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Supersymmetry: what? why? when?

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Introduction

- **The SM, based on the gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$, is in excellent agreement with experimental results.**
- **Strong indications that it is just an effective of fundamental one:**
  1) In the SM we cannot predict a particle mass.
  2) We do not know why there are six kinds of quarks and leptons.
  3) The electroweak and the strong forces are partially unified and gravity is not included.
  4) In the SM neutrinos are massless & why matter dominates antimatter.
  5) In the SM we do not know what the dark matter is made of.

What is Supersymmetry

- **Supersymmetry (SUSY): a symmetry between bosons and fermions.**

$$Q_\alpha|\mathrm{Boson}\rangle = |\mathrm{Fermion}\rangle \qquad Q_\alpha|\mathrm{Fermion}\rangle = |\mathrm{Boson}\rangle$$

- **Introduced in 1973 as a part of an extension of the special relativity.**
- **Super Poincare algebra**

$$\begin{array}{ll} P_\mu & (translation), \\ M_{\mu,\nu} & (rotation\ and\ Lorentz\ tranformation), \\ Q_\alpha & (SUSY\, transformation) \end{array}$$

$$\{Q_\alpha, Q_\beta\} = (\gamma^\mu)_{\alpha\beta} P_\mu$$

- **SUSY = a translation in Superspace**

Space-time (x^μ) $\rightarrow$ Superspace (x^μ, θ)

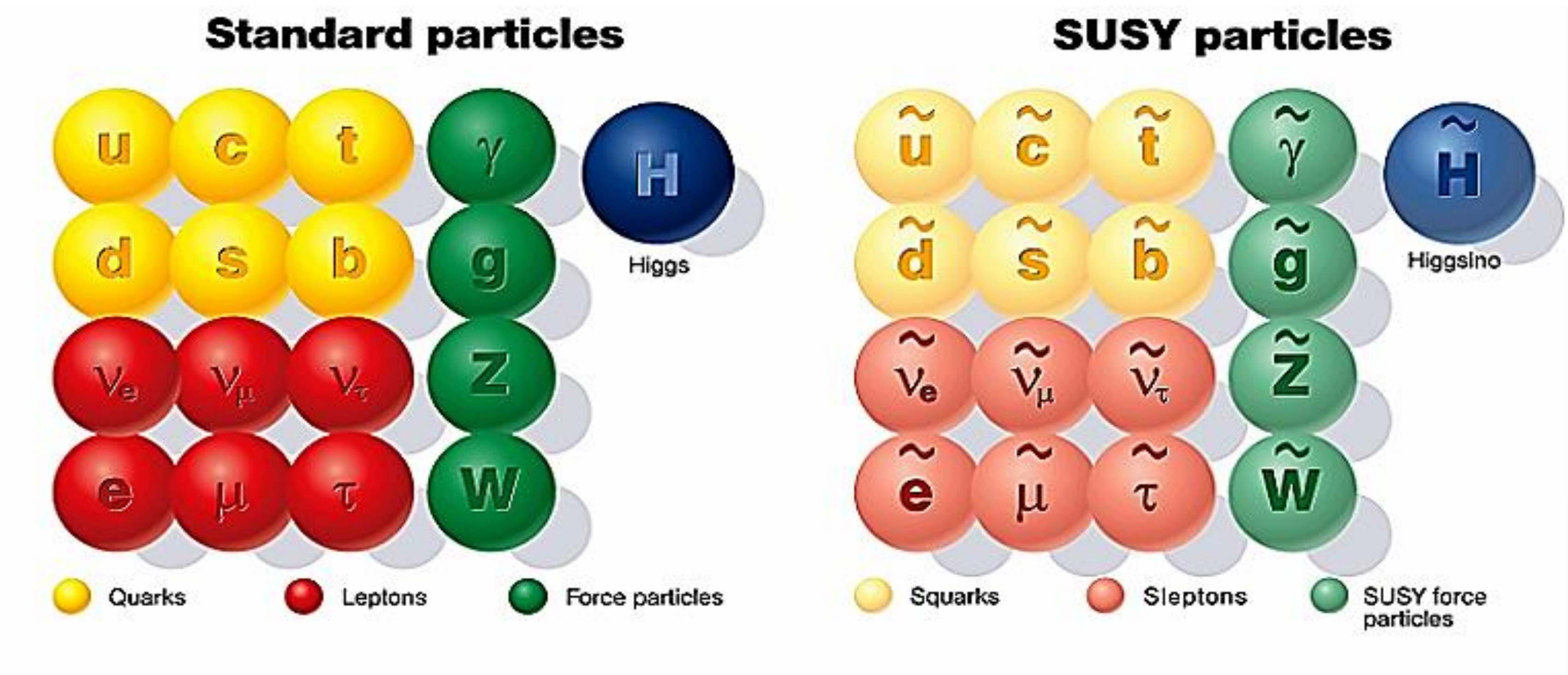
SUSY transformation:

$$\begin{array}{lll} x^\mu & \rightarrow & x'^\mu = x^\mu + \frac{i}{2}\bar{\epsilon}\gamma^\mu\theta \\ \theta & \rightarrow & \theta' = \theta + \epsilon \end{array}$$

SUSY Particle Spectrum

Extends the Standard Model (SM) by predicting a <u>new symmetry</u>:

spin-1/2 matter particles (fermions) $\leftrightarrow$ spin-1 force carriers (bosons)



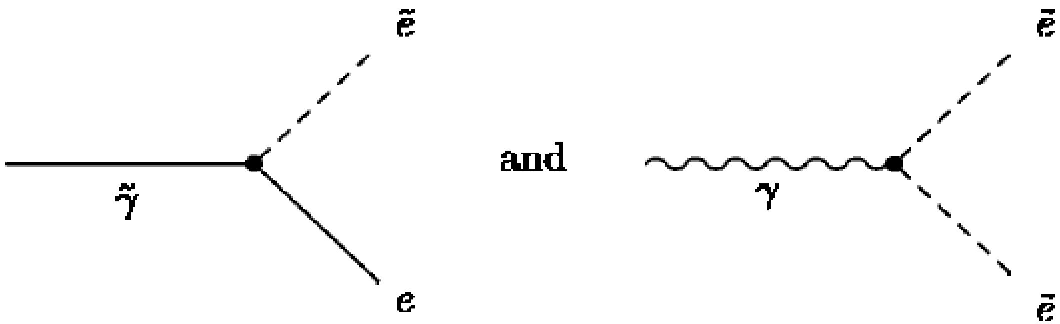
New Quantum Number R-Parity $\Rightarrow R_P = (-1)^{B+L+2s} \begin{cases} +1 & \textit{SM partices} \\ -1 & \textit{SUSY particles} \end{cases}$

If R_p conserved Lightest Sparticle (LSP) stable!

- Each superpartner is written with a tilde: $e \leftrightarrow \tilde{e}$, $\gamma \leftrightarrow \tilde{\gamma}$.

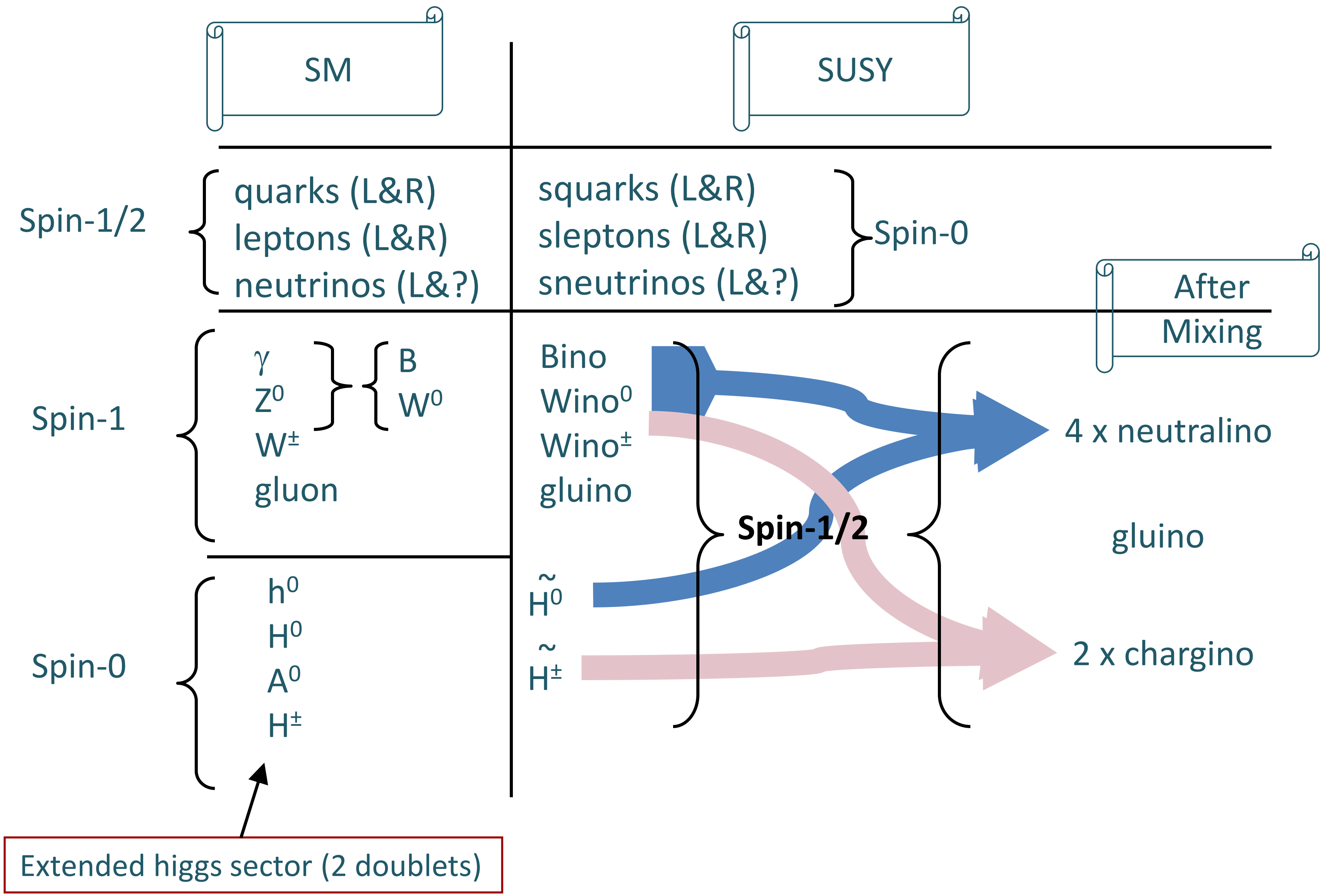
- Standard Model fermions get a pre-s, and Standard Model bosons a suffix-ino, so selectron, squark, photino.

- There are also new interactions . Each of the SM vertices leads to new vertices obtained by replacing SM particles by their superpartners in pairs.



- Each new vertex has the same coupling strength as its SM counterpart. The vertex $ee\gamma$ is of strength *e*, and so are the two associated vertices $\tilde{e}\tilde{e}\gamma$ and $\tilde{e}e\tilde{\gamma}$.

(S)Particles

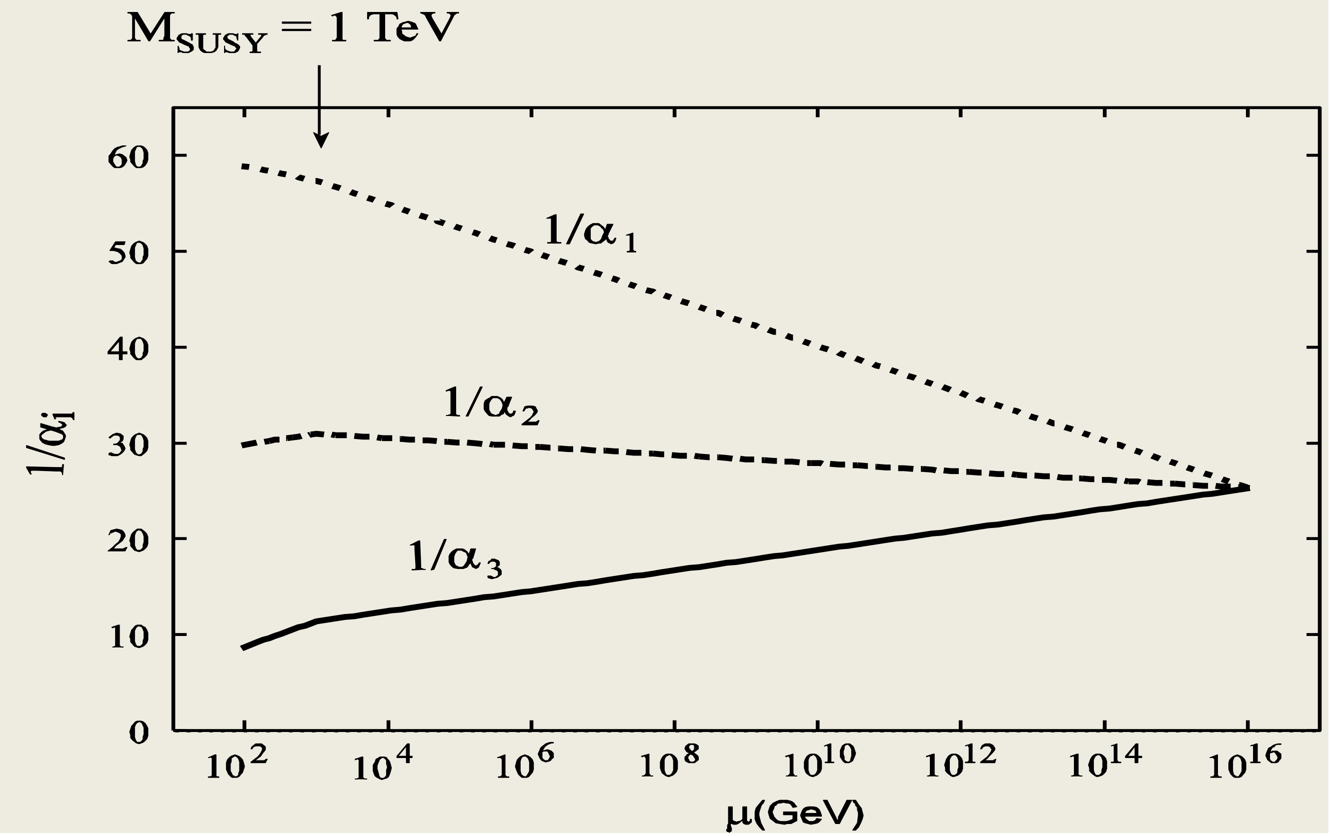


SM
SUSY
Spin-1/2
quarks (L&R)
leptons (L&R)
neutrinos (L&?)
squarks (L&R)
sleptons (L&R)
sneutrinos (L&?)
Spin-0
After Mixing
Spin-1
γ
Z^0
$W^\pm$
gluon
B
W^0
Bino
Wino^0
$\text{Wino}^\pm$
gluino
Spin-1/2
4 x neutralino
gluino
2 x chargino
Spin-0
h^0
H^0
A^0
$H^\pm$
$\tilde{H}^0$
$\tilde{H}^\pm$
Extended higgs sector (2 doublets)

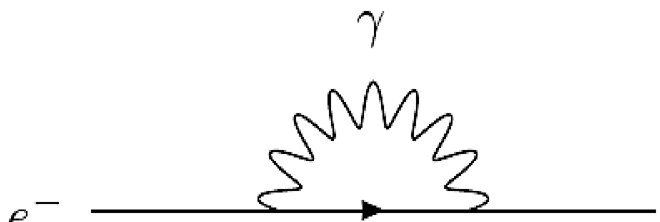
Motivation For Supersymmetry

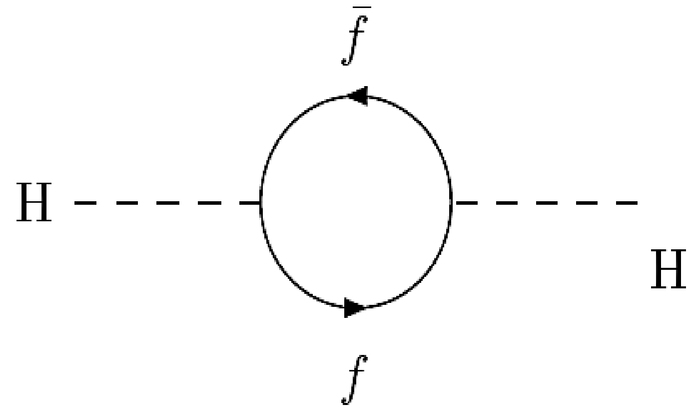
- Supersymmetric theories are promising candidate for unified theory beyond the SM.
- SUSY is a generalization of the space-time symmetries that links the matter particles with the force carrying particles, and leads to additional 'superparticles'.
- SUSY is a new symmetry which relates boson and fermions
- SUSY ensures the stability of hierarchy between the week and the Planck scales.
- Local supersymmetry leads to a partial unificationof gravity of the SM with gravity 'supergravity‘.
- In supersymmetric theories, the mechanism of the electroweak symmetry breaking is natural.

- Supersymmetry is a necessary ingredient in string theory.
- With supersymmetry, the SM gauge couplings are unified at GUT scale MG ≈ 2 x1016 GeV.



Hierarchy problem





$$\delta m_e = 2\frac{\alpha_{em}}{\pi} m_e \log\frac{\Lambda}{m_e} + \ \ldots$$

$$\delta m_H^2(f) = -2N_f \frac{|\lambda_f|^2}{16\pi^2}[\Lambda^2 - 2m_f^2 \ln\frac{\Lambda}{m_f} \ + \ldots]$$

$m_e \to 0$, Chiral Symmetry

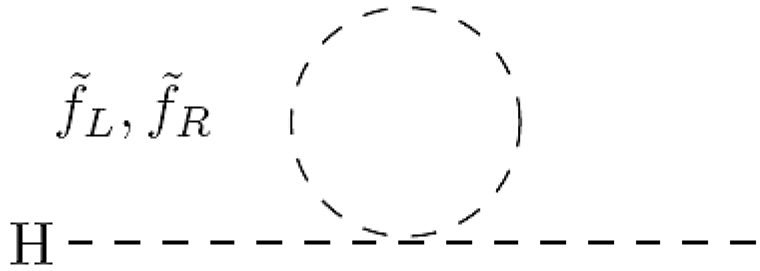
m_H^2 no symmetry, Quadratic div.!!

$$\Lambda = 10^{19} \ \mathrm{GeV}$$

$$\delta m_e \ = \ 0.24 \ m_e$$

$$\delta m_H^2 \ \simeq \ 10^{30} \ \mathrm{GeV}$$

Hierarchy problem and SUSY



$$\delta m_H^2(\tilde{f}) = -2N_{\tilde{f}}\frac{\lambda_{\tilde{f}}}{16\pi^2}[\Lambda^2 - \; 2m_{\tilde{f}}^2 \ln\frac{\Lambda}{m_{\tilde{f}}} \; + \ldots]$$

if $N_f = N_{\tilde{f}}$, $|\lambda_f|^2 = -\lambda_{\tilde{f}}$ and $m_f = m_{\tilde{f}} \Rightarrow \delta m_H^2(f) + \delta m_H^2(\tilde{f}) = 0$

Supersymmetry + Chiral symmetry

solve the hierarchy problem.

But ... there are no scalars exactly degenerate with the SM fermions, SUSY is broken !!

$\boxed{\text{Soft Supersymmetry breaking}}$

- Want to preserve cancellation of Quadratic divergencies. $\Rightarrow$ dimensionless couplings still supersymmetric: $|\lambda_f|^2 = -\lambda_{\tilde{f}}$
- SUSY is broken only in couplings with possitive mass dimension:

Soft Breaking

$$m^2_{\tilde{f}} = m^2_f + \delta^2$$

$$\delta m^2_H(f) + \delta m^2_H(\tilde{f}) \simeq 2N_f \frac{|\lambda_f|^2}{16\pi^2} \delta^2 \ln \frac{\Lambda}{m_{\tilde{f}}} + \ldots$$

If SUSY solves the hierarchy problem

$\delta \lesssim 1$ TeV

Minimal Supersymmetric Standard Model

- Must include all Standard Model particles and interactions
- Supersymmetric partners and SUSY version of SM interactions
- Supersymmetry must be softly broken

Chiral supermultiplets			Vector supermultiplets		
LH SM fermion	$\leftrightarrow$	Scalar partner	gauge bosons	$\leftrightarrow$	fermion part.
$Q, u_R^c, d_R^c, L, e_R^c$		$\tilde{Q}, \tilde{u}_R^c, \tilde{d}_R^c, \tilde{L}, \tilde{e}_R^c$	B_μ, W_μ^i, G_μ^a		$\tilde{B}, \tilde{W}^i, \tilde{g}^a$
2 Higgs	$\leftrightarrow$	fermion part.			
$H_1, \quad H_2$		$\tilde{H}_1, \quad \tilde{H}_2$			

SUSY Particle Spectrum

SM	Superpartner	Quan. Num.	SM	Superpartner	Quan. Num.
Q_L	squark doublet $\tilde{Q}_L$	$(3, 2, \frac{1}{6})$	L_L	slepton doublet $\tilde{L}_L$	$(1, 2, -\frac{1}{2})$
u_R	up squark singl. $\tilde{u}_R$	$(3, 1, \frac{2}{3})$	d_R	sdown singlet $\tilde{d}_R$	$(3, 1, -\frac{1}{3})$
e_R	selectron singlet $\tilde{e}_R$	$(1, 1, -1)$	ν_R	sneutrino right $\tilde{\nu}_R$	$(1, 1, 0)$
H_1	down higgsino $\tilde{H}_1$	$(1, 2, -\frac{1}{2})$	H_2	up higgsino $\tilde{H}_2$	$(1, 2, \frac{1}{2})$
g	gluino $\tilde{g}$	$(8, 1, 0)$	W	Wino $\tilde{W}$	$(1, 3, 0)$
B	bino $\tilde{B}$	$(1, 1, 0)$			

Gauge Interactions

Gauge self-interactions $\quad -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + i\bar{\lambda}^a\bar{\sigma}^\mu D_\mu\lambda^a$

with $\quad D_\mu\lambda^a = \partial_\mu\lambda^a - gf^{abc}A^b_\mu\lambda^c \quad$ for $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$

Covariant derivatives $\quad i\bar{\psi}\gamma^\mu D_\mu\psi + D_\mu\phi D^\mu\phi^*$

with $\quad D_\mu = \delta_\mu + ig'YB_\mu + ig\frac{\tau^i}{2}W^i_\mu + ig_3\frac{\lambda^a}{2}G^a_\mu \quad$ with ψ any of the MSSM left-handed fermions and ϕ any of the scalars

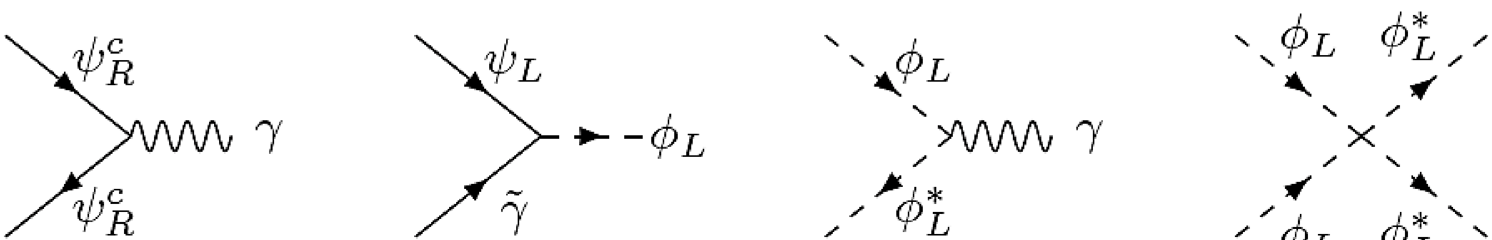
Gaugino interactions and D-terms $\quad ig\sqrt{2}(\phi^*T^a\psi\lambda^a + \bar{\lambda}^a\bar{\psi}T^a\phi) \;+$

$-\frac{1}{2}\,g^2\left(\sum_i \phi^*_i T^a\phi_i\right)$ for $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ and MSSM particles

Gauge Interactions (cont.)

$$
\begin{aligned}
L \;=\; & i\bar{\psi}_L \bar{\sigma}^\mu D_\mu \psi_L \;+\; D^\mu \phi_L^* D_\mu \phi_L + (L \to R^c) \;+\; \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} \\
& - i\,\bar{\lambda}^a \bar{\sigma}^\mu D_\mu \lambda^a \;+\; i\,\sqrt{2}\,g\;\left[\phi_i^* T^a \psi_i\;\lambda^a \;+\; \bar{\lambda}^a\;\bar{\psi}_i T^a \phi_i\right] \\
& - \frac{1}{2} W^{ij}\;\psi_i \psi_j \;-\; \frac{1}{2} W^{ij*}\;\bar{\psi}_i \bar{\psi}_j - \;W^i W_i^* \;-\frac{1}{2}\;g^2\;\left(\textstyle\sum_i \phi_i^* T^a \phi_i\right)^2
\end{aligned}
$$

therefore the gauge interactions are $(U(1))$,



$-\,i\;g\;\gamma_\mu$ $\qquad$ $i\;g\sqrt{2}\;L$ $\qquad$ $-i\;g\;(p+p')_\mu$ $\qquad$ $-\,2\;i\;g^2$

more complete Feynman rules $\qquad$ [Bailin,Haber]

MSSM Superpotential

Includes the Yukawa interactions of the SM

$$W = Y_d^{ij} Q_i H_1 d^c_{Rj} + Y_e^{ij} L_i H_1 e^c_{Rj} + Y_u^{ij} Q_i H_2 u^c_{Rj} + \mu H_1 H_2$$

- Need of two Higgs doublets

$$Y_{Q_i} + Y_{d^c_R} = \frac{1}{6} + \frac{1}{3} = \frac{1}{2} \quad \Rightarrow \quad Y_{H_d} = -\frac{1}{2}$$
$$Y_{Q_i} + Y_{u^c_R} = \frac{1}{6} - \frac{2}{3} = -\frac{1}{2} \quad \Rightarrow \quad Y_{H_u} = \frac{1}{2}$$

in the SM $H_u = H$ and $H_d = H^*$. However, a Superpotential containing H^* would be non-supersymmetric.

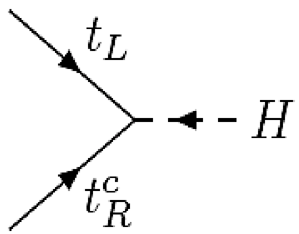
A second doublet also required to cancel the triangle anomaly.

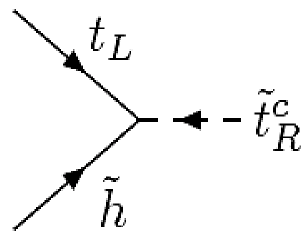
Example: top Yukawa

$$W = h_t\, Q_L H t^c_R$$

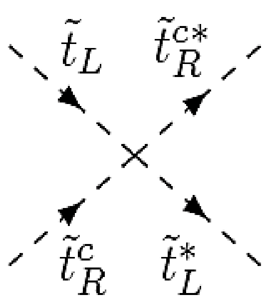
$$L_{int} = -\frac{1}{2} W^{ij}\ \psi_i \psi_j - \frac{1}{2} W^{ij*}\ \bar{\psi}_i \bar{\psi}_j - W^i W^*_i = -\frac{1}{2}\ h_t \Big[H\ Q_L t^c_R + \tilde{Q}_L\ \tilde{h} t^c_R + \tilde{t}^c_R\ Q_l \tilde{h} \Big] + c.c. - h^2_t \left(|H \tilde{t}^c_R|^2 + |H \tilde{Q}_L|^2 + |\tilde{Q}_L \tilde{t}^c_R|^2 \right)$$

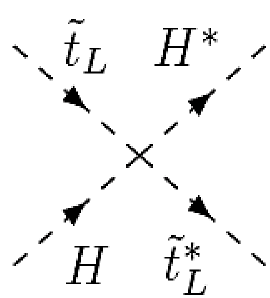
from here the Feynman rules,





$-\ i\ h_t\ L$





$-\ i\ h^2_t$

Soft SUSY Breaking

- SUSY must be broken, $m_{\tilde{e}} \neq m_e$, $m_{\tilde{g}} \neq 0$.
- Solve hierarchy problem, broken by terms of possitive mass dimension

Soft Supersymmetry Breaking, and $M_{susy} \leq \mathcal{O}(1 \text{ TeV})$.

- Gaugino masses

$$L^{(1)}_{soft} = \frac{1}{2}\left(M_1\ \tilde{B}\tilde{B} + M_2\ \tilde{W}\tilde{W} + M_3\ \tilde{g}\tilde{g}\right) + h.c.$$

- Scalar masses

$$L^{(2)}_{soft} = (m^2_{\tilde{Q}})_{ij}\tilde{Q}_i\tilde{Q}^*_j + (m^2_{\tilde{u}})_{ij}\tilde{u}^c_{Ri}\tilde{u}^{c*}_{Rj} + (m^2_{\tilde{d}})_{ij}\tilde{d}^c_{Ri}\tilde{d}^{c*}_{Rj} + (m^2_{\tilde{L}})_{ij}\tilde{L}_i\tilde{L}^*_j + \\ (m^2_{\tilde{e}})_{ij}\tilde{e}^c_{Ri}\tilde{e}^{c*}_{Rj} + (m^2_{H_1})H_1H_1^* + (m^2_{H_2})H_2H_2^*$$

- Trilinear couplings and B–term

$$L^{(3)}_{soft} = (Y^A_d)^{ij}\tilde{Q}_iH_1\tilde{d}_{Rj} + (Y^A_e)^{ij}\tilde{L}_iH_1\tilde{e}^c_{Rj} + (Y^A_u)^{ij}\tilde{Q}_iH_2\tilde{u}^c_{Rj} + B\mu H_1H_2$$

Constrained MSSM

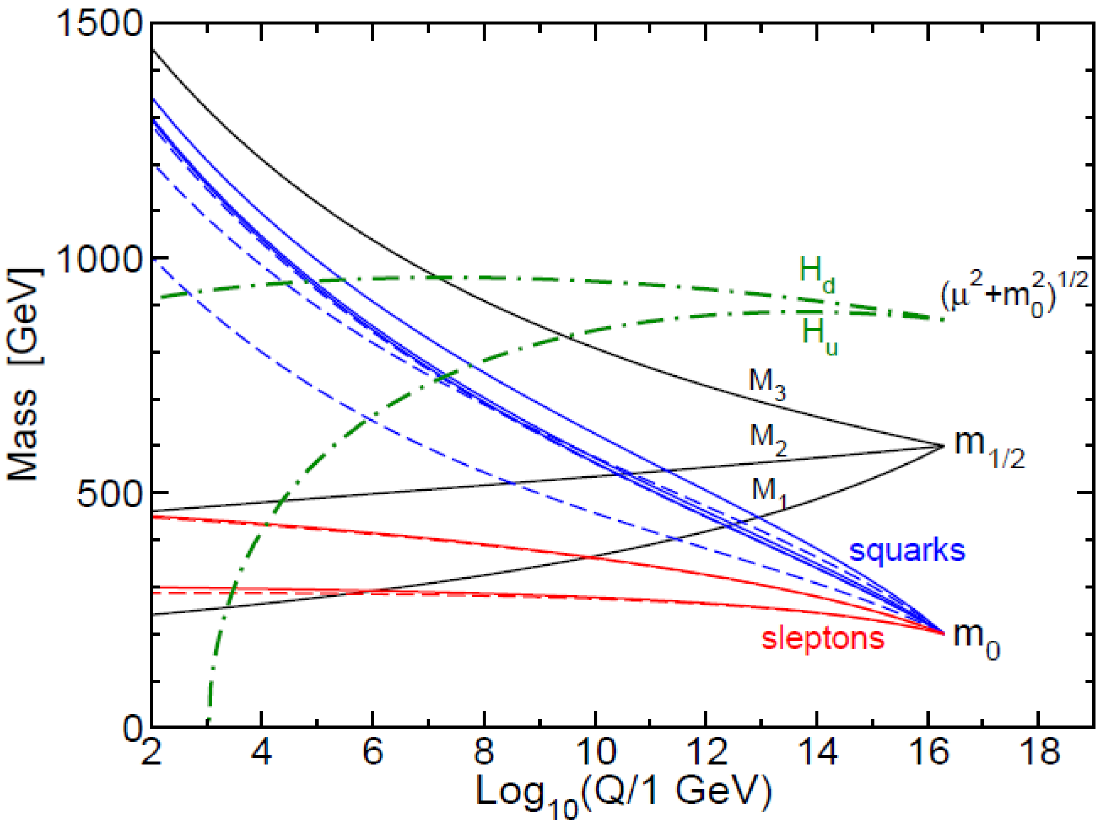
- **Unconstrained SUSY would bring in 105 parameter (on top of the SM ones)**

- **mSUGRA: simple boundary conditions at GUT scale reduce the number of parameters to ~5!**

  – Common scalar mass m_0
  – Common gaugino mass $m_{1/2}$
  – Common trilinear scalar interaction A
  – Ratio of vevs of two Higgs fields *tanb*
  – Sign of Higgs mass parameter *m*

- **Can predict SUSY spectrum at our energies**

- In CMSSM (mSUGRA), universality of soft SUSY breaking terms is assumed at GUT scale.
- The RGE are used to calculate the parameters at the electroweak scale.



Higgs mass in SUSY models

- **SUSY models include at least two Higgs doublets.**
- **This means: 8 degrees of freedom, 3 eaten up by the W± and Z 5 Higgs fields: h0,A0,H0,H±**
- **Connection between Higgs masses and gauge boson masses:**

$$V_{Higgs} = m_1^2|H_1|^2 + m_2^2|H_2|^2 - m_3^2(H_1 \cdot H_2 + \bar{H}_1 \cdot \bar{H}_2) + \frac{g_2^2}{8}(\bar{H}_1\tau^a H_1 + \bar{H}_2\tau^a H_2)^2 + \frac{g_1^2}{8}(|H_1|^2 - |H_2|^2)^2 + \Delta V,$$

$$m_h^2 \leq m_Z^2 \cos^2 2\beta + \frac{6}{(2\pi)^2}\frac{m_t^4}{v^2}\ln\frac{m_{stop}^2}{m_t^2}, \quad \left(\tan\beta = \frac{<H_2^0>}{<H_1^0>}\right)$$

$$m_h < 135\text{GeV}$$

- **Possible vacuum instability is saved by supersymmetry**

Squarks & Sleptons

The squark mass matrices

$$M_{\tilde{u}}^2 = \begin{pmatrix} M_Q^2 + m_u^\dagger m_u + D_{LL}^u & m_u(A_U + \mu \cot\beta) \\ (A_U^\dagger + \mu^* \cot\beta) m_u^\dagger & M_U^2 + m_u m_u^\dagger + D_{RR}^u \end{pmatrix}$$

$$M_{\tilde{d}}^2 = \begin{pmatrix} K^\dagger M_Q^2 K + m_d^\dagger m_d + D_{LL}^d & m_d(A_D + \mu \tan\beta) \\ (A_D^\dagger + \mu^* \tan\beta) m_d^\dagger & M_D^2 + m_d m_d^\dagger + D_{RR}^d \end{pmatrix}$$

Analogous expressions are obtained in the slepton sector, where

$$D_{RR}^f = (m_Z^2 \cos 2\beta \; e_f \sin^2\theta_W)\; \mathbf{1}.$$

Here T_{3f} is the third component of the weak isospin and e_f is the charge.

Charginios

- The mixing of the charged gauginos and charged Higgsinos is described by:

$$M_C = \begin{pmatrix} M_2 & \sqrt{2} M_W \sin\beta \\ \sqrt{2} M_W \cos\beta & -\mu \end{pmatrix}$$

- The chargino masses are given by

$$M^2_{2,1} = 1/2(M_2^2 + \mu^2 + 2M_W^2 \\ \pm \sqrt{(M_2^2 - \mu^2)^2 + 4M_W^4 \cos^2 2\beta + 4M_W^2 (M_2^2 + \mu^2 - 2M_2 \mu \sin 2\beta))}$$

- Within the mSUGRA scenario we have

$$M_{\tilde{\chi}_1^0} \approx 0.45 M_{1/2}$$

$$M_{\tilde{\chi}_2^0} \approx M_{\tilde{\chi}_1^\pm} \approx 2 M_{\tilde{\chi}_1^0}$$

$$M_{\tilde{\chi}_2^0} \approx (0.25 - 0.35) M_{\tilde{g}}$$

Neutralinos

- **The neutralinos □i (i=1,2,3,4) are the physical (mass) superpositions of fermionic partners of bino, wino and Higgsinos.**

$$M_N = \begin{pmatrix} M_1 & 0 & -M_Z \cos\beta \sin\theta_W & M_Z \sin\beta \sin\theta_W \\ 0 & M_1 & M_Z \cos\beta \cos\theta_W & -M_Z \sin\beta \cos\theta_W \\ -M_Z \cos\beta \sin\theta_W & M_Z \cos\beta \cos\theta_W & 0 & \mu \\ M_Z \sin\beta \sin\theta_W & -M_Z \sin\beta \cos\theta_W & \mu & 0 \end{pmatrix}$$

- **The lightest neutralino will be a linear combination of the original fields:**

$$\tilde{\chi}_1^0 = N_{11}\tilde{B}^0 + N_{12}\tilde{W}^0 + N_{13}\tilde{H}_1^0 + U_{14}\tilde{H}_2^0$$

- **The lightest neutralino can be the lightest supersymmetric particle (LSP).**
- **Stable Weakly Interacting Massive Particle (WIMP). Interesting candidate for DM.**

Summary CMSSM Spectrum

- LSP is lightest neutralino $m_{\chi_1^0} \simeq 0.4 M_{1/2}$
- Lightest chargino and second neutralino wino-like $m_{\chi_2^0} \simeq m_{\chi_1^+} \simeq 0.8 M_{1/2}$
- Second chargino and other neutralinos higgsino-like $m_{\chi_2^+} \simeq m_{\chi_3^0} \simeq m_{\chi_4^0} \simeq \mu$
- Gluino much heavier $m_{\tilde{g}} \simeq 2.8 M_{1/2}$
- 1^{st} and 2^{nd} generation squarks $\sim$ gluino $m^2_{\tilde{q}_{1,2}} \simeq 6.5\, M^2_{1/2} + m_0^2$
- $\tilde{t}$ and $\tilde{b}$ lighter, lightest stop $\simeq \tilde{t}_R$: $m^2_{\tilde{t}_1} \simeq 4\, M^2_{1/2} + 0.5\, m_0^2$
- $\tilde{l}$ lighter, $\tilde{\tau}_R$ the lightest slepton $m^2_{\tilde{l}_R} \simeq 0.15\, M^2_{1/2} + m_0^2$
  $m^2_{\tilde{l}_L} \simeq 1.5\, M^2_{1/2} + m_0^2$
- Lightest Higgs below 135 GeV, other Higgs heavier $m_A \simeq m_H \simeq m_{H^+} \simeq \mu$