Beauty 2014, Edinburgh, July 14-18 2014

Lepton Flavor Violation

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EST. 1943

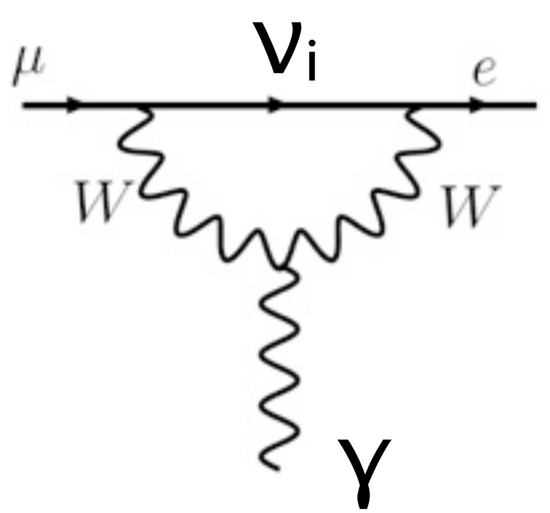
Outline

- Introduction: LFV and new physics
- Effective theory framework for LFV phenomenology
- The reach and model-discriminating power of
    - muon decays
    - tau decays

LFV and new physics

LFV and BSM physics

- ν oscillations ⇒ $L_{e,\mu,\tau}$ not conserved
- In SM + massive "active" ν, effective CLFV vertices are tiny (GIM)



$$Br(\mu \to e\gamma) = \frac{3\alpha}{32\pi}\left|\sum_{i=2,3} U^*_{\mu i} U_{ei} \frac{\Delta m^2_{1i}}{M_W^2}\right|^2 < 10^{-54}$$

Petcov '77, Marciano-Sanda '77 ....

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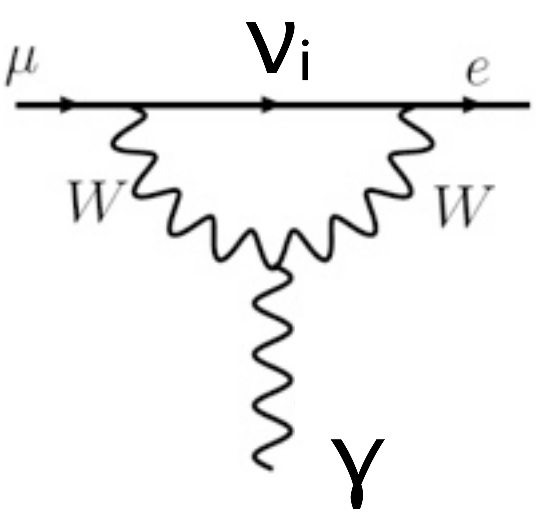

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Petcov '77, Marciano-Sanda '77 ....

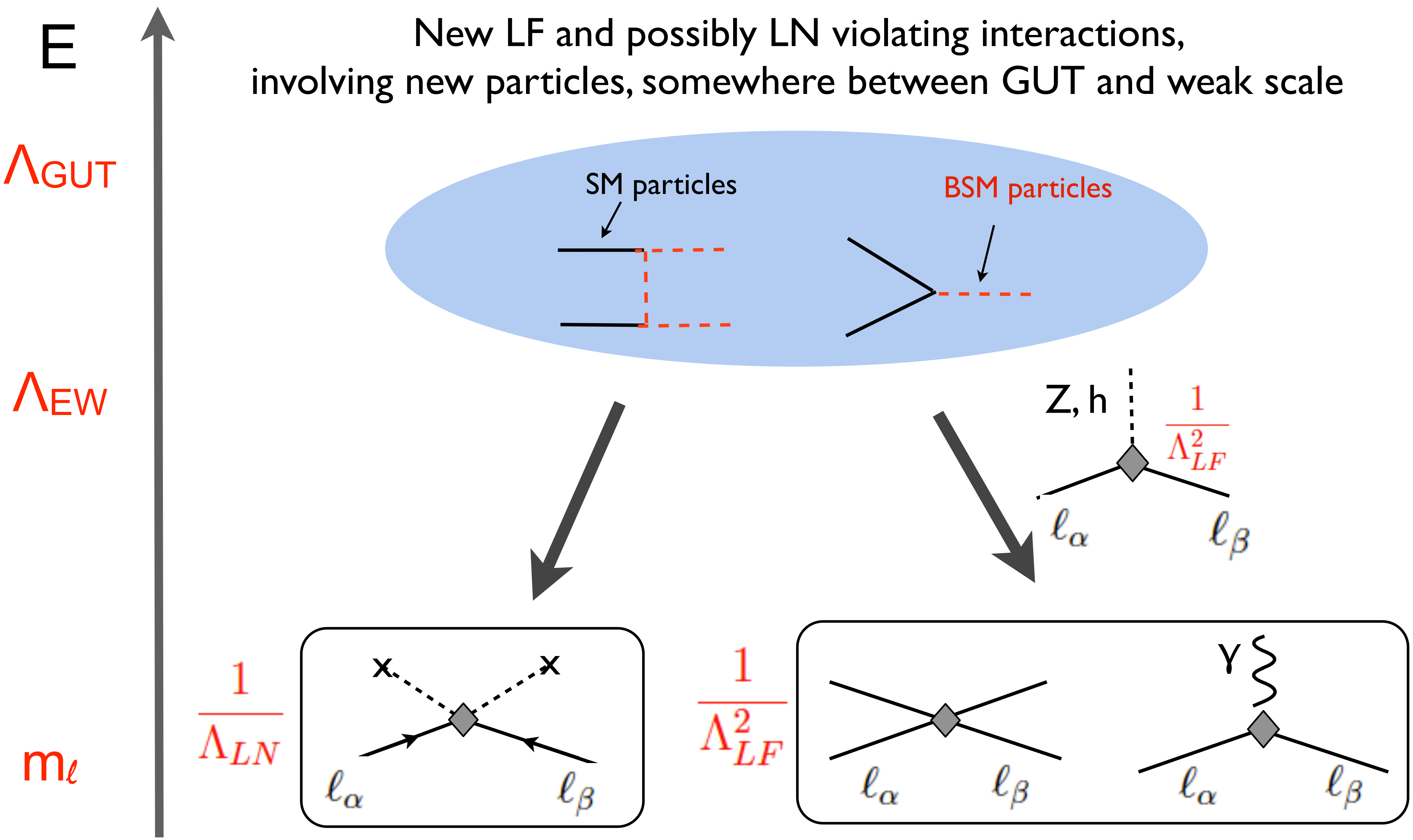
- CLFV processes are an extremely clean probe of "BνSM" physics

$$\mathcal{L}_{\nu \mathrm{SM}} = \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_{\nu-\mathrm{mass}}$$

← dim-4 Dirac or dim5 Majorana

The underlying picture

New LF and possibly LN violating interactions,
involving new particles, somewhere between GUT and weak scale

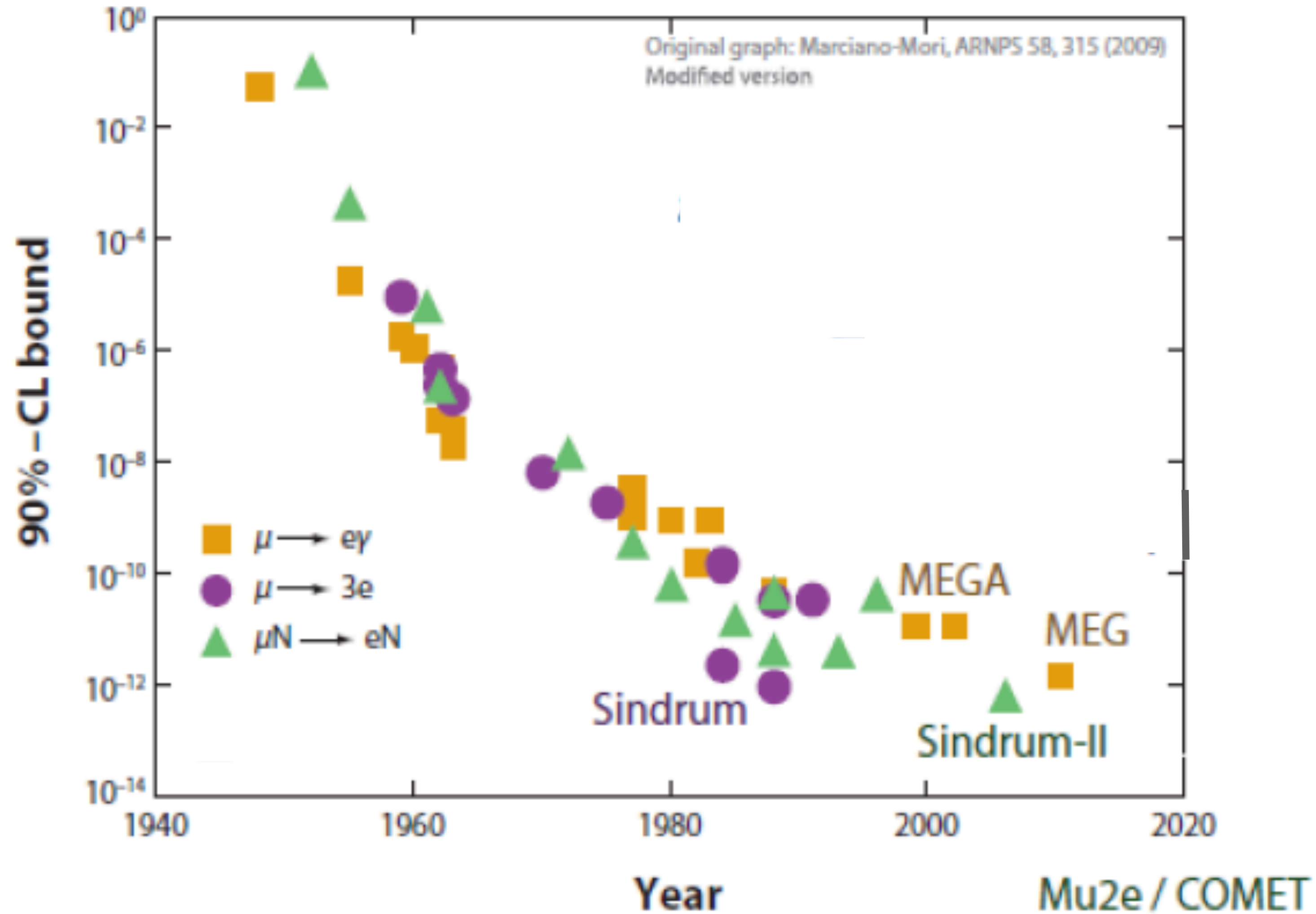


Each scenario generates specific pattern of low-energy operators,
controlling ν mass (dim5) and LFV processes (dim6).

We can probe the underlying physics through a combination of low-energy and collider searches

CLFV processes

- Muon processes : $\mu \to e\gamma, \quad \mu \to e\bar{e}e, \quad \mu\,(A,Z) \to e\,(A,Z)$



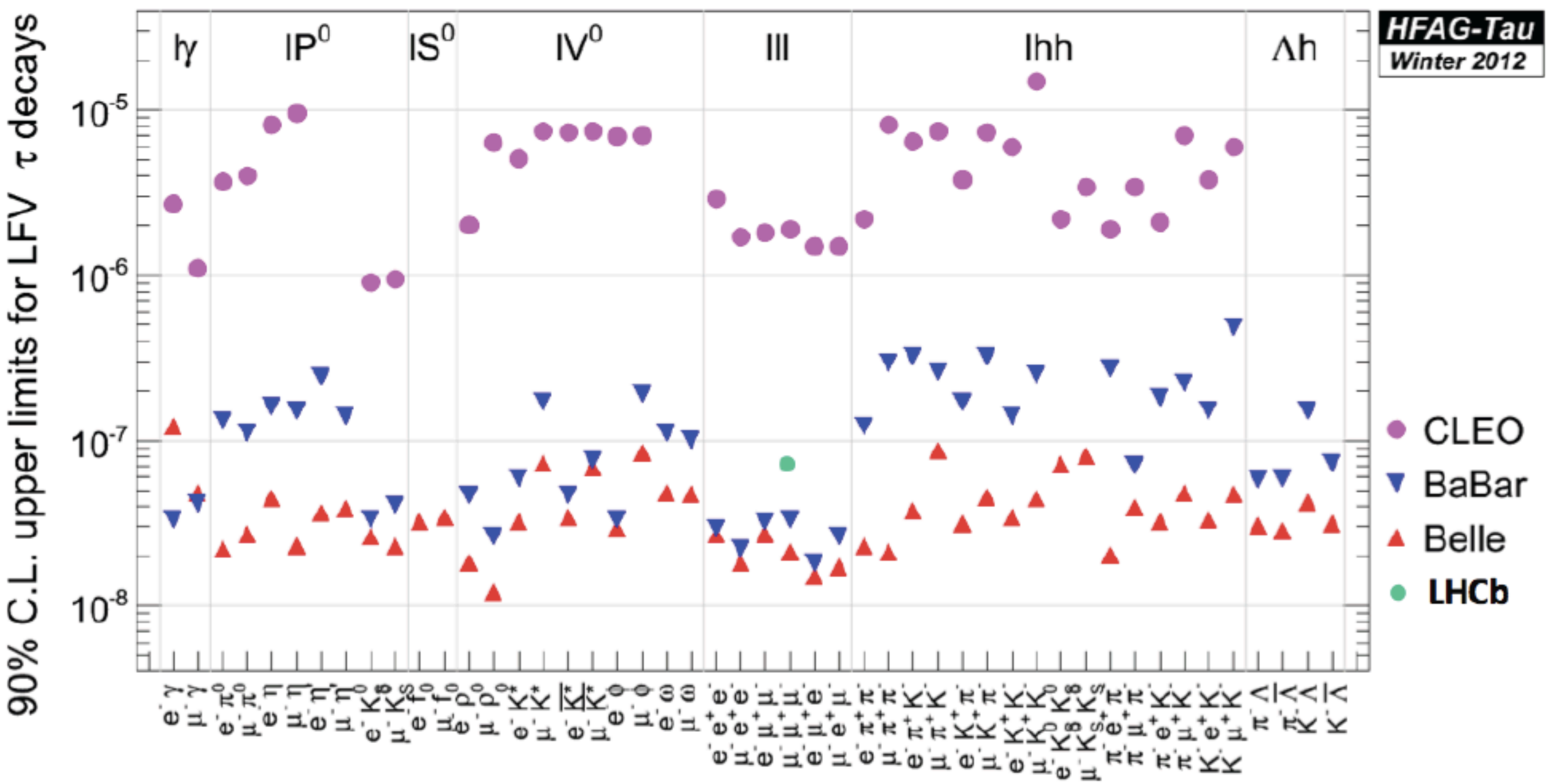
$$B_{\mu\to e\gamma} \;<\; 5.7\times 10^{-13} \quad \longrightarrow \quad 10^{-/14} \text{ (MEG at PSI)}$$

$$B_{\mu\to 3e} \;<\; 1.0\times 10^{-12} \quad \longrightarrow \quad 10^{-15/16} \text{ (PSI)}$$

$$B^{Ti}_{\mu-e} \;<\; 4.3\times 10^{-12} \quad \longrightarrow \quad 10^{-16/17\,\to\,-18} \text{ (Mu2e, COMET)}$$

CLFV processes

- Tau decays: $\tau \to \ell\gamma, \quad \tau \to \ell_\alpha \bar{\ell}_\beta \ell_\beta, \; \tau \to \ell Y \quad Y = P, S, V, P\bar{P}, \ldots$



10^{-9} (or better?) sensitivities at Belle-II, LHCb

CLFV processes

- Great "discovery" tools
  - Observation near current limits $\Rightarrow$ BSM physics
- Great "model-discriminating" tools
  - What type of "mediator"?

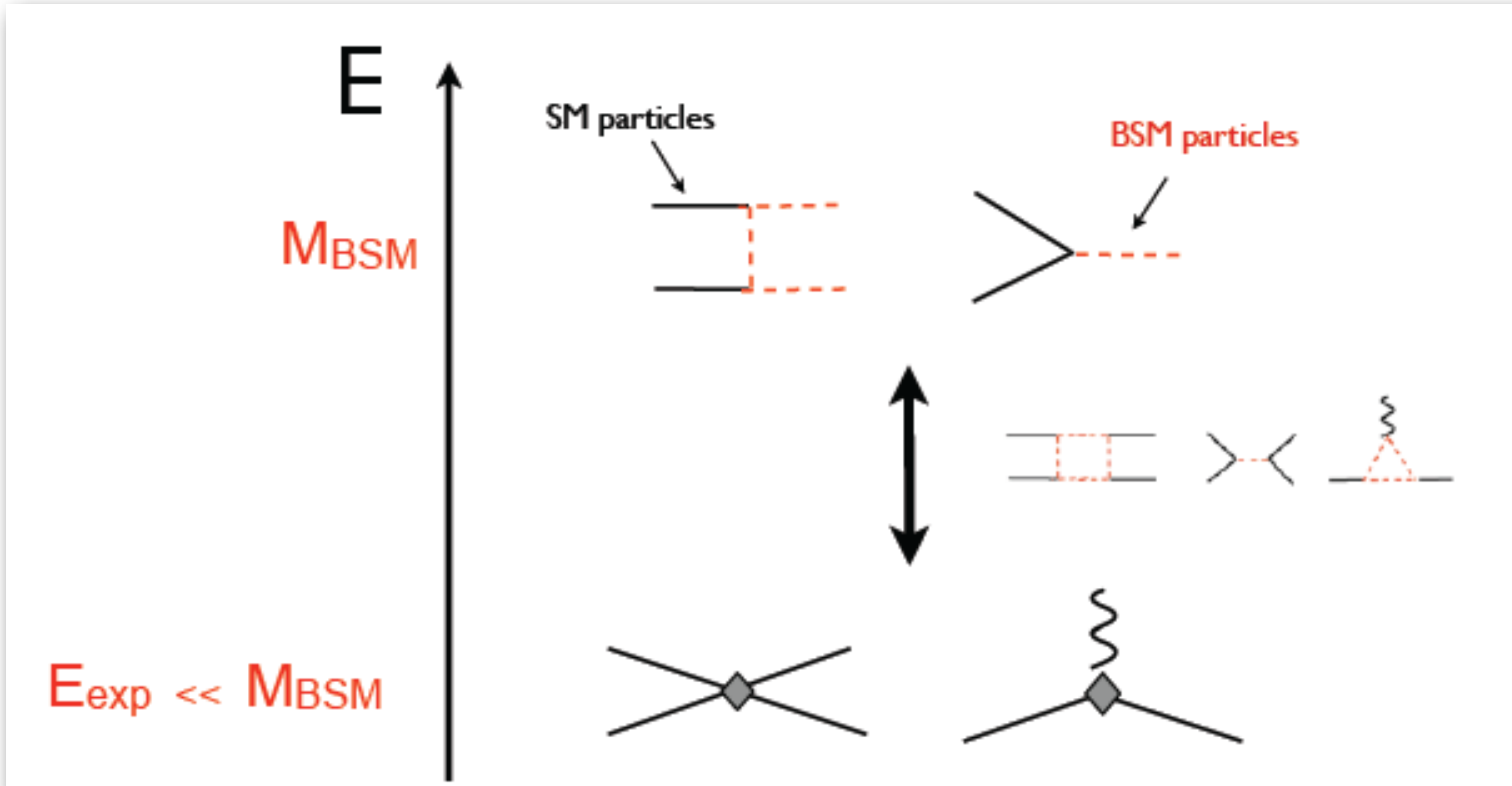
    $\mu \to 3e$ vs $\mu \to e\gamma$ vs $\mu \to e$ conversion

    $\tau \to 3l$ vs $\tau \to l\gamma$ vs $\tau \to l$ + hadrons, $l = e,\mu$
  - What sources of flavor breaking?

    $\mu \to e$ vs $\tau \to \mu$ vs $\tau \to e$

EFT framework

Effective theory framework



- At low energy, BSM dynamics described by local operators

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \ldots$$

$\Lambda \leftrightarrow M_{\rm BSM}$ $\qquad$ $C_i \; [g_{\rm BSM}, \;\; M_a/M_b]$

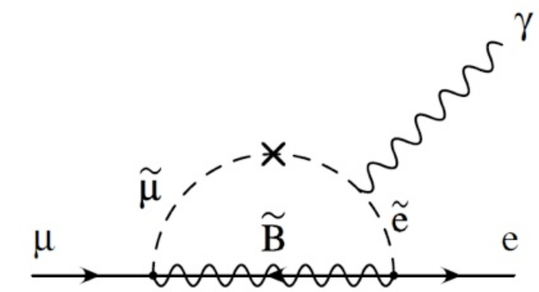
- Each UV model generates a specific pattern of LFV operators

Rich structure at dim=6

- Dipole

$$\frac{v_{\rm ew}\ [\alpha_D]^{ij}}{\Lambda^2}\ \bar{e}^i\,\sigma_{\mu\nu}P_{L,R}\,e^j\ F^{\mu\nu}$$

Dominant in SUSY-GUT and
SUSY see-saw scenarios



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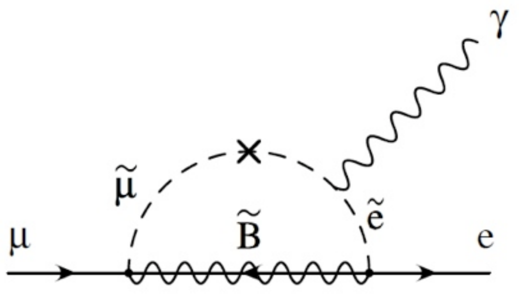
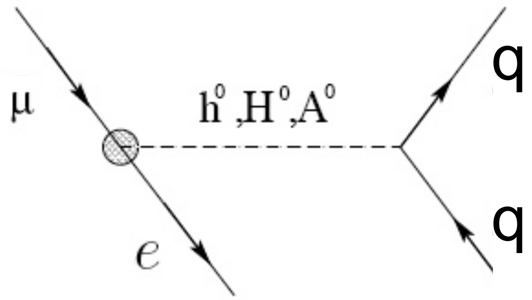


- Scalar (Pseudo-scalar)

$$\frac{[\alpha_S]^{ij}}{\Lambda^2}\;\bar{e}^i\,P_{L,R}\,e^j\;\bar{q}q$$

Dominant in RPV SUSY and RPC SUSY for large tan(β) and low m_A, leptoquarks



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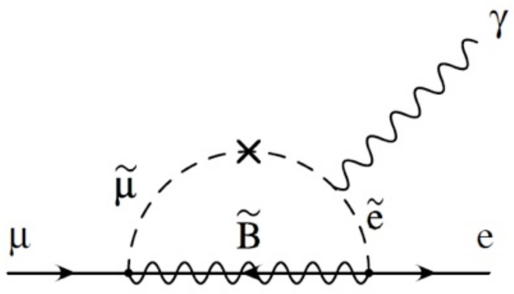

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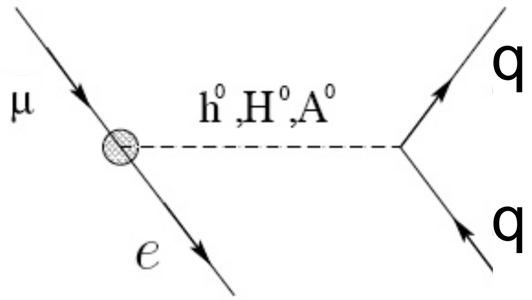

Dominant in RPV SUSY and RPC SUSY for large tan(β) and low m_A, leptoquarks

Integrating out heavy quarks generates gluonic operator:

$$\frac{1}{\Lambda^2}\,\bar{e}^i\,P_{L,R}\,e^j\;\bar{Q}Q\;\rightarrow\;\frac{1}{\Lambda^2\,m_Q}\,\bar{e}^i\,P_{L,R}\,e^j\;G_{\mu\nu}G^{\mu\nu}$$

Rich structure at dim=6

- Dipole

$$\frac{v_{\text{ew}}\,[\alpha_D]^{ij}}{\Lambda^2}\;\bar{e}^i\,\sigma_{\mu\nu}P_{L,R}\,e^j\;F^{\mu\nu}$$

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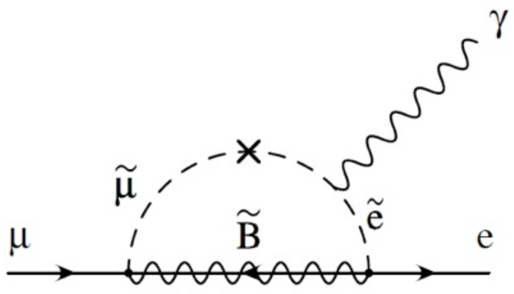

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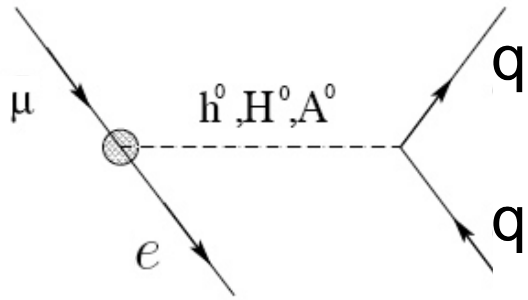
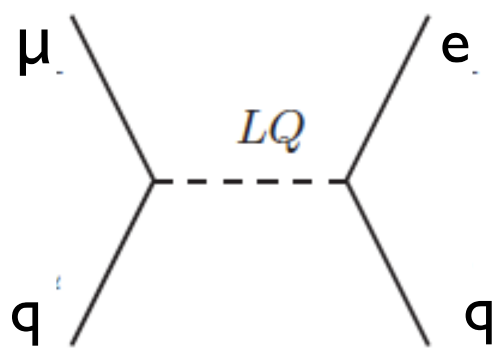


- Vector
(Axial-vector)

$$\frac{[\alpha_V]^{ij}}{\Lambda^2}\;\bar{e}^i\,\gamma_\mu P_{L,R}\,e^j\;\bar{q}\gamma^\mu q$$

Enhanced in Type III seesaw (Z-penguin),
Type II seesaw, LRSM, leptoquarks



Rich structure at dim=6

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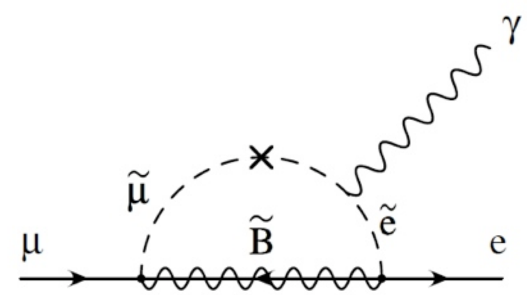

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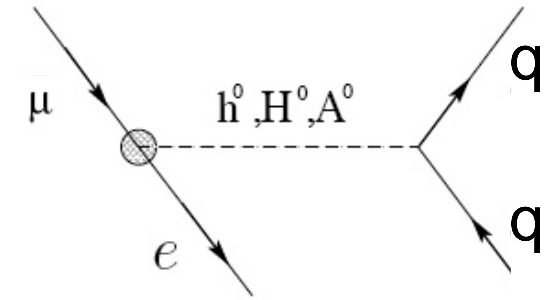

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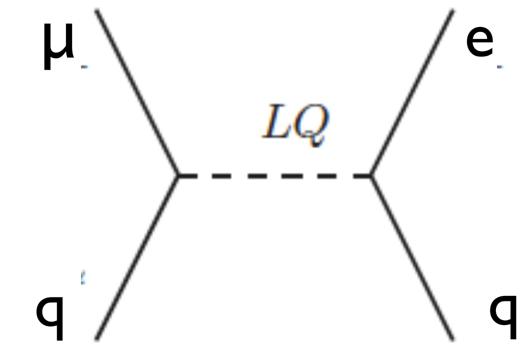
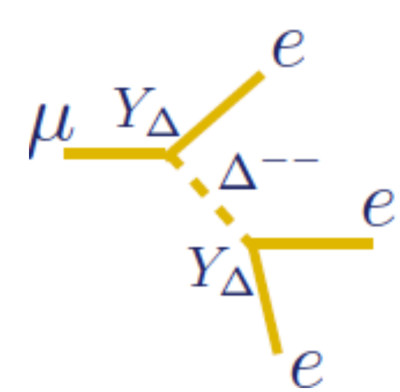


- 4 Leptons, ...

$\Gamma = 1, \gamma_\mu$

$$\frac{[\alpha_\Gamma^{4L}]^{ij}}{\Lambda^2}\ \bar{e}^i\,\Gamma P_{L,R}\,e^j\ \bar{e}^k\,\Gamma P_{L,R}\,e^k$$

Type II and III seesaw, RPV SUSY, LRSM



What can we extract from data

- What effective scale Λ are experiments probing?
- What is the relative strength of various operators (α_D vs α_S ...)? → Mediators, mechanism
- What is the flavor structure of the couplings ($[\alpha_D]^{e\mu}$ vs $[\alpha_D]^{\tau\mu}$...)? → Sources of flavor breaking

↖

(Not discussed in this talk)

Reach in Λ

- LFV BRs scale as

$$BR_{\alpha\to\beta} \sim (v_{EW}/\Lambda)^4 * (\alpha_n)_{\alpha\beta}{}^2$$

- Current limits on $\mu \to e\gamma$ and $\tau\to\mu\gamma$ imply

$$\Lambda/\sqrt{[\alpha_D]^{\mu e}} > 3.4 \times 10^4 \text{ TeV}$$

$$\Lambda/\sqrt{[\alpha_D]^{\tau\mu}} > 5.7 \times 10^2 \text{ TeV}$$

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↓

Assume LFV signals are within reach of planned searches
(e.g. new physics at TeV scale and reasonable mixing parameters)
Ask what can we learn about the underlying mechanism

Model-discriminating power

Discriminating power: μLFV matrix

	$\mu \to 3e$	$\mu \to e\gamma$	$\mu \to e$ conversion
$O^{4\ell}_{S,V}$	✓	–	–
O_D	✓	✓	✓
O^q_V	–	–	✓
O^q_S	–	–	✓

Discriminating power: μLFV matrix

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$O^{4\ell}_{S,V}$	✓	–	–
O_D	✓	✓	✓
O^q_V	–	–	✓
O^q_S	–	–	✓

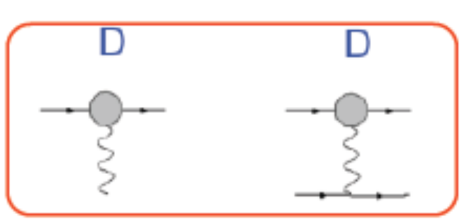
- The notion of "best probe" (= process with largest rate) is model dependent
- Comparing rates of various processes is a key handle on relative strength of operators and hence underlying model

Discriminating power: μLFV matrix

	$\mu \to 3e$	$\mu \to e\gamma$	$\mu \to e$ conversion
$O^{4\ell}_{S,V}$	✓	–	–
O_D	✓	✓	✓
O^{q}_{V}	–	–	✓
O^{q}_{S}	–	–	✓

- $\mu \to 3e$ vs $\mu \to e\gamma$: relative strength of dipole and 4L operators

$$\frac{\Gamma_{\mu\to 3e}}{\Gamma_{\mu\to e\gamma}} = \frac{\alpha}{4\pi} I_{\rm PS} \left(1 + \sum_i \frac{c_i^{(\rm contact)}}{c^{(\rm dipole)}} \right)$$

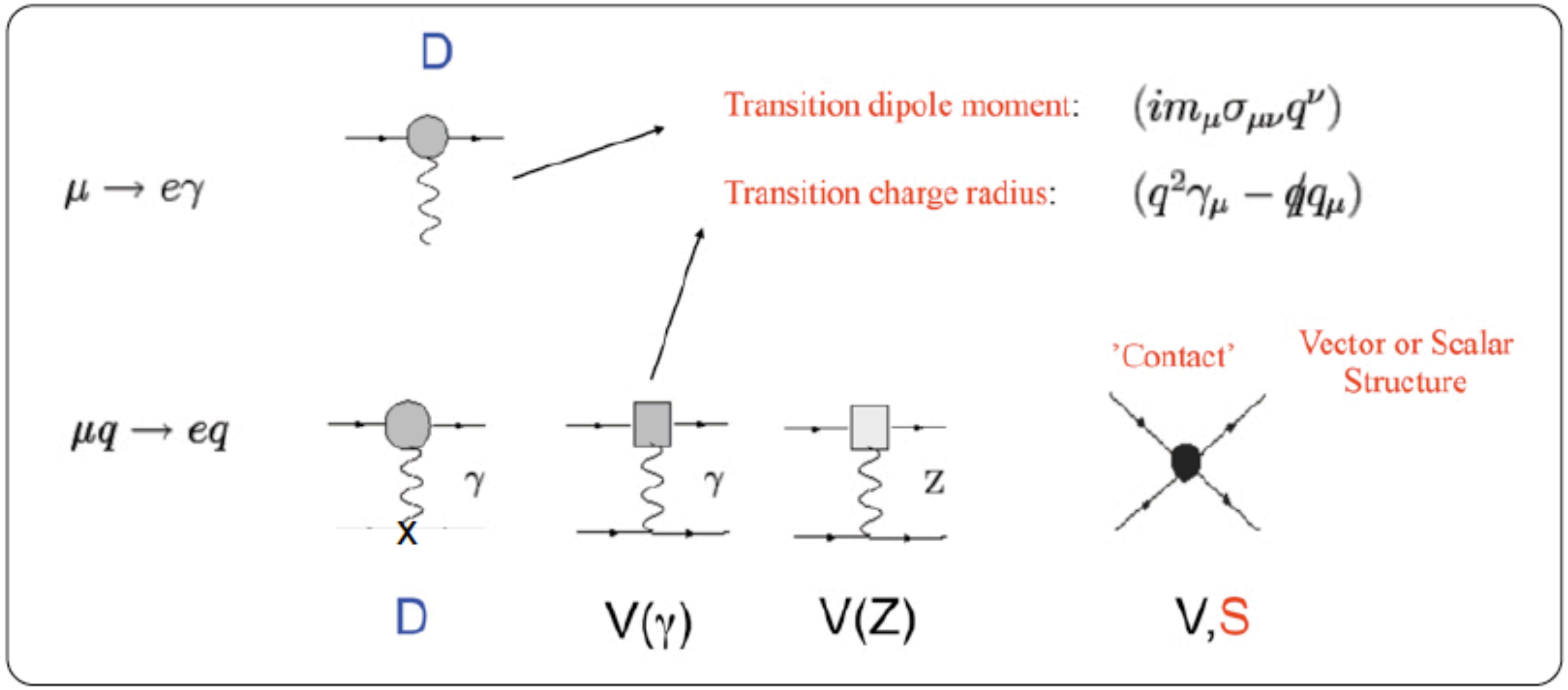


6×10^{-3}

Discriminating power: μLFV matrix

	$\mu \to 3e$	$\mu \to e\gamma$	$\mu \to e$ conversion
$O_{S,V}^{4\ell}$	✓	–	–
O_D	✓	✓	✓
O_V^q	–	–	✓
O_S^q	–	–	✓

- μ →e vs μ →eγ and target-dependence of μ →e conversion: relative strength of dipole and quark operators



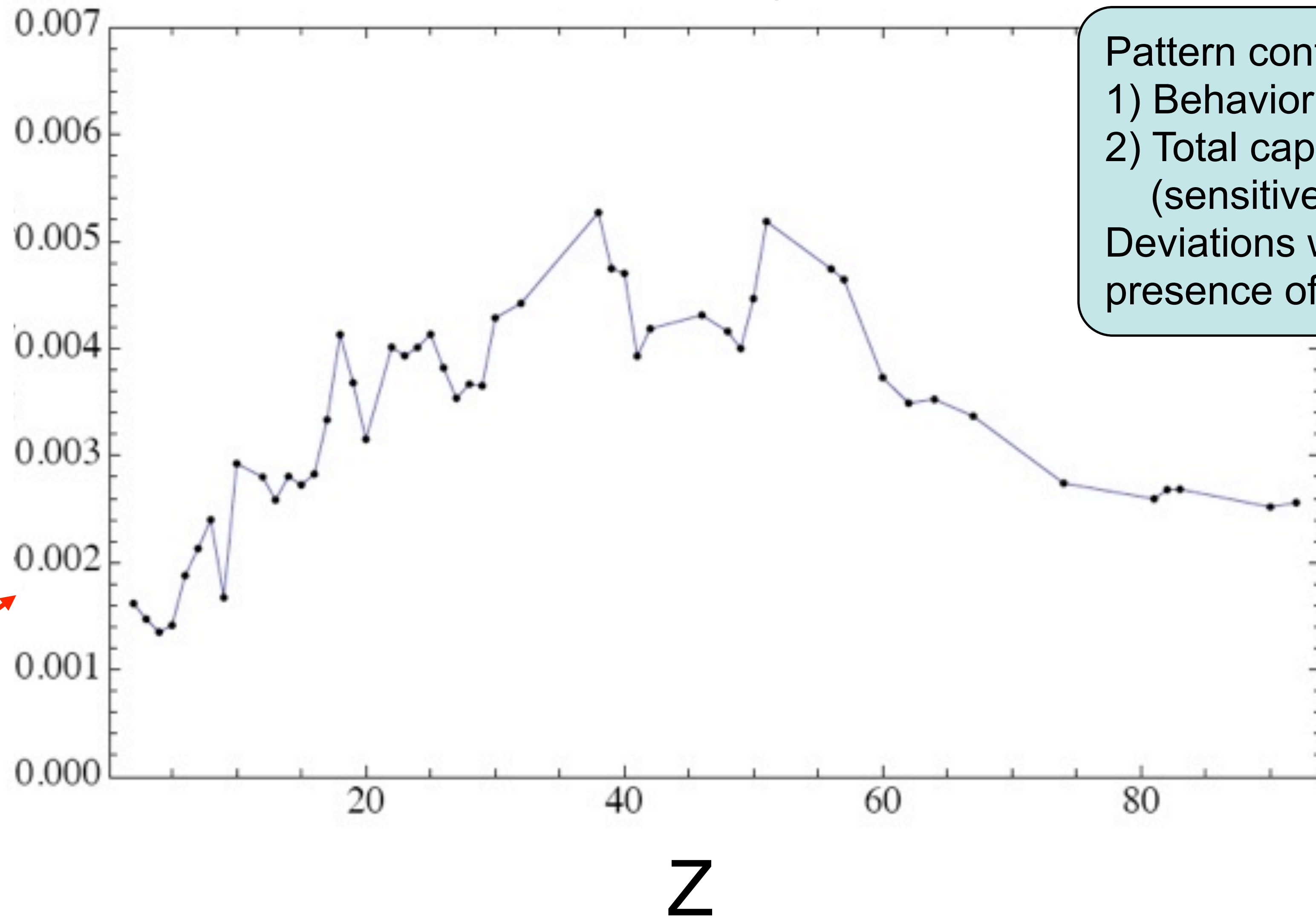
μ→e vs μ→eγ

- Assume dipole dominance:

$$B_{\mu\to e} = \frac{\Gamma(\mu^- + (Z,A) \to e^- + (Z,A))}{\Gamma(\mu^- + (Z,A) \to \nu_\mu + (Z-1,A))}$$

Kitano-Koike-Okada ‘02
VC-Kitano-Okada-Tuzon ‘09

$$\frac{B(\mu \to e, Z)}{B(\mu \to e\gamma)}$$



O(α/π)

Pattern controlled by:
1) Behavior of overlap integrals
2) Total capture rate
(sensitive to nuclear structure)
Deviations would indicate presence of scalar / vector terms

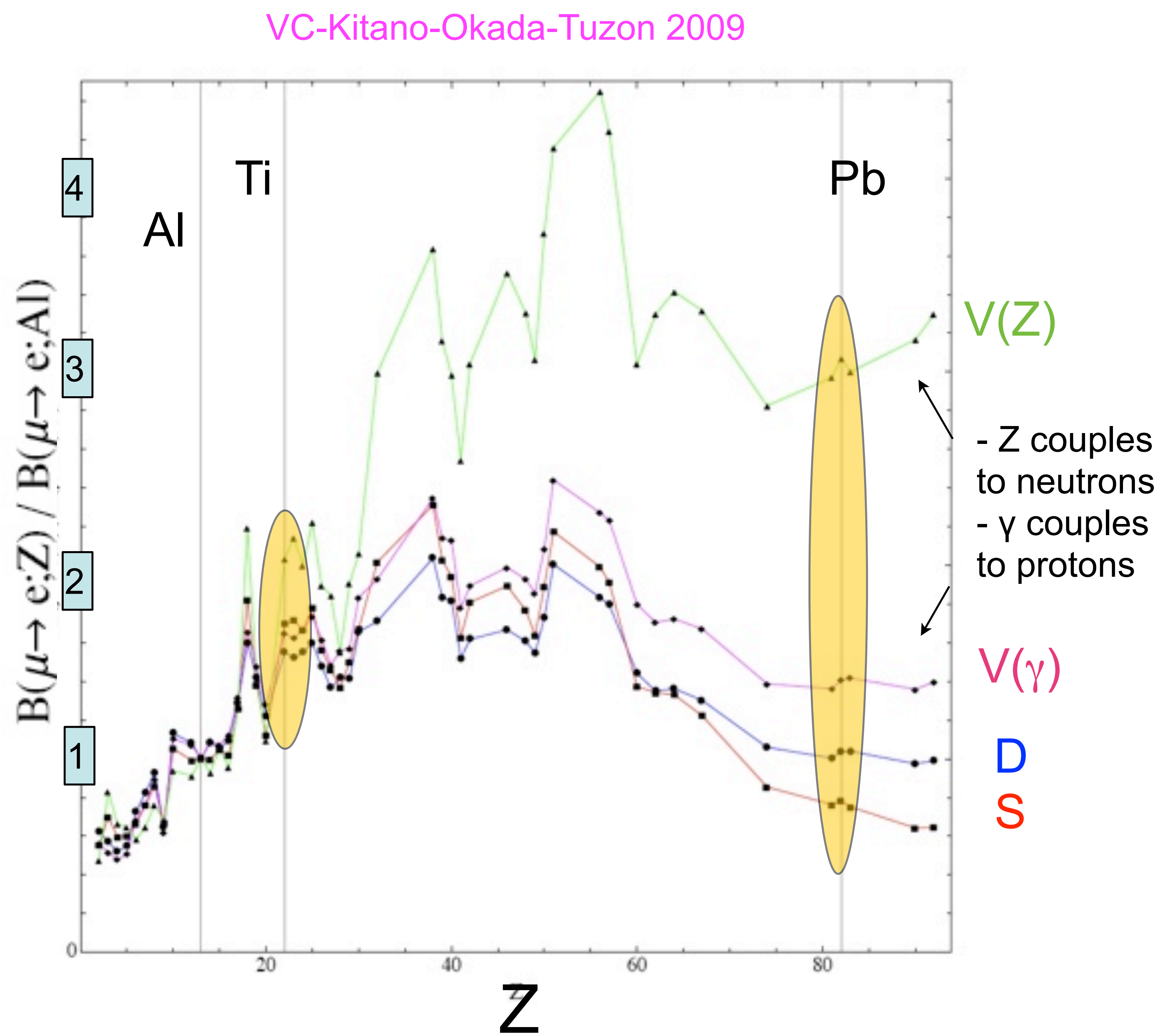
$\mu \to e$ vs $\mu \to e$

- Assume dominance of D, S, or V, and look at $B(\mu \to e, Z_1)/B(\mu \to e, Z_2)$

Target-dependence of the amplitude is different for D, S, V models

Discrimination: need ~5% measure of Ti/Al or ~20% measure of Pb/Al

Ideal world: use Al and a large Z-target (D,S,V have largest separation)



Discriminating power: τLFV matrix

	$\tau \to 3\mu$	$\tau \to \mu\gamma$	$\tau \to \mu\pi^+\pi^-$	$\tau \to \mu K\bar{K}$	$\tau \to \mu\pi$	$\tau \to \mu\eta^{(\prime)}$
$\mathrm{O}_{\mathrm{S,V}}^{4\ell}$	✓	–	–	–	–	–
O_{D}	✓	✓	✓	✓	–	–
$\mathrm{O}_{\mathrm{V}}^{\mathrm{q}}$	–	–	✓ (I=1)	✓(I=0,1)	–	–
$\mathrm{O}_{\mathrm{S}}^{\mathrm{q}}$	–	–	✓ (I=0)	✓(I=0,1)	–	–
O_{GG}	–	–	✓	✓	–	–
$\mathrm{O}_{\mathrm{A}}^{\mathrm{q}}$	–	–	–	–	✓ (I=1)	✓ (I=0)
$\mathrm{O}_{\mathrm{P}}^{\mathrm{q}}$	–	–	–	–	✓ (I=1)	✓ (I=0)
$\mathrm{O}_{\mathrm{G\tilde{G}}}$	–	–	–	–	–	✓

Discriminating power: τLFV matrix

	$\tau \to 3\mu$	$\tau \to \mu\gamma$	$\tau \to \mu\pi^+\pi^-$	$\tau \to \mu K\bar{K}$	$\tau \to \mu\pi$	$\tau \to \mu\eta^{(\prime)}$
$\mathrm{O}^{4\ell}_{\mathrm{S,V}}$	✓	–	–	–	–	–
O_{D}	✓	✓	✓	✓	–	–
$\mathrm{O}^{\mathrm{q}}_{\mathrm{V}}$	–	–	✓ (I=1)	✓(I=0,1)	–	–
$\mathrm{O}^{\mathrm{q}}_{\mathrm{S}}$	–	–	✓ (I=0)	✓(I=0,1)	–	–
O_{GG}	–	–	✓	✓	–	–
$\mathrm{O}^{\mathrm{q}}_{\mathrm{A}}$	–	–	–	–	✓ (I=1)	✓ (I=0)
$\mathrm{O}^{\mathrm{q}}_{\mathrm{P}}$	–	–	–	–	✓ (I=1)	✓ (I=0)
$\mathrm{O}_{\mathrm{G\tilde{G}}}$	–	–	–	–	–	✓

- There is life beyond leptonic and radiative decays!
- Hadronic decays sensitive to large number of operators, but need reliable form factors and decay constants

Discriminating power: τLFV matrix

	$\tau\to 3\mu$	$\tau\to\mu\gamma$	$\tau\to\mu\pi^+\pi^-$	$\tau\to\mu K\bar{K}$	$\tau\to\mu\pi$	$\tau\to\mu\eta^{(\prime)}$
$O^{4\ell}_{S,V}$	✓	–	–	–	–	–
O_D	✓	✓	✓	✓	–	–
O^q_V	–	–	✓ (I=1)	✓(I=0,1)	–	–
O^q_S	–	–	✓ (I=0)	✓(I=0,1)	–	–
O_{GG}	–	–	✓	✓	–	–
O^q_A	–	–	–	–	✓ (I=1)	✓ (I=0)
O^q_P	–	–	–	–	✓ (I=1)	✓ (I=0)
$O_{G\tilde{G}}$	–	–	–	–	–	✓

- Recent progress in $\tau\to\mu(e)\pi\pi$ using dispersive techniques
- Form factors determined by solving 2-channel unitarity condition, with I=0 s-wave meson-meson scattering data as input

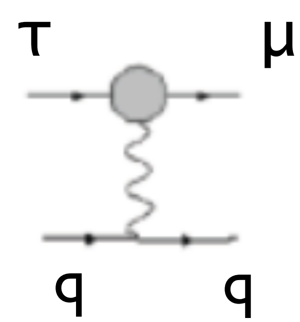
Celis-VC-Passemar 1309.3564, Daub et al 1212.4408

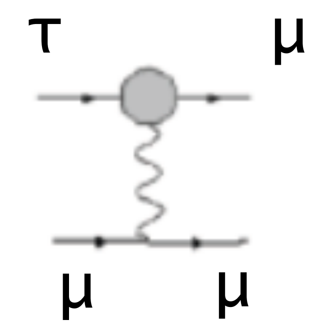
$$\mathrm{Im}F_n(s) = \sum_{m=1}^{2} T^*_{nm}(s)\sigma_m(s)F_m(s)$$

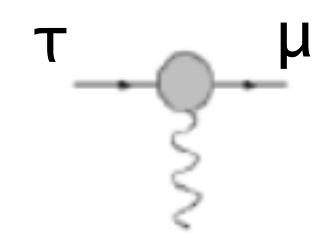
n = ππ, KK

- Two basic handles: 1) Pattern of BRs

		$\mu\pi^+\pi^-$	$\mu\rho$	μf_0	3μ	$\mu\gamma$
D	$R_{F,D}$	0.26×10^{-2}	0.22×10^{-2}	0.13×10^{-3}	0.22×10^{-2}	1
	BR	$< 1.1 \times 10^{-10}$	$< 9.7 \times 10^{-11}$	$< 5.7 \times 10^{-12}$	$< 9.7 \times 10^{-11}$	$< 4.4 \times 10^{-8}$







Illustrative benchmark model

$$R_{F,M} \equiv \frac{\Gamma(\tau \to F)}{\Gamma(\tau \to F_M)}$$

Dominant LFV decay mode for model “M”

Celis-VC-Passemar 1403.5781

- Two basic handles: 1) Pattern of BRs

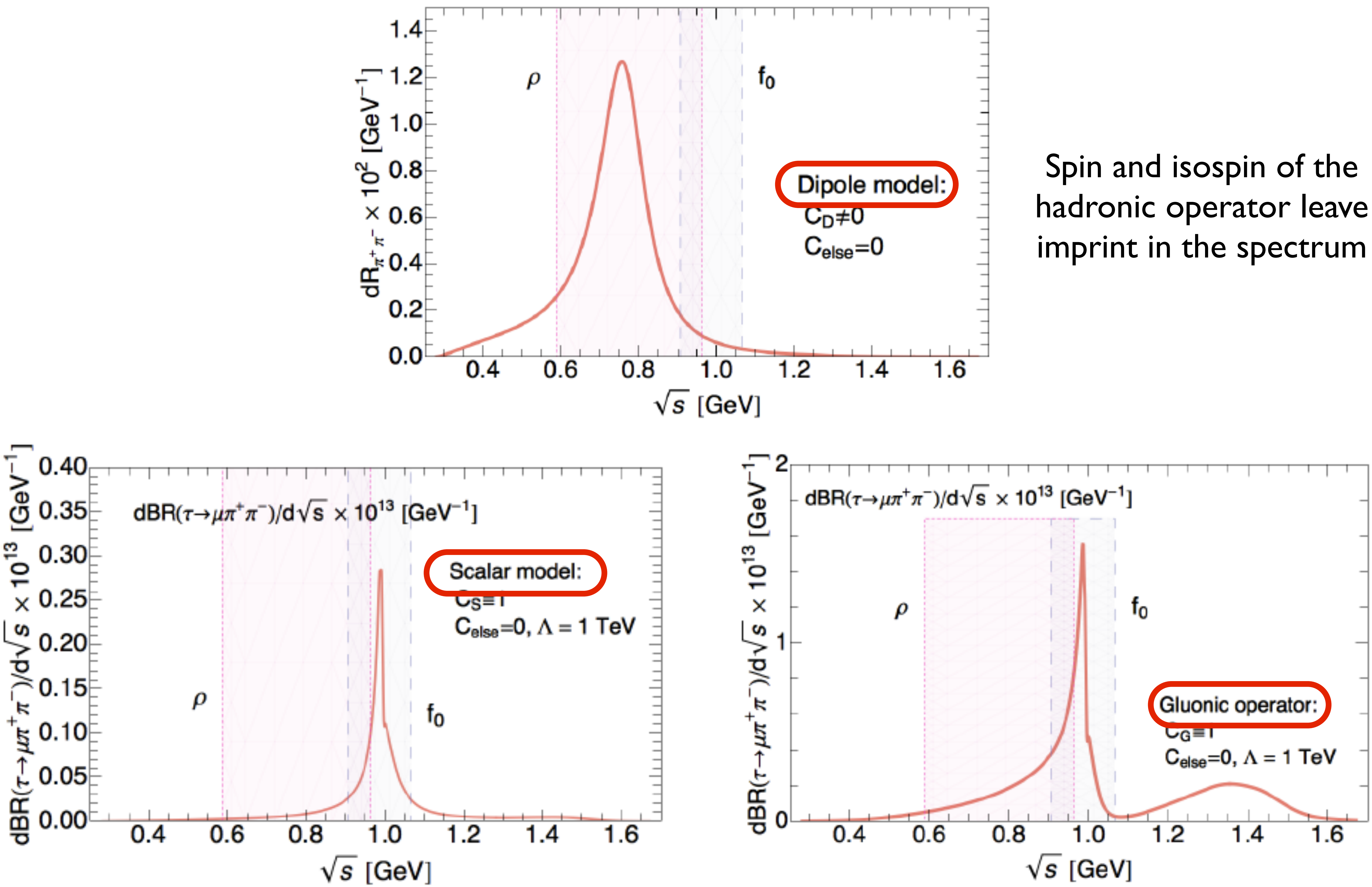
		$\mu\pi^+\pi^-$	$\mu\rho$	μf_0	3μ	$\mu\gamma$
D	$R_{F,D}$	0.26×10^{-2}	0.22×10^{-2}	0.13×10^{-3}	0.22×10^{-2}	1
	BR	$< 1.1 \times 10^{-10}$	$< 9.7 \times 10^{-11}$	$< 5.7 \times 10^{-12}$	$< 9.7 \times 10^{-11}$	$< 4.4 \times 10^{-8}$
S	$R_{F,S}$	1	0.28	0.7	-	-
	BR	$< 2.1 \times 10^{-8}$	$< 5.9 \times 10^{-9}$	$< 1.47 \times 10^{-8}$	-	-
$V^{(\gamma)}$	$R_{F,V^{(\gamma)}}$	1	0.86	0.1	-	-
	BR	$< 1.4 \times 10^{-8}$	$< 1.2 \times 10^{-8}$	$< 1.4 \times 10^{-9}$	-	-
Z	$R_{F,Z}$	1	0.86	0.1	-	-
	BR	$< 1.4 \times 10^{-8}$	$< 1.2 \times 10^{-8}$	$< 1.4 \times 10^{-9}$	-	-
G	$R_{F,G}$	1	0.41	0.41	-	-
	BR	$< 2.1 \times 10^{-8}$	$< 8.6 \times 10^{-9}$	$< 8.6 \times 10^{-9}$	-	-

↑ Illustrative benchmark model

$$R_{F,M} \equiv \frac{\Gamma(\tau \to F)}{\Gamma(\tau \to F_M)}$$

← Dominant LFV decay mode for model "M"

- Two basic handles: 2) Spectra in > 2 body decays



Spin and isospin of the hadronic operator leave imprint in the spectrum

An operator of special interest

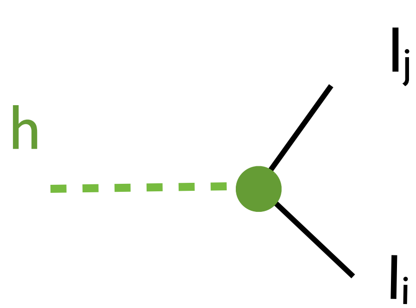
- Non standard (LFV) Higgs couplings

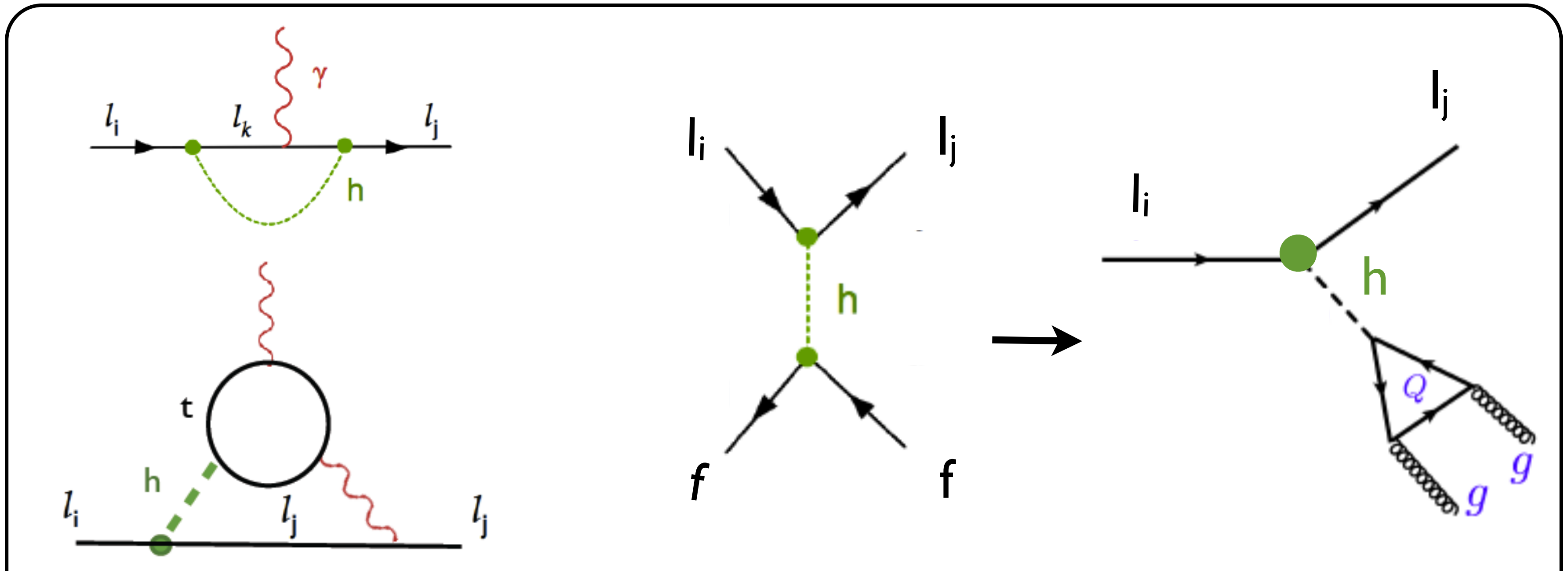
$$\Delta \mathcal{L}_Y = -\frac{\lambda_{ij}}{\Lambda^2} \left(\bar{L}_L^i e_R^j H\right) H^\dagger H \quad \rightarrow \quad -Y_{ij} \, \bar{e}_L^i e_R^j h$$

Goudelis-Lebedev-Park '11
Davidson-Grenier '10
...
Harnik-Kopp-Zupan '12
Blankenburg-Ellis-Isidori 12
McKeen-Pospelov-Ritz '12

High Energy:
LFV Higgs decays

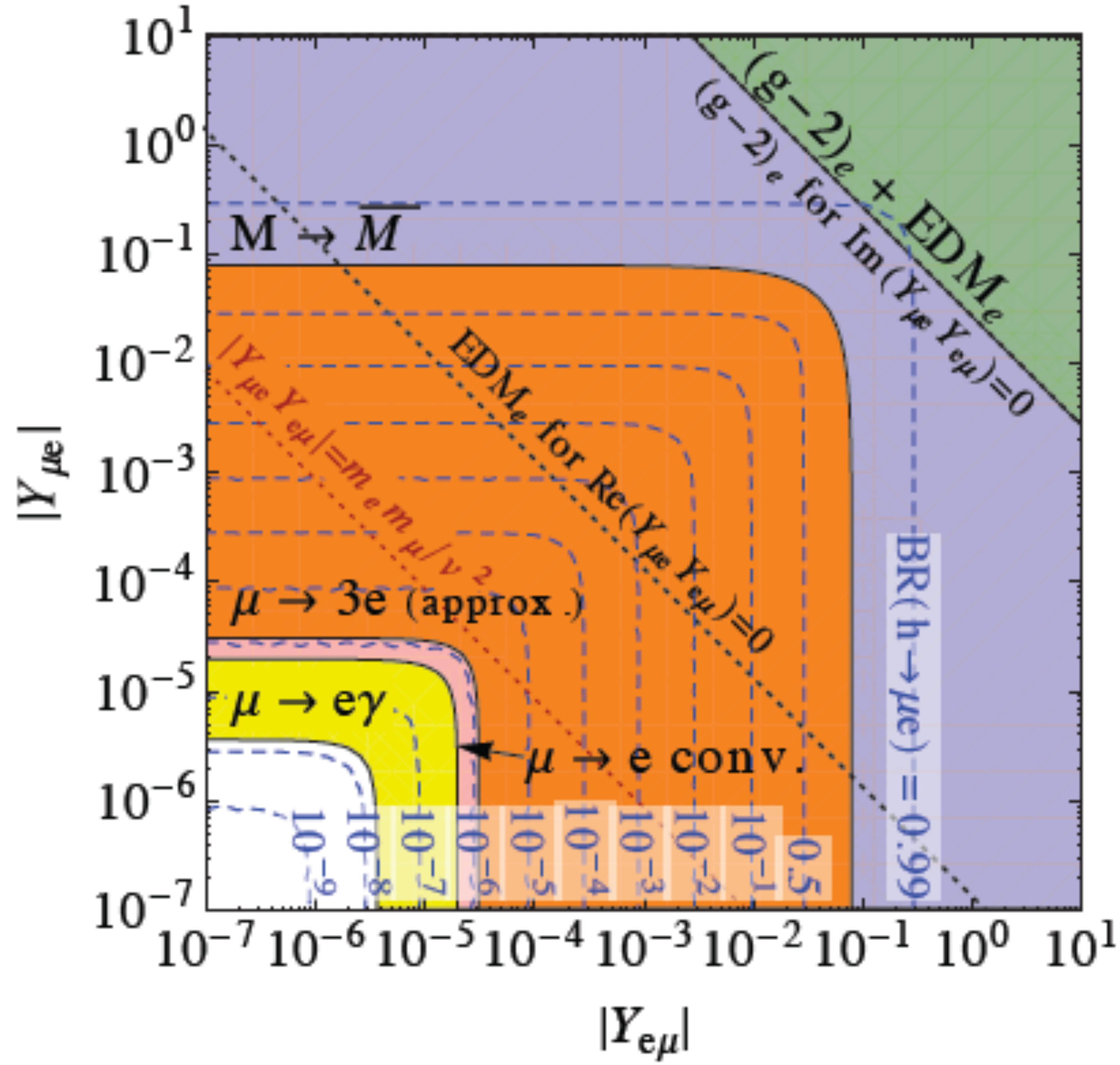
Low Energy:
Dipole, Scalar, Gluon operators





Chang-Hou-Keung 93

- Constraints: Higgs decays vs low-energy LFV and LFC observables
- μe sector: powerful low-energy constraints ⇒ $BR(H \to \mu e) < 10^{-7}$

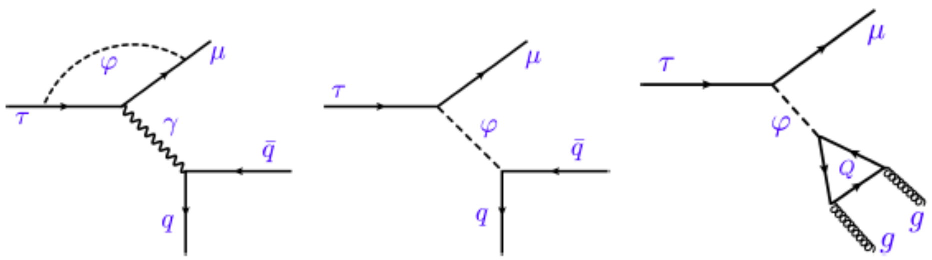


* Diagonal couplings set to SM value

Plot from Harnik-Kopp-Zupan ' 1209.1397

- μτ and eτ sectors:
- Strongest low-energy probes of $Y_{\tau e, \tau\mu}$:

1. $\tau \to \mu\gamma$ via one- and two-loops, sensitive to UV details
2. $\tau \to \mu\pi\pi$ via loops and tree graphs, less stringent but more robust



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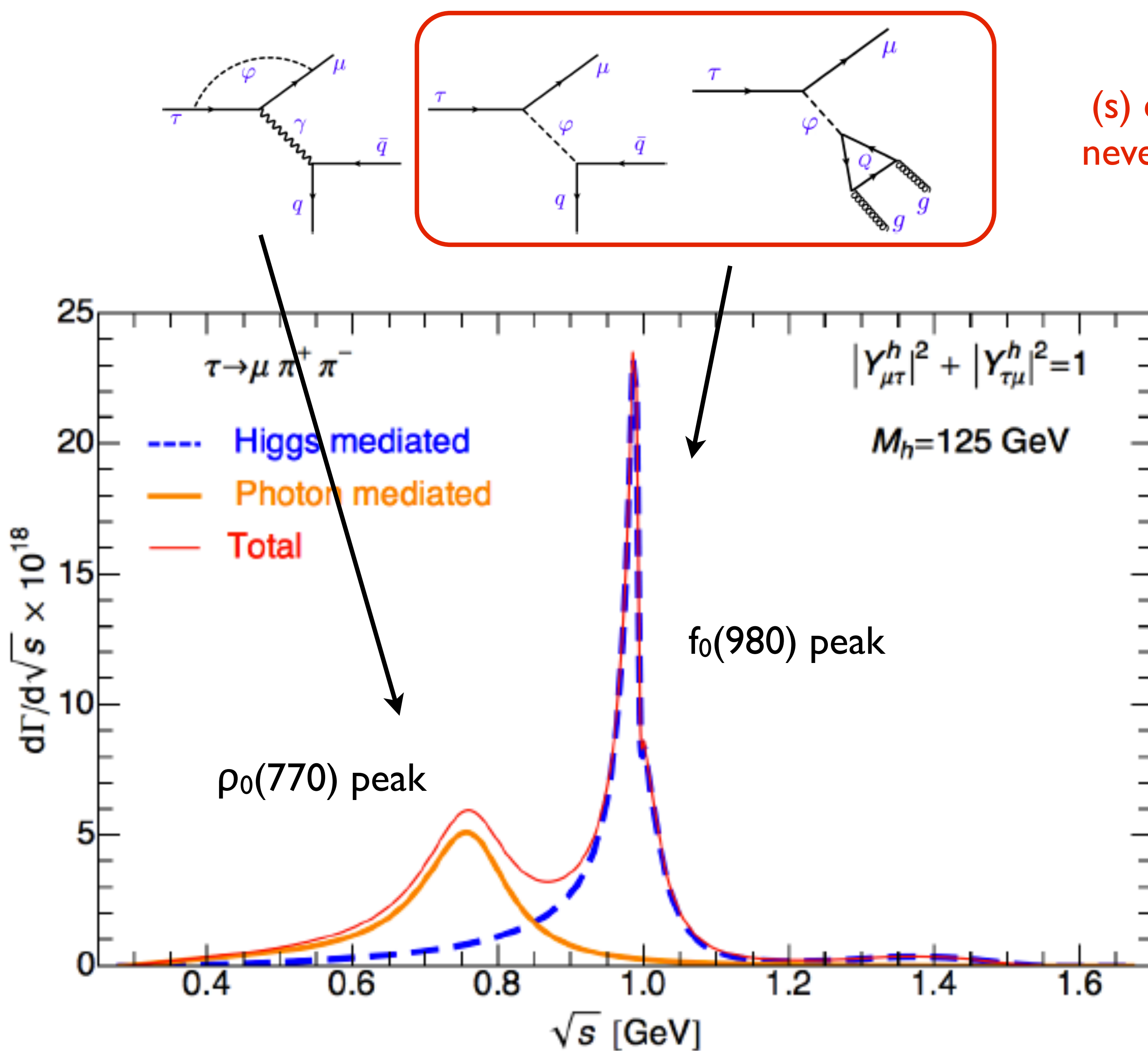
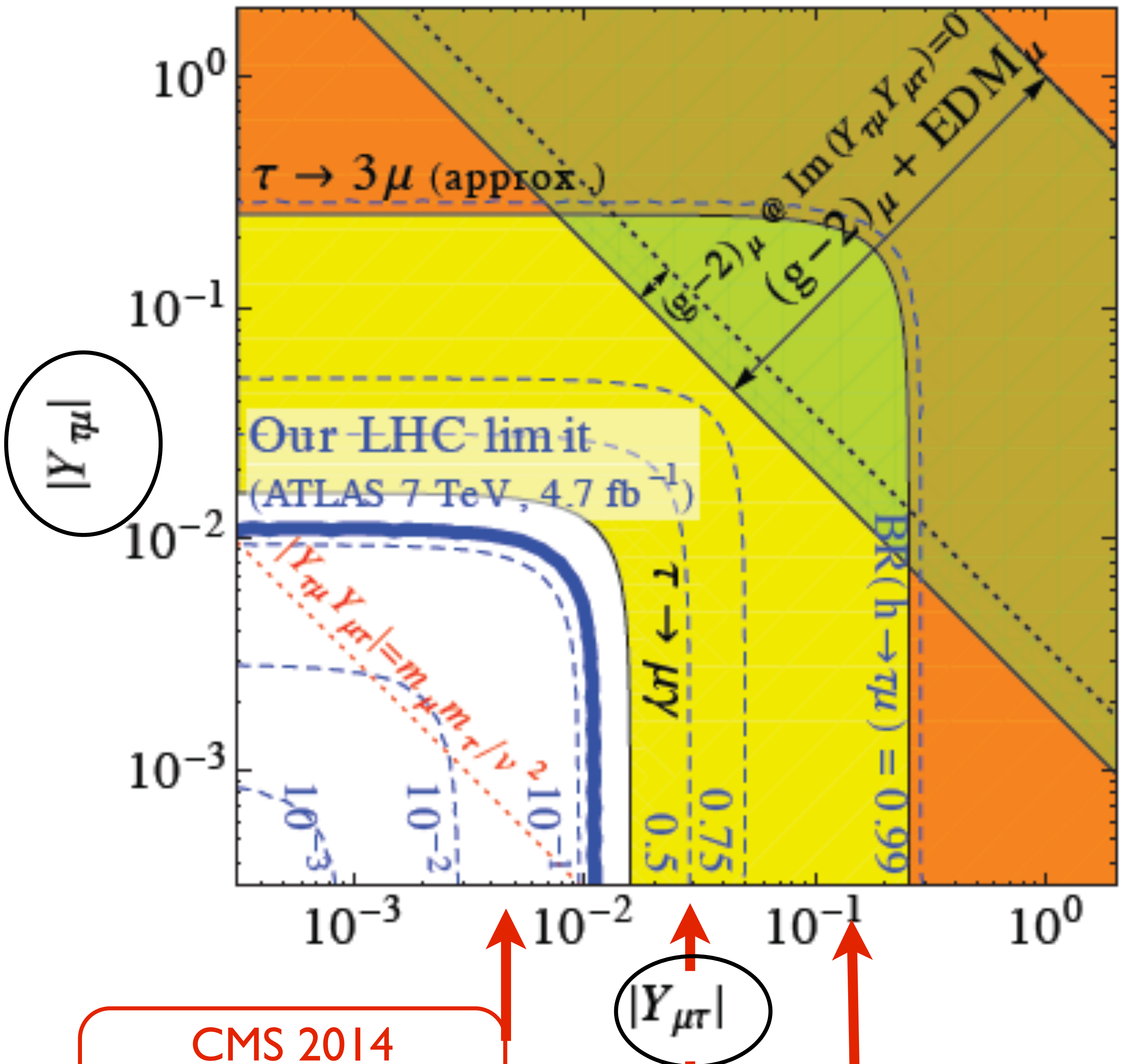
A. Celis, VC, E. Passemar, 1309.3564

ππ spectrum is a great discriminating tool



(s) c,b,t contribution never included before

- μτ and eτ sectors: strongest constraints from Higgs decay at LHC!



Plot from Harnik-Kopp-Zupan ' 1209.1397

Higgs-mediated LFV (and nothing else): CMS results ⇒

B(τ→μγ) < 2.2×10^{-9}
B(τ→μππ) < 1.5×10^{-11}

19.7 fb^{-1}, $\sqrt{s}$ = 8 TeV

CMS PAS HIG-14-005

Conclusions

- Charged LFV are great "discovery" tools: clean, high scale reach
- They are also great "model-discriminating" tools:

  - Operator structure $\rightarrow$ mediators
  - μe vs $\tau\mu$ vs τe $\rightarrow$ sources of flavor breaking

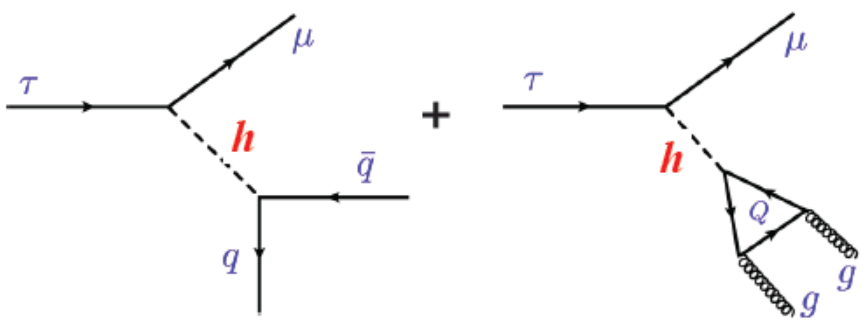
- Hadronic tau decays such as $\tau \rightarrow \mu\pi\pi$ should not be overlooked!

  - Sensitive to many operators, including Higgs-induced ones
  - Model-discrimination via BRs and spectra

Backup

τ →μππ decay (I)

- Tree level Higgs exchange:



$$\mathcal{L}^h_{eff} \simeq -\frac{h}{v}\left(\sum_{q=u,d,s} y^h_q\, m_q\, \bar{q}\, q - \sum_{q=c,b,t} \frac{\alpha_s}{12\pi}\, y^h_q\, G^a_{\mu\nu} G^{\mu\nu}_a\right)$$

- Trade Gluonic operator for trace of energy-momentum tensor

τ →μππ decay (2)

- τ →μππ differential decay rates (scalar and dipole-mediated)

$$\mathcal{K}_\theta = \frac{2}{27} \sum_{q=c,b,t} y_q^h \,, \qquad \mathcal{K}_\Delta = y_s^h - \mathcal{K}_\theta \,, \qquad \mathcal{K}_\Gamma = \frac{m_u y_u^h + m_d y_d^h}{m_u + m_d} - \mathcal{K}_\theta$$

$$\frac{d\Gamma(\tau \to \ell\pi^+\pi^-)_{\rm Higgs}}{d\sqrt{s}} = \frac{(m_\tau^2 - s)^2 \left(s - 4m_\pi^2\right)^{1/2}}{256\,\pi^3\, m_\tau^3} \cdot \frac{|Y_{\tau\ell}^h|^2 + |Y_{\ell\tau}^h|^2}{M_h^4\, v^2} \times \left|\mathcal{K}_\Delta \Delta_\pi(s) + \mathcal{K}_\Gamma \Gamma_\pi(s) + \mathcal{K}_\theta \theta_\pi(s)\right|^2$$

$$\frac{d\Gamma(\tau \to \ell\pi^+\pi^-)_{\rm photon}}{d\sqrt{s}} = \frac{\alpha^2(|c_L|^2 + |c_R|^2)}{768\pi^5\, m_\tau} \cdot \frac{(s - 4m_\pi^2)^{3/2}\,(m_\tau^2 - s)^2\,(s + 2m_\tau^2)\,|F_V(s)|^2}{s^2}$$

$$\left\langle \pi^+(p_{\pi^+})\pi^-(p_{\pi^-})\right|\tfrac{1}{2}(\bar{u}\gamma^\alpha u - \bar{d}\gamma^\alpha d)\left|0\right\rangle \equiv F_V(s)(p_{\pi^+} - p_{\pi^-})^\alpha$$

$$\left\langle \pi^+(p_{\pi^+})\pi^-(p_{\pi^-})\right|m_u\bar{u}u + m_d\bar{d}d\left|0\right\rangle \equiv \Gamma_\pi(s)$$

$$\left\langle \pi^+(p_{\pi^+})\pi^-(p_{\pi^-})\right|m_s\bar{s}s\left|0\right\rangle \equiv \Delta_\pi(s)$$

$$\left\langle \pi^+(p_{\pi^+})\pi^-(p_{\pi^-})\right|\theta_\mu^\mu\left|0\right\rangle \equiv \theta_\pi(s) \,,$$

Form factors

- Two channel unitarity condition (ππ, KK) (OK up to √s ~ 1.4 GeV)

$$\mathrm{Im}F_n(s) = \sum_{m=1}^{2} T^*_{nm}(s)\sigma_m(s)F_m(s)$$

n = ππ, KK

- General solution:

$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}}F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

Canonical solution falling as 1/s for large s
(obey un-subtracted dispersion relation)

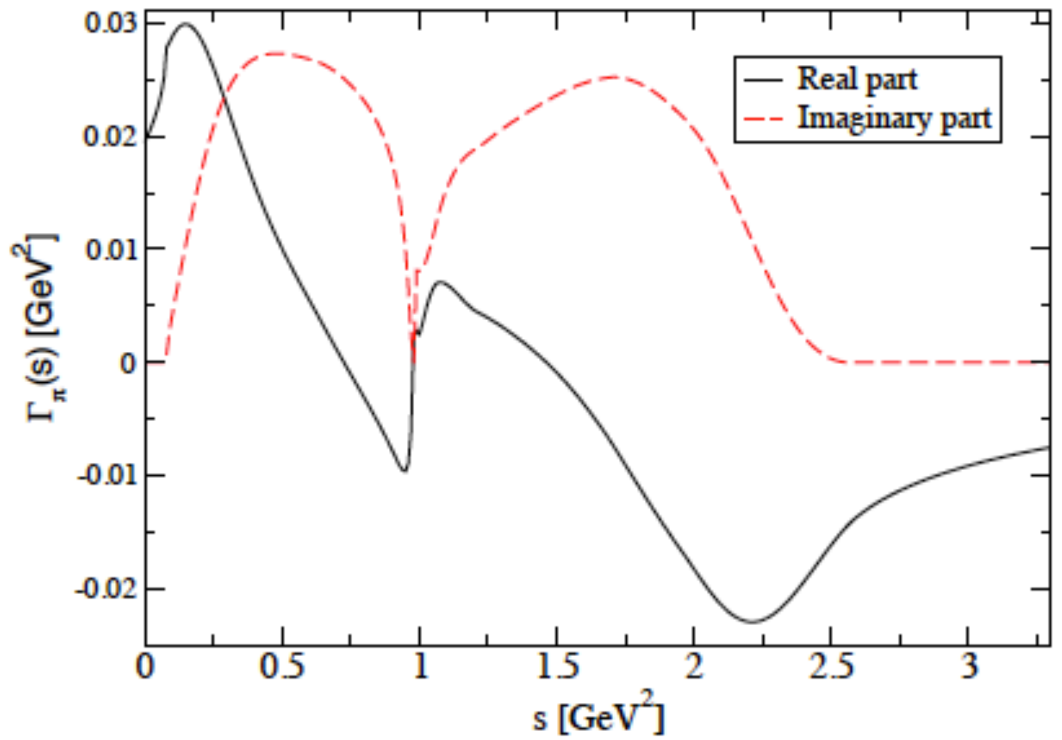
Polynomials determined by matching to ChPT

- Solved iteratively, using input on s-wave I=0 meson meson scattering

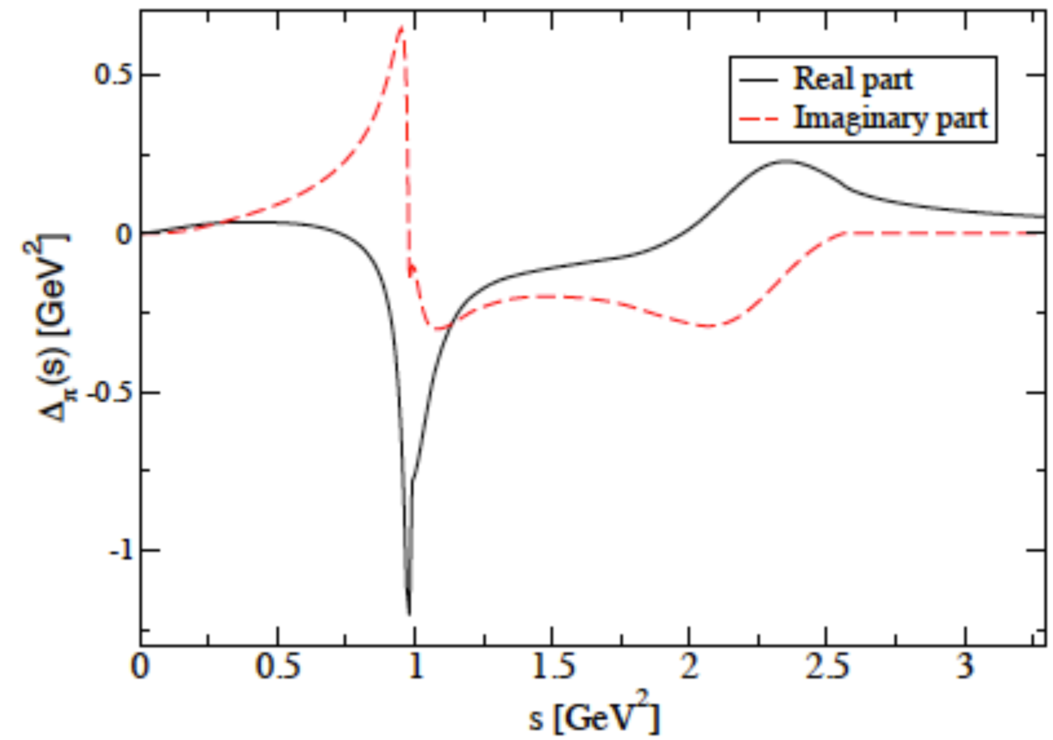
$$X(s) = C(s), D(s)$$

$$X_n(s) = \sum_{m=1}^{2} \frac{1}{\pi} \int_{4M_\pi^2}^{\infty} \frac{dt}{t-s} T^*_{nm}(t)\sigma_m(t)X_m(t)$$

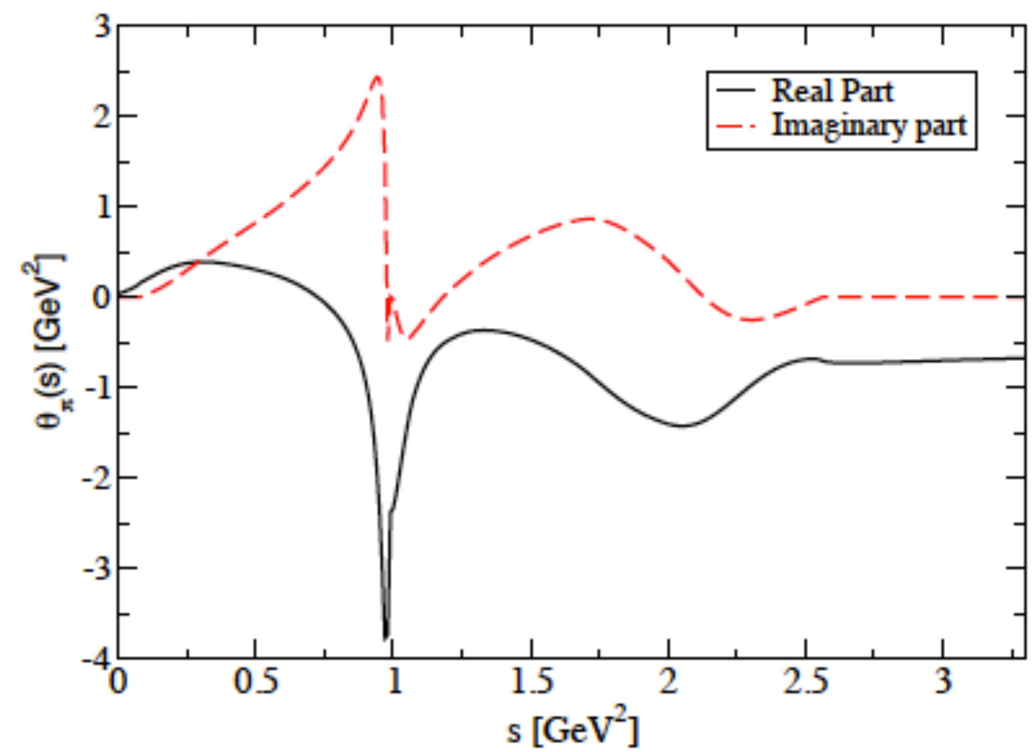
- Results:



Real part
Imaginary part
$\Gamma_\pi(s)$ [GeV^2]
s [GeV^2]



Real part
Imaginary part
$\Delta_\pi(s)$ [GeV^2]
s [GeV^2]



Real Part
Imaginary part
$\theta_\pi(s)$ [GeV^2]
s [GeV^2]

Target dependence of mu-to-e

- Conversion amplitude has non-trivial dependence on target atom, that distinguishes D, S,V underlying operators

$$M_{fi} \sim \langle e^-; A, Z| \int d^3x \, \hat{O}_\ell(x)\, \hat{O}_q(x) \; |\mu^-; A, Z\rangle$$

$$\sim \int d^3x \, \bar{\psi}_e O_\ell \psi_\mu \; \langle A, Z|\hat{O}_q|A, Z\rangle$$

Czarnecki-Marciano-Melnikov

Kitano-Koike-Okada

- Lepton wave-functions in EM field generated by nucleus

- Relativistic components of muon wave-function give different contributions to D,S,V overlap integrals. For example:

$$\bar{\psi}_e \gamma_0 \psi_\mu = \bar{\psi}_e \, \psi_\mu + O(v_\mu/c)$$

- Expect largest discrimination for heavy target nuclei

- Sensitive to hadronic and nuclear properties

$$\langle A, Z| \; \bar{q}\Gamma q \; |A, Z\rangle$$

$$\downarrow$$

$$f_{\Gamma N}^{(q)} \langle A, Z| \; \bar{\psi}_N \Gamma \psi_N \; |A, Z\rangle$$

$$\downarrow$$

$$\begin{aligned} \langle A, Z| \; \bar{\psi}_p(\gamma_0)\psi_p \; |A, Z\rangle &= Z\rho^{(p)} \\ \langle A, Z| \; \bar{\psi}_n(\gamma_0)\psi_n \; |A, Z\rangle &= (A - Z)\rho^{(n)} \end{aligned}$$

** Qualitative behavior of overlap integrals

$\phi_e(x)$ → free outgoing electron wf

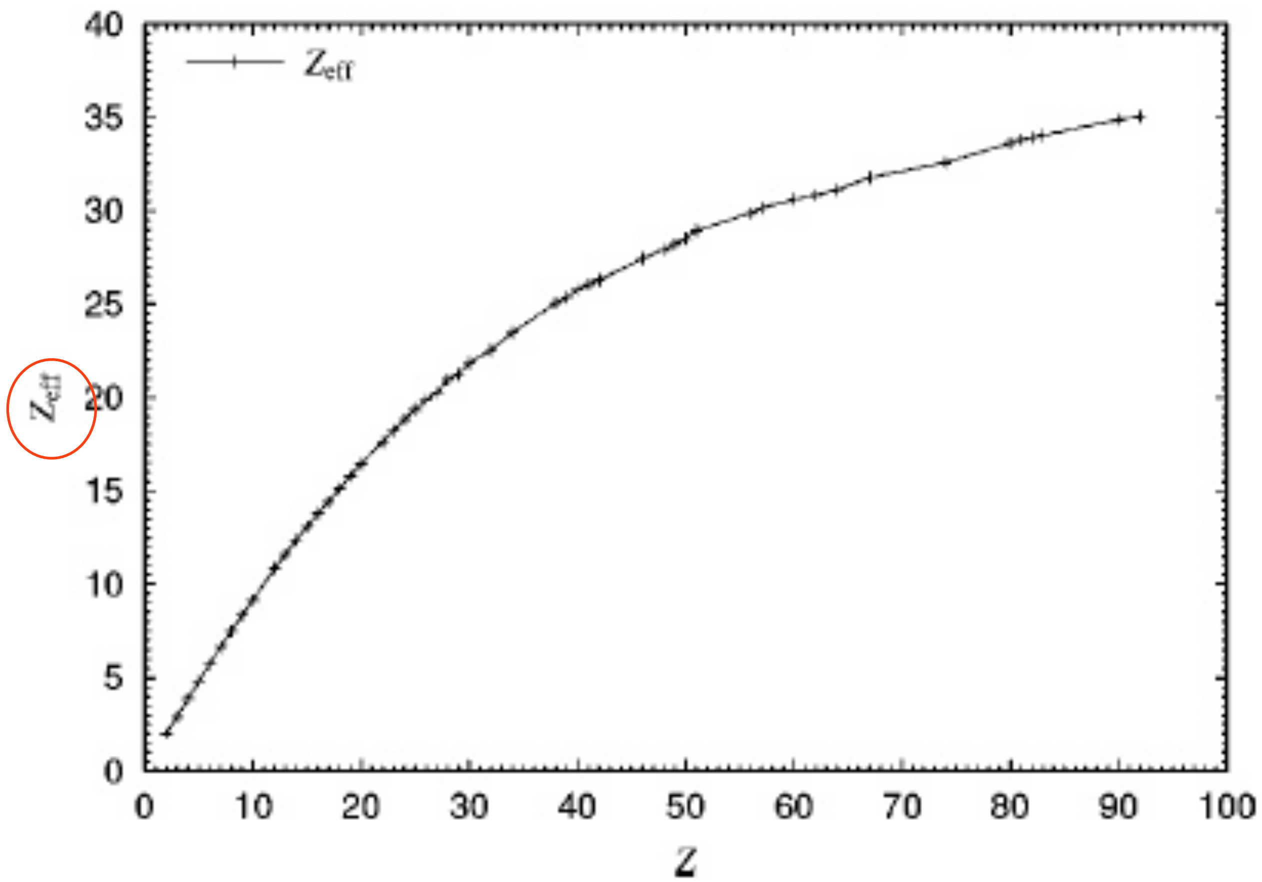
$\phi_\mu(x) \to \langle \phi_\mu(x) \rangle$ (average value)

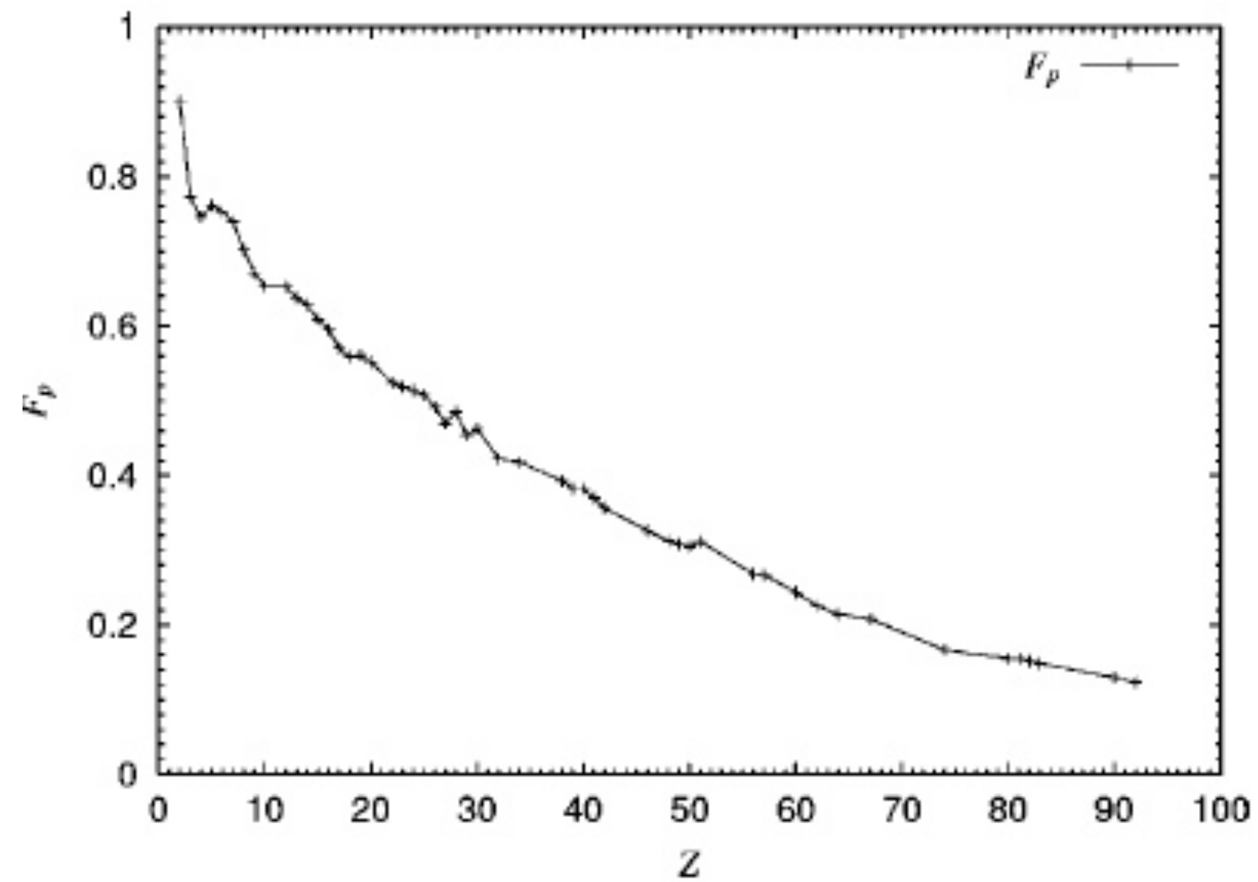
$$I \sim \int d^3x \; \phi_e^*(x) \phi_\mu(x) \; \rho_p(x) \;\; \to \;\; \langle \phi_\mu \rangle \; F_p$$

$p \sim m_\mu$

$$\langle \phi_\mu \rangle^2 = \int_0^\infty dr 4\pi r^2 (g_\mu^2 + f_\mu^2) \rho^{(p)} = \frac{4 m_\mu^3 \alpha^3 Z_{\rm eff}^4}{Z}$$

$$F_p = \int_0^\infty dr 4\pi r^2 \rho^{(p)} \frac{\sin m_\mu r}{m_\mu r}$$





Kitano-Koike-Okada

- Dominant sources of uncertainty:
  - Scalar matrix elements $\langle i|\, m_q q\bar{q}\,|i\rangle = \sigma_q^{(i)} \bar{\psi}_i \psi_i$

$$\sigma_{\pi N} \;=\; \frac{m_u + m_d}{2} \langle p|\bar{u}u + \bar{d}d|p\rangle \;\rightarrow\; 53\,{}^{+21}_{-10}\ \mathrm{MeV} \qquad \boxed{(45 \pm 15)\ \mathrm{MeV}}$$

ChPT — JLQCD 2008 — Lattice range 2012 (Kronfeld 1203.1204)

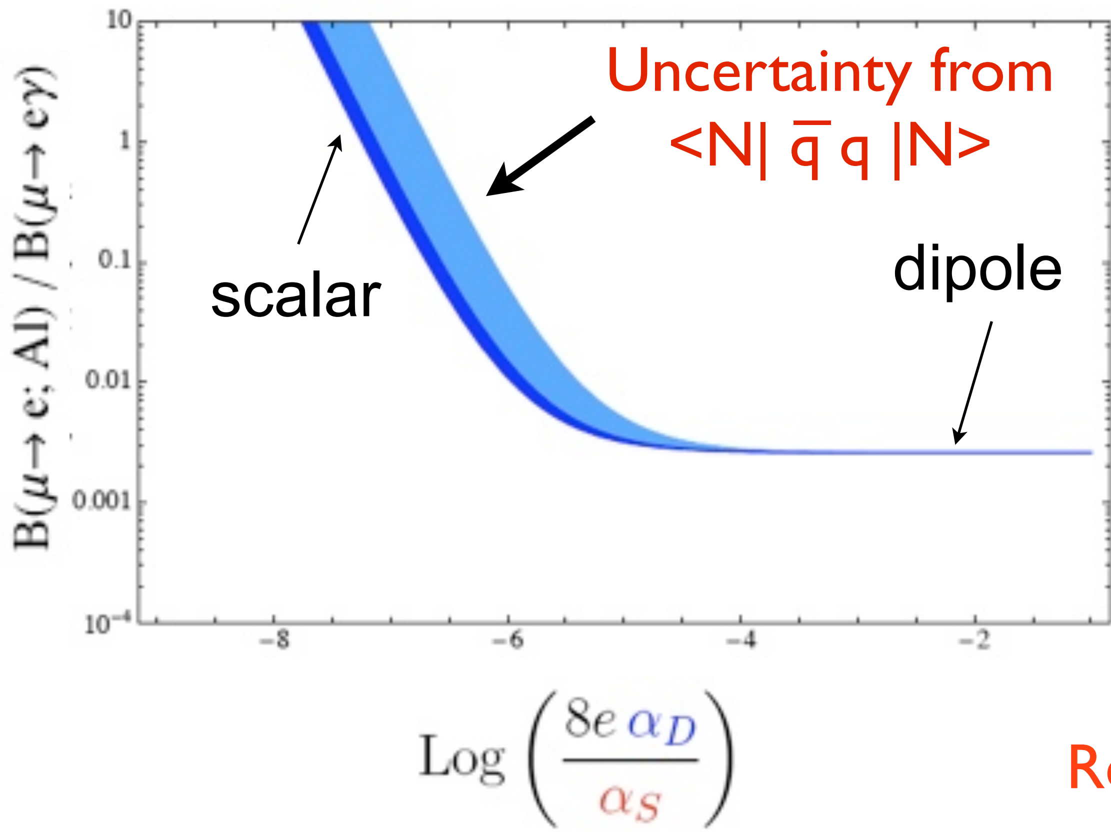
$$y = \frac{2\,\langle p|\bar{s}s|p\rangle}{\langle p|\bar{u}u + \bar{d}d|p\rangle} \;\in\; [0, 0.4] \;\rightarrow\; [0, 0.05] \qquad \boxed{[0.04, 0.12]}$$

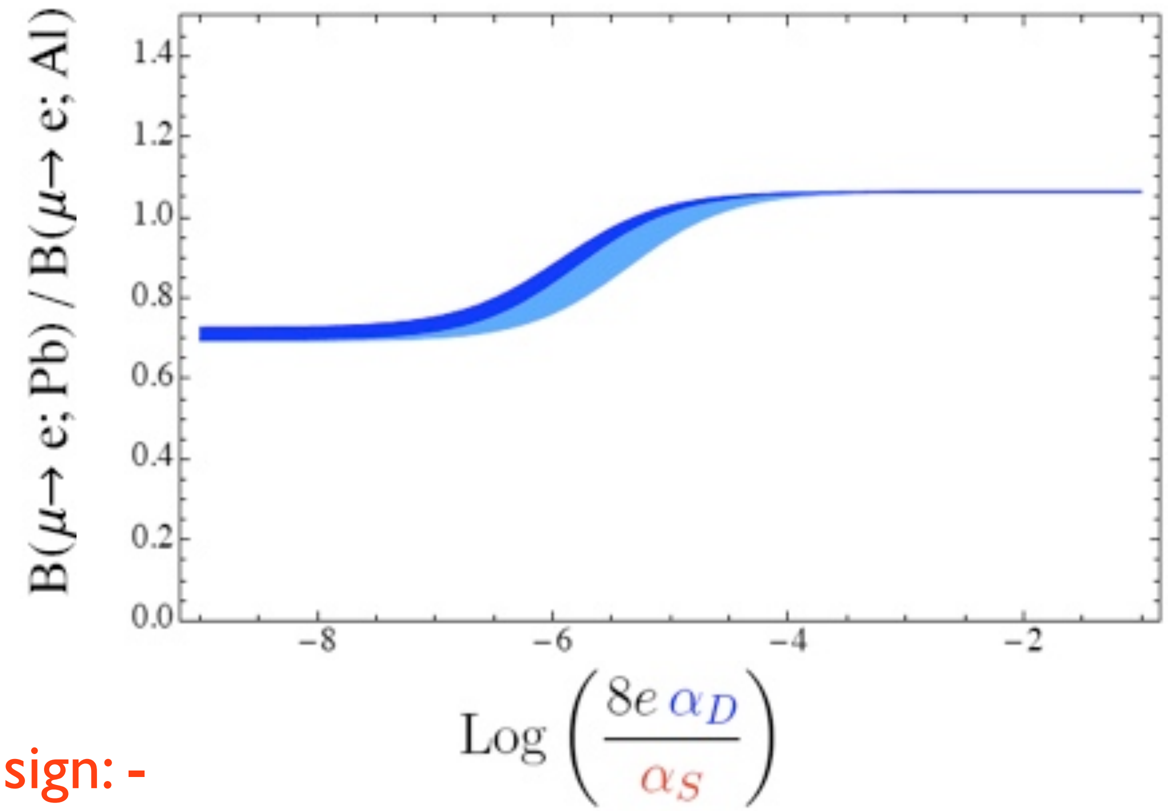
  - Neutron density (heavy nuclei)
  - NLO chiral corrections in matching from quarks to nucleons?

VC-Kitano-Okada-Tuzon ‘09

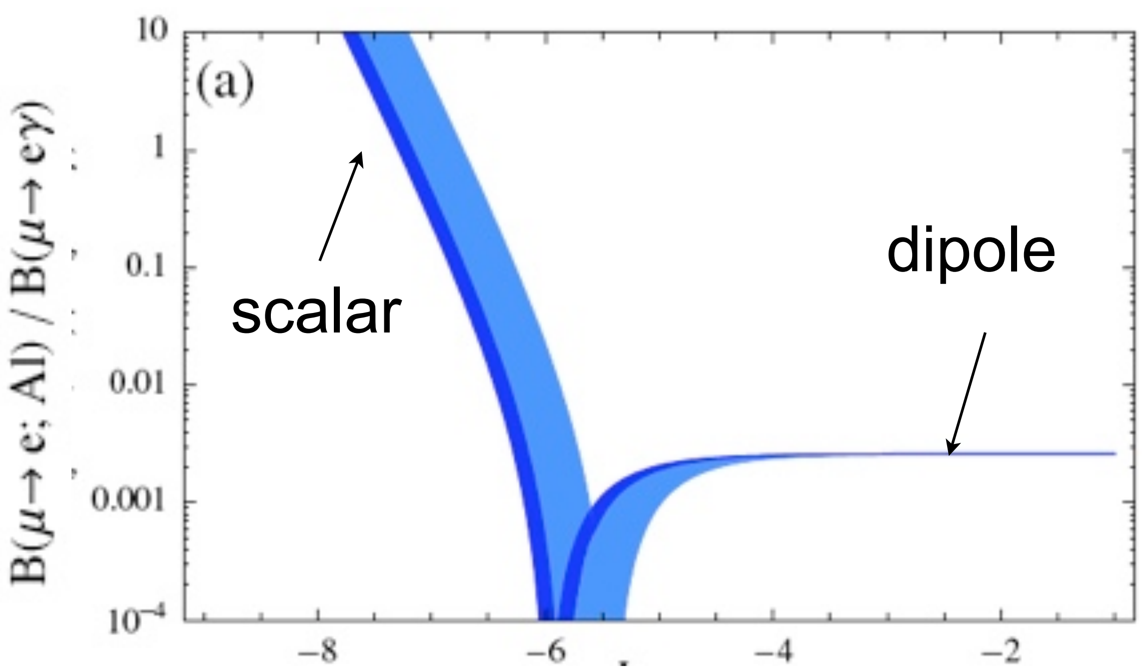
- Beyond single operator dominance: S and D

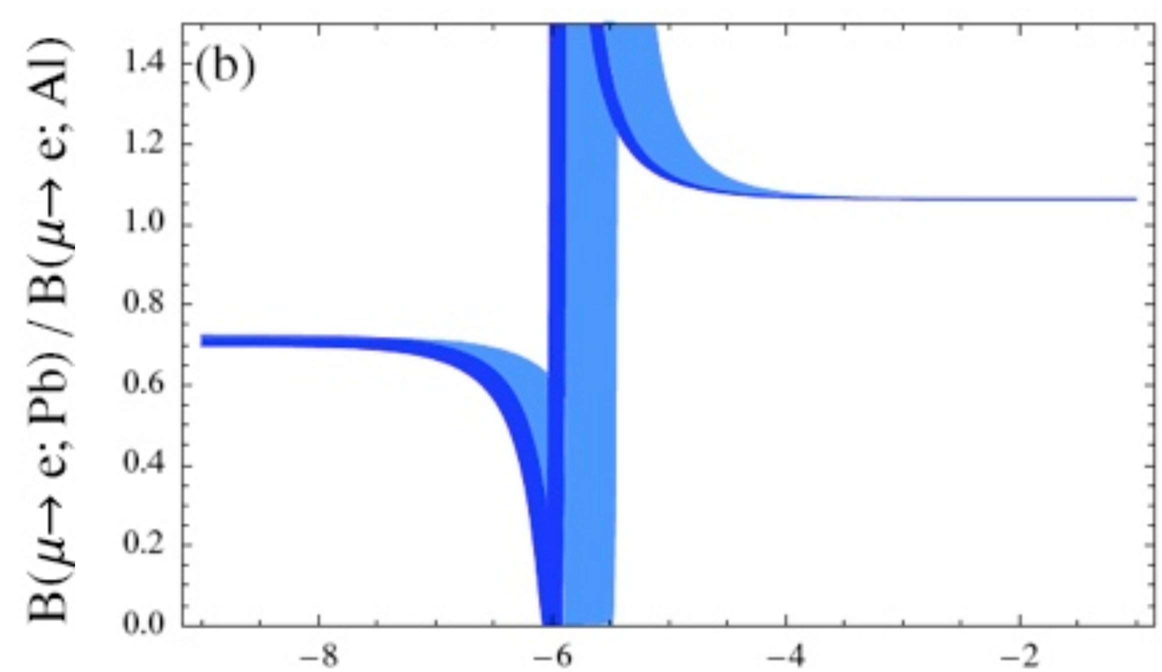
Relative sign: +





Relative sign: -

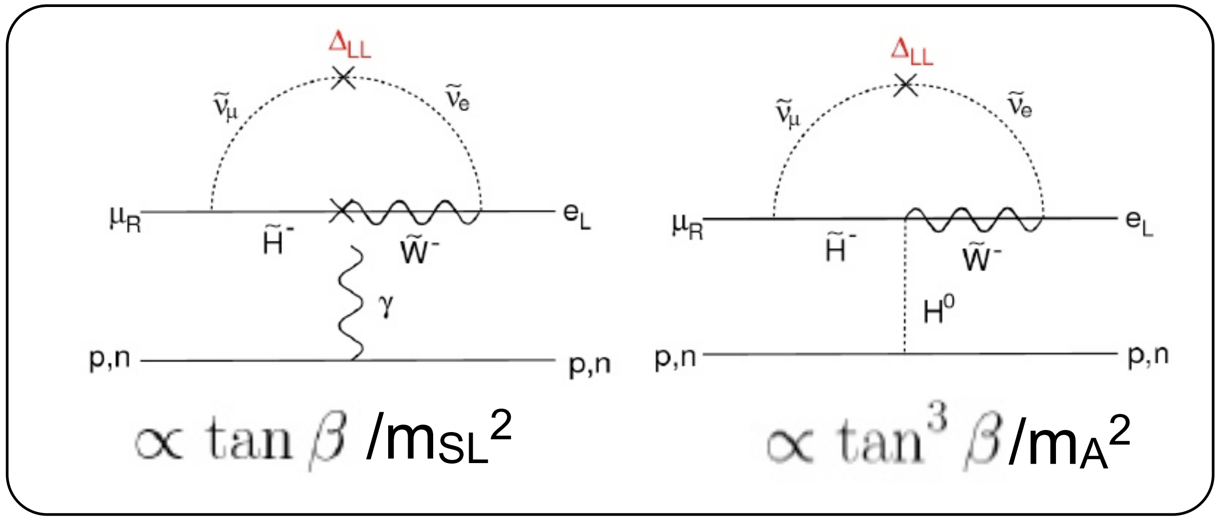


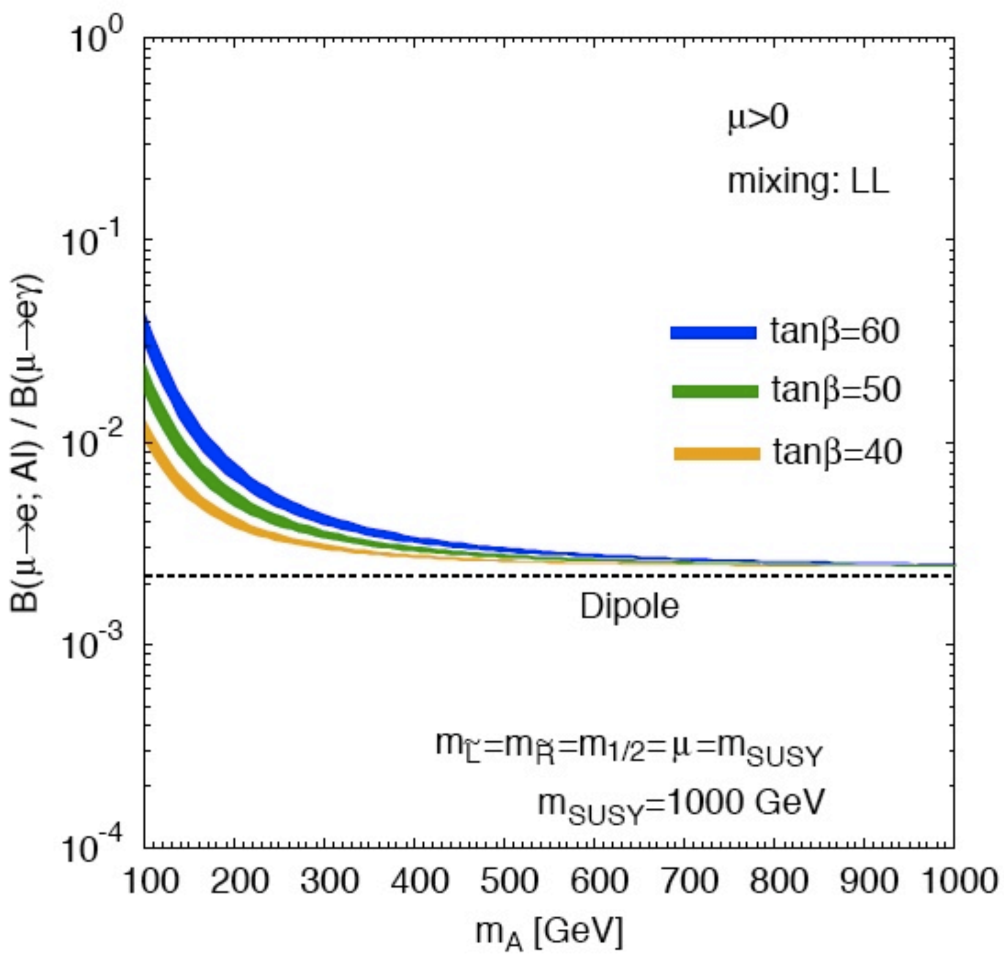


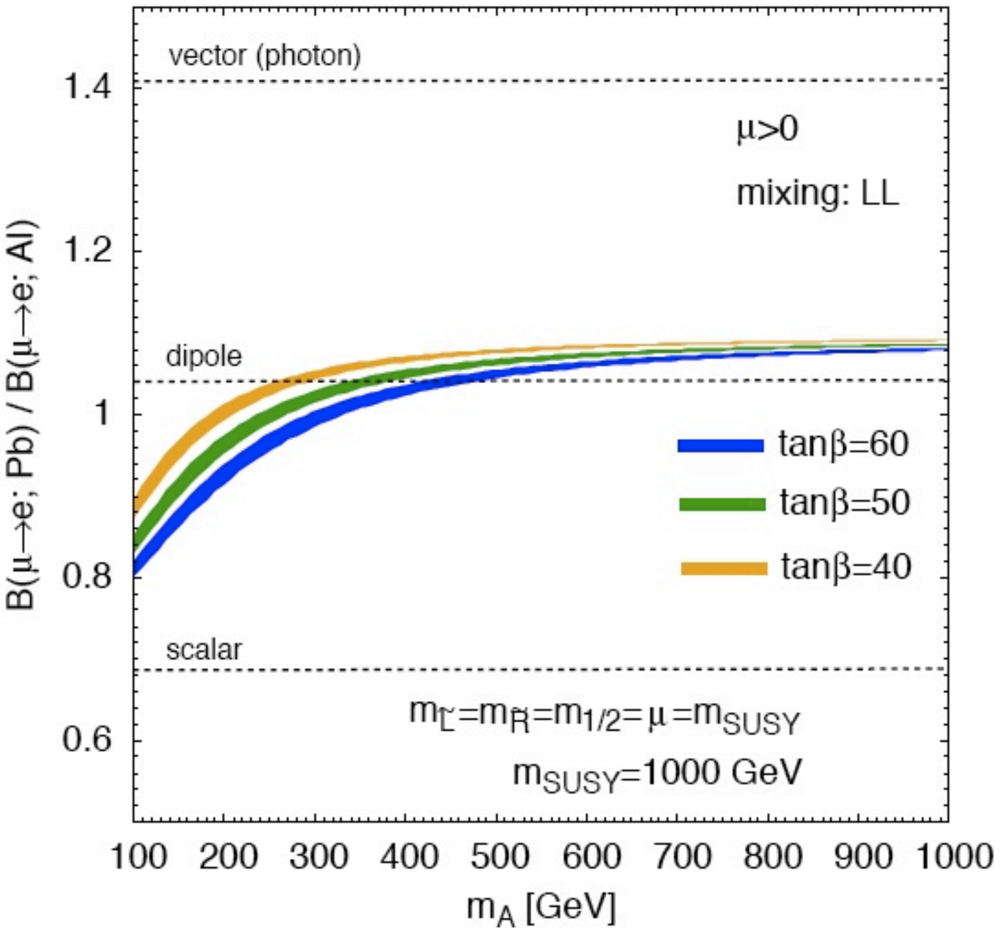
- Beyond single operator dominance: S and D

Kitano-Koike-Komine-Okada 2003

- Explicitly realized in SUSY see-saw models (scalar operator mediated by Higgs exchange)







VC-Kitano-Okada-Tuzon '09

Benchmark models: D, S, V(Z), V(γ)

$$\mathcal{L}_{\rm eff}^{(q)} = -\frac{1}{\Lambda^2}\Big[(C_{DR} m_\mu \bar{e}\sigma^{\rho\nu}P_L\mu + C_{DL} m_\mu \bar{e}\sigma^{\rho\nu}P_R\mu)F_{\rho\nu}$$
$$+\sum_q (C_{VR}^{(q)}\bar{e}\gamma^\rho P_R\mu + C_{VL}^{(q)}\bar{e}\gamma^\rho P_L\mu)\bar{q}\gamma_\rho q$$
$$+\sum_q (C_{SR}^{(q)} m_\mu m_q G_F \bar{e}P_L\mu + C_{SL}^{(q)} m_\mu m_q G_F \bar{e}P_R\mu)\bar{q}q \ + \text{H.c.}\Big]$$

Dipole model

$$C_D \equiv C_{DR} \neq 0, \qquad C_{\rm else} = 0.$$

Scalar model

$$C_S \equiv C_{SR}^{(d)} = C_{SR}^{(s)} = C_{SR}^{(b)} \neq 0,$$
$$C_{\rm else} = 0.$$

Vector model: V(γ)

$$C_V \equiv C_{VR}^{(u)} = -2C_{VR}^{(d)} \neq 0, \qquad C_{\rm else} = 0,$$

Vector model: V(Z)

$$C_V \equiv C_{VR}^{(u)} = \frac{C_{VR}^{(d)}}{a} \neq 0, \qquad C_{\rm else} = 0$$

$$a = \frac{T_{d_L}^3 + T_{d_R}^3 - (Q_{d_L} + Q_{d_R})\sin^2\theta_W}{T_{u_L}^3 + T_{u_R}^3 - (Q_{u_L} + Q_{u_R})\sin^2\theta_W} = -1.73. \qquad \tilde{C}_{VR}^{(n)}/\tilde{C}_{VR}^{(p)} = -9.26.$$

LHC bounds on LFV 4-fermion operators

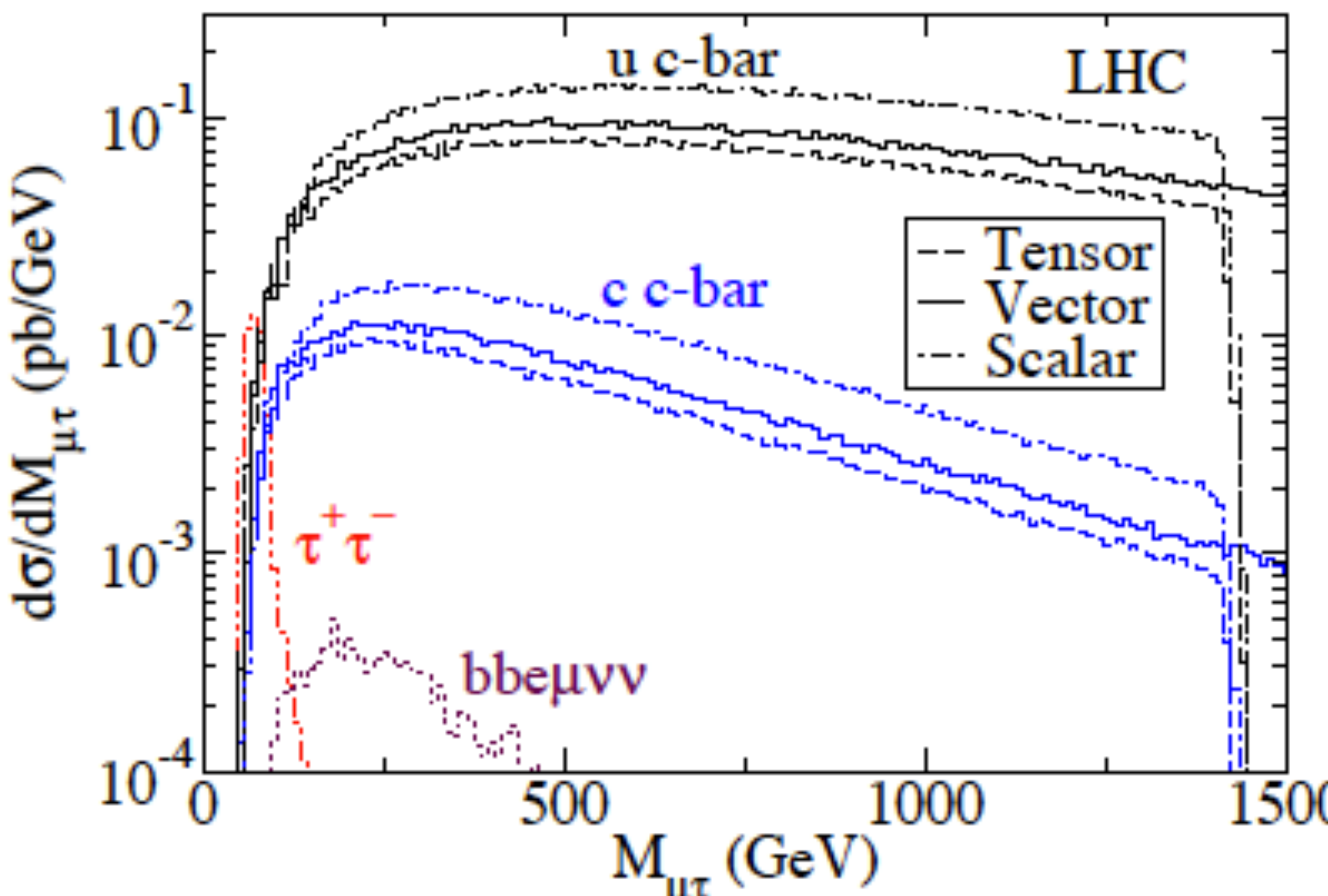
- If Λ >> TeV, EFT description is appropriate at colliders
- 4-fermion operators mediate $p\overset{(-)}{p} \to \ell_\alpha \bar{\ell}_\beta + X$

Han-Lewis-Sher 2010

$$\frac{1}{\Lambda^2} (\overline{\mu}\, \Gamma_j\, \tau) \left(\overline{q}^\alpha\, \Gamma_j\, q^\beta\right)$$

$$\downarrow$$

$$\sigma(\bar{q}_i q_j \to \mu\tau) \propto \frac{s}{\Lambda^4}$$

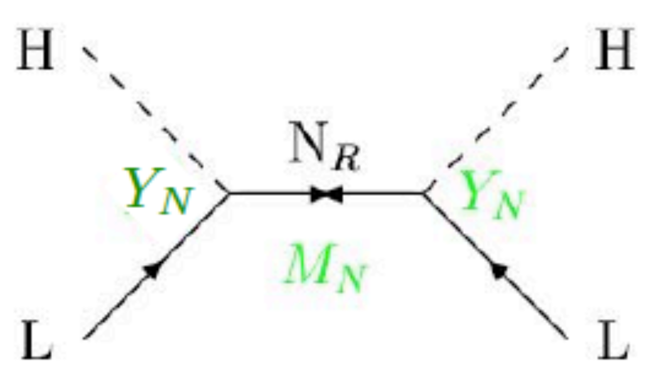


- Probe scales up to Λ ~ 10 TeV for both light and heavy quarks

CLFV in see-saw models

Type I:
Fermion singlet

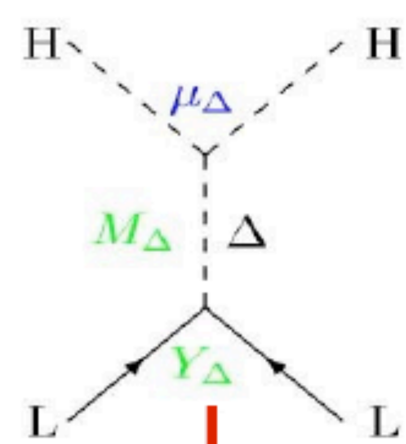
N_{R_i}



$$m_\nu = Y_N^T \frac{1}{M_N} Y_N v^2$$

Type II:
Scalar triplet

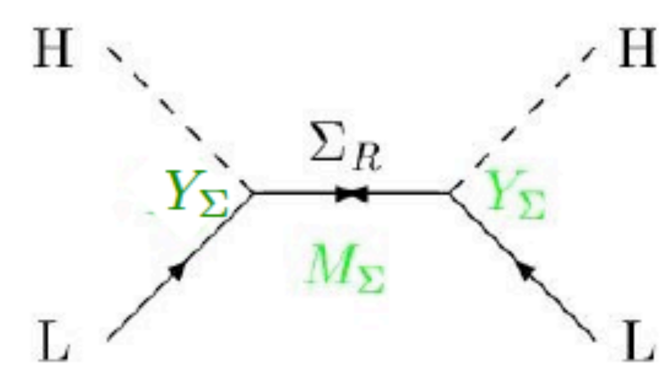
$\Delta \equiv (\Delta^{++}, \Delta^{+}, \Delta^{0})$



$$m_\nu = Y_\Delta \frac{\mu_\Delta}{M_\Delta^2} v^2$$

Type III:
Fermion triplet

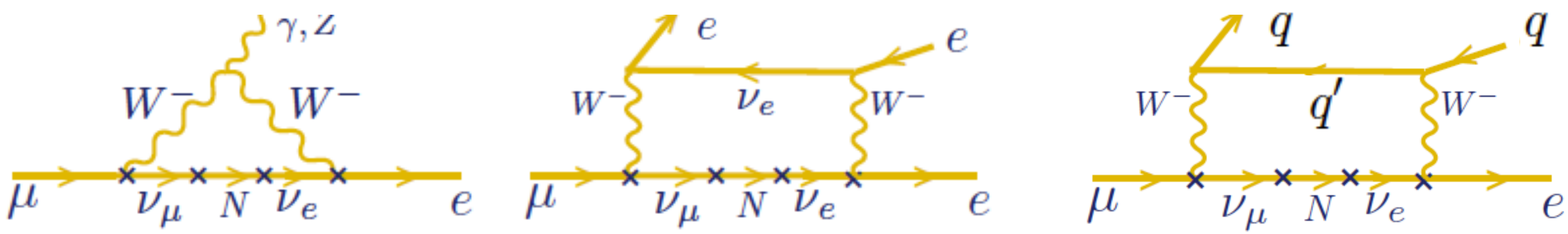
$\Sigma_i \equiv (\Sigma_i^{+}, \Sigma_i^{0}, \Sigma_i^{-})$



$$m_\nu = Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma v^2$$

- Observable CLFV if see-saw scale low (with protection of LN)
- Each model leads to specific CLFV pattern

- CLFV in Type I seesaw: loop-induced D,V operators, coefficients controlled by N_i masses



$$\Gamma(\mu \to e\gamma) = \sum_{N_i} \frac{|Y_{N_{ie}} Y^\dagger_{N_{i\mu}}|^2}{m^4_{N_i}} \cdot [\, c + c' \log(m^2_{N_i}/m^2_W)\,]^2$$

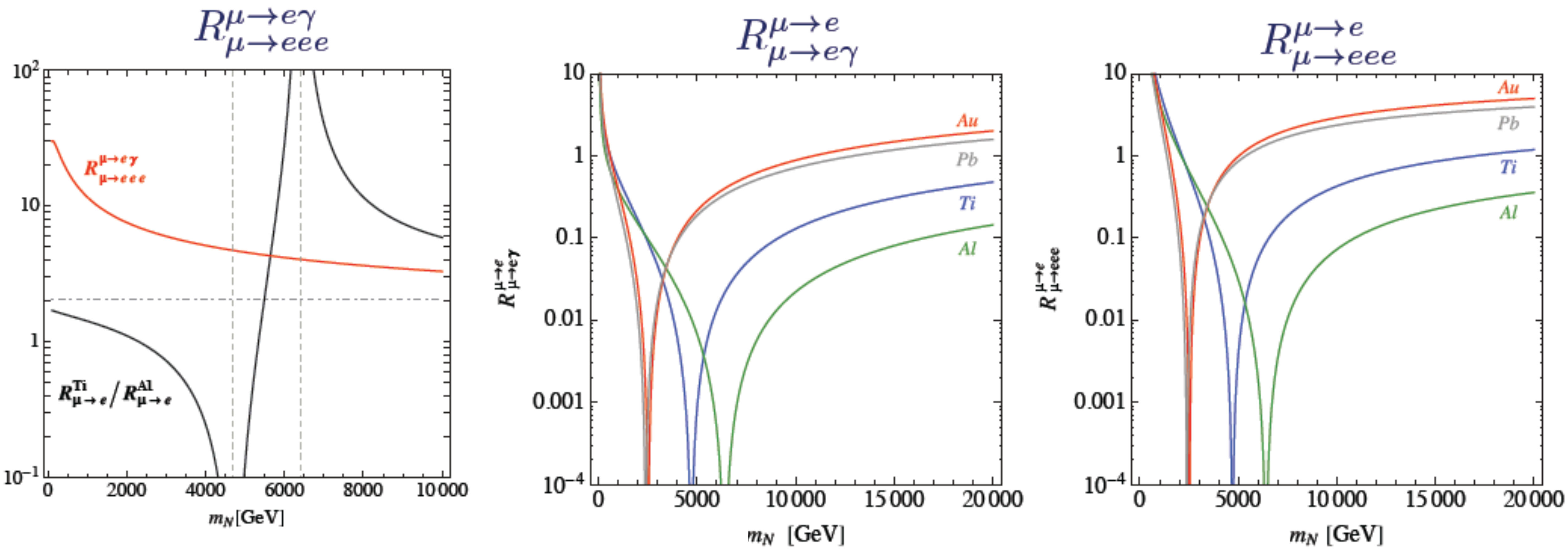
$$\Gamma(\mu \to eee) = \sum_{N_i} \frac{|Y_{N_{ie}} Y^\dagger_{N_{i\mu}}|^2}{m^4_{N_i}} \cdot [\, d + d' \log(m^2_{N_i}/m^2_W)\,]^2$$

$$R^N_{\mu\to e} = \sum_{N_i} \frac{|Y_{N_{ie}} Y^\dagger_{N_{i\mu}}|^2}{m^4_{N_i}} \cdot [\, b^N + b'^N \log(m^2_{N_i}/m^2_W)\,]^2$$

- For ~degenerate N_i masses (suppressed LNV), ratio of 2 rates with same flavor transition depends only on seesaw scale

Alonso-Dhen-Gavela-Hambye '13

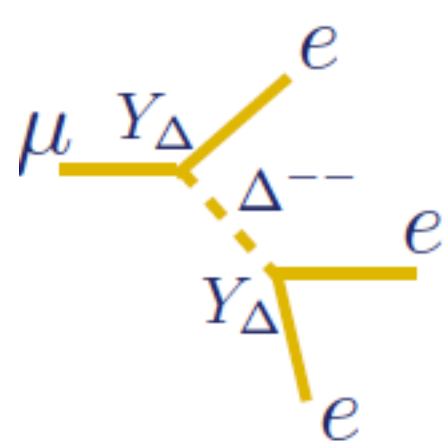
- CLFV in Type I seesaw: loop-induced D,V operators, coefficients controlled by N_i masses

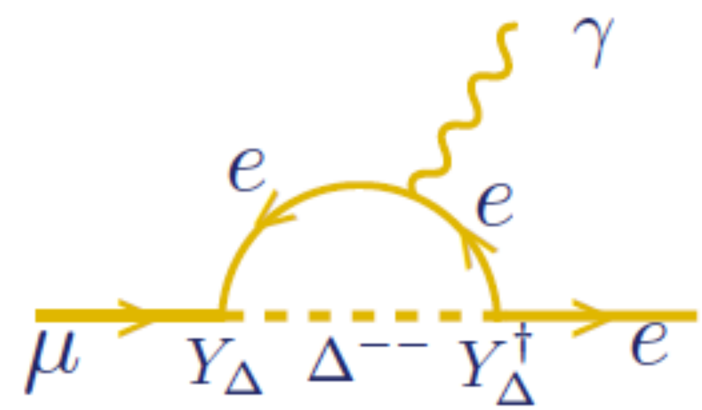


- With three rate measurements (2 ratios):
  - determine seesaw scale or
  - rule out scenario

Alonso-Dhen-Gavela-Hambye ‘13

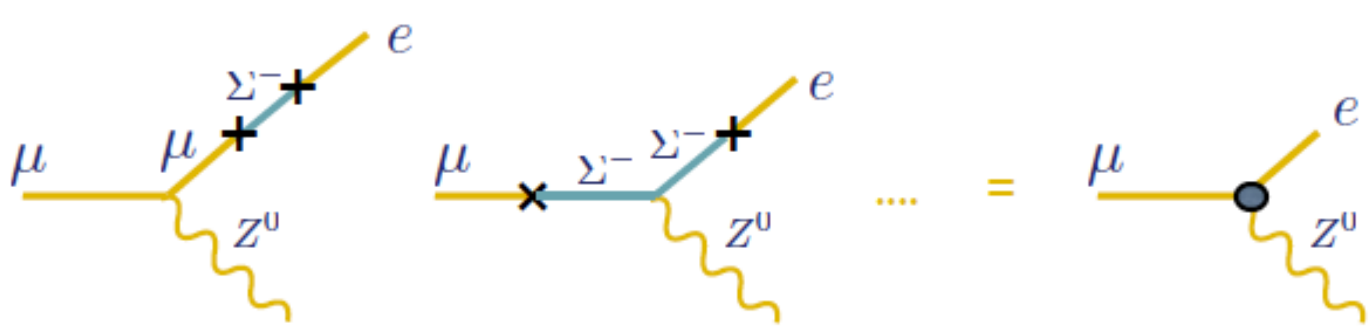
- CLFV in Type II seesaw: tree-level 4L operator (D,V at loop) → 4-lepton processes most sensitive





- CLFV in Type III seesaw: tree-level LFV couplings of Z ⇒

  μ →3e and μ →e conversion at tree level, μ →eγ at loop



Abada-Biggio-Bonnet-Gavela-Hambye '07, '08

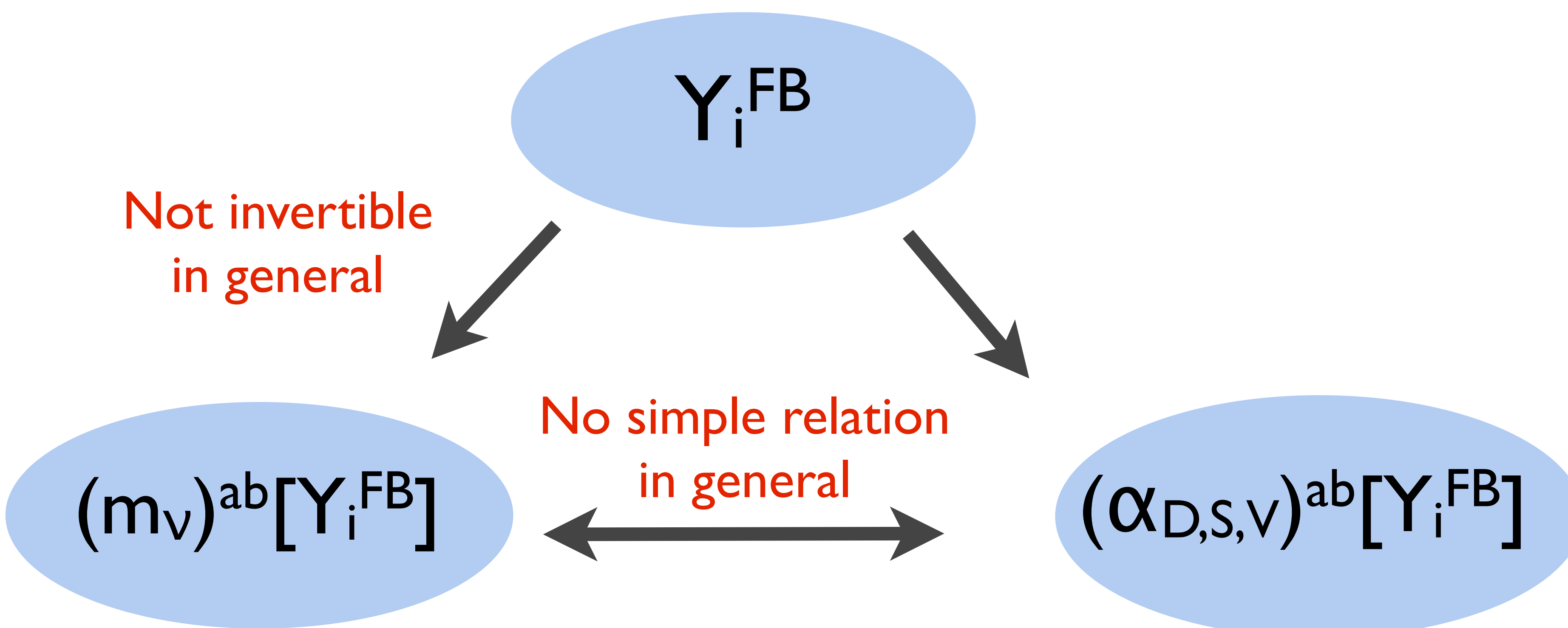
- Ratios of 2 processes with same flavor transition are fixed

$$
\begin{aligned}
Br(\mu \to e\gamma) &= 1.3 \cdot 10^{-3} \cdot Br(\mu \to eee) = 3.1 \cdot 10^{-4} \cdot R^{\mu \to e}_{Ti} \\
Br(\tau \to \mu\gamma) &= 1.3 \cdot 10^{-3} \cdot Br(\tau \to \mu\mu\mu) \\
Br(\tau \to e\gamma) &= 1.3 \cdot 10^{-3} \cdot Br(\tau \to eee)
\end{aligned}
$$

Sources of flavor breaking

- Each model has its flavor group (← field content) and sources of flavor breaking Y_i^{FB} (Yukawa-type, mass matrices of heavy states, ...)

- Y_i^{FB} leave imprint in m_ν and CLFV effective couplings $\alpha_{D,V,S,...}$

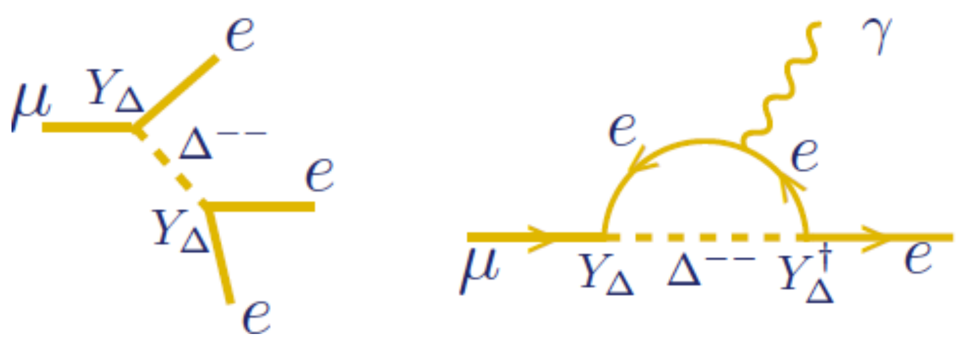


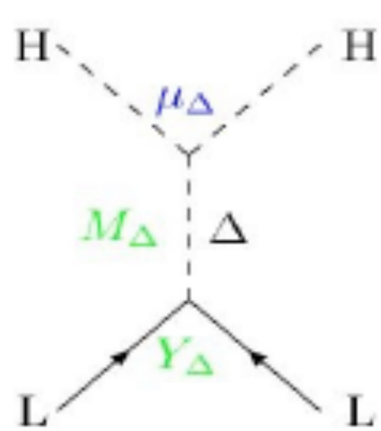
- No general statement is possible in general. However, CLFV provides non-trivial tests of any given ansatz for Y_i^{FB}.
  Cleanest test-ground: $\mu \to e\gamma$ vs $\tau \to \mu\gamma$ ($\tau \to e\gamma$)

- Example: Type II seesaw model (scalar triplet)
  Explicit realization of Minimal Lepton Flavor Violation

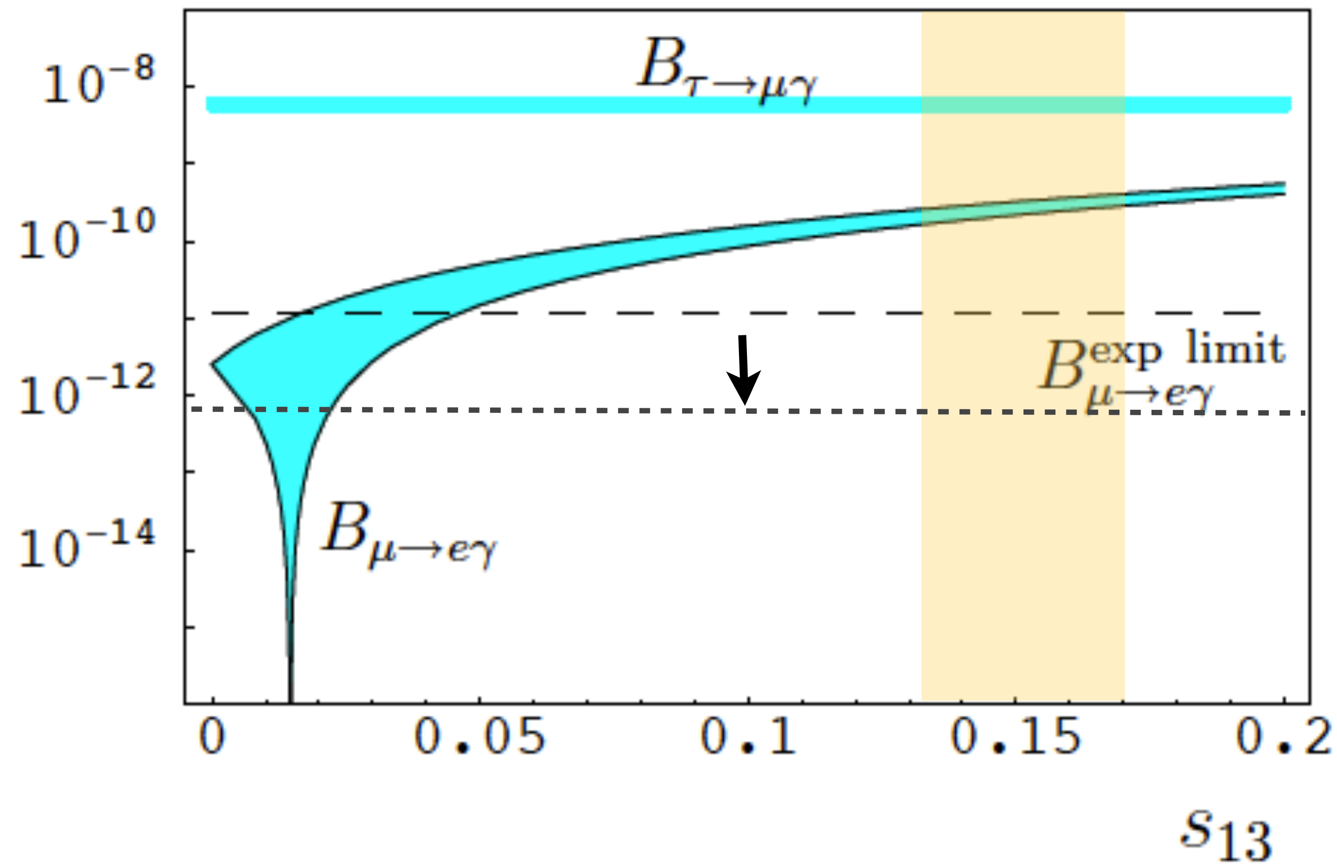
CLFV controlled by

$Y_\Delta \propto m_\nu$





Rossi '02, VC-Grinstein-Isidori-Wise '05



$\tau \to \mu\gamma$ not observable at (super-)B factories

- A different example: SU(5) GUT models (with ~ degenerate N_i)
- Two competing structures:

$$\frac{v}{\Lambda^2}\, \bar{e}^i_R \left(\lambda_e \lambda^\dagger_\nu \lambda_\nu\right)^{ij} \sigma^{\mu\nu} e^j_L \, F_{\mu\nu} \;\longrightarrow\; \text{PMNS mixing pattern} \qquad M_\nu > 10^{12}\ \mathrm{GeV}$$

$$\frac{v}{\Lambda^2}\, \bar{e}^i_R \left(\lambda_U \lambda^\dagger_U \lambda^T_D\right)^{ij} \sigma^{\mu\nu} e^j_L \, F_{\mu\nu} \;\longrightarrow\; \text{CKM mixing pattern} \qquad M_\nu < 10^{12}\ \mathrm{GeV}$$

[~ Barbieri-Hall-Strumia '95]

- CKM ⇒ more hierarchical pattern of BRs: $\tau \to \mu\gamma$ is within reach of (super-)B factories

$$10 - 100 \;:\; 1 \;:\; 1$$

$$B(\tau \to \mu\gamma) : B(\tau \to e\gamma) : B(\mu \to e\gamma) \quad \begin{cases} \mathrm{Min}\left[s_{13}^{-2}\,,\ \dfrac{\Delta m^2_{\mathrm{atm}}}{\Delta m^2_{\mathrm{sol}}}\right] \;:\; 1 \;:\; 1 \\[2ex] \lambda_C^{-6} \;:\; \lambda_C^{-4} \;:\; 1 \end{cases}$$

$\lambda_C \equiv V_{us}$

$$10^4 \;:\; 500 \;:\; 1$$