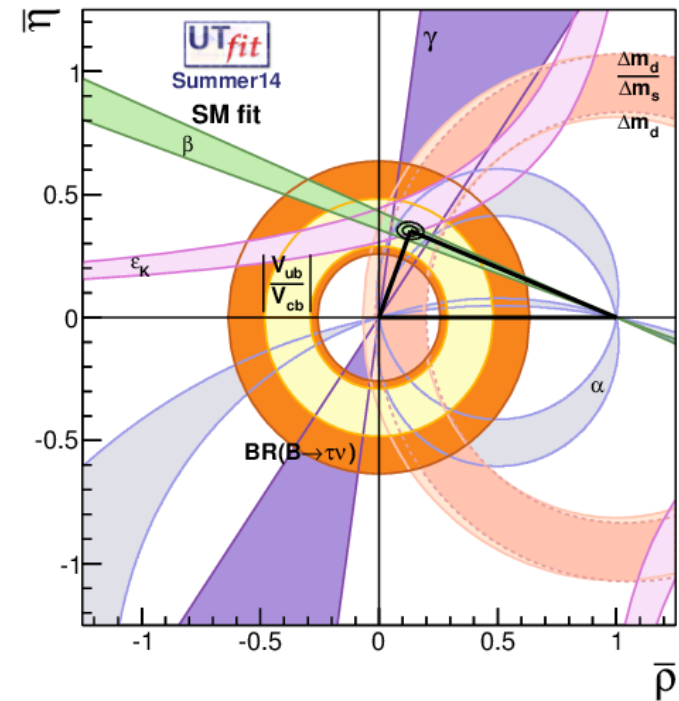
$$\gamma = \arg \left(-\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}} \right)$$

In the CKM phase convention, $\gamma \sim \delta_{CP}$

The SM fit predicts

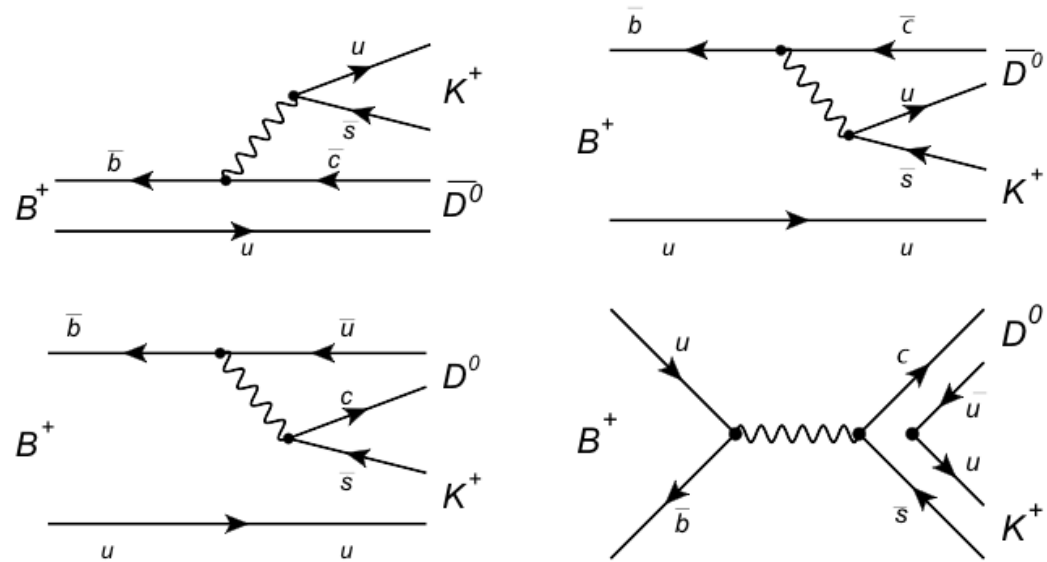
$$\gamma = (69.5 \pm 3.9)^\circ$$

In the presence of NP in the $\Delta F=2$ amplitudes, γ is a crucial input to determine the CKM parameters



Determinations of γ using $B \rightarrow D^{(*)} K^{(*)}$

- no penguins
- the interference between $b \rightarrow c\bar{u}s$ and $b \rightarrow u\bar{c}s$ amplitudes $\sim V_{ub}^* V_{cs} V_{cb} V_{us}^*$



They can interfere in different ways

GLW: $B^+ \rightarrow D_{\pm}^0 K^+ \rightarrow K^+ K^-, \pi^+ \pi^-$ (CP₊), $K_S \pi^0 / \omega / \phi$ (CP₋)

ADS: $B^+ \rightarrow D^0 / \bar{D}^0 K^+ \rightarrow f(\text{CA/DCS}) K^+$
e.g. $f = K^- \pi^+, K \pi \pi^0$

GGSZ: $B^+ \rightarrow \bar{D}^{(-)} K^+ \rightarrow (K_S \pi^+ \pi^-) K^+$

Basic formulae

$$\frac{A(b \rightarrow u)}{A(b \rightarrow c)} \simeq r_B e^{i\delta_B} e^{-i\gamma}$$

GLW: CP-eigenstates

$$R_{\text{CP}\pm} = \frac{\Gamma(B^+ \rightarrow D_{\pm}^0 K^+) + \Gamma(B^- \rightarrow D_{\pm}^0 K^-)}{\Gamma(B^+ \rightarrow D^0 K^+) + \Gamma(B^- \rightarrow \bar{D}^0 K^-)} = 1 + r_B^2 \pm 2r_B \cos \gamma \cos \delta_B$$

$$A_{\text{CP}\pm} = \frac{\Gamma(B^+ \rightarrow D_{\pm}^0 K^+) - \Gamma(B^- \rightarrow D_{\pm}^0 K^-)}{\Gamma(B^+ \rightarrow D_{\pm}^0 K^+) + \Gamma(B^- \rightarrow D_{\pm}^0 K^-)} = \frac{\pm 2r_B \sin \gamma \sin \delta_B}{R_{\text{CP}\pm}}$$

ADS: CA/DCS decays

$$R_{\text{ADS}} = \frac{\Gamma(B^+ \rightarrow [\bar{f}]_{D^0} K^+) + \Gamma(B^- \rightarrow [f]_{D^0} K^-)}{\Gamma(B^+ \rightarrow [f]_{D^0} K^+) + \Gamma(B^- \rightarrow [\bar{f}]_{D^0} K^-)} = r_B^2 + r_D^2 + 2r_B r_D \cos \gamma \cos(\delta_B + \delta_D)$$

$$A_{\text{ADS}} = \frac{\Gamma(B^+ \rightarrow [\bar{f}]_{D^0} K^+) - \Gamma(B^- \rightarrow [f]_{D^0} K^-)}{\Gamma(B^+ \rightarrow [f]_{D^0} K^+) + \Gamma(B^- \rightarrow [\bar{f}]_{D^0} K^-)} = \frac{2r_B r_D \sin \gamma \sin(\delta_B + \delta_D)}{R_{\text{ADS}}}$$

$r_d \exp(i\delta_D) = A(\text{DCS})/A(\text{CA})$ are external inputs taken from D decay studies

GGSZ: Dalitz analysis

$$M_+ = f(m_-^2, m_+^2) + r_B e^{i(\delta_B - \gamma)} f(m_+^2, m_-^2)$$

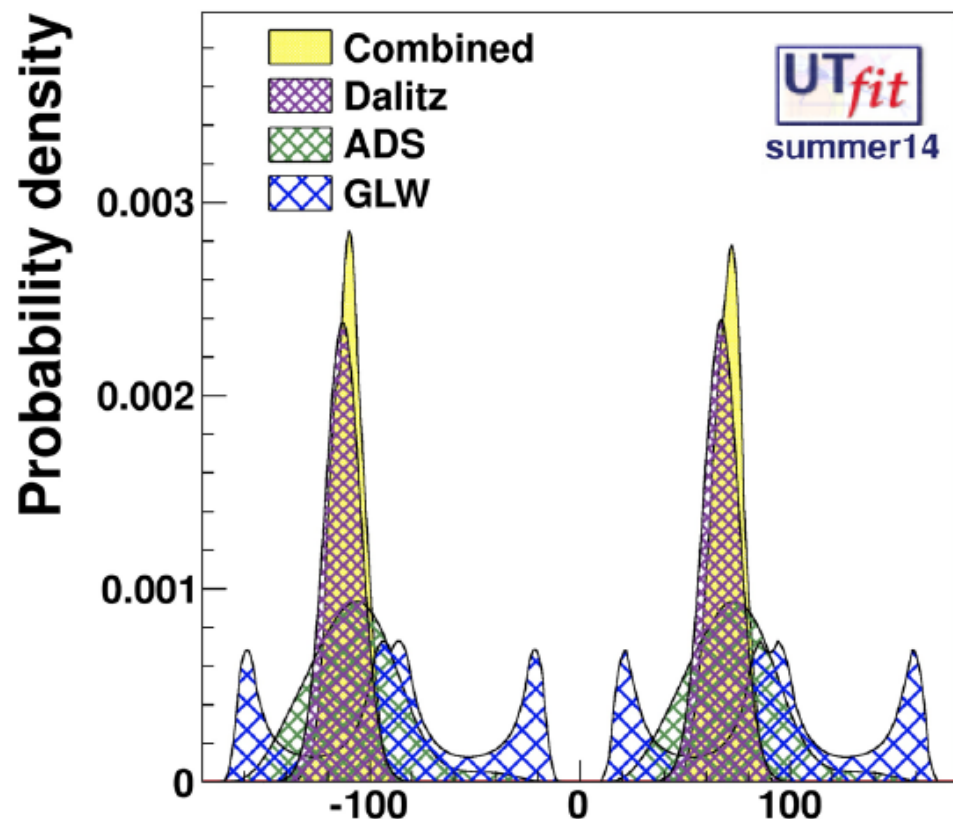
$$M_- = f(m_+^2, m_-^2) + r_B e^{i(\delta_B + \gamma)} f(m_-^2, m_+^2)$$

see next talk ...

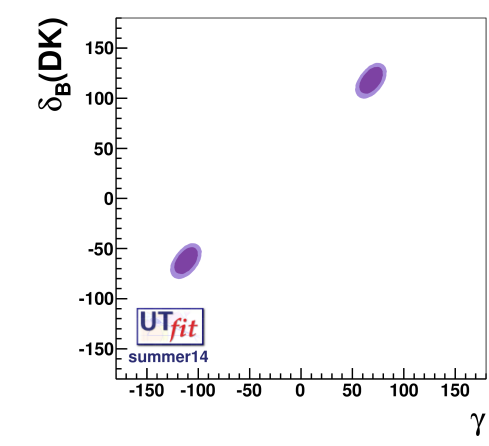
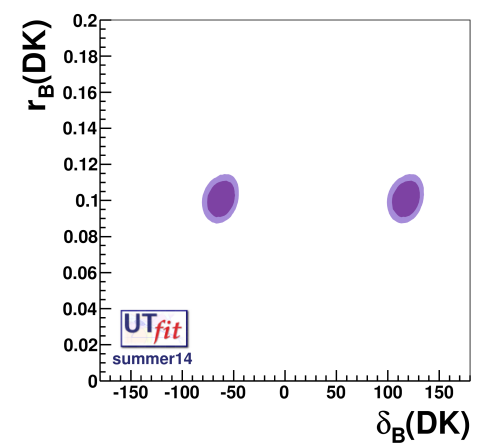
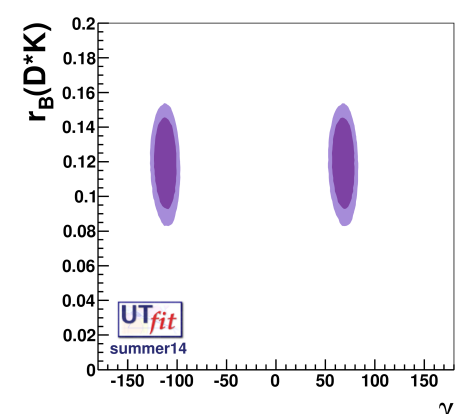
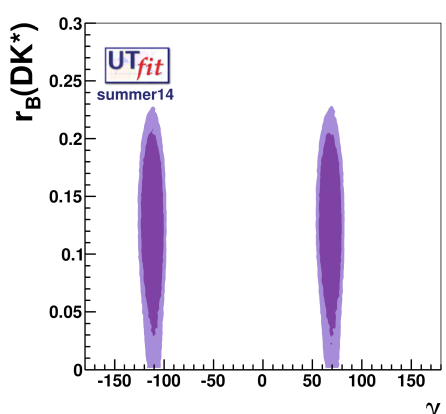
Results

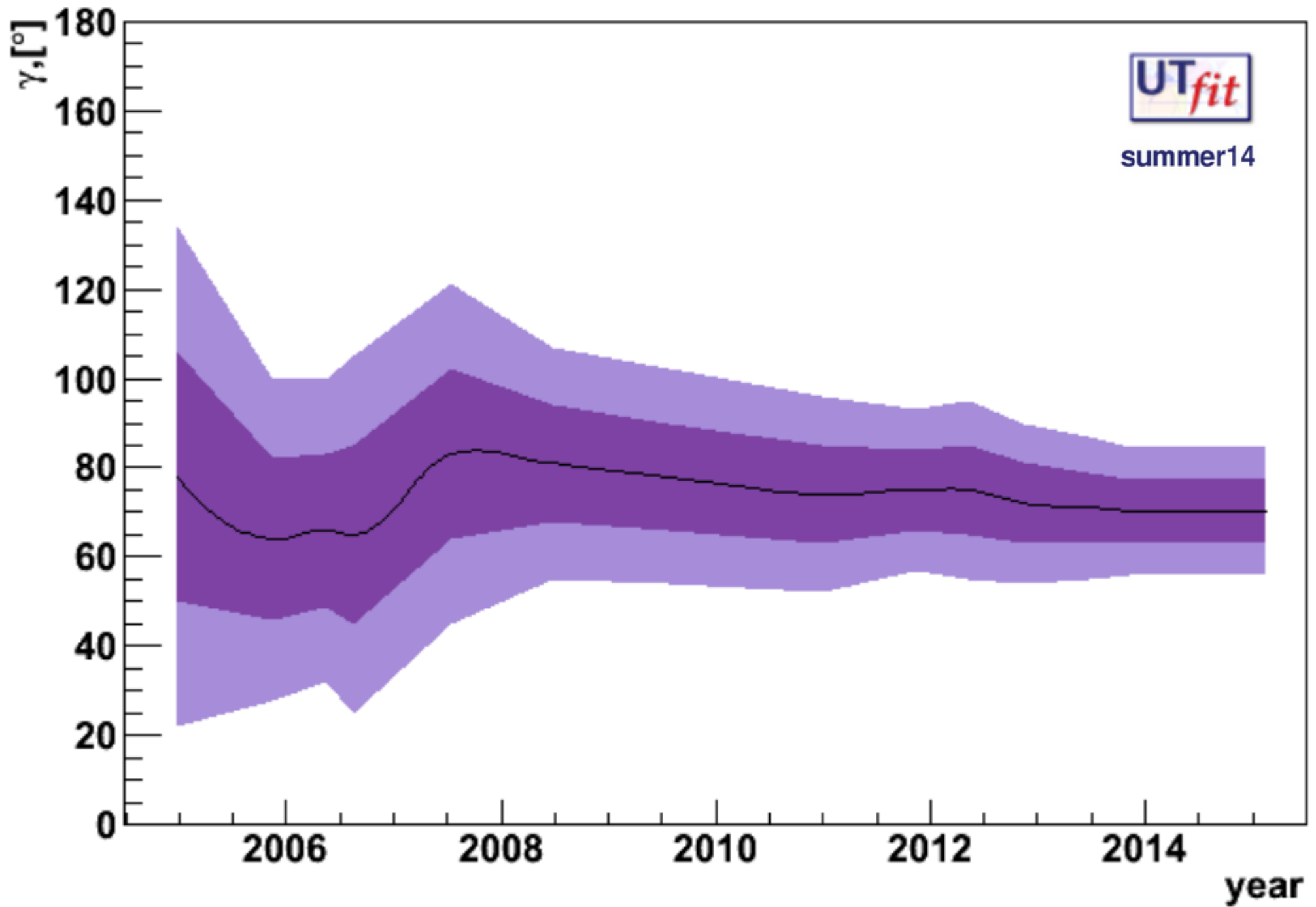
$r_B(\text{DK})$	0.1009 ± 0.0066
$\delta_B(\text{DK})$	$(-62.0 \pm 8.8)^\circ \cup (117.9 \pm 8.8)^\circ$
$r_B(\text{DK}^*)$	0.126 ± 0.056
$\delta_B(\text{DK}^*)$	$(-53 \pm 34)^\circ \cup (126 \pm 34)^\circ$
$r_B(\text{D}^*\text{K})$	0.120 ± 0.018
$\delta_B(\text{D}^*\text{K})$	$(-49 \pm 13)^\circ \cup (131 \pm 13)^\circ$
$r_{B_0}(\text{DK}^*)$	0.25 ± 0.06
$\delta_{B_0}(\text{DK}^*)$	$(-53 \pm 46)^\circ \cup (126 \pm 47)^\circ$

$$+ \delta_D = (18 \pm 14)^\circ$$



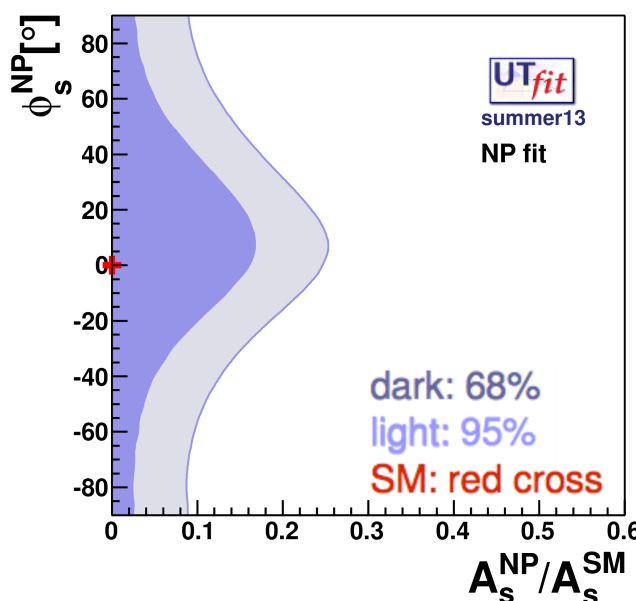
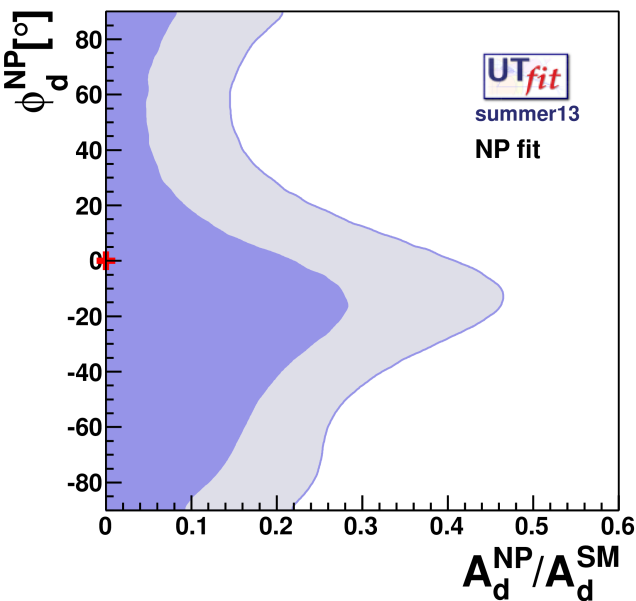
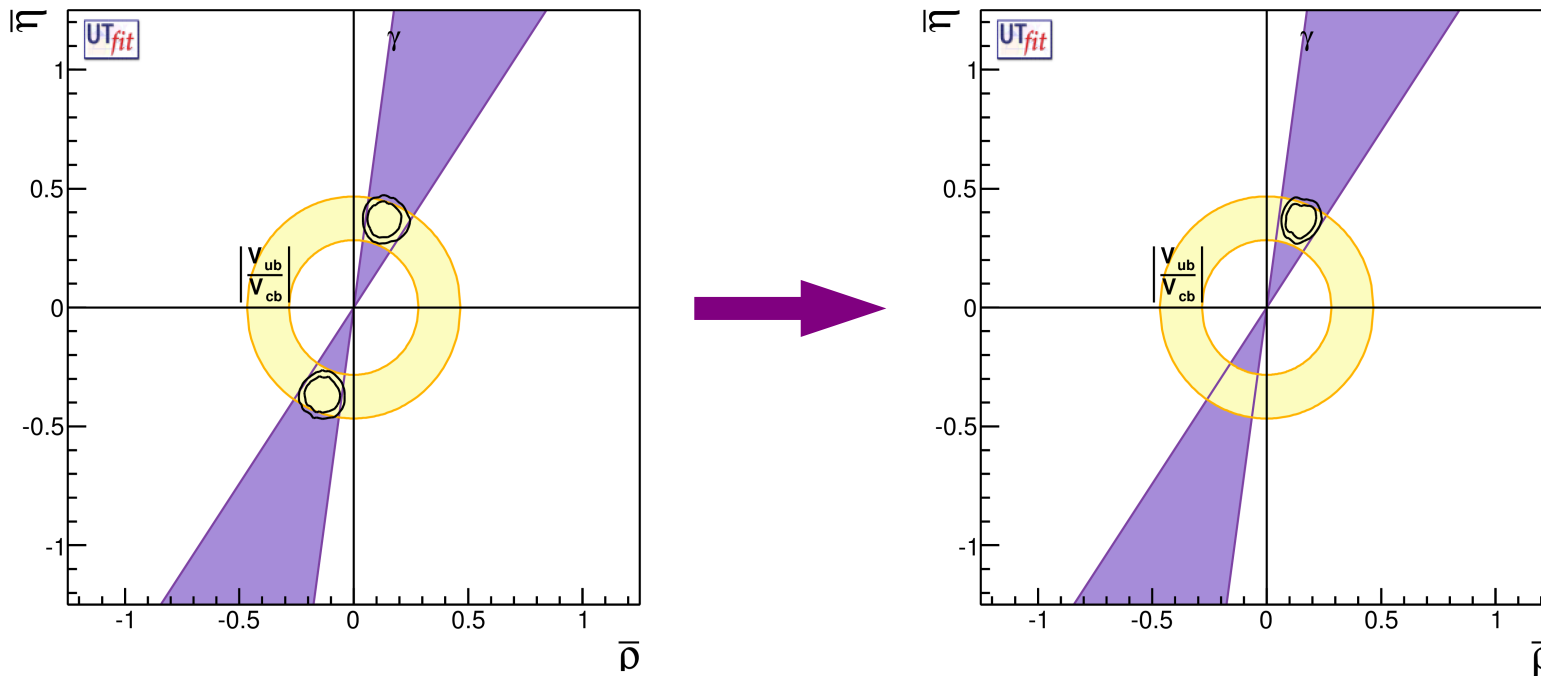
$$\gamma = \begin{cases} (-111.6 \pm 7.4)^\circ & \gamma [^\circ] \\ (68.3 \pm 7.5)^\circ \end{cases}$$





summer14

Unitarity clock and NP UT analysis



Apex coordinates

$$\bar{\rho} = 0.159 \pm 0.045$$

$$\bar{\eta} = 0.363 \pm 0.049$$

$$\gamma = (67.7 \pm 6.0)^\circ$$

NP in $\Delta F=2$ amplitudes

$$A = A^{SM} + A^{NP}$$

Determinations of γ based on U-spin: the Fleischer method

R. Fleischer, hep-ph/9903456

$$A(B_d \rightarrow \pi^+ \pi^-) = C(e^{i\gamma} - de^{i\theta})$$

$$A(B_s \rightarrow K^+ K^-) = C' \frac{\lambda}{1-\lambda^2/2} (e^{i\gamma} + \frac{1-\lambda^2}{\lambda^2} d' e^{i\theta'})$$

In the U-spin limit: $C = C'$, $d = d'$, $\theta = \theta'$ and γ

3 observables for each channel:

$$BR(B \rightarrow MM) = F(B) \frac{|A(B \rightarrow MM)|^2 + |A(\bar{B} \rightarrow MM)|^2}{2},$$

$$\mathcal{A}_{CP} = -\mathcal{C} = \frac{|A(\bar{B} \rightarrow MM)|^2 - |A(B \rightarrow MM)|^2}{|A(\bar{B} \rightarrow MM)|^2 + |A(B \rightarrow MM)|^2}, \quad \mathcal{S} = \frac{2\text{Im} \left(e^{-i\phi_M(B)} \frac{A(\bar{B} \rightarrow MM)}{A(B \rightarrow MM)} \right)}{1 + \left| \frac{A(\bar{B} \rightarrow MM)}{A(B \rightarrow MM)} \right|^2}$$

- all the parameters can be determined up to ambiguities
- a better determination of γ is obtained by taking the value of $\phi_M(B_{d/s})$ extracted from $b \rightarrow c\bar{c}s$ decays

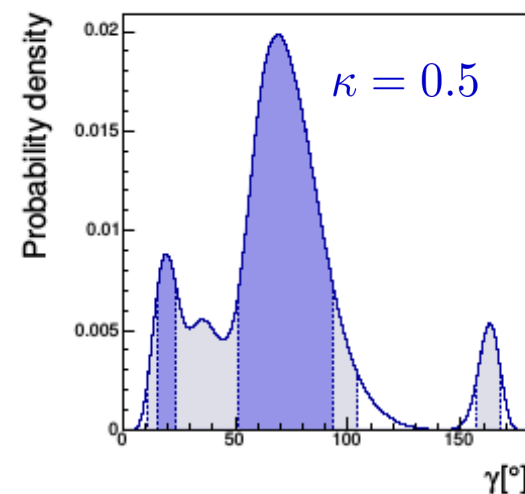
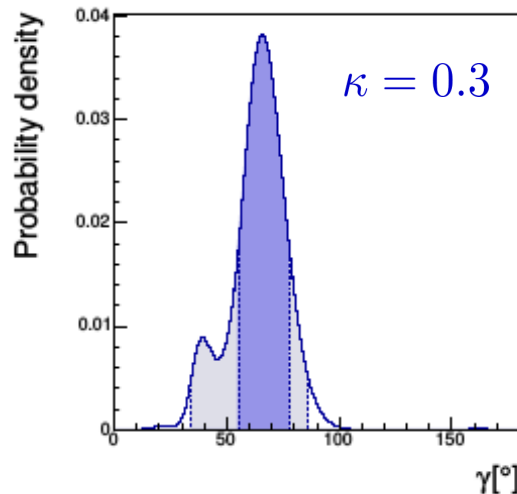
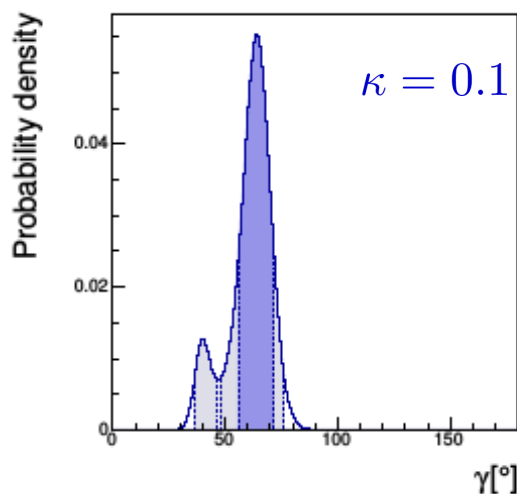
U-spin breaking

MC, Franco, Mishima, Silvestrini, arXiv:1205.4948

U-spin breaking: $C' = r_{\text{fact}} r_C C$, $d' e^{i\theta'} = d e^{i\theta} + r_d d e^{i r_\theta}$

factorizable: $r_{\text{fact}} = |C'/C|_{\text{fact}} = 1.46 \pm 0.15$

non-factorizable: $r_C \in [1 - \kappa, 1 + \kappa]$, $r_d \in [0, \kappa]$, $r_\theta \in [-\pi, \pi]$



Channel	BR $\times 10^6$	$\mathcal{S}(\%)$	$\mathcal{A}_{\text{CP}} (= -\mathcal{C})(\%)$	corr.
$B_d \rightarrow \pi^+ \pi^-$	5.11 ± 0.22	-65 ± 7	38 ± 6	0.08
$B_d \rightarrow \pi^+ \pi^-$	–	$-56 \pm 17 \pm 3$	$11 \pm 21 \pm 3$	-0.34
$B_d \rightarrow \pi^0 \pi^0$	1.91 ± 0.23	–	43 ± 24	–
$B^+ \rightarrow \pi^+ \pi^0$	5.48 ± 0.35	–	2.6 ± 3.9	–
$B_s \rightarrow K^+ K^-$	25.4 ± 3.7	$17 \pm 18 \pm 5$	$2 \pm 18 \pm 4$	-0.1

The determination of γ with the Fleischer method deteriorates for large U-spin breaking

R. Fleischer, arXiv:0705.1121

GL analysis + β from $b \rightarrow c\bar{c}s$

In the isospin limit:

$$A(B_d \rightarrow \pi^+ \pi^-) = C(e^{i\gamma} - de^{i\theta}),$$

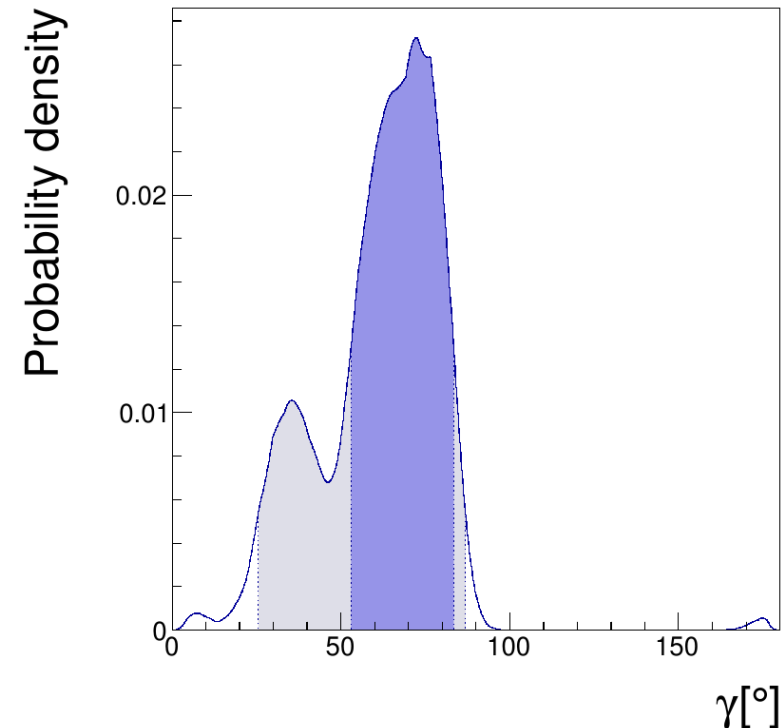
$$A(B_d \rightarrow \pi^0 \pi^0) = \frac{C}{\sqrt{2}}(Te^{i\theta_T} e^{i\gamma} + de^{i\theta}),$$

$$A(B^+ \rightarrow \pi^+ \pi^0) = \frac{A(B_d \rightarrow \pi^+ \pi^-)}{\sqrt{2}} + A(B_d \rightarrow \pi^0 \pi^0)$$

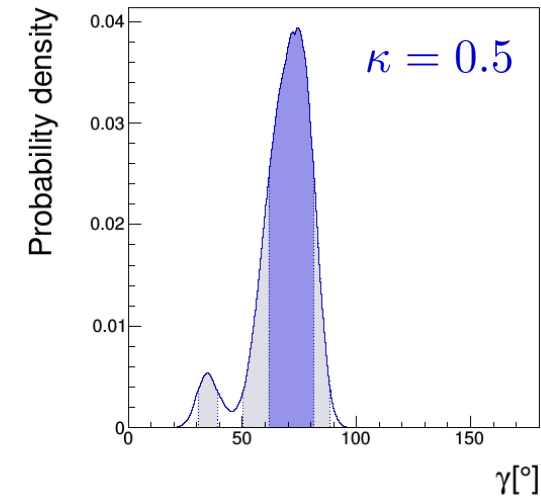
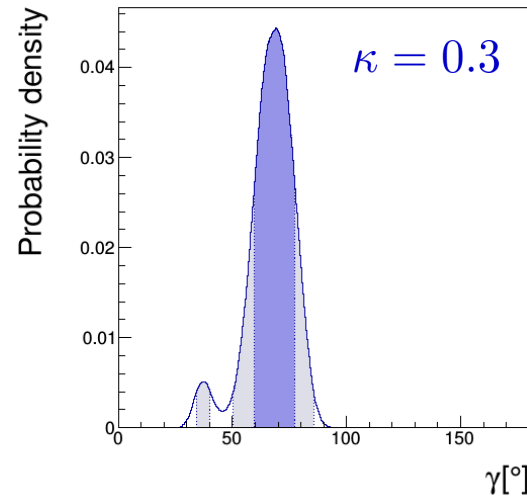
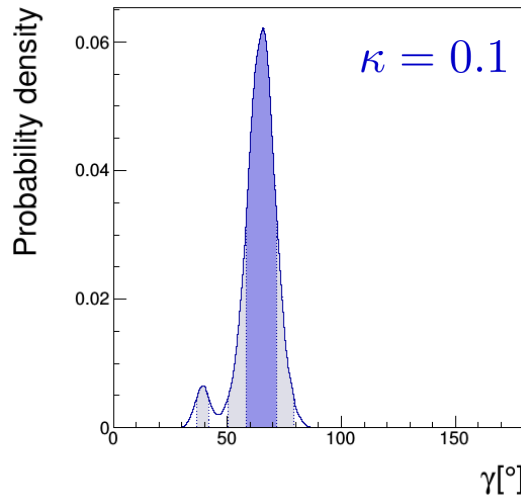
Taking β as an input,
the GL analysis determines γ

$$\gamma = (68 \pm 15)^\circ [25^\circ, 87^\circ]$$

Channel	BR $\times 10^6$	$\mathcal{S}(\%)$	$\mathcal{A}_{CP}(= -\mathcal{C})(\%)$	corr.
$B_d \rightarrow \pi^+ \pi^-$	5.11 ± 0.22	-65 ± 7	38 ± 6	0.08
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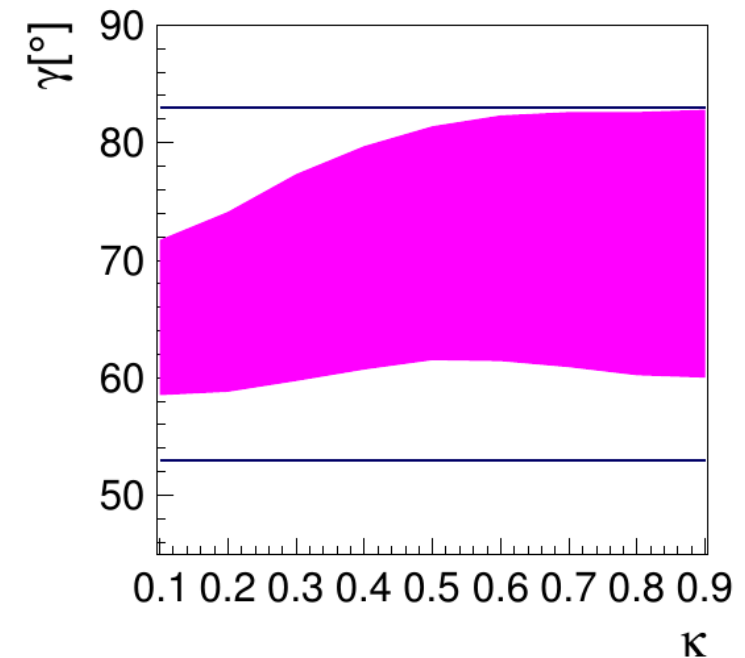


Other determinations of γ : combining the GL and Fleischer analyses



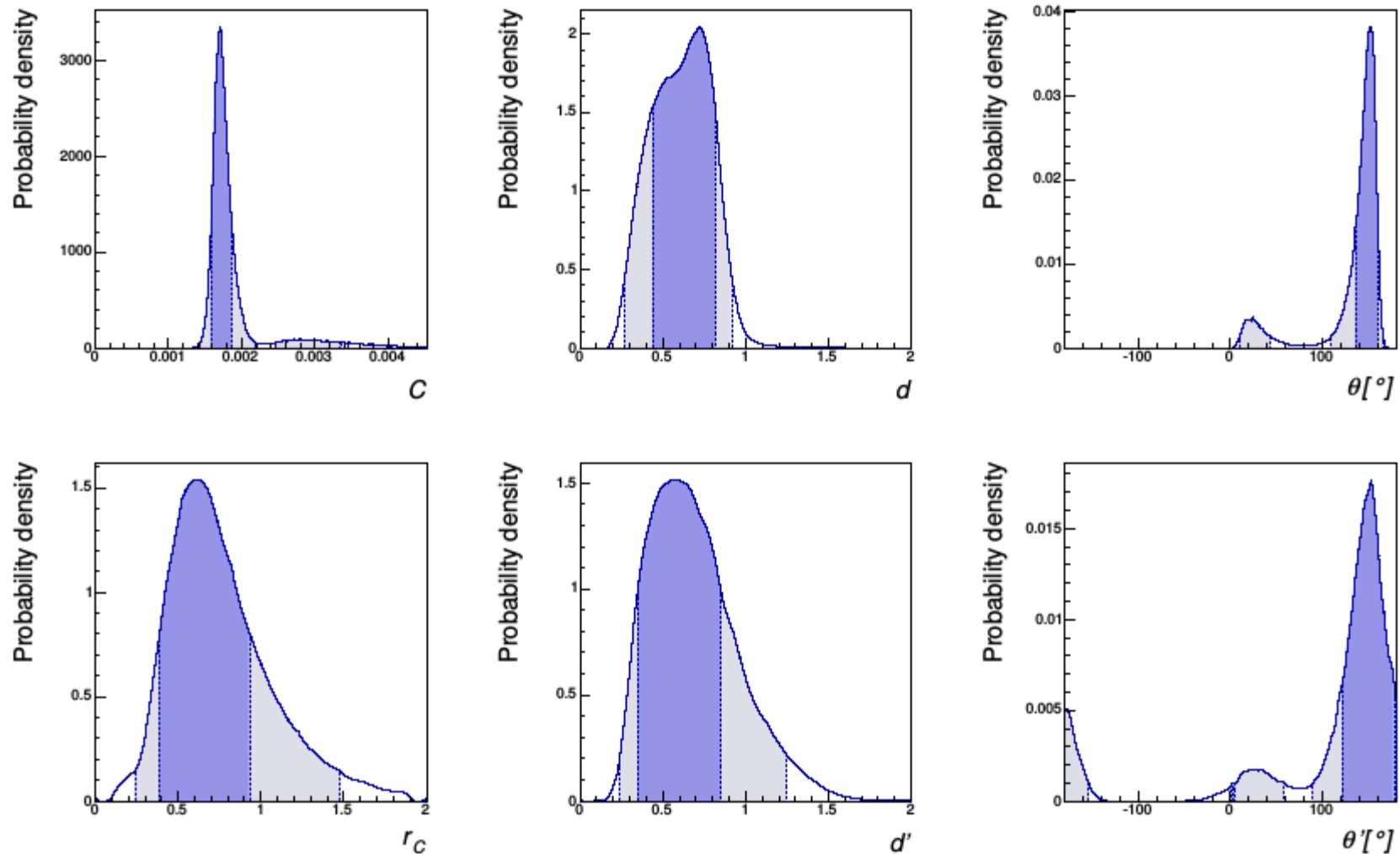
The combined analysis is:

- comparable to the Fleischer method for constraining γ but more stable w.r.t U-spin breaking
- more effective than the GL method for constraining γ



Hadronic Parameters

posteriors, combined analysis, $\kappa = 0.9$



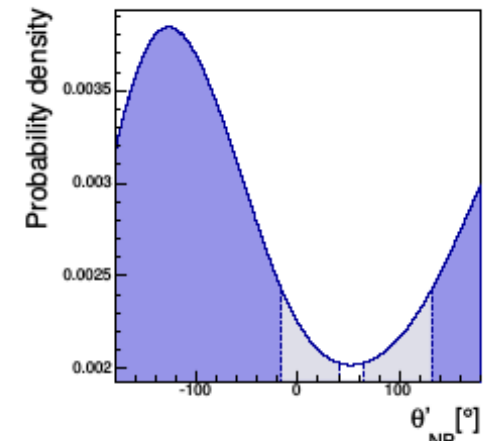
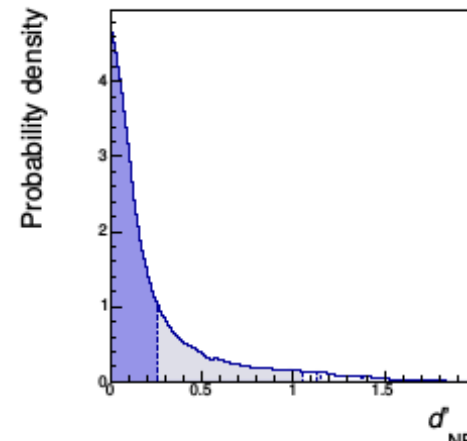
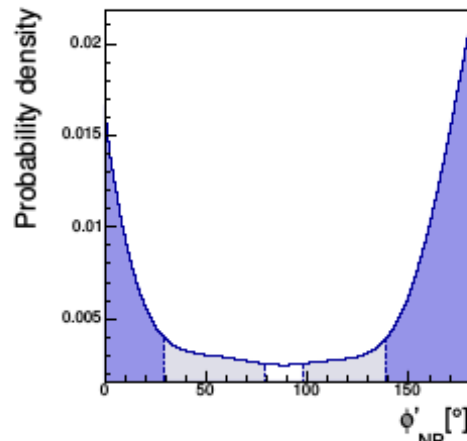
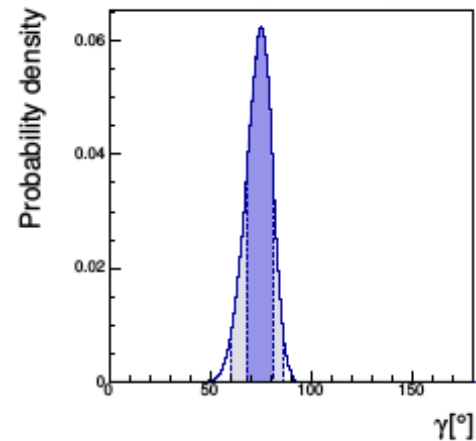
No sign of huge U-spin breaking, but $\kappa \sim 0.5-0.7$ possible

About NP contributions

- NP in the mixing phases is constrained by the UT fit
- NP in $b \rightarrow d$ penguins can jeopardize the analysis, if it introduces new weak phases or breaks isospin
- NP in $b \rightarrow s$ penguins can be accommodated

$$A(B_s \rightarrow K^+ K^-) = C' \frac{\lambda}{1 - \lambda^2/2} (e^{i\gamma} + \frac{1 - \lambda^2}{\lambda^2} (d' e^{i\theta'} + e^{i\phi'_{NP}} d'_{NP} e^{i\theta'_{NP}}))$$

$$\gamma = (74 \pm 6)^\circ$$



Determination of β_s

The combined method can be also used to determine β_s . As a proof of concept, we take as input $\gamma = (69.7 \pm 3.1)^\circ$, instead of β_s , and find for $\kappa = 0.5$:

- $\beta_s = (6 \pm 14)^\circ$ with the inputs used in the analysis
- $\beta_s = (2.6 \pm 2.7)^\circ$ assuming an error of ± 0.02 for CPV observables (S, A_{CP}) in $B_{d/s} \rightarrow \pi^+\pi^-/K^+K^-$

The combined method allows for a competitive determination of β_s at the SM level by providing sufficient control over the subleading amplitude. This requirement applies to any method, including $B_s \rightarrow J/\psi \phi$

Perspectives of this analysis

Data need to be updated (in progress)

The combined method can be more effectively implemented in the framework of the UT analysis (in progress)

The original parametrization of U-spin breaking effects puts a double breaking in the parameter d/d' (alternatives under consideration)

For very large values of the U-spin breaking, additional solutions may appear (under study)

Conclusions

Extracting γ from $B \rightarrow D^{(*)}K^{(*)}$ works and is not limited by theory uncertainties

Methods based on flavour symmetries can provide additional constraints

The Fleischer and the Gronau-London methods can be combined to reduce the sensitivity to U-spin breaking effects

The same method can provide a competitive determination of β_s

Backup

Determinations of γ from $SU(3)$: early proposals with $B \rightarrow K \pi$

Amplitude analysis of

$$BR(B^+ \rightarrow K^0 \pi^+), BR(B^+ \rightarrow K^+ \pi^0), BR(B^+ \rightarrow \pi^0 \pi^+)$$

M. Gronau, J.L. Rosner and D. London, Phys. Rev. Lett. 73 (1994) 21

Issues:

- dynamical assumptions
- electroweak penguins
- rescattering effects
- $SU(3)$ symmetry breaking

N.G. Deshpande and X.-G. He, Phys. Rev. Lett. 74 (1995) 26

M. Neubert and J.L. Rosner, Phys. Rev. Lett. 81 (1998) 5076

A.J. Buras and R. Fleischer, Eur.Phys.J. C11 (1999) 93